Synchronization between delay chaotic systems via matrix projective approach

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INTRODUCTION

Chaos theory is a very active area of research, involving many different disciplines such as Mathematics, Physics, Chemical system, Population studies, Biology, Metrology, Astrophysics, Information theory, etc. In the past few years, chaos synchronization has attracted the attention of many researchers. The idea of synchronization with two identical initial conditions was introduced by Pecora and Carroll [1]. It plays an important role in study of various types of systems such as electromechanical system [2], neuron network system [3], satellite system [4], neural system [5] and a variety of physical and chemical systems. There are several types of synchronization but in this work, we analyze a special type of synchronization namely Matrix projective synchronization. There are various applications of matrix projective synchronization such as Secure Communication [6], delayed fractional order neural networks [7] etc.

A matrix projective synchronization is a type of projective synchronization when a scaling constant is a generalized constant in projective synchronization master and slave system could be synchronized up to that scaling factor. Thus it is interesting to investigate the matrix projective synchronization with the help of control methods applying Lyapunov stability. In order to analyze the behavior of matrix projective synchronization, several results could be found in literatures. Ouannas and Abu-Saris [8] investigated a sufficient condition for matrix projective synchronization and inverse matrix projective synchronization between discrete-time chaotic dynamical systems of identical and non-identical dimensions. Jinman He *et al.* [9] examined the synchronization of disturbed fractional-order hyperchaotic system by the help of fractional matrix and inverse matrix projective synchronization methods. Zhaoyan Wu *et al.* [10] investigated the problem of generalized matrix projective synchronization in general colored networks with different-dimensional node dynamics.

He and Chen [11] found the matrix synchronization, stability, chaotic behavior, and chaos control of a new type of fraction-order Rabinovich system. He *et al.* [12] discussed the global matrix-projective synchronization of time delayed fractional-order competitive neural networks. Khan *et al.* [13] discussed the application of the fractional inverse matrix projective combination synchronization in the field of secure communication.

Motivated by above discussion, author has studied synchronization between delay chaotic systems via matrix projective approach.

MATRIX PROJECTIVE SYNCHRONIZATION

The two chaotic systems with disturbances and hyperbolic nonlinearity are expressed as

$$\dot{x}(t) = Ax + C(x(t-\tau)) + f, \quad t > 0, \qquad x(t) = \phi(t), -\tau \le t \le 0. \tag{1}$$

$$\dot{y}(t) = By + D(y) + \lambda_1(t) + u(x, y), \tag{2}$$

where $x(t) = x_1(t), x_2(t), ..., x_n(t)^T, y(t) = (y_1(t), y_2(t), ..., y_n(t))^T \in R^n$, A and B are the $n \times n$ matrices, C and D are nonlinear parts of the systems, τ denotes time delay, f represents hyperbolic nonlinearity, $\phi(t)$ represent the trajectories of the solutions in the past, λ_1 is disturbances satisfies $|\lambda_1| \le \rho_1$,where ρ_1 is positive constant and $u(x, y)$ is controller.

Definition: System (1) and (2) are said to be matrix projective synchronized if there exist a function $u(x, y) \in R^n$ such that $e(t) = \lim_{t\to\infty} \|y - \omega x\| = 0$, where ω is a $n \times n$ projective matrix.

Theorem: The matrix projective synchronization is achieved between systems (1) and (2) if there exist $u(x, y) \in R^n$ such that

$$u(x, y) = \omega\{Ax + C(x(t-\tau))\} - D(y) - (B+\alpha)\omega x + \alpha y - \lambda_1(t), \tag{3}$$

provided $(B+\alpha)^T + (B+\alpha)$ should be negative.

Proof: The derivative of $e(t)$ is expressed as

$$\dot{e} = \dot{y} - \omega\dot{x} = By + D(y) + \lambda_1(t) + u(x, t) - \omega\{Ax + C(x(t-\tau))\}. \tag{4}$$

Putting the value of $u(x, y)$ from equation (4) in equation (5), we get

$$\dot{e} = -(B+\alpha)\omega x + \alpha y + By = (B+\alpha)e.$$

Now define Lyapunov function as

$$V = \frac{1}{2}e^T(t)e(t) + \frac{1}{2}\int_{-\tau}^{0} e^T(t+\delta)e(t+\delta)d\delta.$$

Its derivative is given as

$$\begin{aligned} \frac{dV}{dt} &= e^T\frac{de}{dt} + \frac{1}{2}\left(e^Te - e^T(t-\tau)e(t-\tau)\right), \\ &= e^T[(B+\alpha)^T + (B+\alpha)]e - \frac{1}{2}\left(e^T(t-\tau)e(t-\tau)\right) < 0 \end{aligned} \tag{5}$$

If $\frac{dV}{dt}$ is negative definite then the error system is globally and asymptotically stable [21], provided $(B+\alpha)^T + (B+\alpha)$ negative definite matrix.

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SYSTEM DESCRIPTION AND MATRIX PROJECTIVE SYNCHRONIZATION BETWEEN CHAOTIC SYSTEMS

Muthuswamy and Chua proposed a new chaotic system. This system with delay is given as

$$\begin{aligned} \dot{x}_1(t) &= x_2, \\ \dot{x}_2(t) &= -\frac{1}{3}x_1(t-\tau) + \frac{1}{2}x_2 - \frac{1}{2}x_2x_3^2, \\ \dot{x}_3(t) &= -x_2 - 0.6x_3 + x_2x_3. \end{aligned} \tag{6}$$

The system (6) represents chaotic behavior with initial condition $(0.1, 0, 0.1)$.

The system with multiple delays is expressed as

$$\begin{aligned} \dot{x}_1(t) &= x_2, \\ \dot{x}_2(t) &= -\frac{1}{3}x_1 + \frac{1}{2}x_2(t-\tau_1) - \frac{1}{2}x_2(t-\tau_2)x_3^2, \\ \dot{x}_3(t) &= -x_2 - 0.6x_3 + x_2x_3, \end{aligned} \tag{7}$$

The system (7) represents chaotic behavior with initial condition $(0.1, 0, 0.1)$.

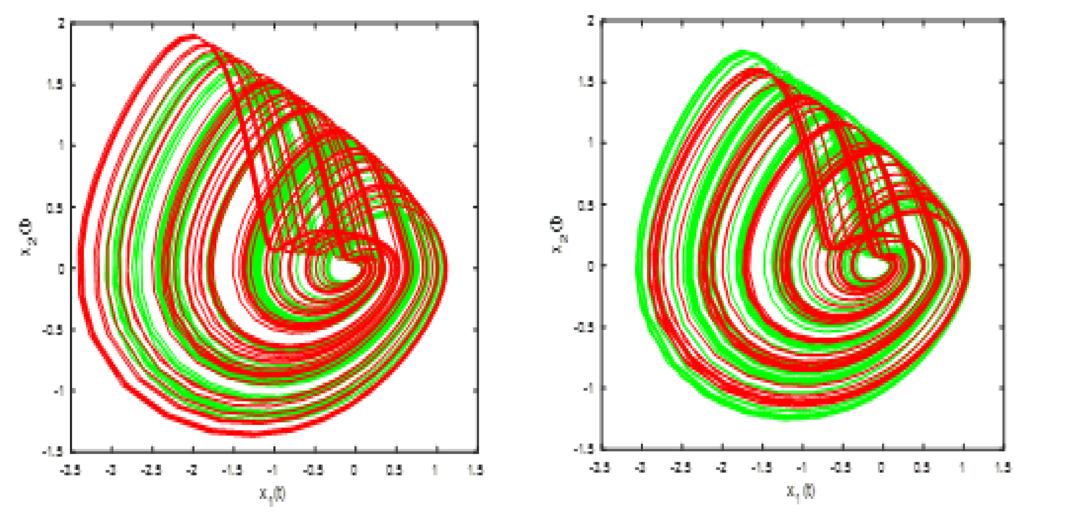

Fig 1. (a) Phase portraits of time delay system (6) in $x_1 - x_2$ plane. Red attractor represents system with delay $\tau = 0.1$ and green attractor show system without delay (b) Phase portraits of multiple delay system (7) in $x_1 - x_2$ plane. Red attractor represents system with $\tau_1 = 0.2$ and $\tau_2 = 0.1$. Green attractor show system without delay.

Jafari and Sprott considered a bunch of chaotic systems which show many features. Here we take one of this system as slave system

$$\begin{aligned} \dot{y}_1(t) &= y_2, \\ \dot{y}_2(t) &= -y_1 + y_2y_3, \\ \dot{y}_3(t) &= -y_1 - 15y_1y_2 - y_1y_3. \end{aligned} \tag{8}$$

The system (8) with disturbances is taken as

$$\begin{aligned} \dot{y}_1(t) &= y_2 + 0.1\cos(50t) + u_1(t), \\ \dot{y}_2(t) &= -y_1 + y_2y_3 - 0.1\cos(50t) + u_2(t), \\ \dot{y}_3(t) &= -y_1 - 15y_1y_2 - y_1y_3 + 0.1\sin(50t) + u_3(t). \end{aligned} \tag{9}$$

The term $\lambda_1(t) = \begin{pmatrix} 0.1\cos(50t) \\ -0.1\cos(50t) \\ 0.1\sin(50t) \end{pmatrix}$ shows disturbances in system (9). The system (9) is chaotic for parametric values with initial condition $(0, 0.5, 0.5)$. Here $(u_1(t), u_2(t), u_3(t))^T$ is a control unction to control slave system.

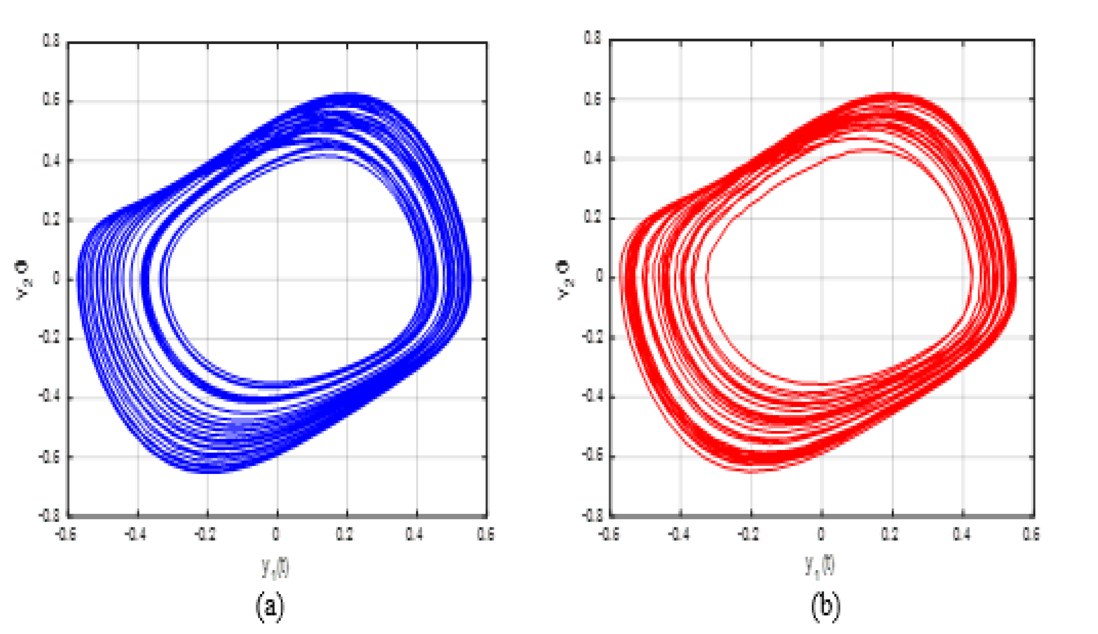

Fig 2. (a) Phase portraits of system (8) in $y_1 - y_2$ plane, (b) Phase portraits of system (9) in $y_1 - y_2$ plane.

Matrix projective Synchronization between chaotic system with single delay and chaotic system with disturbances

To synchronize system (6) and system (9) we consider the projective matrix ω as

$$\omega = \begin{pmatrix} 2 & 1 & 0 \\ 1 & -3 & 0 \\ 1 & 0 & -1 \end{pmatrix}. \tag{10}$$

The coefficient matrix of linear part of slave system is

$$B = \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ -1 & 0 & 0 \end{pmatrix}. \tag{11}$$

There are several possibilities to choose gain matrix, we have considered the gain matrix as

$$\alpha = \begin{pmatrix} -1 & -1 & 0 \\ 1 & -1 & 0 \\ 1 & 0 & -1.5 \end{pmatrix}. \tag{12}$$

Putting the value of ω from equation (10) in equation (4) with consideration of systems (6) and (9), we get the controller as

$$\begin{aligned} u_1(t) &= -0.33x_1(t-\tau) + 2x_1x_2 + 2.5x_2 - 0.5x_2x_3^2 - y_1 - y_2 - 0.1\cos(50t), \\ u_2(t) &= 0.33x_1(t-\tau) + 0.5x_2 + 0.5x_2x_3^2 + x_1 - x_2 + y_1 - y_2 - y_2y_3 + 0.1\cos(50t), \\ u_3(t) &= x_1 + 2x_2 - x_3 + 0.6x_3 - x_2x_3 + 15y_1y_2 + y_1y_3 + y_1 - y_3 - 0.1\sin(50t), \end{aligned} \tag{13}$$

Substituting the value of controller and values of different parameters from equations (11), (12) and (13) in (5), we get the error system as

$$\dot{e}_1 = -e_1, \dot{e}_2 = -e_2, \dot{e}_3 = -e_3. \tag{15}$$

All eigenvalues of error system are negative. According to Theorem1, the matrix projective synchronization has been achieved.

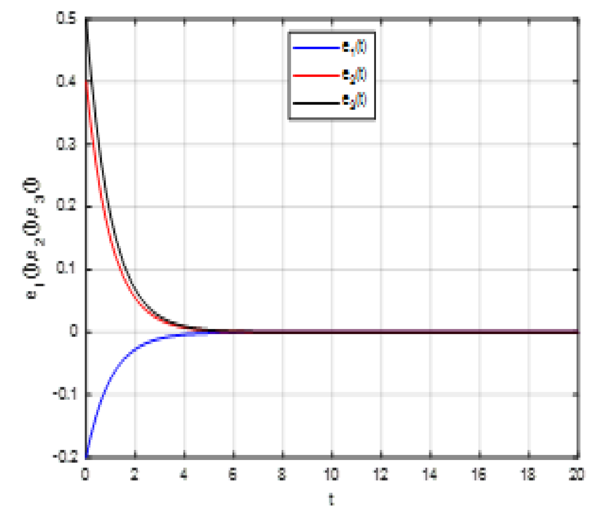

Fig 3. The time evolution of states of errors e_1, e_2, e_3.

NUMERICAL RESULTS AND DISCUSSION

In the numerical simulation of matrix projective synchronization the initial conditions of Simplest chaotic circuit and simple chaotic flows are consider as $(0.1, 0, 0.1)$ and $(0, 0.5, 0.5)$ respectively. Fig. 1 show the phase portraits of simplest chaotic circuit with single delay and multiple delays while Fig. 2 depicts that the phase portraits of simple chaotic flow with disturbance. Fig. 3 depicts state of signal and errors w.r.t. time between chaotic system with single delay and chaotic system with disturbances.

CONCLUSION

In this article, chaos is found in a time delay system and a system with external disturbances. Thereafter matrix projective synchronization between these systems is studied. Further, matrix projective synchronization between time delay chaotic system and chaotic system with external disturbance is obtained. Theoretical analysis and numerical simulations show that these systems can be matrix projective synchronized by choosing proper projective matrix and gain matrix.

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