

Bellman function method in analysis

I was invited to make a talk on this conference in spite of the fact that my field is far from the subject of the conference. Probably the idea of the organizers was to show how far from the original field of interest a former student of M. S. Birman can work. Since I understand that probably almost all participants of the conference hear the term *Bellman function* at the first time in their life, my goal will be not to present the most recent result, but to explain what the Bellman function is, to give some elementary examples, to show how this method works in analysis, etc.

My presentation will be based on examples of some classical inequalities for BMO functions (such as the well-known John–Nirenberg inequality). A prove of such inequality with sharp constants can be obtained by solving some boundary value problem for homogeneous Monge–Ampère equation. The graph of any solution of such equation is a so-called developable surface. Geometrical construction that foliate the domain of solution by special straight line segments is a corner stone of the method of constructing the required developable surface and therefore of finding a solution of the boundary value problem. The found Bellman function immediately supplies us with the desired inequality with the sharp constants.