Investigation of start control problem for one nonlinear model of filtration

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Introduction

A wide class of mathematical physics problems can be considered within the framework of semi-linear Sobolev type equations, which describe various processes (for example deformation processes, processes occurring in semiconductors, processes of oscillatory motion propagation in various media, and so on). The work is devoted to the study of start control problems of one mathematical model of Sobolev type, which is based on the equation describing the process of changing the concentration potential of a viscoelastic fluid filtered in a porous medium (the process of nonlinear diffusion of matter).The problem is to find such a distribution of the state so that the external influence is the closest to the given one with the best achievement of the required state.

Problem

Consider the start control problem

$$J(x,u) = \vartheta\left\|x - x_f\right\|^p_{L_p(0,T;W_p^1(\Omega))} + (1-\vartheta)\|u\|^p_{W_p^1(\Omega)} \to \inf,\ \vartheta\in(0,1) \qquad (1)$$

for solutions

$$(\lambda - \Delta)x_t - \operatorname{div}(|\nabla x|^{p-2}\nabla x) = y \qquad (2)$$

with the initial conditions

$$(\lambda - \Delta)\big(x(s,0) - u(s)\big) = 0,\ \ s \in \Omega, \qquad (3)$$

and boundary conditions

$$x(s,t) = 0,\ \ (s,t) \in \partial\Omega \times [0,T]. \qquad (4)$$

Results

Theorem 1. For all fixed values of the parameter $\lambda \geq -\lambda_1$ and for all $u \in W_p^1(\Omega)$, $T \in \mathbb{R}_+$, $y \in L_p\left(0,T;W_q^{-1}(\Omega)\right)$ exists weak generalized solution there exists a unique weak generalized solution $x \in L_\infty(0,T;\operatorname{coim} L) \cap L_p(0,T;W_p^1(\Omega))$ problem (2)–(4).

As the control space, we take the space $U = W_p^1(\Omega)$ and fix in it a non-empty, closed, and convex subset $U_{ad} \subset U$.

Definition. The pair $(\tilde{x},\ \tilde{u})$ called a solution (1) − (4), if $J(\tilde{x},\tilde{u}) = \inf J(x,u)$, provided that $(x,u) \in L_\infty(0,T;\operatorname{coim} L) \cap L_p(0,T;W_p^1(\Omega)) \times U_{ad}$ and x is weak generalized solution problem (2)–(4).Moreover, the vector function ũ called start control (1)–(4).

Theorem 2. For all fixed values of the parameter $\lambda \geq -\lambda_1$, $T \in \mathbb{R}_+$ exist solution $(\tilde{x},\ \tilde{u})$ problem (1)–(4).

Conclusion

The problem of start control for nonlinear model of filtration is considered.
We find the sufficient conditions under which there exists a solution to the control problem of the model under study.