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3 – 6 July 2017

Dedicated to the memory of M. Sh. Birman (1928 – 2009)

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Programme and abstracts

St. Petersburg, 2017

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Conference website: http://www.pdmi.ras.ru/EIMI/2017/ST/index.html

Speakers

- Volker BACH, *Technische Universität Braunschweig, Germany*
- Kirill CHEREDNICHENKO, *University of Bath, United Kingdom*
- Petru COJUHARI, *AGH University of Science and Technology, Krakow, Poland*
- Andrew COMECH, *Texas A&M University, USA*
- Valeriy IMAIKIN, *LLS Diagnostic NDT Equipment, Russia*
- Victor IVRII, *University of Toronto, Canada*
- Wencai LIU, *University of California, Irvine, USA*
- Alexander NAZAROV, *St. Petersburg State University, Russia*
- Victor NOVOKSHENOV, *Ufa Science Center RAS, Russia*
- Karel PRAVDA-STAROV, *University of Rennes, France*
- Roman ROMANOV, *St. Petersburg State University, Russia*
- Grigori ROZENBLIOUM, *Chalmers University, Sweden*
- Gerald TESCHL, *University of Vienna, Austria*
- Igor VERBITSKY, *Universiry of Missouri, USA*
- Ivan VESELIC, *TU Chemnitz, Germany*
- Dmitri YAFAEV, *University of Rennes, France; St. Petersburg State University, Russia*
- Steven ZELDITCH, *Northwestern University, USA*

Young Researchers

- Ivan GURIANOV, *St. Petesburg State University, Russia*
- Alexander HACH, *Technische Universitt Braunschweig, Germany*
- Kiril RYADOVKIN, *St. Petersburg State University, Russia*
- Nikita SENIK, *St. Petersburg State University, Russia*
- Yunfeng SHI, *Fudan University, China*

Conference programme

MONDAY, 3 July

9:30 – 10:00: REGISTRATION

10:00 – 10:50: Steven Zelditch (Northwestern University, USA). *Counting nodal domains of eigenfunctions on surfaces.*

COFFEE BREAK

11:20 – 12:10: Wencai Liu (University of California, Irvine, USA). *Sharp spectral transitions and growth of eigenfunctions of Laplacians on Riemannian manifolds.*

12:20 – 13:10: Ivan Veselic (TU Chemnitz, Germany). *Unique continuation estimates and lifting of eigenvalues.*

LUNCH

15:10 – 16:00: Igor Verbitsky (Universiry of Missouri, USA). *Global pointwise estimates of positive solutions to linear and non-linear PDE.*

COFFEE BREAK

16:30 – 17:00: Yunfeng Shi (Fudan University, China). *Exponential decay of the lengths of spectral gaps for quasi-periodic operators with Liouville frequencies.*

17:00 – 17:30: Nikita Senik (St. Petersburg State University, Russia). *On homogenization for non-self-adjoint locally periodic elliptic operators.*

18:30 BOAT TRIP WITH WELCOME PARTY

TUESDAY, 4 July

10:00 – 10:50: Volker Bach (Technische Universität Braunschweig, Germany). *Effective mean-field evolution equations for many-fermion systems and their accuracy.*

COFFEE BREAK

11:20 – 12:10: Karel Pravda-Starov (University of Rennes, France). *Generalized Mehler formula for time-dependent non-selfadjoint quadratic operators and propagation of singularities.*

12:20 – 13:10: Kirill Cherednichenko (University of Bath, United Kingdom). *Functional model for extensions of symmetric operators and applications to scattering theory.*

LUNCH

15:10 – 15:40: Alexander Hach (Technische Universität Braunschweig, Germany). *Suppression of decoherence of a spin-boson system by time-periodic control.*

15:40 – 16:10: Ivan Gurianov (St. Petesburg State University, Russia). *Asymptotics of resonant tunneling in quantum waveguides with several equal resonators.*

COFFEE BREAK

16:40 – 17:10: Kirill Ryadovkin (St. Petersburg State University, Russia). *Boundary spectrum for the discrete Laplace operators on graphs embedded into half-plane.*

WEDNESDAY, 5 July

10:00 – 10:50: Dmitri Yafaev (University of Rennes, France, and St. Petersburg State University, Russia). *Quasi-diagonalization of Hankel operators.*

COFFEE BREAK

11:20 – 12:10: Grigori Rozenblioum (Chalmers University, Sweden). *Toeplitz operators with strongly singular symbols and Bergman type spaces of polyanalytic functions.*

12:20 – 13:10: Petru Cojuhari (AGH University of Science and Technology, Krakow, Poland). *Spectral analysis of perturbed Toeplitz operators.*

LUNCH

15:10 – 16:00: Alexander Nazarov (St. Petersburg department of V. A. Steklov Institute, Russia). *Spectral asymptotics for some 1D boundary value problems with self-similar weights.*

COFFEE BREAK

16:30 – 17:20: Victor Ivrii (University of Toronto, Canada). *Spectral asymptotics for fractional Laplacian.*

18:00: CONFERENCE DINNER

THURSDAY, 6 July

10:00 – 10:50: Victor Novokshenov (Ufa Science Center RAS, Russia). *Orthogonal polynomials and Painlevé equations.*

COFFEE BREAK

11:20 – 12:10: Gerard Teschl (University of Vienna, Austria). *Dispersion estimates for one-dimensional Schrödinger equations.*

12:20 – 13:10: Andrew Comech (Texas A&M University, USA). *Spectral stability of small amplitude solitary waves in the nonlinear Dirac equation.*

LUNCH

15:10 – 16:00: Valeriy Imaikin (LLS Diagnostic NDT Equipment, Moscow, Russia). *On continuation to the spectrum of the resolvent of the linearized problem of an electric field and charged particle interaction.*

COFFEE BREAK

16:30 – 17:20: Roman Romanov (St. Petersburg State University, Russia). *Reproducing kernel Hilbert spaces related to determinantal processes.*

Abstracts of the talks

Effective mean-field evolution equations for many-fermion systems and their accuracy

Volker BACH

Technische Universität Braunschweig, Germany

The dynamics of quantum mechanical systems is determined by the Schrödinger Equation. For many-particle systems, however, this is a partial differential equation in 10^{24} spatial variables and one has to resort to approximative solutions. The lecture reviews mean field approximations for systems of identical particles with a special focus on the quantitative validation of the approximation for many-fermion systems.

Functional model for extensions of symmetric operators and applications to scattering theory

Kirill CHEREDNICHENKO

Bath University, United Kingdom

I shall discuss the functional model for extensions of symmetric operators and its applications to the theory of wave scattering. In terms of Boris Pavlov's spectral form of this model, we find explicit formulae for the action of the unitary group of exponentials corresponding to almost solvable extensions of a given closed symmetric operator with equal deficiency indices. On the basis of these formulae, we are able to derive a new representation for the scattering matrix for pairs of such extensions. We use this representation to explicitly recover the coupling constants in the inverse scattering problem for a finite non-compact quantum graph with delta-type vertex conditions. This is joint work with A. V. Kiselev and L. O. Silva.

Spectral analysis of perturbed Toeplitz operators

Petru COJUHARI

AGH University of Science and Technology, Krakow, Poland

A spectral analysis is undertaken on a class of perturbed Toeplitz operators. Emphasis is placed on the structure of their spectra. The problems are treated for the general case in an abstract framework by using direct methods of perturbation theory. Applications to block Jacobi matrices and also to perturbed periodic Jacobi matrices will be considered. In particular, estimate formulas for the number of the eigenvalues created by perturbations in the spectral gaps will be presented.

Spectral stability of small amplitude solitary waves in the nonlinear Dirac equation

Andrew COMECH

Texas A&M University, USA

Joint work with Nabile Boussaïd (Université Bourgogne – Franche-Comte, Besançon, France).

The nonlinear Dirac equation with the scalar self-interaction, known as the Soler model, is given by

$$\mathrm{i}\partial_t\psi = D_m\psi - f(\psi^*\beta\psi)\beta\psi, \qquad \psi(x,t)\in\mathbb{C}^N, \quad x\in\mathbb{R}^n,$$

where the Dirac operator is $D_m = -\mathrm{i}\alpha\cdot\nabla + \beta m$, $m>0$, with the self-adjoint $N\times N$ Dirac matrices $\alpha = (\alpha^\jmath)_{1\le\jmath\le n}$ and β such that $D_m^2 = -\Delta + m^2$. The nonlinearity is represented by a real-valued function $f\in C^1(\mathbb{R})$. Given a solitary wave solution $\phi_\omega(x)e^{-\mathrm{i}\omega t}$, $\omega\in(-m,m)$, we study whether it is *spectrally stable*, that is, whether the linearization at the solitary wave contains eigenvalues with positive real part.

In this talk, we will discuss the possibility of bifurcations of eigenvalues from the essential spectrum. The main result is the absence of embedded eigenvalues $\lambda\in\mathrm{i}\mathbb{R}$ of the linearization operator beyond the embedded thresholds $\pm(m+|\omega|)\mathrm{i}$. As a consequence, when ω changes, the nonzero-real-part eigenvalues could only appear from the essential spectrum bifurcating from an embedded eigenvalue $\lambda\in\mathrm{i}\mathbb{R}$ with $|\lambda|<m+|\omega|$ or from the embedded thresholds $\pm(m+|\omega|)\mathrm{i}$. Our main tool is the Carleman estimates for the Dirac operator [2], which we generalize to the linearization at a solitary wave and to any dimension.

The talk is based on [1].

[1] N. Boussaïd and A. Comech, *On spectral stability of the nonlinear Dirac equation*, J. Funct. Anal. **271** (2016), 1462–1524.

[2] A. Berthier and V. Georgescu, *On the point spectrum of Dirac operators*, J. Funct. Anal. **71** (1987), no. 2, 309–338.

On extension to the spectrum of the resolvent of the linearized problem of interaction of an electromagnetic field and a charged particle.

Valeriy IMAIKIN

LLS Diagnostic NDT Equipment, Russia

Consider the scalar Klein-Gordon field $\psi(x,t)$, $x\in\mathbb{R}^3$, $t\in\mathbb{R}$ interacting with a relativistic charged particle. Let $q(t)$, $p(t)$ be coordinates and momenta of the particle at a time t, then the system reads [1]:

$$\dot\psi(x,t) = \pi(x,t), \qquad \dot\pi(x,t) = \Delta\psi(x,t) - m^2\psi(x,t) - \rho(x-q(t)), \quad x\in\mathbb{R}^3,$$

$$\dot q(t) = p(t)/\sqrt{1+p^2}, \quad \dot p(t) = -\int\nabla\psi(x,t)\,\rho(x-q(t))dx.$$

The system is Hamiltonian, the corresponding symplectic form is defined. Further, the system is invariant w.r.t to translations in $\mathbb{R}^3$ and admits soliton-type solutions

$$Y_{a,v}(t) = (\psi_v(x-vt-a), \pi_v(x-vt-a), vt+a, p_v), \qquad p_v = v/\sqrt{1-v^2}$$

To obtain *soliton-type asyptotics of solutions* we linearize the system at the soliton manifold $\mathcal{S}$: $Z(t) := Y(t) - S(t)$. Here $Y(t)$ is the solution to the system and $S(t)$ is its syplectic orthogonal projection onto the soliton manifold. The linearized equation reads

$$\dot{Z}(t) = A(t)Z(t) + N(S(t), Z(t)),$$

where $A(t)$ is a linear non-autonomous operator and N is the term of the second smallness order. We fix a $t = t_1$ at $A(t)$ and consider the corresponding "frozen" linear autonomous equation $\dot{X} = AX$ with $A := A(t_1)$ instead of $A(t)$. The important step is to prove the decay

$$\|X(t)\|_{-\beta} \leq \frac{C\|X(0)\|_{\beta}}{(1+|t|)^{3/2}}, \qquad t \in \mathbb{R}$$

of the solution $X(t)$ to the frozen equation for any $X(0) \in \mathcal{Z}_{S_1}$, where $S_1 := S(t_1)$, and $\mathcal{Z}_{S_1}$ is the space of vectors X which are symlectic orthogonal to the tangent space $\mathcal{T}_{S_1}\mathcal{S}$; the norm is that of an appropriate weighted Sobolev space.

Apply the Laplace transform and obtain: $\lambda\tilde{X}(\lambda) = A\tilde{X}(\lambda) + X_0$, $\tilde{X}(\lambda) = -(A-\lambda)^{-1}X_0$, $\Re\lambda > 0$, where the resolvent $R(\lambda) = (A-\lambda)^{-1}$ does exist and is analytic for $\Re\lambda > 0$. Formally,

$$\Lambda^{-1}\tilde{X} = X(t) = \frac{1}{2\pi}\int_{\mathbb{R}} e^{i\omega t}\tilde{X}(i\omega + 0)d\omega, \qquad t \in \mathbb{R},$$

within the integral the resolvent is extended from the right half-plane to the imaginary axis, that contains the spectrum of the operator A. To derive the time decay of $X(t)$ we have to prove that the function $\tilde{X}(i\omega + 0)$ 1) is smooth besides the points $\omega = 0$ and $\omega = \pm\mu$, where $\mu = \mu(v) > 0$ are the end points of the continuous spectrum, 2) has a certain decay as $|\omega| \to \infty$, i.e. at the continuous spectrum. 3) admits the Puisaux expansion at the points $\omega = \pm\mu$. 4) is analytic at $\omega = 0$, i.e. at the point of the discrete spectrum, if $X_0 \in \mathcal{Z}_{S_1}$.

This is done by different analytic techniques [1]. Let us stress the two important points: to prove analyticity at 0 it is essential that the initial data are symplectic orthogonal to $\mathcal{T}_{S_1}\mathcal{S}$; at the continuous spectrum the *Wiener condition* $\hat{\rho}(k) \neq 0$ $\forall k \in \mathbb{R}^3$ is essential.

[1] Imaykin V. Soliton asymptotics for systems of "field-particle" type, Russ. Math. Surv., 68, N. 2, 33 – 90.

Spectral asymptotics for fractional Laplacian

Victor IVRII

University of Toronto, Canada

Consider a compact domain with the smooth boundary in the Euclidean space. Fractional Laplacian is defined on functions supported in this domain as a (non-integer) power of the positive Laplacian on the whole space restricted then to this domain. Such operators appear in the theory of stochastic processes. It turns out that the standard results about distribution of eigenvalues (including two-term asymptotics) remain true for fractional Laplacians. There are however some unsolved problems.

Sharp spectral transitions and growth of eigenfunctions of Laplacians on Riemannian manifolds

Wencai LIU

University of California, Irvine, USA

In this talk, we study eigenvalues or singular continuous spectrum of the Laplacian (Δ) embedded in the essential spectrum on either asymptotically flat or asymptotically hyperbolic manifolds.

It is an old result that the essential spectrum is $[\frac{c^2}{4}, \infty]$ if $\Delta r \to c$ as r goes to infinity, where $r(x)$ is the distance function.

Kumura proved that there are no eigenvalues embedded in the essential spectrum $\sigma_{\text{ess}}(-\Delta) = \left[\frac{1}{4}(n-1)^2, \infty\right)$ of Laplacians on asymptotically hyperbolic manifolds, where asymptotic hyperbolicity is characterized by the radial curvature, i.e., $K_{\text{rad}} = -1 + o(r^{-1})$. He also constructed a manifold for which an eigenvalue $\frac{(n-1)^2}{4} + 1$ is embedded into its essential spectrum $[\frac{(n-1)^2}{4}, \infty)$ with the radial curvature $K_{\text{rad}}(r) = -1 + O(r^{-1})$.

The first part of the talk, based on a joint work with S.Jitomirskaya, is devoted to construction of manifolds with embedded eigenvalues and singular continous spectrum. Given any finite (countable) positive energies $\{\lambda_n\} \in [\frac{K_0}{4}(n-1)^2, \infty)$, we construct Riemannian manifolds with the decay of order $K_{\text{rad}} + K_0 = O(r^{-1})$ with $K_0 \geq 0$ ($K_{\text{rad}} + K_0 = \frac{C(r)}{r}$, where $C(r) \geq 0$ and $C(r) \to \infty$ arbitrarily slowly) such that the eigenvalues $\{\lambda_n\}$ are embedded in the essential spectrum $\sigma_{\text{ess}}(-\Delta) = \left[\frac{K_0}{4}(n-1)^2, \infty\right)$. We also construct Riemannian manifolds with the decay of order $K_{\text{rad}} + K_0 = \frac{C(r)}{r}$, where $C(r) \geq 0$ and $C(r) \to \infty$ arbitrarily slowly such that there is singular continous spectrum embedded in the essential spectrum $\sigma_{\text{ess}}(-\Delta) = \left[\frac{K_0}{4}(n-1)^2, \infty\right)$.

In the second part I discuss criteria for the absence of eigenvalues embedded into essential spectrum in terms of the asymptotic behavior of Δr with no conditions on the curvature. Under a weaker convexity of distance function r for certain asymptotic behavior of Δr, we established the growth of the eigensolution of $\Delta u = \lambda u$ for $\lambda > \lambda_0$. As an application, we show that there are no eigenvalues embedded into the essential spectrum if $\Delta r = a + \frac{b}{r} + \frac{o(1)}{r}$ as r goes to infinity and the distance function r satisfies some weaker convexity. The proof is based on the flexible construction of energy functions and a new way to verify the positivity of the initial energy.

Spectral asymptotics for some 1D boundary value problems with self-similar weights

Alexander NAZAROV

St. Petersburg department of V. A. Steklov Institute, Russia

We consider a class of self-adjoint spectral problems

$$\mathcal{L}y := (-1)^{\ell}y^{(2\ell)} + \left(\mathcal{P}_{\ell-1}y^{(\ell-1)}\right)^{(\ell-1)} + \cdots + \mathcal{P}_0 y = \lambda\mathfrak{M}y \tag{1}$$

with proper boundary conditions. Here $\mathcal{P}_i \in L_1(0,1)$, $i = 0, \ldots, \ell-1$, and $\mathfrak{M}$ is a singular self-similar measure on $[0,1]$.

We obtain one-term asymptotics for the counting function of eigenvalues of (1). Then we apply these results to the problem of small ball asymptotics for Gaussian processes in L_2-norm with respect to self-similar measure.

[1] A. I. Nazarov, *Logarithmic asymptotics of small deviations for some Gaussian processes in the L_2-norm with respect to a self-similar measure*, ZNS POMI, **311** (2004), 190–213 (in Russian); J. Math. Sci., **133** (2006), N3, 1314–1327.

[2] A. I. Nazarov, *Log-level comparison principle for small ball probabilities*, Stat. & Prob. Letters, **79** (2009), N4, 481486.

[3] A. I. Nazarov, I. A. Sheipak, *Degenerate self-similar measures, spectral asymptotics and small deviations of Gaussian processes*, BLMS, **44** (2012), N1, 12–24.

[4] N. V. Rastegaev, *On Spectral Asymptotics of the Neumann Problem for the SturmLiouville Equation with Self-Similar Weight of Generalized Cantor Type*, ZNS POMI, **425** (2014), 86–98 (in Russian); J. Math. Sci., **210** (2015), N6, 814–821.

Orthogonal polynomials and Painlevé equations

Victor NOVOKSHENOV

Ufa Science Center RAS, Russia

Asymptotics of the orthogonal polynomial constitute a classic analytic problem. In the talk, a distribution of zeroes to generalized Hermite polynomials $H_{m,n}(z)$ is found as $m = n$, $n \to \infty$, $z = O(\sqrt{n})$. These polynomials defined as Wronskians of classic Hermite polynomials appear in a number of mathematical physics problems as well as in the theory of random matrices. Calculation of asymptotics is based on Riemann-Hilbert problem for Painlevé IV equation which has solutions $u(z) = -2z + \partial_z \ln H_{m,n+1}(z)/H_{m+1,n}(z)$. In this scaling limit the Riemann-Hilbert problem is solved in elementary functions. As a result, we come to analogs of Plancherel-Rotach formulas for asymptotics of classical Hermite polynomials.

Generalized Mehler formula for time-dependent non-selfadjoint quadratic operators and propagation of singularities

Karel PRAVDA-STAROV

University of Rennes, France

We study evolution equations associated to time-dependent dissipative non-selfadjoint quadratic operators. We prove that the solution operators to these non-autonomous evolution equations are given by Fourier integral operators whose kernels are Gaussian tempered distributions associated to non-negative complex symplectic linear transformations, and we derive a generalized Mehler formula for their Weyl symbols. Some applications to the study of the propagation of Gabor singularities (characterizing the lack of Schwartz regularity) for the solutions to non-autonomous quadratic evolution equations are given.

Reproducing kernel Hilbert spaces related to determinantal processes

Roman ROMANOV

St. Petersburg State University, Russia

A class of determinantal processes is described by the reproducing kernel Hilbert spaces with certain divisibility properties which generalize the familiar de Branges spaces of entire functions. In our talk we will discuss the problem of description of these spaces. In particular we will show that the local trace class condition in the axioms of the spaces corresponding to the processes actually follows from other axioms. The talk is based on joint work with Alexander Bufetov.

Toeplitz operators with strongly singular symbols and Bergman type spaces of polyanalytic functions

Grigori ROZENBLIOUM

Chalmers University, Sweden

We present a general approach to defining Toeplitz operators in Bergman type spaces, especially, of analytic functions, with highly singular symbols, such as distributions and hyperfunctions, in each case finding sufficient conditions for the operator to be bounded or compact. With this definition, many important classes of operators become Toeplitz ones. As an application, boundedness and compactness conditions for usual Toeplitz operators in spaces of poly-analytic functions are found. In particular, it turns out that in spaces of poly-analytic functions there are much more finite rank Toeplitz operators than in spaces of analytic functions. Joint work with Nikolai Vasilevski, Mexico.

Dispersion estimates for one-dimensional Schrödinger equations

Gerald TESCHL

University of Vienna, Austria

We show that for a one-dimensional Schrödinger operator with a potential whose first moment is integrable the scattering matrix is in the unital Wiener algebra of functions with integrable Fourier transforms. Then we use this to derive dispersion estimates for solutions of the associated Schrödinger and Klein-Gordon equations. In particular, we remove the additional decay conditions in the case where a resonance is present at the edge of the continuous spectrum. If times permit I will also mention some related results for the radial Schrödinger equation. Based on joint work with I. Egorova, E. Kopylova, and V. Marchenko.

Global pointwise estimates of positive solutions to linear and non-linear PDE

Igor VERBITSKY

Universiry of Missouri, USA

Recent results will be presented on sharp global estimates of positive solutions to linear and non-linear elliptic equations of the type $-\Delta u + \sigma\, u^q = \mu$, for all real $q \neq 0$, where σ, μ are functions, or measures, on a domain $\Omega \subseteq \mathbf{R}^n$, or a weighted Riemannian manifold (M, ω), and $\Delta = \mathrm{div}_\omega \cdot \nabla, \quad \mathrm{div}_\omega = \frac{1}{\omega} \circ \mathrm{div} \circ \omega$ is the weighted Laplacian on (M, ω).

Analogues of these estimates for some nonlocal operators of fractional Laplacian type, as well as integral operators with positive kernels satisfying various forms of the maximum principle will be discussed. This talk is based on joint work with Michael Frazier (to appear in Ann. Inst. Fourier), and Alexander Grigor'yan (to appear in J. d'Analyse Math).

Unique continuation estimates and lifting of eigenvalues

Ivan VESELIC

TU Chemnitz, Germany

Using Carleman estimates we prove scale free unique continuation estimates on bounded and unbounded domains and apply them to the spectral theory of Schroedinger operators. In particluar, we present eigenvalue lifting estimates and lifting estimates for spectral band edges of periodic and similar Schroedinger operators. This is joint work with I. Nakic, M. Taeufer, and M. Tautenhahn.

Quasi-diagonalization of Hankel operators

Dmitri YAFAEV

University of Rennes, France, and St. Petersburg State University, Russia

We show that all Hankel operators H realized as integral operators with kernels $h(t+s)$ in $L^2(\mathbf{R}_+)$ can be quasi-diagonalized as $H = L^*\Sigma L$. Here L is the Laplace transform, Σ is the operator of multiplication by a function (distribution) $\sigma(\lambda)$, $\lambda \in \mathbf{R}$, linked to h by the formula $h = L^*\sigma$. The sigma-function of a Hankel operator contains substantial information about its spectral properties and, in particular, allows one to obtain an explicit formula for the total numbers of its positive and negative eigenvalues.

As an application of this construction, we also find the asymptotic behaviour of singular values of Hankel operators. This leads to new results on rational approximation of functions ω with logarithmic singularities. Thus we obtain an asymptotic formula for the distance in the BMO norm between ω and the set of rational functions of degree n as $n \to \infty$. This part of the talk relies on joint papers with A. Pushnitski.

Counting nodal domains of eigenfunctions on surfaces

Steven ZELDITCH

Northwestern University, USA

An open problem is whether a Riemannian manifold (M, g) possesses any sequence of eigenfunctions for which the number of nodal domains tends to infinity. We prove that the number tends to infinity along almost a full orthonormal basis on a non-positively curved surface with concave boundary or on a real Riemann surface with negatively curved metric. The main point is to prove existence of a lot of zeros of the Cauchy data on the boundary or fixed point set. This requires quantum ergodic restriction theorems and some new results on sup norms of eigenfunctions.

Talks by young researchers

Asymptotics of resonant tunneling in quantum waveguides with several equal resonators

Ivan GURIANOV

St. Petersburg State University, Russia

We consider the Helmholtz equation in a cylindric domain G with several narrows of small diameter ε. We set the Dirichlet boundary condition on ∂G and intrinsic radiation conditions at infinity. The parts of the domain between two neighbouring narrows play the role of resonators. In such a waveguide, the resonant tunneling can occur, i.e. the transmission coefficient $T(E)$ has peaks at some resonant energies. We present asymptotic formulas for these energies as the diameters ε of the narrows tend to zero. The behavior of $T(E)$ near resonances is analysed. This is a joint work with O. Sarafanov.

Suppression of decoherence of a spin-boson system by time-periodic control

Alexander HACH

Technische Universität Braunschweig, Germany

We consider a finite-dimensional quantum system coupled to the bosonic radiation field and subject to a time-periodic control operator. Assuming the validity of a certain dynamic decoupling condition we approximate the system's time evolution with respect to the non-interacting dynamics. For sufficiently small coupling constants g and control periods T we show that a certain deviation of coupled and uncoupled propagator may be estimated by $\mathcal{O}(gt\,T)$. Our approach relies on the concept of Kato stability and general theory on non-autonomous linear evolution equations.

Boundary spectrum for the discrete Laplace operators on graphs embedded into half-plane

Kirill RYADOVKIN

St. Petersburg State University, Russia

The electronic structures of graphene and stanene media have been the subject of intensive research during past years due to their versatile electronic properties. In this talk we will discuss spectral properties of the discrete Laplace operator (tight-binding model) on graphs embedded into half-space with geometrically perturbed boundaries, i.e. with additional vertices and edges. In particular we consider graphene and stanene lattices embedded into the half-plane.

The spectrum of the Laplace operator on the graphene and stanene lattices embedded into the plane is well known. The spectrum remains the same for the Laplace operator on the lattices embedded into the half-plane with the zigzag boundary. Consider the Laplace operator on the same structures with the boundary perturbed by additional vertices and edges. If the number of new edges is finite on the period of the boundary then the spectrum of the unperturbed Laplacian will remain part of the spectrum but the additional boundary spectrum can appear. This additional spectrum may contain bands or eigenvalues of infinite multiplicity. In this talk we show estimates on position and lenght of the boundary spectrum. Also we provide several examples of boundary perturbations of the graphene and stanene lattices and determine spectrum of the Laplacian on these lattices. The main technical tools in finding the boundary spectrum are the direct integral decomposition of the Laplace operator into auxiliary operators on the so-called fundamental graph and further studying their spectra. This talk is based on joint work with E. Korotyaev and N. Saburova.

On Homogenization for Non-Self-Adjoint Locally Periodic Elliptic Operators

Nikita SENIK

St. Petersburg State University, Russia

In homogenization theory one is interested in studying asymptotic properties of solutions to differential equations with rapidly oscillating coefficients.We will consider such a problem for a matrix strongly elliptic operator $\mathcal{A}^\varepsilon = -\operatorname{div} A(x, x/\varepsilon)\nabla$, where A is Hölder continuous of order $s \in [0,1]$ in the first variable and periodic in the second. We do not require that $A^* = A$, so $\mathcal{A}^\varepsilon$ need not be self-adjoint. It is well known that the resolvent $(\mathcal{A}^\varepsilon - \mu)^{-1}$ converges, in some sense, as $\varepsilon \to 0$. In this talk we will discuss results regarding convergence in the uniform operator topology on $L_2(\mathbb{R}^d)^n$, i.e., the strongest type of operator convergence. We present the first two terms of an approximation for $(\mathcal{A}^\varepsilon - \mu)^{-1}$ and the first term of an approximation for $\nabla(\mathcal{A}^\varepsilon - \mu)^{-1}$. Particular attention will be paid to the rates of approximation.

Exponential decay of the lengths of spectral gaps for quasi-periodic operators with Liouville frequencies

Yunfeng SHI

Fudan University, China

In this talk we consider the quasi-periodic operators with Liouville frequencies. Our model contains the quasi-periodic Schrodinger operator and the extended Harper's model. By establishing quantitative reducibility result together with the averaging method, we show that the lenghts of the spectral gaps decay exponentially.