Spectral asymptotics for an elastic strip with an interior crack

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We consider an infinite elastic strip $\Omega := \mathbb{R} \times (-\frac{\pi}{2}, \frac{\pi}{2})$ with zero Poisson ratio and a crack $\Gamma_\ell := [-\ell, \ell] \times \{0\}$. We impose traction-free boundary conditions and consider the existence of trapped modes, i.e., we search for square-integrable solutions $u : \Omega \backslash \Gamma_\ell \to \mathbb{C}^2$ of the eigenvalue problem

$$\begin{cases} (-\Delta - \text{ grad div})\, u &= \omega(\ell) u \qquad \text{in } \Omega\backslash\Gamma_\ell, \\ 2\varepsilon(u)\cdot \mathbf{n} &= 0 \qquad \text{on } \partial(\Omega\backslash\Gamma_\ell). \end{cases}$$

Here $\mathbf{n}$ is the outer normal unit vector, u is the displacement field of the elastic material and $\varepsilon(u) = \frac{1}{2}(\partial_i u_j + \partial_j u_i)_{i,j=1,2}$ the strain tensor.

In [1] the existence of two eigenvalues $\omega_1(\ell)$ and $\omega_2(\ell)$ embedded in the essential spectrum of the corresponding self-adjoint operator was proved. In the present talk we show that these eigenvalues satisfy the asymptotic estimates

$$\omega_1(\ell) = \Lambda - \nu_1 \ell^4 + \mathcal{O}(\ell^5) \qquad \text{as} \quad \ell \to 0,$$
$$\omega_2(\ell) = \Lambda - \nu_2 \ell^8 + \mathcal{O}(\ell^9) \qquad \text{as} \quad \ell \to 0,$$

where Λ is some spectral threshold and $\nu_1, \nu_2 > 0$. The proof is based on the resolvent expansion of the unperturbed problem near the spectral threshold Λ and on an analysis of a suitable Dirichlet-to-Neumann operator. This is a joint work with T. Weidl.

References

[1] Hänel, A. and Schulz, C. and Wirth, J., *Embedded eigenvalues for an elastic strip with cracks*. Quart. J. Mech. Appl. Math. **65**, (2012), 535–554.