# V. A. Rokhlin (23 August 1919–3 December 1984), materials for the biography

# A. M. Vershik

ABSTRACT. This publication presents some facts and documents related to the biography of the remarkable mathematician Vladimir Abramovich Rokhlin.

We present some facts and documents related to the biography of the remarkable mathematician Vladimir Abramovich Rokhlin. Unfortunately, many things are unknown, and Rokhlin himself was not much concerned about preserving his archive. At the end, we give a list of references about Rokhlin's life and work. In brief, one may say that the biography of this scholar intertwines with all difficulties of the previous century and the history of the country where he lived.

\* \* \*

The following concise autobiography turned out to be the only available biography of V. A. Rokhlin written by himself. It was written shortly before he moved to Leningrad, to take up a professorship at the Leningrad State University, which he held for the last 24 years of his life.

### Autobiography of Vladimir Abramovich Rokhlin

I was born in Baku in 1919. My mother, a medical doctor, died in 1923. My father, a public servant, died in 1942. In 1935, I graduated from high school and was admitted to the Department of Mathematics and Mechanics of the Moscow State University. In 1940, I got a master's degree in mathematics and was admitted for postgraduate studies at the same university. In July 1941, I joined the Moscow People's Volunteer Corps. In October 1941, my unit was surrounded by the Germans, I was wounded in both legs and taken prisoner of war. In January 1945, I was liberated by the Soviet Army and then underwent verification. In December 1946, I returned to Moscow and took my postgraduate examinations till June 1947.

Positions held: junior researcher at the Institute of Mathematics of the Soviet Academy of Sciences (July 1947 - January

2020 Mathematics Subject Classification. Primary 01A70.

C2021 American Mathematical Society

1952); professor at the Chair of Higher Mathematics, Arkhangelsk Institute of Forestry (July 1952 - October 1955); professor at the Chair of Mathematical Analysis, Ivanovo Pedagogical Institute (September 1955 - September 1957); professor at the Chair of Mathematics, Kolomna Pedagogical Institute (from September 1957 up till now).

Academic degrees and titles: Candidate of Science in Mathematics (1948), Doctor of Science in Mathematics (1951), Professor (1954).

My family includes a wife, a seven-year-old son, and a fouryear-old daughter. My wife, Anna Alexandrovna Gurevich, is a docent at the Chair of Mathematics, Kolomna Pedagogical Institute.

March 26, 1960

The family of Vladimir Abramovich Rokhlin's mother (Henrietta Emmanuilovna Levinson) lived in the Ukraine. His maternal grandmother, Clara Levinson, was one of the first women doctors in Russia; his mother also received a medical education abroad. The mother's father, Emmanuil Levinson, was a prosperous businessman from a well-to-do Jewish family. The well-known writer and literary critic Korney Chukovsky is Emmanuil Levinson's son (i. e., Rokhlin's mother's stepbrother), but he had never had any relationships with his father (see Lydia Chukovskaya's memoirs about Emmanuil Levinson's visit to Kuokkala in the 1910s). After Rokhlin's family moved to Baku (apparently, during the Revolution), his mother worked as a medical doctor and died in 1923 during the riots caused by an epidemic. The details of her death are unknown. There was a surmise that she had been murdered.

Rokhlin's father, Abram Veniaminovich Rokhlin, came from a family of Belorussian Jews. He was a broadly educated man engaged in social and political activities, was a member of the Socialist party. The well-known communist functionary A. I. Mikoyan, in the part of his memoirs devoted to his activities in Baku during the Revolution, denounces Abram Rokhlin as an ideological foe who entered into polemics with him at some mass meeting. Rokhlin told me that his father once successfully disrupted Mikoyan's speech in Baku at a mass meeting organized by bolsheviks. Clearly, the father's fate after the Revolution was predestined. First he held minor administrative posts in Baku and in the Ukraine, then was sent to work in Kazakhstan, and afterwards, in the late 1930s, was exiled, arrested, and killed in Stalin's "mincer". No reliable information is available. The rehabilitation documents obtained by the family in 1957 informed that "the case is closed due to lack of evidence", and that the death sentence was dated July 13, 1941. Before the war, the father's family was exiled to Siberia and remained to live there. Fortunately, by then Rokhlin was already a student at the Moscow State University. He lost his Stalin scholarship, but somehow escaped serious persecution. I had not been able to find out the details.

Rokhlin, having skipped a class, graduated from a secondary school in Alma-Ata (at first, the family lived in a small town to which his father was exiled) when he was not yet 16 years old. The following document shows Rokhlin's evaluation

 $\mathbf{2}$ 

by his teacher, docent M. Ustimenko (probably, also a political exile; it was hardly normal for a docent to be an ordinary teacher in Alma-Ata), who was apparently very sagacious.

Recommendation Letter

Vladimir Rokhlin is enormously talented in mathematics. He is extremely quick to grasp the material being presented, critically evaluates it and independently verifies a whole chain of logical conclusions. When proving theorems and solving problems, Rokhlin often suggests his own techniques, avoiding clichés and stereotypes. These techniques, while not being exactly new discoveries, still, since they were not known to him, mark him as a gifted young man with creative potential and a taste for research. In spite of his young age (16 years), he is the most earnest student among his fellows and shows an exceptional interest in mathematics. He was especially enthusiastic about learning new material, solving new sophisticated problems, where he indeed revealed much ingenuity, mental speed, force of logic, while showing deep familiarity with and full understanding of techniques and methods of mathematics within the learned material. Rokhlin's interest in mathematics is so strong that he is not satisfied with the content presented by the teacher, but endeavors to deepen it, and often to learn independently new areas of mathematics. Thus, by the time he graduated from high school, he independently studied the beginnings of calculus, analytical geometry, and higher algebra. He also shows great interest in the history of mathematics. For the foregoing reasons, having been his teacher and observed his performance for two years, I am persuaded that Rokhlin is a young man enormously talented in mathematics and possessing great creative potential. It is following the path of quest, the path of research that is characteristic of him, and this made me confident that Rokhlin will grow to become a profound researcher in the field of mathematics and will make his creative contribution to the socialist transformation of his Motherland.

Alma-Ata

Docent M. Ustimenko

July 3, 1935

In order to be admitted to the Moscow State University, Rokhlin, being too young, had to obtain a special permission. By a lucky chance, such a permission was issued by the very then People's Commissar of Education, an appointment with whom Rokhlin managed to get; it was still 1935.

In their accounts, university professors are very enthusiastic about Rokhlin (see the letters and reports by A. N. Kolmogorov, L. S. Pontryagin, A. I. Plessner). They are echoed by his fellow students I. R. Shafarevich, M. A. Shura-Bura, and others. The general opinion is that Rokhlin was the most brilliant and one of the

most advanced students of his generation. The vast number of courses taken by him includes those taught by I. G. Petrovsky, A. G. Kurosh, L. S. Pontryagin, A. N. Kolmogorov, P. S. Alexandrov, D. E. Menshov, N. K. Bari, P. I. Rashevskii, and, of course, A. I. Plessner; Rokhlin also attended lectures in physics by G. S. Landsberg and L. D. Landau. Several chairs recommended him for Stalin scholarship, which he later lost because of his father's arrest. Rokhlin worked much with A. I. Plessner, whom, for a number of reasons, named as his official advisor. The lecture notes of Plessner's course written down by Rokhlin were converted into two huge papers on the spectral theory of operators published in *Uspekhi Mat. Nauk*, the first one in 1941 and the second one in 1946, while Rokhlin was still in a camp. These papers, which were the first publications on this subject in the USSR, served as textbooks for several generations of mathematicians. During his undergraduate years, Rokhlin completed four research papers.

Having graduated from the Moscow University in 1940, Rokhlin was recommended for postgraduate studies by several chairs simultaneously. He enrolled as a postgraduate student at the Institute of Mathematics and Mechanics of the Moscow State University. Rokhlin's master thesis, which later became the foundation of his paper "On the fundamental ideas of measure theory", brought him a student prize.

In July 1941, he joined the Moscow People's Volunteer Corps and got enlisted in the 8th Red Presnya Division, and later, when this division was disbanded, became a private in the 995th Artillery Regiment. Needless to recall how these young civilians from the Volunteer Corps were armed (one rifle for three men) and trained, and what price they paid for Stalin's prewar policy. Near Vyazma, Rokhlin's regiment was surrounded by the Germans, he was wounded in both legs and left in a village in the care of locals. Since his wounds did not heal, he was put into a local hospital in the territory occupied by the Germans. After a denunciation, the Germans arrested him and sent to a prisoner-of-war camp; he became ill with typhoid, recovered, was transferred to a camp in Belorussia, then in Poland; several times he attempted to escape.

I have never managed to find out who he pretended to be in German camps: Rokhlin did not particularly like to speak about the war. According to some accounts, he passed himself off as a native of the Republic of the Volga Germans: he spoke German and, according to him, used his spare time in the camp to improve it. (Later, a German mathematician said that he had not heard such a good German as Rohklin's even in Germany, and that the only man who speaks German better than Rokhlin is P. S. Alexandrov.) On the other hand, Rokhlin knew a little Azerbaijani and supposedly passed himself off as an Azerbaijani. Anyway, it was impossible to verify his Jewishness in the well-known way, and his German had no Yiddish accent. Nevertheless, according to him, he was turned in by one of his fellow campmates, but this had no dire consequences. Rokhlin related that even in the camp he kept a black notebook to write down his future work in measure theory. I saw this notebook, and after Rokhlin's death Anna Alexandrovna suggested that I take it together with the other manuscripts; unfortunately, I did not take the notebook, and after her death it disappeared.

The camp where Rokhlin was being held was liberated by the Red Army in January 1945, and he got enlisted into the 5th Army of the 1st Belorussian front in Germany. For a while, he served as a translator. By some accounts, the circumstances of his arrest by the Soviet security forces are as follows: Rokhlin stood up for a German prisoner whom a drunk Soviet officer wanted to shoot on the spot (cf. Lev Kopelev's story in his book *To Be Preserved Forever*). Anyhow, Rokhlin was sent to a Soviet verification camp. It is within reason to suggest that the fact of his having been a prisoner of war did not become known to the authorities immediately.

In May 1945, he was sent to a camp at the settlement of Vozhael, in the Komi Autonomous Soviet Socialist Republic. Pontryagin mentions in his letters how it became known that Rokhlin was alive. This is yet another miracle. Passing through Poland, Rokhlin managed to throw a postcard, addressed to a former fellow student Ariadna (which later became the wife of the philosopher V. Asmus), out of the window of the prisoner's car in which he was taken to the camp. At the time, almost all mathematicians gave Rokhlin up for lost, and only this message, which was immediately passed to A. N. Kolmogorov, and then to L. S. Pontryagin, revealed the true state of affairs.

The two mathematicians, A. N. Kolmogorov and L. S. Pontryagin, undertook possibly the most courageous act in their lives: they wrote the following letter to the then Minister of Internal Affairs Kruglov and the Head of the State Board of Repatriation Golikov, which was a very unconventional step at the time. (For Golikov's role in the fate of millions of prisoners of war, see *The Gulag Archipelago* by A. I. Solzhenitsyn, Vol. 3.)

Stalin Prize Laureates, A. N. Kolmogorov, Member of the Soviet Academy of Sciences, L. S. Pontryagin, Correspondent Member of the Soviet Academy of Sciences

People's Commissar of Internal Affairs of the USSR Comrade Kruglov, Head of the State Board of Repatriation Colonel General Golikov

We beg your attention to the fate of Vladimir Abramovich Rokhlin, who returned from a German prison camp and is at present in the Komi Autonomous Soviet Socialist Republic. Vladimir Abramovich Rokhlin appears to be the most outstanding and gifted of the young Moscow mathematicians who did their postgraduate studies at the beginning of the war. As professors of the Moscow State University, we were amazed at V. A. Rokhlin's working ability and his great power of thought focused on solving difficult problems. He has already completed some first-class research papers, one already published and the publication of the other ones delayed because in summer 1941 he joined the Moscow People's Volunteer Corps and no information about him was available since the end of 1941. Judging by the start of V. A. Rokhlin's scientific career, he promises to become an outstanding

#### A. M. VERSHIK

scholar; his return to research would undoubtedly lead, within the next few years, to his obtaining new significant scientific results. Completing his research projects initiated before the war would also be a substantial contribution to science. In view of this, we consider that it would be extremely desirable, in the interests of Soviet mathematics, to permit V. A. Rokhlin to return, in as short a time as possible, to his postgraduate studies in order to resume his research under our supervision.

February 13, 1946 A. N. Kolmogorov, Member of the Soviet Academy of Sciences, L. S. Pontryagin, Correspondent Member of the Soviet Academy of Sciences

Rokhlin was not sure that such a measure would be efficient, because sometimes such letters made the life of a prisoner harder rather than easier. But it was written, and Professor V. V. Stepanov, Head of the Institute of Mathematics of the Moscow State University, received the following answer. Other details remain unknown.

Ministry of Internal Affairs of the USSR, Main Administration of Corrective Labour Camps and Colonies, Guard Directorate, December 4, 1946, Moscow Head of the Institute of Mathematics Professor V. V. Stepanov In reference to your letter of September 5, 1946 concerning Vladimir Abramovich Rokhlin's return to the Moscow University, the GULAG of the Ministry of Internal Affairs of the USSR has given the order to release V. A. Rokhlin from his duties as a guard and to place him at your disposal. Head of the Guard and Security Directorate of the Main Administration of Camps (GULAG) of the Ministry of Internal Affairs of the USSR Colonel I. Smirnov

By then, Rokhlin had already been cleared and served as a guard. According to him, he was again denounced, supposedly by the same persons who had denounced him in the German camp. I know two or three unbelievable stories of Jews rescued from a German camp, and it is these Jews who had an especially tough time in Soviet verification camps: they had to convince SMERSH that their survival was not due to treason, explain how they had managed to hide their nationality, etc. Alas, there are many things that we do not know and, quite likely, will never learn. But even the above makes it clear what the life of this man was like. This short correspondence, and all the related grotesque circumstances, must surely remain in the history of science in the USSR.

By the end of 1946, i.e., "merely" a year and a half after the Soviet arrest, Rokhlin finds himself in Moscow. Here is a recommendation letter written by his former scientific advisor A. I. Plessner to give him support.

#### Recommendation Letter

Already as an undergraduate, Comrade Rokhlin showed an outstanding mathematical gift and genuine scientific enthusiasm. In his studies, and later in his research work, he also revealed a high degree of self-discipline. The combination of these qualities allowed him not only to complete his university studies brilliantly, but also to acquire good and all-round knowledge of modern mathematics and its methods. Due to his scientific initiative, Comrade Rokhlin started to participate in research at the very beginning of his undergraduate years, first in topology and then in the spectral theory of dynamical systems. Having obtained important results in the study of the spectral structure of dynamical systems, Comrade Rokhlin continued these most promising investigations during his postgraduate studies up until the day the war tore him away from his work. It should be noted that even in wartime, under extremely tough circumstances, Comrade Rokhlin managed to advance his research and obtain new interesting results. It goes without saying that he also coped brilliantly with his postgraduate studies.

December 31, 1946

A. Plessner

Rokhlin immediately takes up a job at the Steklov Insitute of Mathematics. In this, he received substantial support from L. S. Pontryagin. On the other hand, Rokhlin's role as Pontryagin's research assistant is also considerable. Rokhlin passed his postgraduate exams, defended his Candidate of Science thesis in 1948 and his Doctor of Science thesis in 1951, see the reports by A. N. Kolmogorov, who both times served as a referee. The report on the Candidate of Science thesis:

> Report on V. A. Rokhlin's Dissertation 'Lebesgue Spaces and Their Automorphisms''

The latest development of the metric theory of dynamical systems has led to the necessity for a certain revision of measure theory. Namely, it turned out that in the finer questions it is natural to assume somewhat more of a measure space than was 7

done earlier. These natural additional requirements are introduced by the author as separability and completeness. The requirement of separability is fulfilled in the majority of important special cases and does not much restrict the significance of V. A. Rokhlin's study, though an analogous study of nonseparable spaces would also be of some interest. V. A. Rokhlin's key achievement is his introduction of the concept of completeness. This is in essence not a restriction of the subject of research, but the isolation of regular realizations of various structural types of separable measures. The result about the inextendability of complete separable measures (page 34 of the first part) is very elegant. The limitation to complete spaces implies that the structural isomorphisms of measures are one-to-one pointwise correspondences of the spaces (up to sets of measure zero). Therefore, the classification of complete spaces up to sets of measure zero coincides with the classification of structural types and can be given exhaustively. The first part of the dissertation is crowned with the classification of measurable partitions of separable complete measure spaces. This interesting result serves as a basis for the second part. In the second part, the automorphisms of separable complete measure spaces are investigated. In 1932, von Neumann proved the theorem about the decomposition of automorphisms into transitive components. However, it remained unclear to what degree the structure of the components determines the structure of the original automorphism. This problem was also solved by V. A. Rokhlin. Besides the solution of this important problem, V. A. Rokhlin's exposition has the advantage of methodological purity; in the initial von Neumann's papers, measures were considered in topological spaces with topology induced by distances between points. V. A. Rokhlin undoubtedly deserves to be awarded the Candidate of Science degree.

> A. Kolmogorov, Member of the Soviet Academy of Sciences

The report on the Doctor of Science thesis (the other two referees were N. N. Bogolyubov and I. M. Gelfand; their reports are lost).

> Report on V. A. Rokhlin's Dissertation ''On the Most Important Metric Classes of Dynamical Systems''

The dissertation is a new exposition of the whole metric theory of dynamical systems based on the measure theory earlier developed by the author. Among the new results, of particular note is a proof of the existence of systems with generalized mixing without strict mixing. A number of important problems of the

This is a free offprint provided to the author by the publisher. Copyright restrictions may apply.

theory of dynamical systems, which have not previously been sufficiently elaborated, are fully analyzed by the author (for example, the reduction of the classification of decomposable automorphisms and flows). On the other hand, the dissertation poses a number of important new problems and outlines a way towards discovering new invariants (see especially Section 5 of the first part). Interesting examples are considered in which it is possible to find the spectra of automorphisms and flows. The author deserves to be awarded the Doctor of Science degree.

> A. N. Kolmogorov, Member of the Soviet Academy of Sciences

Actually, in his two theses and several papers published at the same time, Rokhlin made his outstanding contribution to ergodic and measure theory. These were the first works on this subject in the USSR. Besides, in 1946 his two survey papers appeared in the main Soviet mathematical journal *Uspekhi Mat. Nauk*: "The spectral operator theory" (joint with A. I. Plessner; this paper was published when Rokhlin was still in the camp, which is an unparalleled case in the history of mathematics) and "Homotopic groups"; both surveys were pioneering at the time. This was followed by the most intensive collaboration with L. S. Pontryagin in algebraic topology, the results of which became known worldwide.

The next dramatic period of Rokhlin's biography is the story of his expulsion from the Steklov Institute of Mathematics. At the main mathematical institute of the USSR, Rokhlin held a small post of L. S. Pontryagin's assistant. However, he had already got both scientific degrees and by all objective standards, by the authority he had already gained in the mathematical community, he was due to become a full member of the institute. Here is a report on his work at the Steklov Institute written by A. N. Kolmogorov and L. S. Pontryagin.

> Report on the research work of V. A. Rokhlin, Doctor of Science, member of the staff of the Institute of Mathematics of the Soviet Academy of Sciences

Vladimir Abramovich Rokhlin is a prominent mathematician who has made a significant contribution to mathematics. In measure theory and the metric theory of dynamical systems, his studies are among the most interesting and important results of the last decade (they are described in six short notes published in ''Doklady AN SSSR'' and in five large papers published in Mat. Sbornik, Izvestiya AN SSSR, and ''Uspekhi Mat. Nauk'' and constituted his Doctor of Science Dissertation). In topology, V. A. Rokhlin solved the topical and difficult problem of classification of mappings from the (n + 3)-dimensional sphere to the *n*-dimensional sphere. Before V. A. Rokhlin, many leading experts in topology both in the USSR and abroad tried unsuccessfully to solve this problem. The general problem of classification of mappings from a sphere to a sphere is open since 1912. V. A. Rokhlin's results in this area are at the moment the most far-reaching ones. Besides, V. A. Rokhlin is well known as a mathematician of great erudition who is knowledgeable in many areas of mathematics. He is the author of a number of large survey papers (published in ''Uspekhi Mat. Nauk'') which offer the world's first systematic exposition of a number of difficult areas of functional analysis, the theory of dynamical systems, and topology.

December 3, 1951 A. N. Kolmogorov, Member of the Soviet Academy of Sciences, L. S. Pontryagin, Correspondent Member of the Soviet Academy of Sciences

But the longstanding head of the institute, a well-known expert in analytical number theory I. M. Vinogradov, was equally or even more well known for his militant antisemitism. Besides, at the late 1940s and early 1950s, Stalin launched the infamous fight against cosmopolitanism, which was basically a global antisemitic campaign culminated in the "Doctors' Plot" in 1952. Anyhow, the institute had no position for Rokhlin, and he was solemnly dismissed at a session of the Scientific Council. By the time, he had already married Anna Alexandrovna Gurevich, a former postgraduate student of L. S. Pontryagin, and in 1952 their son Vladimir (currently a professor at the Yale University) was born. A. N. Kolmogorov and L. S. Pontryagin provided Rokhlin with excellent recommendations and described him, a man of thirty, as a prominent scholar and one of the best mathematicians of his generation.

#### Recommendation Letter

V. A. Rokhlin is one of the best Soviet mathematicians of his generation. As early as in the middle of his undergraduate studies, he completed several independent research projects in topology and solved a number of problems posed at P. S. Alexandrov's seminar. These results were published in a collection of student research papers of the Moscow State University. V. A. Rokhlin's master thesis ''Unitary rings and dynamical systems'' is a Candidate of Science level research. It was awarded the first prize at the student research papers competition in 1940. That was also when V. A. Rokhlin obtained interesting results on the properties of automorphisms of compact Abelian groups. He studied functional analysis with utmost thoroughness under the supervision of A. I. Plessner and wrote, jointly with A. I. Plessner, the large monograph 'Spectral Theory of Linear Operators''. V. A. Rokhlin's study of combinatorial topology resulted in his writing a large paper ''Homotopic groups'' for ''Uspekhi Mat. Nauk''. These literary-scientific works commend V. A. Rokhlin

10

as an extremely knowing and mature mathematician. Having returned back to research in recent years, V. A. Rokhlin continues the previously initiated studies in general measure theory, metric automorphisms, and dynamical systems. These studies promise to lay a foundation of the metric theory of dynamical systems. They are described in two notes published in 'Doklady AN SSSR' and in two papers accepted to 'Mat. Sbornik'. They also constitute his Candidate of Science Dissertation which is to be defended in October of this year. There is no doubt whatsoever that V. A. Rokhlin, judging from his works, has long ago achieved the level of Candidate of Science and Docent. The fact that his dissertation is to be defended only now is accounted for by wartime conditions. V. A. Rokhlin can undoubtedly succeed in independent teaching as a docent at a university or a technical institute.

August 26, 1947 A. N. Kolmogorov, Member of the Soviet Academy of Sciences

However, in view of the above circumstances, Rokhlin had no chance to find a math job in Moscow, and the Rokhlins had no flat of their own. That is why Rokhlin accepted an offer from the remote Arkhangelsk Pedagogical Institute in the north of Russia.

The Arkhangelsk period of Rokhlin's life is poorly documented. In 1955, he moved to Ivanovo. The work at the Ivanovo Pedagogical Instutute happened during the "thaw" era. Here again, Rokhlin got himself into a precarious situation typical for intelligentsia at that time: he took an active part in a literary discussion at the institute about the modern opinion journalism (the first attempts at timid criticism of the Soviet system). Of course, for obvious reasons, Rokhlin was under special scrutiny. Only a personal intervention of the head of the institute and some city authorities hushed up the affair and saved him from serious trouble. ("Appeal directly to the key persons and do not waste time speaking with mere cogs", was Rokhlin's advice in regard to one of my own problems.) Still, he moved nearer to Moscow, to the Kolomna Pedagogical Institute.

But the 1950s were the most fruitful years for Rokhlin's research in topology. In the late 1950s, he returned to the theory of dynamical systems and runned a seminar at the Moscow State University, the entropy period began, a new collaboration with Kolmogorov and with the new Moscow generation of mathematicians.

The idea to offer Rokhlin a permanent position at the Leningrad State University was apparently due to the family of the Moscow geometer N. V. Efimov, who were friends of Rokhlin's family (Efimov's wife R. Ya. Berri and Anna Alexandrovna Rokhlina had been friends since student days) and kept in touch with A. D. Aleksandrov, the then rector of the Leningrad State University. Aleksandrov, as well as V. I. Smirnov and O. A. Ladyzhenskaya, strongly supported the idea. Surely, the circumstances were favorable: it was a comparatively liberal period by Soviet standard; the rector Aleksandrov, being a true scholar, was well aware of the importance of strengthening Leningrad scientific schools, especially in areas close to his own, such as geometry, topology, measure theory.

#### A. M. VERSHIK

For the last almost quarter of a century of his life, Rokhlin lived in Leningrad and held a professorship at the Chair of Geometry of the Department of Mathematics and Mechanics of the Leningrad State University. The chair was headed by A. D. Aleksandrov until he moved to Novosibirsk in 1964. On the one hand, Rokhlin finally got a professorship at the second most important university in the USSR, which would be impossible without support from prominent administrators (such as A. D. Aleksandrov), taking into account Rokhlin's "dossier" and biography: a son of an executed "enemy of the people", a Jew who passed through German and Soviet camps. The research and pedagogical opportunities opened before him were a well-deserved and tiny compensation for the long years of hardship and injustice. On the other hand, it became clear almost instantly that in him the University and Leningrad mathematics found an outstanding scholar, a representative of the Moscow mathematical school, the leader of several very important areas of mathematics: modern topology and the theory of dynamical systems, which essentially had no representatives in Leningrad. Besides this, and in part due to this, Rokhlin introduced important changes to the mathematical education at the Department of Mathematics and Mechanics and the whole mathematical life in Leningrad. Immediately after he moved to Leningrad, Rokhlin began to teach new courses and organized two seminars, on ergodic theory and topology, which attracted young people and later became centers attracting mathematicians from the whole country and from abroad. Among Rokhlin's pupils and successors who passed through these seminars are M. L. Gromov, Ya. M. Eliashberg, O. Ya. Viro, V. G. Turaev, V. M. Kharlamov, N. V. Ivanov, S. M. Finashin, V. I. Zvonilov, N. M. Mishlachev, L. M. Abramov, A. M. Vershik, S. A. Yuzvinsky, R. M. Belinskaya, etc. This incomplete list should also include pupils of these mathematicians, at least some of them should be rated among the representatives of Rokhlin's school. Rokhlin was a sort of bridge between parts of the Moscow and Leningrad mathematical schools. The most prominent young Moscow mathematicians of the time, V. I. Arnold, S. P. Novikov, Ya. G. Sinai, and many others, were his younger friends and, to some extent, pupils.

Here we will not write about Rokhlin's mathematical achievements, about their importance for mathematics. His outstanding role is already well known and much written about. Let me remind you of the existence of the book V. A. Rokhlin, Selected Works [in Russian], A. M. Vershik (ed.), 2nd edition, MCCME, Moscow, 2010, which contains Rokhlin's main research papers, bibliography, biographic materials, and recollections written by V. I. Arnold, S. P. Novikov, Ya. G. Sinai, A. M. Vershik, as well as short essays on his mathematical work. It should be mentioned that Rokhlin's influence on people around him was enormous. His difficult and eventful life, his firm principles and commitment to science and knowledge served and will serve as a model for subsequent generations. Unfortunately, he did not live to be able to travel over the whole mathematical world, which was impossible for him in Soviet times for reasons clear to everyone who knows the peculiarities of Soviet life. It is not difficult to imagine the enthusiasm with which he would be welcomed in the main mathematical centers. He died of the second heart attack, aged 65, on December 3, 1984, in Leningrad.

## Bibliography of V. A. Rokhlin

- (1) On a property of metrizable spaces, in: *Collection of Student Research Papers*, Vol. 5, Moscow State University, Moscow (1938), pp. 19–22.
- (2) On the theory of condensations, in: Collection of Student Research Papers, Vol. 18, Moscow State University, Moscow (1940), pp. 37–40.
- (3) Spectral theory of linear operators. II (with A. I. Plesner), Uspekhi Mat. Nauk, 1, No. 1(11), 71–191 (1946); English translation: Amer. Math. Soc. Transl., Ser. 2, 62, 29–175 (1967).
- (4) Homotopy groups, Uspekhi Mat. Nauk, 1, No. 5–6(15–16), 175–223 (1946).
- (5) On the classification of measurable partitions, *Doklady Akad. Nauk SSSR*, 58, No. 1, 29–32 (1947).
- (6) On the problem of classification of automorphisms of Lebesgue spaces, Doklady Akad. Nauk SSSR, 58, No. 2, 189–191 (1947).
- (7) Unitary rings, Doklady Akad. Nauk SSSR, 59, No. 4, 643–646 (1948).
- (8) A general measure-preserving transformation is not mixing, *Doklady Akad. Nauk SSSR*, **60**, No. 3, 349–351 (1948).
- (9) On the fundamental ideas of measure theory, *Mat. Sb. (N.S.)*, 25(67), No. 1, 107–150 (1949); English translation: *Amer. Math. Soc. Transl.* 1952, No. 71 (1952), 55 pp.
- (10) On the decomposition of a dynamical system into transitive components, Mat. Sb. (N.S.), 25(67), No. 2, 235–249 (1949).
- (11) On dynamical systems whose irreducible components have a pure point spectrum, *Doklady Akad. Nauk SSSR*, 64, No. 2, 167–169 (1949).
- (12) On the approximation of nonperiodic flows by periodic ones (with A. A. Gurevich), Doklady Akad. Nauk SSSR, 64, No. 5, 619–620 (1949).
- (13) Selected topics from the metric theory of dynamical systems, Uspekhi Mat. Nauk, 4, No. 2(30), 57–128 (1949).
- (14) On endomorphisms of compact commutative groups, Izv. Akad. Nauk SSSR Ser. Mat., 13, No. 4, 329–340 (1949).
- (15) Summary of results in homotopy theory of continuous transformations of a sphere into a sphere, Uspekhi Mat. Nauk, 5, No. 6(40), 88–101 (1950).
- (16) Approximation theorems for measurable flows (with A. A. Gurevich), Izv. Akad. Nauk SSSR Ser. Mat., 14, No. 6, 537–548 (1950).
- (17) On a mapping of the (n + 3)-dimensional sphere into the *n*-dimensional sphere, *Doklady Akad. Nauk SSSR*, **80**, No. 4, 541–544 (1951).
- (18) Classification of mappings of an (n + 3)-dimensional sphere into an n-dimensional one, Doklady Akad. Nauk SSSR, 81, No. 1, 19–22 (1951).
- (19) A three-dimensional manifold is the boundary of a four-dimensional one, Doklady Akad. Nauk SSSR, 81, No. 3, 355–357 (1951).
- (20) New results in the theory of four-dimensional manifolds, Doklady Akad. Nauk SSSR, 84, No. 2, 221–224 (1952).
- (21) Intrinsic definition of Pontryagin's characteristic cycles, Doklady Akad. Nauk SSSR, 84, No. 3, 449–452 (1952).
- (22) Intrinsic homologies, Doklady Akad. Nauk SSSR, 89, No. 5, 789–792 (1952).
- (23) Metric classification of measurable functions, Uspekhi Mat. Nauk, 12, No. 2(74), 169–174 (1957).

- (24) On Pontryagin characteristic classes, Doklady Akad. Nauk SSSR, 113, No. 2, 276–279 (1957).
- (25) The combinatorial invariance of Pontryagin classes (with A. S. Schwarz), Doklady Akad. Nauk SSSR, 114, No. 3, 490–493 (1957).
- (26) Intrinsic homologies. II, Doklady Akad. Nauk SSSR, 119, No. 5, 876–879 (1958).
- (27) Characteristic cycles of smooth manifolds, in: Proceedings of the 3rd All-Union Mathematical Conference (1956), Vol. 2 (1958), p. 55.
- (28) Spectral theory of dynamical systems (with S. V. Fomin), in: Proceedings of the 3rd All-Union Mathematical Conference (1956), Vol. 3 (1958), pp. 284–292.
- (29) Relations between characteristic classes of four-dimensional manifolds, Kolomen. Ped. Inst. Uch. Zap. Ser. Fiz.-Mat., 2, No. 1, 3–17 (1958).
- (30) Entropy of metric automorphism, Doklady Akad. Nauk SSSR, 124, No. 5, 980–982 (1959).
- (31) Intrinsic homology theory, Uspekhi Mat. Nauk, 14, No. 4(88), 3–20 (1959);
  English translation: Amer. Math. Soc. Transl., Ser. 2, 30, 255–271 (1963).
- (32) New progress in the theory of transformations with invariant measure, Uspekhi Mat. Nauk, 15, No. 4(94), 3–26 (1960); English translation: Russian Math. Surveys, 15, No. 4, 1–22 (1960).
- (33) On the entropy of an automorphism of a compact commutative group, *Teor. Veroyatnost. i Primenen.*, 6, No. 3, 351–352 (1961); English translation: *Theory Probab. Appl.*, 6, No. 3, 322–323 (1961).
- (34) Exact endomorphisms of a Lebesgue space, Izv. Akad. Nauk SSSR Ser. Mat., 25, No. 4, 499–530 (1961); English translation: Amer. Math. Soc. Transl., Ser. 2, 39, 1–36 (1964).
- (35) The structure and properties of invariant measurable partitions (with Ya. G. Sinai), *Doklady Akad. Nauk SSSR*, **141**, No. 5, 1038–1041 (1961); English translation: *Sov. Math. Dokl.*, **2**, 1611–1614 (1961).
- (36) Differential topology, in: Proceedings of the 4th All-Union Mathematical Conference (1961), Vol. 2 (1963), pp. 218–219.
- (37) The entropy of a skew product of measure-preserving transformations (with L. M. Abramov), Vestnik Leningrad. Univ., 17, No. 7, 5–13 (1962); English translation: Amer. Math. Soc. Transl., Ser. 2, 48, 255–265 (1965).
- (38) An axiomatic definition of the entropy of a transformation with invariant measure, *Doklady Akad. Nauk SSSR*, **148**, No. 4, 779–781 (1963); English translation: *Sov. Math. Dokl.*, **4**, 188–191 (1963).
- (39) Generators in ergodic theory, Vestnik Leningrad. Univ., 18, No. 1, 26–32 (1963).
- (40) Metric properties of endomorphisms of compact commutative groups, *Izv. Akad. Nauk SSSR Ser. Mat.*, 28, No. 4, 867–874 (1964); English translation: *Amer. Math. Soc. Transl.*, *Ser. 2*, 64, 244–252 (1967).
- (41) Generators in ergodic theory. II, Vestnik Leningrad. Univ., 20, No. 13, 68–72 (1965).
- (42) The embedding of non-orientable three-manifolds into five-dimensional Euclidean space, *Doklady Akad. Nauk SSSR*, **160**, No. 3, 549–551 (1965).
- (43) Diffeomorphisms of the manifold S<sup>2</sup> × S<sup>3</sup>, Izv. Akad. Nauk SSSR Ser. Mat., 29, No. 6, 1386–1387 (1965).

- (44) New examples of four-dimensional manifolds, *Doklady Akad. Nauk SSSR*, 162, No. 2, 273–276 (1965).
- (45) Pontryagin-Hirzebruch class of codimension 2, Izv. Akad. Nauk SSSR Ser. Mat., 30, No. 3, 705–718 (1966); English translation: Amer. Math. Soc. Transl., Ser. 2, 71, 122–134 (1968).
- (46) Area and volume, in: Encyclopedia of Elementary Mathematics, Vol 5: Geometry, Nauka, Moscow (1966), pp. 7–89.
- (47) Lectures on the entropy theory of measure-preserving transformations, Uspekhi Mat. Nauk, 22, No. 5(137), 3–56 (1967); English translation: Russian Math. Surveys, 22, No. 5, 1–52 (1967).
- (48) Embeddings and immersions in Riemannian geometry, in: Meetings of the Moscow Mathematical Society, Uspekhi Mat. Nauk, 23, No. 4(142), 245 (1968).
- (49) Embeddings and immersions in Riemannian geometry (with M. L. Gromov), Uspekhi Mat. Nauk, 25, No. 5(155), 3–62 (1970); English translation: Russian Math. Surveys, 25, No. 5, 1–57 (1970).
- (50) The normal Euler numbers of the projective plane and Klein bottle in four-dimensional Euclidean space, *Doklady Akad. Nauk SSSR*, **191**, No. 1, 27–29 (1970).
- (51) Two-dimensional homology and two-dimensional submanifolds of fourdimensional manifolds, in: Meetings of the Moscow Mathematical Society, Uspekhi Mat. Nauk, 25, No. 3(153), 258 (1970).
- (52) Two-dimensional submanifolds of four-dimensional manifolds, Funktsional. Anal. i Prilozhen., 5, No. 1, 48–60 (1971); English translation: Funct. Anal. Appl., 5, No. 1, 39–48 (1971).
- (53) Proof of Gudkov's hypothesis, Funktsional. Anal. i Prilozhen., 6, No. 2, 62–64 (1972); English translation: Funct. Anal. Appl., 6, No. 2, 136–138 (1972).
- (54) Congruences modulo 16 in Hilbert's sixtieth problem, Funktsional. Anal. i Prilozhen., 6, No. 4, 58–64 (1972); English translation: Funct. Anal. Appl., 6, No. 4, 301–306 (1972).
- (55) Congruence modulo 16 in Hilbert's sixteenth problem. II, Funktsional. Anal. i Prilozhen., 7, No. 2, 91–92 (1973); English translation: Funct. Anal. Appl., 7, No. 2, 163–164 (1973).
- (56) Complex orientations of real algebraic curves, Funktsional. Anal. i Prilozhen., 8, No. 4, 71–75 (1974); English translation: Funct. Anal. Appl., 8, No. 4, 331–334 (1974).
- (57) Recent progress in the topology of real algebraic curves, in: Meetings of the Moscow Mathematical Society, Uspekhi Mat. Nauk, 29, No. 3(177), 180 (1974).
- (58) Beginner's Course in Topology. Geometric Chapters (with D. B. Fuks), Nauka, Moscow (1977), 487 p. English translation: Springer-Verlag, Berlin (1984), xi+519 pp.
- (59) Complex topological characteristics of real algebraic curves, Uspekhi Mat. Nauk, 33, No. 5(203), 77–89 (1978); English translation: Russian Math. Surveys, 33, No. 5, 85–98 (1978).
- (60) The type and signature of a real plane algebraic curve, in: Joint Sessions of the Petrovskii Seminar on Differential Equations and Mathematical

Problems of Physics and of the Moscow Mathematical Society (second meeting, 18–20 January 1978), Uspekhi Mat. Nauk, **33**, No. 3(201), 145 (1974).

(61) New inequalities in the topology of real planar algebraic curves, Funktsional. Anal. i Prilozhen., 14, No. 1, 37–43 (1980); English translation: Funct. Anal. Appl., 14, No. 1, 29–33 (1980).

### About V. A. Rokhlin

- L. M. Abramov and Ya. G. Sinai, Rokhlin's seminar on the metric theory of dynamical systems at the Moscow State University, in: A. M. Vershik (ed.), V. A. Rokhlin, Selected Works [in Russian], revised 2nd edition, MCCME, Moscow (2010), pp. 500–502.
- (2) V. I. Arnold, Vladimir Abramovich Rokhlin, in: A. M. Vershik (ed.), V. A. Rokhlin, Selected Works [in Russian], revised 2nd edition, MCCME, Moscow (2010), pp. 505–516.
- (3) V. I. Arnold, A. M. Vershik, O. Ya. Viro, A. N. Kolmogorov, S. P. Novikov, Ya. G. Sinai, and D. B. Fuks, Vladimir Abramovich Rokhlin, *Uspekhi Mat. Nauk*, 41, No. 3(249), 159–163 (1986); English translation: *Russian Math. Surveys*, 41, No. 3, 189–195 (1986).
- (4) J. J. O'Connor and E. F. Robertson, Vladimir Abramovich Rokhlin, in: MacTutor History of Mathematics, http://mathshistory.st-andrews. ac.uk/Biographies/Rokhlin.html.
- (5) A. Katok, Vladimir Abramovich Rokhlin (23 August 1919 to 3 December 1984): introductory note, *Ergodic Theory Dynam. Systems*, 9, No. 4, 605– 608 (1989).
- (6) S. P. Novikov, Rokhlin, in: A. M. Vershik (ed.), V. A. Rokhlin, Selected Works [in Russian], revised 2nd edition, MCCME, Moscow (2010), pp. 546–557.
- (7) I. R. Shafarevich, Reminiscences of V. A. Rokhlin, Proceedings of the St. Petersburg Math. Soc., 7, 235–238 (2001).
- (8) Ya. G. Sinai, Reminiscences of V. A. Rokhlin, in: A. M. Vershik (ed.), V. A. Rokhlin, Selected Works [in Russian], revised 2nd edition, MCCME, Moscow (2010), pp. 568–572.
- (9) V. N. Sudakov, An unfinished project of V. A. Rokhlin, Proceedings of the St. Petersburg Math. Soc., 7, 239–253 (2001).
- (10) A. M. Vershik (ed.), V. A. Rokhlin, Selected Works [in Russian], revised 2nd edition, MCCME, Moscow (2010).
- (11) A. M. Vershik, Vladimir Abramovich Rokhlin a biographical tribute (23.8.1919–3.12.1984), Ergodic Theory Dynam. Systems, 9, No. 4, 629– 641 (1989).
- (12) A. M. Vershik, Vladimir Abramovich Rokhlin 1919–1984 (on the 70th anniversary of his birth), Algebra i Analiz, 2, No. 2, 227–235 (1990); English translation: *Leningrad Math. J.*, 2, No. 2, 419–426 (1991).
- (13) A. M. Vershik, V. A. Rokhlin's work in ergodic theory, Algebra i Analiz, 2, No. 2, 236–241 (1990); English translation: Leningrad Math. J., 2, No. 2, 427–431(1991).

- (14) A. M. Vershik, Vladimir Abramovich Rokhlin [1919–1984], in: Topology, Ergodic Theory, Real Algebraic Geometry: Rokhlin's Memorial, Amer. Math. Soc. (2001), pp. 1–10.
- (15) A. M. Vershik, V. A. Rokhlin and the modern theory of measurable partitions, in: *Topology, Ergodic Theory, Real Algebraic Geometry: Rokhlin's Memorial*, Amer. Math. Soc. (2001), pp. 11–20.
- (16) A. M. Vershik, V. A. Rokhlin in Leningrad (1960–1984), in: A. M. Vershik (ed.), V. A. Rokhlin, Selected Works [in Russian], revised 2nd edition, MCCME, Moscow (2010), pp. 517–545.
- (17) O. Ya. Viro and V. M. Kharlamov, V. A. Rokhlin's work in topology, Algebra i Analiz, 2, No. 2, 242–248 (1990); English translation: Leningrad Math. J., 2, No. 2, 433–438 (1991).
- (18) B. Weiss, On the work of V. A. Rokhlin in ergodic theory, Ergodic Theory Dynam. Systems, 9, No. 4, 619–627 (1989).
- (19) S. Yuzvinsky, Rokhlin's school in ergodic theory, Ergodic Theory Dynam. Systems, 9, No. 4, 609–618 (1989).
- (20) A.Vershik. The history of V. A. Rokhlin's ergodic seminar (1960–1970), Journal of Mathematical Sciences, 255, No. 12, 175–183 (2021).

St. Petersburg Department of Steklov Institute of Mathematics, St. Petersburg, Russia

Email address: avershik@gmail.com