Complex WKB method for difference equations in the complex plane

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We consider the difference Schrödinger equation

$$\psi(z+h)+\psi(z-h)+v(z)\psi(z)=E\psi(z), \quad z\in\mathbb{C}, \tag{1}$$

where $h>0$ and $E\in\mathbb{C}$ are parameters, and v is a trigonometric polynomial, i.e., $v(z)=\sum_{k=-m}^{n} c_k e^{ikz}$, $m,n>0$, $c_n, c_{-m}\neq 0$. If $v=2\cos z$, equation (1) is called Harper equation. Harper equation with a small h is a model for an electron in a crystal placed in a weak constant magnetic field, see, e.g., [2]. V. Buslaev and A. Fedotov studied quasiclassical asymptotics of solutions of Harper equation in the complex plane, see [1]. It turned out that, as $h\to 0$, solutions have standard quasiclassical behavior in certain canonical domains in the complex plane. We generalized this result to the case where the potential v is a trigonometric polynomial and provide a new, relatively simple proof of it. The talk is based on a joint work with A. Fedotov.

References

[1] Buslaev V.S. and Fedotov A.A. The complex WKB method for Harper's equation. *St. Petersburg Math. J.* 6 (1995), No.3, 495-517.

[2] Guillement J.P., Helffer B. and Treton P. Walk inside Hofstadter's butterfly. J.Phys.France, 50 (1989), 2019-2058.