ASYMPTOTIC AND NUMERICAL STUDY OF RESONANT TUNNELING IN QUANTUM WAVEGUIDES OF VARIABLE CROSS-SECTION

OLEG SARAFANOV

Saint Petersburg State University, o.sarafanov@spbu.ru

We consider an infinite waveguide with two cylindric ends and two narrows of small diameter ε. The electron wave function satisfies the Helmholtz equation with the Dirichlet boundary condition. The narrows of the waveguide play the role of effective potential barriers for the longitudinal motion of electrons. Two narrows form a quantum resonator where a resonant tunneling may appear. It means that electrons with energy in a small range Δ pass through the resonator with probability near to 1. In other words, the transition coefficient $T(k, \varepsilon)$ has sharp peaks at some "resonant"energies k^2_{res}. Various electronic devices (resonant transistors, key devices etc.) can be based on this phenomenon.

To study the resonant tunneling, we calculate the scattering matrix numerically. As approximation for a row of the scattering matrix, we take the minimizer of a quadratic functional. To construct the functional, we solve an auxiliary boundary value problem in the bounded domain obtained by truncating the cylindrical ends of the waveguide at distance R. As $R \to \infty$, the minimizer $a(R)$ tends with exponential rate to the corresponding row of the scattering matrix uniformly on every finite closed interval of the continuous spectrum not containing the thresholds.

The width Δ of resonant peaks is rapidly decreasing as the diameter ε of narrows tends to zero, which presents difficulties for numerical modelling of the resonant tunneling. To give the qualitative picture of the phenomenon when the resonant peaks are "too sharp", we use asymptotic analysis. We give the asymptotics of the corresponding wave function as $\varepsilon \to 0$. Besides, the asymptotics of the resonant energies are presented and the behaviour of the transmission coefficient T(k) near a resonance is analysed. Asymptotic and numerical approaches complete each other and give the full description of the resonant tunneling.

BIBLIOGRAPHY

L. Baskin, P. Neittaanmäki, B. Plamenevskii, and O. Sarafanov. *Resonant Tunneling: Quantum Waveguides of Variable Cross-Section, Asymptotics, Numerics, and Applications* // Lecture Notes on Numerical Methods in Engineering and Sciences, Springer, 2015, 275 pp.