Wandering eigenvalues of the Laplacian with an improper Robin conditions.

Sergei A. Nazarov.

St-Petersburg State University, St-Petersburg, Russia.

The spectrum of the boundary value problem

$$-\Delta u(x) = \lambda u(x),\ x \in \Omega, \quad a(x)\partial_n u(x) + u(x) = 0,\ x \in \partial\Omega,$$

in a smooth bounded domain $\Omega \subset \mathbb{R}^2$ will be considered under the condition

$$a(s) = a_0 s + O(s^2), \quad s \in \partial\Omega,\ a_0 \neq 0, \quad a(s) \neq 0 \text{ for } s \neq 0.$$

In other words, the coefficient a of the normal derivative ∂_n in the Robin condition changes sign on the boundary of the domain. It will be demonstrated that the spectrum of this problem is residual and covers the whole complex plane $\mathbb{C}$. All self-adjoint extensions of the operator A_0 with the domain

$$\mathcal{D}(A_0) = \{u \in H^1(\Omega) :\ \Delta u \in L^2(\Omega), |a|^{-1/2} u \in L^2(\partial\Omega)\}$$

will be described. They have the discrete spectrum but no appropriate choice of the extension is available. Also a skew-symmetric extension of A_0 can be constructed and it will be shown that this one describing wave processes in a finite volume has a clear physical sense. Namely, the problem

$$-\Delta u^\varepsilon(x) = \lambda^\varepsilon u^\varepsilon(x),\ x \in \Omega, \quad a^\varepsilon(x)\partial_n u^\varepsilon(x) + u^\varepsilon(x) = 0,\ x \in \partial\Omega,$$

with the perturbed coefficient

$$a^\varepsilon(s) = a(s) + \varepsilon \operatorname{sign} a(s) \neq 0 \text{ for all } s \in \partial\Omega$$

gets the discrete spectrum but its eigenvalues λ_k^ε depend periodically on $\ln \varepsilon$, i.e., wander (walk aimlessly like a drunkard or insane) when $\varepsilon \to +0$. Such kind of behavior is usually attributed to a skew-symmetric operator with some radiation conditions at the point $s = 0$ on the boundary $\partial\Omega$.

My work in this direction have been performing in cooperation with French mathematicians Lucas Chesnel, Xavier Clayes and Nicolas Popoff.