Non-uniqueness
from molecular equations

St. Petersburg, September 2014
N.N. Uralceva's 80th birthday

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$$\begin{aligned} M_t^{(N)}(d\vec{x}, d\vec{v}) &= \frac{1}{N} \sum_{i=1}^{N} \delta_{(x_i, u_i)}(dx, dv) \\ &= (Id \times \vec{u}_N)_{\#} \mu_N(d\vec{x}, d\vec{v}), \end{aligned}$$

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$$\frac{d}{dt}\int_{\mathbb{R}^3} \phi(x)\, \mu_t^{(N)}(dx) = \int_{\mathbb{R}^3} \nabla_x \phi(x) \cdot u_N(t,x) \mu_t^{(N)}(dx),$$

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&\qquad = \int_{\mathbb{R}^3} \nabla_x \phi(t,x) \cdot u_N(t,x)\, u_N(t,x)\, \mu_t^{(N)}(dx) \\
&\qquad\quad - \frac{1}{N\sigma_N} \int \phi(t,x)\, \Phi'\left(\frac{|x-x'|}{\sigma_N}\right) \frac{x-x'}{|x-x'|}\, n_t^{(N)}(dx,dx').
\end{aligned}$$

Under assumption on positions and velocities can show:

$$\mu_t^{(N)}(d\vec{x}) \Rightarrow \mu_t(d\vec{x}),$$
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$$\int \phi(\vec{x})\vec{u}_N(t,x)\mu_t^{(N)}(d\vec{x}) \to \int \phi(\vec{x})\vec{u}(t,\vec{x})\mu_t(d\vec{x}),\ \forall\ \phi \text{ test}.$$

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weakly.

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$$-\frac{1}{\sigma_N} \sum_{\substack{j=1 \\ j \neq i}}^{N} \Phi' \left(\frac{|x_i(0) - x_j(0)|}{\sigma_N} \right) \leq B_N.$$

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$$\left(B_N := \frac{\min_{1 \leq i \neq j \leq N} (x_i(0) - x_j(0)) \cdot (u_i(0) - u_j(0))}{XT + 3T^2U + 4T^3} \right)$$

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Accelerations are uniformly bounded in N.

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In fact, for the M_t-measure:

$$\frac{d}{dt}\int \phi\,(x,v)\,M_t\,(dx,dv) = \int \nabla_x\phi\,(x,v)\cdot v\ M_t\,(dx,dv)\,.$$

Can solve:

$$\begin{aligned} M_t &= (S_t)_{\#} M_0, \\ S_t(x, v) &= (x + tv, v). \end{aligned}$$

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$N = 4n^2$:

$$x_{ij} = \left(\frac{1}{2n} + \frac{i}{n},\ \frac{1}{2n} + \frac{j}{n},\ 0 \right), \quad -n \leq i, j \leq n - 1$$

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and

$$u(0, x_{ij}) = \begin{cases} (0, 0, 1) & \text{if } i + j \text{ even} \\ (0, 0, -1) & \text{if } i + j \text{ odd,} \end{cases}$$

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Then

$$M_0^{(N)}(dx, dv)$$

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$$M_0(dx, dv) = \Delta_0(dx) \otimes \left(\frac{1}{2} \delta_{(0,0,1)}(dv) + \frac{1}{2} \delta_{(0,0,-1)}(dv) \right)$$

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Therefore: initial macroscopical velocity

$$u(0, x) = 0.$$

$$
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$$

It gives the macroscopic velocity for $t > 0$:

$$
u(t,x) = \begin{cases} (0,0,1) & (x_1, x_2) \in D, \quad x_3 = t \\ (0,0,-1) & (x_1, x_2) \in D, \quad x_3 = -t \\ 0 & \text{otherwise.} \end{cases}
$$

Not even local uniqueness for

$$\partial_t(u\mu_t) + \mathrm{div}(u \otimes u\ \mu_t) = 0$$
$$\partial_t\mu_t + \mathrm{div}(u\ \mu_t) = 0.$$

(μ_t not absolutely continuous on $\mathbb{R}^3$.)

Compatible with macroscopic definition of initial condition for weak solutions:

$$\int \phi(x)u(t,x)\mu_t(dx) - \int \phi(x)u(0,x)\mu_0(dx) = \int_0^t \int \nabla\phi(x) \cdot u(s,x)u(s,x)\mu_s(dx)\, ds$$

given that

$$\lim_{t\to 0} \int \phi(x)u(t,x)\mu_t(dx) = \int \phi(x)u(0,x)\mu_0(dx).$$

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For $n \in \mathbb{N}$ and for $-n \leq i, j, k \leq n-1$, let

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Define

$$u_N(x_{ijk}) = \begin{cases} (0,0,c) & i+j \text{ even} \quad k \text{ even} \\ (0,0,-c) & i+j \text{ odd} \quad k \text{ even} \\ (0,0,-c) & i+j \text{ even} \quad k \text{ odd} \\ (0,0,c) & i+j \text{ odd} \quad k \text{ odd.} \end{cases}$$

and denote the corresponding measure in $\mathbb{R}^6$ with $M^N(dx, dv)$.

As $N \to \infty$,

$$M^N(dx, dv)$$

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$$M_0(dx, dv) = \chi_K(x)dx \otimes \frac{1}{2}\left(\delta_{(0,0,c)}(dv) + \delta_{(0,0,-c)}(dv)\right).$$

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Macroscopic initial conditions:

$$\begin{cases} \rho(0, x) = \chi_K(x) \\ u(0, x) = 0. \end{cases}$$

$$M_t\,(dx, dv) =$$

$$\begin{cases} \frac{1}{2}dx \otimes \delta_{(0,0,-c)}\,(dv) & 0 \le t \le \frac{l}{c} \quad -l - ct \le x_3 \le -l + ct \\ \frac{1}{2}dx \otimes \delta_{(0,0,-c)}\,(dv) \\ \qquad + \frac{1}{2}dx \otimes \delta_{(0,0,c)}\,(dv) & 0 \le t \le \frac{l}{c} \quad -l + ct \le x_3 \le l - ct \\ \frac{1}{2}dx \otimes \delta_{(0,0,c)}\,(dv) & 0 \le t \le \frac{l}{c} \quad l - ct \le x_3 \le l + ct \\ \frac{1}{2}dx \otimes \delta_{(0,0,-c)}\,(dv) & t \ge \frac{l}{c} \quad -l - ct \le x_3 \le l - ct \\ \frac{1}{2}dx \otimes \delta_{(0,0,c)}\,(dv) & t \ge \frac{l}{c} \quad -l + ct \le x_3 \le l + ct \end{cases}$$

Macroscopic density and velocity are

$$\rho(t,x) = \begin{cases} \frac{1}{2} & 0 \leq t \leq \frac{l}{c} \quad -l - ct \leq x_3 \leq -l + ct \\ 1 & 0 \leq t \leq \frac{l}{c} \quad -l + ct \leq x_3 \leq l - ct \\ \frac{1}{2} & 0 \leq t \leq \frac{l}{c} \quad l - ct \leq x_3 \leq l + ct \\ \frac{1}{2} & t \geq \frac{l}{c} \quad -l - ct \leq x_3 \leq l - ct \\ \frac{1}{2} & t \geq \frac{l}{c} \quad -l + ct \leq x_3 \leq l + ct, \end{cases}$$

$$u(t,x) = \begin{cases} (0,0,-c) & 0 \le t \le \frac{l}{c} & -l - ct \le x_3 \le -l + ct \\ 0 & 0 \le t \le \frac{l}{c} & -l + ct \le x_3 \le l - ct \\ (0,0,c) & 0 \le t \le \frac{l}{c} & l - ct \le x_3 \le l + ct \\ (0,0,-c) & t \ge \frac{l}{c} & -l - ct \le x_3 \le l - ct \\ (0,0,c) & t \ge \frac{l}{c} & -l + ct \le x_3 \le l + ct. \end{cases}$$

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Non-zero macroscopical velocity is generated immediately after time 0.
Equations satisfied weakly:

$$\partial_t \rho(t,x) + \operatorname{div}(\rho(t,x)u(t,x)) = 0$$
$$\partial_t(\rho(t,x)u(t,x)) + \operatorname{div}(\rho(t,x)u(t,x) \otimes u(t,x)) = - \underbrace{c^2(0,0,\partial_3 \chi_{B_t}(x))}_{\textit{from averages of fluctuations}}$$

Example Approximate by molecules initial distribution

$$M_0(dx, dv) = C\rho(x) \left\{ \begin{array}{ll} e^{-v^{1/4}} & v > 0 \\ & \\ 0 & v \le 0 \end{array} \right\} dxdv$$

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$$M_0(dx, dv) = C\rho(x) \left\{ \begin{array}{ll} e^{-v^{1/4}} + \varepsilon\, e^{-v^{1/4}} \sin v^{1/4} & v > 0 \\ \\ 0 & v \leq 0 \end{array} \right\} dxdv$$

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Infinitely many macroscopic equations.