DAYS ON DIFFRACTION 2012

INTERNATIONAL CONFERENCE

Saint Petersburg, May 28 – June 1, 2012

ABSTRACTS

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FOREWORD

"Days on Diffraction" is an annual conference taking place in St. Petersburg since 1968. The event is organized in May–June by St. Petersburg State University, St. Petersburg Department of Steklov Mathematical Institute and Euler International Mathematical Institute of the Russian Academy of Sciences.

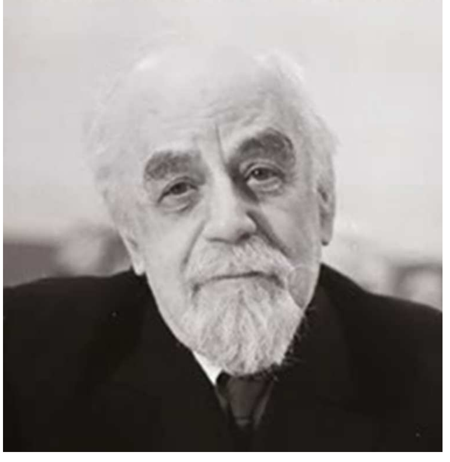
This booklet contains the abstracts of 211 talks to be presented at oral and poster sessions in 5 days of the Conference. Author index can be found on the last page.

The full texts of selected talks will be published in the Proceedings of the Conference. The texts in LaTeX format are due by June 20, 2012 to e-mail `diffraction12@gmail.com`. Format file and instructions can be found on the Seminar Web site at `http://www.imi.ras.ru/~dd/proceedings.php`. The final judgement on accepting the paper for the Proceedings will be made by the Organizing Committee following the recommendations of the referees.

We are as always pleased to see in St. Petersburg active researchers in the field of Diffraction Theory from all over the world.

Organizing Committee

The 125th anniversary of V.I. Smirnov's birthday



The 125th anniversary of Vladimir Ivanovich Smirnov's birthday occurs on 10 June 2012. He is known not only as a prominent researcher in the fields of mathematics and mechanics, but also as a reformer of the mathematical instruction at the Leningrad (now St. Petersburg) University. His interest in mathematics dates back to the years of his education at the St. Petersburg Gymnasium no. 2, where his schoolmates were A. A. Friedmann and J.D. Tamarkin. At the St. Petersburg University, they formed the nucleus of a group of brilliant disciples of V. A. Steklov. Many years later, Smirnov accomplished the creation of the famous Leningrad school of Mathematical Physics that had been initiated by Steklov. A significant role in the formation of this school belongs to the 5-volume "A Course of Higher Mathematics", for which Smirnov was awarded the Stalin prize for 1948 (later renamed as the State prize).

During his long career at the Leningrad University, Smirnov headed many departments. Some of them he organized himself, in particular, the renowned Departments of Mathematical Physics within the Faculties of Physics and of Mathematics and Mechanics. He headed both of them until the death in 1974, when L. D. Faddeev (famous for many discoveries in theoretical physics that include Faddeev equations of the quantum three-body problem, Faddeev–Popov ghosts etc.) and N.N. Uraltseva (well-known for her fundamental results concerning nonlinear partial differential equations) became his successors at the Physics and Mathematics and Mechanics faculties, respectively.

Smirnov's works are classics in various fields of mathematics. Best known are his contributions to complex analysis and the mathematical theory of diffraction. His method of functionally invariant solutions (it was developed in collaboration with S.L. Sobolev), that allowed to obtain explicit solutions for a number of important problems for the wave equation in domains with plane boundaries, is still developing further at present.

List of talks

Workshop on meta-materials

Diffraction of the electromagnetic wave on a grating located nearby semiconductor layer at the flippy of transmitters from a border

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Interest to various aspects of interaction of electromagnetic waves with plasma has considerably increased in the semiconductor the last years. It is connected to learning of fundamental properties of the semiconductor and possibilities of applications in the microwave and terahertz range in micro- and nanostructures [1, 2]. Diffraction of the plane electromagnetic wave on the tape ideally leading grid allocated near to semi-conductor plasma with mirroring of electrons from boundary has been considered.

Fields in the semiconductor it is found from the joint decision of Maxwell's equations and the kinetic equation with a self-consistent field in τ-approach (τ — relaxation time) within the limits of the theory of a small signal. The distribution function satisfies to a boundary condition $f(V_x, V_y, V_z)\left|_{z=-b}^{z=0} = f(V_x, V_y, -V_z)\right|_{z=-b}^{z=0}$, meaning mirroring of electrons at $z = 0$ and $z = -b$. The equilibrium distribution function of electrons is set in a type maxwell allocations drifted on a drift vector. Let on structure presented in a figure 1, falls H — the polarized wave, diffracting on it.

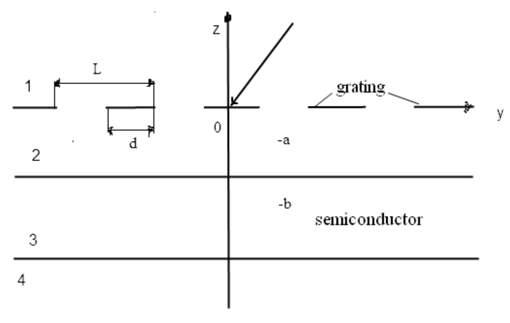

Fig. 1: The electrodynamic model.

The full field of diffraction satisfies to the equation of Helmholtz, a condition the Flock, to a condition on an edge, to a radiation condition. The falling, reflected and transited components of a diffraction field look like.

$$H_x = \left\{ \begin{array}{c} \exp\left[ik\left[y\sin\alpha - z\cos\alpha\right]\right] + \sum\limits_{-\infty}^{\infty} A_n \exp\left[i\left[k_n y + q_n z\right]\right], z \geq 0 \\ \sum\limits_{-\infty}^{\infty} B_n \exp\left[i\left[k_n y - q_n(z+b)\right]\right], z \leq -b \end{array} \right\} \tag{1}$$

where $k = \frac{2\pi}{\lambda}$, $k_n = k\sin\alpha + \frac{2\pi n}{l}$, $q_n = k\left[1 - \left(\frac{\chi\sin\alpha + n}{\chi}\right)^2\right]^{\frac{1}{2}}$, $\chi = \frac{kl}{2\pi}$, λ — length of incident wave. Complex amplitudes of a spectrum $\{A_n\}_{n=-\infty}^{\infty}$, $\{B_n\}_{n=-\infty}^{\infty}$ we find from the decision of the boundary task.

The equilibrium distribution function of electrons is set in the form of Maxwell allocation with offset on a drift vector. On section boundaries tangential components of fields save a continuity everywhere out of grid elements. On a metal surface $E = 0$. Having subordinated fields to boundary conditions on all boundaries of section, we will receive system of the functional equations. Following a method [3] it is reducible the received system of equations to the infinite system of the linear algebraic equations of the second sort.

$$A_m = \sum_{m=\infty}^{\infty} X_{mn} A_n + C_{m,}\ m = 0, \pm 1, \pm 2..... \tag{2}$$

where $\sum_{m=\infty}^{\infty} |X_{mn}|$ — convergent series.

The reduction of system (2) allows to find coefficients of passage and reflection in an analytical type. Complex penetration depth of a field in the semiconductor is defined by a ratio between oscillation frequency ω and ν frequency of collision. It is proportional to the Cherenkov multiplier $(\omega - q_n V_d)$ and consequently can change a sign that in turn leads to signal multiplication.

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On an extremal property of a 2D Gaussian beam under propagation

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It is known that the intensity shape of a 2D Gaussian beam with an arbitrary quadratic phase under propagation in the Fresnel diffraction zone is elliptical one. It may be described by two real parameters that are Gaussian widths in orthogonal directions. In this work it is shown that the ellipse square is minimal only for those planes where a defocusing phase vanishes. Examples of an initial Gaussian beam for some cases of defocusing and astigmatic phases and its propagation are presented.

Exact solutions of nonlinear Klein–Gordon equation

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Exact functionally invariant solutions of nonlinear Klein–Gordon (NKG) equation are obtained

$$\frac{\partial^2 U}{\partial x^2}+\frac{\partial^2 U}{\partial y^2}+\frac{\partial^2 U}{\partial z^2}-\frac{1}{v^2}\frac{\partial^2 U}{\partial t^2}=V'(U) \tag{1}$$

Here $V'(U)$ is nonlinear function U, prime denotes differentiation with respect to the argument. It is proposed to seek solution of the Eq. (1) in the form of the composite function $U=f[W(x,y,z,t)]$. It will be the solution of NKG equation provided that $W(x,y,z,t)$ simultaneously satisfies to two partial differential equations

$$\left(\frac{\partial W}{\partial x}\right)^2+\left(\frac{\partial W}{\partial y}\right)^2+\left(\frac{\partial W}{\partial z}\right)^2-\frac{1}{v^2}\left(\frac{\partial W}{\partial t}\right)^2=P(W), \tag{2}$$

$$\frac{\partial^2 W}{\partial x^2}+\frac{\partial^2 W}{\partial y^2}+\frac{\partial^2 W}{\partial z^2}-\frac{1}{v^2}\frac{\partial^2 W}{\partial t^2}=Q(W), \tag{3}$$

and $f(W)$ is the solution of the ordinary differential equation

$$P(W)\,f''+Q(W)\,f'=V'(f). \tag{4}$$

Here $P(W)$ and $Q(W)$ are arbitrary functions. For particular forms of functions $P(W)$ and $Q(W)$ exact analytical solutions of Eq. (2)–(4) are obtained. Function $W(x,y,z,t)$ is defined on the basis of concepts and methods of construction of functionally invariant solutions developed in the application to the propagation of electromagnetic waves [1] and elastic waves in the continuous media [2]. For the integration of sine-Gordon equation this method was used by authors of [3, 4]. Solutions of Eqs. (2), (3) are expressed by the arbitrary function $F(\alpha)$. Ansatz $\alpha(x,y,z,t)$ is found as the root of algebraic equation linear in (x, y, z, t). Coefficients of the algebraic equations are arbitrary functions $(l(\alpha),\, m(\alpha),\, n(\alpha),\, p(\alpha),\, g(\alpha))$, depending on α. Ansatz α is found nonuniquely. Examples of simple forms of α are given.

Proposed approach to the integration of NKG equation is illustrated by the examples of solutions for special choice of nonlinear function $V'(U)$. Cases of the $V(U)$ in the form of truncated exponential, Taylor, Fourier series and members having the form $\sinh nU$, $\cosh nU$ $(n = 1, 2, \ldots)$ are considered. In the frame of the proposed method considered cases are reduced to the calculation and inversion of algebraic, elliptic, ultraelliptic and Abel integrals. Conditions of elliptic, ultraelliptic and Abel integrals reduction are shown and physical meaning of these transformations applied to the deformation of nonlinear crystal lattice is discussed.

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Scanning periodic grating: diffraction problem and transmission problem

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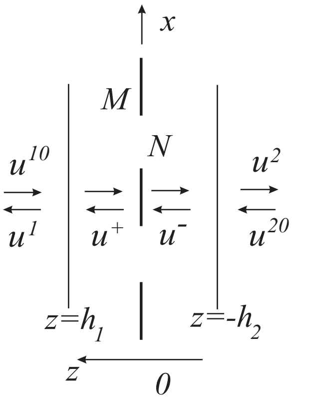
In work [1] it was proposed to use a moving thin conducting screen to get more information on inhomogeneity in a plane waveguide and in the open space in a case when possibilities to measure the characteristics of the reflected electromagnetic field are limited. The infinite periodic grating consisting of thin conducting bands embedded into a dielectric plate can be used as the similar device in the opened waveguide structures.

It is necessary to determine waves outgoing from the layer with grating in **the diffraction problem** when the field u^{10} and u^{20} is given by outside sources. Another problem is used for recalculation of the field characteristics through a plate with a grating. In the case of **the transmission problem** [2] it is necessary to determine the electromagnetic field u^2 and u^{20} on the one side from a layer if the field u^{10} and u^1 is known on its other side.



In the report it will be shown how to transform the summatorial functional equations connecting traces of the oriented waves on media interface into standard form of a kind

$$\sum_{n=-\infty}^{+\infty} c_n\, e^{idnx} = g(x), \quad x \in M, \qquad \sum_{n=-\infty}^{+\infty} c_n\, \gamma_n\, e^{idnx} = 0, \quad x \in N.$$

This dual equation is equivalent to regular infinite set of linear algebraic equations for the coefficients of decomposition of the electromagnetic field by Floquet harmonics. By this set of equations the algorithms are constructed for numerical solving of the diffraction problem and of the transmission problem.

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On wavelet transform in Minkowski space

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We discuss the construction of wavelet transform on the similtitude group of the Minkowski M_1^4 space. The representation in light-cone coordinates has the advantage of the diagonal form of hyperbolic rotations. The construction of the basic wavelets from the localized solutions of the wave equations, earlier proposed by M.V. Perel and co-authors, is considered in light-cone coordinates for the sake of the quantum field theory applications.

Modeling and analysis of resonance scattering and generation of waves on cubically polarisable nonlinear layered structures

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We investigate the scattering and generation of waves on an isotropic, nonmagnetic, linearly polarised nonlinear layered cubically polarisable dielectric structure (a layer in the free space filled with a nonlinear medium) excited by a packet of plane waves. The analysis is performed in the domain of resonance frequencies. We consider wave packets consisting of both strong electromagnetic fields at the excitation frequency of the nonlinear structure (which lead to the generation of waves) and weak fields at the multiple frequencies (which do not lead to the generation of harmonics but influence the scattering and generation process). We show that the propagation of electromagnetic waves in a nonlinear layer with cubic polarisability of the medium can be described by an infinite system of nonlinear boundary-value problems (BVPs). When considering particular nonlinear effects one can reduce this system to a finite number of problems and leave certain terms in the representation of the polarisation coefficients which characterise the physical problem under investigation [1]–[3]. The analysis of quasi-homogeneous electromagnetic fields of the nonlinear dielectric layered structure makes it possible to derive a condition of phase synchronism of waves. If the classical formulation of the problem is supplemented by this condition, we get a rigorous formulation in terms of a system of BVPs with respect to the components of the scattered and generated fields [2], [3]. This system is transformed to equivalent systems of nonlinear problems, namely to a system of one-dimensional

nonlinear Fredholm integral equations (IEs) of the second kind and to a system of nonlinear BVPs of Sturm–Liouville type. We obtain sufficient conditions for the existence and uniqueness of their solutions.

The algorithms of the numerical solution to the nonlinear problems are based on iterative procedures where the approximate solution is described in terms of solutions to linear problems with an induced nonlinear permittivity. The analytical continuation of these linear problems into the region of complex values of the frequency parameter allows us to formulate and analyse spectral problems. Namely, we look for eigenfrequencies and the corresponding eigenfields of the homogeneous linear problems with the induced nonlinear dielectric permittivity in the complex domain of the spectral parameter. We prove that the eigenfrequencies form a discrete countable set of points with the only possible accumulation point at infinity and lie on a complex two-sheeted Riemann surface.

The presented approach demonstrates that in the frequency domain the resonant scattering and generation properties of nonlinear structures are determined by the proximity of the excitation frequencies of the nonlinear structures to the complex eigenfrequencies of the corresponding homogeneous linear spectral problems with the induced nonlinear dielectric permittivity of the medium.

We present some results of calculations that describe important properties of the nonlinear permittivities of the layers as well as their scattering and generation characteristics.

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Weak fields at multiple frequencies and effects of scattering and generation of waves by nonlinear layered media

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We investigate the effects of weak fields at multiple frequencies on the scattering and generation of waves by an isotropic, nonmagnetic, linearly polarised (E-polarisation), layered, cubically polarisable, dielectric structure. In the domain of resonance frequencies we consider wave packets consisting of both strong electromagnetic fields at the excitation frequency of the nonlinear structure, leading to the generation of waves, and of weak fields at the multiple frequencies, which do not lead to the generation of harmonics but influence on the process of scattering and generation of waves by the nonlinear structure. The electromagnetic waves for a nonlinear layer with a cubic polarisability of the medium can be described by an infinite system of nonlinear boundary-value problems in the frequency domain. It is known [1] that in the study of particular nonlinear effects it is possible to restrict this system to a finite number of equations. If the classical formulation of the problem is supplemented by the condition of phase synchronism, we arrive at a self-consistent formulation of a system of boundary-value problems with respect to the components of the scattered and generated fields. It is known that this system is equivalent to a system of one-dimensional nonlinear Fredholm integral equations of the second kind.

The solution of the system of integral equations is approximated numerically by the help of the quadrature method. The numerical algorithms are based on iterative procedures which require the solution of linear systems in each step. In this way the approximate solution of the nonlinear problems is described by means of solutions of linear problems with an induced nonlinear permittivity.

In this work, results of calculations of characteristics of the scattered and generated fields of plane waves are presented, taking into account the influence of weak fields at multiple frequencies on the cubically polarisable layer. We restrict ourselves to the investigation of the third harmonic generated by layers with both negative as well as positive values of the cubic susceptibility of the medium.

Within the framework of the self-consistent system, we show the following. The variation of the imaginary parts of the permittivities of the layer at the multiple frequencies can take both positive and negative values along the height of the nonlinear layer. It is induced by the nonlinear part of the permittivities and is caused by the loss of energy in the nonlinear medium which is spent for the generation of the electromagnetic fields. The magnitudes of these variations are determined by the amplitude and phase characteristics of the fields which are scattered and generated by the nonlinear layer.

Layers with negative and positive values of the coefficient of cubic susceptibility of the nonlinear medium have fundamentally different scattering and generation properties. In the case of negative values of the susceptibility, a decanalisation of the electromagnetic field can be detected. The maximal portion of the total energy generated in the third harmonic is observed in the direction normal to the structure and nearly amounts to 4% of the total dissipated energy. For a layer with a positive value of the susceptibility an effect of energy canalisation is observed. Increasing intensities of the incidents fields lead to an increase of the angle of transparancy which increasingly deviates from the direction normal to the layer. In this case, the maximal portion of energy generated in the third harmonic is observed near the angle of transparency of the nonlinear layer. In the numerical experiments there have been reached intensities of the excitation field of the layer such that the relative portion of the total energy generated in the third harmonic is about 35%. In the paper the effect of weak fields at multiple frequencies on the scattering and generation of waves is investigated numerically. The results indicate a possibility of designing a frequency multiplier and nonlinear dielectrics with controllable permittivity.

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Elastic wave propagation through a layer with graded-index distribution of density

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Let the plane elastic harmonic wave of a type $u_0(x) = A_0 e^{-i\omega/v_1 \cdot x}$ falls on a layer of a thickness L, with density $\rho_2(x)$ and speed $v_2(x)$ from a homogenous isotropic medium. It is necessary to find the diffracted field, to be exact, the reflected, transited waves and a field in a layer $u_2(x)$.

The diffraction problem is reduced to an ordinary differential equation

$$(\rho_2(x)v_2^2(x)u_2'(x))' + \rho_2(x)\omega^2 u_2(x) = 0, \quad 0 < x < L,$$

with boundary conditions

$$\rho_2(0)v_2^2(0)u_2'(0) - i\omega\rho_1 v_1 u_2(0) = -i2\omega\rho_1 v_1 A_0, \quad \rho_2(L)v_2^2(L)u_2'(L) + i\omega\rho_3 v_3 u_2(L) = 0,$$

where $\rho_{1(3)}$ – densities and $v_{1(3)}$ – velocities of media out of a layer. The method of approximation of integral identities is applied to increase of accuracy of the grid solution to the received boundary problem.

The case when speed of elastic wave in a layer is constant is considered, and density continuously changes under the law $\rho_2(x) = \rho_0(1 + A|\sin^m Bx|)$. The diffraction problem on a layer with density $\rho_2(x) = \rho_0(1 + \sin Bx)$ is analytically solved. The characteristic minima and maxima of energy of reflected wave are selected. The case for a layer allocated in water is researched numerically. The fig. 1 shows dependence of the normalized propagate energy ε on an amount of the graded-index layers allocated closely to each other. It is visible that the magnification of number of layers leads to reduction ε for the characteristic frequencies.

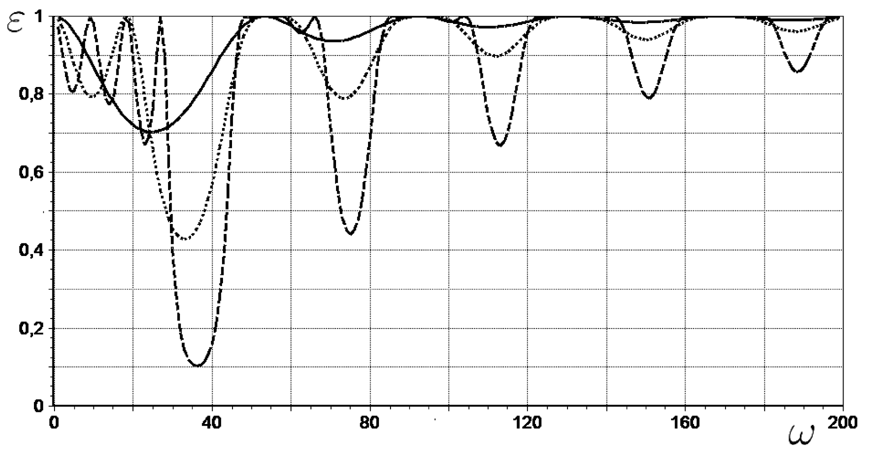

Fig. 1: Dependence of coefficient ε for a structure with thickness of each layer of L =100 m in water from cyclic frequency ω. The distribution of density of a layer $\rho_2(x) = \rho_0(1+\sin^2 \pi x/L)$. The solid line – one layer, a dotted line – two layers, a dashed line – 4 layers.

Graphics showing dependence of the propagate energy from remaining parameters A and m is resulted also. The accuracy of approximate solution for problems with $\rho_2(x) = \rho_0(1 + \sin \pi x/L)$ and homogeneously filled layer is estimated.

Analysis of the plane wave propagation in a thermoelastic half space with regard to a heat flux relaxation constant

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Cattaneo's modification of the classical Fourier's heat law includes a heat flux relaxation constant, a second time derivative of the temperature and also associates a wave propagation of the heat. This model, according to [1], is used to describe a variety of processes such as a the short pulse laser heating of metals, rapidly moving heat sources and travelling waves in systems with a moving phase transition edge.

Equations of coupled thermoelasticity based on the Cattaneo's heat law can be scaled to the classical ones [2] by putting the heat flux relaxation constant to zero. To investigate the problem of the hyperbolic thermoelasticity, the dispersion relations were obtained analytically (in parametric form) and compared to the corresponding results found using Fourier's heat law. The obtained curves coincide with each other by the angles of initial inclines. The analytical forms of horizontal and oblique asymptotes were also obtained and investigated both for classical and non-classical cases. It was discovered that there are two material constant inequalities that dramatically affect acoustic and thermal branch relative position.

The effect of a thermoelastic resonance were investigated in both cases; with regard to the heat flux relaxation and in the classical approach, when a relaxation time constant is excluded.

For periodic and impulse boundary conditions the system of hyperbolic differential equations were solved analytically. The behavior of obtained solutions was investigated depending on frequency and heat flux relaxation value.

In addition, an application field of the model was determined by comparing a wavelength with a lattice constant of appropriate material body. According to exponential solution decay by the coordinate, the wavelength value of non-classical effect appearance is approximately 160 nm for copper, when its lattice constant is about 0,36 nm. Such a ratio between these two values confirms the applicability of the continuum mechanics method.

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An outline of the Smirnov–Sobolev method

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In 1930-th, V.I. Smirnov and S.L. Sobolev presented an original method for solving non-stationary diffraction problems, which is nicely applicable for plane boundaries [1–3]. The talk is devoted to demonstration of the method with considering a simple example. To this end, 2D diffraction of a non-stationary plane wave by a screen is considered. Boundary conditions are either Dirichlet or Neumann. The resulting solution is "very explicit", it involves neither integrals, nor non-elementary functions.

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Diffraction a plane wave by a transparent wedge. Numerical approach

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We consider 2D-scalar problem of diffraction of a plane wave by a transparent wedge. We seek the solution as a sum of layer potentials (see also [1]) which allows to reduce the problem to a system of integral equations. This system is solved numerically. Numerical solution allows to obtain diffraction coefficients of the wave scattered by the vertex. As comparing to the earlier work [2], (where much simpler case was considered when both sides of the wedge were illuminated by a plane wave) we removed a number of limitations and consider a more general case. Nevertheless, some restrictions on the conditions still remain. In particular, the wave velocity in the inner area of the wedge has to be greater than the wave velocity outside. Other approaches to the similar problem are considered in the books [3], [4] and paper [5].

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Spectrum of the Laplace operator on a periodic graph

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In this work we study the spectrum $\sigma(\mathcal{L}_\Gamma)$ of the continuous Laplace operator $\mathcal{L}_\Gamma$ on a periodic graph Γ with the translation group $\mathbf{Z}^2$ and the spectrum $\sigma(\Delta_\Gamma)$ of the discrete Laplace operator Δ_Γ on this graph. It is known that the spectrum $\sigma(\mathcal{L}_\Gamma)$ is related to the spectrum $\sigma(\Delta_\Gamma)$. Particularly if $\sigma(\Delta_\Gamma) = [-1, 1]$ (recall that always $\sigma(\Delta_\Gamma) \subseteq [-1, 1]$), then $\sigma(\mathcal{L}_\Gamma) = [0, +\infty)$. If $\sigma(\Delta_\Gamma) \neq [-1, 1]$, then there is an infinite number of gaps in the spectrum $\sigma(\mathcal{L}_\Gamma)$. So the spectrum of the continuous Laplace operator on a periodic graph either fills the whole semi-axis $[0, +\infty)$ or has an infinite number of gaps. For the general case we prove that

1) the point 1 always is an upper point of the spectrum of the discrete operator,

2) if the fundamental domain of the graph consists of $n \geq 1$ vertices (i.e. the number of orbits is equal to n), then the number of bands in the spectrum of the discrete Laplace operator is not exceeded n.

Consider the periodic graph with the fundamental domain consisting of a single vertex (i.e. the number of orbits is equal to 1). The spectrum of the discrete Laplace operator on this graph consists of exactly one spectral band. This band coincides with the interval $[l, 1]$ for some $l \in [-1, 1)$. We find the necessary and sufficient conditions under which $l > -1$, i.e. the spectrum of the discrete Laplace operator doesn't coincide with the interval $[-1, 1]$ and then the spectrum of the corresponding continuous operator has an infinite number of gaps. For example, if the fundamental domain of the graph consists of exactly one vertex and two edges (as in the case of a square lattice), then the spectrum of the discrete operator Δ_Γ coincides with the interval $[-1, 1]$ and the spectrum of the continuous operator $\mathcal{L}_\Gamma$ fills the whole positive semi-axis.

Consider a graph with fundamental domain consisting of exactly two vertices (i.e. the number of orbits is equal to 2). The spectrum of the discrete Laplace operator on this graph consists of one or two spectral bands. We show that if the edges of the graph connect only vertices from the different orbits, then the spectrum is symmetric with respect to zero. We have found the necessary and sufficient conditions under which the spectrum of the discrete operator on this graph consists of exactly two spectral bands. If the graph contains edges connecting vertices in the same orbit, then the spectrum has the form $[l_1, l_2] \cup [l_3, 1]$ for some $-1 \le l_1 < l_2 \le l_3 < 1$. We have found

1) the necessary and sufficient conditions under which $l_1 = -1$,

2) the necessary and sufficient conditions under which $l_2 = l_3$.

The spectrum of the continuous operator has an infinite number of gaps if and only if at least one of the conditions 1), 2) is not fulfilled.

For a graph with three orbits the necessary and sufficient conditions, when the lower bound l of the spectrum of the discrete Laplace operator is greater than -1, are found.

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Asymptotics of the spectral bands in periodic waveguides with thin and short ligaments

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We address the couple of the spectral boundary-value problems in periodic waiveguides Π^ε depending on the small parameter $\varepsilon > 0$. The parameter ε describes the size of the ligaments connecting the periodicity cells. Taking the limit $\varepsilon \to 0$ the resulting domain is a disjoint union of the countably many translates of a cell ϖ. The spectral structure of the problem on Π^ε is in some sense a pertubation of the discrete spectrum of the corresponding problem on ϖ. On the other hand, according to the Gelfand transform, the spectrum of the problems on Π^ε has a band-gap structure. We calculate the asymptotics of the bands for various boundary-value problems.

For example we consider the spectral problem in the linear theory of water-waves in the cannel Π^ε, consisting of containers with water connected by small apertures. Also we consider the spectral problem for the linearised elasticity system with traction-free boundary conditions.

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Quasi-solutions of ill-posed problems for causal operators on regulated functions in time scales

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This work concerns the equation (E) : $Kz = u$, where u and z are elements of the space of all regulated functions $G(T, X)$ — for a Banach space X — where T is a bounded time scale [that is: T is a nonempty bounded closed subset of the real numbers (see : M.Bohner and A.Peterson "Dynamic equations on time scales, an introduction with applications" — Birkhauser, 2003) and a function is regulated if has only discontinuities of the first kind] and K is a linear continuous causal operator. As shown in B.C. Damasceno and L. Barbanti ("Cauchy–Stieltjes integral equation in time scales and the representation of operators on regulated functions" — Springer Proceedings of the International Conference on Differential & Difference Equations and Applications — Azores, july 2011 — in press) the operator K can be represented by a Cauchy–Stieltjes integral on time scales denoted here by (C-S-K).

The main results in the paper is about the solution of (E) when the problem of solving (E) is ill-posed (in the frame of general time scales T). A quasi-solution for (E) in T is defined in this work (see, for T as the set of all the real numbers, V.K. Ivanov : "On ill-posed problems" — Mat. Sb. 61 (1963)). We will be showing that under certain conditions on the kernel in (C-S-K) the quasi-solution exists and is unique. The Hyers–Ulam stability on the class of the quasi-solution is discussed too. Furthermore, we extend partially the results to a class of problems in which we take K in (E) as a nonlinear operator.

Balance of the hemisphere resonator gyroscope by the neural network algorithm

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The hemisphere resonator gyroscope (HRG) belongs to the class of Coriolis gyroscopes [1, 2], and its operating principle is based on inertial properties of elastic standing waves excited in an axys-symmetric shells due to the Bryan effect [3]. Due to the effect of Coriolis forces, the standing wave begins to precess both with respect to the resonator and in the inertial space. In the general case, due to oscillatory frequency splitting, it is not possible to excite standing waves in the unbalanced gyroscope. It was established that the frequency splitting caused by the fourth harmonic of the defect is proportional to the defect amplitude value, while the splitting for the first, the second, and the third harmonics is proportional to squared values of corresponding defects. Thus at balance, we must pay the main attention to the fourth harmonics of the mass distribution defect. Here, the initially excited standing wave is destroyed and the oscillatory process is represented as the sum of two harmonic oscillations with different frequencies. Two main types of the balancing are distinguished: the static balancing providing coincidence of the mass center with the symmetry axis; the dynamic balancing for removing the frequency splitting. The balancing follows the measurement of effects caused by mass distribution anomalies. Then, these defects are compensated by means of the point-wise or distributed mass removing with using mechanical, laser, ion-plasma, or chemical technologies. One of the main problems connected with the mechanical balancing is the localization of the segment of mass removing and the mass value to be eliminated. The number of such segments can be great enough because, for example, elimination of mass at one point on the resonators edge for removing the first harmonic of the defect can cause appearance of higher harmonics. To avoid this it is necessary to realize balancing at many points with different values of eliminated masses. Approaches considered earlier did not propose the simple universal algorithm for compensation of harmonic components of angular mass distribution. Some methods of artificial intelligence, such as neural network algorithms, genetic algorithms, simulated annealing technique, et al. could be effective for developing the universal algorithm of balance. In the report, for the first time, the neural network algorithm is proposed for balancing the imperfect resonator with inhomogeneous angular mass distribution density. It is based on minimization of the two-dimensional Hopfield network and is analogous to original algorithms for solving some problems of discrete optimization, such as the traveling salesman problem (TSP) etc. Since the energy of the Hopfield network tends to minimum and, according to the statement of the problem, we must minimize the goal function which is the linear combination of the syntax function of the problem, minimal (equal to zero) if and only if each mass is located at one point and at each point only one mass is located, and the quality function (mean squared error with respect to all harmonics), then the correspondence can be established between them. As the result, at the neural network output we obtain an allowable, in the general case quasi-optimal, solution. Both synchronous and asynchronous working regimes of the Hopfield network are considered. Unlike some earlier proposed iteration and analytical balancing techniques, the new approach, due to its flexibility, gives the possibility to realize different statements of the problem. A variant of combining the Hopfield neural network with simulated annealing technique is considered (the Boltzmann machine), which partially allows avoiding the local minima problem.

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Characterization of dynamical inverse data for two velocity dynamical system

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A dynamical system of the form

$$\rho u_{tt} - (\Gamma u_x)_x + A u_x + B u = 0, \qquad x > 0,\ \ 0 < t < T \tag{1}$$

$$u|_{t=0} = u_t|_{t=0} = 0, \qquad x \geq 0 \tag{2}$$

$$u|_{x=0} = f, \qquad 0 \leq t \leq T \tag{3}$$

is under consideration. Here ρ, Γ, A, and B are the C^∞-smooth 2×2-matrix-valued functions depending on $x \geq 0$; $\rho = \operatorname{diag}\{\rho_1(x), \rho_2(x)\}$ and $\Gamma = \operatorname{diag}\{\gamma_1(x), \gamma_2(x)\}$ are diagonal matrices with positive elements. The self-adjointness conditions $A^{\#}(x) = -A(x)$, $\frac{d}{dx}A(x) = B(x) - B^{\#}(x)$ are assumed to be held for all $x > 0$. For a boundary control $f \in L_2([0,T]; \mathbf{R}^2)$, the relevant generalized solution $u = u^f(x,t)$ is well defined [3].

In the system, there are two wave modes propagating with the velocities $c_i = \sqrt{\rho_i^{-1}(x)\gamma_i(x)}$ $(i = 1, 2)$ and interacting with each other. We deal with the case of *separated velocities* that is $0 < c_2(x) < c_1(x) \leq \text{const},\ \ x \geq 0$.

With the system (1)–(2) considered on the time interval $[0, 2T]$, one associates a *response operator* R^{2T}, which acts in $L_2([0,2T]; \mathbf{R}^2)$ on smooth controls f provided $f(0) = 0$ by

$$\left(R^{2T} f\right)(t) := \Gamma(0)\, u^f_x(0,t), \quad 0 \leq t \leq 2T.$$

Analyzing the forward problem (1)–(3), one gets the following facts [3].

Lemma 1. *The representation*

$$\left(R^{2T}f\right)(t) = -\nu\,\frac{df}{dt}(t) + \omega f(t) + \int_0^t r(t-s)f(s)\,ds, \quad 0<t<2T \tag{4}$$

is valid, where $\nu = diag\{\nu_1,\nu_2\}$ *and* ω *are the constant matrices,* $r = r(t)$, $0 \le t \le 2T$ *is a reply matrix-function. Moreover, the matrices satisfy the following conditions*

1. $\nu_1,\nu_2 > 0, \quad \omega_{12} = -\alpha\omega_{21}$ *with* $\alpha > 1$
2. $r \in C^\infty\left([0,2T];\mathbf{R}^2\right), \quad r^\sharp(t) = r(t),\ 0\le t\le 2T$
3. *an operator* C^T*, which acts in* $L_2\left([0,T];\mathbf{R}^2\right)$ *by*

$$\left(C^T f\right)(t) := \nu f(t) + \int_0^t \left[\frac{1}{2}\int_{|t-s|}^{2T-t-s} r(\eta)\,d\eta\right] f(s)\,ds, \quad 0\le t\le T$$

is positive definite i.e., $\left(C^T f, f\right)_{L_2([0,T];\mathbf{R}^2)} > 0$ *for all* $f \neq 0$.

A dynamical inverse problem is: given R^{2T}, to determine the coefficients ρ,Γ,A,B on a relevant interval $0\le x\le x(T)$ (see [1], [2]).

Theorem 1. *The conditions 1–3 are necessary and sufficient for the operator of the form (4) to be a response operator for some dynamical system (1)–(3) with separated velocities.*

For the given response operator, the solution of the inverse problem is not unique. We describe the character of this non-uniqueness and provide the procedure, which recovers the concrete ρ,Γ,A,B on $[0,x(T)]$.

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Calculations of transfer matrix by means of symmetric polynomials

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We consider a wave field, which is described in some plane-parallel layered medium by basic equation of the form $\frac{d}{dz}\Psi(z) = W\Psi(z)$. Here wave field is characterized by n-component column vector Ψ, and $W \equiv \|w_{ij}\|$ is so-called *wave matrix* of order n. The wave is taken to be incident on the multilayered medium boundary $z=0$ from an isotropic semi-infinite medium. Wave field $\Psi(z)$ on the layer depth z is related to those at $z=0$: $\Psi(z) = T\Psi(0)$. $T{\equiv}\|t_{ij}\|$ is known as the *transfer*, or *propagation*, or *scattering matrix.* If $w_{ij}{=}const$, then transfer matrix of layer is found by the theorem on integer powers of matrices [1, p. 274]:

$$T = \exp(Wz) = IS_0 + WS_1 + \ldots + W^{n-1}S_{n-1}, \quad S_g = \frac{z^g}{g!} + \sum_{l=0}^{g}(-1)^{n-l-1}\sigma_{n-l}\sum_{j=n}^{\infty}\frac{z^j B_{j+l-g-1}}{j!}, \tag{1}$$

where $g = 0, 1, \ldots, n-1$, I denotes the unit matrix, z is layer thickness, B_j is defined by the recurrence formulae

$$B_j = 0, \quad j = 0, 1, \ldots, n-2; \qquad B_{n-1} = 1; \qquad B_j = \sigma_1 B_{j-1} - \sigma_2 B_{j-2} + \ldots + (-1)^{n-1}\sigma_n B_{j-n}, \quad j \geq n, \tag{2}$$

and σ_j $(j = 1, \ldots, n)$ are coefficients of the matrix W characteristic equation: $\lambda^n - \sigma_1\lambda^{n-1} + \ldots + (-1)^{n-2}\sigma_n = 0$.

Quantity σ_j equals sum of C_n^j principal minors of j-th order $\det M$. On the other hand, it is an elementary symmetric polynomial in the eigenvalues λ_l of matrix W $(\sigma_1 = \lambda_1 + \lambda_2 + \ldots + \lambda_n$, $\sigma_2 = \sum_{l \neq j} \lambda_l \lambda_j$, $\ldots$, $\sigma_n = \lambda_1\lambda_2\ldots\lambda_n)$. Therefore the recurrence formulae (2) defines functions $B_j = B_j(\sigma_k)$ as *symmetric polynomials of order n*. The main advantage of calculating the matrix exponential by (1) and (2), in comparison with the Lagrange–Sylvester algorithm, is that the method (1–2) does not require the computation of matrix eigenvalues. When $n = 2, 3, 4$, summation of the series $\sum_{j=n}^{\infty} z^j B_{j+l-g-1}/(j!)$ in (1) may be carried out analytically.

Example 1. If $n = 2$, then $T = \exp(\sigma_1 z/2)\Big(\big((W - I\sigma_1/2)/s\big)\sin(sz) + I\cos(sz)\Big)$, where $s = \sqrt{\sigma_2 - \sigma_1^2/4}$.

Example 2. For polar Kerr effect wave and transfer matrices elements depends on propagation wave number in free space k_0, incidence angle θ_0, girotropy factor $\widetilde{g}$ and principal values ε_1, ε_2, ε_3 of the dielectric tensor:

$$\left.\begin{array}{l} w_{11} = w_{13} = w_{14} = 0, \\ w_{12} = ik_0(1 - \sin^2\theta_0/\varepsilon_3), \\ w_{21} = ik_0\varepsilon_1, \\ w_{22} = w_{24} = 0, \\ w_{23} = k_0\widetilde{g}, \\ w_{31} = w_{32} = w_{33} = 0, \\ w_{34} = ik_0, \\ w_{41} = -k_0\widetilde{g}, \\ w_{42} = w_{44} = 0, \\ w_{43} = ik_0(\varepsilon_2 - \sin^2\theta_0), \end{array}\right\} \Rightarrow \left\{\begin{array}{l} t_{11} = t_{22} = S_0 - k_0^2\left(1 - \sin^2\theta_0/\varepsilon_3\right)\varepsilon_1 S_2\,, \\ t_{12} = ik_0\big(1 - \sin^2\theta_0/\varepsilon_3\big)S_1 - ik_0^3\big(1 - \sin^2\theta_0/\varepsilon_3\big)\varepsilon_1 S_3\,, \\ t_{13} = -t_{42} = ik_0^2\left(1 - \sin^2\theta_0/\varepsilon_3\right)\widetilde{g}\,S_2\,, \\ t_{14} = -t_{32} = -k_0^3\left(1 - \sin^2\theta_0/\varepsilon_3\right)\widetilde{g}\,S_3\,, \\ t_{21} = ik_0\varepsilon_1 S_1 + ik_0^3\left[\left((\sin^2\theta_0)/\varepsilon_3 - 1\right)\varepsilon_1^2 - \widetilde{g}^2\right]S_3\,, \\ t_{23} = -t_{41} = k_0\widetilde{g}\,S_1 + k_0^3\widetilde{g}\left[\left((\varepsilon_1 + \varepsilon_3)/\varepsilon_3\right)\sin^2\theta_0 - \varepsilon_1 - \varepsilon_2\right]S_3\,, \\ t_{24} = -t_{31} = ik_0^2\widetilde{g}\,S_2\,, \\ t_{33} = t_{44} = S_0 - k_0^2\left(\varepsilon_2 - \sin^2\theta_0\right)S_2\,, \\ t_{34} = ik_0 S_1 - ik_0^3\left(\varepsilon_2 - \sin^2\theta_0\right)S_3\,, \\ t_{43} = ik_0\big(\varepsilon_2 - \sin^2\theta_0\big)S_1 + ik_0^3\big[(\sin^2\theta_0 - \varepsilon_3)/\varepsilon_3 - (\varepsilon_2 - \sin^2\theta_0)^2\big]S_3, \end{array}\right\} \tag{3}$$

$$S_0 = \frac{\beta_1^2\cos(z\beta_2) - \beta_2^2\cos(z\beta_1)}{\beta_1^2 - \beta_2^2}, \quad S_1 = \frac{\beta_1^3\sin(z\beta_2) - \beta_2^3\sin(z\beta_1)}{\beta_1\beta_2(\beta_1^2 - \beta_2^2)},$$

$$S_2 = \frac{\cos(z\beta_2) - \cos(z\beta_1)}{\beta_1^2 - \beta_2^2}, \quad S_3 = \frac{\beta_1\sin(z\beta_2) - \beta_2\sin(z\beta_1)}{\beta_1\beta_2(\beta_1^2 - \beta_2^2)},$$

$$\begin{Bmatrix}\beta_1 \\ \beta_2\end{Bmatrix} = k_0\left\{\frac{(\varepsilon_2 - \sin^2\theta_0) + (1 - \sin^2\theta_0/\varepsilon_3)\varepsilon_1}{2} \pm \left[\frac{\big((\varepsilon_2 - \sin^2\theta_0) - (1 - \sin^2\theta_0/\varepsilon_3)\varepsilon_1\big)^2}{4} + \left(1 - \frac{\sin^2\theta_0}{\varepsilon_3}\right)\widetilde{g}^2\right]^{1/2}\right\}^{1/2}.$$

Here i is imaginary unit. Comparison of speed of computations T by formulae (3) and (1)–(2) is made.

The propagation of waves in an inhomogeneous medium $(W = W(z))$ is studied by dividing the plate into a large enough number N of layers in each of which $W_i = const$, so that $T_i = \exp(W_i z_i)$ and $T = T_N T_{N-1} \ldots T_1$.

Exact analytical solutions for Bragg diffraction $(n = 2, 4)$ in multilayer periodic structures [2] are found.

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Nonlinear shapes of linear collider. Mathematical aspects

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The principle ideas for synthesizing of quasi-optical electron-positron colliders are formulated in [1]. To maximize the accelerating gradient, a proper structure might represent a coaxial composition of radial-corrugated disks. The profiles of these disks are required to be optimized. The aim is to synthesize a structure where minimum of RF energy would be accumulated within the accelerating channel provided that the accelerating gradient, the drive frequency and the paraxial channel inner diameter are fixed. First of all, an accelerating channel profile would provide the highest wave-electron coupling impedance. Hence, the RF field is composed of the electronsynchronous space harmonic. In other words, the field represents the transverse-magnetic π-mode mainly. The shape of metallic surface keeping this field is presented for the first time in [2]. It seems to be the optimal shape for the paraxial profile. To promote both the structure and maximum amplitude of the suitable field, the opposite sides of metal discs must contain irregularities of a flute type. The optimal parameters of these flutes are presented in [3]. Some results of investigations for the surface, which is optimal for paraxial domain, are presented in [4]. To diminish the electronsynchronous space harmonic within an input part of the collider and to avoid Wood anomaly at the same time, some new ideas are suggested in this paper. Two-dimensional external scattering problem is solved. The model is governed by the Helmholtz type equation for the azimuth component of a magnetic field. Boundary conditions reflect periodicity and symmetry of the structure, equality to zero of the tangential component of an electric field on a metal surface and radiation conditions in infinity. To obtain an approximate solution, the method of discrete sources is developed. To calculate effectively a structure with resonance properties, singular value decomposition technique is used. Various combinations of the boundary profiles are investigated. The results of numerical experiments are presented.

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The differential equations for generalized parametric Chebyshev polynomials

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We continued the investigation of the differential equations for polynomials defined by recurrent relations with periodic coefficients. We consider the case, when the diagonal elements of corresponding Jacobi matrix depend quadratically on parameter α. This parameter take values from the segment $[-1, 1]$. We show that only for values of parameter $\alpha = 0, \pm 1$ these polynomials became the elementary 3-symmetric Chebyshev polynomials connected with compound model of generalized oscillator that authors was discussed at the previous conference.

Reduction of the Ito functional integral associated with two-dimensional non-constant diffusion process with drift to the Wiener type path integral

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In the present paper we propose a method of reduction the functional integral with respect to the Ito measure for two-dimensional drift–diffusion process with variable coefficients to the functional integral over the Wiener measure. The Ito measure functional integral represents the fundamental solution of the backward Kolmogorov equation corresponding to the above drift–diffusion process. Such reduction allows finally to deal only with Wiener functional integrals for which there is a unique relationship with path integrals which can be either computed or effectively analyzed. The main advantages of our approach are following. First, we do not use the momentum representation for the original operator, and thus we do not face the problem of ordering of noncommuting operators, whose solution is not unique. Second, we use the discretization procedure only for Wiener functional integral for which the link with the path integral is known and thus discretization procedure is unambiguous. Third, we do not use the concept of approximating kernels, which is based on various asymptotics of fundamental solutions and the so-called heat kernel expansion. The latter concept being mathematically correct, seems to be not constructive and do not allow to efficiently compute or analyze the corresponding path integrals in the case of nonconstant metrics. The proposed constructions for two-dimensional drift–diffusion processes in general form are applied to the known models of stochastic volatility options and give in this case a number of new results.

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The Rayleigh law of scattering violation peculiarities

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Peculiarities of the Rayleigh wave scattering by a near-surface inhomogeneity connected with the Rayleigh law of scattering violation are presented.

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Interaction of solitons through radiation in optical fibers with randomly varying birefringence

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Propagation of solitons in optical fibers is studied taking into account the polarization mode dispersion (PMD) effect. We show that the soliton interaction caused by the radiation emitted by solitons due to the PMD disorder leads to soliton jitter, and we find its statistical properties. The theoretical predictions are justified by direct numerical simulations.

Acoustic diffusion by an elastic solid. The general Galbrun equations, experimental results and search of a mathematical understanding

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I present results I learnt from workshops with physicists and engineers on the study of the acoustic response of various elastic two dimensional solids (of different nature and shapes) immerged in water. The diedra of the solid creates privileged acoustic directions depending on the angle and on the Lame constants of the solid (number of these directions: 2, 3), permitting the identification of the nature and shape of the solid. Physicists and engineers model the problem by the Galbrun equations which are a linearization of the general equations of continuum mechanics. In some cases they involve products of distributions. Would it be possible to obtain to some extent the observed results from mathematical proofs? As far as I know these results are mainly published in the physics and engineering literature but presumably unknown to mathematicians.

Existence of irregular solutions for some nonlinear PDEs

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We study irregular solutions for systems of conservation laws in an appropriate functional space. To this end, in order to use a version of the Ascoli theorem, we construct a sequence of approximate solutions that provide a numerical scheme which is stable and consistent. We check that the scheme always gives back the known numerical solution obtained by all authors. We further show that classical techniques of calculus of variation permit to obtain existence results for other classes of equations.

Forward- and backward-in-time solutions to parabolic PDE with a small parameter

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The goal of the talk is to present a new approach to the construction of forward- and backward-in-time asymptotic solutions to the Cauchy problem in spite of the fact that the problem is parabolic. The main idea is very simple. It is well known that the solutions of any autonomous ODE (Newton laws) are invertible in time (if they exist). It is also well known that one can construct asymptotic (approximating) solutions to equations with a small parameter. For linear parabolic pseudodifferential equations (of Kolmogorov–Feller type, for example), such solutions have the form $\varphi \exp(-S/\varepsilon)$ almost everywhere. In the local-in-time study, we can assume that $S = S(x,t)$ and $\varphi = \varphi(x,t)$ are smooth functions and S is real valued (nonnegative), the last should be true in any case. Everybody knows that the algorithm for constructing the WKB-like solution representation $\varphi \exp(-S/\varepsilon)$ is based on the use of Hamilton system trajectories. If the symbol corresponding to the equation under study is time independent, then one can use the above remark about invertibility and see that an asymptotic solution in the WKB form is invertible in time till the time instant at which this solution loses its smoothness. Hamilton flows (trajectories in the phase space) are responsible for the dynamics of the Lagrangian manifold Λ^n_t ($p = \nabla S$ in the smooth case) corresponding to WKB-type solutions. The property of smoothness, i.e., invertibility on a time interval, is equivalent to the following: the projection map $\pi : R^n_p \times R^n_x \to R^n_x$ restricted on the Lagrangian manifold Λ^n_t is a diffeomorphism. If this is not true, then the construction that takes into account the fact that the functions S and φ can loose smoothness at the points, where the Jacobian of the projection map of the Lagrangian manifold on R^n_x is zero, was proposed by V.P. Maslov. But this approach is far from the characteristic method. However we can work in the frames of the characteristic method if we take into account that in nonsmooth a solution can also be written in a WKB-like form, but in a (weak) sense. Moreover, it is well known that the function S in this case can be defined as the logarithmic limit, $S = -\lim_{\varepsilon\to 0} \varepsilon \ln u_\varepsilon$, where u_ε is an exact or asymptotic solution to the initial parabolic PDE. The square of the amplitude function φ in the smooth case is a solution of continuity equation (this observation belongs to Madelung). One can define a generalized solution to the continuity equation in a discontinuous velocity field by the characteristic method. This allows one to use the characteristic method in a nonsmooth situation and hence to try to solve backward-in-time problems. A nonsmooth situation typical (of general position) is that the vector ∇S has a jump on a submanifold of codimension 1 and this jump is stable by the generation procedure (it appears because of intersection of characteristics projections). But it becomes unstable in the inverse time direction. So one cannot reconstruct the Lagrangian manifold completely backward in time, and consequently the functions S and φ can be reconstructed not in the entire space R^n_x but only in the domains that are projections of domains that are nonsingular w.r.t. the map $\pi\big|_{\Lambda^n_t}$ and are shifted backwards by the Hamilton flow. But we can do much more using Laplace method. Using this observation one can show that forward and backward in time asymptotic solutions are "equivalent" for momentum calculation.

Rapidly convergent quasi-periodic Green function throughout the spectrum — including Wood anomalies

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Our talk is dedicated to the scattering of a plane wave from a periodic grating Γ of equation $y = f(x)$ (f is a L-periodic smooth function) at Wood anomalies. This problem has been widely investigated from both mathematical and physical points of view because of the variety of associated physical applications (spectrometry, laser, cristallography...). Numerical simulations of this problem are usually carried out by means of integral methods ([3]). However, these methods only work away from Wood anomalies since they are based on the quasi-periodic Green function which is not defined at Wood Anomalies [1].

Using an idea of [2], we propose a family of new quasi-periodic Green functions valid at and around Wood anomalies: for any $j \in \mathbb{N}*$, we define

$$G_j^{\#}(x,y,x',y') := \frac{i}{4}\sum_{n\in\mathbb{Z}} e^{-i\alpha nL}\sum_{m=0}^{j} C_j^m(-1)^m H_0^1(k\sqrt{(x-x'+nL)^2+(y-y'-mhj)^2}), \tag{1}$$

where h is a real parameter satisfying $h > \max(f) - \min(f)$. The general term of Series (1) behaves like $n^{-j/2-1/2}$ if j is even and $n^{-j/2-1}$ if j is odd. Thus, $G_j^{\#}$ converges more rapidly as j increases. We construct associated well posed integral equations, that we discretize by means of a Nyström method (see [3]) to obtain an efficient numerical method that works for any frequency.

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Nonunique continuation for the Maxwell system

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We deal with the unique continuation property of the system

$$\operatorname{rot} u = \varepsilon u, \tag{1}$$

where u is a vector field and ε is a positive real matrix function in $\mathbb{R}^3$.

Theorem 1. *There exist a real nonzero vector field $u \in C_0^\infty(\mathbb{R}^3)$ and a real matrix function*

$$\varepsilon \in \bigcap_{\alpha<1} C^\alpha, \quad \varepsilon_0 I \le \varepsilon(x) \le \varepsilon_1 I$$

$(\varepsilon_0, \varepsilon_1 > 0)$ *satisfying the equality* (1).

Theorem 1 has the following consequences.

- A stationary Maxwell system

$$\operatorname{rot} e = i\omega\mu h, \quad \operatorname{rot} e = -i\omega\varepsilon h, \quad \operatorname{div}(\varepsilon e) = \operatorname{div}(\mu h) = 0 \tag{2}$$

  does not possess the unique continuation property in case of non-smooth coefficients (this property was proved for $\varepsilon, \mu \in C^1$ in [2]).

- A nonstationary Maxwell system
$$\partial_t(\varepsilon E) = \mathrm{rot}\, H, \quad \partial_t(\mu H) = -\mathrm{rot}\, E$$
can have a solution
$$E(t,x) = ie^{-it}u(x), \quad H(t,x) = e^{-it}u(x), \quad \mu(x) = \varepsilon(x),$$
having a fixed compact space support.
- The Maxwell operator in the whole space with periodic coefficients ε, μ can have an eigenvalue of infinite multiplicity. In case of scalar $\varepsilon, \mu \in C^2$ the spectrum of periodic Maxwell operator is absolutely continuous (see [3]).

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Beams dynamics and Lagrangian manifolds

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Under beams we understand here solutions of 3-D wave equation and its dispersive generalizations and Schrödinger type (paraxial optics) approximation localized in the neighborhood of the z-axis. There exist a huge physical and mathematical literature devoted to different problems connected with beams and their propagation. There are well known Gaussian beams, Bessel beams, Airy–Bessel beams etc. Our observation is that one can describe asymptotic solutions corresponding to various beams and their propagation using geometrical objects known as Lagrangian manifold in the phase space and the Maslov canonical operator.

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Seismic wave velocity and attenuation anisotropy analysis for media with one system of parallel fractures

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Oriented fracture systems cause seismic wave velocities and attenuation anisotropy. One system of parallel fractures in isotropic media is described by an effective model of transversally isotropic

(TI) medium with a symmetry axis normal to fracture planes. This model is based on linear slip (LS) boundary conditions for a medium consisting of identical thin layers [1–3]. LSTI model is valid for wave propagation if a wavelength λ much greater than a layer's thickness. Generalization of LSTI model on media with attenuation ($LSTI \to LS\widetilde{T}\widetilde{I}$) was performed and validated by T. Chichinina, for example [4, 5]. Fractures in the model with attenuation $LS\widetilde{T}\widetilde{I}$ are described by matrix of elasticity-attenuation with complex-valued elements including Lamé constants of the isotropic background and the complex-valued normal and tangential weaknesses [5].

Numerical modeling was performed to find optimal variants of estimating complex-valued weaknesses responsible for velocity-attenuation anisotropy in an attenuative linear-slip transversely isotropic (LSTI) model of a fractured medium. Velocities and attenuations of the three wave types (qP, qSV, SH) versus an angle between the symmetry axis and the wave normal were computed for media with different values of the ratio V_S/V_P in the isotropic background and varying values of the real and imaginary parts of the normal and tangential weaknesses. These data were analyzed and then inverted for complex-valued weaknesses. Inversion was made for various combinations of data (velocities, attenuations), wave types and angle intervals (0°–45°, 45°–90°). The results concerning the optimal ways for estimation the complex-valued weaknesses occurred to be as follows. At first, the values of the real parts neglecting the unknown imaginary parts of the weaknesses are to be determined from the velocity anisotropy, namely, for the normal weakness using qP-wave and for the tangential one using SH-wave. Then with the values of the real parts known, one determines the values of the imaginary parts. For the normal weakness, the qP-wave attenuation anisotropy is used, and for the tangential one SH-wave anisotropy. The velocity and attenuation anisotropy of qSV-wave is recommended to use only in the case of weak anisotropy to avoid cusps at the qSV-wave ray velocity surface. Joint inversion of qP- and SH-wave data can also yield good results.

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Early stage dynamics of the regulation network with microRNA

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We consider a mathematical model of a complex network of genes expression, regulating by the microRNA (miRNA), which is the non-coding short RNA molecule containing 10–20 nucleotides only. One of the regulatory mechanisms of gene expression extensively studied in the last years involves miRNA, see [1, 2]. These small regulatory molecules bind a recognition sequence of the target protein-coding matrix RNAs (mRNAs) and preclude them from translation. Recent studies showed that miRNAs participate in buffering of genetic noise in the regulatory systems and in the reduction of the phenotypic variability. It is realized now that this function of miRNA can be explained only via understanding its role in the regulatory network comprising miRNA interacting with transcription factors (TFs). One of the modern interaction models was introduced in [3, 4] in the form of the coupled o.d.e.:

$$\frac{dw}{dt} = k_w - g_w w;\ \frac{dq}{dt} = k_q w - g_q q;\ \frac{ds}{dt} = k_s(q) - g_s s;\ \frac{dr}{dt} = k_r(q) - g_r r;\ \frac{dp}{dt} = k_p(s) r - g_p p \qquad (1)$$

Here g_i are coefficients, w is the number (or concentration) of mRNA transcribed from the TF gene, q the number of TF molecules, s the number of miRNA, r the number of mRNA transcribed from the target gene and p the number of target proteins. Depending on a proper choice of the production functions $k_s(q), k_r(q)$ (the Hill functions) the model describes either coherent or incoherent network. In the *coherent* network both pathways from the TF to the target protein exhibit the same action (repression or activation of the target expression), while in the *incoherent* one the two pathways have opposite actions.

At the steady state approximation ($d/dt = 0$) the system (1) become algebraic, and it was solved in both deterministic and stochastic cases in [3]. The transcription rates of the miRNA gene and of the target gene were assumed to be the Hill functions of the number of TFs (q), while the translation rate of the target gene — a repressive Hill function of the number of miRNAs (s). Evidently, one and the same stationary solution to (1) can fit to solutions, which are completely different at the early stage of development, and some of them may not even have any biological sense. In the algebraic version of (1) there is no way to verify it.

We found the exact solutions to (1) under some biologically relevant restrictions and shown the detailed dynamics of both coherent and incoherent networks. In Fig. 1 the remarkable variations of concentration at early stage become invisible in the steady state solution, since both curves are indistinguishable for $t > 3000$.

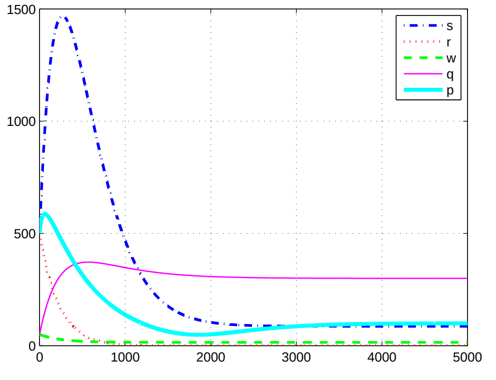

Fig. 1: The concentrations variation in the coherent network in the whole time interval. The steady state occurs at $t > 3000$ (the horizontal axis).

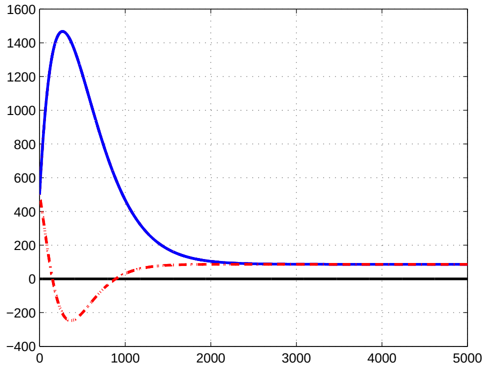

Fig. 2: Under the given initial condition the miRNA concentration s may pass through the negative values (!) at early stage (dotted graph). Sensible result (solid line) also depends on activity of other reagents at early stage. In time both graphs run up to the same value and become indistinguishable.

We have shown that depending on the initial conditions at early stage several concentrations may be negative, and the model becomes inconsistent with biology. Note that the results in [3] are based on steady state solutions and do not contain any details concerning the way on which the system passes to the steady state. These negative non-biological values remain invisible for steady state solutions, while corresponding parameters of the model may be incorrectly considered as appropriate.

We may conclude that the steady state solution obtained in [3] is not enough to select the proper model parameters in order to avoid meaningless concentration values at early stage. Exact solutions found to the coupled o.d.e. may be useful for finding a proper parameter set for modelling.

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An asymptotic model for the Rayleigh wave in elastic half space

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This study is concerned with the development of an asymptotic model for the Rayleigh wave propagating in a 3D elastic half space. The approach is based on the general representation of the surface wave solutions in terms of harmonic functions, see [1], [2]. This research generalizes an asymptotic model for plane surface wave [3] and provides an extension to the 3D formulation for normal loading [4] to a more intricate case of tangential load. The resulting model contains elliptic equations over the interior, and hyperbolic equations and a differential relation at the surface. Thus, the model enables reduction of the vector hyperbolic problem of elastodynamics to a sequence of three scalar elliptic problems, with the dynamical factor appearing only at the boundary.

We consider an elastic isotropic half space given by $x_3 \geq 0$. The boundary conditions are taken in the form

$$\sigma_{33} = 0, \quad \text{and} \quad \sigma_{i3} = -P_i, \quad (i = 1, 2)$$

where $P_i = P_i(x_1, x_2, t)$ are the components of prescribed tangential loading. The load is then decomposed into the gradient and rotational parts P_0 and P_r, respectively, as

$$P_1 = \frac{\partial P_0}{\partial x_1} + \frac{\partial P_r}{\partial x_2}, \qquad P_2 = \frac{\partial P_0}{\partial x_2} - \frac{\partial P_r}{\partial x_1}.$$

The Radon integral transform is applied, see [4], reducing the original 3D problem to a 2D problem of elasticity in terms of the transformed potentials. It is possible to show that the rotational part P_r does not contribute to excitation of surface wave. The slow-time asymptotic perturbation technique [3] leads to the asymptotic formulation for the transformed potentials, which is then inverted to yield the quasi-static elliptic equations over the interior

$$\frac{\partial^2 \phi}{\partial x_3^2} + k_1^2 \Delta_2 \phi = 0, \qquad \frac{\partial^2 \psi_i}{\partial x_3^2} + k_2^2 \Delta_2 \psi_i = 0, \quad (i = 1, 2)$$

with the hyperbolic equation and a relation between the potentials at the surface $x_3 = 0$

$$\Delta_2 \psi_i - \frac{1}{c_R^2} \frac{\partial^2 \psi_i}{\partial t^2} = A \frac{\partial P_0}{\partial x_i}, \qquad \frac{\partial \phi}{\partial x_3} = -\frac{1 + k_2^2}{2} \frac{\partial \psi_i}{\partial x_i},$$

where A is a material constant. The expressions for displacement components in terms of the functions ϕ, ψ_1 and ψ_2 coincide with that presented in [4].

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On the existence and stability of solitary-wave solutions to a class of evolution equations of Whitham type

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We consider a class of pseudodifferential evolution equations of the form $u_t + (n(u) + Lu)_x = 0$, in which L is a linear smoothing operator and n is at least quadratic near the origin; this class includes in particular the Whitham equation, the linear terms of which match the dispersion relation for gravity water waves on finite depth. A family of solitary-wave solutions is found using a constrained minimisation principle and concentration-compactness methods for noncoercive functionals. The solitary waves are approximated by (scalings of) the corresponding solutions to partial differential equations arising as weakly nonlinear approximations; in the case of the Whitham equation the approximation is the Korteweg–deVries equation. We also demonstrate that the family of solitary-wave solutions is conditionally energetically stable.

An asymptotic model for the Rayleigh surface wave in case of mixed boundary value problems

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This study develops an approximate approach to mixed boundary value problems in linear elasticity, reducing the original vector problem to a scalar mixed problem for the Laplace equation. The approach is based on the general representation of the surface wave solutions in terms of harmonic functions, see [1], together with the asymptotic model developed in [2].

We start with the explicit asymptotic formulation obtained in [2] and apply it to a mixed boundary value problem, where the boundary condition is given in two parts: along one part of the surface (S_1) a normal loading is given $\sigma_{zz}(x,0,t) = P(x,t)$, whereas along the remaining part (S_2) a normal displacement is prescribed $u_z(x,0,t) = U(x,t)$.

By employing the relation between the elastic potentials at the surface, first obtained by Chadwick [1] (see also [2]), the original vector mixed problem may be reduced to a scalar Laplace equation with mixed boundary conditions.

The approach is then applied to a steady-state moving stamp problem, where, for the sake of simplicity, the normal loading is assumed zero, and the prescribed vertical displacement at the surface is taken as $U(x,t) = f(x - vt)$. Changing to moving coordinate $\xi = x - vt$ and scaling the elliptic equation for the longitudinal potential, the problem is then formulated in terms of the Laplace equation

$$\frac{\partial^2 \phi}{\partial \eta^2} + \frac{\partial^2 \phi}{\partial \xi^2} = 0,$$

where, now, the mixed surface boundary conditions are given as

$$\phi = f(\xi) \quad \text{at} \;\; \eta = 0, \; \xi \in S_2', \qquad \frac{\partial \phi}{\partial \eta} = 0 \quad \text{at} \;\; \eta = 0, \; \xi \in S_1'.$$

Here S_1' and S_2' correspond to S_1 and S_2 in the moving coordinate system. We remark that the displacement and traction components are expressed in terms of a single harmonic function ϕ.

An illustration of the approach is provided by a semi-infinite stamp of exponential shape. The solution of this problem by the Wiener–Hopf technique can be found in [3].

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Non-linear underwater imaging with realistic models

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Imaging of objects buried in a layered medium constitutes an important class of problems in electromagnetic theory. This is due to the fact that the results of such investigations have various applications in practice in the areas such as detection and location of dielectric mines, non-destructive testing, determination of underground cracks and earthquake zones, detection of underground tunnels and pipelines etc. Although during the last three decades several exact and numerical techniques have been developed, a large number of them are related to the layered backgrounds with planar boundaries. Whereas in most of the real applications the bodies are buried in layered media having rough interfaces where the roughness have a strong effect on the scattering phenomena as well as inversion algorithms. For instance, in the case of objects located in a water reservoir, the roughness of the bottom surface as well as water–air interface can potentially modify the scattering from objects inside flat surface. For that reason the problem has to be considered in its actual conditions. In other words one has to take into consideration the roughness of the interfaces between the layers where the body is located.

The main objective of this paper is to give a method to solve the inverse scattering problems associated with cylindrical bodies located in a water reservoir such as lake, ocean, etc. For the solution of the inverse scattering problem a multi-static measurement configuration where the transmitters and receivers are located just above the water surface on a certain line is considered. The material of the body to be reconstructed is assumed to be inhomogeneous, i.e., its dielectric permittivity and conductivity are the functions of location. The space which is composed of air-water-soil layers is modeled as a three layered medium having rough surfaces at the air-water and water-soil interfaces. Through the Green's function of the background medium with rough interfaces the problem is reduced to the solutions of *data* and *object* equations which are solved iteratively by the application of the *contrast source inversion* method [1]. On the other hand the determination of the Green's function of the background medium containing rough interfaces constitutes a separate and difficult problem. Here it is used the buried object approach (BOA) where the perturbations of the rough surface from the flat one are assumed to be buried objects in a three-part space with planar interface [2]. Modeling the roughness in that way yields us to formulate the problem as scattering of waves from finite number of buried homogeneous bodies, which is solved through a method based on MoM. This approach is very effective for surfaces having a localized roughness, arbitrary rms height and slope. The method yields quite accurate results even with the incomplete data.

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Spectrum and eigenfunctions of the operator of an induction of a magnetic field on a three-dimensional sol-manifold

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The magnetic field in a conducting liquid (in particular, magnetic fields of some planets and galaxies) is described by the operator of an induction L:

$$LB = \varepsilon\Delta B + V, B = \varepsilon\Delta B + (V, \nabla)B - (B, \nabla)V,$$

where B – magnetic field, V – field of velocities and ε – small parameter (resistance). We describe the asymptotic of the spectrum and eigenfunctions of this operator on an three-dimensional sol-manifold, also we describe spatial structure of a magnetic field with $V = (v_1, v_2, v_3)$. We observe an effect of the kinematic dynamo.

Solution to the electrostatic problem for a non-confocal core-mantle spheroid

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The approach recently used in the paper [1] to find the Rayleigh approximation for spheroidal Chebyshev particles is generalized for non-confocal core-mantle spheroids. The electrostatic problem for such particles is formulated as surface integral equations relative to scalar potentials. The potentials are expanded in terms of the eigenfunctions of the Laplace operator in two spheroidal coordinate systems as the external mantle (particle) and core surfaces being non-confocal are coordinate surfaces in different systems. Matching of two different expansions of the internal field potential is discussed. The unknown coefficients of the potential expansions are derived from infinite systems of linear algebraic equations. We also suggest a new explicit solution based on the approximation of the field inside the core by a homogeneous one. The expression obtained for the polarizability resembles that derived in the particular case of the confocal core-mantle spheroids [2, 3]. Our analysis of numerical results shows that for non-confocal core-mantle spheroids the approximate solution suggested gives better results than the solution presented in [4].

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Terahertz plasmonic photogalvanic effects in a planar plasmonic crystal with an asymmetric unit cell

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One of the promising trends of terahertz (THz) detection is utilizing the nonlinear properties of plasma oscillations (plasmons) in two-dimensional electron systems (2DES). Photovoltaic response can exhibit only in 2DES without a center of inversion in 2DES plane. Terahertz plasmonic photovoltaic response in 2DES with a double grating gate (DGG), which is actually a planar plasmonic crystal, was studied in [1]. The DGG is formed by two coplanar subgratings having different widths of their fingers. The asymmetry of a unit cell of such planar plasmonic crystall appears due to latteral shift of the subgratings in respect to each other.

In this paper, the effect of DC current in 2DES on the THz plasmonic photogalvanic response of the planar plasmonic crystal with an asymmetric unit cell is studied theoretically. The photocurrent is found by solving the nonlinear hydrodynamic equations which describe the electron oscillations in 2DES. The solution of hydrodynamic equations within the lowest order of the perturbation series in respect to the powers to external electric field yealds the THz plasmonic responsivity [2].The electric field induced in the 2DES by the incoming THz radiation is calculated in a self-consistent linear electromagnetic approach described in [3].

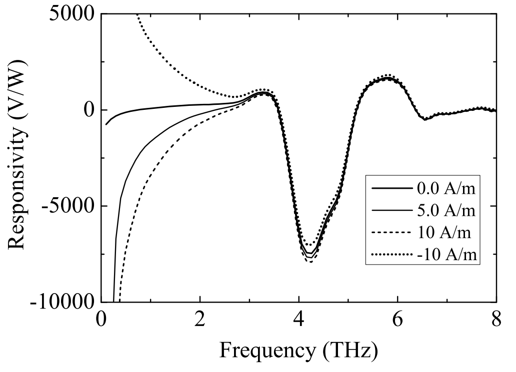

Fig. 1: Dependence of the THz plasmonic respositivity on the DC bias current.

The DC current significantly affects on the THz plasmonic responsivity (Fig. 1) at low THz frequency, when the Doppler corrections to higher plasmon modes are comparable with THz frequency. At high THz frequencies, the THz photogalvanic response is mainly controlled by the geomerical asymmetry of the unit cell of the planar plasmonic crystal.

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The forced oscillations of the cylindrical shell partially submerged into the layer of liquid

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The problem of oscillations of such objects vibrations as different supports, tubes, oil platforms is one of the actual problems of modern techniques and it is important to estimate the parameters of vibrations and acoustical fields in such systems.

The problem of forced oscillations of the cylindrical shell partially submerged into the layer of liquid is considered in the rigorous mathematical statement. The exact analytical solution of the problem is constructed.

Dirichlet-to-Neumann operators in periodic waveguides. Application to the computation of trapped modes

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Locally perturbed periodic media play a major role in applications, in particular in optics for micro and nano technology, because they exhibit interesting properties, such as allowed modes in the spectral band gaps of the non perturbed periodic media. Of course there is a need for efficient numerical methods for computing the propagation of waves inside such structures. For a complete, mathematically oriented presentation, we refer the reader to [2].

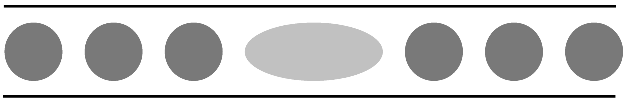

Fig. 1: A locally perturbed periodic waveguide.

We are interested by propagation media which are a local perturbation of an infinite periodic waveguide, namely an infinite structure which is periodic in one priviliged direction (the propagation direction) and bounded in the other transverse directions. We investigate the question of finding artificial (but exact) boundary conditions to reduce the numerical computation to a neighborhood of this perturbation. Our goal was the generalization of the DtN approach to periodic waveguides, which is complicated by the fact that separation variables techniques usually used in the case of homogeneous waveguides can no longer be used. However the notion of guided modes has a natural extension: the notion of Floquet modes. By revisiting the Floquet–Bloch theory [1], we propose a method for constructing DtN operators by solving local problems on a single periodicity cell and an operator-valued Ricatti equation of stationary nature. For more details see [3].

Scattering problem in such media can then be solved naturally using this DtN approach. Characterization and computation of trapped modes can also be handled using this method. Indeed, they are related to a selfadjoint eigenvalue problem associated to the PDE in an unbounded domain, namely the waveguide, which makes both the analysis and the computations more complex. Using the DtN approach, we show that this problem is equivalent to one set on a small neighborhood of the defect. We offer a rigorously justified alternative to existing methods, such as the Supercell Method, which consist simply on truncating the unbounded domain far enough from the perturbation. Conversely to existing methods, this one is exact but there is a price to be paid : the reduction of the problem leads to a nonlinear eigenvalue problem of a fixed point nature.

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The method of parametric representations of integral and pseudo-differential operators in diffraction problems on electrodynamic structures

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Mathematical models of wave diffraction on electrodynamic 2D structures are the external boundary-value problems for the Helmholtz equation with boundary conditions of the first, second or third kind on the boundary surfaces.

One of the effective ways of solving these boundary-value problems consists in their reduction to a singular and hypersingular boundary integral equations by the method of parametric representations of integral and pseudo-differential operators [1, 2].

The numerical solution of integral equations is obtained by using the modifications of discrete singularities method [3].

This approach has proved its effectiveness in solving problems of the electromagnetic waves diffraction by periodic and non-periodic, single-and multi-layered, perfectly conducting and impedance structures [4, 5].

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Diffraction by a subwavelength concaved perfectly conducting wedge

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The fields and current densities behaviors in the vicinity of an isolated infinitely sharp edge or vertex are well known. Many analytic results are available to date for various diffraction problems. Just to name some of the most important ones, among others : Sommerfeld solved the half plane illuminated by a plane wave [1], Mac Donald generalized this result to the wedge [2]. Radlow [3] solved the the quarter plane sector case and Satterwhite and Kouyoumjian [4, 5] found the general solution for any sector and incident field source.

To ensure unicity of the solution in presence of an edge, it has been recognized that a condition was necessary : the so called "edge condition". It states that induced current densities must be integrable to produce finite scattered fields. The edge condition actually selects the solution having the lowest order of singularity at an edge, ruling out any singularities of order greater than $\rho^{-1/2}$, where ρ is the distance to the edge.

By expanding the fields as a power serie of ρ, Meixner characterized their behavior close to an edge surrounded by a combination of metal and dielectric sectors [6, 7]. After the pioneering work of Meixner, several authors analyzed the edge fields and current densities for several more complex configurations [8–10]. The general conclusion of all these works is that the singular fields grow like $\rho^{-\nu}$ very close to the edge, the exponent ν depending both on the geometry and the electromagnetic properties of the wedge-shaped sectors surrounding the edge. Two cases worth remembering are the metallic half plane ($\nu = -1/2$) and the metallic 90° wedge ($\nu = -1/3$) in free space.

More analytically challenging problems involving two or more edges have been tackled by various authors. Exact solutions have been obtained for an infinity [11, 12] or for only two [13] parallel stacked half planes. The so called "thick" half plane has been addressed first in an approximate way by Hanson [14], and later solved exactly by Jones [15] who obtained tractable expressions and numerical results for the far field only for subwavelength thicknesses.

In most of these examples it is found again, or simply assumed based on Meixners work [16], that infinite current densities may arise in regions of the scattering surface where the local radius of curvature is zero, but much less effort is made to obtain the exact behavior of these current densities in the vicinity of the more or less coupled edges. The purpose of this work is to derive explicit laws for the growth to infinity of the current density for the case of a subwavelength perfectly conducting concaved wedge [17].

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Human body surface oscillations remote measurements with use of laser Doppler interferometry

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As is well known, the optical laser vibrometers based on the longitudinal Doppler effect are applied for many engineering solutions associated with the definition of oscillation parameters. In particular, high efficiency of the buildings' vibration remote measurement with use the laser vibrometer was confirmed in our earlier investigation [1].

Present work addresses to the search of hardware and software solutions of the optical laser vibrometer use for medical applications. The starting point of this study was the fact that both muscular tissues and organs of human are produce the mechanical vibrations which possibly to obtain at the skin surface. The process of pulse wave spreading in vascular system is the most visual example of such oscillations. Generally, the pulse wave parameters are reflected not only the heart mechanical function, but they deeply depended on the vascular wall elastic properties. Arterial stiffness change is the most essential predictor of some cardiovascular diseases, e.g. the hypertension. Therefore, the monitoring of arteries mechanical behavior due to the pulse wave propagation has a high diagnostic value.

With use the laser vibrometer “LV-2” (LASER TECHNICS Company. Novosibirsk, Russia) based on the Longitudinal Doppler effect we have demonstrated the fundamental possibility for remote measurement of the pulse wave propagation at the skin surface. Several volunteers were exonerated, and the mechanical oscillations of the vascular wall at the body surface were recorded.

To optimize the signal/noise ratio and to increase the informative value of measurements, the signals were processed using the authors' original software developed by the based on wavelet analysis. The advantages of proposed diagnostics method for vascular wall elastic properties estimation in comparison with the generally accepted one (photoplethysmography) are discussing.

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Lamb wave interaction with through-thickness obstacles of different nature: scatterer characterization

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Layered plate engineering structures along with the defects caused by repeated static or shock loads may initially contain a network of artificial scatterers such as pins, stringers or holes. This circumstance sufficiently complicates the interpretation of guided wave patterns in structural health monitoring (GW SHM).

The aim of the present work is to investigate both theoretically and experimentally particular features of guided wave interaction with small-sized through-thickness obstacles of different nature which can be then taken into account in the GW SHM. Mathematical modeling of the wave phenomena considered is based on the semi-analytical integral approach. The incident field generated by a given source is expressed via the convolution of the Green matrix for the elastic structure considered and the corresponding load vector-function. The scattered field is expanded in terms of laminate elements [1], which are the fundamental solutions for the layered structure as a whole. In the course of joint research work a series of experimental measurements has been performed for an aluminium plate with drilled through-thickness holes of various radii. Guided waves have been excited by a circular piezoelectric actuator and measured by a scanning laser vibrometer.

As an example, Lamb wave diffraction by a cylindrical rigid pin and a hole of arbitrary cross-section is considered. The influence of the defect's type, its size and oscillation frequency on the scattered wavefield intensity is investigated. Specifically, it is shown that in a low frequency range a hole is almost invisible for Lamb waves whereas a rigid pin of the same radius produces strong reflected field. The comparison with the experimental results for holes demonstrates a pretty good coincidence.

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Elastic wave energy trapping in a plate with a crack: theory and experiment

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The research aims at an experimental approval of the trapping mode effect predicted theoretically for a layered plate with a delamination [1]. The geometry of the problem investigated is shown in Figure 1. The effect is featured by a sharp capture of incident wave energy at certain resonance frequency with its localization in the vicinity of the delamination in the form of long-enduring standing waves. The trapping modes are eigensolutions of the related diffraction problem associated with nearly real complex points of its discrete spectrum. The frequency of trapped mode resonance oscillation is well approximated by the value of the real part of the spectral point of the corresponding 2D boundary value problem, see Ref. [1] for more details.

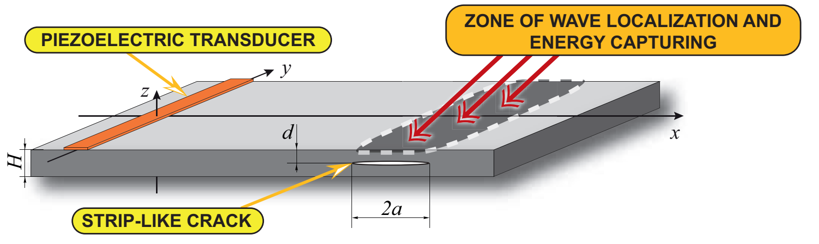

Fig. 1: Geometry of an elastic layer with a strip-like crack and rectangular piezoelectric transducer.

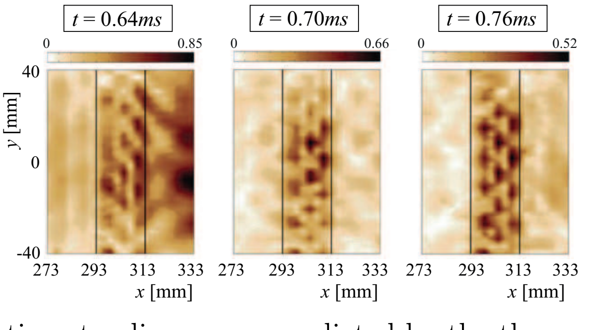

Fig. 2: Snapshots depicting standing waves predicted by the theory above the delamination (experiment).

To detect such resonance motion, a laser-vibrometer based system has been employed for the acquisition and appropriate visualization of piezoelectrically actuated out-of-plane surface motion of an aluminium plate with an artificial strip-like delamination. The laser Doppler vibrometer based experimental measurements of guided wave propagation and diffraction in the specimen have confirmed the trapping mode effect. An example is demonstrated in Figure 2, where the waves trapped at the delamination manifest themselves as dark spots in the snapshots of scanned surface at the carrier frequency predicted theoretically.

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High-velocity surface wave excitation in diamond-based piezoelectric laminate composite structures

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Generation and propagation of surface and pseudo-surface acoustic waves (SAW/PSAW) in two- and three-layer AlN/Diamond and AlN/Diamond/γ-TiAl structures is considered. The waves excited by an electric source (interdigital transducer) are analyzed in the mathematical framework based on the Green's matrix integral representation and guided wave asymptotics derived for laminate anisotropic structures using the residue technique. The attention is focused on the effect of pseudo-surface-to-surface wave degeneration at certain discrete values of h/l (h is the thickness of the piezoelectric layer and l is the wave-length). Earlier such optimal ratios were discovered and experimentally verified for the first pseudo-surface (Sezawa) mode in the AlN/Diamond structure [1]. The present research reveals this effect for higher modes as well as examines its manifestation for three-layer structures with different diamond-to-AlN thickness ratios H/h.

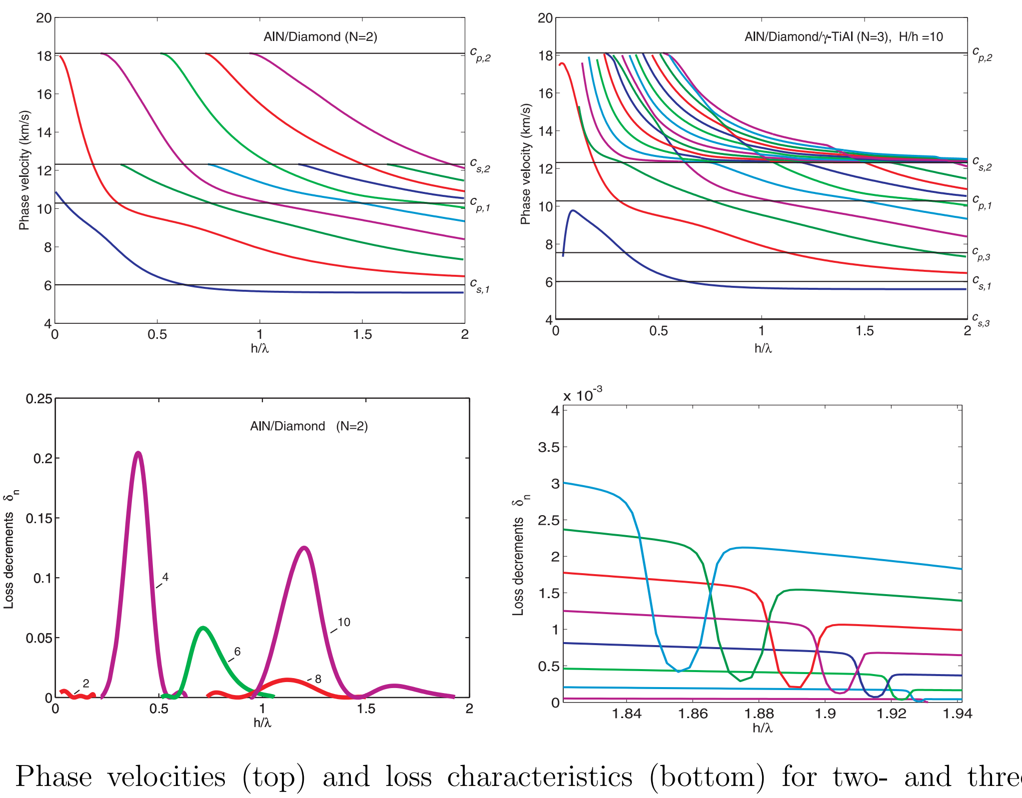

Fig. 1: Phase velocities (top) and loss characteristics (bottom) for two- and three-layer structures (left and right, respectively); typical loss-decrement pattern in a band of PSAW-to-SAW degeneration in the three-layer case (right-bottom).

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On the resolvent of operators with distant perturbations

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Let $\Omega \subseteq \mathbb{R}^n$, $n \geqslant 1$, be an arbitrary periodic domain in $\mathbb{R}^n$ with the unit cell $\square$. We suppose that the domain Ω is invariant with respect to the shifts by discrete parameters $X_i \in \mathbb{R}^n$, $i = 1, \dots, n$, tending to infinity. We introduce arbitrary abstract bounded operators $\mathcal{L}_i$ from $W_2^m(\Omega)$ in $L_2(\Omega)$ and nonnegative functions ξ_i, $\eta_i \in C^m(\overline{\Omega})$ satisfying the following conditions

A1. There exists a function $\varphi \in C^m(\overline{\Omega})$ such that the estimates $|\xi_i| \leqslant C_1|\varphi|$, $|\partial^\alpha \varphi| \leqslant C_2|\varphi|$ hold true, where the constants C_1, C_2 are independent of x, α is a multi-index, $|\alpha| \leqslant m$.

A2. The functions φ, η_i and all its derivatives up to the order m decay at infinity.

We consider an operator $\mathcal{H}_0$ in $L_2(\Omega)$ with domain $W_2^m(\Omega)$, $m \in \mathbb{N}$, which satisfies the following conditions

A3. For all $u \in W_2^m(\Omega)$ the inequality $\|u\|_{W_2^m(\Omega)} \leqslant C(\|\mathcal{H}_0 u\|_{L_2(\Omega)} + \|u\|_{L_2(\Omega)})$ holds true, where the constant C is independent of u.

A4. The identity $\mathcal{S}(-X_i)\mathcal{H}_0\mathcal{S}(X_i) = \mathcal{H}_0$ holds true, where $\mathcal{S}(X_i)$ is the shift operator acting as follows $(\mathcal{S}(X_i)u)(\cdot) = u(\cdot - X_i)$.

A5. For all $u \in W_2^m(\Omega)$ the estimates $\|(\varphi^{-\varepsilon}\mathcal{H}_0\varphi^{\varepsilon} - \mathcal{H}_0)u\|_{L_2(\Omega)} \leqslant \varsigma(\varepsilon)\|u\|_{L_2(\Omega)}$ hold true, where ε is sufficiently small, $\varsigma(\varepsilon) \to 0$ as $\varepsilon \to 0$ and independent of u.

We introduce the operators $\mathcal{H}_X := \mathcal{H}_0 + \sum\limits_{i=1}^{k} \mathcal{S}(-X_i)\xi_i\mathcal{L}_i\eta_i\mathcal{S}(X_i)$ and $\mathcal{H}_i := \mathcal{H}_0 + \xi_i\mathcal{L}_i\eta_i$ in the space $L_2(\Omega)$ with domain $W_2^m(\Omega)$. We assume that the operators $\mathcal{H}_i$ are closed.

The main result is the following

Theorem 1. *Let the set $M := \mathbb{C} \setminus \bigcup\limits_{i=0}^{k} \sigma(\mathcal{H}_i)$ is not empty. Then for sufficiently large X the operator $\mathcal{H}_X$ is closed. For all $\lambda \in M$ and for sufficiently large X the resolvent of operator $\mathcal{H}_X$ exists and one has*

$$(\mathcal{H}_X - \lambda)^{-1} := \Big[\sum_{i=1}^{k} \mathcal{S}(-X_i)(\mathcal{H}_i - \lambda)^{-1}\mathcal{S}(X_i) - (k-1)(\mathcal{H}_0 - \lambda)^{-1}\Big](\mathrm{I} + \mathcal{P}_X)^{-1},$$

$$\mathcal{P}_X := \sum_{\substack{i,j=1 \\ i \neq j}}^{k} \mathcal{S}(-X_i)\xi_i\mathcal{L}_i\eta_i\mathcal{S}(X_i)\Big[\mathcal{S}(-X_j)(\mathcal{H}_j - \lambda)^{-1}\mathcal{S}(X_j) - (\mathcal{H}_0 - \lambda)^{-1}\Big],$$

where $\|\mathcal{P}_X\|_{L_2(\Omega)\to L_2(\Omega)} \to 0$ as $X \to \infty$.

The special case when the operator $\mathcal{H}_0$ is a differential operator was considered in [1], [2].

This is the joint work with D. Borisov.

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Transmission and resonances in layered phononic crystals with damages

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Periodic elastic composites, the so-called phononic crystals, are receiving more and more attention in recent years due to their unique dynamic properties. They have potential applications as multi-functional composites, acoustic and ultrasonic devices, wave filters etc [1]. Wave propagation in periodic composite structures is often accompanied by localization phenomena and band-gaps, which are observed in photonic and phononic crystals [2].

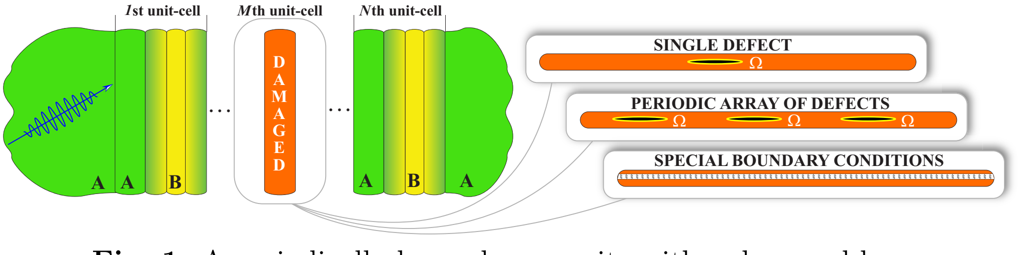

Fig. 1: A periodically layered composite with a damaged layer.

The propagation of 2D time-harmonic plane waves in periodically multi-layered elastic composites with a strip-like crack or a damaged layer as shown in Fig. 1 is investigated in the present paper. The total wave field in the structure is a sum of the incident wave field and the scattered wave field described by an integral representation in terms of the crack-opening-displacements and Green's matrix for the whole elastic structure [3]. The effects of the wave incidence angle, crack location and size are studied using averaged crack-opening-displacements and stress intensity factors. The focus of the study is on the wave propagation, resonances and localization phenomena in a stack of elastic layers containing different types of damages. The damages are simulated as a strip-like crack, a periodic array of cracks or distributed cracks modeled using spring boundary conditions [4]. Numerical results are presented and discussed. Resonant and usual transmissions are illustrated by energy streamlines in the vicinity of the crack to get a better understanding of the physics of wave localization and resonances. Different kinds of the energy capturing by the damages within the pass-bands and band-gaps are demonstrated.

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Energy-absorption calculus for multi-boundary diffraction gratings

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A general energy-balance criterion for multi-boundary lossy periodical objects (general gratings) has been derived and verified numerically. In the general case, the difference $A = 1 - R - T \geq 0$ is called the absorption coefficient in the given diffraction problem with the sums of reflected and transmitted energies R and T, respectively. In addition to being physically meaningful, this expression is useful as one of the accuracy tests for computational codes. The energy criterion in the lossless case says $A = 0$. In the lossy case, one needs an independently calculated quantity to compare with A. For such a quantity, we use the absorption expression defined as the sum of volume or sufface integrals. The equation for the absorption A of an electromagnetic field by a multilayer grating can be derived directly from Maxwell's equations [1], or by the variational principle [2], or by applying the second Green's identity to boundary functions for the contours in the upper and lower media [3]. By definition, the first part of integrals in the expression of A is $1 - R$, and the second, $-T$, vanishes if the lower medium is absorbing or the lower boundary is perfectly conducting. The absorption expression in the explicit form which is based on scattering amplitude matrices has been added to the previous study to treat closed and separated boundaries, e.g. photonic crystals [4]. The sum $A + R + T$ is actually the energy balance for an absorbing grating, and the extent to which it approaches unity is a measure of the accuracy of calculations. Maxwell equations being valid in the sense of distributions, the proposed general energy-balance criterion is valid in the same sense. The connection of the derived expression with the optical theorem and its application to non-periodical surfaces is discussed.

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Poincaré wavelet technique in the depth migration

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A method based on space-time wavelets is developed for the migration problem when the background medium is a smoothly varying medium in depth. The problem is to image reflection bound-

aries and other heterogeneities inside the medium if fields emitted from the surface of the medium and received on the same surface are known. To solve the problem we should solve the boundary value problem for the wave equation separately for emitted and received fields. The reflection (scattering) points inside the medium are determined by maximum of the function which controls the coherence of emitted and received fields at that point [1]. Our contribution deals with the representation of seismic data on the plane surface and the representation of subsurface fields. We apply the technique of continuous space-time wavelet analysis for the representation of data. To compute sub-surface fields we represent the field in terms of the superposition of localized wave-packet solutions suggested in [2], [3] and [4]. Projections of the solutions on the surface boundary are wavelets. The localized solutions are given by the Fourier integrals obtained by the WKB method in vertically varying media. This approach enables us to reduce the amount of data in calculations. Preliminary results are given. Our method has an analogy with the method [1] of summation of Gaussian beams but the decomposition atoms are different. The comparison of methods is discussed.

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Small perturbations of the spherically symmetric prestressed state in a nonlinear isotropic elastic full Cosserat medium: waves and instabilities

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We consider a nonlinear elastic full Cosserat medium under nonlinear isotropic tension or compression. Elastic nonlinear full Cosserat continuum is a medium whose point bodies have both rotational and translational degrees of freedom, and may perform large rotations and translations. The elastic energy of the full Cosserat continuum may depend on all possible kinds of strain, i.e. on the gradient of the position vector, on the turn tensor $\mathbf{P}$ of a point body, and on the gradient of the turn tensor $\overset{\circ}{\nabla}\mathbf{P}$. We consider small perturbations near this prestressed state. It appears that the equations for these perturbations by their structure coincide with the equations for the linear full elastic isotropic Cosserat continuum. The effective constants in these equations depend on the prestressed state and on the type of the strain energy. If they are positive, we have a wave propagation analogous to the wave propagation in the linear elastic Cosserat medium. For simplicity we consider the case of the spherical symmetry of the tensor of inertia of point bodies.

For a wide class of strain energies satisfying the requirement of the strong ellipticity, under sufficiently strong hydrostatic compression or isotropic tension some of these constants become negative, and the medium becomes **unstable** with respect to infinitesimal low frequency shear perturbations or rotational perturbations of a certain "resonance" frequency.

Contrary to this case, it is known that a classical isotropic elastic medium with a strain energy satisfying the requirement of strong ellipticity is **stable** in the spherically symmetric prestressed state with respect to infinitesimal perturbations of any kind.

In previous works (see, e.g., [1]) we considered the case of the **reduced** nonlinear elastic Cosserat medium, where the elastic energy does not depend on the gradient of the turn tensor of the point bodies. We have obtained a similar result on the instability and we supposed that this result was caused by the independence of the elastic energy on $\overset{\circ}{\nabla}\mathbf{P}$. However, this work shows that the phenomenon of the instability is caused by the presence of rotational degrees of freedom and not by the specific type of strain energy.

Comparison of waves and instabilities in different Cosserat-type and classical media are important from the theoretical point of view, and also could be useful for an adequate choice of a model of geomedium in rotational seismology.

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Reconstruction of complex permittivity of a nonhomogeneous body in a rectangular waveguide using the iteration method

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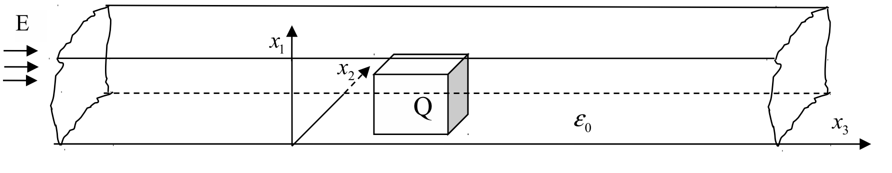
We solve the inverse problem of reconstructing the media parameters (complex permittivity) of a body in a waveguide [1–3]. The aim of this paper is to reconstruct the complex permittivity with help of reflection coefficient. We reduce the problem to the nonlinear volume singular integral equation. We apply the iteration method for solving the integral equation. The results of test computations justify the proposed approach.



The problem is relevant in modern science. It can find the wide practical application for example in nanoelectronics and nanotechnology. These fields interested in reconstructing the parameters of nanomaterials. But the reconstructing of these parameters with help of experimental methods can be difficult. So we find these parameters with help of mathematic modeling. Suggested iteration method allows us to find these parameters numerically.

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Theory of selfrefraction effect of intensive focusing acoustical beams

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The theory of selfrefraction of nonlinear acoustic beams is developed based on some exact and approximate analytical equations and solutions. The system of base equations in geometrical acoustics approximation is sequentially derived from Khokhlov–Zabolotskaya equation for nonlinear focused acoustical beams. The generalized method of extended characteristics allows to set up the simplified closed equation for ray convergence on the beam axis for the most interesting case of small diffraction, when the large amplitudes in the focal area are observed. The exact solution is derived in particular case. For the common case of wave parameters there are suggested some analytical approximations and numerical solution. The amplitude dependencies on longitudinal and transversal distances and other wave parameters are obtained. It is shown, in particular, that at the axis of gaussian beam in the focal area the local minimum of amplitude can be formed. Some initial transversal forms of beam, such as gaussian, and initial phase modulation as parabolic or sinusoidal are analyzed.

The surface impedance tensor and Rayleigh waves

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Rayleigh surface waves in linear elasticity are solutions of zero-traction boundary problems,

$$Lu - \rho\omega^2 u = 0, \quad Tu|_{r=0} = 0,$$

where $(Lu)_i = -(C_{ijk\ell}u_{k;\ell})_{;j}$. Rayleigh waves, if they exist, arise over the subsonic boundary region where no reflection of body waves can occur. In microlocal analysis this region is called the elliptic region because the theory of elliptic boundary problems applies here. In particular, one obtains, microlocally near the elliptic region, a (spectral) factorization

$$h^2 L - \rho \equiv (hD_r - Q^\sharp(hD'))A(hD_r - Q(hD')),$$

$D = -\sqrt{-1}\partial$, $h = 1/\omega$, with pseudodifferential operators Q and $Q^\sharp$ acting tangent to the level surfaces $r = x_3 =$ constant. From the factorization one gets a pseudodifferential operator Z (DN operator) over the boundary which maps the displacements of solutions to their tractions. Rayleigh waves correspond to solutions of $Zv = 0$ via a Dirichlet problem,

$$hD_r u - Q(hD')u = 0, \quad u|_{r=0} = v.$$

The existence and uniqueness of high-frequeny solutions of $Zv = 0$ depends on the characteristic variety of the principal symbol z of Z. In applied physics, z is called the surface impedance tensor. It is well-known that z is Hermitean and positive definite near the static limit. Barnett and Lothe established important properties of z such as an integral formula and the positive definiteness of the real part of z, the latter implying uniqueness of Rayleigh waves. We simplify their argument and outline the construction of Rayleigh quasimodes for (possibly) anisotropic media under the Barnett–Lothe existence condition.

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Free vibration and buckling of functionally graded Euler–Bernoulli and Timoshenko beams using Haar wavelets

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During the last few years, the fields of electronics, biomedicine, automobile, aircraft, and civil engineering have particularly been interested in functionally graded materials (FGM). These are inhomogeneous composites made of different solids with the ultimate aim to improve physical properties, functionality and overall cost of the materials. For instance, FGM made of aluminum and ceramics is noted for its toughness as well as very good thermal resistance which can be applied in antennas or tall building. Nevertheless, the development, application and production of FGM in industry require free static and dynamic behavior analysis.

Thus far, quite a lot research papers are published in literature on uniform and non-uniform constructions made of FGM with varying properties along cross-section; the quantity of the works on FGM beams with variable thickness and material properties oriented in the axial direction is scarce. The latter is explained by the difficulty of mathematical treatment of the governing equation with variable coefficients.

This paper investigates uniform and non-uniform axially graded beams. The theoretical formulations are based on Euler–Bernoulli and Timoshenko beam theories; the second theory includes axial and rotary inertia. The beams are pinned, clamped, or free at its ends.

The aim of the research is to develop a simple numerical approach for general nonlinear functions of material (mass density, Young's modulus) and beam (cross-sectional area, moment of inertia) using the artificial neural networks and the Haar wavelets. The wavelet method is employed to convert the differential equations into a set of algebraic equations with finite number of variables.

The proposed approach has fast convergence; high accuracy is obtained even with a small number of grid points. The latter is validated with the previously published works and found a good agreement with them. Moreover, the present method is capable of solving non-linear problems and applicable to systems with discontinuities. Therefore, the obtained results are of benefit to optimum design used in thermal and structural fields.

Nonlinear total internal reflection of surface plasmons

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We discuss the phenomenon of the total internal reflection of the weak signal plasmon polariton beam from the high-power plasmon-induced inhomogeneity. This phenomenon occurs at the interface between a dielectric with Kerr-type defocusing nonlinearity and metal. Depending on the initial tilt and the intensity of the pump plasmon the signal plasmon can be reflected or refracted with the pump.

We propose a new method of surface plasmon polariton manipulation via the inducing of inhomogeneity by the reference plasmon beam. Due to the high energy concentration intensity of a plasmon beam is higher than the intensity of the bulk laser beam therefore the inhomogeneity can be rather high. The analogous methods were proposed for the bulk laser beams earlier [1, 2].

Surface plasmon polariton wave propagates along the interface between a metal and dielectric (Ox axis) and exponentially decays in the transverse direction (Oz axis):

$$\vec{E}_{spp} = \vec{E}_0 \exp(-\gamma_j|z| + i\beta x), \tag{1}$$

where γ_j is the localization coefficient in metal or dielectric (index $j = m$ or $j = d$ respectively), β is the propagation constant and $\vec{E}_0$ characterizes the amplitude and polarization of the plasmon.

At the interface of the Kerr nonlinear dielectric the strong pump plasmon induces the inhomogeneity of the dielectric permittivity:

$$\Delta\varepsilon_d = \chi\left|\vec{E}_{spp}\right|^2 = \chi\left|\vec{E}_0\right|^2 \exp(-2\gamma_d|z|), \tag{2}$$

χ is the nonlinearity coefficient. In a defocusing dielectric $\Delta\varepsilon_d < 0$ therefore a phenomenon analogous to the total internal reflection from a less dense medium is possible for a signal plasmon beam propagating at a small angle to the reference.

We describe the propagation of both reference and signal plasmon beams with the method of slowly varying amplitude that was proposed in Ref. [3] taking into account diffraction (see Ref. [4]), self-action of the pump beam and the induced inhomogeneity for a weak plasmon beam.

A simple ray theory is used to distinguish the three possible propagation regimes, including refraction, reflection and trapping of the signal. The original diffraction theory was developed taking into account the finite spatial spectrum width of the plasmon beams.

We found the critical parameters dividing regimes of reflection and refraction both using ray and diffraction theories. Due to the finite spatial spectrum signal beam is partially reflected and partially refracted if tilted at the angle near critical. The similar phenomenon of partial transmittance occur in the case of very narrow inhomogeneity (corresponding to focused reference plasmon) and is analogous to the tunneling quantum effect. The effect of the total internal reflection of the signal plasmon beam was analyzed both theoretically and numerically.

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Multiscale boundary layers

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Singular problems a study which required the introduction of the new stretched scale were considered many years ago [1]. Later it was found that the uniform asymptotic behavior of solutions of initial-value problem can be obtained only with the introduction of yet another so-called middleware layer [2]. It is interested that for a some systems of differential equations without introducing an intermediate layer, even the outer asymptotic expansion has the wrong kind.

Now let us consider the following initial value problems for a system of differential equations containing a small parameter ε in the highest derivatives.

$$\varepsilon\frac{d}{dt}U_1 = -{U_1}^2 + {U_2}^3 + t, \quad \varepsilon\frac{d}{dt}U_2 = -{U_2}^2 + {U_1}^3 + t^2. \tag{1}$$

Since the limiting solution has a singularity at the initial point, the asymptotic behavior of solutions can not be described in a standard way as the sum of external and internal expansions. It turns out that the solution to this very simple at first glance, the problem is singular asymptotic behavior is even more complicated than in the papers mentioned above. The correct asymptotic behavior depends on the boundary functions of **four different scales.**

The outer asymptotic expansion has a natural look:

$$U_1(t,\varepsilon) = \sum_{k=0}^{\infty} \varepsilon^k u_{1;k}(t), \quad U_2(t,\varepsilon) = \sum_{k=0}^{\infty} \varepsilon^k u_{2;k}(t). \tag{2}$$

It is easy to see that the principal terms of these series have the following behavior $t \to 0 : u_{1;0}(t) = t^{\frac{1}{2}} + \sum_{j=5}^{\infty} a_{1;0,j} t^{\frac{j}{4}}, u_{2;0}(t) = t^{\frac{3}{4}} + \sum_{j=5}^{\infty} a_{2;0,j} t^{\frac{j}{4}}$. The following members have the more order singularities, so that the outer expansion is certainly not suitable in the vicinity of the origin. In this neighborhood As usual, the right is a new variable $\tau = \varepsilon^{-1} t$.

The inner asymptotic expansion is also a series in powers of ε and $\ln \varepsilon$. Equating coefficients of like powers of small parameter, obtain a system of equations for the principal terms of these series. Just as in the example [2] the outer and inner expansions are not agreed.

The next scale, which is intermediate between the original scale of the t and scale of the initial layer is the scale of the order $\varepsilon^{\frac{2}{3}}$. Only solution of (1) in the variable $\eta = t\varepsilon^{\frac{2}{3}}$ can be ensure alignment with the rows of the internal expansion.

For the final construction of uniform asymptotic solution (1) is sufficient to introduce more an intermediate layer with a new scale and the independent variable $\xi = t\varepsilon^{-\frac{4}{7}}$.

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On hyperbolic type theories of thermoelasticity and thermoviscoelasticity

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We consider the mechanical model of a two-component medium (see Fig. 1) whose first component is the classical continuum and the other component is the continuum having only the rotational degrees of freedom. We show that proposed model can be used for description of thermal and dissipative phenomena. It is the presence of additional rotational degrees of freedom and, accordingly, additional inertia and elastic characteristics which can be interpreted as the thermodynamical constants that distinguish the proposed model among other continual models.

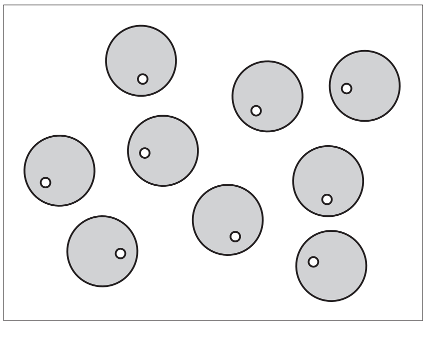

Fig. 1: Elementary volume of a two-component medium

The mathematical description of the proposed mechanical model includes not only the classical formulation of coupled problem of thermoelasticity but also the formulation of the coupled problem of thermoelasticity with the hyperbolic type heat conduction equation. In the context of proposed theory we consider the original model of internal damping.

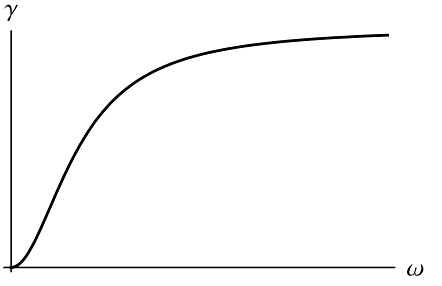

Fig. 2: Dependence of the sound attenuation factor on frequency

We show that the dependence of the sound attenuation factor on the signal frequency obtained with the aid of the proposed theory (see Fig. 2) coincides with the classical dependence in the low-frequency range and is in close agreement with the dependence obtained by means of the phonon theory in the hyper-acoustic frequency range. The lecture is based on [1, 2] and contains some new results.

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Computation of Rayleigh waves in homogeneous anisotropic half space using impedance operator

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In many applications parameters of the medium close to the surface attract a particular interest because of contact with outer environment. To find the parameters we can use physical processes of surface waves propagation. On the other hand the elastic medium close to the surface often is an anisotropic one due to special treatment. So computation of parameters of Rayleigh surface waves, which propagate in the anisotropic homogeneous elastic media become important now.

Consider a homogeneous anisotropic elastic medium, which fills a half space R^3_+ with the boundary plane S normal to $\mathbf{n}$. Any parameters of Rayleigh waves, which propagates in the direction $\mathbf{t}$, $(\mathbf{t},\mathbf{n}) = \mathbf{0}$ depend, in general, on the vectors $(\mathbf{n},\mathbf{t})$, and elastic parameters of the medium, i. e. density ρ and tensors of elastic constants c_{ijkl}. The main parameters of the Rayleigh surface wave are velocity of propagation and polarization on displacement vector on the surface s. The polarization may be characterized by angles ϕ and ψ with the normal $\mathbf{n}$ and direction of propagation $\mathbf{t}$, correspondingly.

As example we present here pictures of velocity of the Rayleigh wave and angles ϕ and ψ for the aragonit. The nonzero elastic constants after dividing on density are equal $c_{1111} = c_{2222} = 16$, $c_{3333} = 8,67$, $c_{1122} = c_{2233} = c_{1133} = 3.73$, $c_{1212} = c_{2323} = c_{1313} = 4.27$. The normal vector $\mathbf{n} = (\sin 30, 0, \cos 30)$.

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Asymptotic analysis of the autoresonance phenomenon

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Autoresonance is a phase locking phenomenon occurring in nonlinear oscillatory system, which is forced by oscillating perturbation with varying pumping frequency. There are many applications of autoresonance in nonlinear physics [1]. The essence of the phenomenon is a strong increase in the response amplitude (or energy) through resonance which holds automatically for a long time interval. In mathematical model the autoresonance is identified with unbounded solutions of the main resonance equations. A problem of such type is considered in the report.

The purpose of the lecture is to show approaches for analyzing the autoresonance phenomenon occured in a mathematical model of nuclear magnetization. The main object of research is a system of three equations

$$\frac{dr}{d\tau} = A(\tau)\sin\psi - \beta_2 r, \quad \frac{dz}{d\tau} = B(\tau)\, r\sin\psi - \beta_1 z,$$
$$r\Big[\frac{d\psi}{d\tau} + \Lambda(\tau) - z\Big] = C(\tau)\cos\psi; \quad (\beta_1, \beta_2 = \text{const} > 0). \tag{1}$$

The unlimited growth at infinity of the time depending factor

$$\Lambda(\tau) = \lambda\,\tau + \mathcal{O}(1), \quad A(\tau) = a\,\tau + \mathcal{O}(1), \quad B(\tau) = b\,\tau + \mathcal{O}(1), \quad C(\tau) = c\,\tau + \mathcal{O}(1), \quad \tau \to \infty \tag{2}$$

is a specific feature of the problem; here $\lambda, a, b, c = \text{const} \neq 0$. It is supposed, that the remainders in these formulas are smooth functions, which are expanded as $\tau \to \infty$ in asymptotic series with integer nonpositiv powers. The leading order terms of asymptotics satisfy the condition on the signs: $\lambda \cdot a \cdot b > 0$. We are interesting for the solutions unboundedly increasing with time. The unbounded solutions of the main resonance equations similar to (1) describe the initial stage of the autoresonance [2].The main achievement of the work is the discovery of the solution with the amplitude unboundedly increasing with time $r^2(\tau), z(\tau) \approx \mathcal{O}(\tau),\ \tau \to \infty$, for which the Lyapunov stability is proved. Such stable solution describes the initial stage of the autoresonance capture phenomenon for the Bloch's equations [3].

The model equations similar to (1) occur in the analysis of different resonance phenomenons. The adiabatic approximation idea is often used in the researches. Application of that approach is supported by a small parameter in the equations, [4]. There is not any small parameter in the equations considered here (1), and we do not deal with any asymptotics in small parameter. Instead of them we are interesting in an asymptotics in the time at infinity $\tau \to \infty$. The problem of both construction and justification of such an asymptotics in the form of power series with constant coefficients is not a difficult task. On contrary, the stability of the isolated solution obtained in such a manner remains often an open problem, especially for nonautonomous systems. Essential part of this work deals just with the stability.

It should be noted, that the system (1) has a lot of solutions, which have both a bounded amplitude $r(\tau), z(\tau) = \mathcal{O}(1)$ and an increasing phase $\psi(\tau) \approx \lambda\tau^2/2,\ \tau \to \infty$. Such solutions correspond to the absence of the autoresonance, and they are not considered here.

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Special functions associated with the Darboux–Dunkl differential-difference operators

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Let $P_k(x)$, $x \in \mathbf{R}$, $k \in \mathbf{Z}_+$ be the Burchnal–Chaundy polynomials which can be defined as solutions of the following recurrence system of differential equations

$$P'_{k+1}(x)\,P_{k-1}(x) - P'_{k-1}(x)\,P_{k+1}(x) = (2k+1)\,P_k^2(x), \quad P_0(x) := 1, \quad P_1(x) := x.$$

It's clear that $P_k(x) \equiv P_k(x;\gamma)$ where $\gamma \in \mathbf{R}^{k-1}$ is the vector-parameter of integrating for this system.

Let $\hat{s}$ be a reflection operator that is $\hat{s}[f](x) = f(-x)$. We consider the Darboux–Dunkl differential-difference operators

$$\nabla = \frac{d}{dx} - \left(\ln\left|\frac{P_k(|x|)}{P_{k-1}(|x|)}\right|\right)' \circ \hat{s}$$

which influences on a set of differentiable functions.

The operators $\nabla|_{\gamma=0}$ are the classical Dunkl operators [1]

$$\nabla\,|_{\gamma=0} = \frac{d}{dx} - \frac{k}{x} \circ \hat{s}.$$

The operators ∇ are also connected to the Lagnese–Stellmacher operators [2]

$$\mathcal{L}_k = \frac{d^2}{dx^2} + \left(\ln P_k^2(|x|)\right)''$$

which are received by the Darboux transformations.

We have the following equalities

$$\nabla^2\,\big|_{\mathcal{F}_+} = \mathcal{L}_{k-1}\,, \quad \nabla^2\,\big|_{\mathcal{F}_-} = \mathcal{L}_k$$

Here $\mathcal{F}_\pm$ are a spaces of even and odd functions.

Now, we consider the family of special functions

$$Sh_k(x) = \sum_{n=0}^{k(k+1)/2} (-1)^n \binom{k(k+1)/2}{n} \mathcal{L}_k^{k(k+1)/2-n}[P_k(|\cdot|)\,sh](x),$$

$$Ch_k(x) = \sum_{n=0}^{k(k-1)/2} (-1)^n \binom{k(k-1)/2}{n} \mathcal{L}_{k-1}^{k(k-1)/2-n}[P_{k-1}(|\cdot|)\,ch](x),$$

which are analogues of hyperbolic function: $Sh_0(x) = sh(x)$, $Ch_1(x) = ch(x)$.

In particular, the functions $Sh_k(x)$ and $Ch_k(x)$ for $\gamma = 0$ are closely connected to the modified Bessel functions

$$Sh_k(x)\,|_{\gamma=0} = \frac{(2k)!!}{(2k+1)!!}\,sgn(x)\,|x|^{k+1}\,J_{k+1/2}(x),$$

$$Ch_k(x)\,|_{\gamma=0} = \frac{(-1)^{k+1}(2k-2)!}{|x|^{k-1}}\,J_{-k+1/2}(x).$$

Theorem. *We have the following ∇-derivations*

$$\nabla[Sh_k](x) = 2k\,sgn(x)\,Sh_{k-1}(x); \qquad \nabla[sgn(\cdot)\,Sh_{k-1}](x) = \frac{1}{2k}\,Sh_k(x);$$

$$\nabla[Ch_k](x) = \frac{1}{2k}\, sgn(x)\, Ch_{k+1}(x); \qquad \nabla[sgn(\cdot)\, Ch_{k+1}](x) = 2k\, Ch_k(x).$$

Corollary 1. *Let $\alpha_j \in \mathbf{R}$, $j = 1, 2, 3, 4$. The space of eigenfunctions of the operator ∇^2 is four-dimensionally. The eigenfunctions of the operator ∇^2 with the unit eigenvalue have the following form*

$$\phi_1^*(x) = \alpha_1 \cdot Ch_k(x) + \alpha_2 \cdot sgn(x) \cdot Sh_{k-1}(x) + \alpha_3 \cdot Sh_k(x) + \alpha_4 \cdot sgn(x) \cdot Ch_{k+1}(x).$$

The subspace of eigenfunctions of the operator ∇ is two-dimensionally. The eigenfunctions of the operator ∇ with the unit eigenvalue have the following form

$$\phi_1(x) = \alpha_1(2k \cdot Ch_k(x) + sgn(x) \cdot Ch_{k+1}(x)) + \alpha_2(Sh_k(x) + 2k \cdot sgn(x) \cdot Sh_{k-1}(x)).$$

Corollary 2. *The eigenfunctions of operators ∇ and ∇^2 with the λ-eigenvalue have the following form ($\lambda \neq 0$)*

$$\phi_\lambda(x) = \phi_1(\lambda x), \qquad \phi_\lambda^*(x) = \begin{cases} \phi_1^*(\sqrt{\lambda} x), & \lambda > 0, \\ \phi_1^*(i\sqrt{-\lambda} x), & \lambda < 0 \end{cases}$$

correspondingly.

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Active control of surface plasmon polaritons pulse dynamics

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Surface plasmon polaritons (SPP), propagating coupled oscillations of electrons and light at a metal surface, have great potential as information carriers for next-generation, highly integrated nanophotonic devices. The control of SPP dynamics has been achieved by making use of transient optical nonlinearities in metals via strong excitation with ultrashort laser pulses (pump) [1–3].

It is experimentally demonstrated that SPP pulse can be used as a pump also, and much smaller intensity for the same nonlinearities values is needed as compared to the laser pump. SPP pulse propagation produces the temporal and spatial changes of metal permittivity. The typical times of changes are of 500 fs order. The permittivity variation leads to the temporal and spatial dependences of the SPP probe's dispersion law. The variation of the dispersion law and therefore of the probe SPP group velocity are determined by the pump intensity and the probe time delay. Thereby, probe SPP pulse can be accelerated or decelerated depending on its central frequency.

Usage of a periodic plasmonic structures or 'plasmonic crystals' opens the new possibilities for active control of SPP dynamics. If the SPP probe propagates in plasmonic crystal, it is possible the probe could be decoupled to light or to be reflected under the influence of the pump, while the absence of the pump could not.

Thus, ultrafast active control of SPP dynamics can be realized by variation of the intensity of the pump SPP and the time delay between the pump and probe SPP pulses.

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Altered approach to shortwave diffraction by a prolate body

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Recently several papers, see [1–3] and references therein, were published in which a new method of shortwave diffraction by a prolate smooth strictly convex body is proposed and developed. The authors consider an axis-symmetrical problem for a body of revolution and construct the wave field in a vicinity of light-shadow boundary on the body surface, the so-called Fock's zone, and in the area where creeping waves appear. The imposed conditions on the wave length of the incident field and obstacle provide possibility to use the classical mathematical technique having been developed in the theory of diffraction on a smooth and strictly convex bodies long ago.

The prolate body is defined as follows. Let ρ and ρ_t be the radiuses of curvature of the body surface in its longitudinal and in the transversal sections, respectively, and defined by k the wave number of the incident wave. It is assumed that $k\rho$ and $k\rho_t$ are large parameters and the prolate body is characterized by the condition $\rho/\rho_t = (kL)^\alpha$ for some $\alpha > 0$ and L is a parameter having the length dimension so that right-hand side of the equation becomes dimensionless. Thus, it turns out that the ratio of the radiuses ρ and ρ_t is considered to be a new large parameter as $k \to \infty$. Note that this characteristic of prolate body links the internal geometric parameters of the scatterer with the wave number of incident wave field. Under these conditions, the authors derived a new parabolic equation in the Fock's zone, which is essentially differs from Fock's parabolic equation. However, the scales of the variables s, the arc length along the geodesic, and n, the normal to the surface, remain the same, i.e. $k^{1/3}s$ and $k^{2/3}n$ are $O(1)$ as $k \to \infty$.

In our opinion, such an approach to the shortwave diffraction by a prolate scatterer does not seem convincing and we propose another, altered approach based on two scaled asymptotic expansion. Namely, we suggest to consider the ratio ρ/ρ_t as independent on k large parameter. As a consequence, we do not need to develop a new theory for diffraction problem for prolate body in the Fock's zone and for the creeping waves as all calculations immerse into classic theory of short wave diffraction by a smooth strictly convex body.

In the presentation we demonstrate our approach on the diffraction problem of incident plane wave by a body of revolution (axis symmetry case). Along with that, we indicate and discuss the real problems which can appear in diffraction problems for prolate body in case of arbitrary incidence angle of a plane wave.

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Further generalizations of the Bateman solution. Novel wave beams and wave packets

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Relatively undistorted solutions of the wave equation $u_{xx} + u_{yy} + u_{zz} - \frac{1}{c^2}u_{tt} = 0$, $c = const$, are defined by

$$u = gf(\theta), \tag{1}$$

where the *amplitude* g and the *phase* θ are fixed functions of x, y, z, t and the *waveform* f is an arbitrary function of one variable. In 1909, H. Bateman presented a particular solution of the form (1) with

$$g = \frac{1}{\beta}, \quad \theta = \alpha + \frac{x^2 + y^2}{\beta}, \tag{2}$$

where $\alpha = z - ct$, $\quad \beta = z + ct$. This solution, having singularities, found no immediate application. In 1980-s its modification by a complex shift $\beta \mapsto \beta - ib$, $b > 0$, was presented, which allows to describe, under a proper choice of the waveform, certain highly localized wave beams and wave packets (see, e.g., a review paper [1]). Replacement of the item $\frac{x^2+y^2}{\beta}$ by a quadratic form $\Phi = p(\beta)x^2 + q(\beta)y^2 + s(\beta)xy$, such that $\operatorname{Im}\Phi$ is positive definite, allows *general astigmatic* beams and packets [2]. This nomenclature is borrowed from the approximate *paraxial* theory, where somewhat similar solutions were earlier described in [3]. Further modification of the phase by adding to θ terms linear in x and y, provides *tilted* beams and packets [4]. *Higher modes*, where the amplitude depends not only on β, but also on x and y, can be considered in a similar way.

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Autoresonant waves in nonlinear dispersive media

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There was constructed a special growing asymptotic solution of nonlinear wave equation with dispersion and small local external autoresonant pumping force. These asymptotic solution is a germ of solitary wave. Properties of the pumped solitary wave depend on the external pumping force. The asymptotic results are confirmed numerically.

Iterative algorithms for computation convolutions of atomic functions including new family $\mathrm{ch}_{a,n}$

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Atomic functions (AF) [1–3] are widely applied in interpolation, boundary-value problems, digital signal and image processing. AF $\mathrm{up}(x)$ is the most known and investigated one. Native generalization of AF $\mathrm{up}(x)$ is $\mathrm{h}_a(x)$ family. Convolutions of these functions $\Xi_n = \underbrace{\mathrm{h}_{n+1} * \cdots * \mathrm{h}_{n+1}}_{n}$ and $\mathrm{cup}(x) = \mathrm{up}(x) * \mathrm{up}(x)$ are important for practice. They are AFs too. In [4–5] an effective iterative algorithms for computation all of these functions were presented.

Consider convolution of two AFs $f(x)$ and $g(x)$ with the same scale parameter and equations

$$f^{(n_f)} = \sum_m c_{f_m} f(ax - b_{f_m}) \qquad \text{and} \qquad g^{(n_g)} = \sum_k c_{g_k} f(ax - b_{g_k}).$$

Fourier transforms of these equations are equations for Fourier transforms of functions $F = \hat{f}$ and $G = \hat{g}$. Multiplication of equations for Fourier transforms gives equation for their product $H(t) = F(t)G(t)$. Reverse Fourier transform according to $\frac{1}{a} e^{\frac{ibt}{a}} F\left(\frac{t}{a}\right) \sim f(ax + b)$ gives equation for convolution $h(x) = f(x) * g(x)$

$$h^{n_f + n_g} = \sum_m \sum_k \frac{c_{f_m} c_{g_k}}{a} h(ax - b_{f_m} - b_{g_k}). \tag{1}$$

Equation (1) means that $h(x)$ is AF. According to [4–5] one can construct iterative algorithm for computation convolution $h(x)$. Algorithm consists of two next steps: iterative construction of self similar sequence with structure based on the form of right-hand side of equation and its summation $n_f + n_g$ times.

A new two-parametric family of AFs $\mathrm{ch}_{a,n} = \underbrace{\mathrm{h}_a * \cdots * \mathrm{h}_a}_{n}$ is a generalization of all enumerated functions $\mathrm{up}(x)$, $\mathrm{h}_a(x)$, Ξ_n and $\mathrm{cup}(x)$. AFs $\mathrm{ch}_{a,n}$ satisfy to the following equation:

$$y^{(n)} = a^{n+1} 2^{-n} \sum_{k=0}^{n} C_n^k (-1)^k y(ax + n - 2k), \qquad a > 1, \quad n = 1, 2, 3 \ldots \tag{2}$$

Note that (2) is a special case of (1). Family $\mathrm{ch}_{a,n}$ allows investigation properties of AFs in unified way. Enumerated AFs are special cases of $\mathrm{ch}_{a,n}$ with fixed values of parameters: $\mathrm{ch}_{2,1}(x) = \mathrm{up}(x)$, $\mathrm{ch}_{2,2}(x) = \mathrm{cup}(x)$, $\mathrm{ch}_{a,1}(x) = \mathrm{h}_a$ and $\mathrm{ch}_{n+1,n}(x) = \Xi_n$. For this family iterative algorithms similar to considered basing on equation (2) can be constructed.

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Inverse problems, trace formulae for discrete Schrödinger operators

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We study discrete Schrödinger operators with compactly supported potentials on $\mathbf{Z}^d$. Constructing spectral representations and representing S-matrices by the generalized eigenfunctions, we show that the potential is uniquely reconstructed from the S-matrix of all energies. We also study the spectral shift function $\xi(\lambda)$ for the trace class potentials, and estimate the discrete spectrum in terms of the moments of $\xi(\lambda)$ and the potential. This talk is jointly with Hiroshi ISOZAKI, Japan.

On a numerical method of solution of a hypersingular integral equation of second kind

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When constructing a mathematical model of diffraction of electromagnetic waves on a lattice which consists of a final number of non-perfectly conducting infinitely thin strips was obtained the following equation

$$hu(y)\sqrt{1-y^2}-\frac{1}{\pi}\int_{-1}^{1}\frac{u(t)}{(t-y)^2}\sqrt{1-t^2}dt+\frac{a}{\pi}\int_{-1}^{1}\ln|t-y|u(t)\sqrt{1-t^2}dt+\frac{1}{\pi}\int_{-1}^{1}K(t,y)u(t)\sqrt{1-t^2}dt=f(y),$$

where h and a are given constants, the function $f\left(y\right)$ is from $C^{1,\alpha}_{[-1,1]}$, the function $K(t,y)$ which has an explicit expression (see [1]) is from the same class for each variable, uniformly with respect to another. The first integral has to be understood in the sense of Hadamard finite part. In this paper was proposed a method of numerical solution of this hypersingular integral equation using quadrature formulas of interpolation type, was proved the existence and uniqueness of solution and the estimation of the convergence rate of approximate solutions to the exact solution was obtained. The proposed method of solution is a modification of the method developed in [2]. The proof of the unique solvability considerably takes into account the expression of $K(t,y)$, uses the technique of parametric representations of pseudodifferential operators [1] and the results obtained in [3].

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Propagation of flexural-gravity waves through multiple straight-line irregularities in an elastic plate floating on water

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The paper continues [1] which was devoted to the propagation of flexural-gravity waves of a small amplitude across a straight-line rigid clamp in an elastic plate floating on water of finite depth.

Exact analytical solution for the motion of flexural-gravity waves was derived. In the present paper we extend our method of solution offered in [1] to the propagation of flexural-gravity waves through multiple parallel straight-line irregularities having arbitrary spacing between them. In comparison with [1], we also expand kinds of irregularities in the plate. In particular, a need of regularization of divergent boundary-contact integrals arises.

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On eigenvalues of the propagation constant for a planar dielectric waveguide

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The propagation of monochromatic polarized electromagnetic TE-waves trough a multilayer planar dielectric waveguide is examined. As is well known if the refractive indexes of layers does'not depend on electromagnetic field strength, the number of electromagnetic waves is finite [1]. We are interested in qualitative characteristics of the spectrum of eigenvalues of effective refractive index (propagation constant) of a waveguide. These eigenvalues can be found from so called dispersion equations, that may be written in different forms [2]. The difficulty while solving a dispersion equation is that its roots — eigenvalues of propagation constant may be very close to each other.

Recently [3] the author has proposed a geometrical interpretation of dispersion equation, obtained by the method of characteristic matrices [4]. This interpretation is based under consideration of families of successive linear transformations of hodograph of a definite vector function in the plain. The dispersion equation is represented as an equality to zero of the scalar product of two such vector functions. Analyzing this geometrical pattern one can estimate distance between nearest possible values of propagation constant. In the case of regular waveguide consisting of two kinds of layers with refraction indexes $n_0 = 1$ and $n > 1$ (the two outer infinite layers have refractive index equal to 1) and thicknesses t_0 and t respectively, this distance diminishes exponentially with the growth of t_0. In this case the following pattern is typical. Let such waveguide consisting of only one wave guiding (with refractive index n) layer admits m TE-modes. Then the waveguide including N wave guiding layers admits mN TE-modes, that can be grouped in m clusters. In each cluster of N modes the propagation constants may be extremely close to each other. The less is the average propagation constant of the cluster the closer are the constants inside the cluster. The distances between the constants in one cluster are approximately equal. If the layers of a waveguide are of different refractive indexes then the difference between eigenvalues in one cluster may differ greatly.

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Anisotropic generalization of the theory of acoustic beams using local ellipsoidal/hyperboloidal approximation for the slowness surface

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The theory of propagation, diffraction, and focusing of acoustic beams in anisotropic media is of interest for the analysis and interpretation of experimental data as well as for the design of acoustic devices in various scientific and technical fields such as acoustoelectronics, acoustooptics, acoustic microscopy, ultrasonic non-destructive testing and seismology. The plane-wave decomposition of acoustic beams is the most commonly used method for these types of studies. This method can give accurate numerical predictions of spreading and focusing of the wave beams. However such numerical solutions do not provide insight into the underlying physics of the wave phenomena. In the present paper a generalization of the parabolic-equation method is proposed. The generalized method allows one to obtain analytical solutions for acoustic Gaussian beams propagating in an arbitrary direction of anisotropic solid media, with one exception of directions in which the curvature of the acoustic slowness surface tends to zero. As a first step to search for anisotropic solution for wave beams, we consider the problem to derive the paraxial wave equation. For this purpose the equation for an ellipsoid is substituted into Green–Christoffel equation that allows us to obtain explicit analytical expressions for the coefficients of the local ellipsoid approximation (hyperboloid approximation in the case of locally concave surface) of the slowness surface for acoustic waves in the case of arbitrary anisotropy. After the ellipsoid coefficients have been found, the problem of anisotropic beam propagation is reduced to an equivalent isotropic case by the following procedure. First, let us rotate the coordinate system in such a way that its axes will coincide with the principal axes of the ellipsoid. Second, the ellipsoid is compressed or stretched along two main axes, so that it becomes a sphere. Third, by an additional rotation of the coordinate system the wave vector is brought into coincidence with one of the coordinate axes. It should be mentioned, however, that the deformation of the coordinate space has to be accompanied by the inverse deformation of the phase space, keeping unchanged the phase shifts of the plane waves which compose the wave beam. Now the well-known solution for the Gaussian beams in the equivalent isotropic media can be used. Then, the anisotropic solution is obtained by performing all of the described transformations in the inverse order. The generalized parabolic equation method has been used by us for simulating two well-known experimental observations of anisotropic beam steering [1] and anisotropic diffraction [2]. The developed method demonstrates good agreement between the calculated and observed wave parameters and the structure of the acoustic beams.

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The main vectors among those used for description of electromagnetic field

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J.K. Maxwell has introduced four vectors to describe electromagnetic field: strength **E** and flux density **D** of electric part of electromagnetic field, as well as strength **H** and flux density **B** of its magnetic part. To solve solutions for Maxwell equations it is necessary and sufficient to use a pair of vectors only: one from electric part, and another from magnetic part. Maxwell did not identify any pair of vectors, therefore the specific pair was selected by his followers in their own way. M. Abraham selected **E** and **H** pair, G. Minkowsky — **D** and **B**, G. Lorentz — **D** and **H**, J. Stratton — **E** and **B**. In this connection the treatise [1] emphasizes that presently the selection of any pair from these vectors in electrodynamics is a matter of taste, at that the author himself prefers **E** and **B** pair.

Recent works [2, 3, 4] note that in tasks associated with field/substance electromagnetic interaction there is a difference in representation of momentum density of electromagnetic field, either in Abraham or Minkowsky form; at that work [2] reveals relativistic noninvariance of tensor of energy and momentum in the Abraham form. Work [5] demonstrates that the representation form of momentum density and tensor of energy and momentum, as well as expressions for electromagnetic forces acting on bodies are completely dictated by the selection of vectors' pair for electromagnetic field description. In general case values of forces acting on bodies differ from each other for each of pairs, but in statics they coincide, and they coincide at the average for the period with respect to periodic processes. However, work [5] fails to identify the preferred pair of field vectors in terms of this 'force' point of view. Definite answer for this question has been found based on energy point of view — from analysis of physical meaning of Pointing formula representations for each of pairs under consideration.

Analysis relying on work outcomes [6] demonstrates that only by description of electromagnetic field with **E** and **H** vectors it is possible to identify field energy density and substance energy density in Pointing formula. The field energy density is agreed with Maxwells' definition. The substance energy density is defined from the most general laws of field/substance interaction assumed in classical electrodynamics, i.e. including effects of relaxation and resonance absorption. Only in the case of using **E** and **H** pair integral electromagnetic energy density can be correctly determined. This integral electromagnetic energy is acting as free energy in statics. Principle of minimum of free energy in statics results in Euler equations, which include all equations of electro- and magnetostatics, in particular London brothers' equation [7] determining current distribution in superconductor.

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Concept of sources in electrodynamics and some of its applications

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The concept of sources in electrodynamics consists in consideration of specific role of vectors determining electromagnetic parameters of the substance, such as electric current density **j**, polarization **P** and magnetization **J**. The specific role of **j**, **P** and **J** values mainly consists in the fact that they may be considered as sources of electromagnetic field. In case these values are specified as limited functions of time t and point of area V, occupied by the substance, then power and induction characteristics of electromagnetic field, i.e. values of vectors **E**, **H**, **D** and **B** in the whole spase, may be expressed in the form of integrals of **j**, **P** and **J** and known functions over V field based on results obtained by J. Stretton [1]. This is implemented in work [2] for the case of periodic electro-magnetic processes.

Representation of field vectors through sources facilitates formulation and solution of a number of electrodynamics' problems, in particular, enables reducing diffraction problem to integral equations over V area occupied by the substance. To this end it is sufficient to use laws of induction interaction between electromagnetic field and substance generally expressed in the form of differential equations, between field sources and vectors. Such approach is different from traditional one [3], when values $\mathbf{j}$, $\mathbf{P}$ and $\mathbf{J}$ are excluded from consideration, to the contrary.

The concept of electromagnetic field sources traces back to magnetostatics' work of S. Poisson [4] considered as fundamental one by Maxwell. Thereafter the concept is developed in the above-mentioned work of J. Stretton investigating periodic process at $\mathbf{P} = 0$ and $\mathbf{J} = 0$, O.V. Tozony and I.D. Mayergoiz [5, 6] and other works of Kiev electrotechnics school.

This work expands results [2] on representation of vectors of electromagnetic field strengths and flux densities in the form of integrals of sources, specifies possibilities of results' practical application.

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Integrated nonparametric estimations of probability density of stochastic processes by atomic functions

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In this report on the basis of atomic functions (AF) [1–4] and results of works [5–7] integrated nonparametric estimations of probability density and its derivatives [5–8] are offered. Such approach allows to receive more smooth estimations of probability density function that improve efficiency of the decision of problems of image classification and recognition. The mathematical apparatus of integrated nonparametric statistics allows to estimate characteristics of sequences more precisely, without having the aprioristic parametrical information.

Integrated probability density estimation. Let $X_1, X_2, ..., X_n$ is the sample of n independent realization of a random variable $X \in \Omega(X)$ with unknown PDF $f(x)$. As approach under the empirical data of required density $f(x)$ take the following statistics:

$$\bar{f}_n(x) = \frac{1}{nh} \sum_{j=1}^{n} \int_{\Omega(u)} K\left(\frac{X_j - x - \beta u}{h}\right) w(u)\, du. \tag{1}$$

Here, the functions $K(x) \in H$ and $w(x) \in H$, where H is functional conditions [5–7], $h = h(n)$ is some sequence of positive numbers $\lim\limits_{n\to\infty} h(n) = 0$, $\lim\limits_{n\to\infty} nh(n) = \infty$. By transforming of (1) will receive expression in the form of functions $K(x)$ and $w(x)$ convolution

$$\bar{f}_n(x) = \frac{1}{nh} \sum_{j=1}^{n} \int_{\Omega(u)} K\left(\frac{X_j - x - \beta u}{h}\right) w(u)\, du = \frac{1}{nh} \sum_{j=1}^{n} \left[K\left(\frac{X_j - x}{h}\right) * \frac{w(x/\beta)}{\beta h}\right] \tag{2}$$

Considering that $w(\infty) = 0$ and properties of H let's open uncertainty $\lim\limits_{\beta\to 0} \dfrac{w(x/\beta)}{\beta h} = \delta(x)$. Therefore at $\beta = 0$ and $\forall\ w(x) \in H$ we receive an estimation identical to Parzen type [5–7]. Choosing $w(x) \in H$ receive wider class of nonparametric estimations. As $w(x)$ the any function, satisfying to conditions H can be chosen for example

$$w(x) = \begin{cases} 1/2b & |x| < b, \\ 0 & |x| \geq b. \end{cases}$$

Here, b characterizes the width of this function. Also as $w(x)$ one of the weight functions [1] or WA-systems of Kravchenko and Kravchenko–Rvachev functions [2, 3] can be used.

Conclusions. On the basis of AF the new constructions of the WF with the compact support are offered and proved. On their basis integrated nonparametric probability density estimations and also its derivative of 1st and 2nd orders are constructed. As has shown numerical experiment using of the integrated approach allows to receive more smooth estimations. The physical analysis of nonparametric estimations of probability density function by means of new WFs constructed on the basis of AF families confirm their efficiency.

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New constructions of digital filters synthesis on base of generalized Kravchenko–Kotelnikov sampling theorem

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In this report the new constructions of digital filters are offered. They are based on the generalized Kravchenko–Kotelnikov Sampling Theorem. Efficiency of the offered low- and high- frequency filters is investigated on an example of digital processing of signal of the various physical nature.

Construction of the filter with the finite pulse response (FIR). For signal processing [1–5] the choice of weight function (WF) is caused by certain requirements. Weight function should provide zero shift of a phase $H(f) = H^*(f)$, i.e. its pulse response is symmetric concerning the beginning of coordinates $h[x] = h[-x]$. The frequency characteristic of the filter $H_0(f)$ is represented by Fourier series of a kind

$$H_0(f) = \sum_{x=-\infty}^{\infty} h_0[x] e^{-jfx}, \tag{1}$$

where the coefficients $h_0[x]$ are defined by means of expression $h_0[x] = \frac{1}{4\pi^2}\int_{-\pi}^{\pi} H_0(f)\, e^{jfx}df$. Here, $h_0[x]$ are the coefficients of the infinite pulse response (PR) of the filter corresponding to the set frequency characteristic $H_0(f)$. For realization of the FIR filter the summation limits in (1) should be limited. For improvement of convergence of truncated series (1) the coefficients $h_0[x]$ it is necessary to multiply on finite WF $w[x]$ $h[x] = w[x] \cdot h_0[x]$. For filters with zero phase shift WF also should satisfy to a condition $w[x] = w[-x]$. Thus the coefficients $\tilde{h}_0[x] = \frac{w[x]}{4\pi^2}\int_{-\pi}^{\pi} H_0(f)\, e^{jfx}df$ and for the frequency characteristic we have $\tilde{H}_0(f_1, f_2) = \sum_{x=N_1}^{N_2} \tilde{h}_0[x]e^{-jfx}$. Here N_1,N_2 are some summation limits. The ideal filter of the low frequencies is Kotelnikov function $w(x) = \operatorname{sinc}\left(\frac{\pi x}{\Delta}\right)$. Where Δ is step of discretization (or smoothness parameter). The more the size Δ then function more wide. The advantages are simplicity of calculations and in the limit the ideal form of the filter. But the ideal filter of the low frequencies turns out only at definition of PR on the infinite interval. At narrowing of the interval of PR definition the Gibbs Effect sharply amplifies.

The Generalized Kravchenko–Kotelnikov Sampling Theorem. On the basis of AF [1] the Generalized Kravchenko–Kotelnikov Sampling Theorem are constructed. This approach meeting all requirements of the Kotelnikov Theorem possesses the best convergence, especially at restoration of localized in time signals. For Kravchenko–Kotelnikov generalized function it is possible to choose the compact support without of appreciable distortion of a spectrum. It does predicted frequency properties of the synthesized filters. Corresponding high-band filter is synthesized by frequency modulation [1–4]. We receive system of two filters which sum completely represent required frequencies area [5].

Conclusions. The new designs of systems of the digital filters based on the Generalized Kravchenko–Kotelnikov Sampling Theorem are offered. Their efficiency is investigated on an example of digital processing of signals of the various physical nature. The similar approach can be realized on basis of WA-systems of Kravchenko and Kravchenko–Rvachev functions [2–5].

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Excitation of electromagnetic waves by a pulsed ring electric current in a magnetoplasma containing a cylindrical density duct

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Electromagnetic radiation from monochromatic sources immersed in homogeneous and inhomogeneous magnetized plasmas has been analyzed in many papers (see, e.g., [1, 2] and references therein).

Over the past decade, there has been shown a substantial degree of interest in the excitation and propagation of nonmonochromatic waves in a magnetoplasma [3–5]. Much previous theoretical work on the subject is focused on studying the fields and radiation characteristics of pulsed sources immersed in a homogeneous magnetoplasma. However, there exists a very little theory of the radiation from nonmonochromatic sources located in cylindrical magnetic-field-aligned density irregularities, commonly known as density ducts [2]. Note that such ducts can arise near electromagnetic sources in a magnetoplasma due to various nonlinear effects and are capable of guiding electromagnetic waves in some frequency ranges.

In this work, we study the energy radiation characteristics of a pulsed ring electric current placed coaxially in a cylindrical density duct that is surrounded by a uniform cold magnetoplasma. At first, we determine the total field excited by such a source. To describe the temporal behavior of the field, the Laplace transform technique is employed. The spatial structure of the source-excited field is represented in the form of an eigenfunction expansion over the discrete- and continuous-spectrum waves of a density duct [2]. Then, using the field representation obtained, we derive an expression for the total radiated energy and analyze its distribution over the spatial and frequency spectra of the excited waves as a function of the source and duct parameters. Detailed numerical calculations of these characteristics have been performed for the case where the frequency spectrum of the current is concentrated in the resonant part of the whistler range and the plasma density inside the duct is enhanced relative to the background medium. It is shown that the presence of a duct with enhanced plasma density can lead to a significant increase in the energy radiated from a pulsed ring current compared with the case where the same source is immersed in the surrounding uniform magnetoplasma. The predominant contribution to the radiated energy is found to be ensured by slightly leaky modes that are guided by such a duct in the resonant part of the whistler range and play an important role in many promising applications [2]. The results obtained can be useful in interpreting the data of space and laboratory experiments on electromagnetic wave excitation by pulsed sources in a magnetized plasma medium containing magnetic-field-aligned density irregularities.

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Homogenization of the twodimensional periodic Dirac operator

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In $L_2(\mathbb{R}^2;\mathbb{C}^2)$ we consider the Dirac operator $\mathcal{D}_\varepsilon$, $\varepsilon > 0$, with a rapidly oscillating magnetic potential:

$$\mathcal{D}_\varepsilon = (D_1 - A_1(\mathbf{x}/\varepsilon))\,\sigma_1 + (D_2 - A_2(\mathbf{x}/\varepsilon))\,\sigma_2 + m\sigma_3, \quad \operatorname{Dom}\mathcal{D}_\varepsilon = H^1\left(\mathbb{R}^2;\mathbb{C}^2\right).$$

Here $\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, $\sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$, $\sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ are the Pauli matrices, m is a real constant. We assume that $\mathbf{A}\left(\mathbf{x}\right) = (A_1(\mathbf{x}), A_2(\mathbf{x}))$ is a vector-valued (magnetic) potential in $\mathbb{R}^2$, which is periodic with respect to some lattice Γ and such that $\mathbf{A} \in L_p\left(\Omega; \mathbb{C}^2\right)$, $p > 2$, $\operatorname{div}\mathbf{A} = 0$, $\int\limits_\Omega \mathbf{A}(\mathbf{x})\, d\mathbf{x} = 0$. Here Ω is the elementary cell of Γ.

We study the behavior of the resolvent $\left(\mathcal{D}_\varepsilon - iI\right)^{-1}$ for small ε. It turns out that $\left(\mathcal{D}_\varepsilon - iI\right)^{-1}$ converges to $\left(\mathcal{D}_0 - iI\right)^{-1}$ in the operator norm, where $\mathcal{D}_0 = D_1\sigma_1 + D_2\sigma_2 + m\sigma_3$ is the "free" Dirac operator.

Theorem 1. *Under the above assumptions, we have*

$$\left\| \left(\mathcal{D}_\varepsilon - iI\right)^{-1} - \left(\mathcal{D}_0 - iI\right)^{-1} \right\|_{L_2(\mathbb{R}^2;\mathbb{C}^2)\to L_2(\mathbb{R}^2;\mathbb{C}^2)} \leqslant C_1\varepsilon. \tag{1}$$

The constant C_1 depends only on m, $\|A\|_{L_p(\Omega)}$ and parameters of the lattice Γ.

Also we study the Dirac operator $\mathcal{D}_{\varepsilon,sing}$ with a singular magnetic potential:

$$\mathcal{D}_{\varepsilon,sing} = \left(D_1 - \frac{1}{\varepsilon}A_1(\mathbf{x}/\varepsilon)\right)\sigma_1 + \left(D_2 - \frac{1}{\varepsilon}A_2(\mathbf{x}/\varepsilon)\right)\sigma_2 + m\sigma_3, \quad \operatorname{Dom}\mathcal{D}_{\varepsilon,sing} = H^1\left(\mathbb{R}^2;\mathbb{C}^2\right).$$

Potential $\mathbf{A}\left(\mathbf{x}\right)$ is the same. It turns out that the resolvent $\left(\mathcal{D}_{\varepsilon,sing} - iI\right)^{-1}$ can be approximated by the resolvent of the effective operator $\mathcal{D}_{0,sing} = \sqrt{\gamma}\mathcal{D}_0$ sandwiched between some rapidly oscillating factors $f^\varepsilon\left(\mathbf{x}\right) = f\left(\mathbf{x}/\varepsilon\right)$. In order to define γ and f we introduce the Γ-periodic solution $\varphi(\mathbf{x})$ of the system $\{\nabla\varphi(\mathbf{x}) = (A_2(\mathbf{x}), -A_1(\mathbf{x}))\,, \int\limits_\Omega \varphi(\mathbf{x})d\mathbf{x} = 0\}$. Then $\gamma = \dfrac{(\operatorname{mes}\Omega)^2}{\left(\int\limits_\Omega e^{2\varphi(\mathbf{x})}d\mathbf{x}\right)\left(\int\limits_\Omega e^{-2\varphi(\mathbf{x})}d\mathbf{x}\right)}$ and $f(\mathbf{x}) = \operatorname{diag}\left\{ e^{-\varphi(\mathbf{x})}\left(\dfrac{\operatorname{mes}\Omega}{\int\limits_\Omega e^{-2\varphi(\mathbf{x})}d\mathbf{x}}\right)^{\frac{1}{2}}, e^{\varphi(\mathbf{x})}\left(\dfrac{\operatorname{mes}\Omega}{\int\limits_\Omega e^{2\varphi(\mathbf{x})}d\mathbf{x}}\right)^{\frac{1}{2}} \right\}$.

Theorem 2. *Under the above assumptions, we have*:

$$\left\| \left(\mathcal{D}_{\varepsilon,sing} - iI\right)^{-1} - f^\varepsilon\left(\mathcal{D}_{0,sing} - iI\right)^{-1} f^\varepsilon \right\|_{L_2(\mathbb{R}^2;\mathbb{C}^2)\to L_2(\mathbb{R}^2;\mathbb{C}^2)} \leqslant C_2\varepsilon. \tag{2}$$

The constant C_2 depends only on m, $\|A\|_{L_p(\Omega)}$ and parameters of the lattice Γ.

Estimates (1) and (2) are order-sharp. We use the results of M. Birman and T. Suslina [1–3] for homogenization problems in $\mathbb{R}^d$, in particular, the result for homogenization of the twodimensional Pauli operator. The key point is representation of the Pauli operator as the square of the Dirac operator.

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Self-similar asymptotics describing nonlinear waves in elastic media with dispersion and dissipation

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Solutions of problems for the system of equations describing weakly nonlinear quasi-transverse waves in an elastic weakly anisotropic medium are studied analytically and numerically. It is assumed that dissipation and dispersion are important for small-scale processes. Dispersion is taken into account by terms involving the third derivatives of the shear strains with respect to the coordinate, in contrast to the previously considered case when dispersion was determined by terms with second derivatives. In large-scale processes, dispersion and dissipation can be neglected and the system of equations is hyperbolic. The indicated small-scale processes determine the structure of discontinuities and a set of admissible discontinuities (with a steady-state structure). This set is such that the solution of a self-similar Riemann problem constructed using solutions of hyperbolic equations and admissible discontinuities is not unique. Asymptotics of non-self-similar problems for equations with dissipation and dispersion were numerically found, and it appeared that they correspond to self-similar solutions of the Riemann problem. In the case of nonunique self-similar solutions, it is shown that the initial conditions specified as a smoothed step lead to a certain self-similar solution implemented as the asymptotics of the unsteady problem depending on the smoothing method.

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Diffraction of terahertz wave packets of a few oscillations of electrical field

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In the past few decades it was the rapid growth of optical systems, laser technology and in particular, femtosecond lasers systems. In many laboratories around the world there are laser systems capable of generating radiation, which consists of few oscillations of the electromagnetic field. The temporal spectra of these pulses are wide enough, but because of the small size of the radiation source or by self-focusing effect, in which the transverse pulse compression occurs, the spatial spectrum can also be very broad. In addition, using the phenomena of photoconductivity of semiconductors one can generate a radiation of terahertz frequencies. Studies of physics and technology of THz radiation have begun long ago, but only with the development of femtosecond optics and microelectronics in this area there has been a significant shift, and increased interest in basic and applied research, published the first monographs [1, 2].

With the growing interest in terahertz radiation due to the emergence of systems of generation and detection of such radiation and the appearance of application perspectives, such as security (scanners) and biomedicine (tomography), in the present work it was done the calculation of the dynamics of electromagnetic fields with the use of equations for calculating the dynamics of spatial-temporal spectra of nonparaxial waves. The spectral equations are derived from the equations of classical optics — from Maxwell's equations. Usability of this particular approach to the analysis and theoretical description of the dynamics of electromagnetic fields is due to the possibility of analysis light beams with ultra-wide spatial spectra as well as ultra-wide temporal spectra.

Objectives:
– Creating the software (MTBeam) that can perform the calculation and visualization of the diffraction of light pulses consisting of a few oscillations with the ability to work remotely on a supercomputer.
– Testing of the resulting software on the experiments on the diffraction of terahertz wave packet on an object with amplitude-phase transmission and also on the terahertz wave-front restoration experiments.

The basic functional of the software system MTBeam:

- cross-platform system (running the program on a PC with any operating system)
- opportunity to present the results in 1D,2D,3D graphs
- the ability to run the calculations remotely on a supercomputer
- extensibility by connecting outside plug-ins with the additional functionality

The developed software is based on a shortened system of equations to describe the spectral diffraction [3] only for the direct wave (backward wave and the associated effects are not included).

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On integral properties of steady gravity waves on water of finite depth

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In the mid-1970s, Longuet-Higgins initiated studies of integral properties of steady gravity waves on water of uniform depth (only solitary and Stokes waves were known at that time), and used these properties in his numerical computations of wave characteristics. An approach described here allows us to derive in a unified manner a certain integral property for all steady waves on water of finite depth. (Notice that periodic waves with more than one crest per period have been discovered by now, whereas Longuet-Higgins considered only solitary waves and Stokes waves on deep water.) A modified form of Bernoulli's equation is applied; this equation has many other applications, in particular, it provides necessary conditions for the existence of non-trivial waves. The main part of material is based on a joint work with Vladimir Kozlov, Linköping University, Sweden.

On trapping of time-harmonic water waves by a system of axisymmetric surface-piercing bodies floating freely

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We investigate the coupled problem that describes the time-harmonic motion of small amplitude executed by the following system. It consists of water that occupies a half-space modelling an open sea and several bodies floating freely, that is, in the absence of external forces other than gravity (for example, due to constraints). The frequency of oscillations, that appears in the boundary conditions as well as in the equations governing the bodies' motion, serves as the spectral parameter of the problem.

In [1, 2], the problem of a single freely floating body was studied, but here we deal with the case of multiple bodies. The approach used in [2] and based on an inverse procedure is developed further, thus allowing us to obtain the following main results: (a) any value of frequency is an eigenvalue of the problem for some system of bodies that have special geometry (the set of admissible structures is infinite); (b) the corresponding eigenfunctions (trapped modes) are constructed explicitly for systems that consist of an arbitrary number of bodies; (c) in the obtained examples all bodies are surface-piercing and have axisymmetric submerged parts, each toroid of a trapping system is either motionless (the only case considered in [2]) or executes time-harmonic heave motion.

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New version of modified method of discrete sources for solving the problem of wave diffraction on a group of impedance bodies of revolution

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The modified method of discrete sources (MMDS), offered in works [1, 2], has been subsequently applied to the solution of a wide class of problems of the theory of diffraction, and high efficiency of the method [1–4] has been shown in all cases. The main idea of the method consists in uniform way of construction of the carrier of discrete (auxiliary) sources by means of analytical deformation of a border of a scatterer. In the mentioned papers the spherical coordinates have been used to construct the carrier of discrete sources. In the present work we use the elongated or flattened spheroidal coordinates or toroidal coordinates to solve the scalar problem of diffraction of the plane wave on two coaxial impedance bodies of revolution. At such approach, first, it is possible to get high accuracy of calculations for the problem of wave diffraction on strongly elongated or flattened bodies and bodies of toroidal form. Secondly, the class of bodies to which MMDS is applicable can be extended, in particular, diffraction on various multi-lobes of revolution is considered in the paper [5].

Let the group of two bodies of revolution is located on one axis and bounded by surfaces S_1 and S_2. We choose the system of coordinates so that the axis z coincides with the axis of revolution of the bodies. Assume, that the impedance boundary condition on the surfaces of the scatterers is satisfied:

$$u = Z_p \frac{\partial u}{\partial n}, \quad p = 1, 2, \tag{1}$$

where Z_p is the impedance on the surface S_p $\partial/\partial n$ is the derivative along the outward normal. The secondary field, everywhere outside of the domains of the bodies, satisfies to homogeneous Helmholtz equation and the attenuation condition at infinity.

The main steps of calculations using standard MMDS are described in [3, 4]. In this paper we explain the choice of the auxiliary surfaces. Let's introduce the local systems of coordinates connected with each of the scatterer. We assume that the surfaces S_1 and S_2 are stated in the elongated spheroidal coordinates by the equations

$$x_p = f \sinh \alpha_p \sin \beta_p \cos \varphi, \quad y_p = f \sinh \alpha_p \sin \beta_p \sin \varphi, \quad z_p = f \cosh \alpha_p \cos \beta_p, p = 1, 2, \tag{2}$$

where $\alpha_p = \alpha_p(\beta_p)$, $\beta_p \in [0, \pi]$, f is the scale coefficient. The auxiliary surfaces are defined by the relations

$$\alpha'_p = \operatorname{Re} \eta_p, \quad \beta'_p = \operatorname{Im} \eta_p, \quad \eta_p(\beta_p) = \alpha_p(\beta_p + i\delta_p) + i(\beta_p + i\delta_p), \tag{3}$$

where $\left(\alpha_p',\ \beta_p',\varphi\right)$ are the spheroidal coordinates of the "image" of the point with coordinates $(\alpha_p,\ \beta_p,\varphi)$ on the initial surface of the scatterer. δ_p is the positive parameter responsible for the degree of deformation of the contour of the p-th body axial cross section. If one need to get the Cartesian coordinates of the point on the auxiliary surface it is necessary to use the formulas

$$x_p' = \operatorname{Im}\xi_p\cos\varphi,\ \ y_p' = \operatorname{Im}\xi_p\sin\varphi,\ \ z_p' = \operatorname{Re}\xi_p$$

where $\xi_p(\beta_p) = f\,\cosh\eta_p(\beta_p)$. The other steps of the algorithm are similar to those described in [3–5].

To test the method we compute the residual of boundary condition on the surfaces of the bodies for the group of two flattened ideal spheroids with the axes ratio 1:25. The maximum level of the residual is not exceed $5\cdot 10^{-6}$ when the number of discrete sources is equal to 250. Using the discribed technique we also compute the pattern of the group of so-called chebyshev particles and toroidal particles [5].

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Zig-zag – armchair junction of nanotubes: the spectrum of quantum graph model

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Carbon nanotubes are the most promising structure for modern nanoelectronics. The quantum graph model of junction of "zig-zag" and "armchair" single-wall carbon nanotubes is considered. The structures of such type (particularly, connection of single-wall nanotubes of zig-zag type $(2N,0)$ and armchair type (N,N)) were synthesized few years ago [1]. The nanotubes coupling causes the disturbance of the crystal structure. As a result, one obtains additional electron refraction at the interface. Hence, the conductivity changes and localized states concentrated near the interface appears. It can be the background for creation of new nanoelectronic devices (diode, transistor, etc.)

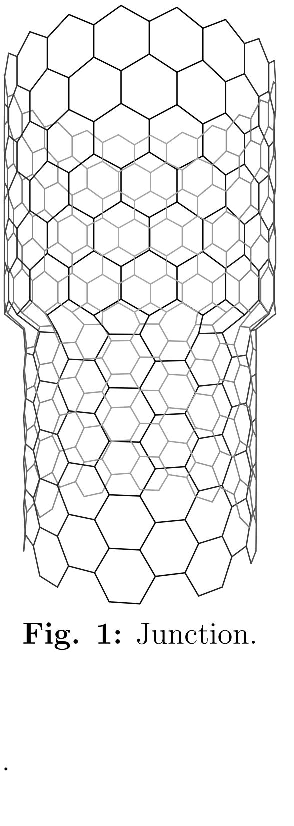

Fig. 1: Junction.

There are few mathematical models for description of transport and spectral properties of such structures. We use the quantum graph model of the system. This model is explicitly solvable and allows us to find the transfer matrix explicitly and to describe the spectrum completely. The model is analogous to that suggested in [2]. A condition on appearance of states localized near the interface is provided. All localized states are explicitly calculated.

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The propagation above the crest of the tsunami waves generated by localized source

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In [1]–[3], formulas for asymptotic solutions of a Cauchy problem with localized initial data for the wave equation with variable velocity were obtained; moreover, in [4]–[6], similar formulas were obtained for the linearized system of shallow-water equations,

$$\frac{\partial \eta}{\partial t} + div(C^2 u) = 0, \quad \frac{\partial u}{\partial t} + \nabla \eta = 0, \quad C(x) = \sqrt{g \cdot D(x)}, \quad x = (x_1, x_2) \in \mathbb{R}^2, \tag{1}$$

$$\eta|_{t=0} = \eta^0 \left(\frac{x}{\mu}\right), \quad \eta_t|_{t=0} = 0. \tag{2}$$

Here $\eta(x,t)$ stands for the height of the free surface of the liquid, $D(x)$ for the depth of the basin, g for the acceleration of gravity, and $\eta^0(y)$ is a given function which decays at infinity more rapidly than $|y|^{-\delta}$, where $\delta > 1$. Problem (1)–(2) arises, in particular, when simulating the propagation of tsunami waves in the ocean ([7], [8]). The asymptotic formulas constructed in [4]–[6] are the base for a fast analytic-numerical algorithm for solving problem (1)–(2). As was shown in these papers that, in the course of time a solution decomposes into two parts, namely, a vortex part describing the motion of a solitary vortex (we do not consider this part of the solution here) and a wave part which describes the propagation of a wave localized in a neighborhood of a closed curve (the front) on the plane R^2. Moreover, if the front is represented for sufficiently small times by a smooth curve (close to a circle), then, in the course of time, turning points (focal points) and self-intersection points can occur on the front, and we shall refer to these points as singular points of the front. Here we consider the propagation of the wave front above the extended crest. In this case the number of the focal points can be indefinitely increased.

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Surface plasmon polariton excitation and interference with femtosecond laser radiation of nontraditional polarization

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According to the universal polariton model under an interaction of fs linear polarized laser radiation the laser-induced damage has the character of linear-polarized gratings ($\boldsymbol{g}$) of multipled periods and orientation $\boldsymbol{g}||\boldsymbol{E}_t$, where is tangential projection of electric field vector of laser radiation on the target surface. Recently a number of experimental publications describe the formation of fs laser-induced nanostructures of various orientation and ordering by radiation having nontraditional polarization: azimuthal, radial and circular one [1–3]. But physical mechanisms behind such nanostructures formation are unknown still.

We suggest mechanism of discussed structures formation based on universal polariton model as an interference of incident laser radiation of either of three above mentioned polarizations with excited surface plasmon polaritons. It is shown that for the radially polarized radiation the produced nanostructures must have azimuthally symmetry and for azimuthally polarized radiation must have nanostructures of radial symmetry. In either case the direction of propagation of excited by incident laser radiation surface plasmon polaritons is along the electric field vector of incident radiation (normal incidence of laser radiation) in closed accordance with theoretic expectations. The experimental examples for nanostructures formations on fused silica [1] and silicon monocrystal [2] for radially and azimuthally polarized laser radiation confirm our theoretical approach.

The interaction of series of pulses of circular polarized femtosecond laser radiation with fused silica followed by chiral three-dimensional helicoidal nanostructures formation having the Segner wheel type cross-section in plane perpendicular to the momentum vector of incident radiation. The wings of this structure were dependant of either clockwise or anticlockwise laser polarization action was used. This result was well explained in the framework of universal polariton model.

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Interaction between electron beam and cavity eigenfields in virtual cathode systems

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Microwave oscillators, in which an electron beam with the virtual cathode (VC) is used as an active medium — the so-called vircators and reflex triodes — are promising devices of relativistic electronics. The VC devices attract an interest of researchers first of all because they are characterized by simple design, high level of power (from hundreds of megawatt to thousands of gigawatt), compactness and the absence of an external magnetic field [1]. Radiation efficiency of the devices depends on interaction between electron beam and cavity eigenfields [2]. In the paper the efficiency of this interaction is investigated from point of view spontaneous radiation theory. This method allows define mode structure and power of radiation taking into account various physical and geometric system parameters. Two configurations of devices, developed in Nuclear Physics Institute of Tomsk Polytechnic University are considered in the work: coaxial and flat-coaxial reflex triode. It is shown that electron beam efficiently interacts with wave E_{01} in coaxial system and with wave H_{11} in flat-coaxial system.

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Theory of sound propagation in suspensions on the basis of the generalized variational principle

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Conception of two interpenetrating continuums, corresponding to the mean mass displacements of solid and liquid phases consisting porous penetrating media, lays in the basis of the Biot's theory [1, 2]. Original Biot's equations are derived with use of Hamiltons variational principle for dissipationless medium and further dissipative terms are introduced by hands into the right parts of these equations. Important element of the Biot's theory is general quadratic forms for kinetic and potential energies, containing the crossing terms. Coefficients of quadratic forms satisfy to some general conditions and finally they are determined by characteristics of waves which are solutions of the Biot's equations. The Biot's theory predicts existence of a single shear wave and a couple of longitudinal waves one of them has a diffusive behavior at low frequencies. In the mentioned form the Biot's theory with its latest modifications was successfully applied for description of porous permeable materials during past five decades.

However, there are questions directly to the basis of the Biot's theory. In particular, why the kinetic and potential energies, which should be additive functions for phases separated in space, contain crossing terms? Formally these terms do not satisfy to this condition. Attempt to answer on this question leads to new interpretation of the Biot's theory.

Consistent approach for description of multiple phase media can be formulated on the basis of generalized variational principle for dissipative continuum mechanics [3, 4]. Important element of this approach is a physical mean of variables in which terms the theory is formulated. The consistent statement has to be formulated in terms of variables dealt with integrals of motion of material points, representing a continuum. The various definitions of mean velocities of phases are available for multiple phase media in dependence on interpretation of temperature. The correct account of this circumstance allows to overcome the mentioned contradiction of the Biot's theory and to evaluate simultaneously some coefficients which should be determined experimentally in the framework of the original Biot's theory.

As illustration of the new approach the simplest double phase medium such as a suspension is considered by uniform way in the report. All coefficients of the developed theory for sound wave propagation in suspension are well determined parameters of constituent phases.

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The interaction of periodic waves with sloping structures

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The results are presented of experimental and numerical study of periodic waves interaction with protective sloping structures. The construction to study is a combination of underwater ledge and a rubble mound sloping wall on it, which ends on a vertical wall. Experimental investigations were produced at the hydro flume with shield wavemaker. Experimental pressure diagrams were obtained along the underwater ledge and the wave run-up values on the vertical wall. There were determined the reduction coefficient and other characteristics of wave interaction with the structure from experimental data. Also numerical studies were performed using models based on linear and nonlinear theories of wave propagation on the water. We analyzed the results and fulfilled the detailed comparison of experimental and numerical data. The relevance of the presented study is justified by the absence of engineering methods for designing of such kind of mixed profile structures.

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Reconstructing the potential for the 1D Schrödinger equation from boundary measurements

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We consider the inverse problem of the determining the potential in the dynamical Schrödinger equation on the interval by the measurement on the whole boundary. Provided that source is *generic* using the Boundary Control method we recover the spectrum of the problem from the observation at either left or right end points. Using the specificity of the one-dimensional situation we recover the spectral function, reducing the problem to the classical one which could be treated by known methods. We adapt the algorithm to the situation when only the finite number of eigenvalues are known and provide the result on the convergence of the method.

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An asymptotic problem for a 2D wave equation with variable velocity and localized right-hand side

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We consider the Cauchy problem for the inhomogeneous wave equation with variable velocity and with a perturbation in the form of a right-hand side localized in space (near the origin) and in time.

$$\frac{\partial^2\eta}{\partial t^2} - \nabla c^2(x)\nabla\eta = \frac{\partial g(t)}{\partial t}V\Big(\frac{x}{\mu}\Big), \quad \eta|_{t=0} = \eta_t|_{t=0} = 0, \qquad t \geq 0, \quad x \in \mathbf{R}^2_x.$$

In particular, this problem is connected with the question about the creation of tsunami and Rayleigh waves. Using abstract operator theory and in particular Maslov's noncommutative analysis, we show that the solution is separated into two parts: the transient one, which is localized in a neighborhood of the origin and decreases in time and the propagating one, which propagates in space like the wave created by the momentary "equivalent source". We present several examples covering a wide range of perturbation resulting in rather explicit formulas expressing the solutions it terms of the error function of complex argument.

Talk is based on the results printed in the paper:

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Properties of the system differential equations describing operators of quantum system conditional evolution

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The evolution of cavity quantum electromagnetic field is under investigation. We consider the situation, when state of atom passing through the cavity detected in ionizing chamber after interaction. The unitary atom-field evolution is assumed. The conditional state of quantum field determined by the result of atomic measurement is of interest. We start from the Shrodinger equation for the atom-field evolution operator $U(t)$:

$$i\frac{d}{dt}U(t) = HU(t), \quad U(0) = I, \tag{1}$$

where $H = H_A + H_F - V_{int}$ – time independent Hamiltonian of the system, H_A, H_F – Hamiltonian operators (with discreet spectra) for pure atomic and field system correspondingly, V_{int} – interaction part and I – identical operator. Let us $K_{i,f}(t)$ – the set of Kraus operators [1], describing the ideal detection process on the field state space and $\Pi_{i,f} = K^{\dagger}_{i,f} K_{i,f}$ are detectors POVM's [2]. We may expand operator $U(t)$ in the following linear combination:

$$U(t) = \sum_{i,f} |f\rangle_A \langle i| K_{i,f}(t), \tag{2}$$

where $|i\rangle_A$ and $|f\rangle_A$ correspond to initial and detected final atomic state. To simplify the situation we suppose the initial state to be known $|i\rangle_A = |g\rangle_A$ and drop the first index of $K_{g,f}(t) = K_f(t)$. Introducing expansion (2) to (1) one may get the following system of operator-valued differential equations:

$$i\frac{d}{dt}K_f(t) = \sum_j \langle f| H |j\rangle_A K_j(t). \tag{3}$$

Here $\langle f| H |j\rangle_A$ – operators in field state space – corresponding matrix element of Hamiltonian in the basis of atomic states. We considered solution of this equations in some special cases of small number of non degenerate atomic states and find the following

Proposition. *Kraus operator $K_g(t)$ commute with photon number operator N_F for chosen measurement bases of atomic states. If N_F is non degeneracy operator then $K_{i,i}(t)$ commute with each other.*

Under this condition one may find the solution of equation (3) in standard form:

$$K_g(t) = \sum_j \widehat{C}_j \exp\left\{i\widehat{\Lambda}_j t\right\},$$

where $\widehat{C}_j$ – operator valued coefficient determined by initial conditions and commutating with $\widehat{\Lambda}_j$ – operator valued eigenvalue of matrix $\mathbf{M}_{i,j} = \langle i| H |j\rangle_A$. This result may be useful in calculation of whole evolution operator $U(t)$ and modeling of quantum optical multiqubit logical gates [3].

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Scholte–Stoneley waves on 1D and 2D phononic crystal gratings

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Sholte–Stoneley waves (SSW) propagate at the interface of a fluid medium and a solid medium. They were found as roots of the Scholte–Stoneley equation for smooth surfaces between solid and fluid [1, 2]. Their velocity is smaller than the transverse and longitudinal velocities in the solid, and smaller than the sound velocity in the fluid. Being the doubly evanescent, they propagate without loss along the interface. SSW energy and particle displacement are mostly localized in fluid, and less in the solid. Nevertheless, theoretically predicted SSW for smooth surface cannot be excited in experiments by plane sound waves and can be excited only through diffraction, i.e., on corrugated surfaces. They were observed for periodically corrugated surfaces in contact with water [3, 4], with the periodic corrugation allowing their conversion from an incident plane wave generated by an ultrasound transducer.

We consider in this work 1D and 2D silicon-water phononic crystals (PC) and study bulk wave to SSW modal conversion at the PC surface, numerically and experimentally. PC are artificial periodic structures composed of at least two different materials. They are generally considered because of their main property — band gaps, i.e., frequency ranges for which wave propagation through the PC is prohibited. As diffraction gratings, they can also be used for the generation of SSW, as we show by looking for the incidence and frequency conditions for which they are excited.

We report on a new theoretical and experimental investigation of conditions for Scholte–Stoneley waves generation on 1D, 2D corrugated silicon plate. The obtained experimental results confirm the SSW generation through diffraction. They appear at cut-off frequencies for diffraction orders from the classical grating law.

Financial support by the Agence Nationale de la Recherche under grant ANR-09-BLANC-0167-01 is gratefully acknowledged.

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The geometry of rays for a wave equation degenerating on the boundary

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Degenerate wave equations of the form $u_{tt} - (c^2(x)u_x)_x = 0$, where the function $c^2(x)$ is positive in a domain Ω and vanishes identically (to the first order) on the boundary $\partial\Omega$, arise, say, when modeling tsunami wave run-up on the shore in the linear approximation [1, 2]. The trajectories of the Hamiltonian system with Hamiltonian $H(x,p) = c(x)|p|$ corresponding to this equation go to infinity with respect to the momenta in finite time when approaching the boundary, and to construct the asymptotics of the solutions by the canonical operator method [3], one should somehow *specify* the subsequent behavior of the trajectories. The "right" law of boundary reflection for the trajectories can be obtained from the condition that the energy integral is finite (this was used for the construction of asymptotics in the one-dimensional case in [4] and [5]) and leads to a very natural extension of the standard phase space $T^*\Omega$, which proves to be a dense open set (the complement of a submanifold Φ_∞ of codimension one) in a symplectic manifold Φ such that the Hamiltonian $H(x,p)$ is a smooth function on Φ and the trajectories of the Hamiltonian vector field are infinitely extendible forward and backward in time. For example, if $\Omega \subset \mathbf{R}^2$ is a domain locally described by the inequality $x_1 > g(x_2)$, then the functions (q, x_2, E, η), where $q = p_1^{-1}$, $\eta = p_2 + p_1 g'(x_2)$, $E = (x_1 - g(x_2))p_1^2 > 0$, and (p_1, p_2) are the momenta dual to the variables (x_1, x_2), can be taken for canonical local coordinates on Φ near Φ_∞. In these coordinates, one has $\Phi_\infty = \{q = 0\}$,

$$dp_1 \wedge dx_1 + dp_2 \wedge dx_2 = dE \wedge dq + d\eta \wedge dx_2, \quad \text{and} \quad H(x,p) = \sqrt{1 + (g'(x_2) + q\eta)^2}\,\gamma(q, x_2, E),$$

where $\gamma(q, x_2, E) = q^{-1} c(g(x_2) + q^2 E, x_2)$ is a smooth function.

The results described in the talk were announced in [6], and a model two-dimensional problem in a half-plane with degeneration on the boundary line was considered in [7].

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Spectral gaps in double-periodic structures

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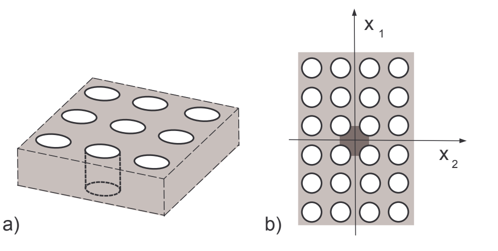
The Dirichlet and Neumann problems for the Laplace operator are considered in the plane with a double-periodic circular perforation (Fig. b),

$$-\Delta u_R(x) = \lambda_R u_R(x), \quad x \in \Pi_R = \mathbb{R}^2 \setminus \bigcup_{\alpha\in\mathbb{Z}^2} \mathbb{B}_R(\alpha),$$

$$u_R(x) = 0 \ (\partial_n u_R(x) = 0), \quad x \in \partial\Pi_R,$$



where $\alpha = (\alpha_1, \alpha_2)$, $\mathbb{Z} = \{0, \pm1, \pm2, \dots\}$ and $\mathbb{B}_R(\alpha) = \{x = (x_1, x_2) : |x_1 - \alpha_1|^2 + |x_2 - \alpha_2|^2 < R^2\}$ is a disk with radius $R \in (0, 1/2)$. The spectra σ_R^M, $M = D, N$, of these problems get the band-gap structure, i.e.

$$\sigma_R^M = \bigcup_{k=1}^{\infty} \Upsilon_R^M(k),$$

where $\Upsilon_R^M(k) \subset [0. + \infty)$ are closed segments. It is proved that there exist numbers $R_+^M \in (0, 1/2)$ such that, for $R \in (R_+^M, 1/2)$, the spectrum σ_R^M has a gap between the bands $\Upsilon_R^M(1)$ and $\Upsilon_R^M(2)$, but at $R = R_+^M$ the gap is closed. The number of opened gaps grows indefinitely as $R \to 1/2$. The asymptotics of bands and gaps are found out as well.

It is also verified that all gaps are closed in the case $R \in (0, R_-^M]$ where $R_-^M \in (0, R_+^M]$ are certain numbers. It is still an open problem to prove or disprove the equality $R_-^M = R_+^M$.

The effect of open spectral gaps is used in engineering of wave dampers and filters. A generalization of the asymptotic analysis of spectral gaps is available for other shapes of perforation and other boundary value problems in mathematical physics, in particular, for surface waves above a water layer with a double-periodic array of circular vertical columns (Fig. a).

The results are obtained in cooperation with Keijo Ruotsalainen and Jari Taskinen.

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The resonant nonlinear scattering theory with bound states in the radiation continuum and the second harmonic generation

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A nonlinear electromagnetic scattering problem is studied in the presence of bound states in the radiation continuum (they are localized solutions of Maxwell's equation whose spectrum lies in the spectrum of diffraction modes and also known as resonances with the vanishing width in scattering theory). It is shown that the scattering amplitudes for the fundamental and higher harmonics are not analytic in the nonlinear susceptibility and the conventional perturbation theory fails. A non-perturbative approach is proposed and applied to the system of two parallel periodic arrays of subwavelength dielectric cylinders with a second order nonlinear susceptibility. This scattering system is known to have bound states in the radiation continuum. In particular, it is demonstrated that, for a wide range of values of the nonlinear susceptibility, the conversion rate of the incident fundamental harmonic into the second one can be as high as 42% when the distance between the arrays is as low as a half of the incident radiation wavelength. The effect is solely attributed to the presence of bound states in the radiation continuum and cannot be achieved otherwise. Furthermore, the suggested mechanism of higher harmonic generation requires neither the phase matching condition nor focusing the incident radiation to increase its amplitude which are inherent to the conventional way to generate higher harmonics in optically nonlinear crystals. The conversion rate depends weakly of the nonlinear susceptibility and is mostly determined by the geometry of the scattering structure. In particular, the conversion rate can be manipulated (or controlled) by adjusting the distance between the two subwavelength periodic arrays.

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Magnetization in thin films and its semiclassical calculation

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We talk about spectrum of electrons in thin curved films placed into electric and magnetic fields, and discuss some physical characteristics of this medium. The model we consider is the one-particle

Schrödinger equation with spin term of Pauli type. Using the method of adiabatic reduction [1] we reduce the original 3-D problem to a 2-D problem on a suitable surface. We construct asymptotics of the spectrum in situations, when the corresponding classical problems on the surface are integrable. The Reeb graphs are used for classification of the spectral series. We apply these asymptotic formulas for calculation of magnetic susceptibility, thermal capacity and other physical features of thin films.

The work was supported by grants RFBR (11-01-00973-a) and DFG-RAS project.

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Hypersingular integral equation of wave diffraction problem on pre-Cantor grating and its discrete mathematical models

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Mathematical model of wave diffraction problem H polarized of a plane electromagnetic wave by a pre-Cantor grating $\Lambda^{(N)}$, consisting of a finite number of perfectly conducting strips have been constructed. In the cross section grating plane perpendicular to the edges of the tapes, there is a system of intervals obtained on the N-th step on the principle of constructing a Cantor set on the interval $[-l, l]$ (see fig. 1).

$$\Lambda^{(N)} = \left\{(x, y, z) \in \mathbb{R}^3,\, y \in L^{(N)},\, z = 0\right\}, \quad L^{(N)} = \bigcup_{q=1}^{2^N} \left(a_q^{2^N}, b_q^{2^N}\right), \tag{1}$$

where $-l < a_1^{(N)} < b_1^{(N)} < ... < a_{2^N}^{(N)} < b_{2^N}^{(N)} < l$.

Cartesian coordinate system is chosen so that the grating is in the xy plane and the edges are parallel to the x-axis.

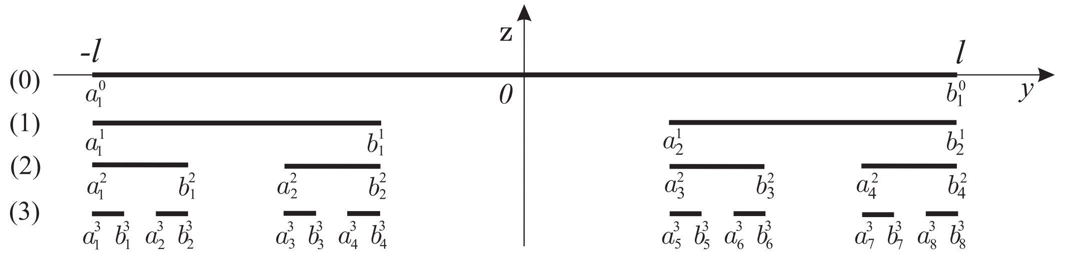

Fig. 1: Pre-Cantor sets $L_{2l}^{(0)}$, $L_{2l}^{(1)}$, $L_{2l}^{(2)}$, $L_{2l}^{(3)}$.

This diffraction problem is reduced to the external Neumann boundary problem for the Helmholtz equation.

As shown in [1, 2], boundary-value problem leads to a boundary hypersingular integral equation for a system of intervals (1)

$$\frac{1}{\pi}\int_{L^{(N)}} \frac{F^{(N)}(\eta)}{(\eta-\xi)^2} d\eta + \frac{A}{\pi}\int_{L^{(N)}} \ln|\eta-\xi|\, F^{(N)}(\eta) d\eta + \frac{1}{\pi}\int_{L^{(N)}} K^{(N)}(\eta,\xi)\, F^{(N)}(\eta)\, d\eta = f^{(N)}(\xi), \quad \xi \in L^{(N)}. \tag{2}$$

For the numerical solution of equation (2) used an effective method of discrete singularities (MDS) [3, 4]. We proceed from the equation (2) to an equivalent system of hypersingular integral equations of the standard interval $(-1, 1)$. Next, we construct a discrete mathematical model of this system, replacing the smooth kernels and the right sides of the interpolation polynomials $(n-2)$ and $(n-1)$

degree of relevant variables and unknown functions found in the form of a polynomial of degree $(n-1)$. And the problem have been reduced to solving a system of linear algebraic equations using the quadrature formulas of interpolation type.

Extensive numerical experiment base on a discrete mathematical model have been carried out. The approximate values of both solutions of integral equations and the vectors of the electromagnetic field have been obtained. The radiation patterns of the field in the far zone have been calculated.

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Generalized and classical solutions of boundary value problem for differential-difference equations

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Boundary value problems for differential-difference equations arises in the control system theory with an aftereffect [1]. In addition, they are used in research of elastic deformations of multilayered plates and shells [2] and other applications. On the other hand the problem might be reduced to differential equations with nonlocal boundary conditions that describe the processes of plasma control [3, 4]. It is known that boundary value problems for differential-difference equations with a shift in the higher derivatives of the arguments may not have classical solutions even for infinitely differentiable right-hand side (see [4, 5]). There are necessary and sufficient conditions for the existence of classical solutions in terms of the orthogonality of the right side of a finite number of functions in [4]. In this paper we consider the boundary value problem

$$-\frac{d^2}{dt^2}R_0u(t)+\frac{d}{dt}R_1u(t)+R_2u(t)=f(t) \qquad (t\in(0,d)) \tag{1}$$

with homogeneous boundary condition

$$u(t)=0 \quad (t\in R\setminus(0,d)), \tag{2}$$

where R_i are difference operators that are defined by the following formula

$$R_iu(t)=\sum_{j=-m}^{m}b_{ij}u(t+j) \qquad (i=0,1,2). \tag{3}$$

Here m is a natural number, $b_{ij}\in R$.

We obtain conditions on the coefficients of the equation under which boundary-value problem (1), (2) has a classical solution for any continuous right-hand side, provided that there exists a generalized solution of the problem. It turns out that a necessary and sufficient condition for this is lack of shifts in the arguments of the derivatives of unknown functions appearing in the equation, i. e., the equality $b_{i,j}=0$ $(i=0,1;j=\pm1,\dots,\pm m)$.

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Surface waves propagation models in semi-infinite systems with gratings

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The two-dimensional problem of surface acoustic wave (SAW) propagation in semi-infinite piezoelectric systems with gratings is considered. Important application of such geometry is investigation of interdigital transducers (IDT) or metallic grating influence on SAW propagation.

We made the comparison between the physical accurate model and the coupling-of-modes (COM) model. The physical model is based on the system of second-order partial differential equations describing the propagation of waves in the piezoelectric media. Because SAWs in many piezoelectric materials used in acoustoelectronic devices are not pure Rayleigh wave, so all three components of the mechanical displacement vector (U_i) have to be taken into account [1]

$$\begin{cases} c_{ijkl}\dfrac{\partial^2 U_k}{\partial x_j \partial x_l} + e_{kij}\dfrac{\partial^2 \varphi}{\partial x_j \partial x_k} = \rho\dfrac{\partial^2 U_i}{\partial^2 t} \\ e_{ijk}\dfrac{\partial^2 U_j}{\partial x_i \partial x_k} - \varepsilon_{ij}\dfrac{\partial^2 \varphi}{\partial x_i \partial x_j} = 0 \end{cases}$$

where c, e, ρ, ε are the material constants; φ is electrical potential; $i, j, k, l = 1, 2, 3$. This system of equations is also valid for metal electrodes when setting zero piezoelectric tensor.

The COM model is based on the system of first-order differential equations assuming that there are forward and backward SAWs propagating in the opposite directions (R and S) [2]

$$\begin{cases} \dfrac{dR}{dx} = -j\delta R + jkS + j\alpha V \\ \dfrac{dS}{dx} = j\delta S + jk^* R - j\alpha^* V \\ \dfrac{dI}{dx} = -2j\alpha^* R + 2j\alpha S + j\omega C V \end{cases}$$

where k, δ, α, C are reflectivity, detuning parameter, transduction coefficient, capacitance per unit length, respectively; I is the current caused when IDT is driven by a voltage V.

The COM parameters obtained from the physical model would be applied to the COM equations. Both of models were simulated numerically. Some effects are detected and analyzed.

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Multi-solitons interaction for generalized KdV equations

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The subject of the talk is the generalized Korteweg – de Vries (GKdV) equation

$$\frac{\partial u}{\partial t}+\frac{\partial u^m}{\partial x}+\varepsilon^2\frac{\partial^3 u}{\partial x^3}=0,\ x\in\mathbb{R}^1,\ t>0 \tag{1}$$

with integer $m\geq 2$ and the parameter $\varepsilon << 1$. We consider (for $m\geq 4$) the character of interaction of the solitary waves, that is of the exact solutions of (1) of the form

$$u=A\cosh^{-2/(m-1)}\left(\beta\frac{x-Vt}{\varepsilon}\right),\quad A=A(\beta,m),\quad V=V(\beta,m), \tag{2}$$

where $\beta>0$ is an arbitrary number.

The GKdV equation has a very long story. Our interest in this equation appeared after the creation of the Weak Asymptotics Method. This approach allowed to construct a formal asymptotic solution for the equation (1) which describes the interaction of two solitary waves (2). During the talk we present the asymptotic description of the two solitary waves interaction, next we describe the finite difference scheme and demonstrate the numerical results which show that the discrepancy between the asymptotics and the exact (numerical) solution remains small for the stable case, whereas the instability effects destroy the solution structure almost immediately. Furthermore, it turns out that three and more solitary waves interact as the solitons. Previously, the same result for two solitons (kinks and/or antikinks) has been discovered for the sine-Gordon type equations.

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Interaction of vector complex source beams with a linear polarizer and their subsequent analytical expansion into VSHs

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Nanoparticles are at the focus of active research in nanosciences. The first description of light interaction with a particle started from the Mie theory and was greatly extended after the invention of the laser. In contrast to the plane wave illumination, the rising interest in highly focused electromagnetic beams is mainly concerned with their polarization, which strongly influences the size and shape of the focal spot of the beam [1]. In recent publications the interaction between such beams and nanoparticles has been investigated [2]. These works have clearly demonstrated that the optical response is strongly dependent on both the particle location relative to the beam in the focus and the polarization state of the beam, and differs notably from that of the classical Mie theory.

To describe highly focused beams of various polarizations, one can start from the exact solution of the scalar wave equation by the complex source beam (CSB), which can be extended to accurately describe highly focused linearly, radially and azimuthally polarized light beams [3].

In this work we theoretically investigate the interaction of the vector CSBs with nonparaxial linear polarizers placed in the focal plane and afterwards expand them analytically into vector spherical harmonics (VSHs). Such an expansion is essential to understand the light-particle interaction with nano-objects such as atoms, molecules or particles. Those nano-objects locally respond to the various multipole components of the incident field. The dipole components are the most important ones, but even objects such as (meta-)atoms already sense quadrupole and even higher order excitations [4]. By knowing the expansion of optical beams into multipoles it is straightforward to describe the interaction of a beam with larger objects, which are conveniently described by a T-matrix. Moreover, the multipole approach also provides an efficient method for calculating the field in the focal region of a lens [5].

We present an analytical study on how a linear nonparaxial polarizer transforms multipole field components of the highly focused beams. In particular, we report on the defocusing of highly focused radially and azimuthally polarized CSBs as they interact with a linear nonparaxial polarizer in the focal plane. The generalized Mie theory is used afterwards to investigate the scattering of the studied beams off a spherical gold particle (Fig. 1).

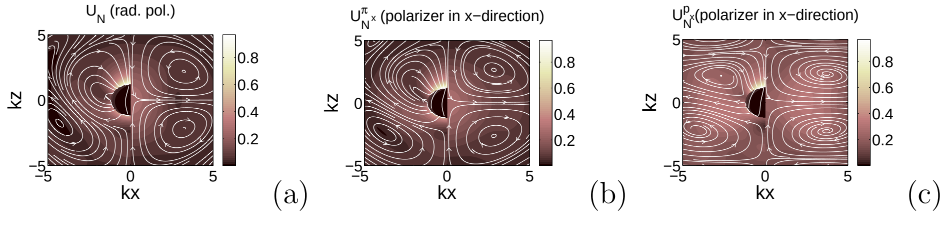

Fig. 1: Modulus of the total (left side) and incident (right side) electric fields for (a) a radially polarized CSB ($\mathbf{U}_N$) scattered off a gold sphere. Corresponding distributions for a radially polarized CSB ($\mathbf{U}_N$) which interacted with (b) a linear polarizer π_x and scattered off a gold sphere and (c) for the same beam after interaction with a linear polarizer p_x. p_x and π_x denote here two different configurations of a linear polarizer. The white arrows depict the direction of the electric field $\mathbf{E}$. The wavelength $\lambda = 780$ nm, the sphere radius $R_{sp} = 75$ nm.

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Radiation of TEM waves from an aperture in a coaxial waveguide

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In the present work, the diffraction of a TEM wave by an aperture on the outer wall of a coaxial waveguide is analyzed rigorously. The formulation of the related boundary-value problem in terms of Fourier integrals leads to a modified Wiener–Hopf equation which is first reduced into a pair of Fredholm integral equations of the second kind and the diffraction coefficient related to the radiated field is determined explicitly in terms of infinite number of unknown coefficients (see e.g. [1]). At the end of the analysis, numerical results illustrating the effects of the cross-sectional area of the coaxial cylindrical waveguide and the aperture length on the radiated field are presented.

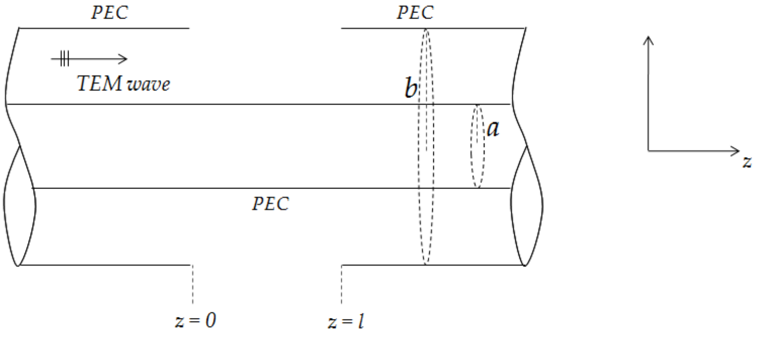

Fig. 1: Geometry of the problem.

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Non linear waves in complex microstructured solids

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A complex system as it is understood nowadays is composed by its constituents that interact with each other resulting in emergent properties of the system as a whole. In mechanics the concepts of complexity has been analysed by Engelbrecht [3] with a focus on wave propagation. This theory is based on some cornerstones like the introduction of internal structures at different scales and the nonlinearity of the models which in other words means incorporating intrinsic microstructural and nonlinear effects. In this case complexity means that we have different scales, with several interaction processes which encompass many physically meaningful phenomena. The pioneering work of Mindlin [5] on microstructured solids is a basic reference, while many papers have appeared where different particular and less particular cases have been described (see the papers by Engelbrecht, Pastrone, Cermelli, Porubov, Samsonov quoted in the references). Usually a microstructured body, as we shall see, is modeled as a solid with an internal structure at a different scale, which is apt to describe the mechanical behaviour of solids with dislocations, polycrystalline solids, ceramic composites, granular media, etc. It is possible, and useful, to develop also models with a hierarchy of microstructures, i.e. a first level micro-structure which contains a second level micro-structure, and so on, as done in Casasso and Pastrone [2], Engelbrecht et al [4]. But it is meaningful also the case of concurrent micro-structures (see Berezovski et al [1]). In this paper we want to analyze the subject, recalling some main results in the theory of complex microstructures, developing new results in the case of multiple microstructures, exploiting hierarchical governing equations and analyzing nonlinear wave propagation, which is crucial to put in evidence the weight of the different scales.

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Zero-Range model for a dissipative operator

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Zero range model of a dissipative operator is obtained as a restriction of it's selfadjoint dilatation onto an appropriate co-invariant subspace selected from the coinvariant subspace of the dilatation. The model is used as an intermediate operator for analytic perturbation of the tectonic plate under a boundary stress.

Vibrations of high-contrast media: eigenmodes, quasimodes and long-time approaches

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We provide an overview ([1]–[6]) on results related to the asymptotic behaviour of the eigenelements of certain singularly perturbed spectral problems arising in models of vibrations of high-contrast structures/media. For certain vibrating systems we construct *standing waves*, which concentrate asymptotically their support at points, along lines, or in certain regions and which approach certain solutions of the evolution problems for long time. We determine this period of time in terms of the small perturbation parameters of the problem.

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Mode parabolic equations for the modeling of sound propagation in 3D-varying shallow water waveguides

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Acoustic waves propagation and scattering in shallow water waveguides is strongly influenced by the sea bottom inhomogeneitites. These inhomogeneities (wedges, seamounts, etc) are essentially three-dimensional and their size is often comparable to the acoustic wavelength. Since the direct solution of Helmholtz equation is usually impossible for typical underwater acoustics problems, it is often approximated by one-way propagation models based on various types of parabolic equations. Existing 3D parabolic equations however usually suffer from the deficiencies related to the incorrect treatment of water-bottom interfaces and lack of systematic approach. In our work the so-called mode parabolic equations (MPE) are derived from the Helmholtz equation using generalized multiple scale method.

Consider a three-dimensional waveguide $\Omega = \{(x, y, z)|0 \leq x \leq L, -d \leq y \leq d, z \geq 0\}$ (where z denotes depth and x is range along the acoustic track). Our goal is to compute acoustic pressure produced by a time-harmonic point source of the frequency f located at $(0, 0, z_s)$. In the MPE model depth-dependent acoustic field at each point (x, y) is represented as a superposition of normal modes (i.e. solutions of acoustic spectral problem) $\phi_{n,x,y}(z)$ (here indices x, y indicate that the set of normal modes may vary from one (x, y) point to another):

$$p(x, y, z) = \sum_{j=1}^{N_m} A_j(x, y)\phi_{j,x,0}(z)e^{i\theta_j(x)} \,. \tag{1}$$

Mode amplitude functions $A_j(x, y)$ hence depend on the both horizontal coordinates and in our work it is shown that they obey mode parabolic equations of the form

$$2\mathrm{i}k_j A_{jx} + \mathrm{i}k_{jx} A_j + A_{jyy} + \alpha_j A_j = 0 \,, \tag{2}$$

where $k_j = k_j(x)$ is local wavenumber corresponding to j-th normal mode and the horizontal variations in refraction index and water-bottom interface depth are incorporated into the coefficient $\alpha_j(x)$ (note that vertical variations are accounted while computing normal modes $\phi_{n,x,y}(z)$). Numerical examples in our work show that these equations allow to compute 3D-varying acoustic fields with high accuracy at low computational cost.

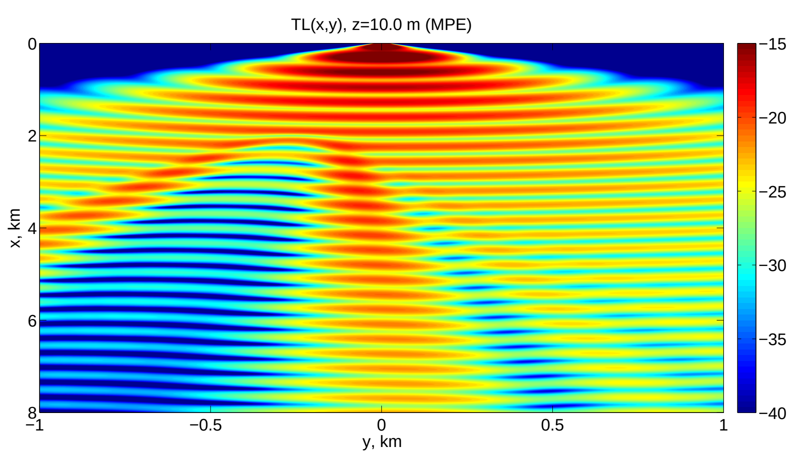

Fig. 1: An example: interference pattern of acoustic field $p(x, y, z_s)$ scattered by a seamount.

Generalized trigonometric transformation

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The well known Fourier, Hartley and Fresnel transformations on the real axis are the particularly cases of the certain general integral transformation.

Theorem 1. *The following transformations on the real axis are mutually inverse:*

$$\Phi[f](\xi) = F(\xi) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \big(a(y)\,\lambda(\xi)\,\cos\xi y + b(y)\,\mu(\xi)\,\sin\xi y\big)\, f(y)\,dy, \quad \xi \in \mathbb{R}; \tag{1}$$

$$\Phi^{-1}[F](x) = f(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \big(c(x)\,\alpha(\xi)\,\cos\xi y + d(x)\,\beta(\xi)\,\sin\xi y\big)\, F(\xi)\,d\xi, \quad x \in \mathbb{R}, \tag{2}$$

where

$$c(x) = \frac{2\,b(-x)}{a(x)b(-x) + a(-x)b(x)}, \quad d(x) = \frac{2\,a(-x)}{a(x)b(-x) + a(-x)b(x)},$$
$$\alpha(\xi) = \frac{2\,\mu(-\xi)}{\lambda(\xi)\mu(-\xi) + \lambda(-\xi)\mu(\xi)}, \quad \beta(\xi) = \frac{2\,\lambda(-\xi)}{\lambda(\xi)\mu(-\xi) + \lambda(-\xi)\mu(\xi)}.$$

Here the coefficients a, b, λ, μ are arbitrary functions on the axis. The function spaces, where (1) and (2) are true, are determined by the properties of these coefficients and the known theorems about the Fourier sine and cosine transformations. If these functions are even, then (2) is simplified:

$$f(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \left(\frac{\cos\xi x}{a(x)\,\lambda(\xi)} + \frac{\sin\xi x}{b(x)\,\mu(\xi)} \right) F(\xi)\,d\xi.$$

As $a = b = \lambda = 1,\ \mu = i$, we have the Fourier transformation. As $a = b = \lambda = \mu = 1$, we have the Hartley transformation. Putting $a(x) = -ib(x) = \exp\{-i\,x^2/2\},\ \lambda(\xi) = \mu(\xi) = \exp\{-i\,\xi^2/2\}$, we obtain the Fresnel transformation.

Theorem 2. *The transformation Φ is selfadjoint in $L_2(-\infty,\,\infty)$ if*

$$a(x) = \overline{\lambda(x)}, \quad b(x) = \overline{\mu(x)},$$

and it is unitary if

$$a(x) = e^{i\,h_2(x)}, \quad b(x) = e^{i\,g_2(x)}, \quad \lambda(\xi) = e^{i\,\gamma_2(\xi)}, \quad \mu(\xi) = e^{i\,\zeta_2(\xi)},$$

where $h_2,\ g_2,\ \gamma_2,\ \zeta_2$ are arbitrary real-valued even functions.

In particular, the operator

$$\Phi[f](\xi) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \left(e^{i(h_2(x)-h_2(\xi))}\cos\xi x + e^{i(g_2(x)-g_2(\xi))}\sin\xi x \right)\, f(x)\,dx$$

is simultaneously selfadjoint and unitary.

There are some matrix generalizations of the formulas (1), (2). For example,

Theorem 3. *Let a and b be arbitrary number invertible $(N \times N)$-matrices. Let $c = a^{-1}$, $d = b^{-1}$. Then the following transformations are mutually inverse:*

$$F_j(\xi) = \frac{1}{\sqrt{2\pi}} \sum_{k=1}^{N} \int_{-\infty}^{\infty} \big(a_{jk}\cos y\xi + b_{jk}\sin y\xi\big)\, f_k(y)\,dy, \quad j = 1\ldots N,$$

$$f_m(x) = \frac{1}{\sqrt{2\pi}} \sum_{j=1}^{N} \int_{-\infty}^{\infty} \big(c_{mj}\cos\xi x + d_{mj}\sin\xi x\big)\, F_j(\xi)\,d\xi, \quad m = 1,\ldots N.$$

The Maxwell system in domains with cylindrical ends

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The stationary Maxwell system with spectral parameter is considered in a 3D domain with several cylindrical outlets to infinity. The boundary of the domain is supposed to be smooth and perfectly conductive. We prove that the corresponding boundary value problem provided with intrinsic radiation conditions is well posed. We also define the unitary scattering matrix and describe the asymptotics of solutions at infinity. To this end, we extend the Maxwell system to an elliptic one and study the latter in detail. The information on the Maxwell boundary value problem comes from that obtained for the elliptic problem.

Diffraction of elementary plane pulse at semi-infinite interface between two dielectric media

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The problem of diffraction by a transparent (dielectric, elastic, etc.) wedge is interesting not only due to a wide range of applications but also due to its deceivingly simple geometry stimulating the development of analytical approaches. However, numerous attempts to construct an analytical solution using Fourier transform or Sommerfeld integral techniques (for references, see e.g. [1–2]) show that it hardly can be obtained in a closed form. Therefore, simplified models allowing for an exact solution are of certain interest.

Following Sommerfeld's idea of branched wave fields we consider a 2D analog of "one-faced" dielectric wedge, mathematically represented by a logarithmic Riemann surface with different propagation velocities on the "positive" and "negative" sheets. Diffraction effects arising at the semi-infinite interface between twoangular half-spaces (cylindrical edge waves and a lateral wave matching their wave fronts) are similar to the case of a realistic dielectric wedge. The difference consists in the absence of the waves diffracted at the second wedge face, which removes complicated interference effects and simplifies the analytical properties of the solution. In order to treat the problem we choose the Smirnov–Sobolev method of functionally-invariant solutions [3]. We assume for definiteness a unit-step initial pulse $\Theta(ct-x)$ coming from the zeroth sheet of the Riemann surface, crossing the branch point $(0,0)$ and propagating along the interface $(y=0, x>0)$ between two semi-infinite angular half-spaces:

(i) $\quad x=r\cos\varphi, \quad y=r\sin\varphi, \quad 0<\varphi<\infty; \quad n_0=1 \qquad$ ("air");
(ii) $\quad x=r\cos\psi, \quad y=r\sin\psi, \ -\infty<\psi<0; \ n=\sqrt{\varepsilon}>1$ (dielectric).

In this formulation, the only free parameter of the problem is refraction index n (square root of the relative dielectric permittivity ε). As there is no dimensional parameter, we are looking for a self-similar solution to the 2D wave equation

$$\Delta E = \frac{\varepsilon}{c^2}\frac{\partial^2 E}{\partial t^2} \tag{1}$$

of the following form

$$E(r,\varphi,t)=a(\nu,\varphi), \quad \nu=\mathrm{Arcosh}\frac{ct}{r}; \quad \varphi>0, \quad 0<r<ct \tag{2}$$

$$E(r,\psi,t)=\tilde{b}(\mu,\psi), \quad \mu=\mathrm{Arcosh}\frac{ct}{nr}; \quad \psi<0, \quad 0<r<\frac{ct}{n} \tag{3}$$

Substitution (2–3) reduces Eq. (1) to the Laplace equations, so the problem can be reformulated as the search for two harmonic functions $a(\nu,\varphi)$ and $\tilde{b}(\mu,\psi)$ in the respective quadrants: $(\nu > 0, \varphi > 0)$, $(\mu > 0, \psi < 0)$, matched at a part of the common boundary $\varphi = \psi = 0$ by continuity and smoothness conditions — linear relations with shifted argument: $\mu = \operatorname{Arcosh}(\cosh \nu / n)$, $\nu > \operatorname{Arcosh} n$. These functions describe the cylindrical diffracted waves penetrating, by transversal diffusion mechanism [4], onto positive and negative sheets of the Riemann surface. In addition, a Cherenkov type lateral wave appears in the $-\arccos(1/n) < \psi < 0$ sector of the "slow" medium. It is governed by a 1D wave equation in variables ψ, $\lambda = -\arccos(ct/nr)$ and has the form of a simple propagating wave $E(r,\psi,t) = c(\psi + \lambda)$.

Eliminating by differentiation the unknown function $c(\psi)$ and applying the interface matching conditions, we relate the wave field at the remaining part of the air-dielectric boundary ($\varphi = 0, 0 < \nu < \operatorname{Arcosh} n$) with the field amplitude at the slow wave front portion ($\lambda = 0, -\arccos(1/n) < \psi < 0$). The resulting mixed boundary value problem for the harmonic functions $a(\nu,\varphi)$ and $\tilde{b(\mu,\psi)}$ can be constructively solved by applying Malyuzhinets' "unifying operator" [5]: $b(\nu,\varphi) = \tilde{b}[\mu(\nu,\varphi),\psi(\nu,\varphi)]$ where real-valued variables (ν,φ) and (μ,ψ) are related by complex Snell's law: $\cos\alpha = n\cos\beta$, $\alpha = \varphi + i\nu, \beta = \psi + i\mu$. Considering $a(\nu,\varphi)$ and $b(\nu,\varphi)$ as the real parts of unknown analytic functions $A(\alpha)$ and $B(\alpha)$ we come to a Hilbert–Privalov matching problem yielding a closed-form exact solution.

This analytical result admits a clear physical interpretation: diffraction by a wedge with one face "blackened", i.e. absorbing any incident wave without reflection and diffraction effects. It reveals the main diffraction phenomena occurring on either side of a real "two-faced" wedge and may serve as a first step of an iterative solution procedure.

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Gaussian Beam Summation Method, mathematical foundations and applications for modeling and migration

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What we call today Gaussian beams was invented by specialists in theory of open optical resonators for gas lasers in the late 1960's. They describe real physical phenomenon, namely, propagation of monochromatic and coherent beam of light generated by a gas laser working in quasi-stationary regime.

Mathematical theory of optical resonators in most general formulation- multi-mirror resonators immersed in inhomogeneous medium- has been developed in Petersburg Mathematical School in wave propagation and diffraction at that time. This theory is based on the so-called parabolic equation of the diffraction theory which is actually non-stationary Schrödinger type equation where arc length of the central ray substitutes the time. In the presentation we give short sketch of appropriate quantum mechanics technique for construction of the Gaussian beams.

The Gaussian beams possess remarkable and mathematically attractive properties: they are concentrated in the vicinity of the central ray and do not have singularities on this ray and its vicinity. Therefore they have been developed for many equations including, naturally, acoustic and elastodynamic equations and used for constructions of different asymptotic solutions of wave propagation problems. This method we call the Gaussian Beam Summation Method.

Now Gaussian beams- we use only so-called main mode of a resonator, or ground quantum state of a quantum oscillator and not the higher order Gaussian beams- loose their physical sense and are being considered solely as a mathematical tool. In order to compute a wave field at an observation point by GBSM we have to proceed as follows: to construct a fan of central rays which covers some vicinity of the observation point; to construct a Gaussian beam propagating along each central ray with appropriate initial data and finally to sum contribution of beams to the observation point. In the presentation we demonstrate the results of GBSM application to a number of wave propagation problems.

Recently we have started to work on migration problems in geophysics with GBSM. Our approach can be visually explained as follows. Assume that we have reflected wave on the seismogram. For a smoothed version of the velocity model we propagate this wave backward in depth together with the direct wave field generated by the explosion and fix it in such a position in migration domain where both fields are coherent, i.e. coincide in phase. This procedure allows restoring reflecting interfaces in subsurface.

It turns out that internal degrees of freedom of GBSM, in particular variation of the imaging condition, enables us to estimate the reflection coefficients on the interfaces and to naturally implement true amplitude approach. In the presentation we present migration results for a number of benchmark models.

It appears that implementation of GBSM into ray-based reflection tomography provides significant improvement of the velocity model reconstruction especially in the presense of massive salt bodies. We demonstrate also some results of application of GBSM for reconstruction of the subsurface velocity model (tomography). In conclusion we briefly discuss main difference between Hill's method and our approach to the depth migration.

It seems worth of noting that GBSM is an invention in mathematical physics and applied mathematics which entirely belongs to the Petersburg Mathematical School in the Theory of Diffraction and Wave propagation. Below we list original papers devoted to this method.

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Localized two-particle states in perturbed nanolayers

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It is known that curved quantum layers can store particles. From mathematical point of view it is related with the existence of eigenvalues of the corresponding Hamiltonian. The increase of curvature leads to increasing of the eigenvalues number. This question is important in various physical problems. For example, to do two-qubit operation in quantum computer based on coupled quantum waveguides (see, e.g., [1] it is necessary to store two electrons in some bounded domain during the operation time. Another interesting application is related with the storage of hydrogen (or protons) in nanolayered structures. It can give effective and safe fuel container for hydrogen engine. One can note that layers with curved boundaries are more effective for particles storage because the increasing of the curvature (or boundary perturbation amplitude) leads to increasing of the discrete spectrum cardinality. Hence, the amount of the hydrogen stored in the layered structure will be greater. Note that the Hamiltonian for the corresponding plane layered structure has empty discrete spectrum.

Local deformations of the nanolayers leads to appearance of eigenvalues of the corresponding one-particle Hamiltonian. The cardinality of the discrete spectrum was used to estimate the maximal number of non-interacting fermions stored in the nanolayers [2]. The discrete spectrum of the Hamiltonian of two interacting particles is considered. Relation between the system parameters (interaction intensity - waveguide deformation) ensuring the existence of non-empty discrete spectrum is studied. Hartree approximation and Finite Elements Method are used. We consider three different types of layers perturbations: deformation of layer boundary, curved layer (bent waveguide),two layers coupled through small window. The efficiencies of particles storage in these systems are compared. It is shown that window coupled layers are the most appropriate.

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On the essential spectra of quantum waveguides

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The main aim of the paper is the study of essential spectra of electromagnetic Schrödinger operators with variable potentials in a cylindric domain $\Pi = \Omega \times \mathbb{R}$ where $\Omega \subset \mathbb{R}^n$ is a bounded domain with a smooth boundary provided by admissible boundary conditions. Applying the limit operators method we obtain explicit estimates of the essential spectrum for a wide class of quantum waveguides.

We also consider a numerical example of calculations of the discrete spectrum of quantum waveguides applying the method of decomposition of solutions of spectral problems for one-dimensional Schrödinger equations as power series with respect to the spectral parameter.

Integral equations of the first kind with a difference kernel on a finite interval and problems of diffraction of waves

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The classic integral equations of the form

$$K \circ g \equiv \int_{-a}^{a} k(x-t)g(t)dt = f(x), |x| \leq a \tag{1}$$

in connection with various problems of diffraction of waves are analyzed. Here K and f are some given functions, g is an unknown function. The equation (1) long times was an object of research of many scientific of different fields of science and technique. However, the explicit solution is retrieved in several special cases well known for the specialists and named classic. This report is dedicated to build-up of the explicit solution of the equation (1) in a situation enough general provisions. A number of problems of diffraction of waves which are reduced to the decision of the equation (1) is discussed.

New type of unstable optical resonators

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The description of a mode structure in optical resonators containing selective units, and also environments, absorptive or strengthening a field is given. In such cases units of a wave matrix of full detour of a resonator (the matrixes of a monodromy) appear complex. The analysis of a natural oscillation in such resonators comes across essential mathematical difficulties, which one were overcome by engaging concepts of modern symplectic geometry and theory of Hamiltonian systems. Has appeared, that the stability conditions for resonators with lossy and amplification have qualitatively diverse kind, than in case of resonators lost-free and amplification. In the given report the generalization of concepts of double-sided (bidirectional) and unilateral (unidirectional) stability on three-dimensional resonators, including with a composite astigmatism and non-coplanar optical contours is conducted. The concrete designs of ring optical resonators having unidirectional stability are adduced. Thus, it is possible to consider existence of optical resonators with unidirectional stability as the fact in evidence assorted as qualitatively new result in of the theory of optical resonators.

Paraxial equation and Bessel functions of fractional order

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Finite energy solutions of the 1D paraxial equation connecting with Bessel functions of the first kind and fractional order are presented. These beams named fractional Bessel beams are investigated theoretically and numerically. It is shown that for some cases these beams, in a sense, are similar to well-known Airy and parabolic beams.

The semi classical Maupertuis–Jacobi correspondence: unstable spectrum

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We investigate semi-classical properties of Maupertuis–Jacobi correspondence for families of Hamiltonians $(H_\lambda(x,\xi), \mathcal{H}_\lambda(x,\xi))$ on $T^*(\mathbf{R}^2)$, when $\mathcal{H}_\lambda(x,\xi)$ is a perturbation of completely integrable Hamiltonian $\widetilde{\mathcal{H}}$ verifying some isoenergetic non-degeneracy conditions. Assuming $\widehat{H}_\lambda$ has only discrete spectrum near E, and the energy surface $\{\widetilde{\mathcal{H}}_0 = \mathcal{E}\}$ is separated by some pairwise disjoint Lagrangian tori, we show that most of eigenvalues for $\widehat{H}_\lambda$ near E are asymptotically degenerate as $h \to 0$. This applies in particular for the determination of trapped modes by an island, in the linear theory of water-waves. We also consider quasi-modes localized near rational tori, associated with certain Reeb graphs. Finally, we discuss breaking of Maupertuis–Jacobi correspondence on the equator of Katok sphere. This is a joint work with S.Dobrokhotov.

Localized vortex in a two-dimensional geophysical model

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As a continuation of our work [1] we study possible trajectories of a long time existing vortex in a model of the atmosphere dynamics, where the vortex can be interpreted as a tropical cyclone. The model can be obtained from the system of primitive equations governing the motion of air over the Earth surface after averaging over the height. We consider approximations of l-plane and β-plane used in geophysics for modeling of middle scale processes and equations on the whole sphere as well. We associate with a cyclone a special class of smooth solutions having a form of a localized steady non-singular vortex moving with a bearing field. We show that the solutions satisfy the equations of the model either exactly or with a discrepancy which is small in a neighborhood of the trajectory of the center of vortex. We show both analytically and numerically that the trajectory of a localized vortex keeps the features of trajectory of vortex with a linear profile of velocity, where the exact solution can be obtained.

In [1] we developed a theory on possible trajectories of a vortex with a linear profile of velocity governed by the system of equations of compressible fluid on a plane. This vortex can be interpreted as a tropical cyclone in the conservative phase of its dynamics, thought it has a number of nonrealistic features. First, the components of the velocity and pressure rise unboundedly as the distance from the center increases (we call these vortices non-localized). Second, the curvature of the Earth surface was not taken into account even in a simplest form. Nevertheless, as it was shown in [1] by comparing with observational data, the theoretical trajectories correspond to the real tropical cyclone paths. In fact, they are a superposition of two circular motions and correspond to the natural circular (or rather parabolic, taking into account the change of the Coriolis parameter with the latitude) trajectories, loops, reversal points, etc. Moreover, the explicit expression for the trajectory was obtained.

It seems very strange that by means of such unrealistic solution one can model a real atmospheric vortex. Here we explain this phenomenon. Namely, we show that under some realistic assumptions on the background field of pressure the trajectories of localized vortices that model the physical process more adequately are very similar to the trajectories of the non-localized vortex with linear profile of velocity considered in [1].

It is worth mentioning that the exact form of equations describing the atmosphere is not known. More exactly, in the primitive system of equations we can take into account some additional forces or sources of energy, e.g. due to phase transitions, and the exact form of these terms can hardly be found, particulary after the averaging procedure.

We base on a special form of solution for the system of equations of the atmosphere dynamics (steady vortex moving in a bearing field), moreover, we assume a smallness of the source term that guarantees the existence of this special solution, at least in a domain of space that we are interesting to control.

We can also look at the problem from another side and call a vector-function the "δ-approximate" solution if the discrepancy term arising after substitution of this function to the system is less than δ in the uniform norm. Thus, we can consider the term either as some source term that guarantees the existence of an exact vortex solution or as a discrepancy, in the latter case the vortex solution can be considered as an approximative one.

Further we perform direct numerical computation of the moving vortex in the 2D models of the l-plane and β-plane and compare the position of the center of vortex with the result obtained by our method. The numerics demonstrates a very good compliance with the theory.

The results are partly published in [2].

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MultiBEAM interference in the kaleidoscopic structures

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The use of the interference field provides the opportunity to carry out the spatial order and control for ensembles of microparticles [1, 2]. In addition to the two-beam interference, one can use more complex schemes of multi-beam interference. This allows us to create two- and three-dimensional order structures of microscopic objects, which offers a promising potential for micro- and nanotechnologies [3]. In this paper, we consider the possibility of formation of multi-beam interference in the kaleidoscopic patterns.

Kaleidoscopes are formed as the intersection of mirror plane surfaces at the angle of π/s, where s is an integer. They can be two-, three- and four-sided, both closed and open. With their help, one can get a set of spatially periodic structures by multiplying a single object image, if this object is placed in the general position inside the kaleidoscope of mirror facets. Each kaleidoscope generates one kind of symmetry, despite on the variety of the resulting structures and properties. Any point inside the kaleidoscope and its mirror images form a system of equivalent points. If one places a source of light to one of them, all remaining points will determine the position of its mirror images, which play the role of auxiliary sources under study of diffraction phenomena by mirror images. The problem is to determine the power and phase of these sources, which satisfy the boundary conditions. In particular, such problems appear under calculation of the radiation patterns of corner antennas and of wave scattering by in homogeneities in a rectangular waveguide. The method of mirror images can be generalized and applied to study of electromagnetic wave scattering and to the construction of Green's functions in any of the kaleidoscopic regions.

In our study, the vector $\{h_x, h_y, h_z\}$ takes the part of an object placed in a kaleidoscope, and the structure is a set of equivalent directions. For kaleidoscopes of cubic systems, they are calculated for the regions $0 \le x \le y \le a - z \le a$, $|x| \le y \le a - z \le a$, $|x| \le y \le a - |z|$. For instance, a kaleidoscope $|x| \le y \le a - z \le a$ generates a set of 48 vectors $\{\pm h_x, \pm h_y, \pm h_z\}$, taking into consideration all possible permutations of indexes and signs. Each direction is associated with a wave beam, and the kaleidoscopic area is connected with the solution of Maxwell's equations as a superposition of wave beams. The beams are interconnected in pairs by mirror reflection on the faces of a kaleidoscope. Therefore, the new beams can't appear during their reflections, and the superposition can be considered as closed in this sense. With the use of such solutions, all possible reflections of the beam in a kaleidoscope of cubic systems are constructed. The multi-beam interference is investigated, and the region of its localization in each of the options is determined. It is shown that increase of the aperture of the incident beam is accompanied by its division and initiation of several options in a kaleidoscope of reflection. By this way, one can increase the number of waves in multi-beam interference up to the maximum value equals to the number of equivalent directions.

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Computation of the band structure for water wave problems in periodic domains

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In this paper the essential spectrum of the linear problem on water-waves in periodic domains will be studied. We show that under certain geometric conditions the essential spectrum has spectral gaps. In other words there exist intervals in the positive real semi-axis which are free of the spectrum, but have their endpoints in it. The position and the length of the gaps are found out by applying an asymptotic analysis to the model problem in the periodicity cell.

Finally, we present a numerical method based on the Finite Element Method (FEM) to compute the band structure of the spectral problem.

Homogenization for second order periodic elliptic differential operators in a strip

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Let $\Pi = \mathbb{R} \times (0,\, a)$ denote a strip on $\mathbb{R}^2$. Given $\varepsilon > 0$, consider the differential expression

$$\mathcal{B}^\varepsilon = \sum_{j=1}^{2} \mathrm{D}_j g_j \left(\frac{x_1}{\varepsilon},\, x_2\right) \mathrm{D}_j + \sum_{j=1}^{2} \left(h_j \left(\frac{x_1}{\varepsilon},\, x_2\right) \mathrm{D}_j + \mathrm{D}_j h_j^* \left(\frac{x_1}{\varepsilon},\, x_2\right)\right) + Q \left(\frac{x_1}{\varepsilon},\, x_2\right) + \lambda Q_* \left(\frac{x_1}{\varepsilon},\, x_2\right). \tag{1}$$

By $\mathcal{B}^\varepsilon_{per}$, $\mathcal{B}^\varepsilon_{N}$, $\mathcal{B}^\varepsilon_{D}$ denote the operators in $L_2(\Pi)$ defined by the expression (1) with the periodic, Neumann or Dirichlet boundary conditions, respectively. Below $\mathcal{B}^\varepsilon_{\#}$ is written in place of $\mathcal{B}^\varepsilon_{per}$, $\mathcal{B}^\varepsilon_{N}$, $\mathcal{B}^\varepsilon_{D}$. All the coefficients in (1) are 1-periodic with respect to the first variable. It is assumed that g_j, $j \in \{1,\, 2\}$, and Q_* are uniformly positive definite, bounded, and belong to $\mathrm{Lip}\,([0,\, a]\,;\, L_\infty(0,\, 1))$. Next, $h_j \in \mathrm{Lip}\,([0,\, a]\,;\, L_2(0,\, 1))$, $j \in \{1,\, 2\}$, and $Q = \overline{Q} \in \mathrm{Lip}\,([0,\, a]\,;\, L_1(0,\, 1))$. Additionally, in the case of the periodic boundary conditions the traces of all the coefficients on $\mathbb{R} \times \{0\}$ and $\mathbb{R} \times \{a\}$ coincide, and in the case of Neumann or Dirichlet boundary conditions $h_2(x_1,\, 0) = h_2(x_1,\, a) = 0$ for a.e. $x_1 \in \mathbb{R}$. The parameter λ in (1) is subject to the restriction $\lambda \geqslant \lambda_0$ which insures positive definiteness of the corresponding operator.

The aim is to approximate the inverse operator $\left(\mathcal{B}^\varepsilon_{\#}\right)^{-1}$ for small ε in terms of $\left(\mathcal{B}^0_{\#}\right)^{-1}$, where $\mathcal{B}^0_{\#}$ is the corresponding effective operator given by

$$\mathcal{B}^0 = \sum_{j=1}^{2} \mathrm{D}_j g_j^0(x_2)\, \mathrm{D}_j + \sum_{j=1}^{2} \left(h_j^0(x_2)\, \mathrm{D}_j + \mathrm{D}_j h_j^0(x_2)\right) + Q^0(x_2) + \lambda Q_*^0(x_2). \tag{2}$$

Boundary conditions for $\mathcal{B}^0_{\#}$ are the same as for $\mathcal{B}^\varepsilon_{\#}$. The effective coefficients in (2) are defined as follows:

$$g_1^0(x_2) = \left\langle g_1^{-1}(\cdot,\, x_2)\right\rangle^{-1}, \quad g_2^0(x_2) = \langle g_2(\cdot,\, x_2)\rangle,$$
$$h_1^0(x_2) = g_1^0(x_2)\, \mathrm{Re} \left\langle \frac{h_1}{g_1}(\cdot,\, x_2)\right\rangle, \quad h_2^0(x_2) = \langle h_2(\cdot,\, x_2)\rangle,$$
$$Q^0(x_2) = \langle Q(\cdot,\, x_2)\rangle - \left\langle \frac{|h_1|^2}{g_1}(\cdot,\, x_2)\right\rangle + g_1^0(x_2) \left| \left\langle \frac{h_1}{g_1}(\cdot,\, x_2)\right\rangle \right|^2, \quad Q_*^0(x_2) = \langle Q_*(\cdot,\, x_2)\rangle,$$

where $\langle f \rangle$ denotes the mean value of a function f over $(0,\, 1)$.

Theorem. *Let $\mathcal{B}^\varepsilon_{\#}$ stand for $\mathcal{B}^\varepsilon_{per}$, $\mathcal{B}^\varepsilon_{N}$ or $\mathcal{B}^\varepsilon_{D}$, and $\mathcal{B}^0_{\#}$ stand for $\mathcal{B}^0_{per}$, $\mathcal{B}^0_{N}$ or $\mathcal{B}^0_{D}$, respectively. Then*

$$\left\| \left(\mathcal{B}^\varepsilon_{\#}\right)^{-1} - \left(\mathcal{B}^0_{\#}\right)^{-1} \right\|_{\boldsymbol{B}(L_2(\Pi))} \leqslant C\varepsilon, \quad 0 < \varepsilon \leqslant 1. \tag{3}$$

The estimate (3) *is order-sharp. The constant C depends only on the problem data: the norms of g_j, g_j^{-1}, $\partial_2 g_j$, Q_*, Q_*^{-1}, $\partial_2 Q_*$ in $L_\infty(\Pi)$, the norms of h_j, $\partial_2 h_j$, $j \in \{1,\, 2\}$, in $L_\infty([0,\, a]\,;\, L_2(0,\, 1))$, and the norms of Q, $\partial_2 Q$ in $L_\infty([0,\, a]\,;\, L_1(0,\, 1))$.*

In the case where $h_j = Q = 0$, $\lambda Q_* = 1$ and boundary conditions are periodic, a similar result has been proved in [1]. Method of the proof of the current result is based upon an operator-theoretic (spectral) approach [2] and is further development of the scheme suggested in [1].

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Effective asymptotics for solutions of the Cauchy problem with localized initial data for linear Boussinesque type equation with variable velocity and small dispersion

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We construct asymptotic solutions of the Cauchy problem with localized initial data for the 2-D linear Boussinesque type equation with variable velocity and small variable dispersion and apply our results in the water wave theory. Under the Boussinesque type equation we understand the wave-type equation perturbed by small dispersion terms. The dispersion effects appear due to the dispersion of the water wave, but also due to rapid oscillations of the basin’s depth. For special class of initial perturbation we present quite explicit formulas for profiles of solutions near its fronts and compare the influence of the “water” dispersion and the dispersion generated by rapid oscillations of the depth to the profiles of solutions.

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Acoustical experiment in layered media

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The talk describes two sets of experiments studying acoustical propagation in layered media: propagation in porous road pavement and propagation along the crowd of listeners of a large pop event. The experimental settings may seem quite different, but physically the situations have many common features. Each of them contains a layer which has some air fraction and solid fraction, some absorption, and which displays waveguiding properties. In both problems the source of the field is assumed to be a point source.

The experimental results are checked againist numerical computations which are held in terms of the Fourier–Bessel integral.

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Delta-shocks in one system of conservation laws

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We study δ-shock solutions in a new type of system of conservation laws:

$$(u_j)_t + \big(u_j\, f_j(\mu_1 u_1 + \cdots + \mu_n u_n)\big)_x = 0, \quad x \in \mathbb{R}, \quad t \geqslant 0, \quad j = 1, 2, \ldots, n,$$

where $f_j(\cdot)$ is a smooth function, μ_j is a constant, $j = 1, 2, \ldots, n$. This class of systems includes some *Temple type system* [4]; in particular, the system of *nonlinear chromatography*: $f_j(v) = 1 + \frac{a_j}{1+v}$, where a_j is Henry's constant, $\mu_j = \pm 1$, $j = 1, 2, \ldots, n$; the system for *isotachophoresis* [1]: $(\rho_j)_t + I\big(\frac{\mu_j \rho_j}{\sum_{s=0}^{n} \mu_s \rho_s}\big)_x = 0$, $j = 0, 1, 2, \ldots, n$, $\sum_{s=0}^{n} \rho_s = 0$, where ρ_j is the charge density of anions of the jth type ($j = 1, 2, \ldots, n$), ρ_0 is the corresponding value for cations, μ_j is the electrophoretic mobilities of the corresponding ions ($j = 0, 1, 2, \ldots, n,$); $\mu_0 < 0 < \mu_1 < \cdots < \mu_n$; $I =$ constant is the current.

A *δ-shock* is a solution whose components *contain Dirac delta functions*. Problems related to δ-shocks have been intensively studied in the last years (see [3] and the references therein). To deal with such singular solutions we use the *weak asymptotics method* developed in [2] and some results from [3]. We introduce integral identities which give the definition of δ-shocks in the above system and derive the corresponding Rankine–Hugoniot conditions. It is proved that the "area" transport processes between the moving singular one-dimensional δ-shock wave front and the region outside the front are going on. Balance relations describing these processes are derived. The Cauchy problem related with the propagation of δ-shock wave is solved.

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Solving diffraction problems on compact scatterers by hybrid approach using continued boundary conditions method

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There are quite obvious computation difficulties, connected in particular with extremely large algebraic systems of equations sizes, to which boundary problem reduces, at numerical solution of wave diffraction problems on scatters with size much larger than wave length. In this case so called hybrid approach is reasoned enough. At this approach in integral equation for the current on the scatterer surface the unknown function (current) represented in form of two summands: known quantity (geometric optical current) and unknown part, localized in the neighborhood of boundary between light and shadow zones. If in this approach we will use the method of continued boundary condition (MCBC) to find unknown summand, the problem reduces to Fredholm integral equations with smooth kernel, which solution algorithm has high speed [1, 2].

Let's consider as an example the diffraction problem on circular cylinder with sufficient large radius a, on which boundary Dirichlet condition is satisfied. Let the plane wave falls on the cylinder at the angle $\phi_0 = \pi/2$. Scattered by cylinder field U^1 in this case can be represent by the following integral

$$U^1(r,\phi) = \frac{ia}{4}\int_0^{2\pi} \frac{\partial U}{\partial r'} H_0^{(2)}(k|\vec{r}-\vec{r}'|)d\phi', \tag{1}$$

where $U = U^0 + U^1$ is the total field (incident U^0 in sum with scattered). Setting the current (value $\frac{ia}{4}\frac{\partial U}{\partial r'}$) on lighted part of boundary surface (i.e. at $\pi+\alpha < \phi' < 2\pi-\alpha$), equal to $\frac{ia}{2}\frac{\partial U^0}{\partial r'}$, equal to zero on the shadow part (i.e. at $\alpha < \phi' < \pi-\alpha$), and equal to unknown values $I_1(\phi')$ and $I_2(\phi')$ on the neighborhood of boundary between light and shadow parts, we obtain for this unknown values the following integral equations

$$\left.\int_{-\alpha}^{\alpha} I_1(\phi')H_0^{(2)}(k|\vec{r}-\vec{r}'|)d\phi'\right|_{S_{1\delta}} + \left.\int_{\pi-\alpha}^{\pi+\alpha} I_2(\phi')H_0^{(2)}(k|\vec{r}-\vec{r}'|)d\phi'\right|_{S_{1\delta}} = $$
$$= -\left.\left[e^{-ik(a+\delta)\sin\phi} + \frac{ka}{2}\int_{\pi+\alpha}^{2\pi-\alpha} e^{-ika\sin\phi'}(\sin\phi')H_0^{(2)}(k|\vec{r}-\vec{r}'|)d\phi'\right]\right|_{S_{1\delta}}, \tag{2}$$

$$\left.\int_{-\alpha}^{\alpha} I_1(\phi')H_0^{(2)}(k|\vec{r}-\vec{r}'|)d\phi'\right|_{S_{2\delta}} + \left.\int_{\pi-\alpha}^{\pi+\alpha} I_2(\phi')H_0^{(2)}(k|\vec{r}-\vec{r}'|)d\phi'\right|_{S_{2\delta}} = $$
$$= -\left.\left[e^{-ik(a+\delta)\sin\phi} + \frac{ka}{2}\int_{\pi+\alpha}^{2\pi-\alpha} e^{-ika\sin\phi'}(\sin\phi')H_0^{(2)}(k|\vec{r}-\vec{r}'|)d\phi'\right]\right|_{S_{2\delta}}. \tag{3}$$

where $S_{1\delta}: \ r = a+\delta, \ -\alpha \leqslant \phi \leqslant \alpha; \quad S_{2\delta}: \ r = a+\delta, \ \pi-\alpha \leqslant \phi \leqslant \pi+\alpha$. By usage of discrete sources [2] method equations (2), (3) are reduced to algebraic equation systems.

Proposed hybrid approach in the couple with MCBC advantages allows essentially increase a solution algorithm calculating capacity for considered problem.

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Scattering of the linear and nonlinear waves in optical waveguide array on the PT-symmetric defects

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It has been shown by Bender in 1998 that non-Hermitian Hamiltonians may have an entirely real eigenvalue spectrum under parity-time (PT)-symmetry constrain [1]. In optics this requirement reduces to the following property of the complex optical potential $V(x) = V^*(-x)$. The systems with such potential have unique properties and give more flexible control of propagating pulses in comparison to the conservative systems.

In the present work, we consider a chain of waveguides that includes a pair of waveguides with energy gain in one of them and loss in another. Within the framework of this model we derive the coefficients of reflection and transmission for linear waves. It was demonstrated numerically that linear theory describes with a good accuracy scattering of small amplitude solitons [2]. For solitons with sizeable amplitude it was shown that passage of the soliton can excite the mode localized on the defect. Interaction of soliton with the localized mode was studied as well. It was shown that within certain range of parameters localized mode can be switched from PT-symmetric regime to the non-PT-symmetric one by the incident soliton.

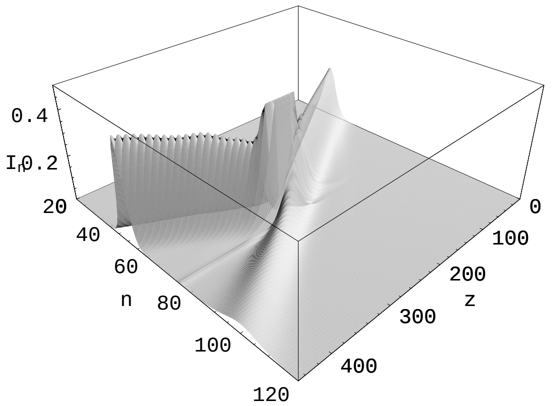

Fig. 1: Example of the soliton scattering with the excited localized mode.

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Non-destructive testing of composite materials through linear sampling method

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Microwave non-destructive testing of composite materials is a challenging research subject with many important industrial applications especially in aviation industry. Deformations in aircraft fuselage due to aging or other reasons threaten the passengers safety and reduce the aircraft lifetime. While vast number of studies have been carried out for the detection and imaging of the deformations in layered structures like composite materials found in aircrafts, it is still required to develop higher resolution techniques to image such deformations for its immense economical and humanitarian consequences.

In this context, we consider the application of the reciprocity gap linear sampling method (RG-LSM) [1] in conjunction with interior transmission eigenvalues [2] for testing of aircraft fuselage. We restrict our analysis to 2D configuration i.e. a layered composite structure which is unchaining in one direction is illuminated by a system of antennas from the accessible sides and the scattered field is measured with the same antennas. As a qualitative inverse scattering method, RG-LSM can reconstruct the boundary of the deformation without a-priori information about the boundary conditions; however the quality of the reconstructions is highly dependent on the measurement configurations. We have investigated several possible measurement settings and source excitations up to 40 GHz to uncover both possibilities and limitations of the sampling method for non-destructive testing purposes. Preliminary numerical results indicate that RG-LSM can be an effective alternative to existing methods.

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Doubling of variables method for some evolutionary equations with rough coefficients

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We prove existence and uniqueness of a weak solution to an ultra-parabolic equation with discontinuous convection term. Due to degeneracy in the parabolic term, the equation does not admit the classical solution. Equations of this type describe processes where a transport is negligible in some directions. Such situation is demonstrated on the figure below.

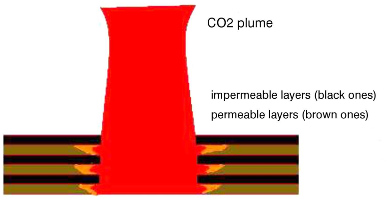

Fig. 1: CO2-plume expansion in a highly stratified surrounding. Transport in the vertical direction can be obviously neglected.

Operator error estimates for homogenization of the elliptic Dirichlet problem in a bounded domain

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Let $\mathcal{O} \subset \mathbb{R}^d$ be a bounded domain of class $C^{1,1}$. In $L_2(\mathcal{O};\mathbb{C}^n)$, we consider a matrix elliptic differential operator $A_{D,\varepsilon}$, $\varepsilon > 0$, given by the differential expression $A_\varepsilon = b(\mathbf{D})^* g(\mathbf{x}/\varepsilon) b(\mathbf{D})$ with the Dirichlet boundary condition. We assume that an $(m \times m)$-matrix-valued function $g(\mathbf{x})$ is bounded, uniformly positive definite and periodic with respect to some lattice Γ. The elementary cell of Γ is denoted by Ω. Next, $b(\mathbf{D}) = \sum_{j=1}^d b_j D_j$ is an $(m \times n)$-matrix first order differential operator (b_j are constant matrices). It is assumed that $m \geqslant n$ and the symbol $b(\boldsymbol{\xi}) = \sum_{j=1}^d b_j \xi_j$ has maximal rank, i. e., $\operatorname{rank} b(\boldsymbol{\xi}) = n$ for $0 \neq \boldsymbol{\xi} \in \mathbb{R}^d$. The simplest example is $A_\varepsilon = -\operatorname{div} g(\mathbf{x}/\varepsilon)\nabla$.

We study the behavior of the solution $\mathbf{u}_\varepsilon$ of the Dirichlet problem $A_\varepsilon \mathbf{u}_\varepsilon = \mathbf{F}$ in $\mathcal{O}$, $\mathbf{u}_\varepsilon|_{\partial\mathcal{O}} = 0$, where $\mathbf{F} \in L_2(\mathcal{O};\mathbb{C}^n)$. It turns out that $\mathbf{u}_\varepsilon$ converges in $L_2(\mathcal{O};\mathbb{C}^n)$ to $\mathbf{u}_0$, as $\varepsilon \to 0$. Here $\mathbf{u}_0$ is the solution of the "homogenized" Dirichlet problem $A^0\mathbf{u}_0 = \mathbf{F}$ in $\mathcal{O}$, $\mathbf{u}_0|_{\partial\mathcal{O}} = 0$. The *effective operator* A^0_D is given by the expression $A^0 = b(\mathbf{D})^* g^0 b(\mathbf{D})$ with the Dirichlet boundary condition. The effective matrix g^0 is a constant positive $(m\times m)$-matrix defined as follows. Denote by $\Lambda(\mathbf{x})$ the $(n\times m)$-matrix-valued periodic solution of the equation $b(\mathbf{D})^* g(\mathbf{x})(b(\mathbf{D})\Lambda(\mathbf{x}) + \mathbf{1}_m) = 0$ such that $\int_\Omega \Lambda(\mathbf{x})\, d\mathbf{x} = 0$. Then $g^0 = |\Omega|^{-1}\int_\Omega g(\mathbf{x})(b(\mathbf{D})\Lambda(\mathbf{x}) + \mathbf{1}_m)\, d\mathbf{x}$.

Theorem 1. (see [2]) *We have the following sharp order error estimate*:

$$\|\mathbf{u}_\varepsilon - \mathbf{u}_0\|_{L_2(\mathcal{O};\mathbb{C}^n)} \leqslant C\varepsilon\|\mathbf{F}\|_{L_2(\mathcal{O};\mathbb{C}^n)}, \tag{1}$$

or, in operator terms, $\|A^{-1}_{D,\varepsilon} - (A^0_D)^{-1}\|_{L_2\to L_2} \leqslant C\varepsilon$.

Now we give approximation of $\mathbf{u}_\varepsilon$ in the Sobolev space $H^1(\mathcal{O};\mathbb{C}^n)$. For this, the first order corrector must be taken into account.

Theorem 2. (see [1]) 1) *If* $\Lambda \in L_\infty$, *then*

$$\|\mathbf{u}_\varepsilon - \mathbf{u}_0 - \varepsilon\Lambda^\varepsilon b(\mathbf{D})\mathbf{u}_0\|_{H^1(\mathcal{O};\mathbb{C}^n)} \leqslant C\varepsilon^{1/2}\|\mathbf{F}\|_{L_2(\mathcal{O};\mathbb{C}^n)}, \tag{2}$$

or, in operator terms, $\|A^{-1}_{D,\varepsilon} - (A^0_D)^{-1} - \varepsilon\Lambda^\varepsilon b(\mathbf{D})(A^0_D)^{-1}\|_{L_2\to H^1} \leqslant C\varepsilon^{1/2}$. *Here* $\Lambda^\varepsilon(\mathbf{x}) = \Lambda(\varepsilon^{-1}\mathbf{x})$.
2) *In the general case, we have*

$$\|\mathbf{u}_\varepsilon - \mathbf{u}_0 - \varepsilon\Lambda^\varepsilon b(\mathbf{D})(S_\varepsilon\widetilde{\mathbf{u}}_0)\|_{H^1(\mathcal{O};\mathbb{C}^n)} \leqslant C\varepsilon^{1/2}\|\mathbf{F}\|_{L_2(\mathcal{O};\mathbb{C}^n)}. \tag{3}$$

Here $\widetilde{\mathbf{u}}_0 = P_\mathcal{O}\mathbf{u}_0$ *and* $P_\mathcal{O} : H^2(\mathcal{O};\mathbb{C}^n) \to H^2(\mathbb{R}^d;\mathbb{C}^n)$ *is a continuous extension operator,* S_ε *is the smoothing operator in Steklov's sense defined by* $(S_\varepsilon\mathbf{u})(\mathbf{x}) = |\Omega|^{-1}\int_\Omega \mathbf{u}(\mathbf{x} - \varepsilon\mathbf{z})\, d\mathbf{z}$.

We use the results of M. Birman and T. Suslina for homogenization problem in $\mathbb{R}^d$: the analogs of estimates (2), (3) in $\mathbb{R}^d$ are of sharp order ε. The problem is reduced to estimating of the discrepancy $\mathbf{w}_\varepsilon$, which is the solution of the problem $A_\varepsilon\mathbf{w}_\varepsilon = 0$ in $\mathcal{O}$, $\mathbf{w}_\varepsilon|_{\partial\mathcal{O}} = \varepsilon\Lambda^\varepsilon b(\mathbf{D})(S_\varepsilon\widetilde{\mathbf{u}}_0)|_{\partial\mathcal{O}}$ (or $\mathbf{w}_\varepsilon|_{\partial\mathcal{O}} = \varepsilon\Lambda^\varepsilon b(\mathbf{D})\mathbf{u}_0|_{\partial\mathcal{O}}$ in the case $\Lambda \in L_\infty$). We show that the norm of $\mathbf{w}_\varepsilon$ in H^1 satisfies estimate of order $\varepsilon^{1/2}$. This leads to (2), (3). At the same time, the norm of $\mathbf{w}_\varepsilon$ in L_2 is of order ε, this allows us to prove sharp order estimate (1).

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Anisotropic diffraction in acoustic delay lines with mosaic transducers

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Based on the Green's function, we built the method for calculating the diffraction fields of mosaic piezoelectric transducers in anisotropic crystalline sound conductor.

We gave the results of numerical research of diffraction fields generated by mosaic piezoelectric transducer on the end surface of the sound conductor and the calculation of the diffraction losses of the main signal, triple and five-time signal passage in acoustic delay line microwave signal operating mode "on the passage".

We investigated the dependence of the diffraction losses on the angle between the crystallographic and the geometrical axes of the sound conductor and diffraction losses at different values of the displacement input and output transducers relative to each other's to determine the allowable for manufacturing value of the displacement transducers.

Based on these investigations we suggested a method for suppression of multipass signals by using the diffraction properties of the transducers.

“Complex source” in 2D real space

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We consider complexified Green function for the 2D Helmholtz operator, $g_* = -\frac{i}{4}H_0^{(1)}(kr_*)$, where $r_* = \sqrt{x^2 + (z - ia)^2}$ with $a > 0$ a free positive constant. The complexified distance r_* is a multivalued function of x, z with branch points at $x = \pm a$, $z = 0$. Under the proper definition of r_*, the function g_* shows a Gaussian beam behavior near the z-axis.

It appears that g_* satisfies a certain inhomogeneous Helmholtz equation in the real 2D space, with source distribution F localized on a certain curve. F depends on the choice of the branch cut for the function r_*. Recently, the source function was calculated for a rather general choice of the branch cut for the 3D analog of g_* (see [1]). We perform a similar calculation in 2D.

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Essential spectrum of a periodic elastic waveguide may contain arbitrarily many gaps

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We study the spectral problem for the linearized elasticity system with traction–free boundary conditions. We show that in the case the elastic body is a suitable infinite waveguide with periodically positioned cells, the essential spectrum contains gaps, for the number of which we are able to prove a lower bound. Moreover, we construct examples where the lower bound can be made larger than any given number.

The construction involves a waveguide depending on a small geometric parameter $h > 0$, which describes the width of ligaments connecting the adjacent cells. On the limit $h \to 0$ the waveguide thus becomes a disjoint union of infinitely many bounded subdomains, and the corresponding elasticity problem has discrete spectrum. The proof of the main result makes use of this property via Gelfand transform techniques.

This presentation reports on the papers [1] and [2]. Finally, we discuss the possibility to generalize the approach to the piezo-electric system.

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Linear approximation resonance curve for pressed wave excited by input grating coupler

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We have calculated earlier [1] the fields generated by input grating coupler in the linear by groove height approximation. It was shown that because of finite dimension of input grating coupler surface-bulk wave is generated on smooth part of the surface in surface electromagnetic wave excitation conditions. This wave was named "pressed wave" by us. In this work we compare calculated resonance curves for pressed wave with experimental ones. The calculated results show that if the width of grating relief Fourier-spectrum is much more (the grating width is small enough) than the width of resonance curve for infinite grating the calculated resonance curve coincides with the Fourier-spectrum of grating relief. However experimental resonance curve may differ significantly from calculated ones obtained in linear by groove height approximation. This difference is enhance with grating width. For grating widths of 1mm or less the experimental resonance curve is almost coincides with calculated ones. As grating width is increased additional peaks and pits are appear in experimental resonance curves. These peaks and pits are observed particularly for grating widths of 4 mm and more. We believe that these peaks and pits are caused by reflections of excited surface wave from the grating edges. In this case the grating with edges acts as peculiar Fabry–Pero resonator and observed peaks and pits are associated with resonances of this resonator. Calculations performed in linear by groove height approximation do not allow to take into account the reflections of surface wave from grating edges. So approximations of second or more orders are needed to obtain correct calculated resonance curves.

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Ultrashort radiation pulses generated by laser wakefield accelerators: A time-domain approach

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A novel experimental technique, in which charged particles are pushed by the electric field of a plasma wave (the wakefield) driven by an intense laser, offer compact accelerators nearly as efficient as huge traditional synchrotrons [1]. The resulting extremely bright betatron radiation sources have a potential to boost numerous uses across the whole spectrum of light-source applications [2]. Due to sufficiently complicated nature of the involved processes, most of the theoretical description of such laser-driven betatrons has been done on the basis of numerical simulation (see, e.g., [3] and references therein). A few published analytical results concerning the electron/ion motion [4] are based on simplified one-dimensional models of laser piston maintained by the radiation pressure. In all circumstances (even if the early stage involves a space-time technique, such as introduction of the Liénard–Wiechert potential), the betatron radiation is described using the frequency-domain approach [5].

The report deals with the time-domain investigation of formation of the laser-betatron radiation. Although being carried out within the framework of a simple (toy) model of a decaying modulated line source current, such an approach can reveal interesting features of the resulting emanated electromagnetic pulse that can hardly, if at all, described using frequency-domain representation.

The problem under investigation is solved using the incomplete separation of variables and the Riemann–Volterra formula. The model differs from those described Sec. 5 of Ref. [6]: (i) transverse charged-particle oscillations play the primary role in the wave emanation, and the transverse component of the source current density cannot be neglected; (ii) the longitudinal period of oscillation k^{-1} is not small with respect to the source-current pulse length l (an illustrative example is given in Fig. 2 of [5]), so the wave potential cannot be assessed via expansion in the small parameter $(kl)^{-1}$.

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Electromagnetic field formed by collimated gamma quanta pulse beam

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In our previous works [1, 2] the electric current pulses formed through the interaction between gamma rays and material of a lengthy target have been computed. It has been shown that the formed current pulses' motions take place at light or superlight velocity depending on the geometry of the simulation experiment. The secondary electromagnetic fields generated by these current pulses (electromagnetic radiation's sources) calculation have been performed within the bounds of the linear current model. But in the real experimental conditions the axial symmetry is often absent and the above linear current model is not applicable. This report is devoted to the presentation of the results of time-space features' study of the electromagnetic emission's superlight sources, formed by free electrons born during the ionization of the lengthy gas target exposed to the gamma quanta shooting from the strait-line segment. In this study current density vectors have been calculated on the basis of an accounting of the motions of the separate electric charges, generated in the issue of the interaction between the gamma quanta beam and the target's medium. The characteristics of the secondary collective radiation have been obtained using the axially symmetric problem's solution and superposition principle. The results of this study let us making the following conclusions:

1. The absence of the axial symmetry lead to the absence of secondary radiation energy spatio-temporal distribution's "cone-shaped" pattern.

2. Secondary electromagnetic radiation's spatio-temporal distribution depends on relationship between geometric parameters of the target and the ionizing gamma beam incidence angle magnitude.

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Modelling dispersion of spin-electromagnetic waves in multilayered ferrite-ferroelectric structure

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In recent years, interest in the fabrication and study of man-made multiferroics has grown rapidly. One way to create a multiferroic material is to combine a ferrites and ferroelectrics into multilayered structures. It was shown, that spin-electromagnetic waves (SEW) propagating in ferrite-ferroelectric (FF) structure may find a variety of applications. In particular, these structures can be used for development of dual-tunable microwave devices [1, 2]. A spectrum of SEW is formed by hybridization between dispersion characteristics of spin waves and electromagnetic waves propagating in the layered structures. Until now a tensorial Green's function method was used to calculate SEW spectra in FF structures composed of one ferrite layer and 1-4 ferroelectric layers [3]. At the same time, it is difficult to use this method in the case of structures composed of a big number of ferrite and ferroelectrics layers. Purposes of the present work are to develop theory of quasi-surface spin-electromagnetic waves in multilayered FF structures consisted of several ferrite and ferroelectric layers and to investigate properties of the such waves. Dispersion equation of SEW in the such structures was found. Numerical modelling of SEW dispersion characteristics in FF structures included several ferrite layers was carried out. The results of modelling in the case of one ferrite layer was in a good agreement with results reported previously [3, 5]. Spectra of SEW in FF structures consisted of several ferrite layers separated by dielectric (included ferroelectric) layers was investigated. It was shown that dispersion characteristic consist of several branches, number of branches is determined by amount of ferrite layers in FF multilayered structures. Influence of different parameters on dispersion characteristics of SEW in FF structures was investigated. Electric field tuning of SEW spectra was demonstrated. An increasing in a number of ferrite layers makes SEW spectra more complicated that provide flexibility for MW device design.

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Numerical simulation of multipactor discharge on a dielectric surface

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At last time extensive attention has been focused on the problem of multipactor discharge on a dielectric surface. This phenomenon is commonplace and influences significantly operation of powerful microwave devices. The secondary avalanche may deposit energy to the window to cause failure. This work proposes numerical simulation of multipactor discharge based on a novel theory of initial stage of single-side multipactor discharge on dielectric surface. We evaluate the combined action of an rf electric field that is parallel to the dielectric surface(as is usually the case near transmission windows), and of a dc electric field normal to the surface. The calculation employs the statistical method based on an exact analytical solution for the probability density of arrival time of secondary electrons. The general integral equation allows to predict the steady-state emission phase distribution and the threshold of multipactor growth in the presence of an external magnetic field is proposed. A computer program has been elaborated to implement this theory for realistic secondary yield curves and arbitrary given distributions in the emission velocities and emission angels of injected electrons. This program allows to calculate the threshold characteristics of discharge, distinguished power on dielectric on stage of saturation, value of limiting concentration of electrons and time of formation of discharge. It is shown that the magnetic field qualitatively changes the multipacting discharge cutoff conditions. It has been found that the shape of susceptibility curves predicted from the statistical model is in a good qualitative agreement with the ones obtained through Monte Carlo simulations.

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Heaviside generalized functions and shock waves in nonlinear problems: a survey

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In the work we study the existence and nonexistence of generalized shock wave solutions for systems of partial differential equations of hydrodynamics with viscosity in one space dimension, for systems of partial differential equations suggested by systems of hydrodynamics without viscosity in a one-dimensional space and for Burger kind equations suggested by the classical inviscid Burgers equation. The study is developed in the Colombeau's theory of generalized functions context. This study uses the equality in the strict sense and the weak equality of generalized functions. The shock wave solutions are given in terms of generalized functions (which are not distributions) that have the Heaviside function, in x and (x, t) variables, as macroscopic aspect. This means that solutions are sought in the form of sequences of regularizations to the Heaviside function, in $\mathbb{R}^n$ and $\mathbb{R}^n \times \mathbb{R}$, in the distributional limit sense. To the considered systems, these solutions have to satisfy part of the equations in the strict sense and part of the equations in the weak equality sense. One of the motivations for introducing Heaviside generalized functions is its use in the study of shock wave solutions of partial differential equations that modeling some physics fhenomenon, for example.

The electromagnetic waves guided by the stratified composite media

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Over the past decade, there has been shown a substantial interest in the propagation characteristics of electromagnetic waves in the composite media. This interest has been motivated by many promising applications of composite materials [1].

We consider the features of propagation of waves guided by the planar anisotropic composite slab located between two dielectric isotropic layers with permittivity ε_1. The composite medium is described by the permittivity and permeability tensors $\hat{\varepsilon}$ and $\hat{\mu}$ with zero off-diagonal elements. We consider single-axis medium with the symmetry axis oriented perpendicular to the interface between composite and dielectric media. In particular we analyze the case of isotropic composite medium. For the monochromatic field, permittivity and permeability constants of the medium can be written in the form $\varepsilon = 1 - (\omega_p/\omega)^2$ and $\mu = 1 - F\omega^2/(\omega^2 - \omega_m^2)$ where parameters F, ω_p, ω_m are determined by the technological properties of elementary cells of composite materials [2]. We investigate a dependence of the dispersion characteristics of direct and backward waves of TE and TM types over the layer width. We obtain reflection and transmission coefficients for the case of oblique incidence of the electromagnetic waves on the considering structure and discuss the possibilities of unreflection regime.

The main attention is given to the nonlinear interaction of the surface waves guided by the isotropic composite slab in the presence of external time-harmonic magnetic field which is perpendicular to the composite layer. The intense external magnetic field may effect on the medium properties and permeability of the composite medium depends on the amplitude magnetic field. The parametric instability has been developed if the space-time conditions between the external magnetic field and surface waves take place. The increments of the parametric instability for some interesting cases have been obtained and discussed.

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Semiclassical analysis of tunneling through a smooth barrier in graphene monolayer

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We present a semiclassical analysis of Dirac electron-hole tunneling in graphene through a smooth Gaussian shape barrier representing electrostatic potential in ballistic regime. For rectangular barrier Kein tunneling we refer to [1], [2], for parabolic and piece-wise constant — to [3], [4]. This 1D scattering problem is formulated in terms of transfer matrix and treated in WKB approximation. In this paper for a skew electron incidence this approximation deals with four turning points for a system of two the fist order ODE. Between the couples of the turning points (not coalescing) we observe two strips of total internal reflection well-known in electromagnetic optics where the field solution exponentially decay. Scattering through a smooth barrier in graphene resembles scattering through a double barrier for 1D Schrödinger operator that is Fabry–Perot resonator. The main results of the paper are very simple WKB formulas for the barrier transfer matrix. These explain the mechanism of total transmission through the barrier for some resonance values of energy of a skew incident electron or hole as well as all different types of WKB solutions describing electron or hole in classically allowed and disallowed domains. Moreover, we show an existence of modes localized within the barrier and exponentially decaying away from it. Namely, these are two groups of quasi-bound (meta-stable) and bound states, with two discrete complex (with small imaginary part) and real sets of energy eigenlevels determined by Bohr–Sommerfeld quantization condition presented in two forms, above and below the cut-off energy, respectively. Similar to [3], [4] and [5], it is shown that the total transmission through the barrier takes place when energy of incident electron or hole coincide with real part of a complex eigenlevel of quasi-bound state. This asymptotic results were confirmed by numerical tests done by finite elements method for one partial example.

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Example of n-ary bialgebra

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In this paper we establish an example of n-ary bialgebra. It is shown that n-ary comultiplication is not a composition of binary comultiplication δ. In this way, it is obtained that for any $n \geqslant 3$, exists $n-1$–dimensional (n,n)-bialgebra, which is a simple $(2,n)$-algebra. This is not possible for $n=2$. It is also shown that the (n,n)-bialgebra is self-dual.

Surface plasmon scattering by one- and two-dimensional defects of metal/dielectric/metal slot waveguide: 3D nanofocusing of light

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Light localization and light control at nanometer scales are currently one of the most developing branch of contemporary nanophotonics, that is caused by a number of possible applications in different fields including medicine, biology, chemistry, nanolasing, integrated optics, etc. Surface plasmon is one of the primary object which is widely discussed in recent years as an electromagnetic excitation enabling to provide light nanofocusing [1–3].

In this report we consider some problems of surface plasmon scattering by one- and two-dimensional defects (narrownesses) of metal/dielectric/metal (MDM) slot waveguide. The plasmon propagation in inhomogeneous MDM slot waveguide we describe by means of the rigorously derived wave equation with effective refraction index written in terms of voltage drop into dielectric slot. We show that in dependence of the defect size in both one- and two-dimensional cases plasmon scattering can accompany by superlocalization of light with 'hot spots' of nanometer scales formation in which the field intensity may exceed intensity of incident plasmon in several orders despite the quite strong Drude energy losses into metal. Furthermore, we obtain an exact solution on surface plasmon scattering in MDM slot waveguide by so-called 'plasmonic black hole' [4], possessing a singularity of the effective refractive index. Bearing in mind a strong electromagnetic field enhancement in local hot spots we also discuss an influence of possible nonlinear effects due to the nonlinearity of dielectric filling the slot of waveguide.

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Differential equation for geometrical spreading on a ray and second derivatives of eikonal matrix structure

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Dynamic ray tracing implies calculation of geometrical spreading and is usually executed by solution of the matrix Riccati equation for a special 2×2 matrix, which represents the part of the 3×3 second derivatives of eikonal matrix in the natural basis, followed by a special integration.The problem arises whether it is possible to deduce the scalar differential equation directly for the geometrical spreading.In other words, Popov's dynamic ray tracing system of differential equations for matrixes Q, P is given and corresponding equation for $\det Q$ is of the interest.This problem has been solved in this paper. With usage of Popov system and identity derived from the Cayley identity for 2×2 matrixes it is shown that the 2×2 second derivatives of eikonal matrix satisfies some algebraic Sylvester equation with inhomogeneity depending on $\det Q$ and $\det P$. It is solved and thereby the special structure is obtained. By such representation of the second derivatives of eikonal matrix and the Riccati equation we got system of 2 differential equations for $\det Q$ and $\det P$. One of them is solved with respect to $\det P$ and using substitution we obtained the sought equation for the geometrical spreading – it is nonlinear and 4$^{\text{th}}$ order. We also showed that the 2×2 second derivatives of eikonal matrix (and hence the 3×3 second derivatives of eikonal matrix) can be represented in terms of geometrical spreading not functionally but with derivatives.

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Meta-session

Electromagnetic characterization of metasurfaces in presence of substrate-induced bianisotropy

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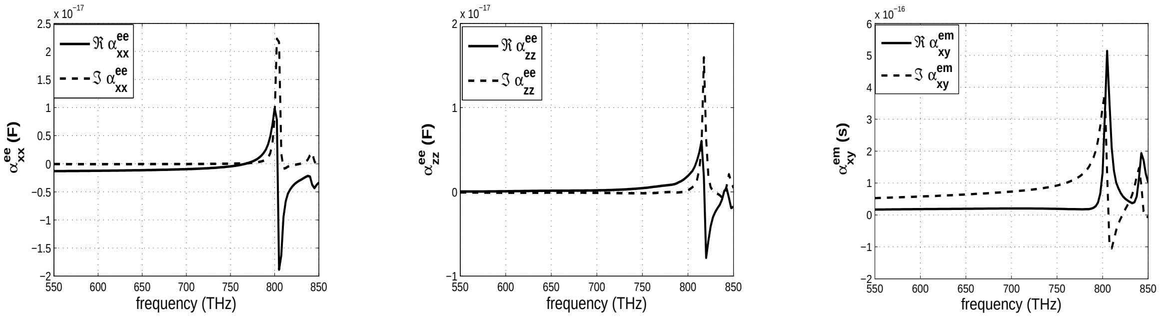
Abstract. This paper is based on the derivation and inversion of Fresnel-like formulas in the presence of an optically dense grid of scattering particles on the interface. Such grids refer to so-called metasurfaces if these particles are resonant. An electro-magnetic and magneto-electric responses of particles arise in the presence of the strongly refracting substrate. The goal of the present study is to demonstrate this effect. Four scalar surface susceptibilities are introduced to characterize the metasurface of simple plasmonic nanoparticles (nanospheres, nanopatches, etc) as a homogenized one. As the grid of particles is very dense, the susceptibilities of the homogenized metasurface turn out to be independent of the angle of incidence, and can be claimed as its characteristic parameters. These introduced parameters can be extracted through measurements/simulations of reflection and transmission coefficients and we developed a rather robust algorithm of this extraction.

Introduction. The problem of an optically dense grid at the interface of two media has been thought for many years [1, 2, 3], however is not closed yet. The dipole grid formed by ion clusters at the dielectric interface was homogenized [1, 2, 3] however these clusters are non-resonant or weakly resonant particles due to their small sizes. Since early 2000 many works have been devoted to characterize optically dense grids which play an important role in the metamaterial science. However, in most cases these surface structures were characterized by bulk material parameters. The inconsistency of this approach has been criticized in several works (e.g. [4] and [5]). To replace bulk tensors of ϵ and μ surface electric susceptibilities (χ_{ES}) and magnetic susceptibilities (χ_{MS}) were introduced and such grids were called metasurfaces (metafilms) in [4, 5, 6]. It was proved that these susceptibilities for optically dense grids do not depend on the incident wave characteristics. The main restriction of the model [4, 5, 6] is the assumption that the grid is placed in a uniformly homogeneous host medium. The goal of the present work is to model a metasurface while it is placed on an arbitrary substrate. To take into account the effect of the substrate we introduced two additional (magneto-electric and electro-magnetic) susceptibilities expressed one through another by the reciprocity condition. This new susceptibility results from the break of the symmetry by the substrate.

Numerical Example. With this example we show that our model is useful and adequate. We consider a square grid of silver spheres of diameter 40 nm with period 80 nm located on a substrate so that the centers of the spheres are distanced by 45 nm from the interface. The relative dielectric permittivity of the dielectric half space is equal ϵ_r=12.8. We have retrieved unknown susceptibilities α^{ee}_{xx}, α^{ee}_{zz}, and α^{em}_{xy} using the results of HFSS simulations for three incident angles (0, 10, and 20 degrees) — see Fig. 1. In the case of so small nanospheres, we can neglect the magnetic susceptibility α_{mm} of the metasurface. However, the electro-magnetic susceptibility α_{em} turns out to be not negligible. If we omit it, the characterization procedure fails.

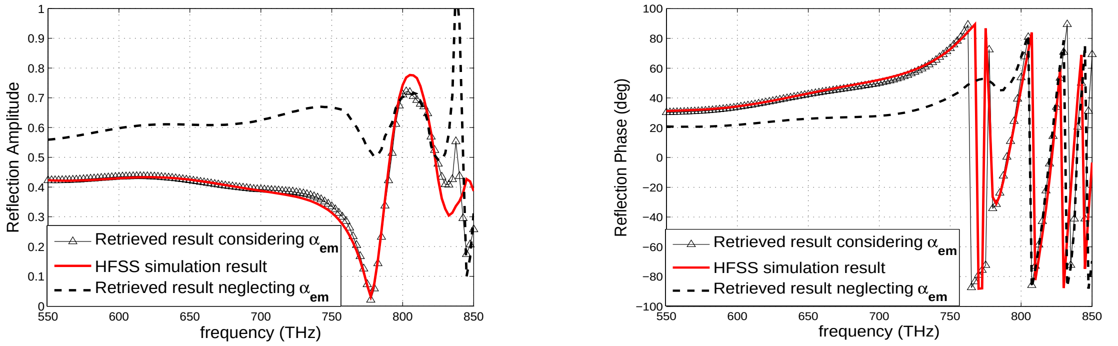
Fig. 2 shows the retrieved and simulated results- magnitude and phase- of the reflection coefficient at 45°. In this figure we can see: if one neglects the substrate-induced bianisotropy letting the magneto-electric susceptibility be zero, the full-wave simulated results would be far from the expected one. This situation is corrected when we introduce the magneto-electric susceptibility into the characterization model. As the figure shows, this model compromise with full-wave simulation results with a small deviation. Notice, that for our retrieval we have selected small incident angles, but the retrieved parameters work for large incident angles.

Conclusion. By introducing a new independent characteristic parameter for a metasurface located on top of a strongly refracting half space we have demonstrated the presence of substrate-induced bianisotropy. Our Fresnel-like formulas at the presence of such a bianisotropy are obtained and inverted for retrieval. The retrieval procedure has been developed. The necessity of the magneto-electric susceptibility has been shown in a numerical example.



(a) (b) (c)

Fig. 1: (a) – $\mathrm{Re}(\alpha^{ee}_{xx})$ (solid) and $\Im(\alpha^{ee}_{xx})$ (dashed), (b) – $\mathrm{Re}(\alpha^{ee}_{zz})$ (solid) and $\Im(\alpha^{ee}_{xx})$ (dashed), and (c) – $\mathrm{Re}(\alpha^{em}_{xy})$ (solid) and $\Im(\alpha^{em}_{xy})$ (dashed) for a metafilm on top of a substrate with relative permittivity $\varepsilon_r = 12.8$.



(a) (b)

Fig. 2: (a) – Reflection amplitude and (b) – Phase characteristics of a metafilm on the top of a substrate for a 45° incident angle.

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Geometrical optics description of radial forces over lossy spherical particles with a negative index of refraction

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During the last years, our group has dedicated a lot of efforts to the theoretical and numerical aspects of the optical trapping and micromanipulation of spherical micro-particles having a simultaneous negative permittivity and permeability [1–6]. We were able to show how these negative refractive index (NRI) scatterers would behave under the influence of some of the most common laser beams in optical tweezers' experiments, specially concentrating our efforts on the prediction of (gradient or scattering) optical forces. Both the ray optics and the Generalized Lorenz–Mie theory have been employed to predict, for example, that depending on the relative distance between a lossless NRI sphere and the beam waist centre of a focused Gaussian beam, the NRI particle could be either attracted to or repealed from that laser beam. However, all predictions so far were based on the weak assumption of negligent (zero) losses, and this certainly poses restriction as one tries to conceive a realizable experiment. Therefore, this work is devoted to the task of presenting an overview of our fundamental results for lossless NRI spheres and our most recent preliminary results on the optical forces over absorbent NRI spheres using geometrical optics. The inclusion of losses may certainly pose difficulties in achieving an effective 3D trap mainly due to the scattering (parallel to the optical axis) forces, although, as we intend to show, radial optical forces may also be unable to push the particle towards the beam. We analyse such a difficulty by looking at some optical force profiles for both focused Gaussian and ordinary Bessel beams. An extension to the Generalized Lorenz–Mie theory will also be outlined. With these results, we expect to be able, in the near future, to extend our results for arbitrary-geometry NRI scatterers, providing the necessary theoretical support towards a realizable experiment and, therefore, to studies relating possible applications of NRI scatterers in biomedical optics and optical micromanipulation.

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Spaser-effect for loss compensation in metamaterials

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It is well known that numerous attracting applications of metamaterials are killed due to high level of losses. To overcome this obstacle it has been suggested to introduce into matrix of metamaterial active media [1–3]. Really, this leads to formation inside these active metamaterials a system spasers. Spaser in this context is a plasmonic nanoparticle surrounded with a system of quantum systems with inverse population [4–5]. It is implied that the inverse population is due to incoherent pumping.

Thus, it is expected that the active metamaterials should behave as usual composites without losses. This means that (1) there is no electromagnetic modes without external coherent field, (2) the wave generated under influence of the external field has the same frequency and amplitude depending on the amplitude of the external field.

Unfortunately, a spaser operating in generation regime is an autonomic (self-oscillating) system, which has its own frequency and amplitude [4, 6]. Though it is possible to synchronize the spaser forcing it to operate with frequency of external field, firstly, this is a threshold process, you need the amplitude of the external field accedes some threshold and, secondly, the amplitude of the spaser dipole oscillations weakly depend on the amplitude of the external field.

In the case of spaser synchronization the loss compensation is possible [7] if the amplitude of the external wave lies on the curve of compensation $E_{com}(\Delta_E)$. If the amplitude is greater than that value the energy will be passed from the wave into the spasers. Thus, the amplitude of the travelling wave will decrease. In the opposite case the energy will be passed from spasers to the wave. We get a gain medium with increase of the wave amplitude. Thus, sooner or later we come to the wave travelling with amplitude independent on the one of external wave. Moreover, if at small wave amplitude (there is no synchronization) a matamaterial is a gain medium too it means that it is unstable system and where could be generated a wave without external forces.

A spaser operating under spasing threshold is real candidate to the usage in metamaterials. Indeed, there is no reply without external field. The spaser is always synchronized. The amplitude of the spaser dipole oscillations nonlinearly depends on the amplitude of the external field. Moreover there is a line of quasi-compensation $E_{qcom}(\Delta_E)$, where the imaginary part of the spaser dipole is negative.

It is shown that in spite of the fact the external wave will loss the energy. Firstly, it will excite a quantum dot that in turn will excite surface plasmons with dipole having negative imaginary part. Thus, the dipole will give back a part of energy to the wave.

All these speculations are based on consideration of a single spaser operation. The coupling among spasers may correct this consideration.

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Collective excitations of spaser chains

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In the present work, we theoretically study the collective interaction of self-oscillationing spasers above the spasing threshold. We consider the simplest example of a spasers array — a regular chain of spasers. In this case, the collective near-field interaction between spasers can significantly change the threshold of the generation and even lead to new phenomena and instabilities in these structures.

We show that depending on the strength of the interaction between the QD and the NP of the nearest spasers, either a synchronized oscillation of all the spasers in the chain or a harmonic autowave travelling along the chain may realize. The pumped QD may either excite its own spaser so that all spasers are synchronized or cooperating with the other QDs, the pumped QD may excite a plasmonic wave traveling along the chain. This is the wave of the NP polarization which dispersion equation $\omega(k)$ is similar to the one predicted in Refs. [1–4] for linear systems such as a chain of plasmonic NPs. This dispersion equation has the form

$$\omega(k) = \frac{\omega_{SP}\tau_a + \omega_{TLS}\tau_\sigma}{\tau_a + \tau_\sigma} + \Omega^{eff}_{NP-NP}\cos kb, \tag{1}$$

where $\Omega_{eff} = 2\Omega_{NP-NP}\tau_a/(\tau_a + \tau_\sigma)$, Ω_{NP-NP} is the coupling constant between neighboring NP, ω_{SP}, τ_a, ω_{TLS}, and τ_σ are frequencies and relaxation times of the surface plasmon and the two-level QD, respectively, b is the distance between neighboring spasers. Unlike the general case of a wave propagating in a nonlinear lattice [1], the nonlinear character of the spasers' response to an external field results neither in soliton nor in kink solutions. Rather, this response is a perfectly harmonic wave. However, unlike harmonic waves in linear systems, in a chain of spasers, this wave has a fixed value of the wavenumber, which depends on the coupling constant between the QD and the NP of neighboring spasers Ω_{NP-TLS}.

Depending on the value Ω_{NP-TLS}, two different scenarios for the stationary behavior of a chain of interacting spasers may be realized: (1) all the spasers are synchronized and oscillate with in phase and (2) a nonlinear autowave travels along the chain. In the latter scenario, the traveling wave is *harmonic* unlike excitations in other known nonlinear systems [5]. The amplitude of this wave and its wave number are strictly determined by pumping and the coupling constants. Due to the nonlinear nature of the system, any initial distribution of spasers' polarization evolves into one of these steady states.

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Coherent plasmonic perfect absorber

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We theoretically demonstrate that a loss indefinite medium can absorb incident electromagnetic radiation at a certain frequency and for a certain incidence angle without reflection from its surface. Unlike isotropic lossy materials where the total absorption occurs for finite-thickness layers only an indefinite material allows this absorption for a half-space. This effect results from the surface plasmon polariton degeneration, namely, this is the very situation, where the Zenneck wave becomes an incident plane wave.

Nonlinear propagation of beams in hyperbolic metamaterials

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Last time, the composed media that possess unusual properties are of great interest. When combining the thin layers of the metal (semimetal or doped semiconductor) with a negative permittivity and dielectric ones with a positive permittivity, it is possible to obtain the (anisotropic) hyperbolic medium [1] with effective (averaged) permittivities, which are positive for tangential components of electric fields and negative for normal ones. When the thicknesses of the alternating layers are $d_{1,2}$, the effective tangential and normal permittivities $\varepsilon_{t,n}$ can be calculated, using the following formulas:

$$\varepsilon_t = (\varepsilon_1 d_1 + \varepsilon_2 d_2)/d > 0; \quad \mathrm{Re}(\varepsilon_t) > 0; \quad 1/\varepsilon_n = (1/d)(d_1/\varepsilon_1 + d_2/\varepsilon_2); \tag{1}$$

$$\mathrm{Re}(\varepsilon_n) < 0; \quad d \equiv d_1 + d_2; \quad \mathrm{Re}(\varepsilon_1) < 0, \quad \mathrm{Re}(\varepsilon_2) > 0. \tag{2}$$

In previous paper, we developed rather general method for modeling the nonlinear field concentrator based on latered inhomogeneous isotropic dielectric medium [1]. Cylindrical and spherical hyperbolic media are effective concentrators of energy, because the electromagnetic wave rays propagate inside the media perpendicularly to the interfaces of the layers. The values of the electric fields near the center of the structure can reach high values, and nonlinearity can manifest there.

In the present paper, the combined method to calculate the nonlinear monochromatic electromagnetic beam propagation within the cylindrical hyperbolic medium is developed. TM wave with the components E_r, E_θ, H_z is investigated. The saturating nonlinearity of the layers with the negative permittivity has been considered as: $\varepsilon_{1NL} = \varepsilon_1 + \alpha_1|\vec{E}|^2/(1+\alpha_2|\vec{E}|^2)$, $\mathrm{Re}(\varepsilon_1) < 0$. Here $\alpha_{1,2}$ are complex coefficients. The thicknesses of the layers are much smaller that the wavelength in vacuum.

The electromagnetic beams are incident onto the surface of the outer cylinder and propagate to the axis of the cylinder. The internal cylinder is made from the hyperbolic medium too but it is assumed as nonlinear. It seems impossible to use the averaged permittivities in the case of nonlinear media whereas in the linear media they are acceptable. The geometrical optics is considered within the outer cylinder, and the effective permittivities $\varepsilon_{t,n}$ are used there. The rays that reach the surface of the inner cylinder are considered as the incident waves in the boundary conditions. Each ray corresponds to the Gaussian wave beam. Within the internal cylinder, the full-wave approach is used for the magnetic field $H \equiv H_z$. The nonlinear wave equation for H has been solved by the expansion on the azimuthal harmonics with iterations due to nonlinearity. The nonlinearity leads to the essential redistribution of the electromagnetic energy within the internal cylinder. Moreover, when the intensities of the incident electromagnetic beams reach some threshold, the switching into the highly nonlinear regime occurs. This can be explained by the accumulation of the electromagnetic energy at the interfaces between nonlinear (with negative permittivity) and linear layers.

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Luminescence in the array of plasmonic antennas

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In recent years the surface plasmonic excitations in the ordered array of metal nanoparticles have attracted significant attention of researchers studying plasmonic metamaterials due to their numerous potential applications in modern optics. In our paper we consider a set of ordered metal particles shaped as horseshoes and arranged inside the layer of actively pumped dielectric medium. Size of the considered nanoantennas is much less than the wavelength of light. On this scale the laser pumping can be treated as a random external force because of the process of photon absorption by the active medium is intrinsically incoherent. Thus, the response of the plasmonic nanoantennae immersed in the active medium is caused by the action of the random electromagnetic field and should be treated at the microscopic level. We have shown that this response is different from the usual plasmon resonances in metal particles. The main result of this work is the prediction of the evolution of the luminescence spectrum in response to the changes in shape of the nanoantennae and level of the inversion. The predicted luminescence spectrum manifests some minima and maxima which correspond to the excitation of various SP resonances and is in a good agreement with our experiments. We have calculated the gain required to achieve the lasing threshold as a function of the antennae structure. Resonant frequencies of the nanoantenane and the field enhancement factor can be conveniently tuned by the variation of the size and shape of the plasmonic nanoantennae. The proposed nanolaser can have many applications, such as the core device for the transmission and processing of optical signals on a scale much smaller than the wavelength.

Strong coupling between surface plasmon and guided modes

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Strong coupling is a particular regime of the light-matter interaction in which an electromagnetic mode interacts strongly with an electronic state in order to create two mixed states. This regime was widely studied in microcavities [1] and more recently in organic semiconductors [2, 3]. It is a phenomenon that is interesting both from the point of view of fundamental quantum physics as well as from the point of view of technology. To show the existence of a strong coupling between surface plasmons and confined guided modes, we consider a stratified medium containing a dielectric waveguide near a metallic film, the whole being embedded in a symmetric medium (Fig. 1).

First of all, the determination of the poles of the S-matrix determinant allows to draw the dispersion relation of the structure and to have access to the imaginary parts of the constants of propagation, which are linked to the absorption. An anticrossing of the dispersion branches is observed, evidencing the existence of a strong coupling (Fig. 2).

The present study is about the consequence of this strong coupling regime. We reveal a decay of the losses which implies that the propagation length of the surface plasmon is increased. Indeed, for the plasmon which lives on the side with $\epsilon_1 = 2.1025$ (the side with the guided mode), this length is multiplied by three.

Besides, another part of this study concerns the demonstration of the existence of temporal oscillations, called Rabi oscillations, between modes at the wavelength of the anticrossing. We carry out FDTD calculations [5] to show that.

In conclusion, this work is about a multilayered structure that demonstrates a strong coupling between a guided mode and a surface plasmon. The numerical study is based on a rigorous computation of the dispersion and loss diagrams. This study is a first stage towards the realization of a transfer of energy between these modes like in a Spaser [6].

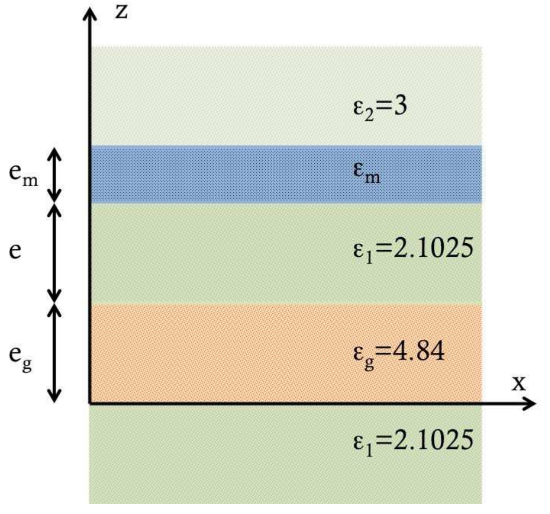

Fig. 1: Multilayered structure with a dielectric waveguide (ϵ_g) and a metallic film (ϵ_m in [4]) inserted in beetween two dielectric hosts (ϵ_1 and ϵ_2) and separated by a dielectric layer. The thickness e is the parameter which is going to change in our simulations.

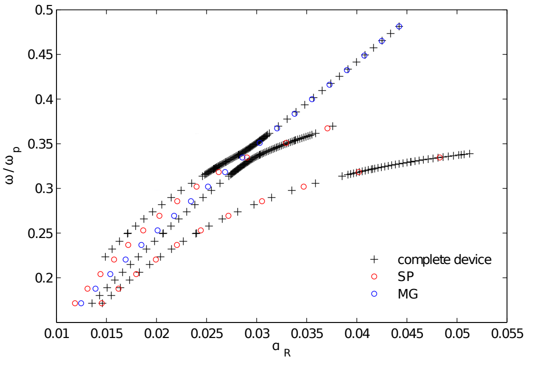

Fig. 2: Dispersion relation: the normalized frequency (to the plasma frequency ω_p) in function of the real part of the propagation constant α. Without the strong coupling, the dispersion would be in red and blue.

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Hyperbolic metamaterials formed by artificial transmission lines

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A hyperbolic medium is an uniaxial system, where the transverse ϵ_{xx}=ϵ_{yy}=$\epsilon_{\perp}$ and longitudinal ϵ_{zz}=$\epsilon_{\parallel}$ dielectric constants have opposite signs [1], [2]. Due to the hyperbolic isofrequency contours in wave-vector space this medium exhibits a number of unusual properties. First, the waves at its boundary may exhibit negative refraction, similarly to the case of double-negative materials. Second, the diverging density of photonic promotes ultra-high spontaneous emission rates [6]. This makes a concept of hyperbolic medium very promising for the broad-band tailoring of light-matter coupling and explains the ongoing intensive attempts to realize hyperbolic plasmonic metamaterials [7]. Still, that only truly conclusive experimental report of the hyperbolic medium we are aware of is restricted to the magnetized plasma in the microwave range [8]. Here we present another way of creation of hyperbolic metamaterials based on transmission lines. A transmission-line approach to synthesis and design of metamaterials has been recently developed in [9], [10]. One, two and three-dimensional transmission-line metamaterials were introduced exhibiting both negative and positive effective material parameters. Nevertheless metamaterial transmission lines revealed uniaxial effective material parameters have not been presented. So, we extend the transmission line approach to design and synthesis of two-dimensional hyperbolic metamaterials.

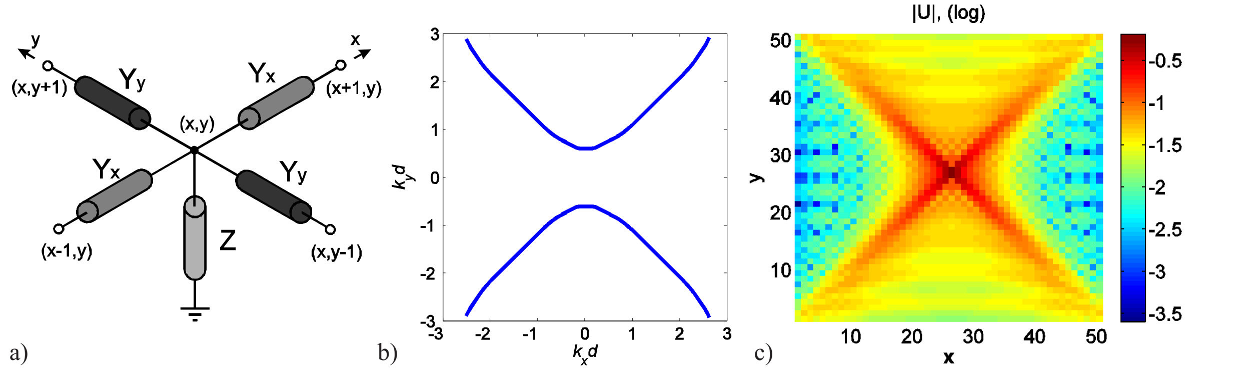

Fig. 1: The unit cell of two-dimensional metamaterial transmission-line structure (a). Isofrequency curve obtained by numerical solution of the dispersion relation (b). The simulation voltage distribution across the two-dimensional hyperbolic medium composed of 51×51 unit cells. The voltage magnitude in logarithmic scale (c).

In this paper we consider an anisotropic hyperbolic medium with a dielectric tensor of size 2×2, where $\epsilon_{xx} > 0$, $\epsilon_{yy} < 0$ and positive constant magnetic permeability μ (Fig. 1). Such a medium supports the propagation of TM polarized waves. To mimic the anisotropic hyperbolic medium we consider two-dimensional grid consisting of artificial transmission line based on lumped elements. The unit cell of the grid is T-circuit composed of series admittances in x and y directions and shunt impedance. We find the dispersion equation of the hyperbolic two-dimensional medium consisting of an infinite number of unit cells, solving the Kirchhoff equations for the currents flowing in and out of site. We show analytically that the solution of the dispersion equation has the form of a hyperbola, if the admittances in the directions x and y have different signs. Next, we find the relationship between material parameters of the anisotropic hyperbolic medium and its counterpart based on artificial two-dimensional transmission lines. In the software package CST Microwave

Studio was produced electrodynamic simulation of hyperbolic structures of a medium consisting of 21×21 elementary cells (Fig. 1). At the central frequency $f = 50$ MHz were calculated electric and magnetic field structure excited by a current source at the center. It was shown that the spatial field distribution has a pronounced "crossform" shape, while in the same direction is the electromagnetic field energy transfer, and in the other direction of the wave decays rapidly, which we expected to observe.

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Electromagnetic wave reflection by bi-isotropic layer

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The task of interaction between electromagnetic waves (EMW) and a plane layer of bi-isotropic composite material is of significant importance in the sphere of applied electrodynamics [1]. It is conditioned by wide-spread usage of different coverings and shields necessary for creation of frequency and polarization-selective filters, protective shields and slightly reflective coatings [2]. Composite materials are used in technology as the backing of printed-circuit antennas [3], they are also used to decrease reciprocal impact among the elements of antenna arrays. Most works present the results that demonstrate obvious benefits of such material application. But the results, as a rule, are given for arbitrary electrodynamic parameters of a composite, such as dielectric permittivity and magnetic permeability, chirality parameter. Composites that contain chiral inclusions (few-coil helices, omega-particles, combinations of a helix coil with attached rectilinear conductors etc.) possess resonance properties in a frequency range [4]. That is why it is important to reliably estimate their electrodynamic parameters when designing any devices on the basis of chiral composites.

This work presents a self-consistent approach to analyzing composite materials and devices on their basis after the example of solving the task of interaction between EMW with a plane layer bi-isotropic composite material. The approach is based on investigating the interaction of electromagnetic field with separate inclusions with no limitations imposed on their geometry and also considering possible magneto-dielectric coating of inclusions or impedance properties of the conductor that the inclusions are made of.

The solution of this task was carried out on step-by-step basis. First, using the modified Pocklington integral equation, that considers the presence of conductor magnetodielectric coating, the task of EMW dissipation on separate conductive particle is solved [5]. The obtained hereby phase-amplitude current distribution allows to calculate electrical and magnetic dipole moments of the particle and its polarizability factor (PF). By the known PF of a single scatterer and using Maxwell–Garnett method effective electrodynamic parameters can be calculated: dielectric permittivity, magnetic permeability and composite material chirality parameter. Then, suggesting that the waves of left-hand and right-hand circular polarization [6] propagate within the chiral layer, the analysis of frequency and angular characteristics of E- and H- polarized waves reflectivity factor for a composite layer on metallic backing was carried out. The composite was formed by integrating the particles of different geometry and identical conductor length into the matrix.

Numerical experiments were carried out in the frequency range that corresponds to the first resonance response, when the length of particle conductor is close to one half of the wavelength of the electromagnetic field that interacts with the particle. At the same time it is ascertained that effective electrodynamic parameters of a composite considerably depend on inclusions geometry with identical conductor length, which the inclusions are made of and indentical concentration of particles in the matrix. For example, by changing the number of coils in a spiral particle leaving the conductor length unchanged it is possible to change the resonance bandwidth and frequency, the value of reflection coefficient for the composite layer. It was determined, that the most effective way to achieve the minimal value of reflection coefficient is the use of the composite, that contains particles in the form of single-coil helices or coils with attached rectilinear conductors. Increase of particle concentration leads to decrease of reflection coefficient and resonance bandwidth widening. At the same time the concentration should remain at such a level that the interaction among the particles could be neglected. At the angles of incidence less than 45 degrees the value of reflection coefficient changes insignificantly in comparison to normal EMW incidence. Cross-polarization component hereby is also insignificant.

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Determination of invisible inhomogeneities of refractive index from eikonal equation

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Basing on eikonal equation we develop a theoretical approach for evaluation of the parameters of refractive inhomogeneities which do not disturb ray pattern of the scalar wave field outside of inhomogeneities. As an example, calculation of the characteristics of isotropic dielectric shell which does invisible the spherical cavity for the fixed supervision direction is presented. It is demonstrated that 2D eikonal equation and 2D wave equation lead to the same expressions for invisible inhomogeneities of refractive index. Finally, we discuss the possibility of designing 3D refractive index inhomogeneities that are invisible in a horizontal plane.

Self-action of single-cycle light pulses

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The recent progress in the area of ultrafast optics has opened a door to the generation of light pulses with durations down to a single optical cycles [1]. The study of propagation of intense ultrashort pulses led to the investigation of novel effects in nonlinear optics. Here we report on a theoretical analysis of spatiotemporal pulse dynamics in cubic nonlinear media with normal group dispersion and reveal a new feature of suppression of third-harmonic generation for single-cycle light pulses.

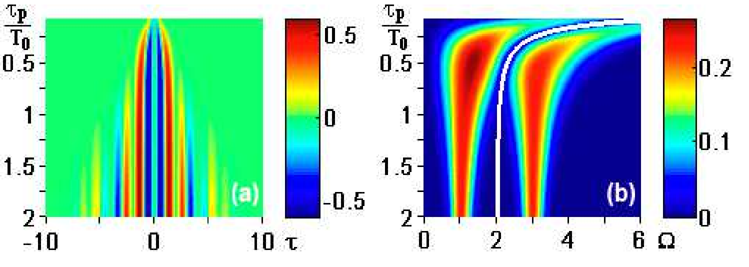

Fig. 1: Nonlinearly-induced corrections to the pulse (a) field and (b) spectrum vs. the normalized duration. τ, Ω – normalized time and frequency, respectively.

We first investigate the harmonic generation depending on the pulse duration by considering a case when the effects of dispersion and diffraction can be neglected. Our results are based on approximate analytical solution of field equation [2] and summarized in Fig. 1. The nonlinearly-induced corrections to the pulse profile and spectrum are shown in Figs. 1(a) and (b), respectively. We see that the spectral correction has two maxima due to combined action of self-phase modulation and high frequency generation, and we show the frequency where the correction completely vanishes with white line in Fig. 1(b). For the normalized on central period T_0 pulse duration $\tau_p/T_0 \cong 0.3$, when the input pulse is single-cycle, the third-harmonic generation completely vanishes. This is a remarkably surprising result, since the third-harmonic generation is one of the fundamental effects in optical media with Kerr-type nonlinearity.

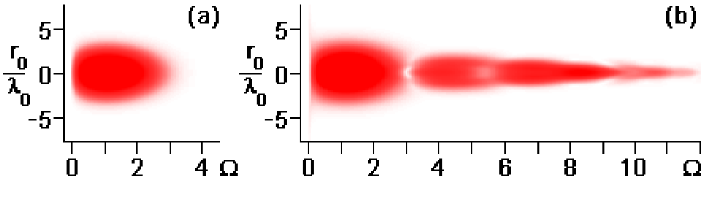

Fig. 2: Modulus of the spectrum for an input single-cycle light wave with Gaussian transverse distribution in the nonlinear medium at distances: (a) 0 mm and (b) 4 mm.

Then we analyze the self-action features of axisymmetric paraxial single-cycle light pulses under the combined effects of cubic nonlinearity, temporal dispersion and spatial diffraction. In Fig. 2 we represent two-dimensional contour plots of the spectral distributions during single-cycle pulse propagation in nonlinear medium. It is shown the formation of high frequency narrow wave tail and suppression of third-harmonic generation [Fig. 2(b)]. Diffraction induces a strong increase in the transverse spectrum size r_0 normalized on central wavelength λ_0 at low frequencies.

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Simulation of symmetric waves passage through biisotropic partition in circular waveguide

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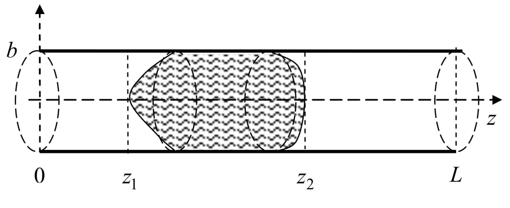
Recently in the literature the enhanced attention is given examination of the waves spreading in wave guides with devices from composite materials: biisotropic, chiral and metamaterials. The majority of examinations is devoted to mathematical simulation with use numerical and analitic methods [1–4]. Publications on making and an experimental research samples of such materials in a unclosed press meet seldom.

In the submitted operation on the basis of computing experiment the electrodynamic problem for a hollow circular cylindrical wave guide with precisely conductive walls in which the azimuthal – symmetric partition from biisotropic material is located is solved. Outside of a partition the wave guide is filled with empty space.



At the left the wave $\vec{E}_0$, $\vec{H}_0$, with the frequency $\omega = 2\pi f$, consisting of a superposition of the partial symmetric E_{0r}-waves and H_{0p}-waves, will run on a partition. The resulting electromagnetic field submits to the dimensionless Maxwell equations for biisotropic mediums $\operatorname{rot}\dot{\vec{E}} = -j\,W\left(\dot{\mu}\dot{\vec{H}} + \dot{\chi}_e\dot{\vec{E}}\right)$, $\operatorname{rot}\dot{\vec{H}} = j\,W\left(\dot{\varepsilon}\dot{\vec{E}} + \dot{\chi}_h\dot{\vec{H}}\right)$, where $W = \omega/\omega_0$, ω – working frequency, ω_0 – the basic frequency chosen for transition to dimensionless variable, $\dot{\mu}$, $\dot{\varepsilon}$ – the relative magnetic and an inductivity of medium, $\dot{\chi}_e$, $\dot{\chi}_h$ – the dimensionless parameters describing biisotropic mediums. In the domain of empty space $\dot{\varepsilon} = 1$, $\dot{\mu} = 1$, $\dot{\chi}_e = 0$, $\dot{\chi}_h = 0$.

The procedure of the solution of problems of a diffraction of the symmetric waves on partition from biisotropic material in the round wave guide designed, allowing to explore the broad audience of problems for lines composite materials which now are intensively explored in many science centers. Efficiency of the given procedure is caused by data of the vector problem for higher the given Maxwell equations to the solution of a boundary-value problem for the bound system of two scalar elliptic equations. Integrated boundary conditions of radiation on input ($z = 0$) and target ($z = L$) sections of a wave guide and direct (not iterative) a grid block matrix run method [5] are used. Thus use of the homogeneous finite-difference plans allows to implement primely enough requirements on interfaces of various mediums. The procedure is applied for the nonuniform partition, in particular schistose and with surfaces of the arbitrary shape.

Reflectivities and passages of partial waves on power are designed. The effect found out as a result of computing experiment practically the complete transformation of the symmetric H_{01}-wave to E-waves (and on the contrary) on a flat partition from chiral material specifies an opportunity to create the simple and effective transformer of modes of the symmetric waves of a round wave guide on a composite materials basis.

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Six- and three-fold axial symmetries in reflection and transmission spectra of opaline photonic crystals

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In this paper, a study has been made of Bragg reflection and transmission spectra of three-dimensional photonic crystal (PhC) films. Particular attention is paid to effects of the multiple Bragg diffraction of light [1] which are discussed in the framework of the dynamical theory of diffraction. By detailed comparison of the reflection and transmission spectra, specific effects caused by light diffraction from non-lateral crystal planes are analyzed. In our calculations we use the previously developed theoretical approach [2] to the analysis of the spectra. This approach, in fact, is analytical and therefore allows a detailed study of physical mechanisms of light interaction with PhC media, as opposed to the methods based on the full-wave electrodynamic simulations using intensive and cumbersome computational procedures.

As a model under numerical study, a thin opaline PhC film assembled from spherical polystyrene particles was considered [3]. The distance between neighboring particles was chosen to correspond to the typical conditions providing observation of the effects of interest in the visible spectrum. Possible energy losses associated with absorption and scattering of light were taken into account by adding an imaginary term to the average permittivity of the PhC. The reflecting (lateral) surfaces of the PhC film are assumed to be parallel to the (111) crystal planes, the crystal direction $[11\bar{1}]$ being in the plane of incidence of s-polarized light. In general, this orientation of the incidence plane corresponds to two possible geometries of light propagation, Γ–L–K and Γ–L–U, within the first Brillouin zone of the reciprocal lattice.

The results of the numerical calculations performed are shown in Fig. 1. We see that the spectra of the specular reflection and transmission demonstrate a complicated structure and are noticeably changed depending on the angle of incidence θ. The reflection spectra (dashed curves) are identical, at the same angle of incidence, both for Γ–L–K (Fig. 1a) and Γ–L–U (Fig. 1b) propagation directions, which is consistent with the time reversal symmetry. So, the reflection spectra have to exhibit *six-fold* axial symmetry. The key feature observed in the reflection spectrum as an intensive broad peak is due to the main photonic stop-band (PSB) which is governed by Bragg diffraction of light from the lateral (111) crystal planes. In the narrow range of incidence angles, $\theta \sim 50$ to $60°$, the reflection spectrum takes the form of the pronounced doublet which is associated with an additional light diffraction from the $(11\bar{1})$ system of inclined planes. This diffraction is accompanied [2] by the excitation of additional low-group-velocity eigenmodes ("slow" light modes), which brings about an additional energy transfer inside the PhC. As a result, the spectrum within the main PSB is shaped by the dip appearing on the "slow" mode frequencies.

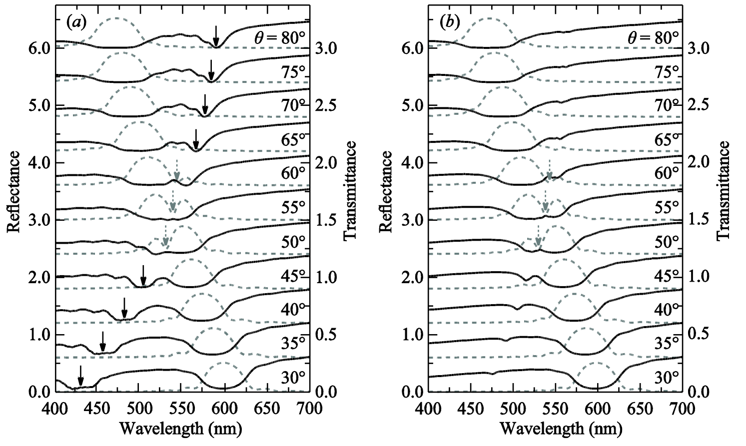

Fig. 1: Calculated transmission (solid curves) and reflection (dashed curves) spectra (s-polarized light) of an opaline photonic crystal film at different angles of incidence θ for the Γ–L–K (a) and Γ–L–U (b) propagation sectors in the first Brillouin zone. Solid arrows mark the position of the extinction band outside of the (111) Bragg resonance. Dashed arrows show the dip positions in the Bragg reflection contours. The film thickness is 20 monolayers, the distance between neighboring particles in the lateral plane is $a_{00} = 280$ nm. The imaginary part of the average permittivity is $\varepsilon_0'' = 0.04$.

The calculated transmission spectra, as expected, show the main extinction band formed by the main PSB. This band shifts to shorter wavelengths with increasing θ and corresponds in its spectral position to the main Bragg reflection peak. However, along with the main band in the transmission spectrum for Γ–L–K propagation direction (Fig. 1*a*, solid curves) an additional extinction band can be seen that shifts to longer wavelengths with increasing θ. This correlates well with the extinction due to light diffraction from the inclined crystal planes $(11\bar{1})$. On the other hand, as an analysis shows, the additional extinction band shifts more rapidly with the incidence angle variation then the dip in the reflection spectrum. A detailed comparison of the transmission spectrum with the eigenmode dispersion curves for spatially confined PhC shows that the additional extinction band arises in the spectral region where incident light and light diffracted from the only $(11\bar{1})$ planes dominate. In the transmission spectrum for Γ–L–U propagation (Fig. 1*b*, solid curves) the additional broad extinction band is not observed. In this case, within the same spectral region, light diffracted from $(11\bar{1})$ planes has low intensity and only incident light dominates. So, for perfect opaline PhCs, the transmission spectra should exhibit *three-fold* axial symmetry in contrast to *six-fold* axial symmetry of the reflection spectra.

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Enhancement of Cherenkov emission inside a nanowire material

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It is well known that a charged particle moving inside a transparent medium can produce Cherenkov radiation [1, 2], provided its velocity v is larger than the phase velocity v_{ph} of the electromagnetic waves inside the medium. This phenomenon has many applications, especially in high energy physics [3]. In general, due to frequency dispersion, when a particle passes through left-handed materials or some photonic crystals [4, 5] it may exhibit both forward and reverse Cherenkov radiation. Moreover, structured media may also allow for Cherenkov emission with no velocity threshold [7].

Here, we study the Cherenkov radiation in arrays of metallic nanowires. It is known that the density of photonic states in nanowire metamaterials can be extremely high [7–9], and it has been shown that this implies an enhancement of the Purcell factor and permits boosting the Casimir interaction between two bodies embedded in a nanowire material [9].

In our work we consider the scenario where an array of charges moves at a constant velocity inside a uniaxial wire medium. We demonstrate that due to the anomalously high number of radiative decay channels inside such a metamaterial, the intensity of Cherenkov radiation is boosted as compared to same phenomenon in the usual dielectrics. Moreover, we prove that in arrays of nanowires the Cherenkov effect has no velocity threshold. This may have some interesting applications in the context of particle detection, as the direction of radiation inside the structure is well defined and depends on the particle velocity. We numerically calculated the amount of energy extracted from the moving particles, which is determined through the stopping power, defined as the average energy loss of the particles per unit of path length. In our study we find that the stopping power of the nanowire arrays can be several orders of magnitude higher than that of natural media, which indicates that this metamaterial strongly enhances the Cherenkov emission.

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Testing the concept of all-dielectric optical nanoantennas at microwaves

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The study of optical nanoantennas is a new and rapidly growing field of research in nanophotonics. Nanoantennas are capable to redirect and transfer freely propagating radiation into localized subwavelength modes, thus being potentially useful for various applications such as solar cells, biological and chemical sensing, quantum communication systems, molecular spectroscopy, etc.

In this work, we introduce a novel concept of all-dielectric optical nanoantennas made of solely dielectric nanoparticles with a high permittivity and low dielectric loss (for example, silicon Si with permittivity $\varepsilon = 16$). First, we analyze the nanoantennas created by a single dielectric sphere excited by a point-like source and demonstrate that at the magnetic resonance a radiation pattern is similar to that of a Huygens element with zero backward and enhanced forward scattering. Next, we demonstrate that this feature can be further enhanced in Yagi–Uda-like optical nanoantennas. We analyze the performance of the Yagi–Uda nanoantenna based on four dielectric spheres, both analytically and numerically using the CST Microwave Studio commercial software. We demonstrate that such a nanoantenna has a good radiation efficiency and adjusted to a high directivity.

A characteristic antenna dimensions are determined by the working wavelength of radiation, which is hundreds of nanometers at optical frequencies. Thus, there are some technological problems to reproduce an object of this size with nanometer accuracy. To study experimentally the all-dielectric antennas, we scale down the optical nanoantenna to the microwave frequency range, keeping all the material parameters, and fabricate a microwave analog of the all-dielectric Yagi–Uda nanoantenna based on four dielectric spheres. The spheres are made of ceramic with the permittivity $\varepsilon = 16$ unchanged up to 20 GHz (mimicking Si at optical frequencies). The antenna was measured at the frequency range 10–12 GHz. The measured data agree well with our numerical results. The measured angular width of the main lobe of radiation is about 60°, with a high directivity of the antenna in both TE and TM planes. The maximum measured value of directivity of the designed antenna in the working frequency range is 7.5, while the numerical results predict the value of 8.5. Our experimental data completely confirm the concept of all-dielectric optical nanoantennas.

Absorption in a finite-thickness array of tilted carbon nanotubes in the terahertz range

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Introduction. Artificial impedance surfaces (AIS) have attracted significant attention in recent years, especially after publication of the seminal paper of D. Sievenpiper et al. [1]. These structures find applications in low-profile antennas, leaky-wave antennas, absorbers, impedance waveguides. Usually, AIS are resonant structures where resonance conditions are achieved by combining the capacitive response of an array of metal patches of various shapes together with the inductive response of an electrically thin grounded dielectric slab. Conventional design of AIS, based on patches and metal pins, for the terahertz range requires fabrication of very small structural elements. As the alternative we propose to use arrays of metallic carbon nanotubes (CNTs) grown on a metal plane. Main properties of both single CNTs and CNT arrays are determined by their complex conductivity, where its imaginary part is comparable with the real part or even exceeds it. That not only considerably changes the waveguiding properties of nanotubes, but causes new effects [2]. Particularly, arrays of metallic CNTs support waves characterized by hyperbolic dispersion, i.e. arrays of metallic CNTs can be referred as indefinite media [3, 4]. It could be expected that arrays of metallic CNTs possess properties, similar to wire media. However, it was shown [4] that the spatial dispersion is totally suppressed in CNT arrays due to a very high kinetic inductance of CNTs and CNT arrays behave as ϵ-negative uniaxial crystals.

In this work we discuss reflection and absorption properties of electromagnetic waves incident onto a grounded slab of tilted CNTs, see Fig. 1. We exploit the plasmonic resonance which takes place if a corresponding component of the permittivity dyadic is close to zero and demonstrate that almost total absorption can be obtained in a wide frequency range at reasonably small thickness of the slab. Tilted CNTs provide interaction of TM-polarized waves with carbon nanotubes under the normal incidence. Additionally, non-symmetry appearing as a difference between wavenumbers of waves propagating upward and downward with respect to an interface under oblique incidence leads to the absence of a thickness resonance.

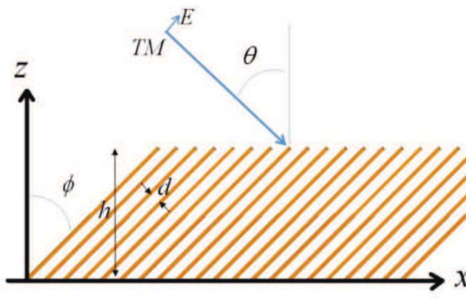

Fig. 1: The cross-section of a slab of indefinite medium with an arbitrary tilted optical axis realized by metallic CNTs. An array of CNTs tilted by angle ϕ with respect to the z-axis is embedded into a host material slab with the thickness h. CNTs are placed in planes parallel to the xOz plane. Each CNT mesh is arranged in a square lattice with the lattice constant d.

Results and discussion. Let us consider first eigenwaves propagating in the CNT array under fixed transversal (k_x-component) of the wave vector. The effective medium model [4] was used for description of the CNT array. Losses were taken into account in framework of the Drude model via the relaxation time τ. Strong difference in their values for opposite directions with respect to the normal to interface is illustrated in Fig. 2.

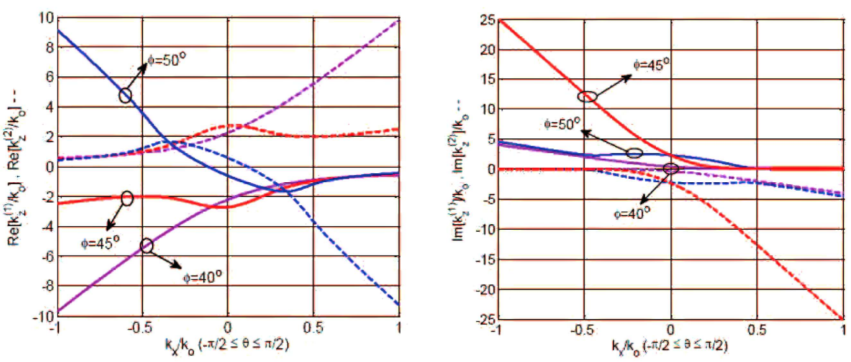

Fig. 2: Real and imaginary parts of normalized z-components of the wave vector, calculated for different tilt angles ϕ. Solid and dashed curves correspond to opposite directions with respect to the z-axis. The incidence angle is 45°, k_0 is the wavenumber in free space.

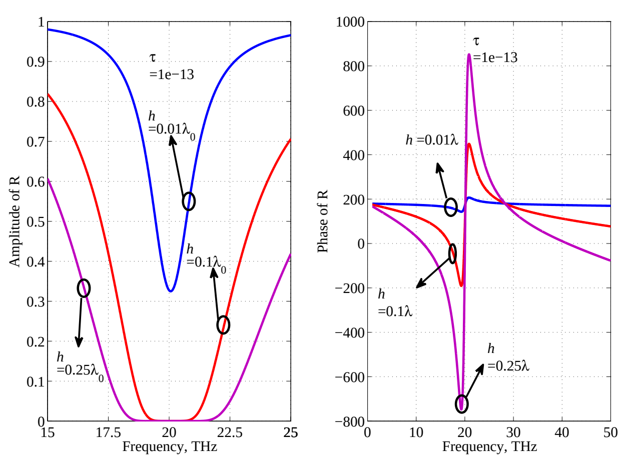

Fig. 3: Amplitude (left) and phase (right) of reflection versus frequency calculated for different thicknesses of the slab. τ is the relaxation time of charge carriers, the incidence angle is 45°, the tilt angle is 45°, λ_0 is the wavelength in free space.

Reflection from the grounded slab was calculated for the TM-waves using modified 2×2 transfer matrix method, which takes into account difference in $k_z^{\pm}$. Results are shown in Fig. 3. It is remarkable, that the thickness resonance is absent due to the difference in propagation constants for upward and downward waves within the slab. The resonant frequency corresponds to the effective plasmonic resonance which is determined by the period of CNT array lattice and the tilt angle. Increase of the thickness causes increase of the absorption band width. Zero phase of the reflected wave is achieved already at $h = 0.1\lambda_0$, so the slab can be considered as a high-impedance surface. However, reflection at the corresponding frequency is accompanied by quite high losses.

Conclusions. In this paper we demonstrated effects of non-symmetry for waves propagating upward and downward with respect to the interface that results in the absence of the thickness resonance. We have demonstrated, that the total absorption bandwidth 15% can be achieved at the thickness $3.75\,\mu$m ($0.25\lambda_0$) and the frequency 20 THz.

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Nonlinear Tamm states in plasmonic metamaterials

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We analyze nonlinear surface modes supported by a metal-dielectric nanostructured metamaterial with a nonlinear surface layer. We demonstrate that such semi-infinite structures can support both TE and TM polarized surface modes with the subwavelength localization, which can be regarded as an optical analogue of the electronic Tamm states. Such *nonlinear Tamm states* may appear even in the cases when linear surface modes do not exist.

The interest in the study of *nonlinear electromagnetic surface waves* has been renewed recently, and it was shown theoretically [1] and experimentally [2] that nonlinearity-induced self-trapping of light may become possible near the edge of a one-dimensional waveguide array leading to the formation of nonlinear Tamm states (see also the review paper [3] and references therein). In particular, it was found that the self-trapped surface modes acquire some novel properties in the nonlinear regime, and they can only exist above a certain threshold power. These modes can also demonstrate bistability when for the same value of the mode power two different surface modes can exist at the same frequency.

Recently, it was shown that strongly confined plasmonic surface modes can be formed at the termination of metal-dielectric metamaterials [4, 5]. Such localized modes demonstrate many properties that can be advantageously for the realization of photonic or plasmonic sources. Indeed, due to their specific dispersion relation and to the hybrid metal-dielectric nature of the structures that support such surface modes, they allow coupling either to the optical mode inside the light-cone or directly to the plasmon mode. More importantly, such surface modes are associated with lower losses than conventional plasmons and can be controlled and laterally confined by a simple patterning of the metal layer. In addition, the surface metallic layer can also allow a simple electrical injection scheme for the realization of plasmonic or photonic integrated sources.

In this work we study the properties of nonlinear surface states that exist at the interface of vacuum and layered nanostructure with a cap nonlinear layer [see Fig. 1(a)]. We find that there may exist up to *four nonlinear surface modes* defined by the same frequency and threshold power for the TM-polarization case.

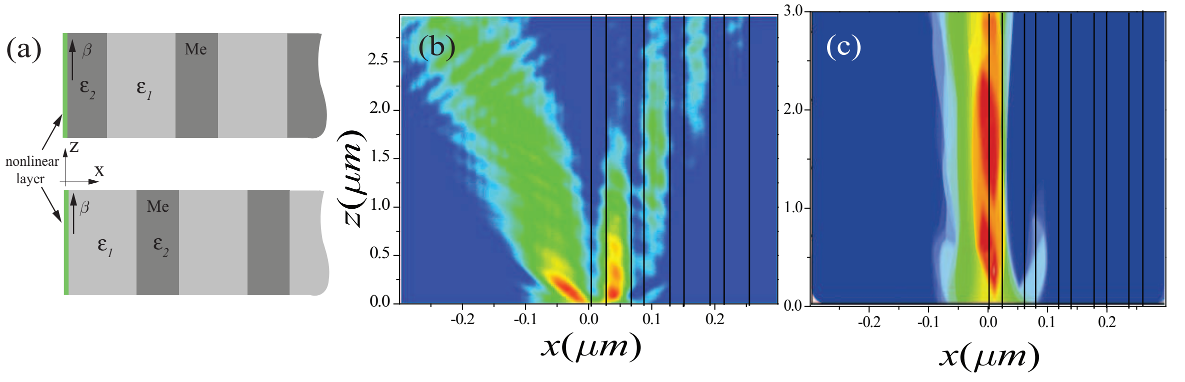

Fig. 1: (a) Geometry of the structure under consideration. (b,c) Beam propagation method simulation results. (b) Field color map for the case when the beam intensity is below a threshold: electromagnetic wave are repelled by the surface; (c) Field color map for the case when the beam intensity near the threshold: wave localizes at the interface.

Next, we analyze the dispersion properties of the nonlinear surface modes which may be supported by a cap nonlinear layer at the interface of vacuum and layered nanostructure . We have shown that these surface states are defined by the finite threshold power. If we launch a Gaussian beam along the surface with power less than the threshold power then the beam would diffract from the surface both in vacuum and into the nanostructure [see Fig. 1(b)]. If the beam intensity reaches the threshold value, the beam localizes at the interface forming a stationary nonlinear surface state [see Fig. 1(c)]

We have also shown that there may exist for the case of TM-polarization both discrete and gap modes may exist in the structure even in the case when the wavelength is much larger than the period of the structure.

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Active metamaterials: theory, experiment, future

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Losses and frequency dispersion are certainly the two biggest obstacles in practical applications of metamaterials. Unfortunately, both naturally arise from the used assemblies of metamaterial unit cells and the use of different geometries, materials and manufacturing technologies has only very limited effect on their reduction. At the present moment it seems that the use of active elements is the only plausible way out of this problem. The first proposal in this direction was published in

2001 [1] and used a Negative Impedance Converter (NIC), which can potentially produce negative capacitances, inductances and resistances and give absolute control over the losses and dispersion of the unit cell. The proposal of [1] was purely theoretical and used ideal operational amplifier. Nowadays, much simpler (two-transistor) designs of NIC are used [2, 3] offering much wider frequency range of operation. In contrary to NIC loads, other authors used two-port amplifiers and variable phase shifters to create active unit cells [4]. These however suffer from coupled input and output, large electrical size and from fabrication difficulties of two-port monolithic amplifier and phase shifter.

This contribution will be focused on active reduction of losses in metamaterials, particularly on metamaterials made of loaded conducting loops, which are behind the practical realization of super-resolution lenses and of cloaking devices. It will be shown that reduction of losses, or even addition of gain, can be achieved by extremely simple one transistor circuit used as a load of conducting loop, see Fig. 1. The circuit is based on negative resistance oscillator scheme tunned out of its oscillating condition. Detailed analysis of this active system will be presented, together with two important issues: stability (instability can completely invalidate the design for practical use) and noise (after all, it is the signal to noise ratio that is important in practice [5]). Finally, a theoretical proposal and analysis of an active quasi-static magnetic lens will be shown as an application of this idea.

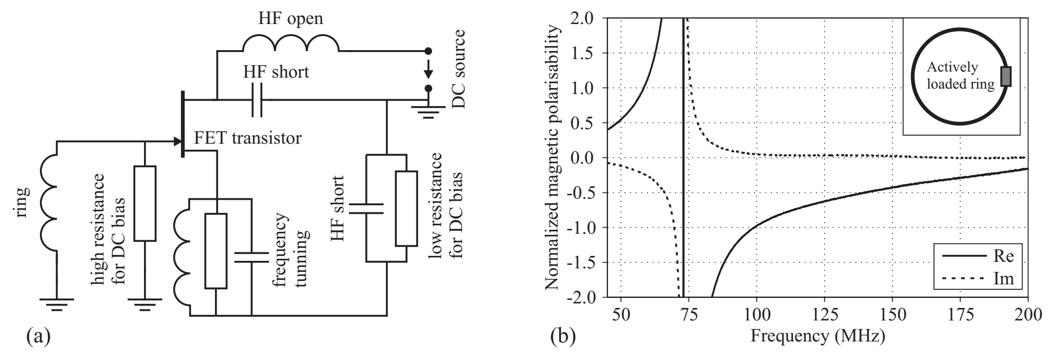

Fig. 1: (a) Schematic of an inductive ring connected to the active FET circuit. (b) Calculated magnetic polarisability based on measured data (imaginary part shows produced gain).

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The spatial frequency separation of surface acoustic waves in a wedge-shaped structures of acoustic metamaterials

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The problem of propagation surface acoustic waves [1] in waveguide of variable thickness is considered. The waveguide consists of a layer surrounded by two half-spaces. Surface acoustic Love wave [2] was considered. Love wave modes can propagate in the waveguide only under certain conditions on the geometry of the waveguide, parameters of the media and frequency. If the condition are not met there can be only so-called leaky waves [3], whose energy is radiated in bulk waves. In particular, the leaky wave will appear with decreasing thickness of the waveguide to the critical value. The position of this section in a waveguide of variable thickness depend on the parameters of media. But if the parameters of media strongly depends on the frequency spatial frequency separation of acoustic waves can be obtained.

Modes propagating in an inhomogeneous waveguide are coupled. Estimate of the energy lost due to mode coupling was carried out using cross-sections method [4]. A field in a certain section of the inhomogeneous waveguide is represented as a superposition of all modes possible in the waveguide. The coupling coefficients of different modes are proportional to the derivative of the layer thickness and have an oscillating factor. In a waveguide with a slow variation of the thickness energy pumped into the other Love wave modes is small.

As a medium with with parameters strongly depend on the frequency acoustic metamaterials considered. An example of an acoustic metamaterial is a matrix with cylindrical inclusions of another material [5]. If the wavelength is much larger than the average distance between inclusions the effective medium approximation can be introduced. The effective medium parameters are calculated in the coherent potential approximation. The dispersion relations of effective material parameters have resonant behavior. This shows the strong dependence parameters of the medium on frequency.

Using metamaterials as the half-spaces of the waveguide of variable thickness the position of the critical section will be strongly dependent on the frequency. That is the spatial frequency separation of surface waves is obtained.

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Light controllable magnetic metamaterials

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Past several years seen a tremendous increase of the interest in the study of tunable, nonlinear and active metamaterial structures. In particular, various efforts are being made to achieve tenability of the metamaterial properties, either externally [1] or by employing their nonlinear response [2]. At microwave frequencies, a split-ring resonator (SRR) loaded with a varactor diode can be considered as a tunable nonlinear meta-atom. The varactor diode can operate in either bias-free or biased regimes [3, 4], and subsequently create a bulk nonlinear metamaterial [5] allowing a power-induced control of the wave transmission and other intrigueing possibilities. The proposed biased regime allows significant tuning of the resonan however, is not practical for application in bulk structures, since the required circuitry becomes too cumbersome.

In this work we overcome this problem with a novel concept for creating light tunable metamaterials. We study experimentally SRRs with varactor diodes, which biasing is supplied by photodiodes operated in the photovoltaic mode, and demonstrate that SRR's magnetic resonance can be tuned by changing the intensity of an external light source. We employ coupled SRRs to demonstrate enhancement of the resonant response and switching between the bright and dark modes.

To demonstrate further effects we design light-controllable metamaterial mirror formed by an array of 24 broadside coupled SRRs placed perpendicular to a metallic screen. Each ring of such resonator contains a varactor. The biasing of the varactors in each SRR is provided by a pair of photodiodes that generate a voltage that increases with illumination intensity. We perform an experimental investigation of the designed sample in a parallel plate waveguide with a possibility of near field scanning. The structure is excited by a beam with a plane-wave front. We demonstrate an efficient control over the angle of the reflected beam by illumination with inhomogeneous light pattern. We are able to achieve beam steering of about 12 degrees. It is also possible to switch between focusing and defocusing regimes for the reflected beam by moving the maximum of illumination from centre to the edges of the metamaterial structure.

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Equipment for visualization of SHF electric field

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We have developed a pioneering experimental equipment [1] for investigation of the properties and measurement of characteristics of SHF metamaterials. The equipment enables in polar coordinate control of the characteristics of the electromagnetic flow in the SFH range in the vicinity of the metamaterial item, that allows reproducing the pattern of its surrounding electromagnetic SHF field.

The measuring chamber of the equipment consists of two horizontal plane-parallel copper discs with a clearance between them where the metamaterial sample is placed. SHF power is supplied from the generator into the measuring chamber by means of a waveguide megaphone aimed at the metamaterial sample placed in the center of the measuring chamber. The value of the SHF field in the vicinity of the sample is analyzed using an antenna-probe that is a coaxial cable ending in a wire loop located in the vertical plane. The wire loop is moved by two computer-controlled step motors along the circular arc as well as in the radial direction that allows taking step-defined measurements the SHF fields in the measuring chamber in polar coordinates.

The SHF field values taken by the antenna-probe are transferred to a computer-connected vector network analyzer. The color image of the electrical component of the SHF electromagnetic field in the vicinity of the metamaterial sample is formed on the computer screen by a special processing program. The picture color corresponds to the SHF field value in a predefined way.

Therefore, this measuring equipment enables get by computer display a visualization of the spatial pattern of the SHF field value in the vicinity of the metamaterial sample. This pattern is depicted in prearranged colors with a predefined step, its value maybe changeable before beginning of each measurement within a certain interval.

For testing of our new equipment we made a metamaterial object following the description given in [2]. The sample was placed in the measuring chamber center of us unit to visualize the SHF electric field pattern.

By the coaxial cable the SHF signal gets to measuring port of the "P4M-18" Vector Network Analyzer that analyzes the signal by measuring its amplitude and phase. Our computer program written specially for our measurements using "Visual Basic 6.0" draws on computer display the SHF electric field pattern inside the measuring chamber in prearranged colors. The aubergine-purple color corresponds to the minima (negative maxima) of the SHF electric field and the red color to the positive maxima. The zero of electric field strength is represented by the aquamarine experimental points on the SHF electric field pattern.

We have studied SHF field patterns in the empty chamber and for the case with a copper cylinder 50 mm in diameter and 10 mm high placed in the center of the measuring chamber at 10 GHz frequency.

We have also received a series of patterns of the SHF field electric component for the metamaterial cloak at different frequencies and a series of similar SHF filed patterns at the same frequencies for the case with the copper cylinder 50 mm in diameter and 10 mm high placed inside the metamaterial cloak. During receiving of both SFH field patterns, the "empty" metamaterial cloak as well as that with the copper cylinder inside, they was placed in the center of the measuring chamber of the our equipment.

We have also obtained the frequency dependences of the SHF electric field strength in the points along the 70 mm radius (the radius of the internal points row on the pattern of the SHF field with metamaterial cloak) at a 15 degree interval, the first curve corresponding to the 30 degree azimuth and the last to the 330 degree azimuth. The curves for twenty-one coordinate points were obtained for the case of "empty" metamaterial cloak as well as that with the copper cylinder inside.

Thus, we have created a fundamentally new automatized equipment that enables to obtain spatial propagation patterns of SHF wave electric field components in prearranged colors that allows investigate SHF field patterns around various objects.

Using our innovative equipment, we have also demonstrated SHF field patterns around the metamaterial cloak suggested in [2] in the frequency range from 7 GHz to 15 GHz and revealed the frequencies at which the camouflage properties of the metamaterial cloak are maximum.

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Experimental verification of electromagnetic cloaking at microwave frequencies with metamaterials

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We have developed a novel experimental technique for investigation of spatial distribution of microwave field components around various prearranged objects. Using this equipment we have demonstrated microwave field patterns around the ring-like metamaterial cloak with copper cylinders inside in the frequency range from 7 GHz to 15 GHz and revealed the frequencies at which the camouflage properties of the metamaterial cloak are maximum.

Fabrication of a flexible Ag-ZnO multilayer fishnet metamaterial

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The field of metamaterials has been intensively developed for the last decade due to their noble properties which is not found in natural material. Specially negative index media (NIM) have attracted much attention for their noble applications, such as imaging beyond the diffraction limit of light and invisible cloaking, just to mention a few. Basically, NIM have been implemented with the composite medium consisted of pairs of the split ring resonator (SRR) and metal wires in the microwave and THz wave regime. For the implementation of NIM working in the optical wavelength regime, a metamaterial with the multilayer fishnet structure is suggested and fabricated. However, in order to apply NIMs to real imaging applications, material energy dissipation, particularly in metal, is a stumbling block to be overcome. One attempt to solve foregoing problem is to incorporate gain medium into the metamaterial to compensate the material energy loss.

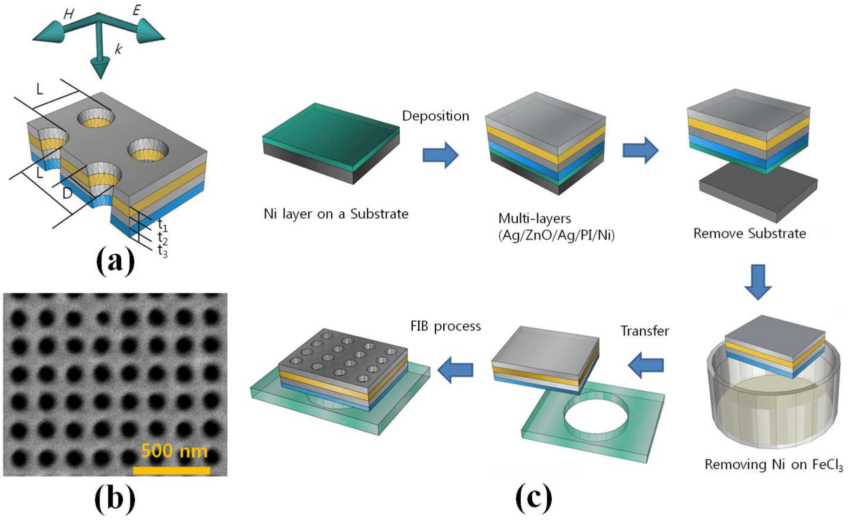

Fig. 1: (a) Schematic of the multi-layer NIM metamaterial. Here, the lengths of a unit cell is set to 196nm, the thicknesses of both metal (t_2), ZnO(t_1) and polyimide layer (t_3) are about 30 nm, and the hole diameter in top (D) metal layer is set to 145 nm. (b) Top-view of the SEM image of the fabricated metamaterial. (c) Schematic of growing process.

In this work, we report a fabrication of a flexible Ag-ZnO multilayer fishnet metamaterial with negative index of refraction working at visible wavelength. The multilayer metamaterial is composed of 4 layers of metal and dielectric (a polyimide layer, a ZnO layer and two silver layers). By using polyimide dielectric material instead of oxides as substrate conventionally used, one can endow flexibility to the metamaterial and easily setup the metamaterial on the system without complex back-etching. Moreover, for the purpose of compensation of the material energy loss, ZnO gain medium, which has a maximum gain at 355 nm wavelength, between silver metal layers is adopted. Patterns on the metamaterial were formed by FIB patterning method. Since the basic spot formed by FIB milling is circular shape, the hole-size is controlled by changing beam current only and the

pitchs of x- and y-direction are controlled by the ion-beam overlapping degree. In the metamaterial, the holes in top metal layer have 145 nm diameter and sidewall slope is 10 degree.
Acknowledgements: This research was supported by the GNT R&D Program (2011-0020206) through the National Research Foundation of Korea funded by the Ministry of Education, Science and Technology.

Engineering of radiation of optically active molecules with chiral nano-meta-particles

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It is well known that nanoparticles influence substantially both fluorescence of molecules and Raman scattering of light by molecules. These effects are especially strong in the case of metallic nanoparticles where plasmon resonances can be excited. When investigating such processes it is usually assumed that only electric dipole transitions are important. However, both electric and magnetic dipole momenta of transitions are important in optically active (chiral) biomolecules. The goal of present work is to investigate influence of nanoparticles made of bi-isotropic metamaterial on spontaneous emission of chiral molecules. We have built general quantum theory, where decay rates were expressed through Green's function of Maxwell equations in presence of nanoparticle. Then we apply our general theory to spontaneous emission of chiral molecule placed near nanospheres made of different materials, including chiral "left-handed" metamaterial. We have found general conditions when radiation of "right" or "left" molecules can be fully suppressed. It paves the way to control radiation of chiral molecules. In Fig. 1 the example of our calculations is shown.

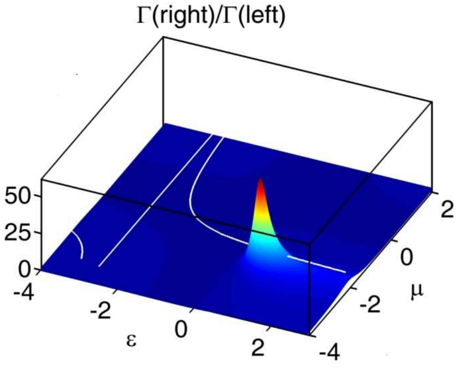

Fig. 1: Ratio of decay rate of "right" molecules and decay rate of "left" molecules as function of ε and μ of sphere. Chirality parameter of sphere $k = 0.1$, size parameter $kR_0 = 0.1$. Electric and magnetic dipole momenta of chiral molecules are oriented along radius and their ratio is equal to $m_0/d_0 = \pm 0.1$.

From this figure one can see the substantial (factor of fifty at $\varepsilon = 1$ $\mu \approx 2$) influence of nanoparticles on spontaneous emission of chiral molecules with different chiralities ("right" and "left" molecules) due to appearance of interference between radiation of electric and magnetic dipoles. Let us stress that "left-handedness" or μ negativeness of chiral sphere is of crucial importance for such discrimination.

An application of results obtained to separate racemic mixtures of drug enantiomers is suggested.

The author thanks the Russian Foundation for Basic Research (grants ## 11-02-01272, 11-02-91065,11-02-92002) for financial support of this work.

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Quantum electrodynamics of atoms and molecules in nanoenvironment

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In this talk I will discuss influence of nanoparticles and nanostructures (including metamaterials) on optical properties of atoms and molecules. In the first part of lecture I will present history and general principles of QED in nanoenvironment. Here I will touch both weak and strong regimes of light-matter interaction. Special attention will be paid to quantization of electromagnetic field near nanoparticles and nanostructures and relation between classical and quantum descriptions. Notion of local density of photon states will be also discussed here. In second part of lecture I will present our results on radiation of atoms and molecules for different specific geometries and materials. In final part of talk I will outline main challenges in this area.

Optimal filling factor of nanorod lenses for subwavelength imaging

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Wire media were suggested and extensively studied as the basis for subwavelength imaging devices [1]–[4]. Arrays of parallel wires can transport near-field images with subwavelength resolution to the distances up to several wavelengths possibly avoiding obstacles. Tapered wire arrays [2, 3] can be employed for magnification of such images. Employing arrays of metallic wires of different radii, it is possible to achieve subwavelength imaging in a wide spectrum of electromagnetic waves ranging from radio and microwaves to THz frequencies [2] or extend this range to optics using plasmonic nanorods.

In this work we analyze the effect of the filling factor on the imaging performance of metallic nanorod lenses. We study numerically a square lattice of nanorods modelled in CST Microwave Studio. The material of the nanowires is silver (Ag). We observe that thicker nanorods allow lower reflection in the canalization regime, and we find optimal values of the filling factor to achieve a transfer function with the characteristics of a perfect lens in a wide range of spatial frequencies.

We have studied numerically the effect of the filling ratio on the characteristics of the nanorod lenses. We have revealed that there exists a range of the relative nanorod radii for which the reflection from the front surface of the lens can be made as small as 0.1. This corresponds to the filling factors in the range $0.12 < f_{\mathrm{r}} < 0.20$. We have studied the properties of the transfer function of the wire lens with thick nanowires and found that higher filling factor can allow for a broader range of operational spatial frequencies. Our numerical simulations have shown a nonmonotonic dependence of the plasmon resonances with the relative nanowire radius. This effect is attributed to a nonvanishing tip capacitance of the thick nanorods.

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Metal-dielectric layered metamaterials for sub-diffraction spatial filtering of the optical wavefront

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Imaging with sub-wavelength resolution with layered metal-dielectric metamaterials has been attracting increasing interest in the recent years [1–9].

While the spectral properties of stratified media had been investigated for a long time and found numerous applications, their use for spatial filtering with sub-wavelength kernel functions is an emerging concept. In fact, layered metamaterials may be described as linear spatial filters [4, 5]. Taking the wavelength and layer thickness as free parameters, the metamaterial may be optimized with respect to selected criteria, particularly for high transmission and resolution [6]. However, the range of attainable point spread functions varies a lot, and apart from direct imaging, also certain image processing operations seem feasible to obtain. In the present paper the possible spatial filtering operations which modify the contrast or high spatial frequency components of the spatial spectrum will be presented.

Some distinct features make spatial filtering with use of layered metamaterials different from spatial filtering investigated in the past within the framework of Fourier Optics. In a rigorous approach, the transfer function and point spread function for such a filter take a matrix form [7]. The width of the point spread function is in some cases an ambiguous measure of resolution [8] due to its rapid phase modulation. Finally, the amplitude transfer function is difficult to measure, however the modulation transfer function can be investigated experimentally [9].

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Active plasmonics: manipulation of light at the nanoscale

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Surface plasmon polaritons (SPPs) offer a unique opportunity to localise and guide photonic signals at the nanoscale, which has attracted a great interest to them as information carriers. In our presentation we will consider various aspects of realisation of nanoplasmonic circuitry, particularly addressing implementation active functionalities into it. Starting from overview of various approaches for guiding the SPP waves, we will concentrate on the most advanced in terms on photonic localisation or technologically robust designs, such as nonowire [1] and dielectric-loaded SPP waveguides [2]. The extreme photonic integration provided by the plasmonic guiding approach comes at a price of the finite propagation length, resulted from the ohmic losses. Indeed, there is a trade-off: the higher the plasmonic mode localisation, the shorter the propagation length. Therefore, the compensation of losses has a crucial importance and various approaches for amplification of the SPP waves, via optical [3] or electric [4] pumping will be considered.

Finally, it will only be possible to speak about nanophotonics in the same way as electronics, if nanoscale switching/modulating plasmonic elements are implemented. Traditional nonlinear materials provide too small optical response to actively control light at such small dimensions and new approaches to enhance light-matter interaction at the nanoscale and the introduction of new nonlinear (meta) materials and component designs are required. We will demonstrate novel approaches in this area, based on the robust all-optical nonlinear effect provided by trans-cis isomerisation of azobenzene molecules [5] and nanoscale drastic electro-absorption effect in degenerate semiconductors [6]. In the latter case it will be demonstrated an active electro-optical field effect nanoplasmonic modulator with a revolutionary small size of $25\times30\times100$ nm^3, providing signal extinction ratio as high as 2 at switching voltages of just 1 V [6].

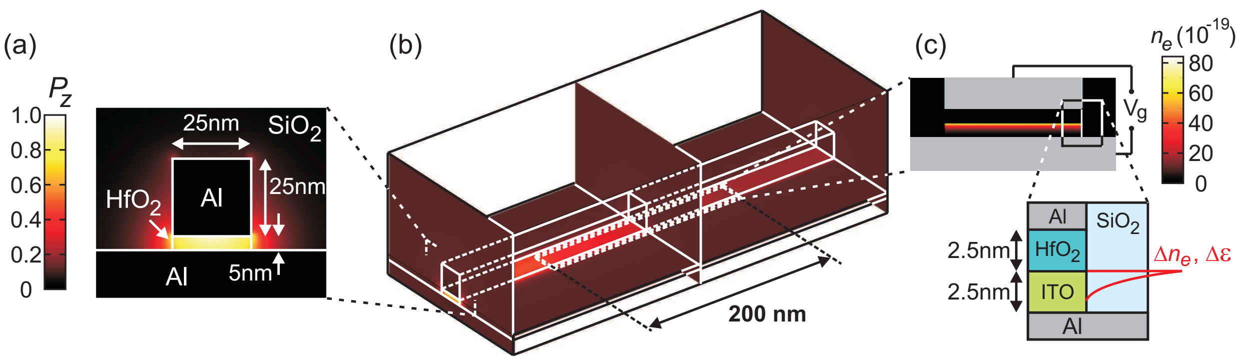

Fig. 1: (a) Power flow profile of an SPP mode in a hybrid 25×25 nm^2 wire-MIM waveguide. (b) Power flow map of the mode transmission through the modulator at V_g=2 V. (c) Charge accumulation layer map with a zoom of the active region structure.

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All-dielectric optical nanoantennas

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The recently emerged field of optical nanoantennas is promising for its potential applications in various areas of nanotechnology [1]. The ability to redirect propagating radiation and transfer it into localized subwavelength modes at the nanoscale makes the optical nanoantennas highly desirable for efficient solar cells, biological and chemical sensing, quantum communication systems, molecular spectroscopy and, in particular, for the emission enhancement and directionality control over a broad wavelength range. We suggest [2] a novel type of optical nanoantennas made of all-dielectric elements. Dielectric nanoantenna have several features. First, dielectric materials exhibit low loss at the optical frequencies. Second, as was suggested earlier [3], nanoparticles made of high-permittivity dielectrics may support both electric and magnetic resonant modes. This feature may greatly expand the applicability of optical nanoantennas for, e.g. detection of magnetic dipole transitions of molecules. In our study we concentrate on nanoparticles made of silicon [4].

By placing a point-like dipole source near a single dielectric particle driven at the magnetic resonance results the radiation pattern similar to that of a Huygens source with the enhanced forward and vanishing backward emission. This feature can be employed in the Yagi–Uda geometry for highly efficient optical nanoantennas. Fig. 1 shows the directivity of the dielectric Yagi–Uda nanoantenna.

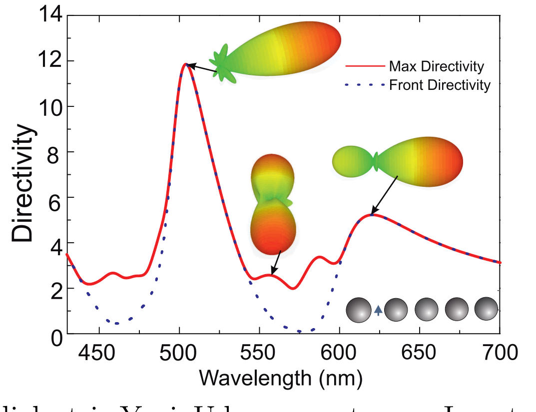

Fig. 1: Directivity of the dielectric Yagi–Uda nanoantenna. Insert demonstrates 3D radiation pattern diagrams at particular wavelengths.

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Specific boundary effects in discrete metamaterials

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In this talk, we report that discrete structure and finite size of metamaterials lead to specific boundary effects which cannot be described with an effective medium approach. We describe the origin and peculiarities of these effects, which are related to an interplay between translational symmetry of a lattice and overall symmetry of a finite sample. This provides useful guidelines for practical assembly of metamaterials so that their behaviour can be brought closer to the predictions of effective medium theory.

Correct calculation of effective material parameters belongs to the most important aspects in theoretical description of metamaterials. By this time, it is commonly admitted that if the grounding requirements for effective medium treatment are observed [1], metamaterial can be reliably replaced with a corresponding piece of continuous medium, allowing for easy and quick analysis. For example, earlier it was shown [2] that a three-dimensional anisotropic system with about thousand sub-wavelength resonators can be almost perfectly described with an effective medium approximation, so that the permeability numerically obtained for a finite sample, coincides with the theoretical curve.

Here we report that an isotropic system of resonators has observable macroscopic properties which are quite different from the predictions of a continuous model. We discuss the physical origin of these effects, which are associated with metamaterial boundaries (Fig. 1), and show how the properties of finite samples can be controlled by choosing an appropriate geometry at the edges.

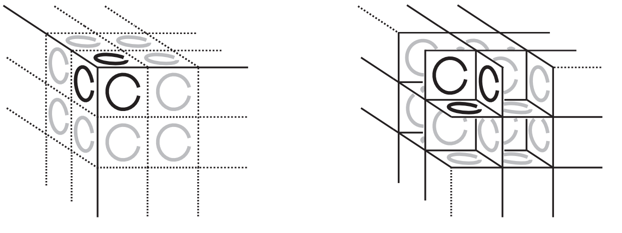

Fig. 1: Ambiguity of the boundary structure.

It should be noted that these phenomena are not analogous to the emergence of non-physical resonances in continuous samples with sharp edges [4], which appear in numerical simulations and can be eliminated by "rounding the corners". They are also more pronounced than the classical effect of transition layers (see [1] and references therein). In real metamaterials, this behaviour is related to specific symmetry requirements, which cause an ambiguity in boundary structure (Fig. 1), and cannot be modelled within a continuous medium approximation.

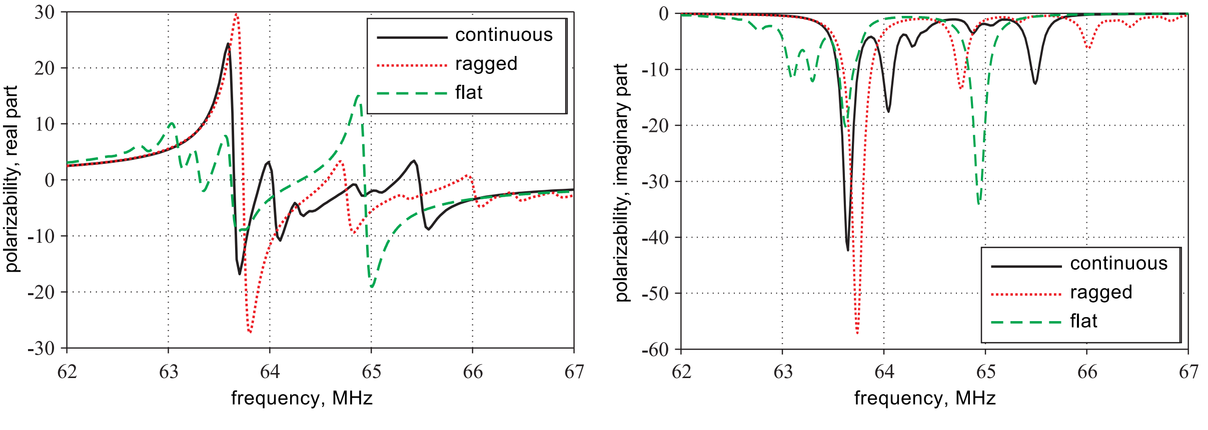

Fig. 2: Comparison between the polarizability of a cubic sample either made of a continuous medium or having a discrete structure with different types of surface arrangement.

To illustrate our findings, we study cubic samples of isotropic three-dimensional metamaterials assembled with strongly sub-wavelength resonant elements (which can be conveniently implemented as capacitively loaded rings in the microwave range). In Fig. 2, we compare the polarizabilities of discrete cubic metamaterial samples with different boundary structure, to that of an equivalent cube made of continuous medium, which has an effective permeability [3] corresponding to such metamaterial. Furthermore, when we compare the quasi-permeability, formally introduced for the discrete cubes based on their polarizability, to the permeability of a continuous cube (Fig. 3), we reveal which boundary configuration is more favourable in terms of effective medium description.

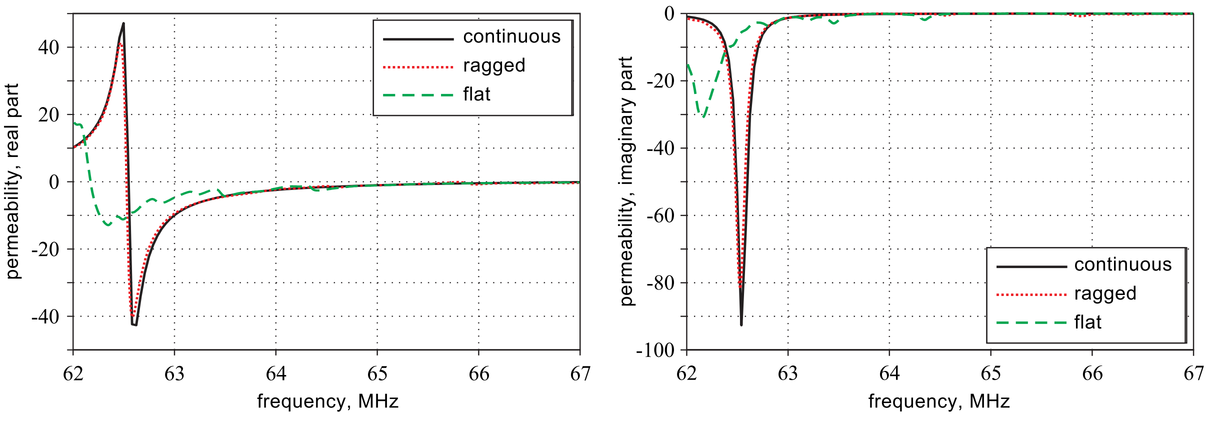

Fig. 3: Comparison between the effective permeability, calculated for cubic samples having a discrete structure with various surface types, to that of a continuous medium.

We should note that the polarizability dispersion of the largest cube we have calculated ($12 \times 12 \times 12$ lattice constants), may suggest that further minor resonances would appear as the cube gets still larger. On the other hand, for sufficiently large size the conditions for macroscopic averaging will be better fulfilled, so the boundary effect will be relatively weak and the response of both the assembly types should converge to that of a continuous sample. However, such an enlargement may also bring the effects of spatial dispersion into play, and on the other hand dissipation, if taken reasonable, would suppress magnetoinductive waves across the entire sample. A consistent analysis of the effects in larger structures would be of interest for future research.

We conclude that for metamaterial samples of moderate size (up to a few thousands elements as far as we have tested), the "ragged" method of boundary assembly is practically useful for the elimination of extra resonances. This corresponds to the size range where most practical metamaterials fit, so our results will provide practical information and design guidelines, allowing for efficient control over the resonance properties in finite metamaterial samples.

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A nanoradar based on a nonlinear plasmonic nanodimer

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The study of optical nanoantennas became a subject of intensive research [1]. Nanoantennas hold a promise for the subwavelength manipulation and control of optical radiation for solar cells and sensing [2]. In a majority of applications, a crucial factor is a control over nanoantenna radiation pattern and its tunability. Spectral tunability and variable directionality have been proposed for bimetallic antennas [3], the Yagi–Uda architectures [4], high-permittivity dielectric nanoparticles [5], and nanoparticle chains [6]. In addition, several suggestions employed the concept of plasmonic nanoantennas with a nonlinear load where the spectral tunability is achieved by varying the pumping energy [7, 8].

In this work, we introduce a novel concept of a *nanoradar* (or nanoantenna with self-tunable directivity) composed of two nonlinear subwavelength silver nanoparticles. More specifically, we consider modulation instability (MI) in this nanodimer and find that MI development allows dynamical exchange of energy between cluster eigenmodes resulting in a periodical rotation of the nanoantenna scattering pattern similar to classical radar systems. We carry out a detailed analysis of the impact of an external field on the width and the rotation frequency of the major lobe of scattering, and show that nanoradars may find applications in positioning systems aimed for dynamical tracing of different nanotargets, including molecules, proteins, viruses etc.

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Laser implantation of ferroelectric nanoparticles into pores of synthetic opal. The phys.-math. models and comparison with experimental data

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The synthetic opal is 3D globular photonic crystals. The laser implantation permit to introduce the functional substances (for example, nanoparticles of ferroelectrics) into pores of such crystals [1, 2]. We are developing some models and numerical codes for the description of optical properties of such materials [3]. The results of numerical simulations have been compared with experimental data.

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Local resonance band gaps in ferromagnetic nanostructured composites

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Nowadays local resonance media for electromagnetic and acoustic waves gain great attention due to possibility of creation metamaterials with negative effective constitutive parameters [1, 2]. In our work we are taking attempt to investigate spin wave propagation in a nanostructured composite ferromagnetic film with cylindrical inclusions in magnetostatic approximation.

Considered nanostructured film is consist of Yttrium-Iron Garnet (YIG) islands, etched on non-magnetic substrate. Space between YIG islands is filled by Nickel. YIG inclusions are distributed at random, have cylindrical shape with radius $R = 5\,\mu m$, generatix of cylinders are parallel, average distance between inclusions is $L = 10\,\mu m$, space between them filled by ferromagnetic matrix, thickness of film is $h = 100\,nm$. Forward volume spin waves (FVSW) propagate in (x, y) plate, bias field is directed perpendicular to film, along Oz axis. Saturation magnetization for Ni is 6 kOe, YIG is 1,7 kOe [3].

To compute values of effective permeability tensor of composite film adapted version of coherent potential approximation is used. The method can take in account structures with local resonances, i.e. structures with areas where wavelength is similar to its size. Due to resonance scattering of FVSW, effective permeability has resonance frequency behavior also. Dispersion of FVSW in the composite film is plotted on Fig. 1. Due to local resonance and destructive interference, some band gap occurs on dispersion. We should note here, that band gap occurs not from Bragg scattering [3], but from Mie-type scatting.

To sum up, in this work we are presenting effective parameters calculation method for ferromagnetic films with random distribution of impurities. Using the method we show that such media exhibit local resonance behavior and frequency band gaps for FVSW can be obtained.

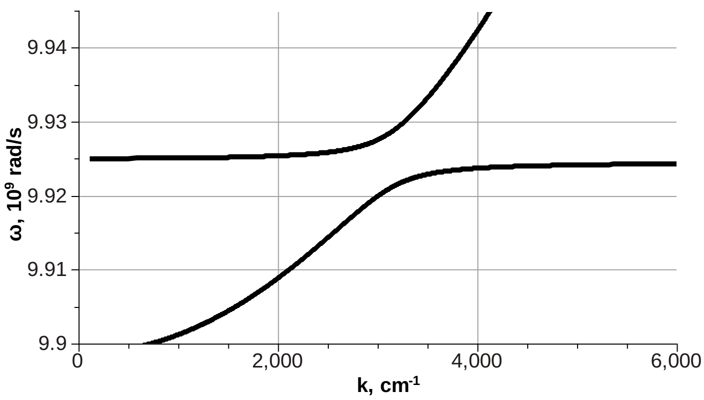

Fig. 1: Dispersion of forward volume magnetostatic wave in Ni/YIG composite.

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Modeling of metamaterials and composites with nonlinear electromagnetic properties

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Composites and metamaterials possess unique and various electromagnetic properties that makes perspective their use in various optical and microwave devices and technologies. Local space distribution of an electromagnetic field in such materials is strongly non-uniform that can lead to various nonlinear effects.

One of perspective directions of the analysis of electromagnetic properties of composites and metamaterials is use of a method of the minimal autonomous blocks (MAB) [1–4]. In its basis decomposition of investigated spatial area on system of blocks in the form of rectangular parallelepipeds lies. For the description of electromagnetic properties of blocks scattering matrixes are used. Calculation of an electromagnetic field distribution in linear mediums is carried out with use recomposition, iterative and combined algorithms.

For the analysis of electromagnetic properties of nonlinear composites and materials it is offered to use iterative algorithm of realization of the MAB method, modeling process of multiple scattering of channel waves in the system of MABs.

Modeling of the nonlinear effects caused by dependence of material parameters of a medium from amplitude electric components of an electromagnetic field is carried out by their step-by-step correction within the framework of iterative process.

For modeling of effect of generation of multiple harmonics it is offered to use a few systems from MAB within the framework of the general iterative process. Each systems of MAB has identical spatial topology, and scattering matrixes of MABs are calculated for the main and several considered multiple frequencies. The mechanism of power interrelation between the blocks corresponding to different decomposition schemes for various frequencies is considered.

Examples of calculation of distribution of electromagnetic fields in various types of nonlinear composites and metamaterials for various models of sources are presented.

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Limitations of the Babinet’s principle at the nanoscale

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Roughly speaking, the Babinet’s principle stays that two complementary conductive screens give rise to equal diffraction patterns and also that the addition of the transmission coefficients of both structures equals 1. In the range of microwaves and RF, there exists a well known demonstration of this principle [1], so that it should be called theorem instead of principle. Why is the Babinet’s principle called principle? This demonstration holds only when the screen is made of perfect electric conductor (PEC), its thickness is negligible, and the metal is not lying on any dielectric substrate. If we were interested in screens working at some optical frequency (infrared, visible, or ultraviolet), every turns more difficult. First, no metal can be approximated as perfect conductor for very high frequencies. Second, since the response of the screen will be weaker than that of a PEC screen, it is convenient to use thick screens. Therefore, the conditions of the Babinet’s theorem for relatively low frequencies do not hold in optics. In order to remark that this demonstration is not always true, lets remain an experiment that almost all physicists have done during some optics lectures in his undergraduate studies in the university. Measure the diffraction pattern of a straight hair and the diffraction pattern of a slot printed on some acetate foil (by using a common printer with black ink). We (if you did it) observed that both diffraction patterns are quite similar. In other words, the Babinet's principle is working well. However, it is worth to note that the hair is not perfect conductor and its radius is not much smaller than the optical wavelength of the used laser, neither the black ink of the printer. Since there is no exact demonstration, we should then call this phenomenon a principle, as it is at the present moment.

By studying the saturation of the resonant frequency of the Complementary Split Ring Resonator (CSRR) and comparing this with the saturation of the Split Ring Resonator (SRR), we found that both saturation frequencies are near in general [2] (see Fig 1 and Fig. 2). After, we realized that this coincidence tends to be exact when the thickness of the screens is bigger and bigger (see Fig. 2). Finally, we found a partial demonstration of the Babinet's principle for thick 2D structures. This theorem is more general, and the example of the SRR and CSRR metasurfaces are only a particular case. In this talk we are going to present this theorem and several applications to complementary metasurfaces and dual optical nanocircuits (in the frame of the metatronics proposed by Prof. Nader Engheta). Also we will discuss the limits of validity for extremely small unit cells, when quantum effects start to arise.

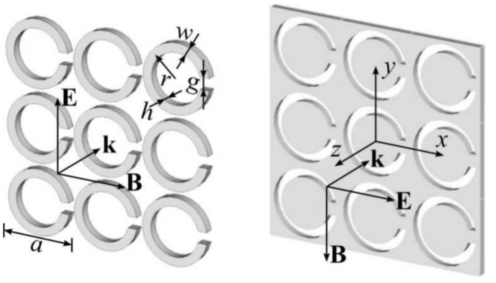

Fig. 1: Metasurfaces made of SRRs (left) and CSRRs (right).

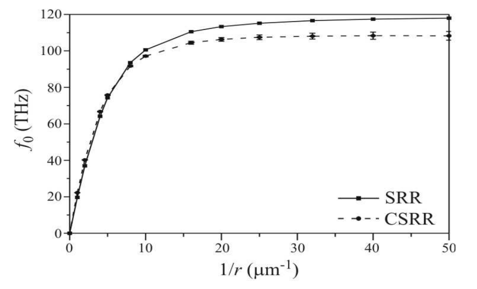

Fig. 2: Resonant frequency versus scaling factor. The ring radius decrease to the right.

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Tunable metamaterials for THz electromagnetic spectrum using piezoelectric cantelivers

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Nowadays terahertz (THz) region of the electromagnetic spectrum reveals huge possibilities for fabrication a variety of different applications. Novel devices for security, medicine, space and earth science became possible to realize by means using THz radiation [1]. THz rays are highly absorbed in water molecules and metal while most dielectrics are transparent for them. This property allows performing high resolution THz imaging. Besides THz radiation is non ionizing hence safe for biological organisms (in contrast x-ray radiation).

Active development of the terahertz technology was initiated relatively recently. Investigations of the THz radiation are attractive due to its possible applications for imaging and sensing [2]. THz radiation can also be used for medical scanning, security screening, quality control, atmospheric investigation, space research etc. Metamaterials (MTM) have been proposed for operating in THz frequency range. Terahertz radiation manipulating is a quiet challenging task. The problem can be solved by a design of metamaterials with tunable or controllable characteristics.

Widely used is control of MTM by tunable or switchable SRR. The design approach is based on creating a bimaterial-cantilever switch with the cantilever sitting above the gap that operates by modifying or shorting the SRR capacitive gap [3]. The effective LC resonance results in a frequency dependent transmission, where, on resonance, a strongly enhanced electric field is concentrated in the gap of the SRR.

We suggest designing a tunable -negative MTM formed as an array of U-shaped resonators with electrically controlled piezoelectric cantilever. Under the biasing voltage the cantilever bent at the angle α. As a result the electric field is concentrated in the formed gap and a tunable capacitance occurs changing the electrical length of the resonator. This in turn leads to a shift of the resonance frequency. The regular metamaterial structure with U-shaped resonators with movable cantilevers was designed. The EM simulation reviled that the cantilevers bent at the angle from $0°$ to$15°$ demonstrated the resonant frequency change from 0.384 THz to 0.586 THz. The structure can be used as electrically controlled band-stop filter.

Another tunable MTM structure for THz applications is an array of MDM cells consisting of two metallic patches sandwiched by a thin dielectric layer [4]. The resonance frequency of the MDM patch array depends on the value of the capacitance between two coupled patches. If the part of the patch metallization is bent, the effective thickness between the patches changes and modifies the capacitance, therefore, the resonant frequency also will be changed. When the angle between the bent part of the patch metallization and substrate is increased, the capacitance decreases and the resonant frequency is blueshifted. The EM simulation demonstrated the low-pass filter behavior of the array of MDM based MTM with tunable cut-off frequency provided by bending one patch. Tunability of the cut-off frequency of the structures with piezoelectric cantilever is about 24% (0.275 – 0.35 THz) for 10 degree angle variation between the movable part of the patch and the substrate.

As suitable piezoelectric materials, PZT ($Pb(Zr,Ti)O_3$) or PVDF ($(C_2\text{-}CH_2)_n$ having a high piezoelectric module are recommended for the cantilever design [5], [6]. The tunable piezoelectric cantilever is mounting at the silicon substrate. A thin layer (1–2 μm) of the silicon substrate situated under piezoelectric layer is removed by etching. An additional membrane layer is used to increase the elastic property of the structure. In case when PVDF is used as piezoelectric, the membrane layer can be missed, because the PVDF demonstrates a good elastic property.

The control voltage is applied to cantilevers by rows of metallic wires, oriented perpendicular to the electric field of the incident electromagnetic wave. The wires do not influence the wave propagation along the structure. The experiment is in progress.

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Sensing by spaser

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The advances of nanotechnology open an opportunity of further development of the spectroscopy and microscopy methods. In the current report we suggest a novel method that combines the ideas of tip-enhanced optical spectroscopy (TEOS) and of laser spectroscopy. The latter, offering no spatial resolution, provides an extremely high sensitivity and reveals even prohibited (non-dipolar) transitions. The combination is enabled by invention of plasmonic laser (spaser) [1], which generates plasmons due to nonradiative energy transfer from the gain medium (quantum dot) to SPP, localized at a nanoparticle. Recently we have suggested a spaser radiating 1D plasmons [2]. The device suggested here is based on 1D spaser generating plasmons on a needle with narrow tip. The plasmonic lasing (spasing) gives much higher field intensity than in the scheme of TEOS, because the plasmon at the tip is excited not by scattering of incident wave but by direct nonradiative energy transfer from quantum dots or other gain medium placed on the needle. The high field intensity favors high sensitivity even for linear methods, nothing to say of tip-enhanced Raman spectroscopy. Additionally, in the case of the current pumping the sample is not exposed to any external radiation. The advantage of a new method over the standard laser spectroscopy, besides the spatial resolution, is the weakness of nano-sized spaser compared to the macroscopic laser, which permits easier suppression of lasing by the analyzed sample and, thus, increases the sensitivity.

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Synthesis of bianisotropic arrays

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In this review presentation we will discuss a possibility of analytically synthesizing different polarization-transforming and absorbing layers realized as arrays of electrically small particles (single-layer grids). The developed method is based on the general relations between the reflection and transmission coefficients and the polarizabilities of arbitrary bianisotropic particles. The transmission and reflection dyadics are presented as functions of the polarizabilities of the most general linear scatterer. From these dyadics one can solve the polarizabilities that are needed in order to obtain the wanted polarization transformation or absorption and reflection properties.

As examples, a novel twist polarizer and a novel circular polarization selective surface (CPSS) are synthesized. The twist polarizer is a device that rotates the polarization plane of a plane-wave and a CPSS that reflects one handedness of CP and is invisible to the other. The synthesized twist polarizer is fitted on a standard printed circuit board, optimized numerically, and finally manufactured and measured. The experimental results for this device show good correspondence with the simulations and the new method is found to be a useful tool for developing any polarizationtransforming devices.

The developed method allows multiple choices for the sets of polarizabilities that fulfill the imposed conditions on reflection and transmission properties, and further analysis (possibly using other criteria) is necessary to come to the optimal design for a particular application.

Furthermore, we will discuss the use of this general synthesis approach for design of thin matched absorbing layers, analyzing possibilities offered by magnetoelectric coupling in the particles.

Details of the study on low-loss devices (polarization transformers) have been submitted for publication [1].

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Spin-wave dynamics in lateral periodic and quasiperiodic magnetic micro- and nanostructures magnonic crystals

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Magnetic periodically structured ferromagnetic films being a microwave analog of photonic and phononic crystals have created a revival interest of spin waves (SW) investigations in periodically micro- and nanostructured magnetic media, which are called magnonic crystals (MC). However SW propagation in such structures has some specific properties untypical for both optic and elastic waves. Namely, they possess strong nonlinearity, anisotropic and nonreciprical propagation characteristics even for the case of wave propagation in isotropic materials. Besides that, ferromagnetic films being a base for MC with strong magnetostriction due to resonance interaction between excitations of magnetic and elastic subsystems lead to a new unusual effects. Such peculiarities of SW propagation in MC are interesting and important both from fundamental and practical points of view as they can be important for microwave signal processing on the basis of magnonic crystals. In this paper we review the recent results of propagating and localized spin-waves excitations in magnetic thin film structures with micro- or nanosize features arranged in lateral one- (1D) or two-dimension (2D) periodic arrays. Such fundamental phenomena as anisotropic Bragg scattering and quantization of SW are discussed as well opportunities to control the magnonic crystals parameters due to metal screening and non- linearity of magnetic system. Some attention will be paid to magnetoelastic interaction in epitaxial yttrium iron garnet/gadolinium iron garnet (YIG/GGG) structures with microstructured surface of ferromagnetic YIG film. We also review results of ferromagnetic resonance (FMR) investigations of localized SW modes in MC crystals based on metallic ferromagnetic films: permalloy (Py) films sputtered on patterned nonmagnetic substrate, bicomponent (Py and Co) MC and antidote lattices.

Ultrafast light switching and routing by nonlinear metal-dielectric nanoantennas

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A recently proposed concept of optical nanoantennas is a subject of an increasing number of researches [1]. Great interest to this realm of contemporary optics is caused by the promise that optical antennas hold for manipulating and controlling optical radiation at subwavelength scales, solar cells, nanomedicine and sensing [2]. In a majority of applications, a crucial factor is a control over nanoantenna radiation pattern and its tunability. Spectral tunability and variable directionality have been proposed and demonstrated by exploiting bimetallic antennas [3], Yagi–Uda architectures [4, 5], mechanical reconfiguration [6], high permittivity dielectric nanoparticles [7, 8], and variation in a refractive index of a host material for nanoparticle-chain antennas [9]. Of special interest for dynamical steering nanoantenna parameters is employment of different nonlinearities inherited in parts of a nanoantenna since such mechanisms of tunability may provide ultrafast all-optical switching unachievable in other schemes. To this end, it has been discussed plasmonic nanoantennas with nonlinear loads allowing spectral tunability by variation in the pumping energy [10, 11].

In our work, we suggest and study theoretically a novel type of metal-dielectric nanoantennas composed of silicon and silver nanoparticles. Utilizing an intrinsic Kerr-type nonlinearity of a metallic nanoparticle and a high permittivity of a dielectric nanoparticle, we demonstrate the possibility of the efficient dynamical control over the scattering pattern of a metal-dielectric dimer by varying the external field intensity. An estimated switching time of 260 fs opens a promising perspective for using nonlinear metal-dielectric nanoantennas in logical devices.

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Self-complementary metasurfaces

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Frequency selective surfaces (FSS) – thin screens made with periodically distributed flat metal scatterers – have the property of filtering incident electromagnetic waves. When scatterers are connected, the FSS behaves as a band-pass filter and when they are unconnected then the FSS is a band-stop filter [1, 2]. Since the proposal of Split Ring Resonator (SRR) by Pendry [3], this and some other resonators used in metamaterial science have been incorporated to FSS designs by some researchers. The newly created devices are often called metasurfaces [4]. Beruete et al. [5] proposed an interesting self-complementary array (using only connected elements), which worked as a polarizer at some frequency in the THz range. That structure had poor frequency selectivity and thus it could not be used as a band-pass filter. As an example, the self-complementary metasurface shown in Fig. 1 behaves as a band-pass filter for polarization with electric field along the vertical direction, while as a band-stop filter for the orthogonal polarization with electric field along the horizontal direction. When the structure was simulated with perfect electric conductor (PEC), it exhibited perfect transmission or rejection just at the resonant frequency. Although the sample of lossy metal shows worse transmission or reflection spectra (see Fig. 2), the properties of band-pass and band-stop filters still remain for their respective polarizations. Up to now, this kind of response has not yet been reported in the literature of frequency selective surface. Therefore, this self-complementary metasurface could open the way for designing a new type of high quality factor FSS made of connected and unconnected elements at once.

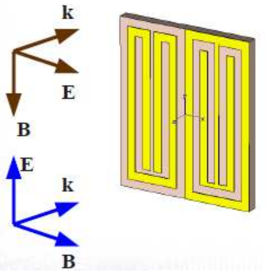

Fig. 1: Unit cell of the self-complementary metasurface.

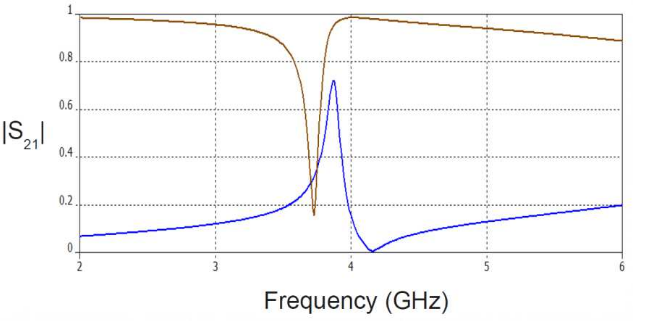

Fig. 2: Transmission coefficient throught the structure of Fig. 1.

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Spaser in above-threshold regime: the lasing frequency shift

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The work presents a theoretical framework describing spaser in above-threshold regime. The main difficulty in this problem lies in the nonlinear behavior of the active medium with respect to the generated electromagnetic field. The key idea is to solve Maxwell's equations in the quasistatic approximation

$$\mathrm{div}(\hat{\varepsilon}(\mathbf{r})\,\mathrm{grad}\,\Phi) = 0, \tag{1}$$

using perturbation theory approach with a small parameter of inverse quality factor (1/Q). In the first-order perturbation theory we found the lasing frequency and the dependence of the stimulated emission intensity of the pumping intensity and, thus, the lasing threshold. Qualitatively, all results are consistent with [1], [2]. In the second-order of perturbation theory we found an interesting effect: the lasing frequency shift with increasing pumping intensity. Nature of this phenomenon is hidden in the change of the spartial structure of the mode due to nonlinear mechanism in permittivity of active medium. This result is interesting by itself, in view of the importance for the frequency of radiation, and in view of its practical application. For example, it allows to determine the change in the structure of the plasmon mode from the frequency shift. This can be useful in problems where it is especially important to maintain the structure of the lasing mode constant.

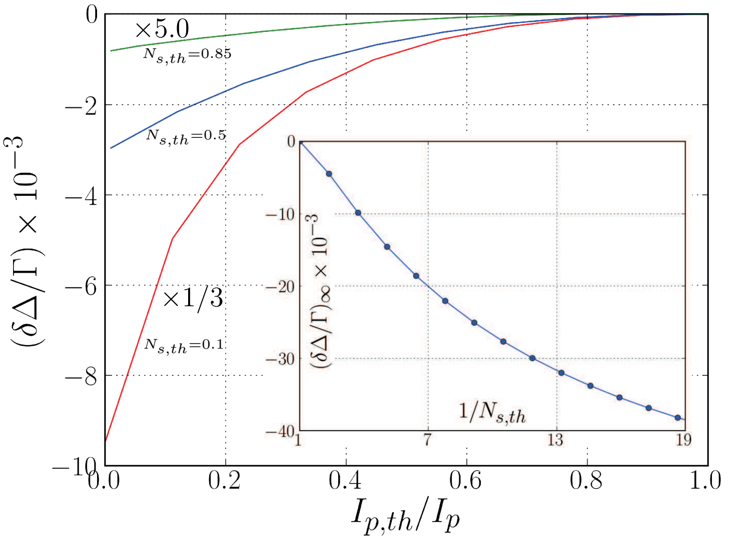

Fig. 1: The lasing frequency shift. Main panel, the dependence of the lasing frequency shift by the inverse of the pump intensity, normalized to the threshold. Insert, the asymptotic value of frequency shift (limit strong above-threshold), depending on inverse equllibrium threshold inversion.

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An explanation of the directed diffraction phenomenon in photonic crystals

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The phenomemon of the directed diffraction of light beams in photonic crystals was first observed numerically in [1]. We present a simple analytic model, which explains this effect. It is shown to be associated with the hyperbolic points of a dispersive surface. We consider a simplest one-dimensional model of the photonic crystal. The hyperbolic points correspond to the lower boundaries of band gaps. We present the results of the numerical simulation obtained by using CST Microwave Studio. The results of the simulation are in a good agreement with the analytic results.

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Purcell effect in hyperbolic medium

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Hyperbolic medium is a uniaxial medium where transverse and longitudinal dielectric constants have opposite signs [1]. It is characterized with hyperbolic isofrequency surfaces in wavevector space (see Fig. 1a), and can be realized in magnetized plasma and in metamaterials. Nowadays hyperbolic medium attracts huge amount of interest due to its unique prospectives for quantum nanophotonics. Indeed, hyperbolic isofrequency surface leads to infinite density of photonic states. This means infinite radiative decay rate of embedded light source, i.e. infinite Purcell factor [2]. Although in realistic case the Purcell factor is limited by a certain cutoff at large wavevectors, experimental reports on Purcell enhancement in hyperbolic metamaterials are already available [3].

In this theoretical work we present study of Purcell factor for several microscopic models. First, we consider finite size light source embedded in homogeneous hyperbolic medium. The cutoff in the wavevector space is then determined by the inverse size of the source [4]. Second, we study Purcell effect in hyperbolic metamaterial realized as layered metal-dielectric structure [5], see Fig. 1b. Importance of nonlocality of dielectric responce due to surface plasmon excitation is discussed. Third, we consider hyperbolic metamaterial based on three-dimensional cubic grid of uniaxial dipole scatterers, see Fig. 1c. We reveal dramatic dependence of Purcell factor dependence on the source position within the metamaterial unit cell. Analytical results for different cases are presented.

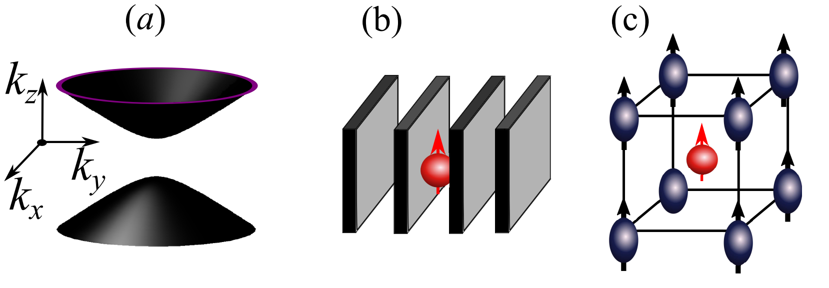

Fig. 1: (a) isofrequency surfaces in hyperbolic medium with $\varepsilon_{zz} > 0, \varepsilon_{xx} = \varepsilon_{yy} < 0$ and schemes of hyperbolic metamaterials based on layered metal-dielectric structure (b) and on lattice of uniaxial dipole scatterers (c).

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Green function for hyperbolic medium

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The hyperbolic medium is a particular case of uniaxial anisotropic medium, where principle values of permittivity tensor have opposite signs. Hence the isofrequency surfaces of wavevectors for given frequency are of hyperbolic shape. This medium can be physically realised in magnetized plasma [1] and in metamaterials [2]. It is now in focus of intensive research due to promising applications for quantum nanophotonics [2,3].

In this theoretical work we obtain Green function of the hyperbolic medium. The Green function describes radiation of point dipole source. We study both longitudinal and transverse orientations of dipole moment with respect to axis of anisotropy (see the Figure). Contrary to the elliptic case, the Green function is highly anisotropic: TM-polarized waves can propagate within the certain cone and are evanescent outside this cone. At the cone edges the Green function has singular behaviour. Due to this singularity, for lossless hyperbolic medium the near field zone extends up to infinity. We also demonstrate that the Green function has singular δ-function term, governing both Purcell factor [4] and Lamb shift of light source.

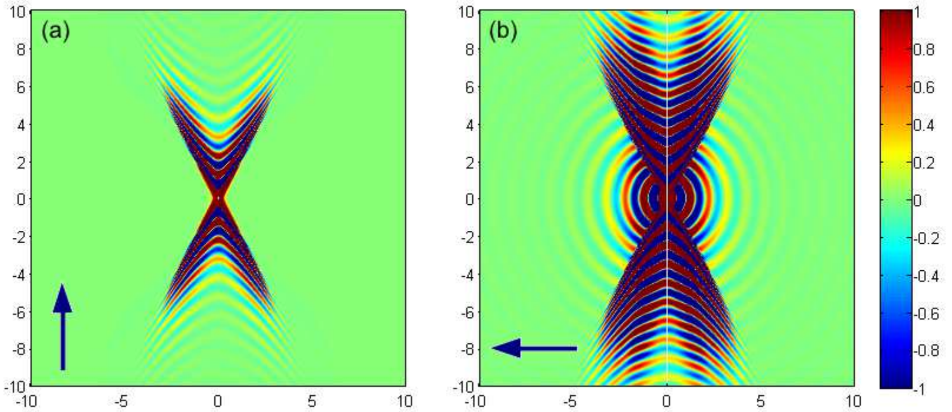

Fig. 1: Real part of electric field along the point dipole in hyperbolic medium with the dipole moment (a) parallel to the axis of anisotropy, (b) orthogonal to the axis of anisotropy.

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Pattern formation in bistable spaser chains

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It has been shown [1] that similar to a laser [2], a spaser exhibits an optical bistability in the external field. We study the phenomenon describing the spaser operation by the optical Bloch equations. We show that the bistability may exist in the below-threshold spasing regime.

The bistability reveals itself in a possibility of two different paths for evolutions of the plasmon field of the spaser's metal nanoparticle (NP). In a spaser, the inversion of the quantum dot (QD) is determined by the pumping and NP fields [3]. In turn, the NP plasmon field is determined by external and QD fields. If the NP field is sufficiently large, it may cause an increase of the QD inversion. The latter leads to the enhancement of the of the NP field until it saturates. On the other hand, if the NP field is small, then the inversion and the field of the QD are determined by pumping only. In this case, the NP field remains small.

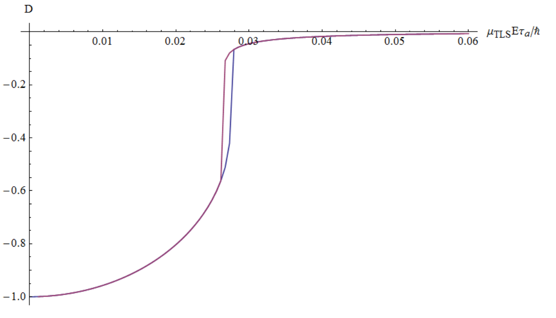
The optical Bloch equation for the spaser [1], [3]–[5] allows one to obtain the following dependence of the intensity, I_{NP}, of the plasmon oscillations in the NP on the intensity, I_{in}, of the external field

$$I_{NP} = T(I_{NP})I_{in}, \quad T\left(I_{NP}\right) = \frac{\left|\beta\right|^2/\tau_a^2}{\left(1 - \frac{D_0\Omega_R^2\tau_a\tau_\sigma}{1+4I_{NP}\Omega_R^2\tau_a\tau_D}\right)^2},$$

where τ_a, τ_σ, and τ_D are relaxation times of NP and QD polarizabilities and the population inversion, respectively, Ω_R is the Rabi frequency of the spaser [3]–[5], $|\beta|^2 = 1/\tau_a^2\Omega_R^2 + \mu_{NP}^2/\mu_{TLS}^2$, μ_{NP} and μ_{TLS} are characteristic values of NP and QD dipole moments. The nonlinear function $T\left(I_{NP}\right)$ plays the role of the transmittance of a nonlinear Fabry–Perot resonator in a theory of bistability [2].

In the present work, we study the manifestation of this effect in a chain of spasers when the external field is perpendicular to the chain so that the wavevector of the incident wave is perpendicular to the chain and the wave amplitude is the same for each spaser [6]. We consider the case when each spaser operates below the threshold of the generation and the mismatch between frequencies of the external field ω and spaser self-oscillation ω_a is arbitrary.

We examine the dynamics of the spaser chain. We show that in our system depending on E such a kink may switch the chain from the low D stable state to the higher D stable state. The velocity of this kink depends on the value of external field. For higher values of τ_a, the quasiperiodic dissipative structure emerges in the spaser chain. The numerical simulation shows that the origin of such structure has a self-assembling character.



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Discrete dissipative topological and knotted solitons in bistable magnetic metamaterials

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In nonlinear fields, knotted conservative solitons were introduced by Faddeev and Niemi [1] as knotted lines without self-crossings embedded into a three-dimensional space. Existence of knot solitons as stationary solutions of nonlinear models was confirmed in numerical experiments [2]. In this report, we predict that nonlinear magnetic metamaterials composed of lattices of resonant elements such as split-ring resonators (SRR's) driven by external coherent radiation can support a variety of localized dissipative patterns including stable dissipative knotted solitons. Solitons in dissipative systems are, in general, far more robust than conservative solitons, and thus they are excellent candidates for the first realization of stable knotted solitons in experiment.

We consider a nonlinear magnetic metamaterial comprised of a cubic lattice of weakly coupled SRR's [3]. Such a system exhibits a wealth of nonlinear dissipative phenomena, including bistability, modulational instability, switching waves and 1D- and 2D-dissipative solitons. Taking this model as an example of a general discrete dissipative system, we analyze topological dissipative solitons in such systems and study their stability. The analysis is based on the system of coupled nonlinear ordinary differential equations for slowly varying amplitudes of the electric currents SRR's [3, 4]. In the first order of the perturbation theory we can approximate solitons as binary distributions with the additional corrections due to the nonzero coupling. We study spatially extended chains of linked (coupled) sites, for example a "bright" soliton along a contour on the background of low-amplitude stable magnetization. The stability of each configuration was checked numerically. We demonstrate various topological solitons, including knots (Fig. 1) and structures where excited SRRs are located on the Moebius loop.

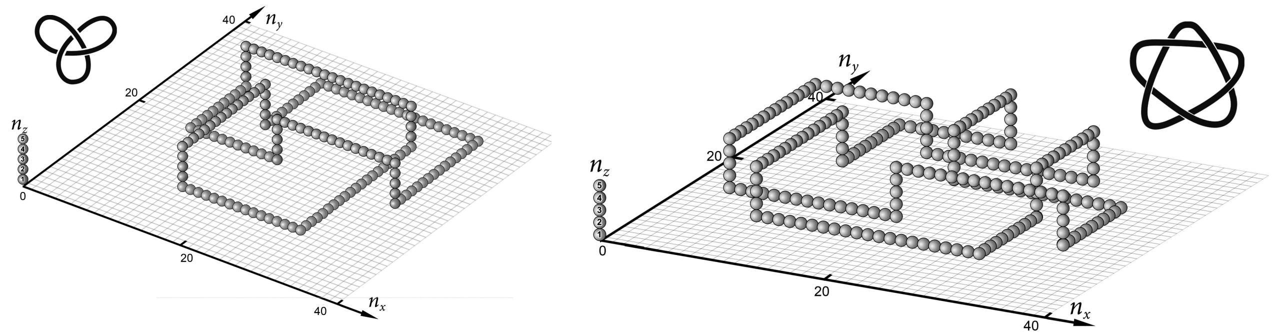

Fig. 1: Knotted discrete dissipative solitons: trefoil (*left*) and figure-eight (*right*).

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Gap soliton characteristics in nonlinear planar Bragg grating structure

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The 2D model of planar nonlinear Bragg grating structure was proposed [1]. The scheme of the structure is shown in the inset of Fig. 1,*a*. The nonlinear Kerr materials were considered. The process of the electromagnetic wave propagation in the nonlinear Bragg grating system has been numerically studied. The possibility of multiple gap soliton generation in this structures was shown by the straightforward numerical modeling of Maxwell's equation. A Finite-Difference-Time-Domain method was used [2].

The transformation of band gap diagram was estimated. The calculation of band gap diagram and estimation of the cut-off frequencies in the periodical structure for the different values of input signal intensities was performed with the FDTD method. The good correspondence between the results of the cut-off frequencies computation and analysis of transmission spectra was shown.

As the amplitude of input signal gets larger the distance between soliton-like pulses decreases and interaction between gap solitons can be observed. The interaction between solitary pulses was studied and discussed.

The efficiency of energy conversion from the energy of continuous wave signal to the energy of pulse signal was computed for different values of the input signal power (Fig. 1,*a*). The maximum value of energy transmission coefficient was equal to 60–70%.

The quantitative characteristics of soliton propagation were obtained. The minimum value of soliton velocity was approximately equal to $0.21c$, where c is the speed of light in vacuum (Fig.1,*b*).

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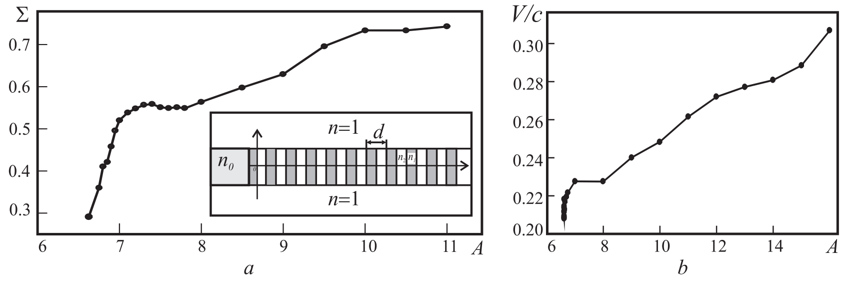

Fig. 1: The efficiency of energy transformation Σ (a) and velocity of solitons' propagation (b) at the different values of amplitude of input CW signal. On the inset – the scheme of periodic structure

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Electrodynamical characteristics of 1d magnonic crystal structure

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In recent years, investigations of the optical, acoustical waves and microwaves propagation periodic structures are of great interest. Due to the possibilities of the formation of complex artificial composite materials (electromagnetic metamaterials) [1] during the last decade the numerical simulation of such structures is very important. In the present work the magnonic crystal (MC) is considered. Such structures are similar to the photonic and acoustic crystals but formed on the basis of magnetic materials in which the magnetostatic waves can propagate. These artificial structures represent different magnetic media in which the magnetic properties are changed periodically. Due to the possibility of propagation of different types of waves these systems can be used in the different signal processing devices such as narrow-band microwave filters or high-speed switches. One of the great advantages of these structures is the possibility to change the wave dynamics by the changing the value and direction of the external magnetic field.

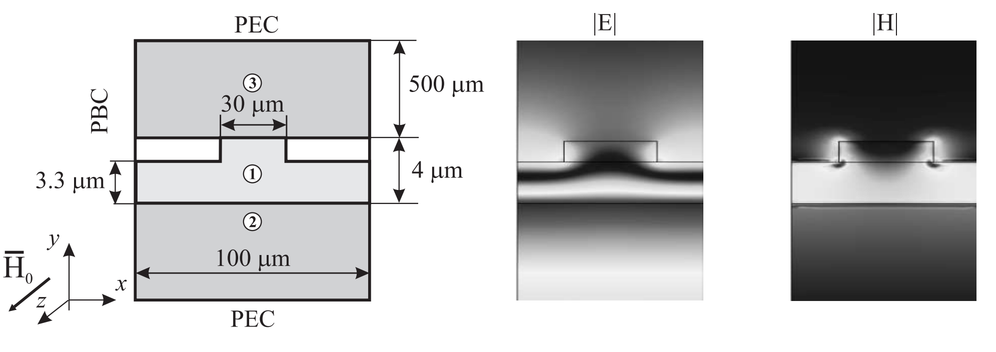

Fig. 1: The primitive cell of 1D MC.

In this work we present the method of computation of electrodynamical characteristics of magnetostatic surface waves (MSSW) propagation in 1D magnonic crystal. In the Fig. 1 the geometrical parameters of primitive cell of considered strucute are shown. Here 1 denotes the yttrium iron garnet(YIG), 2 – gallium gadolinium garnet, 3 – polycor. The period of the structure was 100 microns. The external magnetic field is oriented along the z-axis, the MSSW propagates along x-direction. In this picture also the electric and magnetic field distribution for the definite value of frequency and longitudinal wave number are depicted. The magnetic field is localized in the YIG film and mainly in the regions of the film nonuniformity joints. The frequency ranges of the first and second band gap region obtained from the simulation were compared with the experimental results for the MSSW transmission spectrum in the MC. The detailed study of magnetic and electric field distribution in the primitive cell of periodical structure was provided. The non-reciprocity phenomena was discussed. It was shown that the velocity of MSSW propagation in the highly anisotropic structure(1D MC) was coincided with the group velocity which was derived from the dispersion characteristics. The numerically obtained group velocity values are in good correspondence with the experimental results. The proposed method was based on the vector form of finite element model [2] and can be used for the calculation of the electrodynamical characteristics of highly anisotropic periodic structures with arbitrary geometry of primitive cell.

This work was supported by the Grants (projects no. 2010-1.2.2-123-019-002, 11.G34.31.0030).

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FMR investigation of bicomponent magnonic crystals based on cobalt and Permalloy

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Due to possible application in microwave signal processing artificial materials with periodic spatial modulation of the magnetic properties called magnonic crystals (MC) attract a strong scientific interest (for instance [1]). As a rule, MCs are fabricated from ferrite based structures and only recently results of Brillouin light scattering measurements for periodical structures composed from alternating cobalt (Co) and Permalloy (Py) nanostripes and nanodisks were reported [2].

In this work MCs based on Py and Co with micron sized 1D and 2D elements were investigated by method of cavity ferromagnetic resonance (FMR) at 9.8 GHz for various orientations of the external in-plane magnetic field. Such bicomponent structures were fabricated using magnetron sputtering, photolithography, ion etching.

Measured FMR spectrum showed the presence of additional responses between strong ones corresponding to the uniform FMR in Co and Py elements (see fig. 1). Those additional responses could be attributed to the formation of the standing spin-wave (SW) resonances across the width of the stripes and 2D elements similar to the peaks observed for the tangentially magnetized separate stripes [3]. Such standing SW for Py elements in geometry of magnetostatic surface waves (MSSW) occurred because above the uniform FMR field for Co MSSW can propagate only in Py so that the wave is reflected from boundaries between Co and Py elements and localized in Py. In contrast, in geometry of backward volume magnetostatic waves (BVMSW) localization of SW occurs in Co elements. Performed calculations neglecting the shape anisotropy effect showed a good agreement with the experimental results and, thus, proved the supposition.

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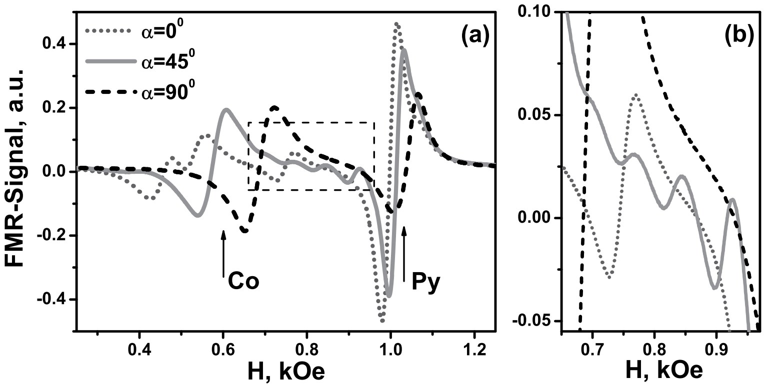

Fig. 1: FMR results for bicomponent striped structure (a) and its zoomed in central part (b) for the various orientation of the in-plane magnetic field with respect to stripes (α). The stripes width is 6.5 and 3.5 μm for the Co and Py, respectively.

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Gain-induced compensation of losses in metal-dielectric metamaterials

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In recent years, we observe a growing interest to study metal-dielectric (MD) multilayered metamaterials, including the multilayered structures with the effective hyperbolic dispersion [1]. However, the use of such structures at optical frequencies is significantly limited due to losses which result from the constituent metal layers. The most promising way to approach this problem is to incorporate gain media in dielectric layers [2, 3]. So far, for such simple single-periodic MD metamaterials, the loss compensation was claimed to be achieved for infinite structures [4].

Here we present the results of the numerical modeling of active MD metamaterials of a finite thickness. We demonstrate that, in contrast to passive finite MD structures with a positive imaginary part of its modal indices (which means absorption), several modes of active system (with gain) can have a negative sign of the imaginary part of their modal indices (which means amplification). As the finite MD structure can have a large number of modes, some of them have large values of the imaginary part of their modal indices, so even using the largest gain coefficient that can be realized in dyes or semiconductor gain, only partial loss-compensation can be achieved in this case. In Fig. 1 we compare two cases: a system only with losses and a system with losses and gain. Gain has a spectral range approximately in the area where $\mathrm{Im}(k_x/k_0)$ is negative in Fig. 1(b); k_x is the x-component of the wave vector $\mathbf{k}$, and the axis x is directed along the layers. As Fig. 1(b) illustrates, there is a possibility of compensation of losses for several modes simultaneously, while other modes remain attenuated.

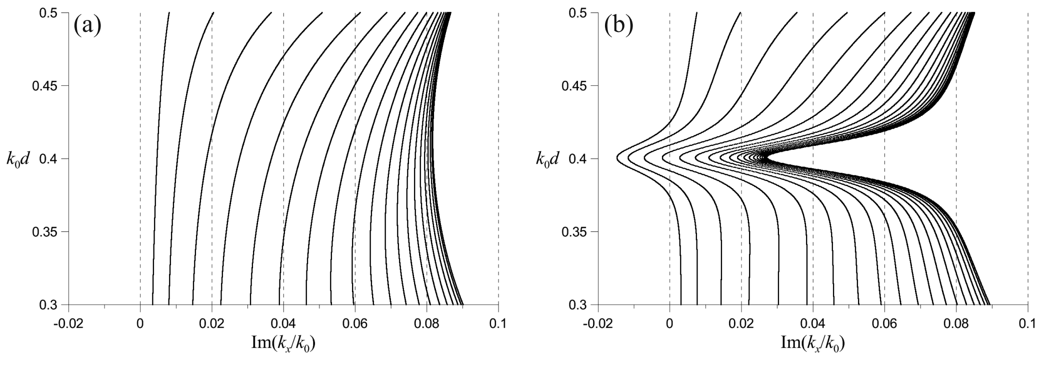

Fig. 1: Dispersion diagrams of the finite MD structure consisted of 20 periods (40 layers). (a) structure with losses, (b) structure with losses and gain.

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Controlling nonlinear properties of metamaterials

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The effective properties of metamaterials depend both on the properties of the individual split-ring resonators (SRRs), and on the arrangement of the SRRs in the lattice, as strong interactions occur between the rings in a lattice due to their complex near-field patterns. This allows us to significantly control the properties of the metamaterial by changing the internal structure of the composite material [1].

The linear properties of individual metamaterial elements can also be controlled by an external signal, by inserting *nonlinear inclusions*. The nonlinear response of metamaterials can be much stronger than that of natural materials, due to their resonant nature, and the local field enhancement which occurs at certain "hot spots". The strength of the local field depends on the design of the resonator, and the choice of an optimal location to place the nonlinear element. However it also depends strongly on the coupling of the wave to the external field, which along with losses determines the quality factor of the resonator. Since modifications of the lattice parameters also influence the quality factor of the resonances, they should affect the local-field enhancement.

In this talk we study *control of the nonlinear response* of two broadside-coupled split-ring resonators by modifying the offset between them, and explain this response by studying how shifting the resonances changes the maximum currents in the resonators.

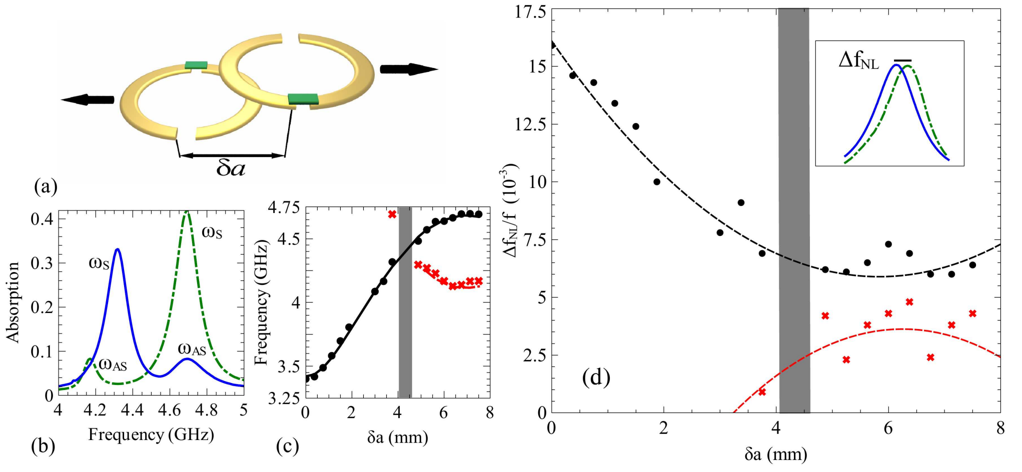

Fig. 1: (a) Schematic of the lateral shifting of SRRs with inserted diodes; δa is a shift between two rings. (b) The measured absorption curves for $\delta a = 3.75$ mm (blue solid) and 7.5 mm (green dashed), at -20dBm input power. Symmetric (ω_S) and antisymmetric (ω_{AS}) modes are highlighted. (c) Resonant frequencies of symmetric (black circle/solid) and antisymmetric (red cross/dashed) resonances, as determined both experimentally (markers) and numerically (lines) for the linear case. (d) Relative nonlinear shift in symmetric (black circles) and antisymmetric (red crosses) resonant frequencies for shifts (δa).

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Plasmonic accelerator for nano-particles

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Currently, interest is on the rise for the development of different ways for manipulation of nanometer- and sub-nanometer-sized objects. One of the promising methods of nano-object manipulation is the optical one based on utilization of electromagnetic forces. In the work of Ashkin [1], it was first demonstrated that a highly focused laser beam can be used for capturing and manipulation of micro-particles. The limit of application of this method in biological and medical areas is determined by the maximum allowed field intensity that is safe for live objects. The intensity of the radiation affecting the objects can be reduced if one uses plasmonic structures. Such structures can create a field enhancement in a limited space around metallic nano-objects that allows one, for example, to effectively hold particles inside optofluidic devices [2].

In the present Report, we suggest a model of an optical nano-accelerator that uses plasmonic focusing of light on dimensions much smaller than a wavelength and allows for the acceleration of nano-objects to the velocities of tens of centimeters per second in air using low electromagnetic fields on the order of $1 \div 2$ mW/μm^2. The accelerating field is highly localized that allows us to use this method in biological and medical applications. We suggest the use of nano-dimensional V-grooves as a nano-accelerator (Fig. 1). Thus, our method of nanoparticles acceleration is expected to possess directionality and high spatial accuracy. V-grooves are known to have high enhancement of the field near the bottom under the certain combination of parameters (the wavelength of the incident field and the depth and the opening angle of the groove) [3]. For a particle to be ejected from the region of high field at the bottom of the V-groove, it should have negative polarizability. We achieved negative polarizability in the visible region by using composite particle with a metal core and a shell made of material with high refractive index.

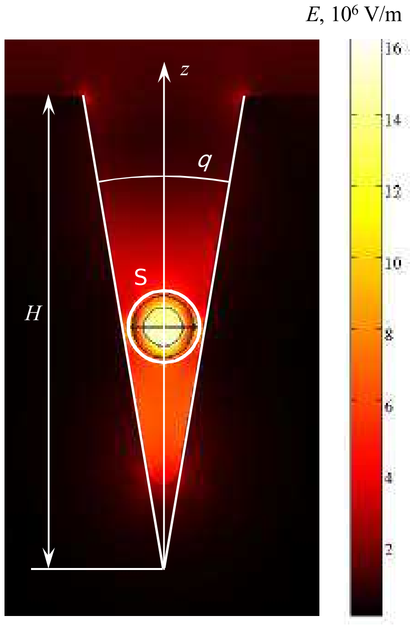

Fig. 1: Calculated distribution of the electric field strength E in a V-groove containing a composite nanoparticle with silver core and a TiO_2 shell. The optical force is calculated by integration of the Maxwell stress-tensor over surface Σ.

Using full electromagnetic calculations (finite element software Comsol Multiphysics), we determined the speed of the particle at the exit of the groove. The maximum velocity of the particle (about 11 cm/s for $E_0 = 10^6$ V/m excitation field) is achieved with a certain set of parameters that tells us about the resonance behavior of the phenomenon.

We show that the V-groove in a metallic surface allows the field to perform positive work over the nano-particle and implement a "gunshot" effect, accelerating the particle up to the speeds of tens of centimeters per second in air. The estimates for the speed and energy of particles accelerated in the discussed groove show larger magnitudes than the ones obtained earlier by other optical methods ($10 \div 10^2$ μm/c).

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Microwave pulse passing through 1D finite magnonic crystal

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In the last years, nonlinear effects related to propagation of magnetostatic waves (MSW) through a delay line based on one-dimensional (1D) magnonic crystal (MC) are actively investigated. For example, a gap solitons formation on surface MSW (MSSW) were observed in MC based on yttrium iron garnet (YIG) films at frequencies above 4 GHz in conditions of four-wave interactions [1]. The investigation of intense wave packets propagation through finite MC at frequencies less than 3 GHz, where three-magnon (3M) decay processes of MSW are allowed, are of great interest. MSW delay line with microstrip antennas was used for measurement. The distance between them was 4 mm. MC was made as a periodic system of etched grooves at the surface of a YIG film with the thickness 4 μm. The grooves have the depth – 0.7 μm, the width – 70 μm and the period – 100 μm. The rectangular microwave pulses with power P_{in} were supplied to the input of the delay line; the pulse duration was 260 ns. P_{in} was changed from -10 dBm to 16 dBm. The carrier frequency of pulses lies inside the first band gap of the MC. Two pulses were observed in the time series of the output signal. These pulses were generated by fronts of the input pulse. It was observed in the whole range of input power. The peak power of these pulses depends linearly on the power of the input signal and their duration is determined by the length of the fronts of the input signal and MSSW dispersion near the carrier frequency. If the value of the power P_{in} of the input signal more than threshold value ($P_{in} > 3$ dBm) an additional pulse is formed between two pulses mentioned above (Fig. 1a). The duration, the peak power and delay time of this new pulse are determined by the power of the input signal. In this case the nonlinear dependence of these values on P_{in} is observed. The dependence of the pulse peak power P_0 on value $1/T_0^2$ is shown in Fig. 1b. T_0 is the pulse duration measured at half the peak amplitude. A linear relationship between these variables may be evidence of the formation of gap soliton [2]. The model of pulse propagation through finite MC based on coupled nonlinear Schrödinger equations was constructed for theoretical investigation of gap soliton. Good agreement is found between the experimental data and numerical results obtained using the constructed model.

This work was supported by the Grants (projects no. 2010-1.2.2-123-019-002, 11.G34.31.0030).

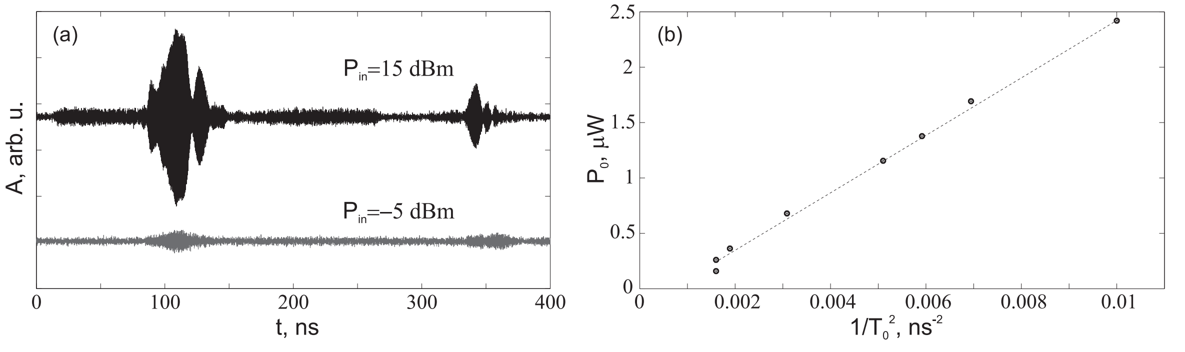

Fig. 1: The amplitude of the transmitted signal at several levels of input power (a); the dependence of the pulse peak power on $1/T_0^2$ (b).

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Metamaterials with extreme parameters: diffraction-free propagation of light and electron waves

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Electromagnetic metamaterials with unusual physical responses have attracted a lot of attention after some seminal works [1] demonstrated that by introducing a new length scale in conventional metals and dielectrics it is possible to mold the flow of light to an unprecedented extent. Even though most of the efforts of the scientific community were initially concentrated on the realization of materials with simultaneously negative permittivity and permeability, other classes of materials — with extreme effective parameters or extreme anisotropy — can lead to exciting new physical phenomena.

In this talk, I will present an overview of our ongoing research work on electromagnetic and semiconductor metamaterials with extreme parameters. I will discuss the tunneling of electromagnetic waves through ultranarrow irregular channels filled with metamaterials with permittivity near zero [2–3], and highlight the unusual potentials of metamaterials with either extreme anisotropy or very large index of refraction [4–6]. In particular, I will show how the extreme anisotropy property can enable waveguiding with no diffraction, and illustrate how this property can be instrumental to boost the interaction between electromagnetic waves with matter, and permit quantum levitation [7] and a dramatic enhancement of the Cherenkov emission by a beam of charged particles [8]. Finally, I will discuss the exotic physics and potential applications of novel quantum metamaterials based on graphene and semiconductor superlattices, wherein the electron waves are super-collimated along a preferred direction of motion [9–10].

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Effective-medium approach to electron waves in graphene superlattices

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Despite some fundamental differences between photons and electrons, such as mass, spin and statistics, there are many formal similarities between photonics and electronics [1, 2], which ultimately result from the wave-particle duality of fundamental particles. For example, an electron is characterized by a de Broglie wavelength and thus can interfere with itself, analogous to interference phenomena in classical electromagnetism. However, the theoretical frameworks typically adopted to describe wave propagation in electromagnetic media and electron waves in semiconductors are usually rather different. Macroscopic electrodynamics is based on the idea of averaging out the strong fluctuations of the microscopic electromagnetic fields in the vicinity of the polarizable particles and on the introduction of effective parameters, so that the dynamics of the wave propagation is formulated in terms of macroscopic fields that vary slowly on the scale of the microscopic unit cell [3]. Quite differently, the computation of the electronic structure of semiconductors is typically based on perturbation methods, usually designated by k·p methods.

Here, as a generalization of our previous studies in the context of electromagnetic metamaterials [3], we propose a general effective-medium framework based on the Hamiltonian formalism that enables describing macroscopic excitations in both electromagnetic metamaterials and semiconductor and graphene superlattices [4]. We apply the method to the case of graphene superlattices, and prove that these structures may be described using simply two effective parameters: a dispersive potential, and an anisotropy tensor that characterizes the pseudospin. Our model predicts that a graphene superlattice characterized by an indefinite anisotropy tensor may permit the perfect tunneling of all the stationary states with a specific value of the energy when it is paired with a dual graphene superlattice with positive definite anisotropy tensor.

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Enhanced radiative heat transfer at microscale in the near infrared

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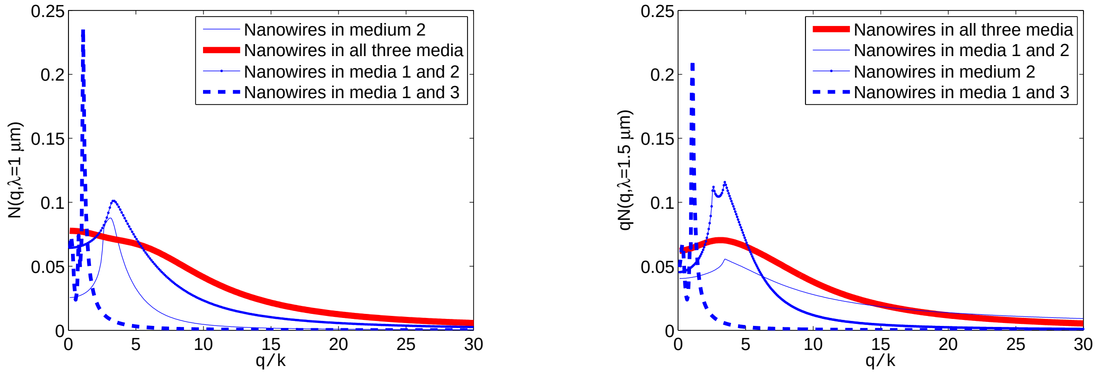
Abstract. This paper continues our previously developed paradigm of the dramatic enhancement of radiative heat transfer at microscale due to the conversion of evanescent waves into propagating ones in the gap separating the hot body from the cold one. This conversion happens due to the presence of the indefinite metamaterial in this gap. Since the study is oriented to thermo-photovoltaic (TPV) applications the metamaterial must ensure the negligible thermal conductive transfer (as compared to the radiative one). This design problem has been solved in our precedent studies for low temperatures i.e. for the mid-infrared frequency range. Here we suggest and investigate the new design solutions for high T i.e. for the near-infrared range. The obtained results promise a technical breakthrough.

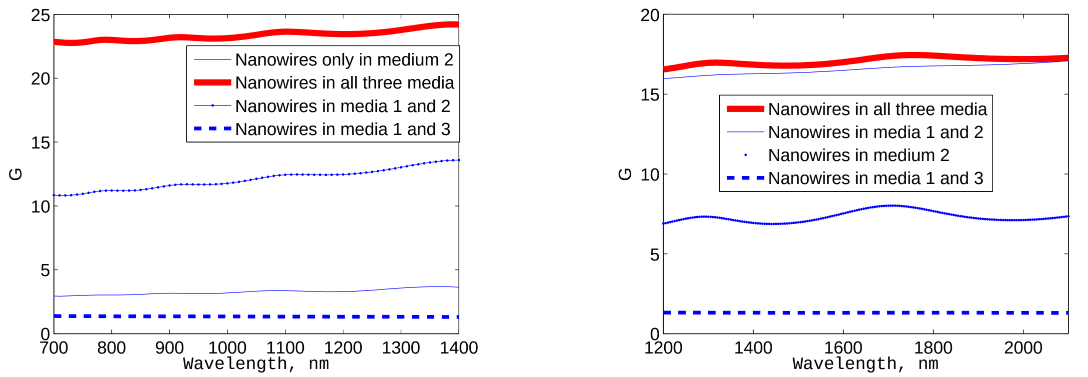
Introduction. Usual TPV cells absorb far fields radiated by the remote hot surface i.e. harvest the radiative part of the waste heat. The operation principle of so-called near-field TPV systems (NF TPVS) is based on the use of the evanescent spatial spectrum, i.e. on the use of the energy of infrared fields stored at nanometer distances from the hot surface. Then the surface of the hot body is modified by the presence of a special superstrate. The radiative heat transfer in NF TPVS is non-comparably larger than that in usual TPV system with a non-optimal distant radiator. Unfortunately, any NF TPVS suffers strong principal drawbacks which restrict their applicability and make them not very prospective for the industrial use. So-called micron-gap TPV systems (MGTPVS) in which the gap d between the cold (photovoltaic) and hot (radiating) surfaces is still smaller than the operational wavelengths but comparable with them occupy an intermediate place between conventional TPVS and NF TPVS. Recently, we have suggested a new approach to the design of MGTPVS which will make such structures promising for the large-area waste heat conversion. In our MGTPVS the radiative thermal transmission is increased by filling the gap with a layer of so-called indefinite medium. We have shown [1] that it can be implemented in the mid-infrared range with carbon nanotubes whereas the thermal conductance is avoided due to the interdigital arrangement. In the present paper we explore the same idea but strongly develop it with the purpose to optimize the design for the near infrared frequency region which is much more prospective for the waste heat transfer.

Numerical Example. With Figs. 1 and 2 we theoretically demonstrate the dramatic impact of tungsten nanowires to the radiative heat transfer between a hot medium (1) and a cold medium (3). Nanowires can be grown in both these media and are partially free-standing in the air gap between them. This air gap modified by the presence of nanowires is denoted as the medium 2. All media in presence of nanowires in them were modelled as indefinite dielectric materials. The definitions of the heat transfer function and its spatial spectrum shown in these figures can be found e.g. in [2] or [3].



(a) (b)

Fig. 1: Spatial spectrum of the heat transfer function between medium 1 (SiC) and medium 3 across the gap $d = 2$ μm versus dimensionless spatial frequency (q/k). (a) – Medium 3 is CIGSS. (b) – Medium 3 is Ge. Four design solutions have been studied, volume fractions of nanowires have been optimized for every design separately.



(a) (b)

Fig. 2: Gain in the heat transfer function versus wavelength λ for the four design solutions with nanowires compared to that in the case when nanowires are absent. (a) – Medium 3 is CIGSS. (b) – Medium 3 is Ge.

The absolute maximum of the spectral density of radiative heat transfer function (that equals to 0.25 [3]) is better approached for the design solution with nanowires in media 1 and 3 and without them in the gap. The mechanism of this enhancement is the coupling of two surface-plasmon polaritons at the interfaces [3]. This mechanism keeps for other design solutions. However, for micron gaps it has a weak impact since corresponds to a quite narrow-band spatial resonance. The really significant enhancement of the heat transfer (two order over the black-body limit) is achieved using the evanescent-to-propagating conversion of waves. The best design solution corresponds to nanowires located in all three media with different volume fractions. The impact of nanowires makes the heat transfer to the p-doped CIGS (this material has the highest efficiency of the photovoltaic conversion in the near IR) better than that to the p-doped germanium which has been previously considered as a best material for MTPVS. This result promises an additional improvement of the electric yield.

Conclusion. In this study we have shown that the radiative heat transfer in prospective MTPVS operating in the near IR range i.e. at temperatures $T = 1000 - 2000^\circ$ can be increased more than one order due to the introduction of nanowires into the system. This effect means a potential breakthrough in the field of the waste heat energy harvesting since promises a very high energetic issue.

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Spiral particles for constructing nonlinear metamaterials

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In this talk, we report a significant experimental improvement in the development of novel structural units for creating nonlinear metamaterials — spiral resonators [1]. From an electromagnetic point of view, it is a resonator with chiral properties, and from a mechanical point of view it is a spring. The nonlinear behaviour of metamaterials based on spiral elements can be explained as its resonance frequency depends on the spiral pitch. When an incident electromagnetic wave induces a current along the spiral conductor, it produces an attractive force between the spiral windings. The spiral is then compressed and the resulting change in spiral geometry shifts the resonance frequency.

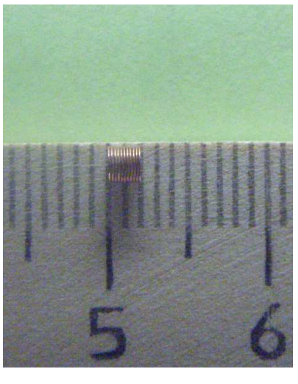

Fig. 1: Photograph: the spiral element.

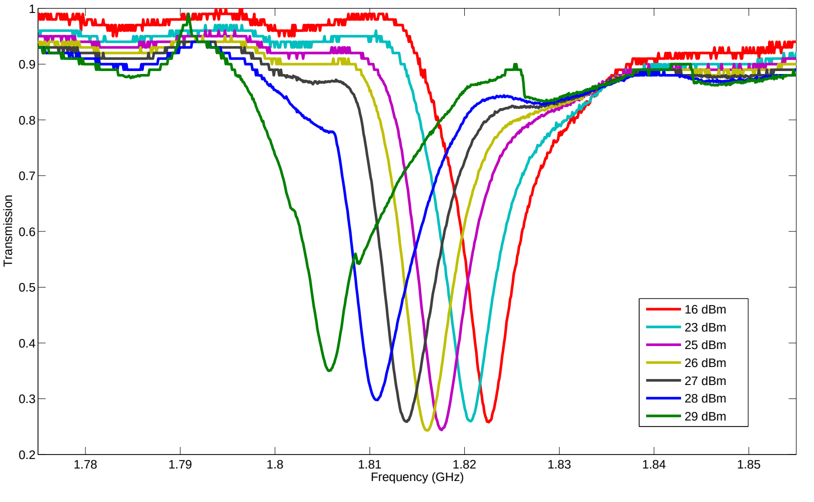

Fig. 2: Experimental result: the curves for the various measured power.

For the experiments in the microwave range, we fabricate spirals with several windings (Fig. 1), which provide a resonant response to an incident electromagnetic wave with appropriate polarization (with the magnetic field being parallel to the spiral axis). To achieve an efficient mechanical response, we use a thin wire and arrange the windings of the spiral close to each other. By optimizing the geometric parameters, we obtain a configuration where mechanic response of the spiral dominates over its thermal expansion. We demonstrate a significant shift of the resonance frequency depending on the incident power (Fig. 2).

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Asymmetric transmission through a structure consisting of two photonic bandgap materials

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Asymmetric transmission through a photonic structure in two opposite directions may result from spectral nonreciprocity associated to breaking the time reversal symmetry [1]. However, also in linear and non-magnetic photonic structures it is possible to achieve a different transmission of optical intensity, depending on the direction of incidence. This happens, when higher order diffraction orders are included in the comparison of the energy transfer balance in the opposite directions of the structure, and the diffraction efficiency for these orders depends on the side of incidence [2]. The system is obviously reciprocal in this case, however the boundary conditions are not symmetric.

In the present paper, we present the numerical analysis of such a structure consisting of two orthogonally oriented one-dimensional photonic crystals consisting of metals and insulators. One of the structures is responsible for supporting a sufficient value of the transverse k-vector to the normally incident light, while the other structure supports directed refraction [3]. The refracted beam propagates in the second medium without diffraction in a similar way as is considered by many authors [4, 5, 6].

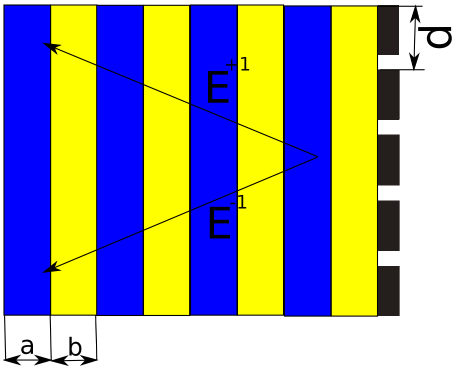

Fig. 1: Schematic view of proposed structure. On the left metamaterial supporting directed refraction, on the right diffraction grating with first order of diffraction matching prefered direction of metamaterial.

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Nonlinear-dispersive interactions of optical pulses in metamaterials

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We present results of numerical simulations and analytical theory of nonlinear interaction of optical pulses under a mismatch and group velocity dispersion in metamaterials. Nonlinearity provides the phase cross-modulation by the pump pulse, and the GVD — the change in frequency and velocity of the signal pulse. Nonlinear-dispersive interaction can lead to total internal reflection of the signal pulse from the pump pulse [1], the discrete diffraction [2] and the Bragg reflection in the moving induced lattice.

In this report two approaches for fast and slow waves are discussed. In the first description of the dispersion law is taken into account in the form of a quadratic dependence on the frequency of the wave number: $k(\omega) = k(\omega_0) + k_\omega(\omega - \omega_0) + 1/2\, k_{\omega\omega}(\omega - \omega_0)^2$. In a cubic medium signal pulse propagation in a given field of the pump pulse is described by the equation

$$\frac{\partial A_2}{\partial z} + \nu_{21}\frac{\partial A_2}{\partial \tau} = ik_{\omega\omega 2}\frac{\partial^2 A_2}{\partial \tau^2} - ik_{20}n_2\left|A_1(\tau)\right|^2 A_2 \tag{1}$$

where $\nu_{21} = u_2^{-1} - u_1^{-1}$ is the GVM. The signal pulse can totally reflect from the pump pulse when [1].

$$|\nu| < \nu_{cr} = \sqrt{2\, k_{\omega\omega 2}k_{20}n_2E_1^2} \tag{2}$$

The reflected pulse changes its frequency and speed (group velocity mismatch changes sign).

The interaction of the signal pulse with a train of pump pulses can result in the discrete diffraction by moving induced lattice [2]. With increasing pump intensity modulation depth increases and the number of generated sub-pulses is reduced. As a result, the signal can be captured by two pump pulses in the soliton. If the pump pulses approach each other, then the signal compression occurs — the signal pulse falls into the “black hole”.

We also investigated the Bragg reflection from a moving induced lattice by setting in (1) pump in a sequence of sub-pulses $A_1 = \sum E_n(\tau - nT)$ (see also [4]).

When analyzing the interaction of slow light at $u_j \to 0$ we specify the dispersion as $\omega = \omega(k_0) + \omega_k(k - k_0) + 1/2\,\omega_{kk}(k - k_0)^2$ [3]. In this representation the equation for the MMA takes the form

$$\frac{\partial A_2}{\partial t} + u_{21}\frac{\partial A_2}{\partial \zeta} = \frac{i}{2}\omega_{kk2}\frac{\partial^2 A_2}{\partial \zeta^2} - i\omega_{20}n_2\left|A_1(\zeta)\right|^2 A_2, \tag{3}$$

where $u_{21} = u_2 - u_1$; $\zeta = z - u_1 t$. By analogy with (2) we found from equation (3) the expression for the critical velocity of the total reflection of slow light

$$\left|u_1 - u_2\right|_{cr} = \sqrt{\omega_{kk2}\omega_2 n_2 E_1^2} \tag{4}$$

where one can put a zero velocity of induced inhomogeneity $u_1 = 0$. The signal may approach the pump pulse with an arbitrarily small speed.

The developed theory is applicable to the analysis of nonlinear dispersive interactions in metematerils such as photonic crystals, Bose–Einstein condensate, and optic waveguides.

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Slow-light enhanced optomechanical interactions in photonic crystals

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Development of reconfigurable photonic circuits utilizing gradient optical forces between micro- and nano-scale optical waveguides opens new possibilities for optical signal shaping and routing based on all-optical tuning of the structure geometry [1]. Optomechanical interactions can be enhanced in photonic-crystal waveguides due to increased optical energy concentration in the slow-light regime [2, 3] or through light trapping in cavities [4].

We overview our recent results on engineering slow-light waveguides to enhance opto-mechanical interactions. We discuss a potential advantages of using slow-light waveguides [2, 3] compared to cavities [4] based on the optical bandwidth and scalability with the length of the waveguide. Then, we present analytical and numerical results demonstrating that slow-light enhanced optically induced forces between side-coupled photonic-crystal wire waveguides can be controlled by introducing a relative longitudinal shift [Fig. 1], enabling the tuning of the transverse force from repulse to attractive, and force suppression for a particular shift value [5]. The shift-induced symmetry breaking can also facilitate longitudinal forces, in contrast to unshifted structures where such forces vanish. We also discuss the design of three-wire structure where broadband slow-light regime can be achieved [6].

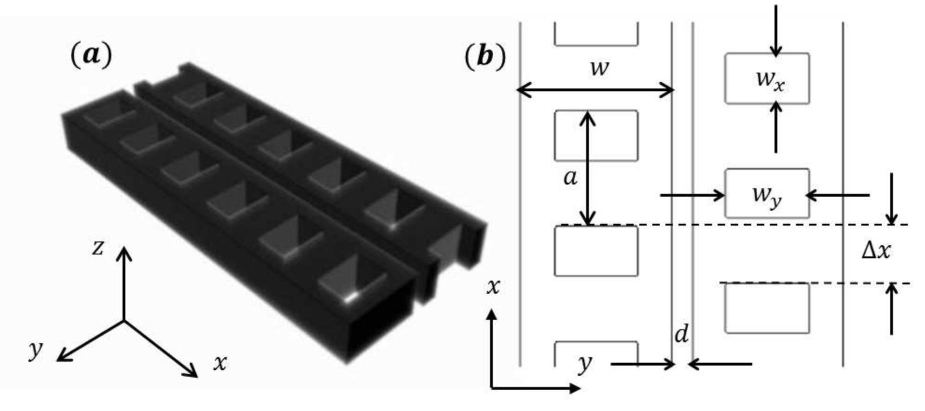

Fig. 1: (a) 3D sketch of the longitudinally shifted side-coupled photonic-crystal waveguides. (b) top view of the structure.

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Application of one-dimensional microwave photonic crystals for measurements of parameters of structures based on thin semiconductor layers

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In development of the technology of producing structures based on thin semiconductor layers (which are used in micro-, acousto- and optoelectronics) the simultaneous measurement of their thickness and electrical conductivity after the production process is done has the outstanding interest.

This work is devoted to investigation of the possibility of application of photonic structures for measurement of thin nanometer semiconductor layer parameters.

One-dimensional microwave photonic crystals are structures with periodic heterogeneity. If one creates an irregularity which disturbs periodicity, the resonant feature is appeared in forbidden photonic gap [1], which is called an impurity mode resonance. The semiconductor layer with sought parameters can play the role of such type of irregularity.

The determination of thickness and electrical conductivity of semiconductor layers is connected with the solving of the inverse problem [2]. The inverse problem solving is based on using frequency dependences of reflection $R(f)$ and transmission $D(f)$ coefficients of microwave radiation interacting with investigated structure inserted into photonic crystal.

With this end in view we offer to carry out measurement at two different temperatures or at fixed temperature at two different positions of heavily doped semiconductor layer on semi-insulating substrate inserted into photonic crystal.

In this case to determine the thickness t and the electrical conductivity σ of the sample under investigation from the frequency dependences $D(f)$ and $R(f)$ one can use the least squares method. In this method one is searching for the such value of parameters t_S and σ_S at which the sum $S(t_S, \sigma_S)$ of squares of differences of calculated and experimental values of the transmission and the reflection coefficients measured at two different temperatures of the sample or at two different positions of thin heavily doped semiconductor layer on semi-insulating substrate inserted into photonic crystal becomes the minimal one.

Computer simulation of one-dimensional waveguide photonic crystal consisted of 11 layers was carried out in the frequency range from 8 to 12 GHz. The odd layers were ceramic (Al_2O_3, $\varepsilon = 9.6$), the even were air gaps ($\varepsilon = 1$). The thickness of odd layers was 1 mm, the thickness of even layers was 12 mm.

The disturbance was implemented as the change of the thickness of the 6$^{\text{th}}$ (central, aerial) layer and the insertion of the two-layer semiconductor structure into this layer. This results in the appearance of low-loss transmission window in the forbidden gap of photonic crystal. The thickness of the disturbed 6$^{\text{th}}$ (air) layer was 2 mm. The thickness of thin heavily doped semiconductor layer in the investigated structure was 100 nm, the electrical conductivity was 1000 $\Omega^{-1}m^{-1}$. The thickness of semi-insulating substrate was 100 μm, the electrical conductivity was 0.1 $\Omega^{-1}m^{-1}$.

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Superconducting metamaterials

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Superconductors can be used for designing metamaterials with extremely low losses, well-controlled nonlinearity, and frequency tunability. Moreover, superconducting metamaterials offer an intriguing and unique possibility of exploring the quantum effects using Josephson quantum circuits. The design flexibility of superconducting thin-film circuits allows for utilizing small sizes down to the nanoscale while maintaining low loss properties. Superconducting metamaterials can be composed of frequency-tunable resonators and Josephson junctions that are characterized by low dissipation and tunability at microwave and millimeter wave frequencies. In the talk, I will present experiments made to date and discuss possible realization of quantum metamaterials based on arrays of superconducting qubits which can be viewed as artificial two-level atoms.

Plasmonic and quantum plasmonic enhancement of magneto-optics

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We consider two scenarios of magneto-optical transmission lines built on the base of an array of plasmonic nanoparticles. In the first case, it is a well-known transmission line of the plasmonic nanoparticles embedded into magneto-optical medium. It is shown that the propagation of the dipole-plasmonic mode travelling along the array is accompanied by rotation of the polarization (the Faraday effect). The angle of rotation is dozens of times greater than one in the uniform MO material. In the second case, we consider a chain of metallic nanoparticles exhibiting MO properties embedded into dielectric matrix. In the case of passive matrix the Faraday rotation enhancement due to small propagation length of the dipole-plasmonic mode is small in spite of the surface plasmon resonance. In the case of an active matrix the metal nanoparticles are surrounded by the active centers (quantum dots or active molecules) and build up MO spasers. The dipole-plasmonic mode in such an array of MO spasers exhibits high values of the Faraday rotation and propagation length.

Defect modes in 1D ferrite magnonic crystals

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One of the interesting issues is how the magnonic gap $\Delta\omega$ is affected by various kind of imperfection which inevitably exist in the real magnonic crystals due technological imperfections or specifically introduced for modificitation of the magnonic spectra. Some of possible discrepancies from ideal periodicity like partial randomization [1]; modulation of several parameters instead of one; nonuniformity in distribution of dissipative parameters; quality, profile and finite thickness [2] of boundaries between regions with different magnetic parameters [3] was discussed theoretically. For experimental investigations the 1D MC with defects of periodic structure were made from YIG film with thickness h $\approx$ 14,7 μm. Lateral periodic structure $\lambda \approx 150$ μm was an array of grooves (or stripes) etched on the depth $\delta h \approx 1.8$ μm and with width $w_{gr} \approx 80$ μm (width of the stripes w_{st} $\approx$ 70 μm). We have investigated several types of 1D MC with structural defect in a form of single groove (or stripe) with width L_d different from that in periodic structure — see insets to fig. 1. In case of MSSW propagation in MC with defects $L_D \approx 150, 300, 600$ μm in width the defect mode (DM) in the first BR gap systematically appeared — see fig. 1 (curve 2). For higher order Bragg resonances appearance of the defect modes was not so evident like for the first stop band. For defects with width $L_D \approx 230, 215, 545$ μm that is not a multiple of the period Λ the defect modes were not detected.

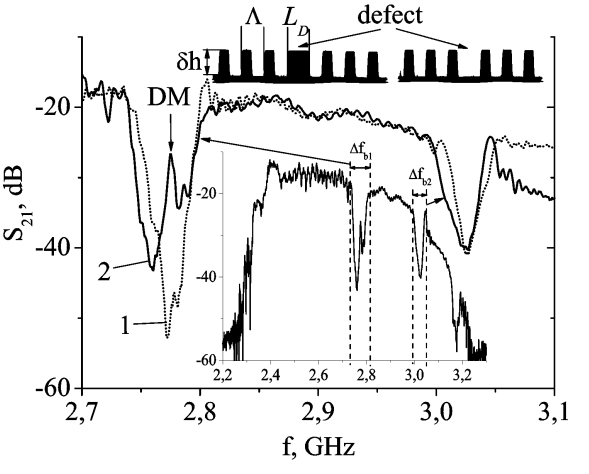

Fig. 1: Frequency dependence of transmission coefficient S_{21} for 1D YIG MC without (curve 1) and with structural defects (curve 2).

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The hybrid waveguide

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The concept of hybrid plasmonic waveguide was proposed in order to solve the problem of high optical signal loss in plasmonic based waveguides. This waveguide consists of a high-permittivity semiconductor nanowire embedded in a low-permittivity dielectric near a metal surface[1]-[2]. The dielectric constant of the cylinder, dielectric and metal are ε_c, ε_d and $\varepsilon_m = \varepsilon'_m + i\varepsilon''_m$. The metal-dielectric-semiconductor architecture redistributes incoming light waves into the low dielectric nanogap so that exiting plasmonic mode is localized within the non-metallic region. Thus this approach integrates dielectric waveguiding with plasmonics and offers ultra-small mode confinement and simultaneously low optical loss.

In the present we propose two theoretical methods of solving eigen modes problem for hybrid plasmonic waveguide. We have calculated the eingen modes dispersion relations for a hybrid waveguide by two different approaches. The perturbation theory is proposed to analyze the problem within quasi-electrostatic approximation. We have found the relationship between permittivities and geometric parameters of the system providing propagation of the strongly confined modes. The subwavelength confinement within quasi-electrostatic regime is possible only on conditions $\varepsilon_d < |\varepsilon_m| < \varepsilon_c$ and $\varepsilon_c < |\varepsilon_m| < \varepsilon_d$.

We also give rigorous analysis of coupling phenomena between circular and planar guiding structures. By employing series expansion of the unknown hybrid modes in the terms of known pure-cylinder modes and surface plasmon-polaritons, the dispersion relation and spatial field distribution are calculated from the numerical simulation for the fundamental HPP mode. Our results are in perfect agreement with the ones reported by other authors[1] where the case $\varepsilon_d < \varepsilon_c < |\varepsilon_m|$ is considered. The estimate for the optimum radius provided strongest coupling is found. Fundamental hybrid mode when $\varepsilon_d < |\varepsilon_m| < \varepsilon_c$ is characterized by more high effective index than in the first case. Thus our result suggests the method of achieving stronger localization by using materials which are characterised by sufficiently small negative real part ε_m' of the permittivity.

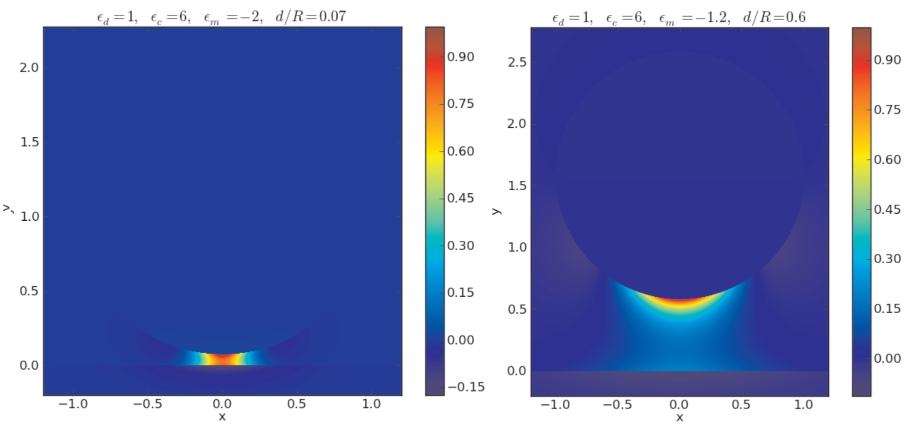

Fig. 1: Time-average z-component of the Poynting vector for the strongly localized fundamental hybrid mode.

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Cherenkov radiation of charged particle bunches in wire metamaterial

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Wire metamaterial represents a periodical volume structure of long parallel wires. In case when wavelengths of the electromagnetic radiation are much greater than the structure period, such material can be described as a medium which possesses both frequency and spatial dispersions. We consider the field of bunch flying through metamaterial perpendicularly to the wires [1, 2]. Analytical and computational investigations are carried out. It is shown that the radiation concentrates in a small vicinity of the determined lines behind the bunch and the Pointing vector is directed along the wires. This phenomenon can be useful for charged bunch examination. Numerical calculations for bunches of different length and width are given. It is shown that the measurements of electrical field intensity and energy flow density allow determining the sizes of the bunch and its velocity.

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Nonlinear subwavelength invisibility cloak

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The problem of invisibility can be solved by means of a cloaking [1]. The complexity of the ideal cloaking caused the origin of variety of simplified versions [2]. One of those is a cloaking of small (compared to the wavelength) objects [3]. It was shown in [4] that a spherical multilayered nanoparticle provides (for definite parameters) the screening of the particle core as well as vanishing of the dipole scattering. Thus, such a core-shell structure displays the cloaking effect.

Similar behaviour is typical also for cylindric core-shell structures, i.e. for the coated nanowires. The analysis of the linear quasistatic problem shows that 1) to screen the wire core we have to cover it with a shell with $\varepsilon_2 \to 0$; 2) to provide the vanishing of dipole scattering we have to cover it with a compensative shell with $\varepsilon_3 = (R_3^2 + R_2^2)/(R_3^2 - R_2^2)$, where R_2 and R_3 — inner and outer radii of the compensative layer (Fig. 1a).

The distinctive feature of this cloak is the effect of the field superlocalization in the screening shell: when the shell width goes to zero, the energy density rises faster than the shell volume decreases. So, in lossless case, an infinitely small volume can store infinite energy.

Significantly localized and increased field in thin screening shell with $\varepsilon \approx 0$ can nonlinearly change the dielectric permittivity in the layer and break up the scattering compensation. These qualitative considerations were confirmed with the numerical modelling, where we used the expansion of TM-polarized field in a multipole series (cylindrical Mie-modes) and iterative algorithm to account for the nonlinearity. Calculations show that there exists some critical intensity (depending on parameters), which leads to an abrupt change in the scattering crossection and also changes the radiation pattern: when the power increases, the forward scattering switches to mainly back scattering (see Fig. 1b).

In conclusion, we have studied the wave scattering on a nonlinear multi-shell nano-particle and demonstrated that in nonlinear regime, the visibility of the particle can be significantly controlled by the intensity of the incident wave.

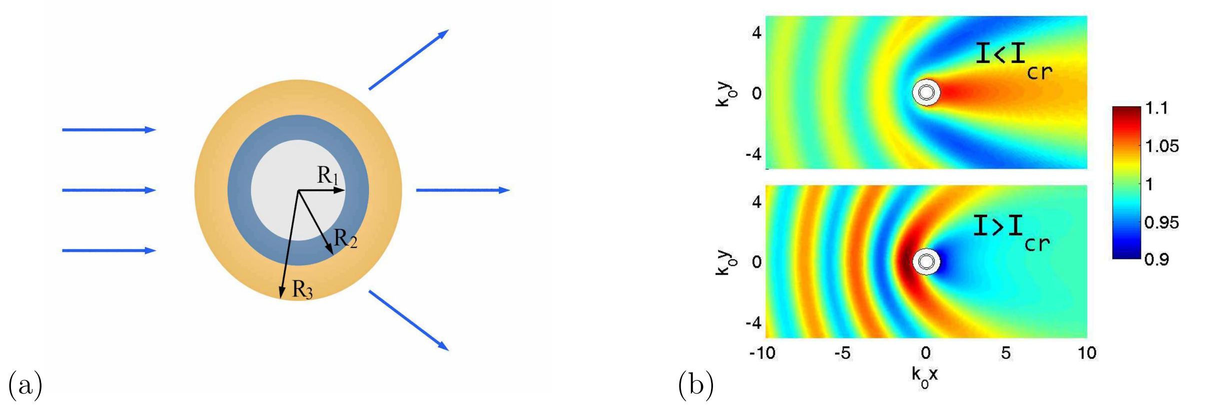

Fig. 1: left – core-shell structure considered; right – nonlinear change of scattering properties

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Radiative properties of 2D array of spasers

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Recently a new idea of self-consistent coherent source of electromagnetic waves, spaser, was proposed [1]. This system generates plasmons localized at nanoparticle by non-radiative excitation from quantum dot. Due to the prospects of loss compensation in metamaterials, a problem considering an array of spasers is of great importance. It is expected that the behavior of the array is significantly different from that of single spaser [2]. We analyze radiative properties of 1D spaser array allowing for both near-field and far-field interactions between the spasers. In particular, we show that the isolation, which is characteristic of a single spaser, disappears with the growth of the array.

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