Spectral asymptotics of operators of the tensor product type with almost regular marginal asymptotics

Rastegaev N. V.
Chebyshev Laboratory, St. Petersburg State University
199178, 14th Line 29B, Vasilyevsky Island, St. Petersburg, Russia
St. Petersburg Department of V.A. Steklov Institute of Mathematics
191023, 27 Fontanka, St. Petersburg, Russia
e-mail: rastmusician@gmail.com

An example of an operator with almost power spectral asymptotics arises naturally when considering an ordinary differential operator with singular self-similar weight. The known formula for the asymptotics of eigenvalue counting function in that case is

$$N(\lambda) = \lambda^D \cdot (s(\ln \lambda) + o(1)), \qquad \lambda \to +\infty,$$

where s is a positive continuous periodic function, dependent on the choice of the weight. Some properties of function s are described in [1], [2] (in particular, the fine structure of s is established for certain classes of weights). It is convenient for the sake of generality to also consider almost regular asymptotics of the form

$$N(\lambda) = \lambda^D \cdot \varphi(\lambda) \cdot (s(\ln \lambda) + o(1)), \qquad \lambda \to +\infty,$$

where φ is a *slowly varying function* (SVF).

As a part of the currently intensive development of the theory of small deviations of Gaussian random functions, tensor products of compact operators with almost regular marginal asymptotics containing periodic functions are considered for asymptotic analysis. We amend the abstract theorems developed in [3] to fit this case.

We infer that the same asymptotic behavior persists for the tensor product. It is also almost regular containing a periodic function. If the asymptotics of the operators are of different powers, it will resemble the stronger of two, only with the periodic term potentially changed. If the powers coincide, the slowly varying function will be the convolution of the original ones (in the case, when slowly varying functions are constant, it means the emergence of a logarithmic term). We establish the cases, where new periodic function could be shown to be non-degenerate under certain circumstances. We also establish cases, where it is guaranteed to degenerate into constant.

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