2D ISING MODEL: CORRELATION FUNCTIONS AT CRITICALITY VIA RIEMANN-TYPE BOUNDARY VALUE PROBLEMS

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In this talk we give a survey of convergence results for correlation functions in the critical 2D Ising model obtained during the last several years. In particular, it includes the convergence (as the mesh size tends to zero, in arbitrary planar domains) of properly rescaled spin expectations to the conformal covariant limits predicted by Conformal Field Theory. We start with reviewing the combinatorics of the nearest-neighbor Ising model considered on a general planar graph and the existence of discrete holomorphic observables in the critical model on a regular grid, which solve some special Riemann-type boundary value problems. Though spin correlations cannot be directly obtained as the values of these discrete holomorphic functions, one can express their spatial derivatives via discrete holomorphic spinors defined in a similar manner. Analyzing the convergence of these spinors as the mesh size tends to zero, one can reconstruct the scaling limits of spin correlations from the asymptotic behaviour at singularities of their continuous counterparts. Interestingly, one can use the same approach to give a short proof of some classical results about the diagonal spin-spin expectations in the full plane via orthogonal polynomials techniques. The core part of the talk talk is based on a joint work with Clément Hongler (Lausanne) and Konstantin Izyurov (Helsinki).