Euler International Mathematical Institute
St. Petersburg Department of Steklov Mathematical Institute of Russian Academy of Sciences

10th St. Petersburg Conference in Spectral Theory

9 – 12 June 2018

Dedicated to the memory of M. Sh. Birman (1928 – 2009)

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Programme and abstracts

St. Petersburg, 2018

Organizers:

Alexandre Fedotov, Nikolai Filonov, Alexander Sobolev, Tatyana Suslina, Dmitri Yafaev

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Conference website: http://www.pdmi.ras.ru/EIMI/2018/ST/index.html

Speakers

- Alexander APTEKAREV, *Keldysh Institute of Applied Mathematics, Russia*
- Roman BESSONOV, *St. Petersburg State University and PDMI RAS, Russia*
- Nikolay FILONOV, *PDMI RAS, Russia*
- Alexander GRIGORIAN, *Bielefeld University, Germany*
- Alexander ITS, *Indiana University – Purdue University Indianapolis, USA*
- Victor IVRII, *University of Toronto, Canada*
- Frédéric KLOPP, *Université Pierre et Marie Curie, Paris*
- Aleksey KOSTENKO, *Universität Wien, Austria, and University of Ljubljana, Slovenia*
- Andrei Martínez FILKENSTEIN, *Universidad de Almería, Spain*
- Boris MITYAGIN, *Ohio State University, USA*
- Sergei NAZAROV, *St. Petersburg State University and Institute of Mechanical Engineering Problems, Russia*
- Georgi RAJKOV, *Pontifical Catholic University of Chile*
- Hermann SCHULTZ-BALDES, *Friedrich-Alexander Universität Erlagen-Nürnberg, Germany*
- Jan Philip SOLOVEJ, *University of Copenhagen, Denmark*
- Andrew SHKALIKOV, *Moscow State University, Russia*
- Tatiana SUSLINA, *St. Petersburg State University, Russia*

Young Researchers

- Matteo CAPOFERRI, *University College London, UK*
- Egor GALKOVSKI, *St. Petersburg State University, Russia*
- Orif IBROGIMOV, *University College London, UK*
- Yulia MESHKOVA, *St. Petersburg State University, Russia*
- Bernard PFIRSCH, *University College London, UK*
- Maria PLATONOVA, *PDMI RAS, Russia*

Conference programme

SATURDAY 9 June:

9:30–10:00: REGISTRATION

10:00–10:50: Frédéric Klopp (Sorbonne Université, Paris). *Resonances for large random samples.*

COFFEE BREAK

11:20–12:10: Georgi Rajkov (Pontifical Catholic University of Chile). *Lifshits tails for randomly twisted quantum waveguides.*

12:20–13:10: Tatiana Suslina (St. Petersburg State University, Russia). *Homogenization of the stationary Maxwell system with periodic coefficients.*

LUNCH

15:10–16:00: Nikolay Filonov (PDMI RAS, Russia). *On the uniqueness of the Leray-Hopf solution for a dyadic model.*

COFFEE BREAK

16:30–17:20: Sergei Nazarov (St. Petersburg State University and Institute of Mechanical Engineering Problems, Russia). *Sharpening and smoothing near-threshold Wood anomalies in cylindrical waveguides.*

18:30 BOAT TRIP WITH WELCOME PARTY

SUNDAY 10 June:

10:00–10:50: Alexander Its (Indiana University – Purdue University Indianapolis, USA). *The Riemann-Hilbert approach to the determinants of Toeiplitz + Hankel matrices.*

COFFEE BREAK

11:20–12:10: Alexander Aptekarev (Keldysh Institute of Applied Mathematics, Russia). *Jacobi matrices on trees and multiple orthogonal polynomials.*

12:20–13:10: Andrei Martìnez Filkenstein (Universidad de Almerìa, Spain). *Asymptotics of multiple orthogonal polynomials for cubic weight.*

LUNCH

15:10–16:00: Boris Mityagin (Ohio State University, USA). *"Arcsine law" for eigenfunctions of anharmonic operators.*

COFFEE BREAK

16:30–17:20: Roman Bessonov (St. Petersburg State University and PDMI RAS, Russia). *Szegö condition and scattering for one-dimensional Dirac operators.*

MONDAY 11 June:

10:00–10:50: Jan Philip Solovej (University of Copenhagen, Denmark). *Spectral flow and zero modes for Dirac operators with magnetic links.*

COFFEE BREAK

11:20–12:10: Alexander Grigorian (Bielefeld University, Germany). *Heat kernels on manifolds with ends.*

12:20–12:50: Yulia Meshkova (Chebyshev Laboratory, St. Petesburg State University, Russia). *Homogenization of hyperbolic equations with periodic coefficients.*

12:50–13:20: Bernhard Pfirsch (University College London, UK). *A Szegö limit theorem for translation-invariant operators on polygons.*

LUNCH

15:10–15:40: Orif Ibrogimov (University College London, UK). *On the spectrum of the spin-boson Hamiltonian with two photons.*

15:40–16:10: Egor Galkovski (St. Petersburg State University, Russia). *A generalization of the trace formula for differential operators on a segment with the lowest-order coefficient perturbed by finite signed measure.*

COFFEE BREAK

16:40–17:10: Matteo Capoferri (University College London, UK). *Global hyperbolic propagators in curved space.*

17:10–17:40: Maria Platonova (Steklov Institute, St. Petersburg, Russia). *On branching random walks.*

18:00: CONFERENCE DINNER

TUESDAY 12 June:

10:00–10:50: Hermann Schulz-Baldes (Friedrich-Alexander Universität Erlagen-Nürnberg, Germany). *The spectral localizer for even index pairings.*

COFFEE BREAK

11:20–12:10: Andrew Shkalikov (Moscow State University, Russia). *Eigenvalue Dynamics for PT-Symmetric Sturm-Liouville Operators with Physical Parameter. Solvable Models and Applications.*

12:20–13:10: Aleksey Kostenko (Universität Wien, Austria, and University of Ljubljana, Slovenia). *Infinite Quantum Graphs.*

LUNCH

15:10–16:00: Victor Ivrii (University of Toronto, Canada). *Spectral asymptotics for Steklov's problem in domains with edges.*

Abstracts of the talks

Jacobi matrices on trees and multiple orthogonal polynomials

Alexander APTEKAREV

Keldysh Institute of Applied Mathematics, Russia

Multiple orthogonal polynomials are known to satisfy recurrence relations on the lattice $(\mathbb{Z}^+)^d$. We use these relations to construct self-adjoint operator (Jacobi matrix) on the tree. This tree is a homogenous infinite rooted tree with homogeneity degree which is equal to $d+1$ (i.e., each vertex has one "parent" (incoming) edge and d "children" (outgoing) edges). The case $d = 1$ gives the polynomials orthogonal on the real line, the tree becomes $\mathbb{Z}^+$, and the Jacobi matrix is the standard three-diagonal matrix. We shall discuss multiple orthogonal polynomials approach to the spectral theory of multidimensional discrete Schrödinger operator and corresponding discrete integrable systems.

It is a joint work with Sergey Denisov and Maxim Yattselev.

Szegő condition and scattering for one-dimensional Dirac operators

Roman BESSONOV

St.Petersburg State University and PDMI RAS, Russia

We prove existence of modified wave operators for one-dimensional Dirac operators whose spectral measures have finite logarithmic integral. This extends previously known results by Christ, Kiselev, and Denisov. Potentials for which the wave operators exist, admit a simple description in terms of their averages on intervals of unit length. The proof is based on an argument from the theory of orthogonal polynomials on the unit circle.

On the uniqueness of the Leray-Hopf solution for a dyadic model

Nikolay FILONOV

PDMI RAS, Russia

We consider the following problem

$$\begin{cases} \dot{u}_n(t) + \lambda^{2n} u_n(t) - \lambda^{\beta n} u_{n-1}(t)^2 + \lambda^{\beta(n+1)} u_n(t) u_{n+1}(t) = f_n(t), \\ u_n(0) = a_n, \qquad n = 1, 2, \dots . \end{cases}$$

The main feature of this system is that it is similar to the system of the Navier-Stokes equations, and it can be considered as a toy model for NSE. It is well known that weak solution (Leray–Hopf solution) always exists. We study the uniqueness of such solutions. We obtain two results:

1) If RHS $f_n = 0$, and the initial data $\{a_n\}$ are "good" enough, then the Leray–Hopf solution is unique.

2) If initial data $a_n = 0$, but RHS f_n are "bad", then it is possible that the Leray–Hopf solution is not unique.

Heat kernels on manifolds with ends

Alexander GRIGORIAN

Bielefeld University, Germany

The heat kernel of a Riemannian manifold is the minimal positive fundamental solution of the heat equation associated with the Laplace–Beltrami operator. Upper and lower estimates of heat kernels play important role in Analysis on manifolds.

A celebrated theorem of Li and Yau provides two sided Gaussian estimates of the heat kernel on a complete Riemannian manifold of non-negative Ricci curvature.

In this talk we present heat kernel estimates on complete manifolds with ends, assuming that the heat kernel on each end satisfies the Li–Yau estimate.

It turns out that the behaviour of the heat kernel on the entire manifold depends on the property of the ends to be parabolic or not (a manifold is called parabolic if Brownian motion on it is recurrent, or, equivalently, if any positive superharmonic function is constant).

The talk is based on joint papers with L. Saloff-Coste and S. Ishiwata.

The Riemann-Hilbert approach to the determinants of Toeiplitz + Hankel matrices.

Alexander ITS

Indiana University – Purdue University Indianapolis

During the last 10-15 years the Riemann-Hilbert technique has been successfully used for solving a number of long-standing problems in the field of asymptotic analysis of the determinants of Toeplitz and Hankel matrices. The technique has been also extended to the determinants of Toelpitz + Hankel matrices generated by the same symbol. In the talk, we will explain how the Riemann-Hilbert framework can be further extended to include the Toeplit+Hankel matrices whose symbols are unrelated. The principal motivation of this extension is the evaluation of the large N asymptotics of the eigenvalues of Hankel matrices. The talk is based on the joint work with R. Gharakhloo, and it is a part of bigger joint project with P. Deift, T. Bothner and I. Krasovsky on the asymptotic theory of Toeplitz, Hankel and Toplitz + Hankel determinants.

Spectral asymptotics for Steklov's problem in domains with edges.

Victor IVRII

University of Toronto, Canada

We derive sharp eigenvalue asymptotics for Dirichlet-to-Neumann operator in the domain with edges and discuss obstacles for deriving a sharper (two-term) asymptotics.

Resonances for large random samples

Frédéric KLOPP

Sorbonne Université, Paris

The talk is devoted to the description of the resonances generated by a large sample of random material. In one dimension, one obtains a very precise description for the resonances that directly related to the description fo the eigenvalues and localization centers for the full random model. In higher dimension, below a region of localization in the spectrum for the full random model, one computes the asymptotic density of resonances in some exponentially small strip below the real axis. The talk is partially based on joint work with M. Vogel.

Infinite Quantum Graphs

Aleksey KOSTENKO

Universität Wien, Austria, and University of Ljubljana, Slovenia

The notion of quantum graph refers to a graph considered as a one-dimensional simplicial complex and equipped with a differential operator ("Hamiltonian"). We will review the basic spectral properties of infinite quantum graphs (graphs having infinitely many vertices and edges). In particular, we will discuss recently discovered fruitful connections between quantum graphs and discrete Laplacians on graphs.

The talk is based on joint works with P. Exner, M. Malamud, H. Neidhardt, and N. Nicolussi.

Asymptotics of multiple orthogonal polynomials for cubic weight

Andrei Martínez FILKENSTEIN

Universidad de Almería, Spain

We consider the type I and type II multiple orthogonal polynomials (MOPs), satisfying non-hermitian orthogonality with respect to the weight $\exp(-z^3)$ on two unbounded contours on the complex plane. Under the assumption that the orthogonality conditions are distributed with a fixed proportion α, we find the detailed (rescaled) asymptotics of these MOPs, and describe the phase transitions of this limit behavior as a function of α. This description is given in terms of the vector critical measure, the saddle point of the energy functional comprising both attracting and repelling forces. These critical measures are characterized by a cubic equation (spectral curve), and their components live on trajectories of a canonical quadratic differential on the Riemann surface of this equation. The structure of these trajectories and their deformations as function of α plays the crucial role.

This is a joint work with Gulherme L. Silva (University of Michigan, Ann Arbor).

“Arcsine law” for eigenfunctions of anharmonic operators

Boris MITYAGIN

Ohio State University, USA

Consider a Schrödinger operator $A = -\frac{d^2}{dx^2} + Q(x)$, where $Q(x) \in C^2(\mathbb{R})$ is a nonnegative, even, convex, slowly changing potential. Let $A\psi_k = \lambda_k \psi_k$, $\|\psi_k\| = 1$, $k \in \mathbb{N}$ be a complete system of eigenfunctions, and let the turning points $x_k > 0$ be defined by $Q(x_k) = \lambda_k$. Assume that $Q(x)$ satisfies

$$\lim_{x\to\infty} \frac{Q(tx)}{Q(x)} = t^\beta, \quad \beta \geq 2.$$

Rescale measures, or their densities, on $\mathbb{R}$ by

$$\varphi_k(x) = x_k \psi_k^2(x_k x).$$

The behavior of measures $\psi_k(x)^2\,dx$ determines [MSV] the asymptotics of the norms of spectral 1D-projections of non-self-adjoint perturbations of A.

For any f in the Schwartz space on $\mathbb{R}$,

$$\lim_{k\to\infty} \int_{-\infty}^{\infty} f(x)\varphi_k(x)\,dx = c(\beta) \int_{-1}^{1} f(x) \frac{dx}{(1-|x|^\beta)^{1/2}}$$

where $c(\beta) = \frac{\Gamma(\frac{1}{2}+\frac{1}{\beta})}{2\pi^{1/2}\Gamma(1+\frac{1}{\beta})}$.

Such statements, in the context of the theory of orthogonal polynomials, are well known (Rakhmanov, Mhaskar–Saff, Lubinsky).

These are preliminary results of a joint work of the speaker, Petr Siegl (Queen’s University Belfast, UK) and Joseph Viola (University of Nantes, France).

[MSV] B. Mityagin, P. Siegl, and J. Viola, *Differential operators admitting various rates of spectral projection growth*, **J. Funct. Anal.** 272, no. 8 (2017), 3129 – 3175.

Sharpening and smoothing near-threshold Wood anomalies in cylindrical waveguides

Sergei NAZAROV

St. Petersburg State University and Institute of Mechanical Engineering Problems, Russia

Gently sloped perturbation of the wall of an acoustic or elastic waveguide can lead to Wood’s anomalies which realizes as disproportionately rapid changes of the diffraction pattern near thresholds of the continuous spectrum. By means of an asymptotic analysis certain restrictions on the profile of the wall perturbations are found that provide the appearance of the anomaly, its sharpening or extinction. Secveral ways are found out to avoid the anomaly, namely either to keep the threshold resonance which itself provokes the anomaly, or to provide an embedded eigenvalue, both require a fine tuning of the profile of the perturbed wall. At the same time, violation of the fine tuning procedure usually leads to the anomaly.

This work is supported by the grant 17-11-01003 of Russian Science Foundation.

Lifshits tails for randomly twisted quantum waveguides

Georgi RAJKOV

Pontifical Catholic University of Chile

I will consider the Dirichlet Laplacian on a three-dimensional twisted waveguide with random Anderson-type twisting. I will discuss the Lifshits tails for the related integrated density of states (IDS), i.e. the asymptotics of the IDS as the energy approaches from above the infimum of its support. In particular, I will specify the dependence of the Lifshits exponent on the decay rate of the single-site twisting.

The talk is based on joint works with Werner Kirsch (Hagen) and David Krejcirik (Prague). The partial support of the Chilean Science Foundation Fondecyt under Grant 1170816 is gratefully acknowledged.

The spectral localizer for even index pairings

Hermann SCHULZ-BALDES

Friedrich-Alexander Universität Erlagen-Nürnberg, Germany

Even index pairings are integer-valued homotopy invariants combining an even Fredholm module with a K_0-class specified by a projection. Numerous classical examples are known from differential and non-commutative geometry and physics. Here it is shown how to construct a finite dimensional selfadjoint and invertible matrix, called the spectral localizer, such that half its signature is equal to the even index pairing. This makes the invariant numerically accessible. The index-theoretic proof heavily uses fuzzy spheres.

Spectral flow and zero modes for Dirac operators with magnetic links

Jan Philip SOLOVEJ

University of Copenhagen, Denmark

The occurence of zero modes for Dirac operators with magnetic fields is the cause of break down of stability of matter for charged systems. All known examples of magnetic fields leading to zero modes are geometrically very complex. In order to better understand this geometry I will discuss singular magnetic fields supported on a finite number of possibly interlinking field lines (magnetic links). I will show that the occurence of zero modes is intimately connected to the twisting and interlinking of the field lines. The result will rely on explicitly calculating appropriate spectral flows for the Dirac operators. This is joint work with Fabian Portmann and Jeremy Sok.

Eigenvalue Dynamics for PT-Symmetric Sturm–Liouville Operators with Physical Parameter. Solvable Models and Applications

Andrew SHKALIKOV

Moscow State University, Russia

We consider PT-symmetric Sturm-Liouville operators

$$T(\varepsilon) = -\frac{1}{\varepsilon}\frac{d^2}{dx^2} + P(x), \quad \varepsilon > 0,$$

in the space $L_2(-a,a)$, $0 < a \leqslant \infty$, where the potential P is subject to the condition $P(x) = -\overline{P(-x)}$. The spectra of these operators are symmetric with respect to the real axis and discrete, provided that the interval $(-a,a)$ is finite and the potential P does not have high order singularities. The problem is to clarify the dynamics of the eigenvalues when the parameter ε changes near zero, in the middle zone and near the infinity. An important problem is to find the interval for the values of the parameter ε (or to evaluate the boundary of this interval) such that $T(\varepsilon)$ is similar to a self-adjoint operator for $\varepsilon \in (0, \varepsilon_0)$. We present several results for some classes of analytic potentials and present models when the problem can be solved explicitly.

We will discuss the applications of the obtained results to the celebrated Orr-Sommerfeld problem in hydrodynamics.

The talk will be based on joint works with S.N.Tumanov

Homogenization of the Stationary Maxwell System

Tatiana SUSLINA

St. Petersburg State University, Russia

We study homogenization of a stationary Maxwell system with periodic coefficients in $\mathbb{R}^3$ and in a bounded domain $\mathcal{O} \subset \mathbb{R}^3$. Assume that the dielectric permittivity and the magnetic permeability are given by $\eta(\mathbf{x}/\varepsilon)$ and $\mu(\mathbf{x}/\varepsilon)$, $\varepsilon > 0$. Here η and μ are positive definite and bounded (3×3)-matrix-valued functions, periodic with respect to some lattice. The classical results show that, as $\varepsilon \to 0$, the solutions of the Maxwell system with such coefficients converge weakly in L_2 to the solution of the homogenized Maxwell system with the effective coefficients η^0 and μ^0. We improve the classical results and find approximations for the solutions in the L_2-norm. The approximations involve not only the solution of the homogenized Maxwell system, but also some correctors of zero order. For the problem in $\mathbb{R}^3$, the error of approximation is $O(\varepsilon)$, while for the problem in a bounded domain the error is $O(\sqrt{\varepsilon})$. This is explained by the boundary influence.

Talks by young researchers

Global hyperbolic propagators in curved space

Matteo CAPOFERRI

University College London, UK

In [1] and [2] Laptev, Safarov and Vassiliev showed that it is possible to write the propagator of a class of hyperbolic operators on manifolds as one single oscillatory integral with complex-valued phase function, global both in space and in time. In my talk I will discuss a more refined, geometric version of the method, in the Riemannian setting. In particular, the adoption of a distinguished complex-valued phase function, naturally dictated by the geometric framework, will allow us to visualise the process of circumventing topological obstructions. The calculation of the subprincipal symbol of the propagator will enable us to recover asymptotic spectral properties of the operators at hand. I will discuss explicit formulae and recent results for the special case of the wave operator.

[1] A. Laptev, Yu. Safarov and D. Vassiliev, On global representation of Lagrangian distributions and solutions of hyperbolic equations, Communications on Pure and Applied Mathematics 47 (1994) 1411 – 1456.

[2] Yu. Safarov and D. Vassiliev, The Asymptotic Distribution of Eigenvalues of Partial Differential Operators (Translations of Mathematical Monographs, American Mathematical Society, 1997).

A generalization of the trace formula for differential operators on a segment with the lowest-order coefficient perturbed by finite signed measure

Egor GALKOVSKI

St. Petersburg State University, Russia

A new trace formula for differential operator on a segment was obtained in case where the lowest-order coefficient perturbed by finite signed measure. Arbitrary regular boundary conditions were considered for the operator of order $n \geqslant 2$. In case of $n = 2$ the terms corresponding to jumps of the antiderivative of the measure appear. For the Dirichlet boundary condition this term was found earlier by A. Shkalikov and A. Savchuk. A new phenomenon was discovered in case of even $n \geqslant 4$: a new term generated the jump of the antiderivative of the measure in the middle point of the segment appears.

On the spectrum of the spin-boson Hamiltonian with two photons

Orif IBROGIMOV

University College London, UK

I will discuss the spectrum of the spin-boson model with two photons for arbitrary interaction constant under "minimal" regularity conditions on the photon dispersion relation and the coupling function.

Homogenization of hyperbolic equations with periodic coefficients

Yulia Meshkova

St. Petersburg State University, Russia

$$-\mathrm{div}\, g(\mathbf{x}/\varepsilon)\nabla u_\varepsilon(\mathbf{x}) + u_\varepsilon(\mathbf{x}) = F(\mathbf{x}), \quad F \in L_2(\mathbb{R}^d),$$

in the small period limit $\varepsilon \to 0$. And a typical result is the convergence $u_\varepsilon \to u_0$ in $L_2(\mathbb{R}^d)$ with the error estimate

$$\|u_\varepsilon - u_0\|_{L_2} \leqslant C(F)\varepsilon.$$

Here u_0 satisfies the homogenized equation

$$-\mathrm{div}\, g^0\nabla u_0(\mathbf{x}) + u_0(\mathbf{x}) = F(\mathbf{x})$$

with the constant effective matrix g^0.

M. Birman and T. Suslina (2001) suggested an abstract operator-theoretic (spectral) approach to homogenization problems and proved the estimate

$$\|u_\varepsilon - u_0\|_{L_2} \leqslant C\varepsilon\|F\|_{L_2}.$$

Since the right-hand side F is an arbitrary function from $L_2(\mathbb{R}^d)$, this inequality can be written as convergence of the resolvents:

$$\|(A_\varepsilon + I)^{-1} - (A^0 + I)^{-1}\|_{L_2\to L_2} \leqslant C\varepsilon.$$

Here $A^0 = -\mathrm{div}\, g^0\nabla$ is the effective operator. Approximation in the $(L_2 \to H^1)$-norm was also obtained by M. Birman and T. Suslina (2006):

$$\|(A_\varepsilon + I)^{-1} - (A^0 + I)^{-1} - \varepsilon K(\varepsilon)\|_{L_2\to H^1} \leqslant C\varepsilon.$$

Here $K(\varepsilon)$ is the corrector. It involves rapidly oscillating factors, that is why $\|\varepsilon K(\varepsilon)\|_{L_2\to H^1} = O(1)$. For parabolic problems, i. e., for e^{-tA_ε}, approximations in $(L_2 \to L_2)$- and $(L_2 \to H^1)$-norms were found by T. Suslina (2004, 2010). In the last joint paper of M. Birman and T. Suslina (2008), the L_2-approximation of the solutions of hyperbolic problems was obtained. But, up to now, no corrector-type estimates for hyperbolic problems were known. Our main goal is to fill this gap somehow. (More details can be found in the preprint arXiv:1705.02531.)

A Szegő limit theorem for translation-invariant operators on polygons.

Bernard PFIRSCH

University College London, UK

We present Szegő-type trace asymptotics for translation-invariant operators on polygons of growing size. Here, the assumption on the translation-kernel is super-polynomial decay away from the diagonal. The corresponding asymptotic formula for domains with smooth boundary has been known for more than 30 years. In the latter case, the power-like trace asymptotics consist of infinitely many terms that reflect the geometry of the regular domain. For polygonal domains only a two-term asymptotic formula is known, with coefficients as in the smooth case. We provide full asymptotics consisting of three terms. In particular, we show that the constant order term differs from the one in the smooth boundary case.

On branching random walks

Maria Platonova

PDMI RAS, Russia

We consider a continuous time symmetric irreducible branching random walk on a multidimensional lattice with a periodic set of particle generation centres, i.e., branching sources. Despite the probabilistic background of the problem, the work is essentially based on the functional analytic methods and, more precisely, on the methods of spectral theory. The main object of study is an evolution operator for the mean number of particles at an arbitrary point. The existence of the positive spectrum of the evolution operator leads to an exponential growth of the number of particles in branching random walks, called supercritical in a such case. We also calculate the leading term of an asymptotic behaviour for the mean number of particles at an arbitrary point as $t \to \infty$.

This work is supported by the Grant RSF 17-11-01136.