DAYS ON DIFFRACTION'2006

INTERNATIONAL SEMINAR

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ABSTRACTS

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FOREWORD

The Seminars "Day on Diffraction" are annually held since 1968 in late May or in June by the Faculty of Physics of St.Petersburg State University, St.Petersburg Branch of the Steklov's Mathematical Institute and Euler International Mathematical Institute of the Russian Academy of Sciences.

This booklet contains the abstracts of talks to be presented at oral and poster sessions in 4 days of the Seminar. Author index can be found on the last page.

The full texts of selected talks will be published in the Proceedings of the Seminar. The texts in LaTeX format are due by September 15, 2005 to e-mail `iva@aa2628.spb.edu`. Format file and instructions can be found on the Seminar Web site at `http:/math.nw.ru/DD`. The final judgement on accepting the paper for the Proceedings will be made by the Organizing Committee following the recommendations of the referees.

We are as always pleased to see in St.Petersburg active researchers in the field of Diffraction Theory from all over the world.

Organizing Committee

The seminar is sponsored by

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List of talks

Computation of eigenvalues for the Sturm-Liouville boundary value problem with an interior double pole

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Asymptotic solutions for the Sturm-Liouville boundary value problem with interior singularities were obtained using asymptotic forms of the Whittaker functions for higher order modes and Titchmarsh-Weyl m-functions for low order modes. An attempt is made in this paper to compute the eigenvalues for the Sturm-Liouville boundary value problem when the Titchmarsh-Weyl m-function technique is employed.

Difference equations with periodic coefficients in diffraction theory

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An analytical method for scalar second-order difference equations with meromorphic periodic coefficients and systems of difference equations of the first order is proposed. The technique is based on the reduction to a scalar Riemann-Hilbert problem on a hyperelliptic surface and its solution by quadratures. The final step of the procedure is the solution of the associated Jacobi's inversion problem.

This technique is applied for the solution of two diffraction problems. The first problem concerns electromagnetic (E-polarization) scattering by a right-angled magnetically conductive wedge [1]. The physical problem reduces to a second-order difference equation with 2π-periodic coefficients and with the shift π. A rigorous procedure for constructing the general solution is proposed. It consists of two steps. First, an auxiliary equation with the shift 2π and the period π is derived and solved by the method of the Riemann-Hilbert problem on a torus. Next, necessary and sufficient conditions for the solution of the auxiliary equation to satisfy the governing equation are derived. These conditions separate the general solution of the main equation from those solutions of the auxiliary equation which fail to satisfy the governing difference equation. In addition, the particular case of no branch points is analyzed by the machinery of the Riemann-Hilbert problem for a segment on the complex plane. A high-frequency asymptotic expression for the electric field is presented.

The second problem analyzes scattering of a plane electromagnetic wave from an anisotropic impedance half-plane at skew incidence [2]. The two matrix surface impedances involved are assumed to be complex and different. The problem is solved in closed form. The boundary value problem reduces to a system of two first-order difference equations with periodic coefficients subject to a symmetry condition. The main idea of the method developed is to convert the system of difference equations into a scalar Riemann-Hilbert problem on a finite contour of a hyperelliptic surface of genus 3. A constructive procedure for its solution and the solution of the associated Jacobi inversion problem is proposed and described in detail. Numerical results for the edge diffraction coefficients are reported.

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Interaction of kinks and anti-kinks for semilinear wave equations with a small parameter

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We consider a class of semi-linear wave equations with a small parameter ε

$$\varepsilon^2(u_{tt} - u_{xx}) + F'(u) = 0, \quad t > 0, \quad x \in \mathbb{R}. \tag{1}$$

The nonlinearities $F(u)$ are assumed to be such that equations (1) have self-similar exact solutions of the so-called "kink" / "antikink" type

$$u(x,t,\varepsilon) = \omega\left(\beta\frac{x - Vt}{\varepsilon}\right), \qquad \beta^2 = \left(1 - V^2\right)^{-1}, \qquad \omega(\eta) \in C^\infty\left(\mathbb{R}^1\right), \tag{2}$$

where $\omega(\eta) \to 0$ for $\eta \to -\infty$, $\omega(\eta) \to 1$ for $\eta \to +\infty$.

More precisely we assume that:

A) $F(z) \in C^\infty\left(\mathbb{R}^1\right)$, $F(z) > 0$ for $z \in (0,1)$,

B) $\left.\frac{d^i F(z)}{dz^i}\right|_{z=z_0} = 0, \quad i = 0, 1, \ldots, k, \quad \left.\frac{d^{k+1}F(z)}{dz^{k+1}}\right|_{z=z_0} > 0,$
where $z_0 = 0$ and $z_0 = 1$ and $k = 1$ or $k = 3$.

An example of such nonlinearities presents the function $F(z) = (1 - \cos(2\pi z))/4\pi^2$, for which Eq. (1) corresponds to the sine-Gordon equation. It is well known that the kinks of the sine-Gordon equation collide without changing their form and the unique result of the interaction is a phase shift appearance.

The main subject of the work consists in obtaining sufficient conditions for the nonlinearities under which the interaction of kinks preserves (in the leading with respect to ε term) the sine-Gordon scenario. This means that the interaction occurs without changing the waves shape of the leading term of an asymptotic solution and with small shifts of trajectories.

The research was supported by CONACYT under grant 43208 (Mexico).

Feynman–Kac–Ito formula for infinite dimensional Schroedinger equation with scalar and vector potentials

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Schroedinger equation with scalar and vector potentials in Hilbert space is considered. Vector potential plays the same role as magnetic field in finite dimensional case. The existence of the solution is proved. The solution is local with respect to time and space variables and is given by probabilistic Feynman–Kac–Ito type formula.

Localization of radiation near border of the stochastic discrete medium

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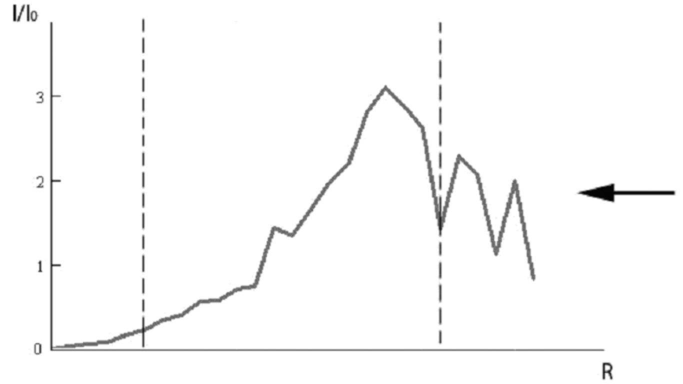
Earlier in work [1] it has been shown that in a stochastic strongly fluctuating discrete medium spatial distribution of full intensity of the electromagnetic field created by a source immersed into this medium has irregular character. In particular, in the two-dimensional medium consisting of ideally conducting infinitely long cylinders, considered in work, near to the source of field - infinite filament of electric current - arose ring area, within the limits of which the full intensity of radiation defined as $I = \langle |E|^2 \rangle$ (where E is intensity of a field in a point of supervision), considerably surpassed intensity of a field on the same distances from the source for the absence of elements of medium. Thus, the opportunity of localization of radiated energy in casual discrete structures has been proved. The degree of display of the given effect and a site of maxima of full intensity were defined by ratio between length of a wave and density of medium.

In the present message results of consideration of interaction of electromagnetic waves with spatially limited stochastic discrete structures are given. The purpose of investigation consists in revealing the effects, similar the aforesaid, at external accommodation of the source of field relatively of system of scatterers. The technique of investigates remained former [1]. The method of separation of the variables not imposing restrictions on mutual position of scattering elements and a source of the field was used. The considered structure represented set of ideally conducting infinitely long cylinders of small electric radius $ka = 0.1$ (a is radius of cylinders, $k = 2\pi/\lambda$, λ is length of wave) oriented parallel to each other. In the plane perpendicular to their axes cylinders are randomly distributed with an average number density σ within a region of restricted area $20\ \lambda \times 5\ \lambda$. The probability that a cylinder occurs within a finite part of this region is described by the Poisson law. The structure was excited by the plane wave incident upon greater party of structure on normal to its edge. The averaging was performed over an ensemble of 500 realizations of the arrangements of elements.

In figure results of calculation of spatial distribution full intensity fields at presence of structure in relation to intensity in free space are presented. The length of a wave was equal 3 cm, density of structure $\sigma = 1.15\ \mathrm{cm}^{-2}$. Borders of volume of space, within the limits of which there were stochastic located cylinders, are represented by a dotted line; the direction of propagation of the wave is shown by the arrow. Obtained data show that excitation of a limited stochastic discrete structure from the outside, as well as in case the source of field is immersed into structure, in the certain area there is a localization of energy. Apparently, the physical mechanisms leading observable effect consist in affinity of phases of the single scattering on elements of structure waves as it is obvious that increase of relative intensity of a field occurs on distances on which mutual influence of cylinders can be neglected. If the depth of immersing is increasing than the multiple scattering on elements of structure leads to decrease of full intensity of a field.



Work was supported by Russian Foundation for Basic Research, Grant 05-02-97205.

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Attenuation of waves in the forest medium

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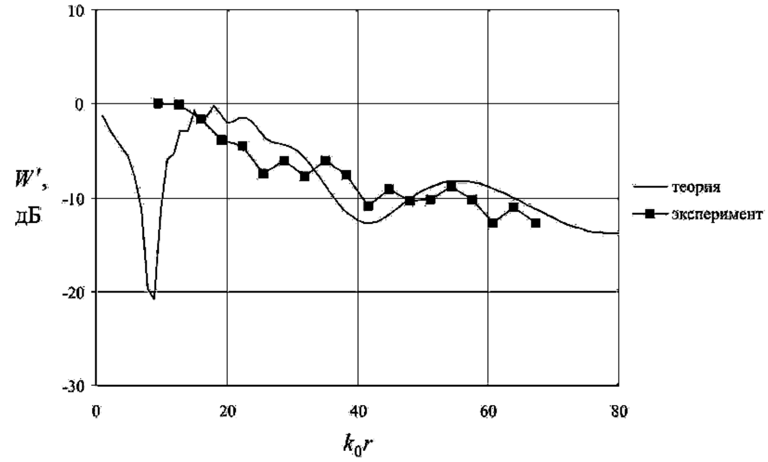
Forest foliage and other vegetation significantly restrict the communication range when the radio waves propagate through vegetation. Quantitatively detailed knowledge of the radio wave propagation mechanism and radio transmission loss is essential both for creating the system of communication and for their EMC[1,2].

The electrodynamic model proposed of forest, within the framework of which is undertaken the attempt to consider all basic factors, which influence shaping of average field in the forest medium: the random arrangement of trees, diameter it is trunk, the height of forest, the dimensions of crown, the electrophysical parameters of wood, and also the presence of the earth's surface.

It is assumed during the construction of the mathematical model of average field in the random discrete medium that the scattering elements are placed in accordance with Poisson's distribution. In this case, the distances between the elements are random and probability density distribution is subordinated to Raleigh's distribution.

Trees were simulated by the composite cylindrical elements, which are consist of an internal cylinder, describing the trees trunks, with the complex permittivity, which conforms with electrophysical parameters of wood in the meter range of electromagnetic spectrum, and external cylindrical. The external cylinder simulates the crown of trees. It is the heterogeneous medium, which consists of air and filiations. Thus discrete scatterers formed by components with different electrophysical parameters, the resulting permittivity of composite elements is introduced. It is further considered that the trees have final height, and it is assumed that they are located on the interface air the earth's surface. The electric dipole is examined as the source of field. The source and observation point of field are located at the some heights relative to the interface of media indicated. Source emits harmonic waves, vector E is parallel to the axes of cylinders. The solution of nonhomogeneous wave equation searches for in the form of a number of multiple scattering taking into account the presence of the earth's surface by the way of the introduction to function of Sommerfeld.

On the assumption that the scattering elements have identical geometric and physical properties are independent, a number of multiple scattering easily is summarized. Namely, after averaging over the ensemble of realizations is introduced the unknown coefficient, which is determined from the equality of expression for the average field, recorded for the case of the maximally close-packed arrangement of elements, to field expression of dipole in continuous medium with the resulting permittivity. As a result we obtain the locked analytical expression, which makes it possible to determine average electromagnetic field in the forest medium.



The results of calculating the average field in the dependence on the distance between the corresponding points, made employing the procedure given above, are shown on Figure.

The parameters of the Earth's surface are the following: $\varepsilon_3' = 2.2$, $\varepsilon_3'' = 0.1/\varepsilon_0 2\pi f$. Parameters of the forest are: the mean diameter of the trunk is $a_1 = 0.05$m, the mean diameter of crown is $a_2 = 0.5$m, the medium altitude of forest is $h = 7$m, average density is $\sigma = 0.38\text{m}^{-2}$. Emission frequency is $f =$

152MHz, the height of the transmitting and receiving antennas $h_1 = h_2 = 2$m. There are the averaged experimental results were given, obtained in the coniferous scaffolding with the analogous parameters.

Work was supported by Russian Foundation for Basic Research, Grant 05-02-97205.

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Sub-wavelength imaging without negative refraction and amplification of evanescent waves

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Original regime of sub-wavelength imaging by flat superlenses is proposed. It does not involve negative refraction and amplification of evanescent waves in contrast to the perfect lenses formed by left-handed media.

The resolution of common imaging systems is restricted by the so-called diffraction limit, since these systems operate only with propagating spatial harmonics emitted by the source. The conventional lenses can not transport evanescent harmonics which carry sub-wavelength information, since these waves exhibit exponential decay in natural materials and even in free space. One of the options how the diffraction limit can be overcame was suggested by Sir John Pendry in his seminal paper [1]. Pendry proposed to use left-handed materials, isotropic media with both negative permittivity and permeability. A planar slab of such a metamaterial provides an opportunity of imaging with sub-wavelength resolution due to effects of the negative refraction and amplification of evanescent wave. Developing this direction of studies, Luo et. al. introduced a flat superlens formed by a slab of photonic crystal [2], and studied theoretically the possibility for sub-wavelength imaging by this device [3]. Recently, it has been shown that with photonic crystal based structures neither negative refraction nor surface plasmon excitation is required to obtain a point source image whose size is smaller than half of the wavelength [4]. In this case the sub-wavelength image size has theoretically been achieved when the crystal isofrequency contour has a special (flat) shape. Such contours are available for different types of photonic and electromagnetic crystals. For example, at microwave frequency range 2d electromagnetic crystal formed by capacitively loaded wires [5], the so-called capacitively loaded wire medium (CLWM), has required flat isofrequency contours. The sub-wavelength imaging by superlens formed by CLWM was experimentally demonstrated in [6].

In the present work we study (theoretically and experimentally) sub-wavelength imaging by a slab of simple wire medium (WM), 2d electromagnetic crystal formed by conducting (not loaded) wires [7]. WM supports so-called transmission line modes [7], which have completely flat isofrequency contours and exist at very low frequencies as compared to the periods of the lattice. In order to achieve sub-wavelength imaging, we do not involve negative refraction and amplification of evanescent modes. We propose to transform the most part of the spatial spectrum of the source radiation into the propagating transmission line modes of the crystal having the same group velocity (directed along wires and across the slab, respectively) and the same longitudinal components of the wave vector. The spatial harmonics produced by a source (propagating and evanescent) refract into the crystal eigenmodes at the front interface. These eigenmodes propagate normally to the interface and deliver the distribution of near-field electric field from the front interface to the back interface without disturbances. This way the incident field with sub-wavelength details is transported from one interface to the other one. The problem of strong harmful reflection from the slab is solved by choosing its thickness appropriately so that it operates as a Fabry-Perot resonator. In our case the Fabry-Perot resonance holds for all incidence angles and even for incident evanescent waves [8]. We call the described regime as canalization with sub-wavelength resolution [4]. We also suggest an opportunity to create a structure operating in the similar regime in the optical frequency range using layered metal-dielectric material [9].

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Symmetric and skew-symmetric solutions arising in an accelerating channel of a supercollider

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An electrodynamical system of energy accumulation (accelerating section of a supercollider) is considered. The principle ideas for synthesizing of an optimal structure are formulated in [1]. The aim is to maximize a coupling resistance and to minimize maximum of an electric field at the same time.

This optimization problem generates some model ones. Among them is the "zero eigenvalue problem" (ZEP) [2]: to find profiles of the accelerating channel providing the zero eigenvalue for Helmholtz type operator under corresponding boundary conditions. This zero eigenvalue is multiple a priori. The corresponding eigenfunctions are either symmetric or skew-symmetric ones.

A constructive numerical algorithm for the ZEP basing on the method of discrete sources together with singular value decomposition technique is developed in [2]. Some results concerning the ZEP are published in [2], [3].

Here we present results of advanced studies for the ZEP. In particular, a convergence procedure for obtaining of numerical solutions is suggested.

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Split-step Fourier Method for Nonlinear Schroedinger Equation

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The nonlinear Schroedinger equation (NLSE) describes a wide class of physical phenomena. In this paper we consider the well-known cubic NLSE, which occurs in nonlinear optics, fluid dynamics, plasma physics, etc. This equation was investigated analytically and numerically by some authors (see, e.g., [1]). There also exists a number of papers on the numerical solution of NLSE. From the practical user's viewpoint it is important to choose the effective numerical algorithm for NLSE. This choice can be based on the comparison of schemes applied to some test problems. The reasonable approach for comparison is to (a) fix the accuracy for computations beginning at $t = 0$ and ending at $t = T$; (b) leave mesh parameters $(\Delta t, \Delta x)$ free and compare the computing time required to attain such accuracy for various choices of the parameters. In [1] the well-known numerical methods (8 schemes in total) for the cubic NLSE were compared in this way. Among these methods were: finite difference methods (explicit and implicit) and finite Fourier transform (FFT) or pseudospectral methods. One-soliton solution and collision of two solitons were considered.

The results of [1] suggest that the most efficient time integration scheme is the split-step method of Hardin and Tappert [2]. It should be natural to develop these investigations since there exist other (more accurate) versions of the split-step method: the operator exponential scheme (OES) [3] and the simplified one (SOES). We use the numerical tests from [1] and compare some versions of the split-step method. The results of numerical tests are presented.

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On a Srödinger operator with a large potential concentrated on a small set

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We consider a singular perturbation of the Dirichlet value problem for

$$\mathcal{H}_\varepsilon := -(\Delta + \mu_0) + \varepsilon^{-\alpha} V\left(\frac{x}{\varepsilon}\right), \quad \alpha < 3/2, \quad 0 < \varepsilon \ll 1$$

in 3-dimensional cylinder $\Pi = (-\infty, \infty) \times \Omega$, where $\Omega \subset \mathbb{R}^2$ is a simply connected bounded domain with C^∞-boundary. We indicate by μ_0 the minimal eigenvalue of $-\Delta' := -(\frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_2^2})$ in Ω, with the Dirichlet boundary condition. It is known that unperturbed boundary problem have no eigenvalues. At the same time eigenvalues can emerge under perturbations. We study the question on existence of such emerging eigenvalues and constructing their asymptotic expansions.

Asymptotic soliton-type solutions of the Hartree equation

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For the nonstationary Hartree equation with a smooth self-action potential $U(x-y)$, $x, y \in R^n$, we construct a new class of localized asymptotic solutions. In the case of a below convex potential, these asymptotics determine a $4n$-parameter family of *soliton-type solutions* with Gaussian profile. In the quantum-mechanical language, these solutions are nondiffusing wave packets. We show that, under a certain choice of parameters, the constructed solutions form an asymptotics of self-similar solutions whose existence was proved in 1979 by V. P. Maslov and M. V. Karasev for the Hartree equation with the external electromagnetic field taken into account.

Nonstationary problem of thin elastic plate oscillations

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We consider the oscillations of elastic plate $-\infty < x, y < \infty$, $-h < z < h$ excited by a vertical forces applied to the both sides of plate with the same distribution $f(t, x, y)$ where $f \equiv 0$ for $t < 0$. Let $\boldsymbol{G}(t, x, y, z) = \{u_x, u_y, u_z\}$ where u_x, u_y, u_z is the displacement vector, be the Green function for this problem. After passing to dimensionless coordinates: $t' = t/(h\sqrt{\rho})$, $x' = x/h$, $y' = y/h$, $z' = z/h$ where ρ is the density of plate we obtain (the superscripts $'$ are omitted below.)

$$\frac{\partial^2 \boldsymbol{u}}{\partial t^2} = (\lambda+\mu)\mathrm{grad}\left(\mathrm{div}\,\boldsymbol{u} + \frac{\partial w}{\partial z}\right) + \mu\Delta\boldsymbol{u}; \quad \frac{\partial^2 w}{\partial t^2} = (\lambda+\mu)\frac{\partial}{\partial z}\left(\mathrm{div}\,\boldsymbol{u} + \frac{\partial w}{\partial z}\right) + \mu\Delta w \qquad (1a)$$

at $z = \pm 1$

$$\boldsymbol{\sigma}_{gz} = 2\mu\left(\frac{\partial \boldsymbol{u}}{\partial z} + \mathrm{grad}\, w\right) = 0; \quad \sigma_{zz} = \lambda\,\mathrm{div}\,\boldsymbol{u} + (\lambda+2\mu)w'_z = \mathrm{sign}\, z\delta(t)\delta(x,y) \qquad (1b)$$

Here $\boldsymbol{u}(x, y, z) = \{u_x,\ u_y\}$ and $\boldsymbol{\sigma}_{gz} = \{\sigma_{xz}, \sigma_{yz}\}$ are the horizontal components of displacement vector and z-component of stress tensor; the operators grad div are taken along horizontal coordinates and λ, μ are the Lame coefficients. It follows from the boundary conditions (1.1b) that w is even and $\boldsymbol{u}$ – odd functions z.

We find the asymptotics of $\boldsymbol{G}$ in the far zone i.e. for $r = \sqrt{x^2+y^2} \gg 1$ and (or) $tc_s = t\sqrt{\mu} \gg 1$ where c_s is the shear wave velocity. For this end we obtain the expression for $\boldsymbol{G}$ in the form of the integral over defined in (2d) dispersion surface $P(\sqrt{\xi^2+\eta^2}, \omega) = 0$ [1]).

The solution of our problem is expressed by the integral

$$\boldsymbol{G} = \begin{Bmatrix} u_x \\ u_y \\ u_z \end{Bmatrix} = \frac{1}{16\pi^3}\int\limits_{-\infty+i0}^{\infty+i0} e^{-i\omega t} d\omega \int\limits_{-\infty}^{\infty}\int\limits_{-\infty}^{\infty} e^{i(px+qy)} \begin{Bmatrix} ipu(k,\omega,z) \\ iqu(k,\omega,z) \\ w(k,\omega,z) \end{Bmatrix} \frac{dpdq}{P(\omega,k)} \qquad (2a)$$

where

$$k = \sqrt{p^2+q^2}; \quad u(k,\omega,z) = -A\gamma_-\sin\gamma_- z + B\frac{\sin\gamma_+ z}{\gamma_+}; \quad w(k,\omega,z) = k^2 A\cos\gamma_- z + B\cos\gamma_+ z; \quad (2b)$$

[1]) The same approach can be used for the case of given symmetric and antisymmetric (when w is odd and u_x, u_y are even functions z) tangent surface stresses at $z = \pm h$.

$$\gamma_- = \sqrt{\frac{\omega^2}{\mu} - k^2}; \quad \gamma_+ = \sqrt{\frac{\omega^2}{\lambda + 2\mu} - k^2}; \quad A = -2\mu\cos\gamma_+; \quad B = (2k^2\mu - \omega^2)\cos\gamma_- \qquad (2c)$$

and

$$P(\omega, k) = (\omega^2 - 2k^2\mu)^2 \cos\gamma_- \frac{\sin\gamma_+}{\gamma_+} + 4k^2\mu(\omega^2 - k^2\mu)\cos\gamma_+ \frac{\sin\gamma_-}{\gamma_-} = 0 \qquad (2d)$$

is the dispersion equation. If k is real then this equation does not has complex roots ω. Further, if $P(\omega, k) = 0$ and $k, \omega \neq 0$ then $\partial P/\partial\omega \neq 0$. To prove these properties of dispersion equation we use the energy considerations.

The functions u, w and $P(\omega, k)$ are entire functions ω, k and if $t < 0$ then the integrands in (2a) are analytic in half-plane $\Im\omega > 0$ and tend to zero exponentially for $\Im\omega \to \infty$, from which follows that $\mathbf{U} \equiv 0$ at $t < 0$. Closing at $t > 0$ the contour of the integration over ω in lower half-plane, we obtain

$$\begin{Bmatrix} u_x \\ u_y \\ u_z \end{Bmatrix} = \sum_{n=1}^{\infty} \frac{1}{8\pi^2} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{i(px + qy)} \sin\omega_n t \begin{Bmatrix} ipu(k, \omega_n, z) \\ iqu(k, \omega_n, z) \\ w(k, \omega_n, z) \end{Bmatrix} \frac{dpdq}{\partial P(\omega_n, k)/\partial\omega};$$

where $\omega_n = \omega_n(k^2)$ is the n-th positive root of equation $P(\omega, k) = 0$. Introduce polar coordinates $r, \varphi : x = r\cos\varphi, y = r\sin\varphi$. Evidently, the angular component u_φ of horizontal displacement is equal to zero. After passing to the integration variables $k, \psi : p = k\cos\psi$, $q = k\sin\psi$ and integrating over ψ we obtain

$$\begin{Bmatrix} u_r \\ u_z \end{Bmatrix} = \sum_{n=1}^{\infty} \frac{1}{4\pi} \int_0^{\infty} \sin\omega_n t \begin{Bmatrix} J_1(kr)u(k, \omega_n, z) \\ J_0(kr)w(k, \omega_n, z) \end{Bmatrix} \frac{kdk}{\partial P(\omega_n, k)/\partial\omega}. \qquad (3)$$

The asymptotic of (3) for $t \to \infty$, $x = ct$, $c = \text{const} > 0$ is calculated by standard method of stationary phase. The case $t \to \infty, x/t \to 0$ needs additional considerations depending on conduct of $\omega_n(k^2)$ at small k.

As $k \to 0$ then with the accuracy $O(k^4)$ we have $\omega_1(k^2) = 2k^2\sqrt{\mu(\lambda + \mu)/3(\lambda + 2\mu)}$. The first term of sum (3) using this expression for $\omega_1(k^2)$ gives the solution in thin elastic plate approximation. All following values of $\omega_n(0)$ i.e. the positive zeros of $P(\omega, 0)$ coincide with the zeros of $\cos(\omega/\sqrt{\mu})$ and $\sin(\omega/\sqrt{\lambda + 2\mu})$ and constitute two sets: $\Omega_m = (m + 1/2)\pi\sqrt{\mu}$ and $\Omega_l = (l + 1)\pi\sqrt{\lambda + 2\mu}$. Obviously, $\omega_n(0)$ are the ordered values Ω_m, Ω_l and there can exist such m, l that corresponding $\omega_n(0)$ and $\omega_{n+1}(0)$ coincide or are close. There arises the problem of asymptotic of the sum of n-th and $(n+1)$-th terms of sum (3) uniform with respect of $\omega_{n+1}(0) - \omega_n(0)$.

Mention that if $\omega_{n+1}(0) - \omega_n(0)$ is sufficiently small then there is the vicinity of $k = 0$ such that $\partial\omega_n/\partial k < 0$ and therefore the phase velocity ω_n/k and the group velocity have different signs.

Scattering of a flexural plane wave by the thin elastic quarter plane in the case of pinched boundary

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We consider diffraction of plane flexural wave $u_{inc} = \exp[-ik(x\cos\psi + y\sin\psi)]$ by the vertex of thin elastic quarter plane $0 < x,\ y < \infty$. The vertical displacement $u(x, y)$ satisfies to the equation

$$\Delta^2 u - k^4 u = 0. \qquad (1)$$

At the boundary of quarter plane $x = 0, y > 0$ and $x > 0,\ y = 0$ the condition of *pinched boundary* is posed: $u = \partial u/\partial n = 0$. At the vertex $x = y = 0$ we pose the *Meixner condition* which for our problem has the form $u(x, y) = O(r^{1+\alpha})$ at $r = \sqrt{x^2 + y^2} \to 0$ where $\alpha > 0$ and we can differentiate this estimate. At infinity we pose the *limiting absorption condition.* Namely, present the solution u in the form

$$u = u_{GO} + w \tag{2}$$

where the GO field u_{GO} is the sum of incident wave and its reflections (including double reflections) from the sides of boundary. We require the exponential decaying of w as $r \to \infty$ in the case $0 < \Im k \ll 1$ (which corresponds to the little absorption in the media).

The uniqueness of the solution of this problem and the equivalence of the Meixner condition to the condition of boundedness of potential energy of oscillations in vicinity of vertex are proved analogously to the considerations of [1] where the case of simply supported boundary is considered.

We reduce this problem to the integral equation of second kind. Our approach is close to the method used in [2].

Choose u_{GO} in (2) in the form $u_{GO} = \Phi(x, y, \psi)$ where

$$\begin{aligned}\Phi(x, y, \psi) = {} & e^{ik(-x\cos\psi - y\sin\psi)} + R_1 e^{ik(-x\cos\psi + y\sin\psi)} + T_1 e^{-ikx\cos\psi - ky\sqrt{1+\cos^2\psi}} \\ & + R_2 e^{ik(x\cos\psi - y\sin\psi)} + T_2 e^{-iky\sin\psi - kx\sqrt{1+\sin^2\psi}} + R_1R_2 e^{ik(x\cos\psi + y\sin\psi)} \\ & + R_1T_2 e^{iky\sin\psi - kx\sqrt{1+\sin^2\psi}} + R_2T_1 e^{ikx\cos\psi - ky\sqrt{1+\cos^2\psi}}\end{aligned}$$

Here the first member is the incident wave u_{inc}; the second and third members are the waves excited at the incidence of u_{inc} at the boundary $y = 0$; fourth and fifth are the waves excited at the boundary $x = 0$; $R_{1,2}$ and $T_{1,2}$ are the corresponding reflection and transformation coefficients. The sixth member is doubly reflected propagating wave. At last, the seventh and eighth members are the decaying waves excited at the incidence at sides of boundary of the wave reflected from opposite side.

Further we consider fields symmetric and antisymmetric with respect of variables permutation $x \leftrightarrow y$. Introduce the parameter $\delta = \pm 1$ and let $u_{inc} = \exp(-ikx\cos\psi - iky\sin\psi) + \delta\exp(-ikx\sin\psi - iky\cos\psi)$; $u_{GO} = \Phi(x, y, \psi) + \delta\Phi(y, x, \psi) = \Phi(x, y, \psi) + \delta\Phi(x, y, \pi/2 - \psi)$. Presenting the solution in the form (2), obtain boundary conditions for w:

$$\begin{aligned} w(\xi, 0) = \delta w(0, \xi) = T_1T_2[e^{-k\xi\sqrt{1+\sin^2\psi}} + \delta e^{-k\xi\sqrt{1+\cos^2\psi}}]; \\ \frac{\partial w(\xi,0)}{\partial y} = \delta\frac{\partial w(0,\xi)}{\partial x} = -kT_1T_2[\sqrt{1+\cos^2\psi}e^{-k\xi\sqrt{1+\sin^2\psi}} + \delta\sqrt{1+\sin^2\psi}e^{-k\xi\sqrt{1+\cos^2\psi}}]\end{aligned} \tag{3}$$

Thus we seek in the domain $x > 0$, $y > 0$ the solution $w(x, y)$ of the equation (1), satisfying to the boundary conditions (3), to the Meixner condition at vertex and to the limiting absorption condition at infinity.

Let $q=(k^2+\Delta)w/2k^2$; $p=(k^2-\Delta)w/2k^2$. Then $w=p+q$ and p, q satisfy to the equations $\Delta p + k^2 p = 0$, $\Delta q - k^2 q = 0$. Denote $p(\xi, 0, \delta) = \delta p(0, \xi, \delta) = f(\xi, \delta)$; $\partial p(\xi, 0, \delta)/\partial y = \delta\partial p(0, \xi, \delta)/\partial x = g(\xi, \delta)$ Then the Fourier transforms $\widetilde{f}(s)$ and $\widetilde{g}(s)$ are analytic in upper half-plane $\Im s > 0$ and satisfy to the functional equation (see [2]):

$$\widetilde{f}(s) + \widetilde{f}(-s) = -i[\widetilde{g}(s) + \widetilde{g}(-s) + 2\delta\widetilde{g}(\sqrt{k^2 - s^2})]/\sqrt{k^2 - s^2} \tag{4}$$

Analogously if $q(\xi, 0, \delta) = \delta q(0, \xi, \delta) = h(\xi)$; $\partial q(\xi, 0)\partial y = \delta\partial q(0, \xi)\partial x = l(\xi)$; then the Fourier transforms $\widetilde{h}(s)$, $\widetilde{l}(s)$ satisfy to the equation

$$\widetilde{h}(s) + \widetilde{h}(-s) = -[\widetilde{l}(s) + \widetilde{l}(-s) + 2\delta\widetilde{l}(i\sqrt{k^2 + s^2})]/\sqrt{k^2 + s^2} \tag{5}$$

Since $w = p + q$, from boundary conditions (3) follows $\widetilde{f}(s) + \widetilde{h}(s) = A(s)$; $\widetilde{g}(s) + \widetilde{l}(s) = B(s)$ where $A(s)$ and $B(s)$ are the Fourier transforms of right-hand members of (3). Excluding $\widetilde{f}(s)$, $\widetilde{h}(s)$, $\widetilde{l}(s)$, from these equations and (4)-(5) and seeking $\widetilde{g}(s)$ in the form: $\widetilde{g}(s) = T_1T_2(\widetilde{v}(s, \psi) + \delta\widetilde{v}(s, \pi/2 - \psi))$ obtain for $\widetilde{v}(s)$:

$$[\widetilde{v}(s, \psi) + \widetilde{v}(-s, \psi)] + \delta[[P(s)\widetilde{v}(\sqrt{k^2 - s^2}, \psi) + Q(s)\widetilde{v}(i\sqrt{k^2 + s^2}, \psi)] = r(s, \psi);$$

where we let $\Re k > 0$, $\Im k > 0$. The functions $P(s)$, $Q(s)$, $r(s)$ have branch points at $s = \pm k, \pm ik$ and $r(s)$ has poles at $s = \pm ik\sqrt{1+\sin^2\psi}$. The function $\widetilde{v}(s,\psi)$ has the pole at $s = -ik\sqrt{1+\sin^2\psi}$ with the same residue as $r(s,\psi)$. Extracting this pole: $\widetilde{v}(s,\psi) = R(\psi)/(s+ik\sqrt{1+\sin^2\psi}) + \widetilde{\rho}(s,\psi)$, obtain the equation for $\widetilde{\rho}(s,\psi)$:

$$\widetilde{\rho}(s,\psi) + \widetilde{\rho}(-s,\psi) + \delta[P(s)\widetilde{\rho}(\sqrt{k^2-s^2},\psi) + Q(s)\widetilde{\rho}(i\sqrt{k^2+s^2},\psi)] = r_0(s,\psi); \tag{6}$$

The functions $P(s)$, $Q(s)$, $r_0(s,\psi)$ are one-valued functions s at plane s with cuts $\mathcal{L}^{i,r}_{+}$ shown at

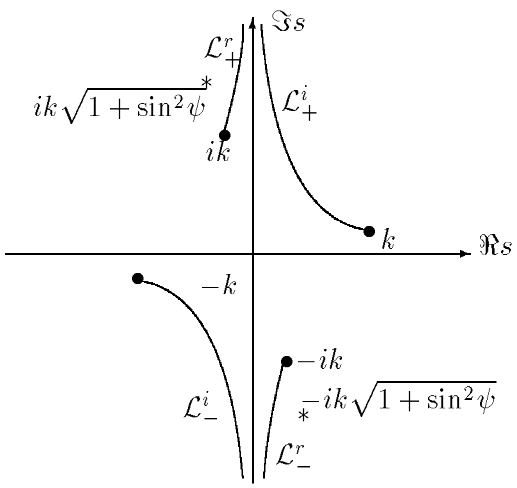

Fig 1

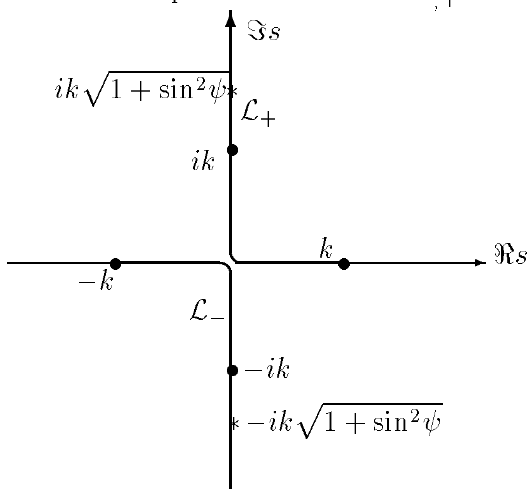

Fig 2

Fig 1. Since $\widetilde{\rho}(s)$ and $\widetilde{\rho}(-s)$ are analytic correspondingly in upper and lower half-planes, their only singularities are the cuts $\mathcal{L}^{i,r}_{-}$ and $\mathcal{L}^{i,r}_{+}$ and therefore these functions can be expressed as the integrals over these cuts:

$$\widetilde{\rho}(-s,\psi) = \frac{1}{2\pi i}\int\limits_{\mathcal{L}^r_+} \frac{w_r(\sigma)d\sigma}{\sigma - s} + \frac{1}{2\pi i}\int\limits_{\mathcal{L}^i_+} \frac{w_i(\sigma)d\sigma}{\sigma - s}; \quad \widetilde{\rho}(s,\psi) = \frac{1}{2\pi i}\int\limits_{\mathcal{L}^r_+} \frac{w_r(\sigma)d\sigma}{\sigma + s} + \frac{1}{2\pi i}\int\limits_{\mathcal{L}^i_+} \frac{w_i(\sigma)d\sigma}{\sigma + s}. \tag{7}$$

To obtain the equations for unknown $w_r(\sigma)$, $w_i(\sigma)$, we equate the jumps at cuts $\mathcal{L}^{r,i}_{+}$ of left- and right-hand members in (7). It is reasonable to pass to limit $\Re k > 0$, $\Im k \to +0$. Then the contour $\mathcal{L}^i_+$ passes to contour $L_1 :\ s = i\xi;\ k < \xi < \infty$ and the contour $\mathcal{L}^i_+$ passes to $\mathcal{L}_+ = L_1 \bigcup L_2$ where the consisting of two segments $[\,0, k\,]$ and $[\,0, ik\,]$ contour L_2 bypasses the origin $s = 0$ as shown at Fig.2. The equation (7) for $\omega(s)$ becomes:

for $s \in L_2$:

$$\omega(s) = [t(s)] + \frac{\delta}{2\pi i}\left(P_+(s)\int\limits_{\mathcal{L}_+} \frac{\omega(\sigma)d\sigma}{\sigma - \sqrt{k^2-s^2}} - P_-(s)\int\limits_{\mathcal{L}_+} \frac{\omega(\sigma)d\sigma}{\sigma + \sqrt{k^2-s^2}} + [Q_(s)]\int\limits_{\mathcal{L}_+} \frac{\omega(\sigma)d\sigma}{\sigma + i\sqrt{k^2+s^2}}\right)$$

for $s \in L_1$:

$$\omega(s) = [t(s)] + \frac{\delta}{\pi i}\left(P(s)\int\limits_{\mathcal{L}_+} \frac{\sqrt{k^2-s^2}\ \omega(\sigma)d\sigma}{\sigma^2 - k^2 + s^2} + Q(s)\int\limits_{\mathcal{L}_+} \frac{\sqrt{-k^2-s^2}\ \omega(\sigma)d\sigma}{\sigma^2 + k^2 + s^2}\right), \tag{8}$$

where $[t(s)], P_\pm(s), [Q(s)]$ are known functions. These equations are not the integral equations of the second kind since in the integrals in right-hand members the poles of integrand lie at the integration contour. But it is simple to overcome this difficulty changing a little the integration contour.

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Energy conservation law within the framework of the parabolic equation

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The parabolic equation

$$Lu = \left(2ik\frac{\partial}{\partial x} + \Delta_\perp\right) u(x,\rho) = \left(2ik\frac{\partial}{\partial x} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right) u(x,y,z) = 0 \tag{1}$$

describes diffraction of a wave at arbitrary distance from the plane $x = 0$, if an obstacle with the transverse size of a (where $a \gg \lambda$, and λ is the wavelength) is situated near the plane $x = 0$. In the near zone of the obstacle $x \ll a^2/\lambda$, the diffraction can be ignored, and Eq. (1) is reduced to the truncated equation

$$2ik\,\frac{\partial u}{\partial x} = 0$$

that can be called the straight-ray approximation.

For any wave field, the energy conservation law is as follows

$$P = \int_S \mathbf{jM}dS$$

(3) where P is the power of the sources surrounded by a closed surface S, $\mathbf{M}$ is the external normal to this surface, and $\mathbf{j}$ is the energy flux density.

An explicit expression for the energy flux density $\mathbf{j}$ can be found for any wave equation. In our case, if we start from an initial exact wave equation, the both parabolic and straight-ray equations describe the wave field approximately. What energy flux density $\mathbf{j}$ should be used for these approximate wave fields? We state that we should use the expressions for the value $\mathbf{j}$ that are physically justified. Namely, the energy flux density for both the parabolic and straight-ray equations should be taken as follows

$$\mathbf{j}(x,\boldsymbol{\rho}) = |u(x,\boldsymbol{\rho})|^2\hat{x}$$

where $\hat{x}$ is the unit vector along the x-axis. Then the energy conservation law appears as the property of being unitary relative to the transverse variables y, z for the surface Green function of the parabolic equation $G_S = 2ikL^{-1}$.

Such a physically obvious treatment of the energy conservation law allows us to prove a number of properties inherent to small-angular scattering by large obstacles. In particular, we prove that extinction cross section is equal to double area of the obstacle projection in the basic case. In general, this extinction cross section varies from zero to 4 areas of the obstacle projection depending on interference between two exactly defined components of the scattered field [1].

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On some solutions of temporal dependence of the velocities of the pulse sources

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In this report we represent the explicit axisymmetric solutions of the wave equation, supposing that the source pulse starts at a certain fixed moment of time and moves with an acceleration along a strait line. We construct two specific solutions of the initial - value problem of wave formation for sub-superluminal hyperbolic motions of the source pulses .

We give an algorithm for constructing a solution for an arbitrary temporal dependence of the velocities of the front and back of the pulse source (see also [1] for details).

This permits us, in particular, to obtain the desired solutions of the wave equation for the pulse sources travelling slower or faster than the wave-front. Here, the scalar solutions are independent of the length of the radiating segment as well as of the location of the back. The application of the scalar solutions to a description of electromagnetic waves is discussed.

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Coherent states of Krawtchouk oscillator and beyond

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We continue the investigation of the generalized coherent states for the oscillator-like systems connected with the known classes of the orthogonal polynomials. In our talks at the previous seminars "Day on Diffraction" (DD'02-DD'05). we discussed the cases of the classical polynomials in a continuous as well as in a discrete variables and also their deformed analogues. In this talk we consider some properties of the coherent states for the Krawtchouk oscillator in finite dimensional Hilbert space in the frames of our construction of generalized oscillators connected with the given family of orthogonal polynomials. In considered case the usual definition of the Barut-Girardello coherent states as eigenstates of annihilation operator falls so we use a modification of Glauber-Perelomov construction with truncated exponential operator. It turns out that resulted coherent states can be considered as the variant of spin coherent states. In the limit, when the dimension N of Hilbert space tends to infinity, our coherent states became the Glauber-Perelomov coherent states for the generalized oscillator. We also compare our results with the results of other authors.

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Propagation in periodic medium : use of tools of quantum mechanics

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Resolution of Maxwell equations in periodical medium are very useful for many problems in electromagnetism like propagation in optical waveguides, diffraction by gratings, photonics band gap crystals and many others. Now, one of the most popular method is the coupled wave method CWM that consist to solve a large problem by the use of modal solutions in some layers and then consider a large problem as a stratified media. We show in this paper that it is possible to improve this numerical method with the use of tools initially developed for quantum mechanics, solving Maxwell equations in term of operators and not in term of functions.

Now the numerical modeling of the phenomena of diffraction and propagation do not use the quantification at all and this point of view do not seem of any interest. In fact for numerical modeling all these ideas may be really useful to obtain the best modeling with good operators. This is not surprising because the photon, particle associated with the electromagnetic interaction, satisfied rules of quantum physics.

The goal of this work is to show on the canonical example of the propagation in a periodic medium, how, certain difficulties encountered during the numerical simulation, can be solved very simply by using the principles of quantum physics like uncertainty principle of Heisenberg, and Dirac operators.

After that it appear that the main problem for modeling propagation is to know how to associate an operator to the permitivity of medium. We propose here a new approach derived of techniques initially used in C Method where quantum approach is really obvious.

Wave scattering on dyon in nonlinear (Born-Infeld) electrodynamics

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Nonlinear electrodynamics model in hypercomplex form is considered [1,2]. Its linearization around a solution is obtained. The appropriate problem for linear waves around static dyon solution (SDS) of Born-Infeld electrodynamics [3] is investigated. Two types of wave scattering on SDS are considered: dissipative (with momentum transmission from plane wave to SDS) and non-dissipative (for SDS imbedded to an equilibrium wave background). Resonance phenomenon in the problem is discovered and some resonance frequencies are obtained by using a numerical method. The form of resonance wave modes is discussed. For details see [4].

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Geometrical properties and semi-classical asymptotics for Shroedinger equations on quantum graphs

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We study Shroedinger equation on a compact geometrical graph. We describe kernels of Laplace operators acting on functions and 1-forms in terms of topological characteristics of graph. Also we construct algorithm for computing semiclassical eigenvalues for the Shroedinger operator, by means of generalized Bohr-Sommerfeld quantization conditions.

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Kirchhoff-Fresnel diffraction on a conducting strip

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The problem of waves scattering on the infinite conducting strip is the classical diffraction problem to which many works were devoted. Besides rigorous and asymptotic methods of solution of this problem, in practice in view of its simplicity and clarity finds wide application the approximate solution based on Kirchhoff method. On usual application of this method diffraction field behind the opaque bodies does not depend on polarization and in the direction, which coincide with the direction of the incident field it is determined by the projection of body on the wave front. Hence at the grazing incidence on the strip the wave can not "observes" it. In [1] was proposed new approach for the solution of the electromagnetic waves diffraction problem on the strip at grazing incidence, which was not earlier covered by Kirchhoff theory in its usual form. Method is based on the successive application of Huygens-Fresnel principle and uses the principle of mirror images taking into account the wave polarization.

In this work the generalization of the results [1] is considered for the case of arbitrary strip orientation in the space.

Geometry of the problem is shown in Fig. 1. Ideally conducting strip of width $2a$ is located between the point source A and the receiver B and oriented in an arbitrary manner. The overall length of path

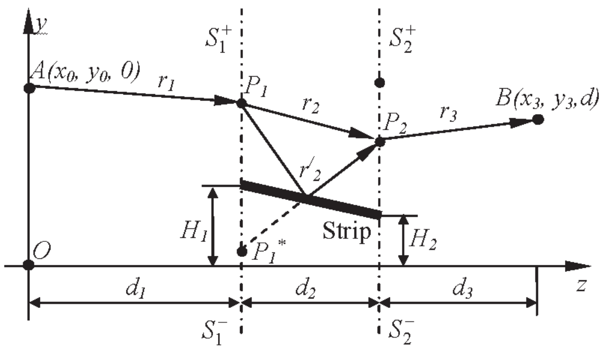

Figure : Geometry of the problem

is $d = d_1 + d_2 + d_3$.

The method proposed earlier in [1] is used for solution of the problem. According to this method the field at point B can be written in the following form:

$$U(B) = \frac{1}{(i\lambda)^2}\frac{\exp(ikd)}{d_1 d_2 d_3}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} dx_1 dx_2 \left(\int_{-\infty}^{H_1}\int_{-\infty}^{H_2} + \int_{H_1}^{+\infty}\int_{H_2}^{+\infty}\right) dy_1 dy_2 \times$$

$$\times \exp\left\{\frac{ik}{2}\sum_{j=1}^{3}\frac{(x_j - x_{j-1})^2 + (y_j - y_{j-1})^2}{d_j}\right\}\left[1 + \Phi\exp(2iky_1y_2/d_2)\right] \qquad (1)$$

where Φ is reflection coefficient, λ is wave length of the incidence field, $k = \frac{2\pi}{\lambda}$ is wave number. Integrating last expression on x_1 and x_2 and carrying out a number of conversions, one can find:

$$U(B) = U_0\left(V_1 + \Phi\, V_2 \exp\left(\frac{2iky_0y_3}{d}\right)\right)$$

where

$$U_0 = \exp\left(ikd\left(1 + \frac{1}{2}\frac{|\vec{r}_A - \vec{r}_B|^2}{d^2}\right)\right) d^{-1}$$

is the field at point B when strip is absent, $\vec{r}_A$, $\vec{r}_B$ are radius-vectors, determining the position of source and receiver in planes $z = 0$ and $z = d$ respectively, and

$$V_1 = 1 - \frac{1}{\sqrt{\pi i}}Qs\left(h_1 - h_A^+, h_1 - h_A^+, \beta\right) + \frac{1}{\pi}Ps\left(h_1 - h_A^+, h_1 - h_A^+, \beta\right),$$

$$V_2 = 1 - \frac{1}{\sqrt{\pi i}}Qs\left(h_1 - h_A^-, h_1 - h_A^-, \beta\right) + \frac{1}{\pi}Ps\left(h_1 - h_A^-, h_1 - h_A^-, -\beta\right),$$

and

$$h_1 = \frac{\sqrt{\pi}H_1}{b_1}, \quad h_2 = \frac{\sqrt{\pi}H_2}{b_2}, \quad b_1 = \sqrt{\frac{\lambda d_1 d_2}{d_1 + d_2}}, \quad b_2 = \sqrt{\frac{\lambda d_2 d_3}{d_2 + d_3}}$$

are radiuses of the first Fresnel zones in the plane of first and second edges of strip respectively,

$$h_A\pm = \frac{\sqrt{\pi}(y_0(d_2 + d_3) \pm y_3 d_1)}{b_1 d}, \quad h_B\pm = \frac{\sqrt{\pi}(y_3(d_1 + d_2) \pm y_0 d_3)}{b_2 d},$$

$$\beta = \sqrt{\frac{d_1 d_3}{(d_1 + d_2)(d_2 + d_3)}}, \quad Qs(x, y, z) = F\left(x\sqrt{1 - z^2}\right) + F\left(y\sqrt{1 - z^2}\right)$$

and

$$Ps(x, y, z) = G\left(x - yz, \sqrt{1 - z^2}y\right) + G\left(y - xz, \sqrt{1 - z^2}x\right)$$

are auxiliary functions, which are expressed through Fresnel integral

$$F(x) = \int_x^\infty \exp(it^2)\, dt$$

and generalized Fresnel integral

$$G(z, \alpha) = \alpha\int_z^\infty \frac{\exp(i(t^2 + \alpha^2))}{t^2 + \alpha^2}dt.$$

In the report it is shown that expression (1) in the limiting cases of perpendicular to the plane of strip and grazing incidence passes into the known classical solution and the solution [1]. Also the results of experimental study are given which satisfactorily agreed with the calculations.

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Waveguide-bianisotropic metamaterial with DNG and DPS pass-bands

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It is presented novel characterization of a periodic composite medium containing interleaved metallic films, dielectric layers and planar gratings of arrays of bianisotropic (chiral) conductive nonmagnetic inclusions. Metallic films form waveguide structures for electromagnetic wave transmitting along films. In the case when electric field E is parallel to them waveguide structures may be cutoff and considered as a medium with negative permittivity. New realization of double negative composite medium [1] on basis of cutoff waveguide structures has been presented at microwave using planar gratings of double split rings [2] and bianisotropic helices [3,4]. Bianisotropic gratings are both artificial magnetic with effective resonant permeability (under h-excitation by microwave magnetic field) and dielectric with resonant permittivity (under E-excitation by microwave electric field).

In this paper it has been shown that such waveguide-bianisotropic metamaterials are attractive objects with multifarious transmission spectrums depending on composite geometry, parameters of elements and conditions of resonant excitation. It is observed both DNG (double negative) pass-band where effective permittivity and permeability are simultaneously negative and DPS (double positive) pass-band where these effective parameters are positive. DNG and DPS pass-bands as well over-forbidden bands are separated and identified by experiment and analytical theory of effective refractive index, permittivity and permeability. It has been shown that fundamental eigen-wave is backward wave at DNG pass-band. Here we investigate microwave resonant spectrum in comparison of effective parameters using waveguide-bianisotropic metamaterial, containing gratings of the well known conductive planar double split rings PDSR [2,5,6] to clear up features of composite media on basis of waveguide structures with bianisotropic (chiral) inclusions.

It is presented theory of waveguide-bianisotropic metamaterial. It has been find the components of permittivity and permeability as well as the chirality tensors of an artificial medium on basis of a periodic structure of PDSR in both free space and in rectangular waveguide to investigate transparency properties. It has been find fundamental wave of rectangular waveguide containing periodic structure of PDSR (waveguide-bianisitropic medium) and analyzed pass-bands properties including boundary frequencies of pass-bands. It has been theoretically investigated the effective refractive index, permittivity and permeability and analyzed DNG and DPS pass-bands as well as over-forbidden bands defined by positive or negative values of refractive index n^2 depending on the type of the resonance excitation. It has been shown that the waves in waveguide-bianisotropic media are backward waves at DNG pass-band

It has been experimentally investigated the transmission coefficients depending on rings orientation with respect to the components of the microwave field providing necessary type of the resonance excitation (by magnetic or electric components of microwave field) using both cutoff and traditional regime of waveguide structures at microwave. It has been observed the pass-bands and over-forbidden bands, connected with different type of resonance excitation, if bianisitropic inclusions are added into cutoff waveguide structures.

Comparing the experimental and theoretical results it has been separated and identified DNG and DPS pass-bands. DNG pass-band is observed higher the ChR (chiral resonance) frequency, this effect is provided by h-excitation of the ChR. DPS pass-band is observed lower the ChR frequency and provided by E-excitation.

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Weak approximation of nonlinear wave interaction

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In this report we discuss a new method for studying interaction of solitary waves such as kinks and solitons. These objects appear as solutions of nonlinear equations with a small parameter ε at the highest-order derivatives (equations with small viscosity, small dispersion, etc.). Although these exact solutions are different, all of them are approximations as $\varepsilon \to 0$ of generalized functions of $\varepsilon\delta$ type, where δ is the Dirac delta function, and of Heaviside function type. In several papers, the author and V. M. Shelkovich developed a technique, which allows one to present smooth functions of linear combinations of these generalized functions in the form of linear combinations of these generalized functions themselves up to quantities small in the weak sense. For example, for $a, b, c, d = \text{const}$ we have $f(a + bH_\varepsilon(x) + cH_\varepsilon(x + d)) = f(a) + B(d/\varepsilon)H(x) + (1 - B(d/\varepsilon))H(x + d) + O_{D'}(\varepsilon)$. Here $H_\varepsilon(x)$ is an approximation of the Heaviside function $H(x)$, $B(z) \in C^\infty$, and $B'(z) \in \mathbb{S}(\mathbb{R}^1)$.

In fact, such formulas mean that the principle of nonlinear superposition can be written explicitly. In turn, this permits describing the interaction of above-mentioned solitary nonlinear waves by explicit formulas up to terms small in the weak sense.

In the report, we discuss the applications of this method to problems of interaction of solitons [1,2] in the nonintegrable case, shock waves [3], and δ-shock waves [4], as well as formation of shock waves [5] and δ-shock waves.

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Asymptotics of linear problems with pure imaginary characteristics

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The plane wave expansion $\delta(x) = (2\pi h)^{-n/2}\int_{R^n} e^{(i/h)\langle p,x\rangle}\,dp$ of the delta function is the central point in constructing parametrices (asymptotics of Green's function) for equations with real characteristics by the Hadamard–Lax technique. The symbol of the parametrix is obtained from the solution to a Cauchy problem with the initial data of the form $e^{(i/h)\langle p,x\rangle}$ by integrating w.r.t. p. Note that integration w.r.t. p commutes with the operators occurring in the original problem, because it is applied w.r.t. parameters.

We consider linear equations with a small parameter ε at the highest-order derivatives and pure imaginary characteristics. Roughly speaking, this means that the substitution of $\varphi(x,t)e^{-S(x,t)/\varepsilon}$ into such an equation leads to a real Hamilton–Jacobi (HJ) equation for the function $S(x,t)$. Clearly, in this case, the substitution of a function of the form $\varphi(x,t)e^{iS(x,t)/\varepsilon}$, corresponding to the plane wave expansion, lead to a complex HJ equation. To avoid this, we propose the formula $\delta(x) = 2^{n/2}(\pi\varepsilon)^{-n}e^{-|x-a^+|^2/2\varepsilon}e^{-\zeta^2/\varepsilon}|_{\zeta=0}$ [1,2], representing the δ-function as a superposition of Gaussian packets, $\delta(x) = \mathcal{L}(x,a^+)e^{-\zeta^2/\varepsilon}|_{\zeta=0}$, where $\mathcal{L}(x,a^+)$ is an operator with constant coefficients, $[x,a^+]=0$, $a_i^+ = \zeta_i - \varepsilon\frac{\partial}{\partial\zeta_i}$.

We use this formula to represent the fundamental solution of the Cauchy problem for the equation $-\varepsilon u_t + H(\overset{2}{x}, -\varepsilon\overset{1}{\frac{\partial}{\partial x}}, t)u = 0$ as $\Gamma(x,t,\xi,\varepsilon) = 2^{n/2}(\pi\varepsilon)^{-n}V(x,t,a^+ + \xi,\varepsilon)e^{-\zeta^2/\varepsilon}|_{\zeta=0}$, where the function $V(x,t,y+\xi,\varepsilon)$, called the symbol of a fundamental solution, satisfies the Cauchy problem

$$-h\frac{\partial V}{\partial t} + H\left(\overset{2}{x}, -h\overset{1}{\frac{\partial}{\partial x}}, t\right)V = 0, \qquad V = \Big|_{t=0} = e^{-(x-\xi-y)^2/2\varepsilon},$$

where $y \in R^n$ is a parameter.

If $H(\overset{2}{x}, -\varepsilon\overset{1}{\frac{\partial}{\partial x}}) = \varepsilon^2 a(x)\frac{\partial^2}{\partial x^2}$, this construction gives the well-known integral representation of the Cauchy problem $u|_{t=0} = e^{-S_0/\varepsilon}$ solution of equation with small diffusion and, in nonsingular charts (where the critical points are nondegenerate [3]), gives the well-known small diffusion expansion. But, at singular points (as in hyperbolic problems) it is, in general, impossible to calculate the integrals. But, in applications, it is often interesting to calculate the logarithmic limit $\lim_{\varepsilon\to 0+0}\varepsilon\ln u$. In our example, the global in time smooth approximation of this limit can be calculated explicitly. This calculation is based on constructing L^1-global approximation of the solution of the equation $\frac{\partial\varphi}{\partial t} + \frac{\partial}{\partial x}(a(x)\varphi^2) = \varepsilon 2\frac{\partial}{\partial x}(a(x)\frac{\partial\varphi}{\partial x})$ which can be constructed by the methods [4], i.e., as a solution describing the interaction of nonlinear waves.

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Unstable closed trajectories, librations and splitting of the lowest eigenvalues in quantum double well problem

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We discuss the structure of asymptotic splitting formula for the lowest eigenvalues of multidimensional quantum double well problem. We show that the change of instanton by closed unstable trajectory of appropriate hamiltonian system gives more natural and simpler preexponential factor (amplitude) in splitting formula. The projection of this trajectories onto configuration space are well know librations in classical mechanics.

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Left-handed materials based upon photonic crystals

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It has been shown recently that some photonic crystals might be the Left-Handed Materials where the propagating modes are described by negative ε and μ. These modes provide negative refraction at the interface with a regular material and they may be used for creation of the Veselago lens. These properties however are rather properties of the separate modes than the properties of the whole crystal. For example the amplification of the evanescent waves in photonic crystal does not follow from the fact that propagating modes with the same frequency in the same crystal have negative ε and μ. The reason is that the spatial dispersion makes ε and μ for propagating modes completely different from those for the evanescent modes. We consider the case when evanescent waves do not play any role in creation of the image of Veselago's lens and find some unusual features of this image, including perfect imaging in the lateral direction of the phase shifted part of the point source. Simple analytical calculations of the field near focus are in a perfect agreement with the results of computer simulation. On the other hand, photonic crystals may support surface waves that are not plasmons and that are not due to the Left-Handed properties of the bulk material. Those waves may improve the sharpness of the focus beyond the diffraction limit. The construction of the multi-focal Veselago lens predicted earlier is proposed on the basis of a uniaxial photonic crystal consisting of cylindrical air holes in silicon that make a triangular lattice in a plane perpendicular to the axis of the crystal. The object and image are in air. The period of the crystal should be 0.44 μm to work at the wavelength 1.5 μm. The lens does not provide superlensing but the half-width of the image is 0.5 wavelengths.

Acoustic scattering of a rotating transversely isotropic cylinder insonified by a plane obliquely incident wave

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It is known that vibrating patterns of an isotropic cylinder, subjected to inertial rotation over the symmetry axis, precess in the direction of rotation with a different angular rate. The proportionality coefficients, or the so-called Bryan factor, equal to the ratio of vibrating patterns rate to the inertial angular rate, depend on particular thickness modes as well as circumferential wave numbers. This effect is retained in the case of a transversely isotropic cylinder if the axis of anisotropy coincides with the axis of the cylinder. In the present paper we consider the case when the vibrating patterns of a rotating transversely isotropic cylinder are generated by an obliquely incident plane acoustic wave. In this case the capture effect exists, which demonstrate finite angular declinations of particular vibrating patterns at different angles depending on modes and circumferential numbers. The resulting scattered acoustic field is very specific, because the lower mode components, corresponding to the lower circumferential wave numbers are found to be turned over the cylinder's axis at higher angles compare to the higher modes. The angles of relative rotations of the modes are proportional to the abovementioned Bryan factors. These changes in the scattered patterns of the solid cylinder due to the inertial rotation are investigated and the corresponding figures are drawn.

The phenomenon of linear acoustic anisotropic absorption observed on a model with an angular unconformity with elastic symmetry elements

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The acoustopolariscopy allowed determining elastic and non-elastic properties of minerals, rocks, concrete, plastics, woods, ceramics, metals and other solid bodies [1]. New phenomena (effects) have been revealed in samples of heterogeneous anisotropic media by this method. The first one is named the effect of linear acoustic anisotropic absorption (LAAA). Effect LAAA is very often registered in media of linear and planar texture [2]. The essence of the phenomenon is the following: if the polarization vector (PV) coincides with the direction of a sample's texture elements then the shear waves amplitude is transmitted practically without losses. If the PV is turned perpendicularly to this direction then propagating waves are intensively attenuated. The effect of LAAA is an analogue of optical dichroism (pleochroism). Our observations mainly on rock samples and minerals allowed one to reveal a wide distribution of the LAAA phenomenon. Among them we have often observed the phenomenon of anomalous increase of shear waves amplitude in position with crossed vectors of polarization. In this case the LAAA effect will probably be displayed in combination with the effect of elastic anisotropy.

We have taken model measurements of the mutual influence that the mentioned effects exert on the acoustopolarigram shape [3]. The model prepared for measurements was made of two plates (Figure). One of the plates was made of high anisortropic ceramics of transverse-isotropic symmetry type. The plate was sawn in such a way that its surfaces were parallel to the elastic symmetry axis of the ceramics. It was 4.25 mm thick. The second plate was made of wood with regular foliation, since

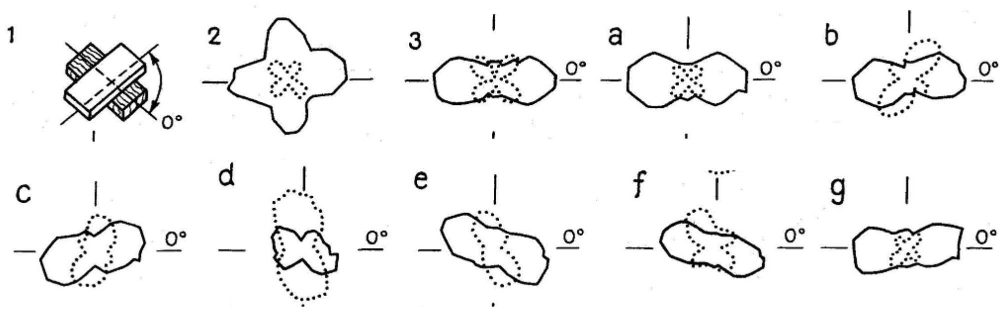

Figure : Sketch model and acoustopolarigrams obtained at different angles between the symmetry elements of the ceramic and wooden plates. 1 - sketch model, - the upper plate is ceramic, the lower - wooden. 2 - acoustopolarigram for the ceramic plate. 3 - acoustopolarigram for the wooden plate. Acoustopolarigrams for the model with angles τ between the symmetry elements of the plates, accordingly: - 0°, b - 15°, c - 30°, d - 45°, e - 60°, f - 75°, g - 90°. The angle τ count was performed between the LAAA symmetry element of the wooden plate and the elastic symmetry element of the ceramic plate. VP - solid line; VC - dotted line.

it has a strong LAAA effect [2]. The wooden plate of 2.25 mm thickness was sawn in such a way that the direction of its fibres coincided with the symmetry axis along elongation. The measurements were taken at the basic frequency $f_0 = 1.12$ MHz. The phase shift along the axis and the symmetry plane in the ceramic plate is 88° and in the wooden plate is 56°.

The amplitude measurements were conducted both at crossed (VC) and parallel (VP) polarization vectors. Acoustopolarigrams for the ceramic and wooden plates are presented in the Figure. In a sequence of measurements the mutual orientation of the model plates was successively changed. At first the angle between the plates was 0°. Then the angle between the plate axes was changed to 15°. After that the mutual angle was increased by further 15°. Thus, a set of acoustopolarization measurements was performed with the angles between the model symmetry elements of 0, 15, 30, 45, 60, 75 and 90°.

An analysis of the experimental figures shows that the VC-acoustopolarigrams b, c, d, e do not have classical forms [1]. The maximum amplitude of the VC acoustopolarigrams, Fig. d, is more than that registered by the VP position. Thus, the observed forms of the model acoustopolarigrams can explain the forms obtained for rock samples. In these rocks the spatial orientation of elastic symmetry elements and structural elements that cause the LAAA effect do not coincide. The most characteristic feature of the angular divergence between the symmetry elements of LAAA and elastic anisotropy is inequality of the petal size and area of the VC acoustopolarigram. These peculiarities of the acoustopolarigram forms can help analyse the structure of natural solid media.

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Radiation of a dipole source in the presence of a rough surface: A new formulation based on curvilinear complex coordinates

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The problem of radiation of an antenna in the presence of a scatterer is a very classical one in electromagnetic wave theory. Here, we consider the canonical problem of radiation of a dipole source above a rough surface that can be the earth. This problem has been investigated by many authors like Leontovich or Baos [1] [2]. We have to find a Green function that satisfies radiation conditions at infinity and boundary conditions on the surface. For that purpose we use a spectral approach in a curvilinear coordinate system.

The so-called curvilinear co-ordinate method C.C.M. was introduced by Chandezon et al. [3] to study the scattering of plane waves by a grating. It was also applied to study the diffraction by non-periodic rough surfaces[4]. The main important point of this method is the choice of a co-ordinate system such that the boundaries conditions are written in a simple way. Furthermore, in the case of a translation curvilinear coordinate system, the Maxwell's equations lead to an eigenvalue equation. Numerically, the eigenvalue equation is solved by using the method of moments in Fourier space. In case of non-periodic rough surfaces, it has to be noted that the spectrum of the scattered field is continuous. Unfortunately, any numerical method introduces discretization. It is well known that discretization of a continuous spectrum results in its periodization. Hence, due to the implementation of the method of moments in the spectral domain, it turns out that a grating like problem is solved instead of a non periodic one. The periodicity of the expansion basis introduces couplings that do not exist in the initial problem. In principle they cannot be removed although their effect can be attenuated by increasing the artificial period. However, the larger the period, the larger the size of the matrices involved in computation.

The introduction of absorbers layers of PML kind [5] allows to dramatically reduce these couplings. It has been shown that it is then possible to calculate with a great accuracy the free space Green function in curvilinear coordinates, by using periodic functions.

PML can also be considered as a change of coordinates that maps real space onto a complex space that has no physical meaning. Hence, our way of solving the problem of radiation of a dipole over the earth is based on changing coordinates and using Maxwell's equation under the covariant form in the discrete Fourier space.

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On the polarized ultrasound distribution along elements of symmetry of the polarized anisotropic media

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Natural and artificial solid mediums were studied by means of acoustopolariscopic method [1], which represents an analogue of an optical polarizing method, where instead of light linearly — polarized ultrasonic oscillations are used. Cubic samples turns on the platform with a given (10°) step within the limits of 360° around the axis of sounding. Measurements are carried out for all three pairs of sample facets at crossed (VC) and then at parallel (VP) vectors of polarization of the source and the receiver of oscillations. By results of measurements circular diagrams — acoustopolarigrams are constructed. Acoustopolarigrams defines spatial position of elements of elastic symmetry of samples. If a sample is isotropic, acoustopolarigram is of a circle form, if anisotropic — figures with usually even number (2-4) of petals are obtained according to the order of symmetry axis.

Earlier, on a sample made of artificial quartz crystal [1, 2] acoustopolarigram (VP) with three petals was obtained in the direction of symmetry axis of the 3-rd order. That is, the amplitude of a signal is maximal in one position of coincidence of elastic symmetry plane of the sample with the polarization plane of gauges, and at turn on 180° — full damping of the signal is observed though the same plane of the sample coincides with the polarization plane of gauges. Petals of the acoustopolarigram coincide with the position of quartz piezo-axes [3]. Matching of the signal polarization plane with piezo-axes of crystal creates a potential difference stipulated by perturbations, caused by ultrasonic S-waves [4]. It was considered earlier, that polarization of rocks is caused by there own asymmetry, local dipole moment of an elementary cell and cooperative effects of rock texturing [5]. Piezo-effect can be observed everywhere where there are polar directions [3]; so during rock deformation and disclosing of cracks, their coast get, as a result of rupture of connections, opposite charges and, accordingly — polar directions.

Earlier, in the work [6] the situation was discussed in which the plane of polarization of ultrasonic signal was represented as "the charged plate" (dipole), which one surface bears positive, and another - negative "charges". The main reason of elastic symmetry occurrence in rocks is systems of equally oriented microcracks. For example, three of such systems, being crossed under a corner of 120°, form an axis of the 3-rd order in the place of crossing. These systems of cracks divide a sample on six sectors "charged" sequentially opposite. During sounding in the direction of the 3-rd order axis and coincidence of a signal dipole "charges" with "charges" of sample sectors, the amplitude of the signal passing through the sample, will be maximal. At turn of this sample around of the axis of sounding on 180 the dipole of the signal will be turned to sectors with opposite "charges", therefore there will be absorption of the signal. Such situation will be characteristic when sounding is held along axes of symmetry of odd order (1, 3, ...) and is impossible when sounding is held along axes of symmetry of even order (2, 4, ...), because in case of even axes at matching of a signal dipole with a plane of microcracks system one half of the system always coincides by "charge" signs with the dipole.

During acoustopolariscopic measurements of metamorphic rocks asymmetrical acoustopolarigrams are frequently obtained with different size of opposite petals, or one of petals is considerably larger than others. Such situation can be explained by presence of shearing dislocations [7] that result in changes of arrangement of the charged sectors, which influences the symmetry of acoustopolarigrams. Thus, the polarized signal passing through the polarized medium in directions of the given medium symmetry axes will have maximal amplitude only at coincidence of "charge" signs of the signal dipole with "charges" of medium sectors. The amount of such maxima, at a full revolution of studied object around the axis of sounding, is equal to the order of the symmetry axis in which direction sounding was carried out.

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Extended wave propagators as pulsed-beam communication channels

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Let $P(x_r - x_e)$ be the causal propagator for the wave equation, representing the signal received at the spacetime point x_r due to an impulse emitted at the spacetime point x_e. Such processes are highly idealized since no signal can be emitted or received at a precise point in space and at a precise time. We propose a simple and compact model for extended emitters and receivers by continuing P to an analytic function $\tilde{P}(z_r - z_e)$, where $z_e = x_e + iy_e$ represents a pulsed-beam emitting antenna dish centered at x_e and radiating in the direction of y_e while $z_r = x_r - iy_r$ represents a pulsed-beam receiving antenna dish centered at x_r and receiving from the direction of y_r. The space components $\mathbf{y}_*$ of y_* $(* = e,\ r)$ give the spatial orientations and radii of the dishes, while their time components s_* represent the time a signal takes to propagate along the dish from the center to the rim. The analytic propagator $\tilde{P}$ represents the transmission amplitude between the emission and reception dishes, forming a communication channel. Causality requires that the extension/orientation 4-vectors y_e and y_r belong to the future cone, so that z_e and z_r belong to the future tube and the past tube in complex spacetime, respectively. The "retarded time" $T_e = s_e - |\mathbf{y}_e|/c$ represents the duration of the emitted pulse, and $T_r = s_r - |\mathbf{y}_r|/c$ represents the integration time for the pulse reception. The bandwidths of the dishes are $1/T_e$ and $1/T_r$, respectively. The invariance of $\tilde{P}$ under imaginary spacetime translations $(z_e \to z_e + iy,\ z_r \to z_r + iy)$ has nontrivial consequences.

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Precession of elastic waves in vibrating isotropic spheres and transversely isotropic cylinders subjected to an inertial rotation

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It was found by G. Bryan in 1890 that vibrating pattern of a rotating ring follows to a direction of the inertial rotation of this ring with an angular rate of the vibrating pattern smaller than the inertial rate. In 1979 E. Loper and D. Lynch proposed a hemispherical vibrating bell gyroscope utilising the Bryan's effect, which can measure an inertial angular rate and angle of rotation about the symmetry axis of the hemispherical shell. All these works exploited the precession properties of thin vibrating shells subjected to an inertial rotation around their axes of symmetry. In 1985 V. Zhuravlev generalized the abovementioned results and shown that the Bryan's effect has a three dimensional nature, i.e. that a vibrating pattern of an isotropic spherically symmetric body, arbitrary rotating in 3-D space, follows the inertial rotation of the solid body with a proportionality factor depending on the vibrating mode. This result had a qualitative nature without classification of vibrating modes and calculation of the corresponding proportionality factors. Radial and torsional vibrating modes are considered on the basis of an exact solution of 3-D equations of motion of an isotropic body in spherical coordinates. The solutions are obtained by means of a three potential method in the spherical Bessel and associated Legendre functions. It is shown that the Bryan's effect exists for the radial modes only. The proportionality factors of corresponding vibrating modes are calculated. Furthermore, the effects of gyroscopic forces on wave propagation in a transversely isotropic cylinder due to the inertial rotation are considered. The solutions are expressed in Bessel functions for different modes and the corresponding Bryan's proportionality factors are calculated.

A probabilistic approach to function approximation

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In this talk we will investigate a new and novel approach to function approximation for functions defined over a bounded real interval which, without loss of generality is assume to be the unit interval [0, 1]. This approach is based upon an idea by J. Kolibal and C. Saltiel. We will give a derivation of their technique and show how to interpret it statistically using the Gaussian distribution. Since Bernstein polynomials lie at the foundation of the Kolibal-Saltiel technique which, they call the Bernstein function approximation, we will call our technique the Gauss-Bernstein approximation technique.

These ideas are quite general and have wide applicability in function approximation, high frequency filtering, data interpolation and multidimensional data regularization.

In order to narrow the focus for this investigation we concentrate on the following problem: Given a function defined over the real interval [0, 1], how does one selected a finite number of points x_j and accordingly the data set $\{(x_j, y_j = f(x_j),\ j = 1, \ldots, n)\}$ so that the Gauss-Bernstein approximation, called $k(x)$, interpolates the data set exactly over the interval [0, 1] so that the difference between $f(x)$ and $k(x)$ in within a preassigned tolerance.

We will develop an interactive adaptive algorithm that selects the data points in a way that attempts to maximize the efficiency of the computation leading to $k(x)$. We do this in three ways.

Firstly we discuss the general philosophy behind the Gauss-Bernstein function approximation technique. The general idea, different from Taylor and Fourier series which use a finite number of basis of functions with constant coefficients that can be evaluated each argument x, is to generate a basis Gaussian distributions with mean x and standard deviation $\sigma(x)$. For an arbitrary x, the Gauss-Bernstein approximation for $f(x)$ is of the form $k(x) = \sum_{j=0}^{n} y_j p_j$, a finite weighted sum of y_j with weights p_j. The weights are constants, probabilities determined by the Gaussian distribution with mean x and standard deviation $\sigma(x)$.

Secondly we observe that it is possible to derive a scheme of choosing $\sigma(x)$ so that the Gauss-Bernstein approximation of a straight line, passing through two points, is exact, that is the difference between $f(x)$ and $k(x)$ is guaranteed to be exactly zero. Hence a technique is developed to uniformly approximate any function of bounded variation which is based on approximating such a function by piecewise straight line segments. Our scheme again attempts to do this in such a way as to minimize the number of straight line segments used while guaranteeing that the error is uniformly distributed over the interval [0, 1] and bounded everywhere by a preassigned tolerance.

Finally we address the classical problem of the Gibbs' phenomenon occurring to the left and right of a jump discontinuity and demonstrate that the Gauss-Bernstein approximation has no Gibbs' effect.

Electromagnetic waves propagation in the half-space with a homogeneous smooth disturbance of the boundaries impedance

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In this paper we consider the influence of a one-dimensional disturbance of the boundaries impedance $\eta(x)$ of a half-space on the field of a two-dimensional source in the form $\vec{j}(x,z) = \vec{z^0} J\delta(x)\delta(z-z_0)e^{i\omega t}$, where (x, y, z) is the Decarte coordinate system, $\delta(\xi)$ is the Dirac function. We assume that on the plane boundary $z = 0$ the normalized on $\sqrt{\varepsilon_0/\mu_0}$ impedance $\eta(x)$ with a positive real part is determined. The reasons of the appearance of such impedance inhomogeneities could be internal gravitational waves, the area of the terminator, turbulent flows in the ionosphere and so on. We consider the case when the space scale of the change of the function $\eta(x)$ to be much greater than the length of the wave $\lambda = \omega/c$. In this report, as well as in the works [1,2], we investigate the case of the sliding propagation of electromagnetic waves. However the aim of this research is to find a solution expressed not in quadrates as in [1,2], but in an analytic form. For solving the formulated problem, we will use the method described in the work [2]. For this goal, using a conformal transformation, we go over to a half-space with a curved boundary possing a constant impedance. Then, by mean of a differential operator defined in the above mentioned work [2], we come to the problem of the calculating an additional function satisfying the Dirichle zero boundary condition on a curved boundary, which, evidently, will be preserved while going over to the initial coordinates. Thus the initial problem of calculation of the field with an impedance boundary condition in the Neiman form is reduced to a problem with an essentially more simple zero boundary condition in the Dirichle form. After that using the method of the geometric optics we obtain the attenuation function [1]. Particularly, in the case of the observation point's height equal to zero, we obtain the rather simple formula for attenuation function: $|\Pi(x,0)/\Pi_0(x,0)| = \exp\left((\arg(\xi'(u)|_{u=x+i\cdot 0} - \arg(\xi'(u)|_{u=x+i\cdot 0})\,\eta_0^{-1}\right)$, valid for $\ln(|\xi'(u)|)\eta_0^{-1} \ll 1$. Here the function Π is the vertical component of the Hertz vector, which determines the components of field, the value $\xi'(u)$ is the derivative of the conformal transformation on the variable $u = x + iz$, η_0 is the undisturbed value of the boundary impedance.

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On nonlinear Ginzburg–Landau boundary value problem for a superconducting plate in a magnetic field

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The results of [1] relating to the analytic–numerical study of nonlinear Ginzburg–Landau (GL) one–dimensional boundary value problem (BVP) for a superconducting plate in a magnetic field are presented. In terms of dimensionless variables, this BVP has the form (see [2], [3])

$$a'' - \psi^2 a = 0, \tag{1}$$

$$\psi'' + \kappa^2(\psi - \psi^3) - a^2\psi = 0, \qquad 0 \le x \le D, \tag{2}$$

$$a(0) = 0, \qquad \psi'(0) = 0, \tag{3}$$

$$a'(D) = h, \qquad \psi'(D) = 0. \tag{4}$$

Here a is the potential of the magnetic field, the quantity ψ^2 is equal to the concentration of the superconducting electrons in the metal, $0 \le \psi^2 \le 1$; $0 < D$ is the half–width of the plate, h and κ are positive parameters, where h is the magnetic field strength and κ is the parameter of the GL theory that characterizes the material of superconductor.

The following results, obtained in [1] for this well–known BVP (1)–(4), are new: 1) on the basis of preliminary theoretical analysis of the BVP and the estimates of parameters, a new numerical method is proposed allowing efficient retrieval of all its solutions in a wide range of physical parameters, including dynamically unstable and nonisolated solutions (these types of soluions were not found so far using alternative methods [3]); this makes possible to form more exact continuous bifurcation diagrams that describe the plate transfer from superconduction to normal state (and vice versa) under amplification (reduction) of the magnetic field that leads to more detail study of magnetic hysteresis phenomenon, the evolution of diagrams for varying parameters of superconductor material, plate width et. al.; 2) spectral problems for dynamic (in)stability analysis of solutions in frame of linear perturbation theory and for a priori detection of threshold levels of magnetic field were posed and studied; in particular the reasons why iteration methods of relaxation type [3] could not detect solutions lying inside hysteresis loops were clarified, namely these solutions are dynamically unstable.

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Diffraction of plane short-time variable on the front shock wave on a semi-infinite screen

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The exact solution of the two-dimansional automodel problem of special form plane wave diffraction on a plane semi-infinite screen is constructed by the method of functionaly-invariant solutions (Smirnov–Sobolev method). The screen is assumed acoustically soft: the field of pressure $P(x, y, t)$ satisfied Dirichlet boundary condition $P(x, 0, t) = 0$, $x > 0$ on it. The incident wave $P_0(x, y, t)$ has delta-type form and its amplitude linearly varies along the front

$$P_0(x, y, t) = (-x \sin \varphi_0 + y \cos \varphi_0)\delta(ct - x \cos \varphi_0 - y \sin \varphi_0), \qquad t < 0.$$

Diffraction by a strongly elongated object illuminated by an electromagnetic plane wave propagating in the paraxial direction - application to a prolate ellipsoid

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Following the asymptotic theory of creeping waves on a strongly elongated object, developed by Andronov and Bouche [1], the authors have derived explicit formulas for the asymptotic currents on a strongly elongated object excited by an electromagnetic plane wave propagating in the paraxial direction.

The problem is first solved in the penumbra region close to the lit-shadow boundary on the surface. Analytical solutions of the bi-confluent Heun equation verified by the Fourier transform of the dominant term of the asymptotic expansion of the magnetic field component along the bi-normal to the geodesic, have been derived for an observation point located very close to the surface. By applying the boundary conditions on the surface, with an incident field derived from the solution of the eikonal equation in the usual semi-geodesic co-ordinate system, analytic expressions for the total field on the surface have been derived. They involve new Fock functions depending on the curvature of the wave front transverse to the geodesics followed by the creeping waves.

In a second step this solution has been extended in the usual way to the lit region and also matched with the solution verified by the creeping waves in the deep shadow region. The field away from the object is obtained by evaluating numerically the radiation integral of the asymptotic currents.

The general asymptotic solutions coming out from our theoretical investigations have been applied to a strongly elongated prolate spheroid, illuminated by a plane wave propagating in its axial direction. Numerical results for the asymptotic currents along the geodesics followed by the creeping waves and for the bi-static radar cross section, will be shown and compared to the results obtained by solving the EFIE.

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On stability of nonlinear equation

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Energy growth in time of the solutions of linearized systems is investigated. For the stability of nonlinear system it is sufficient that its linearization does not have a considerable energy growth. Sometimes the principal symbol of the linearized system has a Jordan block. Existence of Jordan block provides a possibility of energy growth. The energy growth depends on the low-order terms of the linearized equation. This situation occurs for the equation of hydrodynamics for ideal compressible fluids and for the equation of magnetohydrodynamics. As a rule the Jordan blocks arise when the characteristics of main symbol change their multiplicity. The stability of linearized system is considered with respect to the high frequency initial data.

As a rule the solutions of linearized systems have some energy estimates. These estimates determine the wave modes that occur in such systems. The main symbol can have Jordan block only if the energy form in the a priori estimate is degenerated.

Semiclassical asymptotics of eigenvalue for vector Sturm-Liouville problem

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Vector Sturm-Liouville problem

$$\left(-ih\frac{d}{dx}\right)^2 y + A(x)\, y = Ey,\ x \in \mathbb{R}^1,\ y \in L_2\left(\mathbb{R}^1\right),$$

where $y \in \mathbb{C}^n$, and $h \in [0,1)$ is a small parameter is considered. Let A be a selfadjoint matrix, i.e. $A(x) = A^*(x)$ and $A(x) = U(x)\,\|\delta_{ij}\lambda_j(x)\|\,U^*(x)$, where $U(x)$ is a unitary matrix. Suppose that $\|\delta_{ij}\lambda_j(x)\|, U(x) \in C^\infty\left(\mathbb{R}^1\right)$. The semiclassical asymptotics of eigenvalues of E is constructed for the case when the roots $\lambda_j(x)$ have variable multiplicity. Resonance phenomenon in the continuous spectrum is investigated.

On the propagation of surface electromagnetic waves, similar to Rayleigh waves in the case the Leontovich boundary conditions

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Let Σ be a surface, which is a boundary of a domain containing electromagnetic wave field. We assume, that Leontovich boundary conditions take place on Σ. Surface waves (an electromagnetic version of classical Rayleigh surface waves in elasiticy theory) can exists, only if the coefficient in the Leontovich boundary conditions is pure imaginary. The ray theory of surface electromagnetic waves is developed in this paper.

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Influence of extensional movements of face elastic walls on free oscillations of a cylindrical acoustic resonator

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The frequencies and forms of free oscillations of cylindrical volume filled with compressible irrotational fluid bounded by two thin elastic flat face plates and a rigid circular cylindrical wall are searched. The acoustic pressure in fluid satisfy the Helmholtz equation. The bending and extensional vibrations of elastic walls are described by the Kirchhoff and Fylon equations accordingly. The edges of plates are clamped. Both bending and extensional oscillations of plates in interaction of elastic walls with fluid are taken into account.

The exact equation and approximate formulae for calculation of natural frequencies of the resonator are constructed. The analytical and numerical comparison of the found frequencies with frequencies of two simplified adjacent models of oscillations is conducted. These models do not take into account the extensional movements of plates, and the bending movements satisfy the Kirchhoff equation and Timoshenko-Mindlin equations respectively.

Solution of the inverse dynamical problem for reconstruction of thin-layered strata

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In the report the effective method for reconstruction of thin layered structure of geological strata is suggested on the basis of dynamical inversion of low frequency surface seismic records and account of data of well acoustic logging.

Presently the attempts to solve the inverse problem on determination of properties of thin layered strata on the base of dynamical inversion of surface seismic records. There are several variants for such inversion. In particular, one of such variants is so called "annealing" - the variant of finding of global minimum of distortion functional by statistical matching of parameters of multi parametric problem with special behavior of statistics. However the stability and speed of that algorithm is not always acceptable.

In the suggested algorithm the structure of thin layered stratum and its variation in lateral direction is described in terms of boundaries separate the layers with different properties. The smoothness of variation of thin layered stratum in lateral direction will correspond to smooth and slow variation of separate boundary. At such conditions the spline representation is convenient for mathematical description of boundaries.

The data of well logging determining the exact structure of thin layered stratum in a well vicinity is considered as the principle regularization of the inversion problem

The results of reconstruction of real thin layered structures, showing the efficiency of the developed approach, are presented in the report.

Parametric refraction and reflection of optical beams

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All-optical switching is well investigated in soliton driven photonics. Parametric solitons, for one, exist due to three-wave interactions in quadratic media. They can repulse and attract each other depending on phase difference. Such a method needs beams trapping into solitons that is not always convenient and achievable. One way around this problem is to look at parametric interactions of tilted signal wave with pump beam. We first discuss novel mechanisms of all-optical switching: refraction and reflection under mismatched three-frequency noncollinear interplay. Signal wave refraction arises due to cascaded modulation of a nonlinear refractive index by pump beam. It results in a repulsion of signal wave.

We consider noncollinear parametric interaction of three waves at frequencies $\omega_3 = \omega_1 + \omega_2$. The slowly varying amplitudes of diffracting pump (ω_1), signal (ω_2), and idler (ω_3) beams obey the equations:

$$\frac{\partial A_1}{\partial z} + iD_1\Delta_\perp A_1 = -i\gamma_1 A_2^* A_3, \quad \frac{\partial A_2}{\partial z} + iD_2\Delta_\perp A_2 = -i\gamma_2 A_1^* A_3,$$
$$\frac{\partial A_3}{\partial z} + iD_3\Delta_\perp A_3 = i\Delta k A_3 - i\gamma_3 A_2 A_3. \tag{1}$$

The input envelopes are given by $A_1 = E_1(x,y)$, $A_2 = E_2(x,y)exp(-ik_2\theta_2 x)$, $A_3 = E_3(x,y) = 0$, where θ_2 is the tilt angle of signal beam at $z = 0$. The idler beam is inclined by the angle $\theta_3 = -k_2\theta_2/k_3$. We have performed numerical simulation of vector parametric interaction of Gaussian beams.

In order to better appreciate the physical mechanism for parametric reflection we have developed the original analytical theory. First, the effective mismatch owing to the beam tilts must be considered in (1) as

$$\Delta k = -k_1 k_2 \theta_2^2/(2k_3).$$

Assuming the mismatch is large, we can write in the cascaded limit the amplitude of the idler wave as $A_3 = (\gamma_3/\Delta k)A_1(x,y,z)A_2$, and obtain the equation for the signal wave

$$\frac{\partial A_2}{\partial z} + iD_2\Delta_\perp A_2 = ik_2 n_{nl}(x,y,z)A_2, n_{nl} = -(\gamma_2\gamma_3/(k_2\Delta k))|A_1(x,y,z)|^2. \tag{2}$$

The equation (2) describes signal wave propagation in the channel created by the pump beam. This problem is typical for inhomogeneous media optics. In the geometric optics approximation the ray coordinate $x(z)$ obey the equation in the form

$$\frac{dx}{dz} = \pm\sqrt{-n_{nl}(x) + n_{nl}(x_0) + \theta_2^2/2}. \tag{3}$$

The trajectory is drawn parallel to the z-axis at the turn point where $dx/dz = 0$. It follows that we can find the turn point coordinate x_t and the critical angle:

$$\theta_2 = \theta_{cr}(E_1(x_t)/E_{1max})^{1/2}, \theta_{cr} = \left(\frac{4\gamma_2\gamma_3 k_3 E_{1max}^2}{k_1 k_2^2}\right)^{1/4}.$$

The parametric reflection occurs for tilt angles less than the critical value, $\theta_2 < \theta_{cr}$. Signal beam with larger inclination passes through the pump beam not being reflected.

In bulk medium propagation dynamics of beam cross-sections resembles potential scattering of particles. If beams axes are in the same plane, interaction can be called "central". In this case all trajectories lie in the same plane. If initial signal wave-vector is not exactly directed to the pump axis "non-central" interaction takes place. In such case the result depends on the magnitude of axis deviation (aiming parameter). If this parameter is small enough parametric reflection can be observed. Interaction dynamics also depends on beam shape. If pump beam is axial-symmetric and its width is comparable with signal width parametric inhomogeneity possesses a considerable curvature. Interaction process looks like a reflection from the convex mirror and reflected signal beam is divergent. If curvature is small reflected beam conserves its shape.

An interesting effect takes place if signal beam contents the optical vortex. Vortex is the phase singularity: the amplitude in the center of vortex is zero and phase is uncertain. The amplitude of vortex is

$$A = E[x - x_v + i\text{sign}(m)(y - y_v)]^{|m|} exp[-(x - x_v)^2 - (y - y_v)^2], \tag{4}$$

where m is topological charge, which describes the order of singularity, x_v and y_v are coordinates of vortex. The process of reflection is rather complicated: pairs of vortices with opposite topological charges appear and annihilate during the parametric interaction. Reflected signal beam also contains a vortex, but topological charge changes its sign.

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Berry phases for the multidimensional nonlocal Gross-Pitaevskii equation with a quadratic potential

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The Berry phase [1], well-known in non-relativistic quantum mechanics, is modified for the nonlocal Gross-Pitaevskii equation (GPE)

$$\{-i\hbar\partial_t + \hat{\mathcal{H}}_\varkappa(t,\Psi)\}\Psi = 0, \quad \hat{\mathcal{H}}_\varkappa(t,\Psi) = \mathcal{H}(\hat{z},t) + \varkappa\int_{\mathbb{R}^n} d\vec{y}\,\Psi^*(\vec{y},t)V(\hat{z},\hat{w},t)\Psi(\vec{y},t),$$

Here $\Psi \in L_2(\mathbb{R}^n_x)$, and operators

$$\mathcal{H}(\hat{z},t) = \frac{1}{2}\langle\hat{z},\mathcal{H}_{zz}(t)\hat{z}\rangle + \langle\mathcal{H}_z(t),\hat{z}\rangle,$$
$$V(\hat{z},\hat{w},t) = \frac{1}{2}\Big\{\langle\hat{z},W_{zz}(t)\hat{z}\rangle + 2\langle\hat{z},W_{zw}(t)\hat{w}\rangle + \langle\hat{w},W_{ww}(t)\hat{w}\rangle\Big\}$$

are functions of non-commuting operators

$$\hat{z} = \Big(-i\hbar\frac{\partial}{\partial\vec{x}},\vec{x}\Big), \qquad \hat{w} = \Big(-i\hbar\frac{\partial}{\partial\vec{y}},\vec{y}\Big), \qquad \vec{x},\vec{y} \in \mathbb{R}^n.$$

A countable set of space-localized asymptotic solutions is constructed for the GPE (1) in an adiabatic approximation in $1/T$, where $T \gg 1$ is a time of the adiabatic evolution. For the solutions constructed, the Berry phases are found in explicit form, corresponding to the formulas obtained in [2] in the 1D case.

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Eigenfunction expansions of source-excited electromagnetic fields in cylindrically stratified unbounded gyrotropic media

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This paper extends methods, initially developed for representing source-excited electromagnetic fields in the presence of cylindrical guiding structures immersed in a magnetoplasma [1, 2], to the case where spatially bounded, given sources are located in a gyrotropic medium whose permittivity and permeability are both describable by tensors with nonzero off-diagonal elements. It is assumed that the medium is radially inhomogeneous inside a cylinder of given radius, and is homogeneous outside it. The axis of symmetry of such a cylindrically stratified structure, here taken as z axis, is parallel to the gyrotropic axis of the medium.

The total field is sought in terms of the vector modal solutions of homogeneous Maxwell's equations. The transverse (with respect to the z axis) components of the field of each mode are expressed via the axial components which satisfy two coupled second-order partial differential equations. By demanding completeness of the modal spectrum and using a continuity argument, we determine the content of the spectrum, which comprises both the discrete and continuous parts, and derive the set of vector eigenfunctions describing the spatial distributions of the modal fields in cylindrical coordinates. Note that in the considered case the continuous-spectrum modes are grouped into two categories related to the presence of two normal waves in a homogeneous background gyrotropic medium, which makes the obtained solution substantially different from that in the case of an isotropic background region [3]. We establish orthogonality relations for modes of the discrete and continuous parts of the modal spectrum and then calculate the mode excitation factors due to given sources. To demonstrate the applicability of the method, representations of total fields due to given sources will be provided for special cases. It is shown that the developed theory makes it possible to immediately obtain source-excited fields in the form of an eigenfunction expansion in terms of the found modal solutions, which allows one to readily perform field evaluations in unbounded gyrotropic media.

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Evolution of wave packet in quantum box: novel approach to the problem

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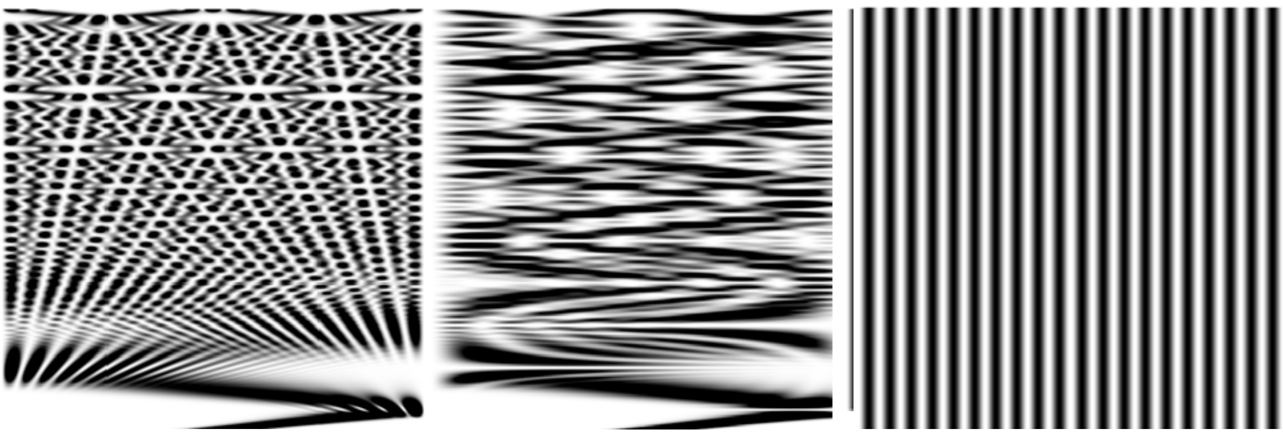
Evolution of wave packet in quantum well with infinite potential barrier (quantum box) is one of the corner problems in Quantum Physics. Recently this problem has drawn attention of many scientists because of studying of spatio-temporal dynamics of QM particles in a quantum box. Such phenomenon as wave packet revival, fractional revival, so-called quantum carpets have been revealed while studying this problem. Quantum carpet is a spatio-temporal pattern formed by intensity of wave function describing evolution of a wave packet in quantum box of a certain size. It has been shown that quantum carpets have fractal structure. Dispersion of wave packet and its self-interference are the phenomena, which lay in the basis of quantum carpet. Representation of the wave function as a superposition of eigen states of the quantum box is a standard approach to solution of the problem. Equivalent method consists in using Green's function for the quantum box. Essentially different solution has been suggested by Max Born (1953), which consists in solving initial boundary problem for Schroedinger's equation for free space, but with periodical boundary condition. The latter is the consequence of multiple mirror reflections of a wave packet by quantum wells forming the quantum box. Much later the same problem has been solved applying Finemann path integral approach (Goodman, 1981).

In the paper, we suggest a new approach to solve this "classical" problem of Quantum Physics. Analysis of spatio-temporal evolution of wave packets during finite intervals of time is the basic principle for the approach suggested. This principle enabled constructing of the solution as a superposition of wave packets formed by successive reflections of initial packet by the walls of quantum box. Solution of the problem is represented as a sum of finite number of the partial wave packets, propagation of which is described in terms of Green's function of Schroedinger's equation for free space. For infinite number of reflections this solution coincides with solution obtained with help of standard QM approach. Computer simulation of spatio-temporal dynamics of wave packets in quantum box enabled observing of quantum interference phenomena mentioned above. Advantage of the approach suggested consists in possibility of its generalizing for the case of finite barrier quantum well and also for the case of nonlinear transformation of the wave packet when reflecting by the walls. The suggested approach enabled us also to study an interesting process of quantum states forming.

Wave packet Evolution in a Box



Quantum Carpet | Diffusive interference part | Wave interference part

Dipole currents and interstellar propagation of electromagnetic signals

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Most of the information we have obtained about the universe beyond the planet Earth came from electromagnetic (EM) waves, and most of the information obtained from EM waves came from steady state waves. Whenever received EM waves are decomposed into sinusoidal waves with various frequencies one thinks and acts in terms of steady state waves theory. For instance, the Doppler shift frequency - implied every time the term red-shift is used -is a steady state concept even though the Doppler effect is not. A power spectrum density (Fourier frequency spectrum) of received waves is a steady state concept as well. However, in reality there are many electromagnetic events and phenomena which can not be described using a steady state concept, and one have to apply a different approach for their proper description and understanding. For example, a sunrise and sunset being considered as single events provide us with non steady-state events since we can determine their beginning and ending times. In this way, we are coming to the necessity of use of a different concept in describing of EM waves, namely to the concept of EM signals. Electromagnetic signal is an electromagnetic wave that is zero before a certain finite time and has finite energy. In terms of mathematics a signal can be represented by a function of a real variable with the usual topology of the real numbers and denoted time, the function being zero below a certain finite value of the variable and quadratic integrable. Pulsars provide us with electromagnetic signals since we can observe the increase of the electric and magnetic filed strength from zero or the decrease to zero.

In the paper, we present results of our investigations on propagation of EM signals through interstellar medium that is supposed to be formed by neutral atomic hydrogen gas with rather low density. We describe the model under consideration and obtaining of the partial differential equation for one of the EM field components, as well as formulation of initial-boundary value problem for that equation and the method for its solution in time domain. Step-like changing of the EM signal field requires solving of the problem in time domain and application of a non-conventional description of the above medium. Propagation of EM waves is governed by Maxwell equations. However, those equations should be modified according to the media properties where EM waves propagate. In the case under consideration, EM signals have to propagate through the medium consisting of atomic hydrogen gas with a very low density. Since the atoms are neutral and signals are supposed to be rather weak they cannot carry electric monopole current, but only dipole currents. Electric field strength will pull the positive proton and the negative electron slightly apart and produce an electric dipole. A dipole current flows while this pulling apart is in progress and also when the electric field strength drops to zero and the hydrogen atom returns to its original, non-polarized state. Similarly, the hydrogen atom has a magnetic momentum like a little bar magnet. A magnetic field strength will rotate the atoms to make them line up with the field strength. Magnetic dipole current flows while this rotation is in progress and also when the magnetic field strength drops to zero and the magnetic dipoles return to their original random orientation. So, we have to take into account the reaction of the medium onto the action of the EM field of the propagating signal. Conventional approaches to solution of that problem suppose that both intrinsic characteristic time of the media and its relaxation time are much smaller the characteristic time of the EM field variation. Besides, they also suppose performing of space averaging of the EM fields introducing into consideration both electric and magnetic flux densities. Since in our case we have both very low density of the neutral atomic hydrogen and very fast variation of the EM signal fields those methods are not applicable anymore. In order to solve the problem we modify Maxwell equations for "empty space" using both electric and magnetic dipole current densities rather then electric and magnetic flux densities [1]. This implies description of the medium in the frame of microscopic approach using representation of a hydrogen atom as a combination of electric and magnetic dipoles. Those dipoles produce electric and magnetic dipole currents under the EM field action that is to be calculated in a self-consistent way. EM signals propagation over Billions light years distance through such medium is studied for various combinations of its parameters. The related results are presented and discussed.

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Scattering of electromagnetic impulse on the standing ion-acoustic wave forced by the powerful electromagnetic wave in the liner plasma layer

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In the recent experiment with the ionospheric plasma, disturbed by the powerful electromagnetic wave, the interesting space – time characteristics have been obtained for the probe electromagnetic impulse amplitude and phase. We suggest the simple physical model, which explains the most of the features of experimental data.

We assume the scattering take place on the ion – sound wave induced by the powerful electromagnetic wave, reflecting in nonuniform plasma with dielectric permeability $\varepsilon = -z/L$, $z \geq -L$ ($z = 0$ — reflection level coordinate, L — plasma density scale). In the most simple case of short-term powerful wave action (T — small)with respect to ion - sound wave period (but at the same time $T \gg L/c$, c— speed of light), the expression for the plasma concentration perturbation $\delta N(z,t)$ may be obtained:

$$\frac{\delta N(z,t)}{N_0(z)} = -\frac{8\hat{k}^2 V_s^2 T}{\sqrt{-z/L}} \frac{|E_*|^2}{E_p^2} \cdot \frac{\sin(t\sqrt{(2\hat{k}V_s)^2-\gamma^2})}{\sqrt{(2\hat{k}V_s)^2-\gamma^2}} \mathrm{e}^{-\gamma t} \sin\left(\frac{4}{3}\hat{k}z\right), \qquad \hat{k} \equiv k \cdot \sqrt{-\frac{z}{L}}.$$

Where $N_0(z)$ — undisturbed plasma concentration, $k = \omega/c$— wave number, V_s,γ— ion sound wave velocity and decrement, E_*— amplitude of powerful wave, E_p — so could "plasma field".

This simple expression take place at some distance from reflection level. At any z $\delta N(t)$ looks like relaxation of impulse — single impulse near the reflection level and oscillating relaxation of impulse at large distance. The specific time scale diminishes when the distance increases. The single-scattered probe impulse amplitude is analogues to the $\delta N(t)$ behavior and is in agreement with the experimental data.

The phase of the probe impulse is associated with the scattering structure too. The structure coordinates are of importance. In the single impulse-like relaxation region the coordinate of the maximum is the constant and no phase evolution take place. At the same time in the region where $(2\hat{k}V_s)^2 - \gamma^2 \geq 0$, the maximums and minimums of scattering structure exchange each another at the time, when the first sine in the expression above is equal to zero. So the probe impulse phase path difference arise at this moment and it is equal to the spatial period of the reflecting structure. As far the structure was created by the powerful wave, the phase difference of it is equal to π number at the length equal to period. The same phase addition π takes the probe quasimonochromatical impulse with the same frequency. The fast step-wise phase changes of the order of π takes place at the experiment.

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Study of porous materials by acoustic microscopy

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Acoustic microscopy is a technique of nondestructive characterization; it makes it possible to determine starting from the acoustic signature $V(z)$ different velocities various propagation of the waves in isotropic, anisotropic and porous materials. From where one deduces the various elastic parameters from materials such as (Young modulus, modulus of rigidity, Poisson's ratio); by applying two types of models of waves propagations: A) model of Shoch (isotropic, and anisotropic material); B) Model of Biot (porous materials). One deduces from the two models two various types of acoustic signatures $V(z)$. To show the differences between these two signatures we developed a program that one will use for the determination the elastic properties of porous material.

Realization of impedance surface in terms of a slab of metamaterial

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Impedance boundary is generally considered as an approximate model for material interfaces. Considering wave propagation in a wave-guiding anisotropic metamaterial it is shown that a slab of such a material backed by a PEC plane can be exactly represented by impedance-boundary conditions. As a result of the analysis, a novel explicit relation is derived between the surface admittance dyadic of the impedance boundary and the parameters of the material slab which can be used to realize a given admittance dyadic. The relation is verified with results known for the perfect electromagnetic (PEMC) boundary.

As a particular example, the generalized soft-and-hard surface (SHS) boundary conditions defined by $a \cdot E = 0$ and $b \cdot H = 0$ are considered where a and b are two complex vectors tangential to the boundary. Potential application of a generalized SHS boundary as a polarization transformer has been pointed out previously. It is shown that analytic expressions can be found for the material parameters and thickness of the slab as functions of given complex vectors a and b.

About one nonlinear spectral problem

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Spectral equation, type $Lu - \lambda u + T(\lambda, u)$, where λ is a spectral parameter is examined in some important practical problems. This equation is considered to be bifurcation and its choice allows to use different topological approach to get bifurcation results. In the supposition that nonlinear operator $T(\lambda, u)$ is like gradient (on potential) operator) it is stated that on the spectrum of selfadjoint operator L always appears bifurcation. The examples of different types of nonlinearity on λ are given.

About existing of an inertial manifolds for one class of evolution equation

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The global properties of nonlinear dynamic system solution, that is, dissipative type of evolutional equation in infinitely functional space were studied. The conditions of stable invariant set - inertia manifold are stated in the supposition of exponentional dihotomy existing on attractor of evolution equation. The most important of these conditions is the cone condition (availability of spectral gap). This means that for the evolution equation Hoph hypothesis is correct and is applicable so called Galerkin dynamic method. The examples are given.

The fields generated by finite size Input grating coupler in conditions of SEW excitation

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The problem of surface electromagnetic waves (SEW) excitation by input grating coupler of finite dimension is considered in linear approximation of groove height. The finite grating is described as the sum of infinite gratings by means of Fourier integral. The SEW is excited by the directed plane bulk wave onto the input coupler at resonant angle of incidence. The resulted field for SEW is presented as a sum of partial SEW fields, generated by infinite gratings. The obtained results were quite unexpected. It was discovered that in conditions of SEW excitation a wave packet consisted of evanescent and bulk waves is generated. This wave packet is conventionally called "pressed wave" by us.

The wave packet is demonstrated to satisfy Maxwell equations and boundary conditions on smooth part of surface outside the grating coupler. Thus the generated radiation is a new type of surface-bulk electromagnetic wave on metal surface. It follows from our results that as "pressed wave" propagates along the metal surface the bulk radiation is emitted generated from smooth part of metal surface.

This bulk radiation was observed experimentally [1].

Another essential difference is that as "pressed wave" travels along the surface, its profile changes unlike that of the SEW.

In the present work we investigate by numerical methods and analytically the changes of "pressed wave" field profiles. The results of calculations for "pressed waves" of the middle infrared range are presented. The wavelength choice allows to compare the obtained results with experimental data on SEW excitation by CO_2 laser radiation.

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Negative dispersion cylindrical surface plasmon-polaritions in layered media

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The dispersion relation for surface cylindrical plasmon - polaritons in layered left - handed media is considered and solved. The model of the infinite length circular cylinder with the layered structure: core metal - thin film of material - air is proposed. The dielectric permittivity $\varepsilon(\omega)$ of the thin film material is near $\mathrm{Re}\varepsilon(\omega) \sim -1$.

The numerical solutions of dispersion relation were obtained for model case when dielectric function of metal are taken from experimental data and dielectric function of thin film material are taken to be constant. This is the real situation for the middle IR range, where the metal-like thin oxide film (native metal oxide) is considered as a thin film [1] or for the visible spectral range where the metal silicides and some nonstoichiometry metal oxides are considered as a thin film layer [2].

In all considered cases the dispersion curve behavior is a similar: it contains the upwards and downward branches and has a maximum in ω-k coordinates (see [1,3] for the case of plane geometry). The upward branch has $k_s \sim k_0$, and the downward branch has $k_s \gg k_0$ and negative dispersion (the group and phase velocities have opposite directions). The two branches existence in the surface polariton dispersion curve are due to the evanescent electromagnetic field penetration through the thin film layer with small negative value of dielectric permittivity.

The effect of the cylinder radius, the thickness of thin film material and dielectric function variation of thin film material on the surface plasmon dispersion behavior has been calculated. An examples of numerical solutions of dispersion equation for $m = 0$, 1 quantum numbers illustrates the dispersion relation behavior for two cylinder's radius $r = 25$ nm and $r = 10$ mcm.

The effect of the mutual interference of two coherent waves ($k_1 < k_2$) excited by laser radiation on the character of optical damage of the surface active media boundary has been analyzed. The small value of wavelength of the large k-value branch allows one to predict metal surface structuring with typical dimension of the order $10^{-2}\lambda$ during femtosecond repetitive pulses action on some metal surfaces, nanowires formation and excitation and interference of cylindrical surface polaritons with incident radiation.

The work was partially supported by INTAS/Belgium, project #03-51-4924.

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On variational principle in dissipative hydrodynamics

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A variational principle for dissipative hydrodynamic is formulated in the form of generalization of Hamilton's variational principle for mechanical systems. The Lagrangian in the developed approach is built as a difference of kinetic energy and free energy of a continue medium, as well as time integral of dissipation function. Two fields - the displacement field and the heat displacement field are considered as variation variables. Following to Biot the field of heat displacement is introduced so that its divergence determines variation of temperature. The extreme condition of the action functional at variation by the both mentioned fields results in a system of the motion equations of the two fields - the displacement one and the heat displacement. In the linear by amplitude approximation these equations occur to be analogous to the linearized equation of Navier-Stockes and energy balance equation in the entropy form, what reduces in linear approximation to the heat conduction equation. Thus it is shown that the formulated variation principle allows to obtain the usual system of hydrodynamics equations.

The fulfilled analysis of the dispersion relations for thermo-acoustic and acousto-thermal modes, as well as for shear mode, shows that on the high frequencies their behavior has a diffusion character, that does not provide a finite velocity of signal or short pulse propagation. This fact is well known and reflects the low frequency, approximate character of usual hydrodynamics equations.

Correction of the hydrodynamics equations, provided a finite propagation velocity of small perturbations could be fulfilled by introduction of finite relaxation time of a heat flow, as by account of relaxation of viscosity.

In the report it was shown that the introduction of the necessary corrections to the Navier-Stokes system of hydrodynamic equations, accounting relaxation of viscosity and heat flow, can be naturally achieved on the basis of generalized variation principle, if to take into account inertion of heat displacement field and existence of additional internal parameters in a medium by analogy to Mandelshtam - Leontovich approach, responsible for relaxation of the order parameters in a medium. In that case the kinetic equations for the order parameters are derived from the variation principle together with motion equations for the displacement field and for the heat displacement field.

Thus on the basis of variation principle for dissipative hydrodynamics it is possible to obtain the natural generalization of the Navier-Stokes equation system, describing in the linear approximation propagation of termo-acoustic and acousto-termal modes as well as shear mode with finite propagation velocity at high frequencies, provided finite speed of signal propagation.

Strong asymptotic completeness for a quantum wire: time-dependent approach

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A time-dependent approach to scattering theory is applied to a quantum system describing a curved quantum wire in two dimensions. Existence and strong asymptotic completeness of the (localized) wave operators are proven and the singular spectrum is shown to be discrete away from thresholds.

Nonlinear paraxial reflection of few-cycle pulses from dielectrics

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It's supposed that light falls on the interface of dielectric homogeneous and isotropic linear and nonlinear media along positive direction of the z axis from linear medium. Then, the paraxial dynamics of the electric field E of linearly polarized extremely short pulses for incident, reflected and transmitted radiation along z axis can be described by following equations [1]:

$$\begin{cases} \frac{\partial E_i}{\partial z} + \frac{N_1}{c}\frac{\partial E_i}{\partial t} - a_1\frac{\partial^3 E_i}{\partial t^3} + b_1\int\limits_{-\infty}^{t} E_i dt' = \frac{c}{2N_1}\Delta_\perp\int\limits_{-\infty}^{t} E_i dt', \\ \frac{\partial E_r}{\partial z} - \frac{N_1}{c}\frac{\partial E_r}{\partial t} + a_1\frac{\partial^3 E_r}{\partial t^3} - b_1\int\limits_{-\infty}^{t} E_r dt' = -\frac{c}{2N_1}\Delta_\perp\int\limits_{-\infty}^{t} E_r dt', \\ \frac{\partial E_{tr}}{\partial z} + \frac{N_2}{c}\frac{\partial E_{tr}}{\partial t} - a_2\frac{\partial^3 E_{tr}}{\partial t^3} + b_2\int\limits_{-\infty}^{t} E_{tr} dt' + gE_{tr}^2\frac{\partial E_{tr}}{\partial t} = \frac{c}{2N_2}\Delta_\perp\int\limits_{-\infty}^{t} E_{tr} dt', \end{cases}$$

where E_i, E_r and E_{tr} are electric field of incident, reflected and transmitted waves accordingly, N, a, b are the empirical dispersion constants of the medium, which describe the dependence of the linear index of refraction on frequency:

$$n(\omega) = N + ca\omega^2 - c\frac{b}{\omega^2} \tag{1}$$

$g = \frac{3\varepsilon_{nl}}{2cN_0}$ characterizes the magnitude of the instantaneous nonlinearity of the medium polarization response, c is the velocity of light, z is the direction of propagation of light pulses, $\Delta_\perp$ is the transverse Laplacian, t is the time. Using boundary conditions

$$E_i + E_r = E_{tr}, \qquad \frac{\partial E_i}{\partial z} + \frac{\partial E_r}{\partial z} = \frac{\partial E_{tr}}{\partial z}$$

and taking into account expression (1), we can write for spatial-temporal spectrum of radiation

$$\iiint E(t,x,y)exp[i(\omega t - k_x x - k_y y)]dtdxdy$$

the following relation between spectrums of reflected and incident radiation:

$$G_r = \frac{n_1 - n_2}{n_1 + n_2}\left(1 + \frac{c^2}{n_1 n_2}\frac{k_x^2 + k_y^2}{\omega^2}\right)G_i - \frac{gcG_1}{n_1 + n_2}, \tag{2}$$

where G_i, G_r are spectrums of incident and reflected waves accordingly,

$$G_1(\omega, k_x, k_y) = \frac{1}{(2\pi)^4}\iiiint\!\!\iint G_i(\omega - \omega', k_x - k'_x, k_y - k'_y)G_i(\omega' - \omega'', k'_x - k''_x, k'_y - k''_y)G_i(\omega'', k''_x, k''_y).$$

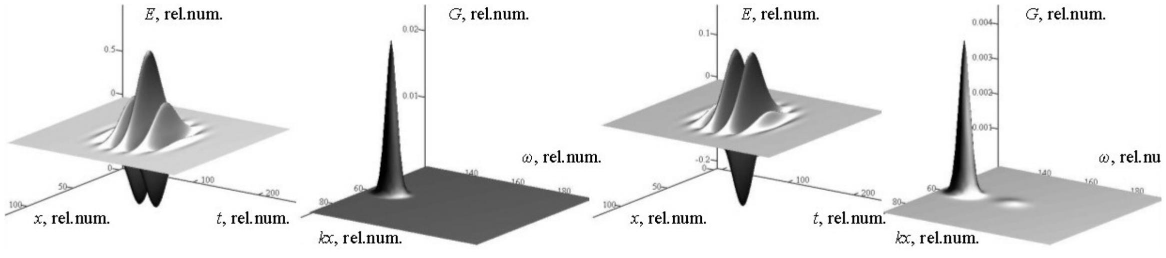

Figure : The electrical field and spectrum of incident(left) end reflected (right) impulse.

Linearized equation (2) is equivalent to Fresnel formulae in approximation of paraxial radiation. The reflection of extremely short Gaussian pulses is simulated for the interface air - fused silica. Typical results of numerical modelling are shown in Fig. 1, which depicts modifications of spatial-temporal spectrum and electric field of radiation.

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Theory of three-dimensional sub-diffraction imaging in metamaterial super-lenses

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After the seminal papers of Veselago [1] and Pendry [2], the imaging properties of metamaterial slabs have been a subject of growing interest for the scientific community. Specifically, the possibility of obtaining sub-diffraction images [2] soon attracted the attention of the physics community and the public in general. Sub-diffraction image formation by a perfect lens [2] substantially differs from the standard (i.e., over-diffraction) image formation reported by Veselago in left-handed slabs [1]. The main differences come from the fact that information for sub-diffraction imaging is mainly carried by the evanescent spatial Fourier harmonics of the source field [2]. Thus, despite of the reproduction of the source fields at the image plane and beyond, nothing similar to a focusing of energy around the image can be expected in sub-diffraction imaging devices. However, more recently, some papers claiming for the obtention of three dimensional sub-diffraction images have been published. These papers include experimental observations in coupled magneto-inductive arrays [3], and in electromagnetic band-gap slabs [4]-[5]. The reported images (of sub-wavelength sized sources) consisted in three-dimensional spots having a size substantially smaller than the free space wavelength. Such results, which clearly seem to violate uncertainty principle [5]-[6], deserve a theoretical explanation.

In this contribution, following some previous analysis [6]-[7], the formation and measurement of sub-diffraction images is analyzed. It is concluded that focusing of energy into three-dimensional spots of sub-diffraction size can not be obtained from any imaging device. However, in some sense, three-dimensional sub-diffraction images can be obtained from the measurement of the transmission coefficient through some metamaterial devices, a as a consequence of a tunneling/matching process. New experimental and theoretical results supporting this conclusion will be provided.

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Resonant enhancement of evanescent electromagnetic waves in passive structures

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A review of recent results in the field of sub-wavelength imaging will be given. The presentation will be focused on resonant amplification of evanescent modes in passive structures. The "super-lens" or "perfect lens" proposed by Pendry [1] is one of such structures. In Pendry's lens the amplification is achieved in a slab of backward wave medium whose $\varepsilon(\omega)$ and $\mu(\omega)$ are both negative at certain frequency (Veselago medium [2]). It may seem that the presence of a backward wave material is crucial for the amplification, but an accurate analysis shows that it is not so. In [3] we proposed a non-linear passive analogy of Pendry's lens which uses a couple of phaseconjugating planes. Such planes may be thought as very dense arrays of small antennas where every antenna is connected to a non-linear element that provides the necessary wave mixing. In [4] we showed that a couple of resonating grids or arrays of particles working in a linear regime can also "amplify" evanescent waves. The theoretical predictions in [4] were supported by preliminary experiments. Later similar results were obtained for arrays of magnetic resonant particles, [5]. Future experiments will most likely make use of plasmonic resonances which can help to shift operating frequencies up to terahertz or even optical frequencies.

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Near and far field diffraction by highly conducting wire gratings

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We present a modal approach for calculating the near and far fields diffracted by gratings made of highly conducting wires that have a rectangular shape. Because of the conductivity, the calculations are made using the approximated surface impedance boundary condition (SIBC). The Poynting vector is used to show the behavior of the field within and in the vicinity of the wires. In addition, far field spectra are obtained as a function of the opto-geometrical parameters and compared with those obtained from a perfect conductor.

On the connection between stationary and nonstationary diffraction problems on wedges

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We continue to investigate a nonstationary scattering of plane waves by a wedge $W := \{y = (x, y) : x = \rho\cos\theta, y = \rho\sin\theta, \rho > 0, 0 < \theta < \phi\}$ (see [2]-[4]). Usually, in the diffraction problems on wedges it is supposed that the dependence of solution on time t has the form $w(x, y, t) = u(x, y)e^{-i\omega_0 t}$. In this case, for the diffraction of a plane wave of the form

$$w_{in}(x, y, t) = e^{i\omega_0(\cos\alpha x + \sin\alpha y - t)} \tag{1}$$

on an ideal wedge, the amplitude u is a solution of a homogeneous boundary value problem for the Helmholtz equation. This solution is the sum of the optic and diffracted parts while the diffracted part satisfies the radiation conditions [1]. With the additional Meiksner condition such solution is unique. In this paper we justify this classic solution in the following sense. We obtain this solution u as the *limit* of the amplitude of the solution of some nonstationary scattering problem on wedge. For that we consider an incident plane wave $u_-(x, y, t)$ of the form

$$u_-(x, y, t) = e^{i\omega_0(\cos\alpha x + \sin\alpha y - t)} f(t - \cos\alpha x - \sin\alpha y), \quad (x, y) \in Q := \mathbb{R}^2 \setminus W.$$

We assume that $\max(0, \phi - \pi/2) < \alpha < \min(\pi/2, \phi)$. In this case $u_-(x, y, 0) = 0,\ (x, y) \in \partial Q$. The profile $f \in C^\infty(\mathbb{R})$, $f(s) = 0, s < 0$, and $f(s) = 1, s > \mu$, for some $\mu > 0$. At the moment $t = 0$ this plane wave begins to interact with W and converges to the plane wave (1) when $t \to \infty$. For this plane wave u_- we pose the mixed Cauchy problem and we find the explicit solution of this problem for Dirichlet and Neumann boundary conditions. We prove that this solution is unique in some functional class. Finally, we prove that the *Limiting Amplitude Principle* holds. Namely, we prove that the amplitude of this solution converges to the solution of the classic diffraction stationary problem of the plane wave by a wedge of the Sommerfeld-Maljuzhinetz type described in detail in [1], section 1.4.

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The analysis of full spectra of normal elastic waves in homogeneous and inhomogeneous anisotropic cylindrical waveguides of complex cross-section

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Obtained results of theoretical researches and the numerically-analytical analysis of dispersion equations for running and edge stationary normal waves in axes-anisotropic cylindrical waveguides with several types of cross-sections are presented. Sections of considered waveguides are represented by circular and ring areas, or ring areas with internal circular inclusion and have sector cut of an any angular measure. External cylindrical boundary surfaces of waveguides of circular or ring section are free from stresses or are fixed, and thin not extensible coverings without inertia are placed on rectilinear boundary surfaces of sector cut. Conditions of ideal mechanical contact on a cylindrical contact surface of external ring components of a compound wave guide and internal circular cylindrical inclusion are satisfied. Dispersion functions for the basic sets of investigated normal waves are obtained in the analytical form and are presented by functional determinants of the third, sixth and twelfth orders for each value of the generalized circle wave number. Elements of dispersion determinants are expressed through special cylindrical functions of different types.

The real, imaginary and complex branches of full dispersion spectra are estimated on the basis of the numerical analysis of the dispersion equations for waveguides with various physico-mechanical and geometrical characteristics. Leading effects of spectra transformation by change of sector cut angular measure are investigated. Problems of obtaining wave spectra with necessary characteristics are investigated at a corresponding choice of angular cut value. Diagrams of dispersion curves and the basic kinematic and power characteristics of running normal waves from various modes of a spectrum are investigated in details. Effects of high-frequency short-wave localization of normal elastic waves for waveguides of investigated type are described.

Integrable models of the longitudinal motion of electrons in curved 3D-nanotubes

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In the framework of the adiabatic approach [1,2], we find several model examples of curved 3D-nanotubes and configurations of a homogeneous magnetic field for which the one-dimensional matrix equation can be integrated exactly on a subregion of dimensional quantization. For periodic tubes (with the Born–Karman conditions), we obtain explicit (analytic) formulas of the electron spectrum and the time-domain Green function of the corresponding initially boundary-value problem. In the electron spectrum, we distinguish the corresponding contributions of the Aharonov–Bohm and Berry phases and the Maslov "geometric" potential.

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Investigation of wave field in effective model of layered elastic-fluid medium

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For the medium consisting of alternating elastic and fluid layers, the effective model was constructed [1] and investigated [2]. In the wave field of this model there are leading and "triangular" fronts. The model is a special case of the Biot medium. In this model, the wave field exited by an impulse point source is represented in a half-plane as Fourier and Mellin integrals. In the Mellin integral we replace contour integration by a stationary contour. In the obtained expressions, we rearrange the integrals and calculate the inner integral. The external integral is equal to two residues. The corresponding poles are roots of two equations of fourth order. These roots are situated at the right half-plane and can be complex or real. If these roots go out at the imaginary axis then the corresponding points of observation are on the fronts or inside of "triangular" front. In these cases the displacements are equal to zero. The obtained representation for the wave field corresponds to the expressions derived by the method of Smirnov-Sobolev. For calculation of the complex and real roots determining the wave field, we suggest an effective method.

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A criterion of uniqueness for linear water-wave problems

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Problems, which describe radiation of waves by forced motion of rigid bodies or diffraction of waves by fixed bodies in an ideal unbounded fluid, are considered in the framework of the linearized surface wave theory. Uniqueness for the linear problems is known to be equivalent to non-existence of the so-called trapped modes, satisfying the equation $\Delta u = 0$ in the fluid domain W, the condition $\partial_y u - \nu u = 0$ on the free surface $y = 0$, the homogeneous Neumann condition $\partial_n u = 0$ on the surface of bodies S, and the condition of finiteness of energy $u \in H^1(W)$. Very few criteria of uniqueness are known for the seemingly simple problems (good review can be found in the book [1]; a new criterion was recently suggested in [3]). The existence of trapped modes for some special geometries was first proved in [2].

In the present work a new criterion of uniqueness in two- and three-dimensional problems for totally submerged bodies of arbitrary shape is found. A self-adjoint operator $T + T^* - T^*T$, where T is an operator of a boundary integral equation on S, is introduced. It is shown that its maximum eigenvalue $\lambda_1 = \max\{\lambda_i\} \leq 1$ characterizes uniqueness property, so that $\lambda_1 = 1$ indicates existence of trapped modes and the inequality $\lambda_1 < 1$ guarantees uniqueness. Since the operator is self-adjoint, the criterion allows us to find effective and mathematically justifiable procedure for numerical verification of uniqueness property. Numerical results are obtained and compared with computations by the criterion [3] and with known examples of non-uniqueness.

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On a solvability of multibody contact problem with Coulomb friction for a wave equation in visco-elasticity

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In seismology, geomechanics as well as in mining and building industry in regions with great earthquakes, there are problems whose investigations lead to solving model problems describing propagations of elastic waves, propagated through broken up media (rocks). Such problems are modelled by dynamic contact problems with Coulombian friction for visco-elastic materials for the case of N-dimensional bodies ($N = 2,\ 3$) of arbitrary shapes being in mutual contacts.

The dynamic contact problem with friction in visco-elasticity, where the contact problem with given time-independent friction and contact condition formulated in velocities, was firstly studied in Duvaut, Lions (1976). For die physically well defined contact condition with Coulombian friction in displacements no proofs of sufficient regularity of velocities to the auxiliary solutions are at present solved completely, and therefore, the problems with given friction are solved only. From this reason the contact conditions for the solvability of contact problems with Coulombian friction are formulated in velocities.

In this contribution the solvability of unilateral hyperbolic contact problem with Coulomb friction and damping in visco-elasticity with short memory, describing the propagation of elastic waves in visco-elastic media occupying by N-dimensional bodies ($N = 2,\ 3$) of arbitrary shapes being in mutual contacts, wilt be investigated and discussed. The contact conditions will be based on the non-penetrability of mass. For the analysis (he contact condition will be formulated in velocities, which limits the applicability of the result to small time interval only. We will assume that the coefficient of friction may depend on the solution, but that it will be bounded by a certain constant.

Moreover, the numerical analysis of the initial problem in displacements will be also investigated and discussed. It will be shown that the semi-implicit scheme is suitable for application of such model problems.

The contribution represents extension of problems solved in Nedoma (1998), (2000), (2001).

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Modelling stable and unstable shear layers as infinitesimally thin

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Shear layers are unstable if the frequency of the disturbance is low enough. More precisely it is the Strouhal number s, based on the shear layer thickness, that controls this Kelvin-Helmholtz instability. The flow is stable for s larger than a critical Strouhal number and unstable for smaller s.

If the thickness of the shear layer is acoustically thin, i.e., the thickness is much smaller than the wavelength, it would be appropriate to model the shear layer as infinitesimally thin, a sheet. Two types of arguments have been used for this purpose, both with the result that pressure and normal displacement are continuous across the sheet. Such a sheet is then unstable for all frequencies, reflecting that the shear layer thickness is vanishing.

The physical argument, requires quite simply that there can be no discontinuity in the normal displacement, while the mathematical argument consists of integrating the wave equation across the shear layer and then take a vanishing shear layer thickness. Both these arguments have deficiencies since it is possible to construct models of finite shear layers and then go to the limit of vanishing shear layer thickness keeping the Strouhal number constant. Two questions arise. What is the weak point in the mathematical argument above? Can a thin shear layer be modelled as infinitesimally thin, allowing a variation with Strouhal number to get both stable and unstable layers? It is the purpose of this paper to answer these questions.

An analysis reveals that the limiting process in the mathematical argument above amounts to calculating the product of Dirac's delta function and Heaviside's step function as half the delta function. The generalized functions have their singularities on the sheet. However, this product is not uniquely defined. This ambiguity can be used to construct coupling conditions depending on the Strouhal number.

To this end coupling conditions are derived for an infinitesimally thin shear layer. One condition is continuity of pressure. The other condition varies continuously with the Strouhal number from continuity of normal displacement, via normal velocity to normal derivative of pressure. This model, that has the important property of being analytic in the wave number in the flow direction, has been analysed in great detail and the results are reported in this paper. It is demonstrated that the variation with Strouhal number of the unstable pole in the complex wave number plane is near that of the linear shear layer. Therefore, the designed model for the sheet is appropriate for modelling acoustical problems with thin shear layers.

On quasiphotons of Rayleigh waves (anisotropic elastic body case)

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The construction procedure of quasiphotons analytic formulas is developed. Quasiphotons are special asymptotic solutions of linear equations, that describes wave processes. These asymptotic solutions correspond to concentrated wave packets propagating along rays on a surface of elastic body.

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A physical picture of wedge diffraction: generalization of transversal diffusion method

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A promising generalization of the Leontovich-Fock parabolic equation has been proposed in a classical paper by Malyuzhinets [1]. It assumes description of the diffraction field as a superposition of different type waves, each of them being governed by a corresponding parabolic equation expressed in ray coordinates. Physical concept of diffraction consists in diffusion of the complex wave amplitude along curvilinear wave fronts through the common border of adjacent partial waves. This method provides perfect agreement with the exact solution in a number of canonical problems (diffraction by impedance cylinder and by perfectly conducting wedge) yielding wave field asymptotics not only in a narrow vicinity of the shadow boundary but also in deep shadow. However, the transversal diffusion method has not attained wide-spread development because of a number of unsolved basic and computational problems, such as ambiguity of dividing the entire wave field into partial components and lack of the boundary conditions to define a unique solution. Moreover, the coefficients of the parabolic equation written in curvilinear ray coordinates may have singularities which requires a special study of the solution behavior and creation of specific computational procedures.

These issues are considered in our work by an example of the classical problem: wave radiation by a continuous source nonuniformly distributed on the face of a wedge. Its particular case of smooth amplitude and phase variation along the wedge surface is physically attractive, for it is easy to predict the existence of just two different quasioptical waves: a primary wave radiated by the given source and a cylindrical diffraction wave emerging from the primary wave shadow boundary. At a first glance, it is perfectly suitable for solving with the transversal diffusion method. Nevertheless, our attempts to implement Malyuzhinets' algorithm [1] have failed. Although the primary wave front and shadow boundary can be uniquely determined by the phase distribution on the wedge surface and parabolic equations for all partial waves can be written down, the surface boundary conditions and smoothness of the resulting wave field are not sufficient to formulate a correct boundary value problem.

In order to find a physically justified approach we turned to the exact solution of this problem given in another paper by Malyuzhinets [2] in the form of Sommerfeld integral. Being rather complicated due to multiple Fourier transforms involved, this solution is difficult to use for immediate applications. However, by analyzing it we can answer some of the aforementioned questions, not solved in the transversal diffusion method.

It turns out that it is better to abandon the idea of describing the primary wave by a parabolic equation. Instead we construct it as an exact superposition of ordinary and evanescent plane waves caused by separate complex Fourier harmonics of the given distributed surface source. Than we truncate each plane wave component at its own shadow boundary defined by the condition of its phase synchronism with the cylindrical diffraction wave born at this boundary. In the case of a discrete set of plane waves, an amplitude jump of the primary wave arises at each partial shadow boundary which is compensated by the solution of the radial parabolic equation describing the cylindrical diffraction wave. In a continuous case, the multiple jumps merge in a blurred shadow boundary. The primary wave constructed in such a way does not satisfy the Helmholtz equation but its discrepancy has the phase of a cylindrical wave in the whole wedge exterior. This is just what allows us to build up an accurate approximate solution by adding some diffractive wave satisfying a nonhomogeneous radial parabolic equation with the right-hand side begotten by the primary wave discrepancy.

Probably this procedure is a true mathematical expression of the transversal diffusion concept described in early Malyuzhinets' works. Comparison of the proposed approach with the exact solution of the wedge radiation problem in a number of particular examples demonstrates its good accuracy and computational efficiency.

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Asymptotic Inverse Problem for Almost-Periodically Perturbed Quantum Harmonic Oscillator

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Consider the operator $A = -\frac{d^2}{dx^2} + x^2 + q(x)$ in $L^2(\mathbb{R})$ with the perturbation $q(x)$ from the class $\mathcal{B} = \{q : \|q'\|_\infty + \|Q\|_\infty < \infty\}$, where $Q(x) = \int_0^x q\, dt$ and $\|\cdot\|_\infty$ denotes the norm in $L^\infty(\mathbb{R})$. It was proved in [1] that the spectrum $\{\mu_n\}_{n=0}^\infty$ of A has the asymptotics $\mu_n = \mu_n^0 + \mu_n^1 + O(n^{-\frac{1}{3}})$, where $\mu_n^0 = 2n+1$ and $\mu_n^1 = O(n^{-\frac{1}{4}})$.

Let the perturbations be the sum of almost-periodic and decaying terms. We recover its almost-periodic part from the first asymptotic correction μ_n^1. Specifically, we consider the perturbations

$$q = p + r \in \mathcal{B}, \quad p \in B^1, p(-x) = p(x) \quad \text{and} \quad \|r\|_{B^1} \equiv \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} |r(x)|dx = 0, \tag{1}$$

where B^1 is the Besikovitch space of almost-periodic functions [2]. Here is the main result.

Theorem 1 *Let $\{\Delta\mu_n\}_{n=N}^\infty$ approximates the first asymptotic correction to the spectrum of A with perturbation (1): $\Delta\mu_n = \mu_n - \mu_n^0 + o(n^{-\frac{1}{4}})$. Then the spectrum and the Fourier coefficients of the almost-periodic part p are given by*

$$\lim_{L\to\infty} \frac{1}{x_L} \sum_{n=0}^{L-1} \Delta\mu_n G_\nu(x_n, x_L)(x_{n+1} - x_n) = \lim_{T\to\infty} \frac{1}{T} \int_0^T p(t) \cos \nu t\, dt, \quad \nu \geq 0, \tag{2}$$

where

$$x_n = \sqrt{\mu_n^0} = \sqrt{2n+1}, \quad G_\nu(x,T) = -x \int_x^T \frac{\varphi'_{\nu,T}(t)dt}{\sqrt{t^2 - x^2}}, \quad \varphi_{\nu,T}(t) = \eta(t-T)\cos\nu t$$

and $\eta \in C^2(\mathbb{R})$ is a smoothed step function such that $\eta(t) = 1$ for $x \in (-\infty, -1]$, $\eta(t) = 0$ for $x \in [0, \infty)$ and $\eta'(0) = 0$.

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Multilayer piezoelectric actuators

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In recent years, multilayer piezoelectric actuators have received wide acceptance as actuators and sensors.

In this paper piezoelectric actuators composed of piezoceramic layers, PVDF film, and elastic layers are studied. The actuators in the shape of cylindrical bars and cylindrical shells are composed of thin layers normal to their faces. The layers are arranged in definite repetitive sequence. They present a regular structure with some period t. This kind of the actuator is referred to as the piezoelectric stack and its electro mechanical coupling coefficient is high.

The theory of these actuators is constructed by two different methods. One of them is the method of homogenization with the use of the small period of layers structures as a small parameter [1] and the second one is the asymptotic method with the actuator's thickness as a small parameter [2]. In the theory of homogenization the period t compared to the dimensions of the overall domain (actuator's length) and assumed to be very small. Hence the characteristic functions of these highly heterogeneous media will rapidly vary within each interval t. This fact inspires the consideration of two different scales of dependencies for all quantities: one on the macroscopic or global level which indicates slow variations and the other on the microscopic or local level which describes rapid oscillations.

Calculations are done for different cylindrical stack. Analysis of electroelastic state for composite actuators demonstrates that soft layers normal the middle surface of the shell decrease actuators stiffness essentially.

For example the result of calculation of tangential displacement for the thin walled cylindrical stack is presented in Fig. 1.

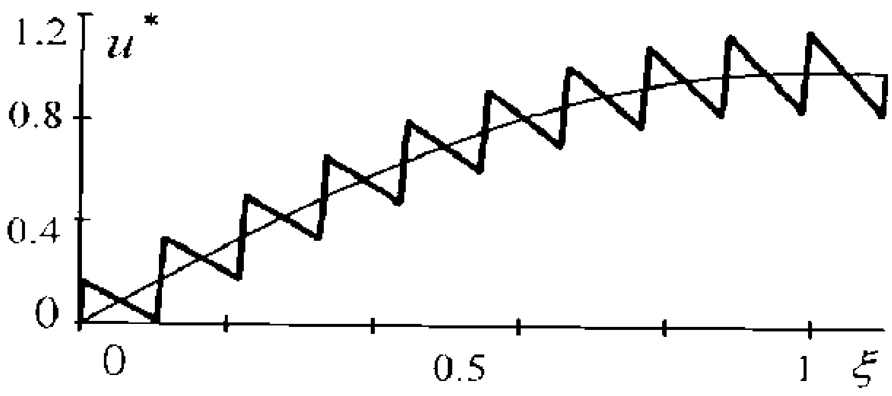

Figure : Distribution of the displacement u^* along the length of the shell.

Calculations were done for the thin walled cylindrical stack made of PZT-5 and the adhesive with Young's modulus equal $2.35 \cdot 10^9 \mathrm{N/m}^2$. The sizes are $r = 0.02$m, $l = 0.022$m, $t_p = 0.001$m, $t_a = 0.0001$m, where t_a is the adhesive thickness and t_p is the thickness of the piezoelectric layer. Figure 1 presents the distribution of the dimensionless displacement $u^* = u/u^0_{max}$ along the cylindrical stack The smooth line corresponds the solution with a small variability and the polygonal line is the complete displacement.

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RCS computation in near field

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Evaluation and reduction of Radar signatures of military targets are largely investigate by weapon industries since more than thirty years. Evaluation of Radar signatures allows enhancing the simulation process of electronic warfare. Reduction of Radar signatures, by using stealth techniques, assures a better chance of success for penetration missions in adverse territories. This domain of research deals with a value called Radar Cross Section (RCS) expressed in square meter, which is, most of the time, defined in far field condition.

Therefore, in some kind of scenario, it can be more appropriate to deal with the near field scattering characteristics of the targets. For example, if we consider a naval platform with a length of 120 meters, at a frequency of 10 GHz, according to Fraunhoffer condition the far field condition arises at distances greater or equal to 1920 kilometres. This limit is very far from the ranges of interest for operational scenario. Under this limit, the RCS of the target is dependant of the distance and needs to be compute versus this parameter.

In this communication the Stratton-Chu relations, simplified by Physical Optics (PO), are used to compute the electric field scattered by a target illuminated by a spherical electromagnetic wave. Then, a relation is proposed to express an equivalent to the RCS in near field condition, taking into account the distance between the transmitter/receiver and the phase centre of the target and the tension voltage at the transmitter.

Some remarkable near-field phenomena are isolated and described for several simple targets. For example, the specular monostatic RCS of a perfectly conducting disc of radius a exhibits a phenomenon of periodic arches with minima appearing at frequencies given by relation

$$F_n = nCD/a^2$$

where n is an integer, D the distance berween the target and the radar and C the light speed. One observes also that monostatic maxima RCS values are only dependent of the distance D and can be expressed by

$$\sigma_{max} = 4\pi D^2.$$

Near field RCS of more complex targets will be also presented. For example, other remarkable phenomena will be analyzed for dihedral, trihedral and convex targets whose multiple reflected fields are computed using Iterative Physical Optics algorithm. Computation results of near field RCS of greater and more complicated shapes will be given. Finally, in order to validate this near field RCS computation method, some comparisons will be presented with measurements performed in the RCS measurement facilities of CELAR.

Semiclassically-concentrated solutions for the multidimensional nonlinear Fokker–Plank–Kolmogorov Equation with Quadratic Potential

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Starting from the Maslov's complex germ theory, we propose a method for the semiclassical asymptotics construction for the Fokker–Plank–Kolmogorov equation (FPK) with quadratic potential and quadratic nonlocal nonlinearity

$$D\frac{\partial u(\vec{x},t)}{\partial t} = \langle \hat{\vec{\pi}}, T\hat{\vec{\pi}}\rangle u(\vec{x},t) + \langle \hat{\vec{\pi}}, \Big[V_{\vec{x}}(\vec{x},t)u(\vec{x},t) + \\ + \varkappa u(\vec{x},t)\int\limits_{\mathbb{R}^2} W_{\vec{x}}(\vec{x},\vec{y},t)u(\vec{y},t)d\vec{y}\Big]\rangle, \quad \vec{x}\in\mathbb{R}^{2n}. \tag{1}$$

Here D is a "small" parameter; T is a constant matrix, $V(\vec{x},t)$ and $W(\vec{x},\vec{y},t)$ are smooth functions with polynomial growth at $|\vec{x}|, |\vec{y}| \to \infty$, and

$$V_{\vec{x}}(\vec{x},t) = \frac{\partial V(\vec{x},t)}{\partial \vec{x}}, \qquad W_{\vec{x}}(\vec{x},\vec{y},t) = \frac{\partial W(\vec{x},\vec{y},t)}{\partial \vec{x}}, \quad \hat{\vec{\pi}} = D\frac{\partial}{\partial \vec{x}}.$$

An exact solution of the Cauchy problem for the FPK (1) is constructed in the class of semiclassically concentrated functions [1]. An illustrative example is considered for the 2D FPK eq. (1).

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Motion of complexes of laser solitons

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Reviewed are features of spatial soliton-like structures of light in wide-aperture lasers with saturable absorber described by generalized complex Ginzburg-Landau equation. We demonstrate found by numerical simulations bound states (complexes) of solitons with weak and strong coupling, and different types of their transverse motion which can be curvilinear even in transversely homogeneous schemes.

Limitations on the effective parameters of resonant and low loss negative refractive index metamaterials

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One of the approaches used to realise negative refractive index (nri) media consists in the use of Split-Ring Resonators (srr) and wire media introduced by Pendry et al. [1, 2] and Smith et al. [3]. The resonant metallo-dielectric srr and wire composite is considered homogeneous; its effective parameters are calculated using extraction procedures whereby the effective permittivity and permeability (ε, μ) are calculated from the complex reflection and transmission coefficients (r, t).

In a recent publication [4], we have shown the existence of a frequency band where effective parameters cannot be defined and a second one where effective parameters can be correctly defined, based on physical considerations. In [4], we proposed a numerical study of the well-known low-loss nri metamaterial introduced by K. Li et al. [5].

In this paper, we propose to further analyse the limitations on the definition of effective parameters for low-loss resonant metamaterials.

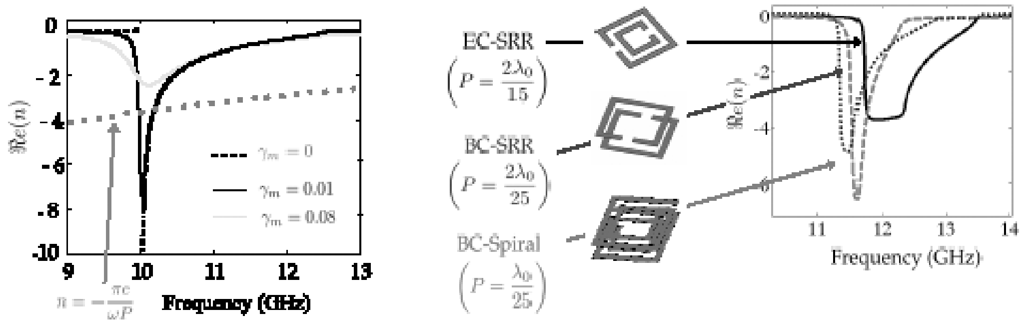
To this end, we first consider a dispersion model for a resonant and continuous media. A Lorentz model is considered for the permeability and a Drude model for the permittivity. The index of refraction is plotted [Fig. 1(a)] for different values of the dissipation factor γ_m. We then consider the influence of an intrinsic periodicity P of the composite.

We show that the limitation on the refractive index due to wave propagation in periodic media is given by the equation:

$$\mathrm{Re}(n) = m\pi e/\omega P, \qquad \text{where} \quad m \in \mathbb{Z} \tag{1}$$

and is depicted on Fig. 1 for a given value of the periodicity, $P = 3.3$ mm (a). We consider a negative refractive index media and only the fundamental mode, i.e. $m = -1$. The maximum value $n_{\max}$ reached by $\mathrm{Re}(n)$ is dependent on the parameters (P, γ_m). As shown on Fig. 1(a), the limitation on $n_{\max}$ is of greater relevance for low-loss media, i.e. for smaller values of γ_m.

Secondly, since the only route to obtain high refractive indices is to decrease P, we propose to study two different nri metamaterials having resonant frequencies much lower than the one [or edge-side coupled srr based metamaterial (ec-srr)] proposed by K. Li et al., namely a broad-side coupled srr (bc-srr) based metamaterial or a broad-side coupled spiral (bc-spiral) based metamaterial. It should



(a) (b)

Figure : (a) Real part of the index of refraction for continuous resonant media for different values of the dissipation constant γ_m. The limitation for periodic media is given by the dotted line ($P = 3.3$ mm). (b) Refractive index for different resonant nri metamaterials. The highest index is achieved by the bc-spiral metamaterial which is the one showing lower spatial dispersion.

be noted that the value of P is generally determined by the dimensions of the srr; to decrease P, one should be able to design srrs having smaller electrical dimensions at resonance.

The refractive index of these metamaterials have been numerically determined using two methods: (i) a finite element computation of $(r,\, t)$ followed by an effective parameter extraction procedure, and (ii) a phase velocity calculation from a dispersion diagram (calculated by a finite element eigenvalue determination). Fig. 1(b) shows that for metamaterials having a lower spatial dispersion (defined by the ratio P/λ_0), the values of nmax reached can indeed be much higher.

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Wave packet propagation by the Faber polynomial approximation in electrodynamics of passive media

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Maxwell's equations for propagation of electromagnetic waves in dispersive and absorptive (passive) media are represented in the form of the Schrödinger equation $i\partial\Psi/\partial t = H\Psi$, where H is a linear differential operator (Hamiltonian) acting on a multi-dimensional vector Ψ composed of the electromagnetic fields and auxiliary matter fields describing the medium response. In this representation, the initial value problem is solved by applying the fundamental solution $\exp(-itH)$ to the initial field configuration. The Faber polynomial approximation of the fundamental solution is used to develop a numerical algorithm for propagation of broad band wave packets in passive media. The action of the Hamiltonian on the wave function Ψ is approximated by the Fourier grid pseudospectral method. The algorithm is global in time, meaning that the entire propagation can be carried out in just a few time steps. A typical time step Δt_F is much larger than that in finite differencing schemes, $\Delta t_F \gg \|H\|^{-1}$. The accuracy and stability of the algorithm is analyzed. The Faber propagation method is compared with the Lanczos-Arnoldi propagation method with an example of scattering of broad band laser pulses on a periodic grating made of a dielectric whose dispersive properties are described by the Rocard-Powels-Debye model. The Faber algorithm is shown to be more efficient. The Courant limit for time stepping, $\Delta t_C \sim \|H\|^{-1}$, is exceeded at least in 3000 times in the Faber propagation scheme.

The talk is based on the joint works with A.G. Borisov [1], [2].

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Unusual laws of the refraction and reflection

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For description of the electromagnetic waves in periodical structures it is convenient to use Similarity of the electromagnetic wave and de Broglie wave at physic of the solid state.

One of the branches the periodic structures science is studying the laws of reflection and refraction on the boundary of such crystals and searching of the media with unusual behavior of the waves. For example in it is shown that ray incident from the free space on a medium, in which the group and phase velocities are counter directed, is refracted in the direction opposite to the direction of refraction in a conventional dielectric. Many articles about unusually laws reflection and refraction, mainly concentrating on media that propagates waves in two dimensions such as 2D-periodic media containing metal elements, ferrite films propagating magnetostatic waves, cholesteric liquid crystals and plasma.

Ray paths in a medium can be calculated basing on the isofrequency method. This method use surfaces on that terminated wave vectors for every possible direction normal waves at clamp frequency. The use of the isofrequency method led to the discovering of series of physical phenomena, which were unknown in classical optic. This phenomena is devoted in this revive.

Direction and quality of the phase velocity is defined by of the wave vector $\boldsymbol{\beta} = \omega \mathbf{v}/v^2$, where ω is circular frequency, so as a ray, energy flow and information are run in direction of the group velocity $v_g = \mathrm{grad}_\beta \omega$. They are orthogonal to the tangent isofrequency and directed along rise of the frequency. We call them *forward waves* if the angle θ between phase and group velocities is less then $\pi/2$ ($|\theta| < \pi/2$). The wave is named *backward* if $\pi/2 < |\theta| < \pi$. It is important to know phase velocities, wave vectors and amplitudes of the space harmonics because they design direction and intensive of diffraction maximum (in diffraction and aerial lattices for example). Backward waves and corresponding examples of the media and laws refraction are known once long ago. They are widely used in traveling wave tubes and in backward wave antennas. It is important to know them for investigation of interaction of electrons with the wave (Vavilov - Cherenkov effect). We are investigated same cubic cells for two artificial media and coinciding suitable isofrequencies in the zero zone in the second pass band. Media are consisted by ideal conducing cubic, frame and rod.

It is considered a refraction wave at an interface between two isotropic media and course of the ray through a plate by means of isofrequencies. Note that frequencies of the incident and reflected waves are equal, energy of the refracted wave travels away from the interface and projections of the wave vectors of the incident (β_i), reflected (β_{rl}) and refracted (β_r) waves are equal. From this is follows Snelli law: $\beta_i \sin\theta_i = \beta_{ra} \sin\theta_{ra}$, that is true for wave vectors, but do not for group velocity vectors (rays). There θ_i and θ_{ra} are angles of the incident and refraction correspondingly.

Above mentioned only the isotropic media whose isofrequencies have shape the sphere or the circle behave as isotropic only at the some of the frequency. In practice there is not exit isotropic media. They are behave as isotropic media in narrow pass band of frequency. We are investigated of the wave vectors and direction of the rays, i.e. of the group velocities for different forms of the isofrequencies.

Here it may be that the object and the imaginary disposed on the different sides of plate in spite of the forward wave. A contrary situation have to be if correspond isofrequency belong to the backward wave. In this case the ray deflects from the normal as in a usual dielectric and object and imaginary are disposed in the same side of the lower side of the plate.

Here it may be that for the little angles of the incident the wave does not propagate in to the artificial media because there are no isofrequencies on which wave vector can end. As far as the angle incident increases the wave begin to pass in to media and while increasing of the angle incident the angle refraction falls down at first and changes the sign after that.

Here it may be that two refracted rays corresponding to one incident ray (birefringence). As a result there appear two images for one object. The polarization of the both rays becomes the same in contrast to usually birefringence.

Unusual are laws reflection in artificial media. It is possible inequality angles incident and reflection in anisotropic media. In particular both wave vectors may direction to boundary or both wave vectors may direction out the boundary. Most interesting two case of reflection. At first of them exist two reflected waves when exist one incident wave. At second of them It is exist in the ferrite films between metal plates encased in to magnetic field. They have isofrequencies, which the direct line crosses one time if it is orthogonal to the interface. From this follows the fact that no reflected wave and no refracted wave at the presence of the incident wave. It may seem unbelievable. This effect is predicted in theory and discovered in experiment. The energy is taken away by means of the edge wave.

Electromagnetic waves in artificial periodic structures

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In the investigations of the electromagnetic waves in periodical structures (artificial crystals) it is easy to use zone theory, developed in physics of the solid body for de Broglie waves. Properties of the artificial crystals are described by the similarity of the de Broglie and electromagnetic waves that are given in table 1.

Table 1: Similarity of the de Broglie and electromagnetic wave

de Broglie waves	Electromagnetic waves
Electron energy, $E = z\omega$	Frequency, ω
Quasi impulse, p	Wave vector, $\boldsymbol{\beta}$
Electron velocity $v_e = \mathrm{grad}_p\omega$	Group velocity $v_{gr} = \mathrm{grad}_\beta\omega$, equal to the velocity of the energy transfer
Dispersion characteristic, $E(p)$	Dispersion characteristic, $\omega(\boldsymbol{\beta})$
Energy zone	Pass band
Energy gap or forbidden zone	Stop band
Isoenergetic surface	Isofrequency
Impurity or surface levels	Local oscillations

We have begun the investigation of such structures in the year of 1959 for electron devices, including structures with backward waves and including unusually law of refraction and reflection. But Smith et al. had described artificial crystal guided backward wave in one dimension on second pass band only in 2000. In the eighties artificial crystals was named photonic crystals.

Resonators, translation lines, filters, dividers of the signals and other radio devices can be done on the basis of the photonic crystals. The discontinuous are used in order to create local oscillations similarity impurity levels in solid body. The obstacle at a point allows to make the resonators, and obstacles along the line permits to make the transmission lines and other radio devices.

One of the branches of the photonic crystal science is studying the laws of reflection and refraction on the boundary of such crystals and searching of the media with unusual behavior of the waves. For example there is shown that ray incident from the free space on a medium, in which the group and phase velocities are counterdirected, is refracted in the direction opposite to the direction of refraction in a conventional dielectric. Since that time a lot of articles about unusually laws reflection and refraction, mainly concentrating on media that propagates waves in two dimensions such as 2D-periodic media, ferrite films propagating magnetostatic waves, and cholesteric liquid crystals have appeared. Plasma also exhibits unusual laws of reflection and refraction.

Present report is dedicated to account of the unusually laws refraction and reflection investigated by means of isofrequencies.

The Riemann problem admitting δ-, δ'-shocks and vacuum states

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A solution of the Riemann problem for the system of conservation laws

$$u_t + (u^2)_x = 0, \quad v_t + 2(uv)_x = 0, \quad w_t + 2(v^2 + uw)_x = 0 \tag{1}$$

with the initial data

$$(u(x,0), v(x,0), w(x,0)) = \begin{cases} (u_-, v_-, w_-), & x < 0, \\ (u_+, v_+, w_+), & x > 0, \end{cases} \tag{2}$$

is constructed by the vanishing viscosity method. This problem admits δ-*, δ'-shocks and vacuum states*. δ'-Shocks are new type singular solutions to systems of conservation laws first introduced in [1] (see also [2], [3]). It is a solution of the Riemann problem such that for $t > 0$ its second component v may contain Dirac measures, the third component w may contain a linear combination of Dirac measures and their derivatives, while the first component u has bounded variation.

To solve this problem, using the Hopf-Cole transformations, we construct solutions of a parabolic approximation of our system

$$u_{\varepsilon t} + (u_\varepsilon^2)_x = \varepsilon u_{\varepsilon\, xx}, \quad v_{\varepsilon t} + 2(u_\varepsilon v_\varepsilon)_x = \varepsilon v_{\varepsilon xx}, \quad w_{\varepsilon t} + 2(v_\varepsilon^2 + u_\varepsilon w_\varepsilon)_x = \varepsilon w_{\varepsilon xx} \tag{3}$$

with the initial data (2). Next, a solution of the Riemann problem (1), (2) is constructed as the vanishing viscosity limit of the solution of problem (3), (2) [4].

In fact, we describe the formation of the δ'-shocks and the vacuum states from a smooth solutions of the parabolic problem (3), (2).

These results as well as those of the paper [1]– [3] show that solutions of systems of conservation laws can develop not only Dirac measures (as in the case of δ-shocks) but their derivatives as well.

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A novel approach to the Helmholtz integral equation solution by Fourier series expansion for acoustic radiation and scattering problems

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Conventional methods of solution of the Helmholtz integral equation consist in discretization of a radiating/scattering boundary on multiple boundary elements, assumption of a smooth distribution of the velocity potential on every element and transforming of original problem to the solution of a system of linear algebraic equations. This method needs a large amount of time consuming calculations in the case when the radiating/scattering surface is large and the frequency of the acoustic field could not be considered as low or high and belongs to an "intermediate" frequency range. Analysis of these effects is practically impossible on a conventional PC due to the number of boundary elements, which are necessary to spread over the surface of the large-scale structure to guarantee the numerical accuracy of solution. In the present paper we propose to use a novel method of solution of the Helmholtz integral equation, which is based on expansion of the integrands in double Fourier series. The main difficulty of realization of the Fourier series approach is that the kernels of this equation do not satisfy to the Dirichlet's theorem and hence, could not be directly expanded into Fourier series. To overcome the abovementioned difficulty we represent the Helmholtz integral as sum of the integral with modified kernel, which satisfy the Dirichlet's theorem and so could be expanded in the Fourier series, and an additional integral in the vicinity of the point of singularity. This approach helps to substantially reduce the volume of calculations, take advantage of fast discrete Fourier transformations and achieve a substantial progress in solution of acoustic radiation and scattering problems. The typical example of scattering of an obliquely incident plane wave by a large-scale structure composed by a cylinder with two flat end caps is considered.

Transient waves produced by a periodically expanding sphere

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Here we construct the solutions of the initial value problem to inhomogeneous wave equation in terms of spherical harmonics for specific sources distributed on a pulsate spherical surface expanging with the velocity v. The sphere has the fixed radius at the initial moment of time and then is expanding. The time of sphere expansion is fixed. In the same time the sphere repeats its expansion. The number of expantion is finite. We consider three cases when the velocity of sphere expansion v is equal to the velocity of light, greater and less than the velocity of light.

The wavefunction and electromagnetic field components are constructed by using general expressions for the radial current obtained in [1] and [2] where the problem are reduced to the scalar one. The obtained equation is solved with the help of the Smirnov method of incomplete separation of variables [3]. Separating the polar-angle variable we represent the solution as the Legendre polynomial series whose coefficients are function of the radial and time variables. The analytical expressions for the above coefficients are obtained by means of the Riemann formula.

We investigate the space-time structure of formed waves and compare the results of the cases of different velocities of sphere expansion.

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On the homogenization of artificial lattices

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The Lorentz concept of the homogenization of 3D dipole lattices is based on a relation between the local and the averaged (mean) fields depending on the local polarization of the lattice unit cell and its geometry. The formula relating these two fields can be different but the result is the local permittivity (or permeability if scattering centers are magnetic) that is not affected by the shape of the macroscopic body formed by these dipoles, e.g. by the layer thickness. This concept was developed in many works and obviously refers the origin point (to which the local field is attributed) to a scattering center (lattice node). After obtaining in this way the effective permittivity or (and) permeability of the lattice referred to this point one usually attributes the same result to an arbitrary point of the lattice. This logical operation is usually thought to be strict due to the invariance of the dipole moment of any real particle with respect to its origin that allows attribute the dipole to any point of a lattice unit cell. It is, however, shown that this method is not strict if the averaged field is the same as the Ewald mean field. As a result, the Lorenz concept gives different results from those obtained in an accurate way. Te accurate approach is based on the quasi-static simplification of the dispersion equation of lattice that describes its eigenstates. The difference between two results is significant for resonant scatterers (within the resonant band, of course). For the resonant case the Lorentz approach is either not self-consistent with the Ewald concept of the mean field (and therefore does not allow fit the boundary conditions) or predicts the non-physical frequency behavior of one of two material parameters (the violation of Foster's theorem and the wrong sign of losses). If the lattice comprises the electric scatterers the wrong result is obtained for its permeability and vice versa (for magnetic dipoles and permittivity). Another commonly adopted approach to the homogenization of lattices is the transfer matrix method based on the transmission line (TL) model [1,2]. It also gives the same wrong results. These effects happen at low frequencies (much lower than the first lattice resonance) and cannot be explained in terms of hidden spatial dispersion (HDS) as was wrongly claimed in [3] and in all known works that followed [3]. There is a quasi-static effect ignored in the TL model as well as in the Lorentz model. We explain how to take it into account theoretically and correct the material parameters until the frequency of the scatterer individual resonance (where the HDS is obvious and local material parameters lose their physical meaning). This correction requires the dramatic modification of the known Pendry-Smith's procedure for extraction of material parameters through plane-wave reflectance-transmittance of finite layers [1-4].

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Bohr–Sommerfeld–Maslov conditions in Riemann surfaces, spectral graphs and pseudospectrum of Schrödinger operator with complex periodic potential

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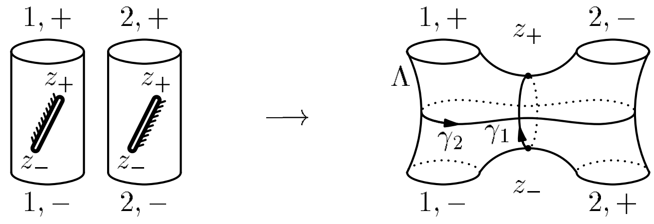
Statement of the problem. Let us consider differential expression $D = -h^2\frac{d^2}{dx^2} + iV(x)$ for an entire holomorphic function $V : \mathbb{R}/T\mathbb{Z} \to \mathbb{R}$, $T \in (0, +\infty)$. This differential expression generates an unbounded nonself-adjoint operator $\mathfrak{D}$ on $L_2(\S^1 = \mathbb{R}/T\mathbb{Z})$. We are interested in pseudospectrum and in asymptotic specturm of operator $\mathfrak{D}$ for $h \to 0+0$.

Pseudospectrum. Let us consider function space Φ with a scalar product $(\cdot,\cdot)$ and corresponding norm $\|\cdot\|$. Further, let there be an unbounded linear parameter-dependent operator $A = A(x,\varepsilon)\colon \Phi_0 \to \Phi$ ($\overline{\Phi_0} = \Phi$) for parameter $\varepsilon \in (0, +\infty)$. Point λ belongs to ε–pseudospectrum of operator $A = A(x,\varepsilon)$ if and only if there exists function $\varphi = \varphi(x,\varepsilon)$ in unit sphere of space Φ_0 for any fixed $\varepsilon \in (0, +\infty)$ such that asymptotic equality $\|A(x,\varepsilon)\varphi(x,\varepsilon) - \lambda\varphi(x,\varepsilon)\| = O(\varepsilon)$ holds for $\varepsilon \to 0+0$.

Theorem 1 *For a nonconstant holomorphic $V(x)$ and for any natural number N h^N–pseudospectrum of operator $\mathfrak{D}$ for periodic $V(x)$ is a half-string* $[0, +\infty) + i[\min V, \max V]$.

Spectrum. For spectrum we consider only case $V(x) = \cos x$. Asymptotic spectrum can be expressed in terms of integrals of holomorphic forms over cycles in Riemann surface Λ defined in $\mathbb{C}^2/T\mathbb{Z}$ by equation $p^2 + iV(x) = E$ ($p \in \mathbb{C}$, $x \in \mathbb{C}/T\mathbb{Z}$). This (noncompact) surface is homeomorphic (for $E \neq \pm i$) to sphere with four holes and is obtained by gluing of two cylinders $\mathbb{C}/T\mathbb{Z}$ along the segment connecting zeros of function $iV(x) - E$.



(1)

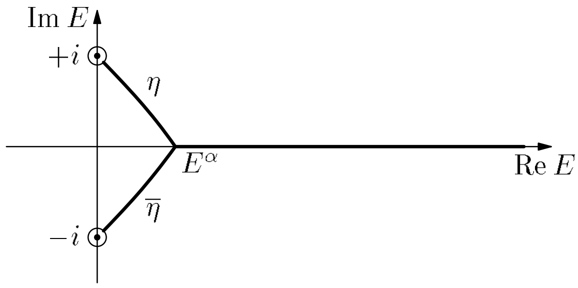
Theorem 2 *Let $E \in \mathbb{C}$ be such that there exists a cycle γ in Λ and natural m satisfying*

$$\frac{1}{2\pi h}\int_\gamma p\,dx = m + \frac{\mu}{2}\,. \tag{2}$$

Then there exists eigenvalue λ of operator $\mathfrak{D}$ such that $\lambda - E = O(h^2)$. Here $\mu = 0$ if cycle γ becomes contractible when some hole in Λ is glued and $\mu = 1$ otherwise.

Remark 1 *Conditions (2) look like Bohr–Sommerfeld–Maslov conditions of quantization of Lagrangian manifolds giving spectrum of operator $H\left(x, -ih\frac{\partial}{\partial x}\right)$. However, in theory of quasiclassical quantization quantization conditions for all cycles in Λ should be satisfied. In our case, it is sufficient to satisfy condition (2) for only one cycle.*

Remark 2 *In Λ there is three basis cycles: γ_1, γ_2 (as in figure (1)) and any cycle γ_3 around one hole in Λ. Conditions (2) can be rewritten: $\frac{1}{2\pi h}\int_{\gamma_j} p\,dx = m_j + \frac{1}{2}$ (for $j \in \{1,2\}$) and $\frac{1}{2\pi h}\int_{\gamma_3} p\,dx = m_3$. Thus, for $h \to 0$ eigenvalues are concentrated in $O(h^2)$–neighborhood of the following graph in $\mathbb{C}$:*



This graph is called asymptotic spectral graph of operator $\mathfrak{D}$.

Electromagnetic surface waves guided by the Isotropic plasma - chiral medium boundary

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Electrodynamics of new composite materials especially artificial chiral media for the microwave technical equipment has been the subject of many studies [1-4]. The chiral materials can be applied to the coverings absorbing electromagnetic radiation, for microwave antennas and so on. The propagation of electromagnetic waves in chiral media has been studied extensively in many works during past decades (see [4] and references therein). In particular, the processes of reflection, transmission and depolarization of plane electromagnetic waves on the dielectric - chiral medium plane boundary have been examined in [5]. Here we consider the surface electromagnetic waves guided by the isotropic plasma - chiral medium interface. It should be noted that the similar problem was discussed for low frequency surface waves guided by the dielectric - magnetoactive plasma boundary in the case when external static magnetic field to be aligned with the propagation direction of surface waves [6].

The expressions of electromagnetic fields of surface waves containing six components in chiral medium are obtained from Maxwell's equations and material equations for isotropic chiral medium. The transverse wave numbers of surface waves in chiral medium are complex. The dispersion equation of surface waves guided by isotropic plasma -chiral medium can be obtained from boundary conditions. This equation was examined by numerical methods. The parameters for chiral medium (magnetic and dielectric permeability, chiral parameter) have been chosen according to the experimental results submitted in [4]. It is established that frequency of surface waves has to be grater than plasma frequency. In the case when surface waves frequency is more grater than plasma frequency, the value of longitudinal propagation constant poorly depends on surface wave frequency and increases with growth of dielectric and magnetic permeability of chiral medium. If the chiral coverings are used for microwave antennas we take into account that the presence of surface waves can change the antenna compatibility especially in the case when the special-temporal synchronism between the external electromagnetic field and the high- frequency surface waves is realized.

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Synthesis of phase corrector sequence forming the desired wave beam structure

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A new procedure for synthesis of phase corrector sequence forming a desired paraxial field is proposed. The procedure includes a consecutive correction at each step of a wave beam phase up to a phase determined by a desirable field at the system output. It is proved, that each step increases the coupling coefficient of the received field with the desirable one. Such a procedure is used for designing of gyrotron quasi-optical systems and other reflector antennas. Phase correctors synthesis procedure is very useful tool for microwave wave beam operation. The Katzenelenbaum - Semenov synthesis is well-known one provides the high-efficient beam pattern conversion by means of two phase correctors [1]. Phase-correcting mirrors are developed for high-power microwaves in gyrotrons and transmission lines [2].

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Negative dispersion cylindrical surface plasmon-polaritons in spatially inhomogeneous dielectric permittivity media

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The investigation of surface electromagnetic waves properties and propagation in left-handed media is significant problem in area of surface physics for the last years. It has been investigated the existence and propagation of surface plasmon polaritons in the plane interface for media one of which has spatially inhomogeneous dielectric permittivity in comparison with homogeneous one [1].

In this work the cylindrical surface plasmon-polaritons in the case of air-metal boundary and metal having spatially inhomogeneous dielectric permittivity has been considered. The dispersion relation for cylindrical surface plasmon-polaritons has been obtained for the frequency with respect to the wave number of excitation, cylinder radius, dielectric permittivity taking into account its gradient from border surface. One of the considered examples of the suggested inhomogeneous media with $\mathrm{Re}[-\varepsilon(\omega)] \sim 1$ was the native metallike oxides of variable contents on some transition metals in the middle IR spectral range. The numerical calculations allow us to obtain the behavior of rising and falling branches of the dispersion relation and the positions of the maxima for particular values of chosen parameters.

Special attention was devoted to cylinder radius of about 50 nm and excitation wavelengths $\lambda = 10,6$ and 1 mcm. The numerical calculations of surface polariton attenuation lengths along the nanowires of suggested structure has been made.

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Complex singularities of fields on metamaterial wedges

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Recent theoretical and experimental studies have shown the possibility of creating novel types of microstructured materials, which effective response is characterized by magnetic permeability and dielectric permittivity with the negative real parts [1, 2]. These materials are often referred to as left-handed metamaterials or negative-index materials, and they possess a number of peculiar properties. One of the intriguing phenomena is negative refraction for waves at an interface between positive and negative-index materials [3], suggesting that the process of wave scattering on wedges made of negative-index materials should also be fundamentally modified.

We study wave scattering by 90° metamaterial wedges, as illustrated in Fig. 1. According to the general mathematical theory [4], there appears a singularity in the amplitudes of field components of the order $\rho^{\gamma-1}$, where ρ is the distance from the corner. However, we find that the type of singularity defined by the value of γ nontrivially depends on the material parameters. When the magnetic permeability is $\mu > -1/3$ or $\mu < -3$ for TE polarized waves, then γ is real, defining monotonic field decay in the vicinity of the corner, similar to the case of dielectric wedges.

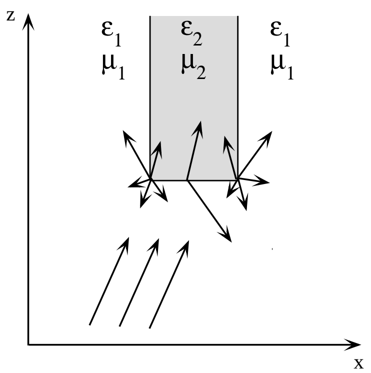

Figure 1: Geometry of the scattering problem.

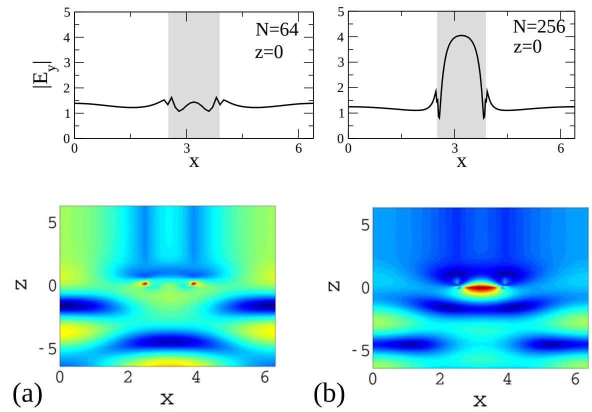

Figure 2: Scattering of a normally incident TE polarized plane wave calculated with (a) $N = 64$ and (b) $N = 256$ discretization points. Top: electric field amplitude at the interface; bottom: spatial field profile. Parameters are $\varepsilon_1 = \mu_1 = -1$, $\varepsilon_2 = -1.2$, and $\mu_2 = -1.5$.

We find that a fundamentally different field structure emerges for $-3 < \mu < -1/3$, when the singularity parameter γ becomes complex. In this case, the field amplitudes not only grow, but also oscillate with an increasing frequency as the corner is approached. However, infinitely fast oscillations cannot be supported in real metamaterial structures, and the overall scattering process should be sensitive to the effects of spatial dispersion. We perform finite-difference numerical modeling and find that the scattered field structure is indeed very sensitive to modification of dispersion for waves with large wavevector components, which depends on the number of discretization points (N) as illustrated in Fig. 2. We also show that, for losses above a certain threshold, the wave scattering becomes almost insensitive to dispersion at high spatial frequencies for sufficiently large N, similar to the case of dielectric wedges. Finally, we note that the same features appear for TM wave scattering on metal wedges with $-3 < \varepsilon < -1/3$. These results suggest new possibilities for probing the fine structure of metamaterials, and show that it is critically important to take into account spatial dispersion and losses in FDTD numerical simulations of wave scattering on metamaterial wedges [5].

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Spectral problem for the Hartree type operator

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The spectral problem is considered for the Hartree type operator

$$\hat{\mathcal{H}}_{\varkappa}(\Psi) = \mathcal{H}(\hat{z}) + \varkappa \int\limits_{\mathbb{R}^n} d\vec{y}\, \Psi^*(\vec{y},t) V(\hat{z},\hat{w}) \Psi(\vec{y},t).$$

Here pseudo-differential operators $\mathcal{H}(\hat{z})$ and $V(\hat{z},\hat{w})$ are functions of non-commutative operators

$$\hat{z} = (-i\hbar\nabla_x, \vec{x}), \qquad \hat{w} = (-i\hbar\nabla_y, \vec{y}), \qquad \vec{x}, \vec{y} \in \mathbb{R}^n.$$

The operators $\mathcal{H}(\hat{z})$ and $V(\hat{z},\hat{w})$ are determined by the smooth symbols $\mathcal{H}(z)$ and $V(z,w)$, respectively. A function Ψ^* is complex conjugate to Ψ, $\varkappa$ is a real parameter, $\hbar > 0$ is a small parameter.

Using the complex germ method [1, 2], we have obtained the spectral series related to a rest point of the Hamilton-Ehrenfest system accurate to $O(\hbar^{3/2})$ in the class of semiclassically concentrated functions [3]. In the linear case ($\varkappa = 0$) the expressions obtained correspond to the well-known results.

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Metawaveguides and metaresonators formed by arrays of resonating particles

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In this review presentation we will discuss our recent results on studies of electromagnetic properties of regular arrays formed by small resonant particles. Depending on the operational frequency, these particles can be complex-shaped metal inclusions (such as split rings) or plasmonic nanospheres.

We start from a study of propagation and radiation properties of a line array of electric or magnetic dipoles. These structures may be used as "metawaveguides", supporting guided modes in a narrow or broad frequency band depending on the particle response. In our earlier studies we considered the properties of axially oriented dipole line arrays. In this presentation, also transversely oriented dipole line is considered, which may be more practically interesting. Very recently, there has been much more activity and interest in this research area especially in the optical regime because of new potential applications in wave guiding structures with forward and backward wave propagation. A line of metal nanospheres or a line of small scatterers would be examples of such structures.

Denoting the distance between the dipoles d and the polarizability of the dipole by α, we can write a linera relation between the dipole moment of the particle and the local electric field: $p(0) = \alpha E_{\rm loc}$. Electromagnetic interaction between the dipoles is taken into account by the interaction constant. Because the structure is periodical, according to the Floquet theorem $p(nd) = e^{-jqnd}p(0)$. The propagation factor q in the lossless case is a real number. In the case when $q < k$ leaky waves exist and in the case when $q > k$ guided waves exist. If there are losses due to dissipation in particles or material losses, obviously the propagation factor q becomes a complex number.

The imaginary part of the eigenvalue equation is related to the energy conservation in arrays of lossless particles: The power received by the array is radiated back by the dipoles in array. For example, in the guided wave region, $q > k$, the imaginary part of the interaction constant is $k^3/6\pi\epsilon$. The equation for calculating the propagation factor along the structure reads

$$Re\left\{\frac{1}{\alpha}\right\} = \frac{1}{2\pi\epsilon d^3}\sum_{n=1}^{\infty}\left[\frac{k^2d^2\cos knd}{n} - \frac{kd\sin knd}{n^2} - \frac{\cos knd}{n^3}\right]\cos qnd.$$

For plasma spheres, for example silver spheres in the optical region and neglecting losses, the relative permittivity inside the sphere is $\epsilon_s = 1 - \omega_p^2/\omega^2$. Using the polarizability of a dielectric sphere, we obtain the solution for the propagation factor q along the line.

Very interesting properties are found near the plasma resonant range. There exist two guided waves, one is forward and the other one is backward. Calculating the power density distribution around the line we have found that in the close vicinity of the chain axis the energy propagates backward, while outside it propagates forward. Also at a certain frequency there exists a standing wave with no power transportation. One of the most interesting applications of this structure is in the realization of the perfect lens proposed recently by Pendry. The ideal operation requires rather flat regions on the dispersion curves. These optimal conditions can be achieved by choosing the distance and the diameter of the resonating spheres properly.

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Full wave analysis of imaging by the Pendry-Ramakrishna single-layer and stackable lenses

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The energy transfer through a single-layer Pendry lens [1] is considered. It is shown that a part of energy transmitted by evanescent waves is totally evoked by losses and causes a phase shift between the evanescent waves forming source and image fields. In the case of a lens made of a single-negative material whose permittivity has negative value, this phase shift depends on the transverse wave number of the evanescent wave. This results in destructive interference of the evanescent waves forming the image and, as a consequence, in deterioration of the image. The role of losses in the lens and detector is analyzed.

We also produce full wave analysis of a stackable lens which was suggested in [2] to improve subwavelength imaging. It is shown that (i) this lens, obtained by splitting a single-layer lens into a set of thinner layers, which form 1D photonic crystal, is a resonator cavity for the traveling Bloch waves which can not leave this PC resonator due to total internal reflection; (ii) the imaging is possible outside the band gaps only, there is also no imaging in the vicinity of the eigenstates of the resonator; (iii) the expected advantage is due to thinning of the layers which results in the shifting of both the band edge and the eigenstates toward higher values of the wavenumbers; (iv) nevertheless, a single-layer lens has the broadest working range of all the stackable lenses which have the same elementary layer thickness.

It is shown that a slight deviation of a permittivity of external medium to one side of the stackable lens leads to a rapid shift of resonance states because of the change of boundary conditions. This effect can, in principle, be used for measuring permittivity.

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Optical screw phase dislocations in space-time domain

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We present a new insight on the spatiotemporally localized waves - entangled spatio-temporal vortices (ESTVs), drawing on analogy with spatial screw phase dislocations [1, 2]. The singular point of zero amplitude in such structures is defined by a spatial coordinate and a certain time moment:

$$A_v = A_0(x + i\tau)\exp\left(-x^2 - \tau^2\right).$$

Our starting point is the linear equation for the slowly varying field amplitude

$$\frac{\partial A}{\partial z} + iD_\perp \frac{\partial^2 A}{\partial x^2} + iD_\tau \frac{\partial^2 A}{\partial \tau^2},$$

from which, taking into consideration $A(x,\tau,z) = |A(x,\tau,z)|\exp(i\phi)$, we obtain the following energy-transport equation:

$$\frac{\partial |A|^2}{\partial z} - 2D_\perp \frac{\partial}{\partial x}\left(|A|^2 \frac{\partial \phi}{\partial x}\right) - 2D_\tau \frac{\partial}{\partial \tau}\left(|A|^2 \frac{\partial \phi}{\partial \tau}\right) = 0.$$

Then, we investigated energy flow, and, at small distances $z << l_{dif}$, $z << l_{dis}$ in particular case $D_{\perp} = D_{\tau} = D$ it's integrals of motion look like:

$$z = \frac{\tau^2 + x^2}{D} \arcsin \sqrt{\frac{\tau^2}{\tau^2 + x^2}} + C_1, z = \frac{\tau^2 + x^2}{D} \arccos \sqrt{\frac{x^2}{\tau^2 + x^2}} + C_2.$$

We continue describing the methods of ESTVs generation by pulsed beams mixing. First, single entangled vortex can be obtained via superposition of two pulsed beams with $\pi/2$-phase difference. One of them has spatial Gaussian-Laguerre mode (1,0) and temporal Gaussian envelope, while another is a bipolar pulse:

$$A_1 = E_1 x A_G(x, \tau) + iE_2 \left(A_G(x, \tau + \tau_0) - A_G(x, \tau - \tau_0)\right).$$

Next, guided by the method which we suggested earlier for spatial vortices creation [3], we report on how several ESTV modes can be formed by superposition of two non-complanar phase modulated pulsed beams. We show that two crossing beams with time delay

$$A_2 = E_1 A_G(x, \tau + \tau_0, z) \exp(i\theta x/a) + E_2 A_G(x, \tau - \tau_0, z) \exp(-i\theta x/a)$$

give birth to the long train of ESTVs observed over a given time period at a fixed spatial coordinate x_v (see left part in Fig.):

$$x_v = \ln\left(E_2/E_1\right) a^2/(4x_0), \tau_{vn} = \pi(1 + 2n)T/(2\Omega), n = 0, \pm 1, \pm 2...$$

We also propose dynamic interferogram method for ESTVs registration. When superposed with a tilted reference wave, a time series of interferogram snapshots along the X-direction can be obtained. These intensity distribution patterns indicate that the number of maxima increases by one as the dislocation appears. In order to judge the ESTV location a dynamic interferogram should be recovered (see right part in Fig.).

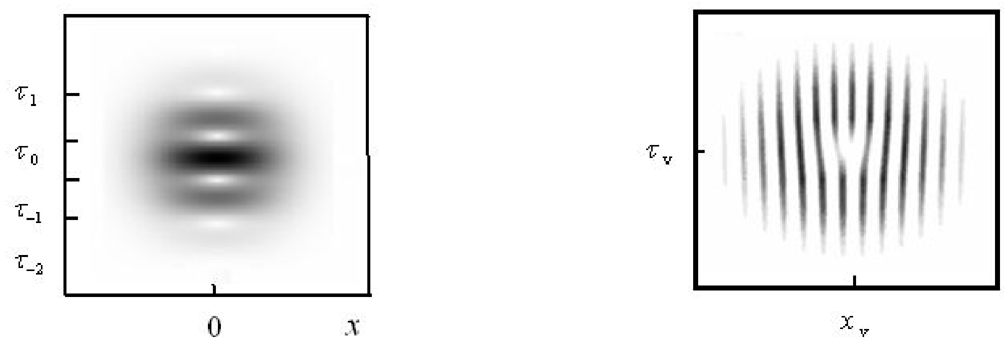

Figure : *Left:* Intensity pattern of ESTV periodical train obtained as a result of two tilted delaying beams superposition. *Right:* Dynamic interferogram of a single STV on the plain (x, τ).

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Method of generalized eikonal as a new approach to diffraction process description

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Recently a new method (method of generalized eikonal) for analytical solving of 2-D diffraction problems was introduced in [1 - 3]. Method of generalized eikonal allows one to obtain analytical solutions of diffraction problems for a wide range of scatterers, including scatterers with dimensional parameters. The analytical solution may be comparatively simply found after determining a conformal mapping of a complex number half-plane onto the region external to the scatterer.

As an example analytical solution for diffraction of electromagnetic wave on 2-dimentional perfectly conducting finite thickness half-plate based on this method has been made. The electric thickness of the half-plate is kh where $k = 2\pi/\lambda$, h is half-plate thickness, λ is the wavelength. Method of generalized eikonal allows finding solution on a special curve r_{d0}.

The solution has been compared with another analytical solution obtained by method of successive diffractions. At first, plane wave was scattered by wedge coincident with "lighted" vertice of the half-plate. Then the scattered signal was scattered by wedge coincident with "shadowed" vertice of the half-plate. And finally the signal scattered by "shadowed" vertice of the half-plate was again scattered by wedge coincident with "lighted" vertice of the half-plate. In this approach we get two "light-shadow" boundaries (Fig. 1). Signal discontinuities on these boundaries are eliminated by special functions that are well known in diffraction theory.

On the contrary, solution obtained by method of generalized eikonal may be interpreted as a direct rounding of half-plate face by the signal scattered from lighted edge (Fig. 2). In spite of the fact that the approaches are quite different comparison of the solutions has shown good agreement in range of dimensional parameter values $h/\lambda = 1/4 \div 1/8$.

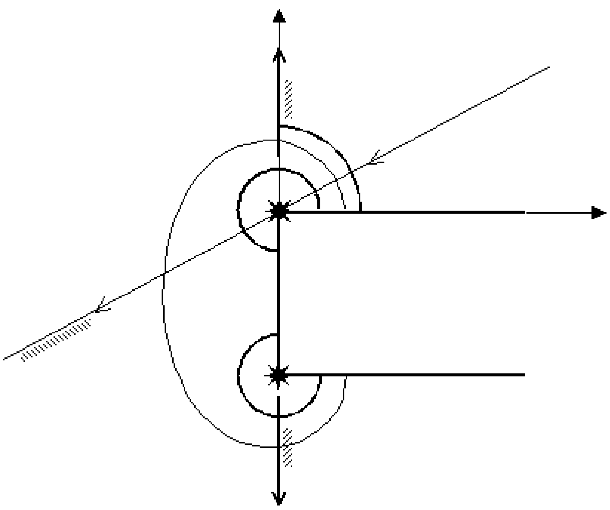

Figure : Method of successive diffractions

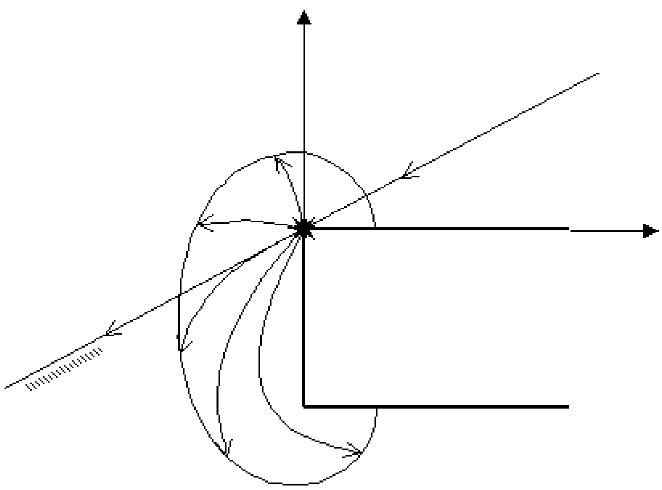

Figure : Method of generalized eikonal

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Electromagnetic waves generated by line exponentially decaying current pulses

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Two exponentially decaying pulses of opposite polarity provide a feasible first-order approximation of real source-current pulses in different sources of electromagnetic radiation. Thus, in the absence of significant non-linear effects, the space-time structure of EM waves emanated from such sources can be evaluated on the basis of a linear combination of exponentially decaying current pulse solutions.

The structure of electromagnetic waves generated by exponentially decaying pulses travelling along a finite straight line with subluminal and luminal speed is derived in the space-time domain by:

(i) expressing the electric and magnetic field vectors via the Whittaker-Bromwich potential, reducing the vector electrodynamics problem to the scalar one;

(ii) solving the scalar problem by application of the Smirnov method of incomplete separation of variables (the cylindrical coordinate system is used and the radial variable is separated by the Fourier-Bessel transform);

(iii) analysing and simplifying the resulting solution that in the general case has a semi-analytical quadrature form.

Due to the predefined shape of the source current pulse, the formulas describing the wave structure are much more concrete, illustrative and analysable than those obtained for the general case, so the results of this work can readily be used for optimisation of current pulses in EM radiators.

Increasing of complexity in nonlinear systems

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We consider stability and evolution of some biological, chemical and physical nonlinear systems in particular, genetic networks. These systems exhibits a very complicated behaviour, for example, time oscillations, chaos, wave propagation, pattern formation etc. To describe complexity of this behaviour, for such systems and their states some complexity characteristics can be introduced. We focus our attention on the problem of stability of such complicated regimes with respect to random fluctuations of an external medium. We show that a generic system with fixed parameters is unstable, i.e., the probability to support a complicated structures converges to zero as time $T \to \infty$. However, if we consider systems, which are capable to evolve (change their parameters), then such a system can be stable as $T \to \infty$. We show, for a general class of systems, and under some conditions on random fluctuations, that, in generic situation, this time evolution can be successful (the system structure conserves), only if the system complexity increases during evolution processes. This class of system contains, in particular, genetic and neural networks, reaction diffusion systems with polynomial non-linearities. Results for networks are in a good accordance with experiments. Mathematical methods use some new ideas, in particular, the theory of pfaffian nad noetherian functions.

Modelling isotropic artificial media with simultaneously negative permittivity and permeability

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A composite medium consisting of two sublattices of dielectric spherical particles of high permittivity and of different radii embedded in a dielectric matrix of smaller permittivity is considered. Results of analytical and numerical simulation of effective permittivity and magnetic permeability are compared. Attenuation of the electromagnetic wave outside the spheres is calculated. Influence of distribution of the sphere size and permittivity is estimated.

To realize a perfect lens one should use three-dimensional, homogeneous, isotropic left-handed material with simultaneously negative dielectric permittivity and magnetic permeability, so-called double negative (DNG) material. One of the ways to produce such a media is using a structure consisting of two arrays of purely dielectric spheres of different radii embedded into a host matrix [1], [2]. Considered media is isotropic due to isotropic structure of the constituent particles. The dielectric permittivity of the spheres is much higher than the permittivity of the host material. At the same time, the magnetic permeability of the both materials is the same and equal to unity. The diameters of the spheres are adjusted so that the lowest Mie resonance mode (TE_{011} in smaller spheres) and the second Mie resonance mode (TM_{011} in larger spheres) are excited at the same frequency. In this structure, the TE_{011} and TM_{011} resonant modes magnetic and electric dipole moments respectively, from which the DNG-response is obtained.

Due to very high permittivity of the spheres ($\varepsilon_r = 400$–1000), the interaction between the spheres is considered to be very small due to a high concentration of the electromagnetic field inside the spheres. We consider a diffraction of the plane wave on the spheres of the structure. The problem of scattering of the plane wave on a sphere is formulated in [3]. Using the same approach, the electric and magnetic field distribution inside and outside the spheres was described in closed form. The calculation shows a remarkable attenuation of the electromagnetic field outside the spheres and is sufficient to assume that there is no remarkable interaction between the spheres.

Numerical calculation gives the resonant frequencies of the spheres, which are close to their eigenfrequencies for chosen modes. Analytical expression for resonant frequency dependence on the sphere radius was obtained.

The numerical simulation of double-sphere structure was implemented. Extraction of the dielectric permittivity and the magnetic permeability is based on the results of calculation of propagation characteristics of effective transmission line[4]. The results obtained were compared with analytically calculated.

The dispersion of spherical particle size and permittivity caused by manufacturing inaccuracies influences the DNG-media properties. Analytical and numerical simulation shows that very high accuracy of sphere size and permittivity should be provided [4].

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Susceptibility and resonant scattering of intensive electromagnetic fields of waves by the weakly nonlinear dielectric layer a Kerr-like nonlinearity

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The layer with weakly Kerr-like dielectric nonlinearity is considered. The results of a numerical analysis of the diffraction problem of a plane wave on the weakly nonlinear object with positive and negative value of the susceptibility are shown. The effects: non-uniform shift of resonant frequency of the diffraction characteristics of a weakly nonlinear dielectric layer; itself the channeling and de-channeling of a field are found out.

The transverse inhomogeneous, isotropic, nonmagnetic, linearly polarized, weakly nonlinear (a Kerr-like dielectric nonlinearity) dielectric layer is considered. The algorithms for the solution of nonlinear diffraction problems (with use of approaches developed in [1-3]) and the results of a numerical analysis of the diffraction problem of a plane wave on the weakly nonlinear object with positive and negative value of the susceptibility are shown. The effects: non-uniform shift of resonant frequency of the diffraction characteristics of a weakly nonlinear dielectric layer; itself the channeling of a field - increase of the angle of the transparency of the nonlinear layer when growth of intensity of the field (at positive value of the susceptibility); de-channeling of a field (at negative value of the susceptibility) are found out. These effects are connected to resonant properties of a nonlinear dielectric layer and caused by increase at positive value of the susceptibility or reduction at negative value of the susceptibility of a variation of dielectric permeability of a layer (its nonlinear components) when increase of intensity of a field of excitation of researched nonlinear object, see [4, 5].

Let's note also, that the analysis of an investigated problem at strong intensive electromagnetic fields of excitation exceeding critical admissible values, within the limits of mathematical model considered by us, inevitably leads us to the account of redistribution of energy of a field of excitation between harmonics on the combined frequencies. Such physical problems can be approximately solved on the basis of the considered mathematical model of process which is reduced to the decision to the connected system of the boundary problems which have been written down for each considered combined frequencies.

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Natural oscillations of a rectangular acoustic volume with elastic walls

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The two-dimensional problem on search of frequencies and pulse forms of natural oscillations of rectangular volume of compressible irrotational fluid bounded by four thin elastic plates is considered. The acoustic pressure in fluid satisfy the Helmholtz equation. The bending and extensional vibrations of elastic walls are described by the Kirchhoff and Fylon equations accordingly. The adjacent plates are rigidly soldered.

The expression for fields of free acoustic oscillations in fluid and vibrations of plates is constructed. Dependence of natural frequencies on geometric sizes of the resonator is investigated. The approximate formulae for natural frequencies are constructed and numerically tested for case of small thicknesses of elastic walls. The frequencies under consideration are compared with frequencies of simplified model that does not take into account the extensional movements of elastic walls.

Controllable bandgap structures in an EIT periodic material

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The most interesting phenomenon in the control of phase coherence of multilevel atomic system is electromagnetically induced transparency (EIT). EIT results from both destructive quantum interference and quantum coherence in the atomic transition process from the ground states to the excited ones. It is such a quantum optical phenomenon that if one resonant laser beam propagates through a medium, the beam will get absorbed at once; but if two resonant laser beams instead propagate through the same medium, neither will be absorbed. Thus the opaque medium is turned into a transparent one. By making use of the EIT property that the optical behavior of an EIT vapor medium at probe frequencies can be dramatically modified by the intensity of an external control field, we propose a new scheme to controllably manipulate the photonic bandgap structures of a three-level EIT periodic material by an external field. The EIT periodic material under consideration is fabricated periodically from both regular dielectric medium (classical electromagnetic medium) and EIT atomic vapor (this can in principle be realized by filling the hollow holes of a host medium with a certain kind of three-level coherent vapor, e.g., alkali atomic vapor). It can be shown that the bandgap structures of such a periodic material, where a weak probe light is propagating, can be manipulated by both the frequency detuning and the field strength of the external control light. An ideal photonic bandgap structure can be achieved by choosing proper frequency detuning and intensity of the control field. Such a scenario may be believed to be useful for the development of new technologies in quantum optics.

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Modern classes of fractal antennas and fractal frequency selective surfaces and volumes

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This paper was prepared within the limits of perspectives of creating of bursting radar technologies in context of new interdisciplinary scientific direction "Fractal Radio Electronics and Fractal Radio Physics", which was suggested and developed on the base of theory of fractals and deterministic chaos in IRE RAS are considered [1]. To date, substantial positive results have been obtained in the substantiation and development of various methods for fractal filtering of weak multidimensional stochastic signals (see [1 - 7] and references herein). In particular, such a filtering can be directly performed by values of fractal dimension D, fractal cepstral, and fractal signatures, as well as by probabilistic distributions of the instantaneous fractal dimension of the analyzed sample [1].

The elaboration of the first standard dictionary of fractal features of target classes and permanent improvement of the algorithmic software are the main stages of the development and prototyping of a fractal nonparametric detector of radar signals designed as a dedicated processor. On the basis of these results, we can speak about the design of not only *fractal units (devices)* but also the entire *fractal radio system* [1, 3, 5, 7]. Such fractal radio systems contain (starting from the input) fractal antennas and digital fractal detectors and use fractal data-processing methods; future devices will use fractal methods for modulation and demodulation of radio signals [1]. In the design of fractal radio systems, it is expedient to use fractal antennas in this case.

The information on fractal antennas appeared in foreign publications in the late 1980s [1]. Currently, the number of foreign publications on the development of various designs of fractal antennas is rapidly increasing (see [1], Chapter 11, "Fractal Antennas and Methods for the Design of Such Antennas"). Constructions based on classical fractal sets, in particular, the Sierpinski universal triangular curve, are very often used in the analysis and synthesis of fractal antennas. The multiband operation of Sierpinski fractal monopoles and dipoles is demonstrated and discussed in [1, 3, 5, 6, 7].

Application of fractal structures allow to create media, witch exhibit complex dispersive and pass properties in a broad range of frequencies and capable modeling of 3D photon and magnon crystals, being a new media of information transmission (see in detail [4]).

Fractal antennas can be used for the design of fractal absorbing and reflecting coatings and fractal radar barriers. The results obtained allow us to extend the calculation technique based on numerical solution of hypersingular integral equations to a wide class of electromagnetic problems arising in the course of studying fractal magnon crystals [1, 4], fractal screens, and other fractal frequency-selective surfaces and volumes required for implementation of fractal radio systems. The fractal radio systems proposed in this paper open new prospects for modern radio electronics and can have wide possibilities for practical applications.

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Spatially dispersive FDTD method for numerical verification of sub-wavelength imaging by wire medium

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This paper presents the modelling of wire media (lattices of parallel metallic wires) using a Finite-Difference Time-Domain (FDTD) method [1]. The flat sub-wavelength lenses formed by wire media have been modelled in FDTD using effective medium approach. The Auxiliary Differential Equation (ADE) method [1] has been applied in FDTD model in order to take into account both the frequency dispersion effect and the spatial dispersion effect which was found recently in [2]. According to the authors' knowledge, this is the first time when the spatial dispersion effects are taken into account in the FDTD modelling.

The stability study [1] shows that the conventional Courant condition applies to the spatially dispersive FDTD method for modelling of wire medium when the standard second-order discretisation scheme is used. By applying the Generalized-Material-Independent Perfect Matched Layer (GMIPML) [3], infinite wire medium slabs can be modelled in FDTD along arbitrary directions and the convergence of the FDTD model can be significantly improved.

The developed formulas have been implemented in both two-dimensional (2-D) and three-dimensional (3-D) FDTD models. Simulation results have verified the canalization regime [4] and show excellent sub-wavelength imaging even for enough low plasma frequencies and small transverse dimensions of the wire media lens. Furthermore, an internal focusing phenomenon has also been found for the wire medium slab through the FDTD modelling.

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Light Propagation in Helicoidal Media with Large Periodicity

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Recently, the wave processes in liquid crystals are intensively studied. Both, the semiclassical and diffraction methods are used. The helicoidal liquid crystals are of special interest due to their application in the systems of mapping information.

We consider the propagation of electromagnetic waves in twisted liquid crystals with large scale one-dimensional periodicity. These structures are uniaxial ones with optical axis periodically rotating along definite direction. Solution of the Maxwell equation in such media presents ordinary and extraordinary waves. Due to variation of refraction index extraordinary wave with large angle of incidence experiences return back which is analogous to total internal reflection. As a result the wave guide channel and forbidden zones appear. The reason is the presence of turning points in the wave equation. The propagation of such rays are investigated. The vector nature of the problem is important as far as it leads to the solving of the set of differential equations. Far from the turning points the set is solved by the WKB method. The description of the vicinity of the turning point is based on the model equation method. We analyze in detail the case of critical ray corresponding to zero forbidden zone and the case of large incidence angle corresponding to the wide forbidden zone.

It is shown that for the critical ray mode transformation effect and elliptical polarization of waves take place. For large angles of incidence the percolation through the forbidden zone is analyzed. Amplitudes of reflected and transmitted waves are calculated and mode transformations are quantitatively analyzed. For critical ray three waves, two extraordinary and one ordinary, take part in the transformation process. In the case of wide forbidden zone there is a set of turning points where the waves are transformed in pairs.

Asymptotic solutions to 2-D wave equation with time-depending localized sources and their application to tsunami problems

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We consider the Cauchy problem for 2-D wave equation

$$\eta_{tt} - \nabla C^2(x)\nabla\eta = f(\frac{x}{\mu})g(t), \quad \eta|_{t=0} = 0, \quad \eta_t|_{t=0} = 0,$$

where $\mu << 1$, and $f(y)$ decays as $|y| \to \infty$, $g = 0$ for $t < 0$ and $g \to 0$ as $t \to \infty$. We present the asymptotic solution to this problem and show that its solution η approximately coincides with the solution u to the Cauchy problem

$$u_{tt} - \nabla C^2(x)\nabla u = 0, \quad u|_{t=0} = u^0_{f,g}(\frac{x}{\mu}), \quad u_t|_{t=0} = 0,$$

where $u^0_{f,g}(y)$ decays as $|y| \to \infty$. We discuss the application of these result to tsunami problems.

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The behavior near the focal points of solutions to the 2-D wave equation with fast decaying initial data

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We consider the Cauchy problem for 2-D wave equation

$$\eta_{tt} - \nabla C^2(x)\nabla\eta = 0, \quad \eta|_{t=0} = \eta^0(\frac{x}{\mu}), \quad \eta_t|_{t=0} = 0,$$

where $\mu << 1$, and $\eta^0(y)$ decays fast as $|y| \to \infty$. For $t > t_0 > 0$ the solution of these problem is localized in the strip of the fronts γ_t which are the closed curves on the plane (x_1, x_2). We study the behavior of the solution η in the neighborhood of the focal (or turning) points on γ_t. We present ad discuss the explicit formulas describing the solution dependence on the structure of the corresponding focal point and on the initial perturbation η^0.

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Temperature dependence of surface electromagnetic wave attenuation on nickel

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The temperature dependence of surface electromagnetic wave (SEW) attenuation coefficient on nickel has been experimentally studied in temperature range 20 – 120°C. The sample under consideration was flat polished super invar mirror with thermally evaporated nickel layer and thickness 150 nm. Two mechanically ruled diffractive gratings on the mirror surface with distance 10 mm between them were used as input and output gratings for transformation of bulk wave to SEW and vice versa. The SEW was excited on input grating by the beam of single frequency and single mode CO_2 laser ($\lambda = 10.6$ μm). On the basis of experimental results the value of nickel surface absorptivity temperature coefficient was obtained.

In experiment the phenomenon of full cancellation of the first diffraction order on grating was detected and profile of this grating was examined.

Influence of the input grating width and laser beam divergence on the properties of near surface bulk wave ("pressed" wave) [1], which excitation was observed together with SEW excitation, was discussed.

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Inverse problem of acoustic wave propagation in a structure with weak lateral inhomogeneity

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Acoustic wave equation with two spatial variables, x and z,

$$Au_{tt} = \operatorname{div}(B\operatorname{grad}(u)) \tag{1}$$

is considered in the half plane $z > 0$. The solution satisfies the boundary conditions

$$u|_{t<0} = 0, \quad u\,|_{t=0} = \delta(x - x_0). \tag{2}$$

Assuming that the function

$$u_z|_{z=0} = R(x, x_0, t) \tag{3}$$

is known, we seek the coefficients A and B in (1). It is done in the assupmption that their dependencies on the lateral variable x is much weaker than on the vertical variable z, which is typical for sound propagation in oceans etc. We quantify the assumption by

$$A = A(z, \varepsilon x) = A_0(z) + \varepsilon x A_1(z) + \ldots, \quad B = B(z, \varepsilon x) = B_0(z) + \varepsilon x B_1(z) + \ldots, \tag{4}$$

where $\varepsilon \ll 1$ is a small parameter.

The general BC (Boundary Control) method by M. I. Belishev [1] for solving (1)-(3) does not take into account the specific structure of coefficients (4).

We are concerned with approximate finding $A_0(z)$, $A_1(z)$, $B_0(z)$, $B_1(z)$, with error $O(\varepsilon^2)$).The further result is based on [2] where either A or B was constant. It was shown in [3] that A_0, A_1, B_0, B_1 can be expressed via four moments of the function $R(x, 0, t)$: $r^{(k)}(t) = \int\limits_{-\infty}^{\infty} x^k R(x, 0, t)\, dx$, k=0,1,2,3 and the constant $\frac{dB_1}{dz}|_{z=0} = C$. We demonstrate that A_i, B_i, $i = 0, 1$, are determined by the Fourier transform $\tilde{r}(\xi, t) = \int e^{ix\xi} R(x, 0, t)\, dx$ at two fixed values of ξ and C. We reduce the problem under consideration to successive solving of the standard inverse problem for a layered structure [1] and to solving linear Volterra equations.

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Conservative and dissipative fiber Bragg solitons

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We analyze solitons in fibers with Bragg gratings for the cases (i) fibers with nonlinear refractive index (conservative solitons) and (ii) fibers with saturable gain and absorption (dissipative solitons). Set of solitons is found, their bifurcations and stability are investigated. Special attention is paid to the development of the theory beyond the standard approach of slowly varying amplitudes.

Localized solutions of the wave equation: paraxial and exact theories

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We compare the results of the approximate time-harmonic parabolic-equation paraxial approach [1] with those found within an exact theory based on Bateman-type non-stationary solutions of the wave equation [2,3].

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Correctness of the diffraction problems in case of angle domains

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The subject of the study is the proof of correctness of the problem of diffraction a plane wave by a transparent wedge. The problem is scalar, the wave numbers inside the wedge and outside it are different. The method is analogous to the one of the book [1]. The original idea of the method is to seek a solution of the problem as the sum of two layer potentials, one for each face of the wedge. Let us consider a distribution $\alpha_1(x)$ belonging to the special functional class A. Then the distribution $u_1(x,y) = (\Delta + k_1^2)^{-1}\alpha_1(x)\delta(y)$ is a solution of the Helmgolz equation. We define $u(x,y)$ as the superposition of the distributions $u_1(x,y)$ and $u_2(x',y')$ supported by corresponding faces of the wedge. According to [1] the solution can be represented as the incident wave and so called 'outgoing solution'. To get the definition of the 'outgoing solution' let's use the complex wave number $k_1 e^{-i\varepsilon}$, $\varepsilon > 0$. The 'outgoing solution' is defined as the limit of layer potential when $\varepsilon \to 0$. After some manipulations the problem can be formulated for new unknown called the spectral function. The spectral function $\Sigma(\xi), Im\xi < 0$ is the Fourier transformation of layer potential densities. The decomposition of the spectral function is $\Sigma(\xi) = y(\xi) + X(\xi)$. The function $y(\xi)$ is meromorphic and contains poles of the spectral function. Then we consider the problem for regular part $X(\xi)$ of the spectral function. By applying the 'isomorphism theorem' we prove the existence of the solution of the problem for $X(\xi)$.

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Two-side distribution function and WKB solutions for ultrasound at stratified gas

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The system of hydrodynamic-type equations[1,2], derived by two-side distribution function for a stratified gas in gravity field is applied to a problem of ultrasound. The theory is based on BGK or Gross-Jakson kinetic equation, which solution is built by means of locally equilibrium distribution function with different local parameters for molecules moving "up"and "down". The background state and linearized version of the obtained system is studied and compared with the Navier-Stokes one at arbitrary Knudsen numbers. The problem of a generation by a moving plane in a rarefied gas is explored. The regime of the propagation of sound dramatically changes from a typically hydrodynamic to the free-molecular one. The WKB solutions for ultrasound in a stratified medium are constructed in explicit form, evaluated and plotted.

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Love waves effects on microtremor H/V spectra

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It was suggested by Japanese seismologists that the H/V spectral ratio of microtremors corresponds roughly to the amplification factor (resonance frequencies) of subsurface sediments. For a simple model of homogeneous layer above homogeneous half space it can be obtained from so called quater wave law:

$$f_{shear,k} = (2k+1)\frac{V_s}{4h}, \quad f_{long,k} = (2k+1)\frac{V_p}{4h}, \tag{1}$$

where V_p and V_s are velocities of P- and S-waves, respectively; h is thickness of subsurface sediments.

However there is the disagreement about wave content of the microtremor. Number of works assumed that microtremor consists mainly of body waves. On the other hand, it was indicated that the variation with frequency of the microtremor corresponds to that of fundamental Rayleigh mode. Also there is a point of view that the difference between H/V values of microtremors and those of fundamental Rayleigh mode may be caused by the influence of higher Rayleigh modes and Love waves in microtremors.

For the model of homogeneous layer above homogeneous half-space the numerical modeling of microtremors was realized. It was shown that H/V spectral ratio gives approximately amplification factor of the model for different types and locations of source. Dispersion characteristics of the synthetic were compared with theoretical dispersion characteristics for Love waves. It was concluded that H/V ratios of surface waves are in good agreement with those of microtremors.

Amplified self-induced transparency and few cycle pulses and dissipative solitons

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We analyze propagation of light pulses with very short duration in a fiber with two types of dopped atoms: active (with pumping) and passive (without pumping). We show that under the standard approximation of two-level schemes for both types of atoms, there are pulses with duration decreasing and amplitude increasing without limits during propagation. Taking into account the third level for passive atoms, we demonstrate stable dissipative solitons with extremely short duration and extremely large maximum amplitude.

Use of decomposition of the wave equations and pseudo-differential operators for the description nonparaxial beams and broad-band packages of waves

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The common approach to a getting of the equations for the description of wave beams and packages in a wide range of angles and frequencies is discussed by decomposition of the initial equations on the connected equations of normal waves with using of the pseudo-differential operators [1]. It is offered using of method of the moments for the description of pulses with a wide spectrum. The transition from the received equations to known is described. As example the application of the received equations it is theoretically investigated the broadband modulation instability on combinational frequencies in media having selffocusing cubic nonlinearity in a combination with selfdefocusing nonlinearity of higher order. It is supposed that dispersion of media does not suppose the modulation instability with only cubic nonlinearity. In this case if the power of beam is more critical power of self focusing there is a collapse of a wave field [2,3]. The nonlinearity of higher order limits a field in nonlinear focus and does to possible the development of instability on combinational frequencies. The increments of combinational frequencies in wave beams with such nonlinearity are found. They provide increase of combinational fields from noise up to meanings comparable to a field of a powerful beam. Such increase takes place in the event when the power of beam in some times exceeds critical one. The advanced theory can find application for an explanation of effect super broadening of a spectrum [4] having a place at selffocusing enough of short pulses of light and generation of high harmonics.

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Electromagnetic wave diffraction on the set of periodic shields located in the vicinity of plasma-like medium

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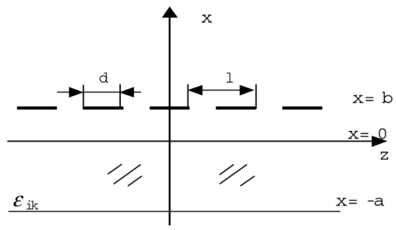
Problems and requirements of update physics demands studying of electromagnetic wave behavior in structures with anisotropic medium. Primarily there are problems of wave scattering in natural and artificial medium, including metamaterials, plasma and semiconductor electronics, optoelectronics [1]. Let's note that problem of electromagnetic wave diffraction on ribbon shields in the presence of anisotropic medium has been solved previously, however medium anisotropy was described by means of diagonal tensor or gyrotropy was taken into account (magnetoactive plasma), moreover in the majority works normal or oblique wave incidence was considered. However many problems require solving of vector task of electrodynamic.

In the work the problem of electromagnetic wave diffraction with arbitrary angle on set of ribbon perfect conducting shields is solved лентами ($x=b$; $-\infty<y<\infty$; $sl-d/2<z<sl+d/2$; $s=0,\pm1,\pm2$; l – structure period; d – width of ribbon shield). The medium is characterized by tensor of dielectric permittivity that describes its full anisotropy.

$$\varepsilon_{i,j}=\begin{pmatrix}\varepsilon_{xx} & \varepsilon_{xy} & \varepsilon_{xz}\\ \varepsilon_{yx} & \varepsilon_{yy} & \varepsilon_{yz}\\ \varepsilon_{zx} & \varepsilon_{zy} & \varepsilon_{zz}\end{pmatrix}$$



The incident field on structure

$$E_0=\mathrm{e}e^{ik(hr)}e^{-i\omega t},\ H_0=\mathrm{h}e^{ik(hr)}e^{-i\omega t},$$

$$e=(e_1,e_2,e_3);h=(h_1,h_2,h_3);\ k=\frac{\omega}{c};\ h=\{-\alpha,\beta,\gamma\},\ (\alpha>0,\alpha^2+\beta^2+\gamma^2=1)$$

Let us relate to the scattered field the Hertz vector which determines through a_{jn} - Fourier terms of linear current density on ribbon shields.

Taking into account that fields are subject to boundary conditions we obtain the set of functional equations for a_{jn}, that reduces to integral equations with logarithmic kernel in the following form

$$\int_{-\frac{d}{2}}^{\frac{d}{2}} j_m(\xi)\ln\frac{1}{2\left|\sin\frac{\pi}{2}(z-\xi)\right|}d\xi=1F$$

where $j_m(z)$ - linear current density.

Fourier coefficients j_{mn} of function $j_m(z)$ are determined by means of Crane formulae [2].

Integrating reduces the task to infinite set of linear algebraic equations.

In the case of narrow slots between shields the current Fourier coefficients are determined and consequently the fields are reflected (transmitted) from structure. The various particular cases are considered.

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Many particles problems for quantum layers

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In the present paper we study the Green function for the simplest cases of two noninteracting particles in two-dimensional infinite straight waveguide and three-dimensional infinite layer with Neumann boundary conditions. We use the convolution method based on known representation of the one-particle Green function.

Following this idea, adding of the new particle in waveguide is equivalent to increasing of dimension of the associated three-body problem. For instance the quantum problem of three noninteracting particles in two-dimensional waveguide (three-dimensional layer) is equivalent to the three-body problem in $\mathbf{R^6}$ (in $\mathbf{R^{12}}$). For two noninteracting particles in the domain of infinite planar waveguide with Neumann boundary conditions one has:

$$G(\overline{X},\overline{X}',E)\sim\frac{i\sqrt{E}}{8\pi}\frac{1}{\sqrt{R^2+(y_1-y_1')^2+(y_2-y_2')^2}}+$$
$$+\ln(R\sqrt{E})\left[\frac{2i}{\sqrt{E}d^3}(\delta(y_1-y_1')+\delta(y_2-y_2'))-\frac{2\pi}{d^2}\delta(y_1-y_1')\delta(y_2-y_2')-\frac{8}{\pi d^2}\right]+$$
$$+\frac{1}{\pi d}\left[\frac{1+\frac{i}{4}\delta(y_1-y_1')}{\sqrt{R^2+(y_2-y_2')^2}}+\frac{1+\frac{i}{4}\delta(y_2-y_2')}{\sqrt{R^2+(y_1-y_1')^2}}+\frac{1+\frac{i}{4}\delta(y_1-y_1')}{\sqrt{R^2+(y_2+y_2')^2}}+\frac{1+\frac{i}{4}\delta(y_2-y_2')}{\sqrt{R^2+(y_1+y_1')^2}}\right].$$

Here (x_i,y_i), $i=1,2$ is the Cartesian coordinates of the i-th particle, $0\le y_i\le d$, d is the width of waveguide, E is total energy of both particles ($\Im E = const > 0$), $R=\sqrt{(x_1-x_1')^2+(x_2-x_2')^2}$.

The analogous asymptotics was constructed for the case of three-dimensional layers.

One can choose as initial approximation the problem of one particle in the field (mean field) created by another one. So, the two-particles problem reduces to the problem of electron transport in the system of quantum waveguide in a weak transverse electric field. It's known that the resonant phenomena take place when electron passes through the the system of weakly coupled quantum waveguides. The external electric field allows us to control the resonance parameters, i.e. the electron transport. Here we deal with the two-qubit operation in the corresponding element of the quantum computer.

Particularly, we consider the electron in weak transverse electric field in the system of two waveguides coupled through small window of width $2a$. Using the method of matching of asymptotic expansions we founded the asymptotic (in small a) for quasi eigenvalue k_a for the Neumann boundary condition:

$$\sqrt{\pi^2 d_+^{-2}-k_a^2}=\tau_1\ln^{-1}a+\tau_2\ln^{-2}a+o(\ln^{-2}a),$$

where

$$\tau_1\sim\frac{\pi}{2d_+}+\frac{5\cdot 7\cdot 11}{48^2}\cdot\frac{F^2\hbar^2 d_+^2}{2m\pi^2},\qquad \tau_2\sim\tau_1\ln 2+0.5\pi\tau_1\left(g^+(0)+g^-(0)\right).$$

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6D resolvent to initial problems for Maxwell's equations

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To consider problems with parameters that change in time it needs to solve the time domain Maxwell's equations. If a medium is inhomogeneous than a problem becomes the initial and boundary value one. To solve such a problem it is convenient to transform the differential equations into integral equation, which contain initial and boundary conditions. This transformation can be done by virtue of the Green's function for the Maxwell equations. The free spatial Green's function for the time domain Maxwell equations in the 6D formulation has been obtained in [1]. Derivation of the integral equation and construction of the resolvent operator for it in a simple case is given in the presented paper.

The Maxwell equations in the general matrix form for 6D vector $\vec{F} = \begin{pmatrix} \vec{E} \\ \vec{B} \end{pmatrix}$ are

$$\begin{pmatrix} \frac{1}{v^2}\frac{\partial}{\partial t}\times\hat{1} & -\nabla\times\hat{1} \\ \nabla\times\hat{1} & \frac{\partial}{\partial t}\times\hat{1} \end{pmatrix}\begin{pmatrix} \vec{E} \\ \vec{B} \end{pmatrix} = -\mu_0\mu\begin{pmatrix} \frac{\partial}{\partial t} & rot \\ 0 & 0 \end{pmatrix}\begin{pmatrix} \vec{P} \\ \vec{M} \end{pmatrix} \tag{1}$$

where $\hat{1}$ is 3×3 identity matrix and $\vec{P} = \chi(\vec{P}_1 - \vec{P}_{ex}) + \vec{P}_{ex}$, $\vec{M} = \chi(\vec{M}_1 - \vec{M}_{ex}) + \vec{M}_{ex}$ .are vectors of electric and magnetic polarizations. Index 1 means that these vectors are considered in the depending on time region *V(t)*, which is given by the characteristic function χ , and the index *ex* denotes the outer region.

The Green's function for equation (1) is obtained and has the following form

$$G_0 = \frac{1}{4\pi}\begin{pmatrix} \mu\partial_t\times\hat{1}\frac{1}{R}\delta(T-\frac{R}{v}) + \mu\nabla^2\times\hat{1}\frac{v^2}{R}\theta(T-\frac{R}{v}) & \nabla\times\hat{1}\frac{1}{R}\delta(T-\frac{R}{v}) \\ -\mu\nabla\times\hat{1}\frac{1}{R}\delta(T-\frac{R}{v}) & \partial_t\times\hat{1}\frac{v^{-2}}{R}\delta(T-\frac{R}{v}) - \nabla^2\times\hat{1}\frac{1}{R}\theta(T-\frac{R}{v}) \end{pmatrix} \tag{2}$$

where $T = t - t'$, $R = |\mathbf{r} - \mathbf{r}'|$. This operator in accordance with equations (1) contains the time derivative of the first order only. By means of the Green's function (2) the general matrix equation (1) can be reduce to the Volterra integral equation of the second kind that is given by the convolution

$$\begin{pmatrix} \vec{E} \\ \vec{B} \end{pmatrix} = -\frac{\mu_0}{4\pi}\begin{pmatrix} \mu\partial_t Dl - v^2\mu\nabla\nabla Hs & rotDl \\ -\mu rotDl & v^{-2}\partial_t Dl - \nabla\nabla Hs \end{pmatrix}\otimes\begin{pmatrix} \partial_t\chi(\vec{P}_1 - \vec{P}_{ex}) + rot\chi(\vec{M}_1 - \vec{M}_{ex}) \\ 0 \end{pmatrix} \tag{3}$$

where $Dl = \frac{1}{R}\delta(T-\frac{R}{v}),\quad Hs = \frac{1}{R}\theta(T-\frac{R}{v})$.

The main features of a time electromagnetic process in non-stationary medium can be described by the key initial problem when the medium properties change by jump at some moment of time. In this case one can reckon the field before this moment to be known. When the medium parameters change from ε, μ to ε_1, μ_1, i.e. the constitutive equations are $\vec{P}_1 - \vec{P}_{ex} = \varepsilon_0(\varepsilon_1 - \varepsilon)\vec{E}$ and $\vec{M}_1 - \vec{M}_{ex} = \frac{1}{\mu_0}\left(\frac{1}{\mu_1} - \frac{1}{\mu}\right)\vec{B}$ then the solution to (3) can be found exactly by means of a resolvent $\hat{R}$. In this case the kernel of the resolvent operator has the form

$$\langle x|\hat{R}|x'\rangle = \frac{1}{4\pi}\begin{pmatrix} -\frac{\varepsilon-\varepsilon_1}{\varepsilon_1}\left(\partial_t^2 + v_1^2\Delta\right)\times\hat{1}\frac{1}{R}\delta(T-\frac{R}{v_1}) & -\frac{\mu_1-\mu}{\mu_1}\cdot v_1\partial_t\nabla\times\hat{1}\frac{1}{R}\delta(T-\frac{R}{v_1}) \\ \frac{\varepsilon-\varepsilon_1}{\varepsilon_1}\partial_t\nabla\times\hat{1}\frac{1}{R}\delta(T-\frac{R}{v_1}) & \frac{\mu_1-\mu}{\mu_1}\cdot v_1^2(\Delta-\nabla\nabla)\times\hat{1}\frac{1}{R}\delta(T-\frac{R}{v_1}) \end{pmatrix} \tag{4}$$

This operator gives a single compact expression describing the transformation both electric and magnetic fields. It is worth to note also that the resolvent operator contain the time derivative of the second order in spite of the first order in the equation (1) and in the Green's function.

The obtained resolvent can be used for calculation of the radiation of given sources or transformation of the electromagnetic field in a non-stationary medium.

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On the history of the parabolic-equation method

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62 years ago M. A. Leontovich first proposed an approach to high-frequency diffraction, based on separating out a rapidly oscillating exponent $u = e^{ikz}W$ and omitting the second derivative W_{zz} in the equation for W. This approach was applied [1] to a problem of point-source wave field propagation near an impedance plane (exact solution to this problem was known by that time). Two years later, M. A. Leontovich and V. A. Fock [2] (see also [3]) applied this approach to description of the penumbra for diffraction by a convex body. Besides, an application of the boundary-layer technique also starts from [2] which remains up to now one of fundamental papers in diffraction theory.

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Gaussian beam migration of multi-valued zero-offset data

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We develop a method for migration of zero-offset data based on Gaussian beam approach and explore it in situations where input data are multi-valued because of caustics of different geometrical structure and physical nature. To this end we consider the following two models. The first one can be called as "exploding sine" in inhomogeneous media where caustics appear mainly due to curvature of the reflector. The second one contains a low velocity lens immersed in a constant gradient velocity model. The results of imaging of the reflectors occurs to be different in those cases. For the first model the migration image of the reflector is clearly observed for extended range of parameters of the medium and geometrical characteristics of the reflector. The reflection coefficient is restored within 2% accuracy everywhere on the reflector in the true amplitude migration approach which we developed on the base of Gaussian beam method.

For the second model, we observe dimming of the migration image of the reflector underneath the low velocity lens and caustics appear due to the lens. The dimming phenomenon is the stronger the lower value of the velocity is inside the lens. The reflection coefficient cannot be restored on the part of the reflector underneath the lens with the help of the true amplitude approach (the relative error is around 70%).

However, our method of zero-offset migration with Gaussian beams proves to be efficient and does not meet additional theoretical and programming problems in application to relatively complex models with caustics.

Use of asymptotic solutions of differential approximations for improvement of the schemes

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Numerical solutions of fast-changing processes or problems containing discontinuities may possess non-physical perturbations in the vicinity of singularities in the form of oscillations or smoothing of the profile. These defects of a scheme may be studied employing the theory of differential equations.

The non-linear advection equation is considered. Simplest differential approximations correspond to the simplest schemes. More efficient schemes give rise to nonlinear differential approximations whose exact solutions are unknown. However, they may be considered as perturbed Burgers equation. It is shown that travelling wave asymptotic solutions of the perturbed Burgers equation may describe dispersive/dissipative features of the difference schemes used for simulations of the non-linear advection equation. Also these solutions may be employed to improve numerical modelling by choosing suitable artificial non-linear additions to the scheme.

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