# Spectral properties of the half-line Schrödinger operator with slowly decaying Wigner-von Neumann potential

Sergey Simonov

St. Petersburg Department of V. A. Steklov Institute of Mathematics of the Russian Academy of Sciences

We will discuss new results [1] on asymptotic behavior of the spectral density of the operator $\mathcal{L}_\alpha$ in $L_2(\mathbb{R}_+)$,

$$\mathcal{L}_\alpha = -\frac{d^2}{dx^2} + q_{per}(x) + \frac{c\sin(2\omega x+\delta)}{x^\gamma} + q_1(x),$$

$$\operatorname{dom}\mathcal{L}_\alpha = \{u \in H^2_{loc}(\mathbb{R}_+) : u, \mathcal{L}_\alpha u \in L_2(\mathbb{R}_+), u(0)\cos\alpha = u'(0)\sin\alpha\},$$

near critical points which lie inside the absolutely continuous spectrum. Here $q_{per}$ is a periodic background potential, $q_1 \in L_1(\mathbb{R}_+)$ and $\gamma \in (\frac{1}{2}, 1)$. Each spectral band of $\mathcal{L}_\alpha$ contains two critical points locations of which are determined by the frequency $\omega$ and the potential $q_{per}$. Spectral density $\rho'_\alpha$ of $\mathcal{L}_\alpha$ has exponential zeros at these critical points $\nu_{cr}$:

$$\rho'_\alpha(\lambda) = \text{const}\cdot\exp\left(-\frac{2c_{cr}}{|\lambda-\nu_{cr}|^{\frac{1-\gamma}{\gamma}}}\right)(1+o(1)) \text{ as } \lambda\to\nu_{cr}$$

with

$$c_{cr} = \frac{(2\beta_{cr})^{\frac{1}{\gamma}}}{4\gamma}B\left(\frac{3}{2},\frac{1-\gamma}{2\gamma}\right)\left(\frac{a}{2\pi k'(\nu_{cr})}\right)^{\frac{1-\gamma}{\gamma}},$$

$$\beta_{cr} = \frac{|c|}{2a|W\{\psi_+,\psi_-\}(\nu_{cr})|}\left|\int_0^a \psi_+^2(x,\nu_{cr})e^{2i\omega x}dx\right|,$$

where $\psi_\pm$ are Bloch solutions of the unperturbed periodic equation and $k$ is the quasi-momentum. Earlier the case $\gamma = 1$ was studied for which power type zeros of $\rho'_\alpha$ take place.

## References

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