DAYS ON DIFFRACTION 2013

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ABSTRACTS

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FOREWORD

"Days on Diffraction" is an annual conference taking place in St. Petersburg since 1968. The event is organized in May–June by St. Petersburg State University, St. Petersburg Department of Steklov Mathematical Institute and Euler International Mathematical Institute of the Russian Academy of Sciences.

This booklet contains the abstracts of 196 talks to be presented at oral and poster sessions in 5 days of the Conference. Author index can be found on the last pages.

The full texts of selected talks will be published in the Proceedings of the Conference. The texts in LaTeX format are due by June 16, 2013 to e-mail `diffraction13@gmail.com`. Format file and instructions can be found on the Seminar Web site at `http://www.pdmi.ras.ru/~dd/proceedings.php`. The final judgement on accepting the paper for the Proceedings will be made by the Organizing Committee following the recommendations of the referees.

We are as always pleased to see in St. Petersburg active researchers in the field of Diffraction Theory from all over the world.

Organizing Committee

List of talks

Workshop on meta-materials

Nonlinear Klein–Fock–Gordon equation and Abelian functions

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Exact functionally invariant solutions of nonlinear Klein–Fock–Gordon equation

$$\frac{\partial^2 U}{\partial x^2}+\frac{\partial^2 U}{\partial y^2}+\frac{\partial^2 U}{\partial z^2}-\frac{1}{v^2}\frac{\partial^2 U}{\partial t^2}=V'(U) \tag{1}$$

have been obtained. Here $V'(U)$ is nonlinear function of U, and prime denotes differentiation with respect to the argument. Earlier cases are most studied when $V'(U)$ is a piece of the Taylor series (Landau–Ginzburg equation), Fourier series (sine-Gordon, double sine-Gordon etc.) and exponents (Toda's chain). Solutions of the equation (1) are searched in the form of a superposition $U = f[W(x,y,z,t)]$ [1]. The function $W(x,y,z,t)$ at the same time satisfies to two differential equations with partial derivatives — eikonal's type and wave equation. The function $W(x,y,z,t)$ is found by the method of construction of functionally invariant solutions [2]–[5] and has the form of an arbitrary function of the ansatz $\alpha(x,y,z,t)$. Ansatz can be found as the root of an algebraic equation, and the function $f(W)$ is given as inversion of the integral

$$u(W)=\int_0^f \frac{df}{\sqrt{V(f)+E}}, \tag{2}$$

where E is a constant of integration. For special cases of $V'(U)$, mentioned above, the subradical expression is transformed by suitable change of variable in polynomial. The polynomial degree n depends on length of a piece of the corresponding series and defines the genus p of the integral (2). Integral (2) is elliptic ($p = 1$), ultraelliptic ($p = 2$) or hyperelliptic ($p > 2$). The inversion formulas for the integral (2) of genus p are expressed through theta functions of p-variables. They are $2p$-periodic Abelian functions. The problem of the integral (2) inversion had been solved by outstanding mathematicians of the past (Jacobi, Weierstrass, Riemann) both by their pupils and followers (Göpel, Rosenhain, Prym). Unfortunately, in literature there is no available statement of the inversion algorithm for the ultra- and hyperelliptic integrals. The inversion algorithm for ultraelliptic integral ($p = 2$) is given here. The peculiarities of the nonlinear Klein–Fock–Gordon equation solutions are discussed in dependence of $V(U)$.

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Approximate paraxial solution of the Helmholtz equation, based on lateral diffusion concept

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Analytic methods of the variable wave number Helmholtz equation play fundamental role in the development of theoretical and applied research in optics, acoustics, seismic, electrodynamics, quantum mechanics. The most general analytic approach to solution is multiplicative separation of variables. This method reduces 2D Helholtz equation to the system of two second order ordinary equations. After the works of Robertson (1927) and Eisenhart (1934–1935) on the separating coordiante systems, it became clear that the exact solution can be found only for very view classes of variable wave number. The lack of perspective of solving the equation in the general case turned to be the main reason of decrease of the interest to the “exact” analytic methods. The various asymptotic, high-frequency methods of the wave motion and diffraction approximation became widespread. During the evolution of these methods the heuristic concept of lateral diffusion of Fok and Leontovich (1944–1945) was developed. In the framework of the lateral diffusion concept the solution structure in the boundary layers has the multiplicative form: the product of the 1D transfer equation solution and 1D parabolic lateral diffusion equation solution.

Later, the theory of tip waves [1] allowed to transform the parabolic equation to the Kummer equation, by using the dimensionless bieikonal variable. The variable turns to be the difference between two different eikonals: “diffraction” and “geometrical” one. This techniques allows to parameterize the boundary layers in the lateral direction invariantly with respect to the medium inhomogeneities. In the paper [2] is shown that the analytic solution of 2D Helmholtz equation with variable wave number, called Rubinovitch integral, could be expressed in turms of special functions of bieikonal variables, that are the sum and the difference of “geometrical” and “diffraction” integrals.

We have derived the separation of biekonal coordinates for 2D Helmholtz equation with smooth variable wave number. It is shown that bieikonal coordinates in the local neighborhood of any point correspond to the elliptic coordinates for Helmholtz equation with the frozen wave number in the considered point.

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Problems of diffraction and scattering by strongly elongated bodies

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Problems of high-frequency diffraction by bodies that are very elongated can not be described with usual asymptotic formulae. These problems contain two asymptotic parameters, the usual large parameter $k\rho$, where k is the wave number and ρ is the characteristic radius of surface curvature in the plane of incidence, and the ratio of ρ to the radius of transverse curvature ρ_t. We consider

$$\frac{\rho}{\rho_t} = O\left((k\rho)^{\delta}\right).$$

It was shown in [1] that dependently on the value of δ the two cases are possible. In the case of moderately elongated body, when $\delta < 2/3$, the structure of the high-frequency asymptotics both in Fock domain and in deep shadow is preserved, and the asymptotics can be obtained from the results of [2] by moving correction terms containing ρ/ρ_t to the leading order approximation.

For strongly elongated bodies, when $\delta \geq 2/3$ the structure of the asymptotics completely changes. Classical approach is no more possible because the entire body lies in the Fock domain and locality property is partly lost.

In the approach started in [3, 4, 5, 6] for the description of the diffraction on strongly elongated bodies, the rotational symmetry of the surface was assumed and the incident wave was running along the axis. The derived asymptotic expansions provide a very accurate approximation for the fields which was checked by comparing with test results [7]. Further progress was in the description of creeping waves reflection from the shadowed end of the body [8]. Further we generalized the approach to deal with waves incident at an angle to the axis of the body [9, 10] and with fields of point sources [11]. Recent results deal with far fields [12].

We shall present the formulae for forward and backward scattering amplitudes for acoustic problems. Where possible asymptotic approximations will be compared to results provided by A.A. Kleschev.

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Multifunctionality models of resonant scattering and generation of oscillations for nonlinear layered media

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Nonlinear dielectrics with controllable permittivity begin to find broad applications in device technology and are subject of intense studies in the range of both radio and the optical frequencies. Here we study problems of resonant scattering and generation of waves by an isotropic, nonmagnetic, linearly polarised (E-polarisation), nonlinear dielectric layered structure with a cubic polarisability which is excited by a wave packet consisting of plane waves at multiple frequencies.

We focus on the development of a mathematical model, an effective algorithm and a self-consistent numerical analysis of the multifunctional properties of resonant scattering and generation of oscillations by nonlinear, cubically polarisable layered structures. The polyfunctionality of the nonlinear layered media will be caused by the interference mechanism between interacting oscillations — the incident oscillations exciting the nonlinear layer from the upper and lower half-spaces as well as the scattered and generated oscillations at the frequencies of excitation/scattering and generation. The study of the resonant properties of scattering and generation of oscillations by a nonlinear layered structure with a controllable permittivity in dependence on the variation of the intensities of the components of the exciting wave package is of particular interest. Here we extend our former results and analyse the realisability of multifunctional properties of nonlinear electromagnetic objects with controllable permittivity.

The results of the investigations [1–3]: (i) demonstrate the possibility to control the scattering and generation properties of the nonlinear structure via the intensity of the incident field, (ii) indicate the possibility of increasing the multifunctionality of electronic devices, of designing frequency multipliers and other electrodynamic devices containing nonlinear dielectrics with controllable permittivity.

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Negative eigenvalues of Laplacian for the Y-bent chain of weakly coupled conglobated resonators

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The spectral problem for the Y-bent chain of weakly coupled conglobated resonators is studied. Y-bent system can be described as the central ball linking three chains consisted of balls of the same radius. Two types of coupling are under investigation: the δ-coupling and the δ'-coupling conditions with parameter α in each contact point. Specifically, it is supposed that for each branch there is an axis passing through all sphere's coupling points and these three axes lie in the same plane. And it also supposed that centers of balls that are the closest to the central ball are the vertices of an equilateral triangle. The transfer-matrix approach and the theory of operator extensions are employed to solve the spectral problem for this system. The upper estimates for negative eigenvalues are derived.

Peculiarities of propagation of a plane elastic wave through a gradient layer

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Nonhomogeneous elastic media layers with continuous distribution of density and elasticity modula are often encountered in problems of geophysics. In particular, problems of reflection and propagation of sound through heterogeneous alloys, composite materials and spatially confined porous structures were investigated by a number researchers earlier. Layers with gradient distribution of speed are also encountered in some hydroacoustic problems. Similar layers arise at different depths of the oceans. Sound speed in such layers can change continuously within the wide range.

In this study we investigated the problem of diffraction of an elastic wave by the inhomogeneous isotropic layer with constant elastic parameters in the direction of the axis of a waveguide with continuous distribution of parameters in the section. The case of normal incidence of the wave on the layer corresponds to the one-dimensional diffraction problem, which was considered in [1].

Differential equations for describing the diffraction problem are considered separately for half-planes and for the layer. Problems in the half-planes are overdetermined, which allow establishing a connection between traces of the required functions at media interfaces. Thus, the original problem reduces to the boundary value problem for the Lame system with the boundary conditions of the third type. The Fourier transformation is applied with respect to the variable for which homogeneity of the problem is preserved [2]. The obtained system of ordinary differential equations is solved using the grid method.

Results of numerical calculations show distinctive points of extrema in barrier-transmission coefficients for elastic waves. The differences between the diffraction by homogeneous and nonhomogeneous layers are indicated.

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Heat wave propagation processes in a layer with regard to the heat flux relaxation constant

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A generalization of the classical Fourier's law (1) proposed by Cattaneo [1] and Vernotte [2] includes a heat flux relaxation constant τ, which means that the heat flux does not disappear instantly, but gradually fades if the temperature gradient is suddenly unloaded.

$$\tau\dot{\boldsymbol{h}} + \boldsymbol{h} = -\lambda\nabla T \tag{1}$$

The hyperbolic heat equation (2), which is derived by means of the Maxwell–Cattaneo law, associates the wave propagation of the heat with the finite velocity $c_h = \sqrt{\lambda/(\rho c_v \tau)}$.

$$\rho c_v(\tau\ddot{T} + \dot{T}) = \lambda\Delta T \tag{2}$$

This model, according to [3], is used to describe a variety of processes such as a short pulse laser heating of metals, rapidly moving heat sources or travelling waves in systems with a moving phase transition edge.

In this work the heat wave propagation processes in the layer under the laser irradiation was investigated. The laser irradiation was modelled by defining the heat flow at the layer boundary for the opaque media and by defining the intensity of the internal heat sources according to Bouguer's law for the semi-transparent media. The laser pulse intensity depends on time as the δ-Dirac function, or as the Heaviside function, which allowed us to simulate instant and continuous laser exposure on the medium.

The temperature distributions in the layer with the internal heat sources were obtained as a series by the means of Green's functions method. The behavior of the temperature curves near the irradiated boundary and the reflection of the temperature waves from the opposite boundary of the layer were examined both for the boundary conditions of thermal insulation and for the homogeneous boundary conditions. The temperature curves were compared respectively with the ones obtained by the means of the classical Fourier's theory and the pure wave equation as the limiting cases. The temperature distribution in the opaque medium was compared with the temperature distribution in the semi-transparent medium.

In the case if the homogeneous boundary conditions were considered it was found that the temperature in the vicinity of the irradiated boundary drops below its initial value for a short time period of the order of τ. The numerical analysis showed that for a fixed τ it is possible to find such an optimal value of the laser attenuation in the medium, that the value of the absolute temperature minimum would be the lowest.

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Trace formulas and sharp asymptotics for the fourth order operators

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We consider trace formulas for the fourth order operators. Trace formulas for the second order operators are well known: the potential is expressed in terms of the periodic eigenvalues and the eigenvalues of the Dirichlet problem. Its and Matveev, 1975, used these trace formulas to solve the periodic Korteweg – de Vries equation. Our goal is to obtain similar trace formulas for the fourth order operator. The proof of these formulas is based on the asymptotics of the Fredholm determinant and on the sharp asymptotics of the periodic eigenvalues and the eigenvalues of the Dirichlet problem.

The method of discrete singularity in problems of diffraction by a system of closed cylindrical surfaces

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Let $Ł = \bigcup\limits_{q=0}^{l} L_q$ be the system of the cylindrical surfaces whose generators are parallel OX_3 and

$$L_q : \begin{cases} x_{1q} = x_1(s) + a \cdot q, \\ x_{2q} = x_2(s), \quad s \in [0, L_q], q = 0, 1, ..., l. \end{cases} \tag{1}$$

The scattered field is found in the form of double layer potential [2]:

$$u(y) = \frac{-1}{2\pi}\int\limits_L \frac{\partial}{\partial n_x}G(x,y)\cdot u_f(x)ds_x, \qquad y=(y_1,y_2)\,\epsilon\, C\overline{\Omega}, \qquad \Omega=\bigcup_{q=0}^{l}\operatorname{int}L_q, \tag{2}$$

$$G(x,y) = \frac{\pi}{2i}H_0^{(1)}(\chi|x-y|). \tag{3}$$

Consider the case of H-polarization, for the unknown $u_f(x)$ function we have a system of equations ($u_0(x)$ – incident field), $q_0 = 0,1,...,l$:

$$\frac{\partial}{\partial n_y}u_0(y_{q_0}) = \frac{1}{2\pi}\int\limits_{L_{q_0}}\frac{\partial^2}{\partial n_y\partial n_x}G(x_{q_0},y_{q_0})\cdot u_f(x_{q_0})ds_x + \sum_{q=0,q\neq q_0}^{l}\frac{1}{2\pi}\int\limits_{L_q}\frac{\partial^2}{\partial n_y\partial n_x}G(x_q,y_{q_0})\cdot u_f(x_q)ds_x. \tag{4}$$

These boundary integral equations have the integral with a logarithmic kernel and the hypersingular integral, which must be understood in the sense of Hadamard finite part. Turning to the problem of the approximate solutions, we use the method of discrete features, move to a discrete mathematical model of the problem. We have a system of the linear algebraic equations [1]:

$$\frac{1}{2n+1}\sum_{k=0}^{2n}\left[\frac{\sin^2\frac{n}{2}(\varphi_{0j}^n-\varphi_k^n)}{\sin^2\frac{1}{2}(\varphi_{0j}^n-\varphi_k^n)} - \frac{n\cdot\sin(n+\frac{1}{2})(\varphi_{0j}^n-\varphi_k^n)}{\sin\frac{1}{2}(\varphi_{0j}^n-\varphi_k^n)}\right]\cdot u_f(x_{q_0}(\varphi_k^n)) + \left(\frac{\chi\cdot|L_{q_0}|}{2\pi}\right)^2\cdot$$

$$\cdot\frac{1}{2(2n+1)}\sum_{k=0}^{2n}\left[\ln 2+\sum_{p=1}^{n}\frac{\cos\ p(\varphi_{0j}^n-\varphi_k^n)}{p}\right]\cdot u_f(x_{q_0}(\varphi_k^n)) + \frac{1}{2n+1}\sum_{k=0}^{2n}Q_1(x_{q_0}(\varphi_k^n),x_{q_0}(\varphi_{0j}^n))\cdot$$

$$\cdot u_f(x_{q_0}(\varphi_k^n)) + \sum_{q=0,q\neq q_0}^{l}\frac{1}{2n+1}\sum_{k=0}^{2n}\frac{\partial^2}{\partial n_\varphi\partial n_{\varphi_0}}G(x_q(\varphi_k^n),x_{q_0}(\varphi_{0j}^n))\cdot u_f(x_q(\varphi_k^n)) = \frac{|L_{q_0}|}{\pi}\frac{\partial}{\partial n_{\varphi_0}}u_0(x_{q_0}(\varphi_{0j}^n)),$$

$$j=0,1,...,2n;\ q_0=0,1,...,l, \tag{5}$$

where the function Q_1 is smooth kernel. The numerical experiment has been made. It was also considered the case of E-polarization.

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Conductance in the sandwich structures

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In the case of a one-dimensional periodic lattice Bloch waves are obtained as linear combinations of standard solutions of the Cauchy problem for the corresponding 1D Schrödinger equation. While the Cauchy problem approach to calculation of the dispersion function fails in the case of the multidimensional Schrödinger equation, the approach based on the boundary problem and the Dirichlet-to-Neumann map on the period is efficient and and even admits extension to periodic sandwich structures, see [1]. To make the approach efficient for low energy, we substitute the quasi-periodic matching condition for the Bloch functions on the mutual boundary of the neighboring periods by an appropriate partial matching on selected contact zones in an appropriate finite-dimensional contact spaces and substituting the Dirichlet-to-Neumann map, for low temperature, by an appropriate rational approximation taking into account only the eigenvalues of the Dirichlet problem for the Schrödinger operator on the period which are situated on the temperature interval centered at the Fermi level. The rational approximation of the DN-map permits to calculate the approximate dispersion function which can be interpreted as a fitted dispersion of the appropriate solvable model. The result presented in the form of an explicit formula, providing the approximate dispersion function in dependence of the shapes of the resonance eigenfunctions of the Dirichlet problem on the period and the characteristics of the selected contact zones and contact spaces, automatically fits, with controllable precision, the exact dispersion function of the periodic structure. The spectral characteristics of the model can be used as a first step of a convergent analytic perturbation procedure leading to calculation of the spectral characteristics of the original periodic structure. In particular it permits to reveal the Landau–Zener enhancing of BCS gap as a cause of the corresponding High Temperature Superconductivity phenomenon in multidimensional sandwiches, see the discussion of the 1D version of the phenomenon in [2], and experiment in [3]. Another interesting application of the theory is connected with low-threshold field emission from cathodes covered with carbon nano-structures, see [4].

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Models of scaling and shapes complication in biological morphogenesis

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Scaling (restoration of a proportional structure under different linear dimensions) and a progressive complication of shapes (reduction of symmetry order) are the fundamental, interconnected and still non-comprehended features of embryonic development. We attempt to interpret them with the use of a model approach, implying mechano-sensitivity of embryonic cells. The main idea is that each tissue piece or an individual cell senses the deviations of its mechanical stress imposed by the active deformations of surrounding tissues and reacts by hyper-restoring (restoring with an overlap) its initial stress value. A set of models based upon this assumption permits to reproduce the scaling capacities within a large enough range of linear dimensions and to imitate several crucial steps of shape complication starting from the shapes of a different curvature.

Symmetric polynomials in the transfer matrix scaling

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Expression of the transfer matrix $T = \exp(Wz)$ through symmetric polynomials of n-th order matrix W was considered in [1]. Among dozens of techniques that were used to calculate the matrix exponential, the method of symmetric polynomials (MSP) has significant advantages. One of the most important of them is carrying out calculations using analytical estimates. Another is that these estimates do not depend on the eigenvalues of the matrix. Using the fundamental property of the exponential function $\exp A = [\exp(A/m)]^m$, we represent the transfer matrix as follows: $T = X^m$, where

$$X \simeq \sum_{l=0}^{n-1} \left(\frac{Wz}{m}\right)^l \frac{1}{l!} \left[1 + l! \sum_{g=0}^{l} (-1)^{n-l+g-1} \sigma_{n-l+g} \sum_{j=n}^{n+N} \frac{\mathscr{B}_{j-l-g}(n)}{j!}\right], \tag{1}$$

$W \equiv ||w_j l||$ is wave matrix of order n, z is layer thickness, m is integer; σ_j, $j = 1, 2, \ldots, n$, are elementary symmetric polynomials of matrix (Wz/m), and symmetric polynomials of n-th order $\mathscr{B}_l(n)$ are defined by σ_j using recurrence relations:

$$\mathscr{B}_l(n) = 0,\ l = 0, 1, \ldots, n-2; \quad \mathscr{B}_{n-1}(n) = 0; \quad \mathscr{B}_l(n) = \sum_{i=1}^{n} (-1)^{i-1} \sigma_i \mathscr{B}_{l-i}(n), \quad l \geq n.$$

Approximate equality in (1) is replaced by the exact if N goes to infinity.

Accuracy of the transfer matrix $T = \exp(Wz)$ calculation by the method of scaling is determined by the truncation errors in the calculation of matrix X and roundoff errors in the computation of the matrix X^m. MSP allows to control the first error and minimize the latter.

Theorem. *The relative truncation error of the matrix X calculation according to the formula* (1) *does not exceed*

$$\epsilon < \frac{\xi^{N+1}}{(N+n)\prod_{i=1}^{N}(n+i)}, \quad \textit{provided that} \;\; \xi = (2n-1)\frac{\max|w_{jl}|z}{m} < 1,$$

and $\max|w_{jl}|$ *are largest absolute elements value of the matrix* W.

Example. Let the scaling factor m for matrix Wz of order $n = 4$ is the smallest integer satisfying the condition $m \geq 10(2n-1)z\max|w_{jl}|$. In this case $\xi < 0.1$, and the calculation by formula (1) with the value $N = 2$ gives the matrix X with a relative error less than 10^{-5}.

When $m > n+1$, the most efficient way of calculating the matrix X^m, as shown in [1], is to use the theorem on integer powers of matrices

$$X^m = \sum_{l=0}^{n-1} X^l \sum_{g=0}^{l} (-1)^{n-l+g-1} \sigma_{n-l+g} \mathscr{B}_{m-1-g}(n), \tag{2}$$

where σ_j and $\mathscr{B}_l(n)$ are symmetric polynomials of matrix X.

When $m = 2^j$ and $j > n$, algorithm for computing the matrix X^m based on the use of (2) with a corresponding replacement $m \to 2^j$ is also more effective in comparison with repeated squaring: $X^{2^j} = (\ldots((X\underbrace{)^2)^2\ldots)^2}_{j}$.

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Method of discrete sources vs. plane scattering problem with singularities

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The classical plane scattering problem in the strip region with sharp ledge is considered. The specific feature of the problem considered is the singularities at the edges of the boundary. The problem is governed by the Helmholtz equation. To solve this equation, the method of discrete sources (MDS) is used. Effectiveness of the MDS depends essentially on the way of source placement. An algorithm, permitting us to find the optimal source placement, based on the singular value decomposition technique is presented. Moreover, the regularization procedure for the resonance case is suggested. A number of the analytical tests illustrates our investigations.

Realization of the Hamiltonian of bivariate Chebyshev–Koornwinder oscillator by differential operators

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The orthogonal system of the generalized Chebyshev polynomials (Chebyshev–Koornwinder polynomials) was defined in [1] by T. Koornwinder. There it was constructed an commutative algebra of all finite-order differential operators which admit the Chebyshev–Koornwinder polynomials as eigenfunctions. This algebra was generated by two basic differential operators of order two and three.

The algebra of generalized oscillator connected with Chebyshev–Koornwinder polynomials was defined by authors in [2]. In our talk we present some realization of ladder operators and Hamiltonian of considered Chebyshev–Koornwinder oscillator by some infinite order differential operators. It is possible to connect proposed differential expressions with Koornwinder's basic differential operators.

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Structural study of a cracked porous surface by using acoustic microscopy

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The determination of fundamental parameters characterizing mechanical properties of materials, by dynamical methods, is based on the propagation of elastic waves in the solids. This last is

studied in our work to determine the porous silicon elastic parameters in the case of a cracked surface by using acoustic microscopy. Attenuation of surface acoustic waves (SAWs) occurs when a wave loses some of its energy while propagating through a fluid or solid medium. The presence of a cracked surface may be the result of the losses within the medium, which may be due to a combination of reflection refraction, diffraction and scattering. The attenuation coefficient of SAWs on a liquid/solid structure is characteristic of the material bulk elastic properties as well as the surface state of substrate, topographic variations, grain size and distribution. Thus, SAW attenuation is a sensitive means of surface characterization. Therefore, the scanning acoustic microscopy technique, based on the emission and reflection of ultrasonic waves, would be a very useful and promising tool for attenuation investigations. In this work, we use at first Schoch model using a numerical simple approach to calculate the variation of the reflection coefficient modulus and at second, we calculate Young and bulk modulus by analyzing the obtained acoustic signatures curves, V(z).

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Light scattering in the medium with nanosize inhomogeneities

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We investigate the light propagation in scattering and absorbing medium using diffusion approximation. The method of nanoparticle size estimation based on decreasing of light intensity in the medium was suggested. The comparing the results of calculation with experiment demonstrates the good agreement. The beam broadening in the homogeneous and inhomogeneous medium was calculated.

On the new laws of the Rayleigh scattering

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The new laws of the Rayleigh scattering, violating the Rayleigh law, are considered in details. It is obtained that oscillations of the Rayleigh wave coefficient of scattering on a statistical near-surface inhomogeneity in dependence on incident wave frequency in the Rayleigh limit and an appearance of the arbitrary number of coefficient of scattering zeroes in angle of scattering in the Rayleigh limit are possible. These new effects are determined by the form of inhomogeneity correlation function in the plane parallel to the surface approximated by sum of the Gaussian exponents and independently by the deterministic form of inhomogeneity in the direction perpendicular to the surface. Both these effects violate the Rayleigh law of scattering for the frequency dependence of the coefficient of scattering and the Rayleigh law about isotropy of scattering in dependence on the angle of scattering in the Rayleigh limit.

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Oscillations in classical boundary layer for flow with double-deck boundary layers structure

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We consider a two-dimensional flow of an incompressible Newtonian fluid past a fixed flat plate with small periodic irregularities for large Reynolds numbers. This process is governed by the system of equations

$$\varepsilon^{-2/3}\tfrac{\partial \mathbf{U}}{\partial t} + \langle \mathbf{U}, \nabla\rangle \mathbf{U} = -\nabla p + \varepsilon^2 \triangle \mathbf{U}, \qquad \langle \nabla, \mathbf{U}\rangle = 0, \tag{1}$$

where $\mathbf{U} = (u, v)$ is the velocity vector, p is the pressure and $\varepsilon = \mathbf{Re}^{-1/2}$ is a small parameter. We assume that the plate surface is given by the equation $y = \varepsilon^{4/3}\mu(x, x/\varepsilon)$. The reason of such a special time scale in (1) is that, in the double-deck structure [1] corresponding to the chosen spatial scale, the flow near the surface is developing in the usual time scale (of order 1). The boundary conditions have the form

$$\mathbf{U}\Big|_{\substack{y=\varepsilon^{4/3}\mu(x,x/\varepsilon)\\ x>0}} = \left(\begin{smallmatrix}0\\0\end{smallmatrix}\right), \quad \tfrac{\partial u}{\partial y}\Big|_{\substack{y=0\\ x<0}} = 0, \quad v\big|_{\substack{y=0\\ x<0}} = 0, \quad \mathbf{U}|_{y\to\pm\infty} \to \left(\begin{smallmatrix}1\\0\end{smallmatrix}\right), \quad \mathbf{U}|_{x\to-\infty} \to \left(\begin{smallmatrix}1\\0\end{smallmatrix}\right). \tag{2}$$

The flow in a thin boundary layer is described by the classical Prandtl equation with induced pressure [2], [3], see also [1]. In the classical Prandtl boundary layer, we have the standard flow as averaging and oscillations which are described by the time-dependent Rayleigh-type equation of the form

$$\varepsilon^{1/3}\tfrac{\partial}{\partial t}\triangle\widetilde{u} + u_0\triangle\widetilde{v} - \widetilde{v}\tfrac{\partial^2 u_0}{\partial \tau^2} = 0, \qquad \tfrac{\partial \widetilde{u}}{\partial \xi} = \widetilde{v}. \tag{3}$$

Here $\widetilde{u}$ and $\widetilde{v}$ are the oscillating velocity components in classical boundary layer with zero average, $u_0(x,\tau) = f'\big(\frac{y}{\varepsilon\sqrt{x}}\big)$, $f(\gamma)$ is the Blasius function, $\xi = x/\varepsilon \in [0, 2\pi]$ and $\triangle = \triangle_{\xi,\tau}$, $\tau = y/\varepsilon$, x is a parameter here. The solution $\widetilde{v}_{st}$ of the corresponding stationary problem exists and is unique for $x > M$, where $M = \max\limits_{\gamma\in[0,\infty)} |f'''(\gamma)/f'(\gamma)|$.

Theorem. *Let $x > M$, then the stationary solution of Eq.* (3) *exists and it stable, i.e., for a small perturbation $\delta\widetilde{v}$ of the stationary solution $\widetilde{v}_{st}$ of* (3) *it follows that*

$$\frac{\partial}{\partial t}\int\limits_0^\infty\int\limits_0^{2\pi}\left(-x\frac{f'\big(\frac{\tau}{\sqrt{x}}\big)}{f'''\big(\frac{\tau}{\sqrt{x}}\big)} - 1\right)\Big(|\triangle\widetilde{V}_\delta|^2 + |\widetilde{V}_\delta|^2\Big)\,d\xi d\tau \le 4\int\limits_0^\infty\int\limits_0^{2\pi}\left(-x\frac{f'\big(\frac{\tau}{\sqrt{x}}\big)}{f'''\big(\frac{\tau}{\sqrt{x}}\big)} - 1\right)\Big(|\triangle\widetilde{V}_\delta|^2 + |\widetilde{V}_\delta|^2\Big)\,d\xi d\tau,$$

where $\widetilde{V}_\delta$ is the primitives of $\delta\widetilde{v}$ with zero average.

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Theoretical calculations of reflection coefficient for poroelastic media

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Pourous materials are mainly used as acoustic absorbants to reduce noise levels in transport industry. Many theoretical models using a large number of parameters have been develpped to predict the acoustic behaviour of porous medias. Modelling biot proelastic type requires the introduction of numerical simulation codes. In this work, we use two different methods to calculate reflection coefficient for the glass wool in 2D and 3D systems. The first method we simulate Biot–Allard model that allows to obtain analytic formulations. In the second method, we put biot's set of equations of mix formulation in a simulating software based on finite elements method called Comsol multiphysics 4.2. the obtained results shows very good concordance.

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On a new representation of the Maslov canonical operator and its application to waves in graphene

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We discuss the new representation of the Maslov canonical operator in the neighborhood of caustics and focal point working in some wide class of important physical problems. Using this representation we construct in particular asymptotic solution of the scattering problem and the asymptotics of the Green function for 2-D Dirac equation for graphene. This work was done together with T. Tudorovskiy and G. Makrakis.

Early stage dynamics modeling of the feed-forward loops with miRNA

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We consider several mathematical models of the feed-forward loops of a complex network of gene expression. This network is regulated by the miRNAs, which are small non-coding RNA molecules, containing about 20 nucleotides only. We consider the loops regulated by the miRNAs and by special proteins, called TFs, simultaneously, which can be divided into coherent and incoherent loops, depending on whether their two regulators have the same effect on target or an opposite one, respectively.

We have obtained exact solutions for number of molecules of components for one of the models studied. We found that dynamics at early stage may differ remarkably for different initial conditions, whereas the steady state values are the same.

We consider different models based on various ideas about mechanism of regulation for each loop. We found that different early stage dynamics may lead to one and the same steady state values. In Fig. 1 one can see temporal dependence of relative number of molecules for different models of the same loop. We demonstrate the different protein molecules maxima at early stage for three models, having nonlinear production of the target protein, nonlinear degradation of target mRNA, and the model with miRNA and target mRNA binding, resp., however the steady state values are the same. It should be noted that these graphs correspond to models with different degradation coefficients of components, i.e. different biological cases.

We may conclude that consideration of various models at the whole time interval is very important for better understanding the process of regulation. Experimental data may give us the steady state values for number of molecules, while the early stage dynamics may show the biologically sensitive set of parameters, and therefore, an appropriate model.

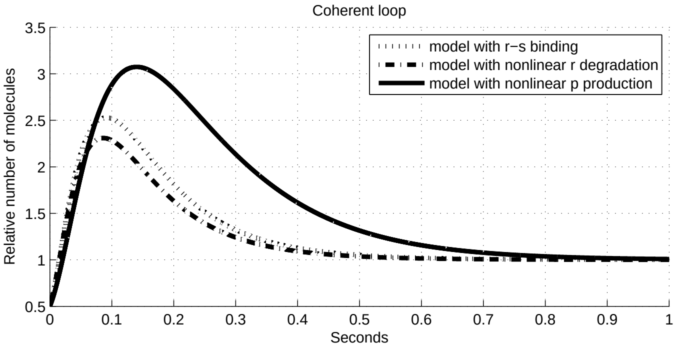

Fig. 1: Relative number of molecules of target protein for three models. The following parameters were used for model with nonlinear protein production $g_r \approx 3g_s, g_p \approx 1.2g_s$, and $g_r \approx 1.6g_s, g_p \approx 11g_s$ for two other models.

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An approximate solution of the 3D moving load problem for an elastic half space based on the explicit model for the Rayleigh wave

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We consider the dynamic response of an elastic half-space $-\infty < x_1, x_2 < \infty$, $x_3 \geq 0$ loaded by a vertical point force moving at a constant speed c along the Ox_1 coordinate axes. The problem is formulated within the framework of the asymptotic hyperbolic-elliptic model for the Rayleigh wave developed in [1], see also [2]. The obtained solutions are therefore valid for loads moving with the speeds close to the resonant speed $c = c_R$. The governing equations within the model [1] are expressed as a set of scalar 3D elliptic problems, with the boundary conditions at the surface $x_3 = 0$ given by a 2D hyperbolic equation and relations between the potentials.

In case of the steady-state motion the above mentioned 2D hyperbolic equation at the surface reduces to a 1D hyperbolic equation in the super-Rayleigh regime ($c > c_R$) and the 2D elliptic equation in the sub-Rayleigh regime ($c < c_R$), which already provides a drastic distinction of the 3D steady-state motion from the corresponding 2D setup, analysed in [3], see also [4] for the approximate treatment within the model for the Rayleigh wave. One more expected feature of the 3D solution is a Mach cone generated in the super-Rayleigh regime. Curiously enough, the associated discontinuities occur not only behind but also in front of the moving super-Rayleigh load. The reason is that the hyperbolic-elliptic model cannot be just reduced to a hyperbolic equation on the surface, involving also "non-wave" phenomena. It is worth noting, that the 3D steady-state solution is uniquely defined in contrast to its 2D analogue, containing no rigid body motion components. In addition, instead of a pole at the Rayleigh wave speed within the 2D setup, the 3D solution has a square root branch point in the denominator. In this case the associated transient solution grows in time also as a square root.

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Electromagnetic wave scattering by an array of axially magnetized parallel plasma columns

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Scattering of electromagnetic waves by periodically spaced elements demonstrates many interesting features that are of considerable practical and scientific importance (see, e.g., [1, 2] and references therein). Despite significant progress in the theory of multiple scattering by arrays of isotropic elements [3], arrays consisting of scatterers filled with a magnetoplasma have not yet been studied in sufficient detail.

In this work, we study scattering of an electromagnetic H-polarized plane wave by an equidistant array of axially magnetized plasma columns located perpendicular to the incidence plane in free space. At first, we obtain a system of equations for the scattering coefficients of the array and find the dipole scattering coefficients in explicit form in the case where the radii of the columns are electrically small. It is shown that the splitting of the individual dipole resonance of a single plasma column due to the presence of an external dc magnetic field can produce a significant change in the scattering field of the entire array of such columns compared with the case of periodically spaced isotropic scatterers. Conditions have been determined under which the individual and collective resonant effects in the considered system lead to the formation of an electromagnetic field with a 2D-periodic structure of chessboard type near the array. The array parameters have been found for which a giant enhancement of the scattered field takes place such that the field possesses a quasi-1D layered structure with alternating sharp maxima and deep minima in this case. It is also shown that for certain parameters, the scattered field can propagate predominantly towards the incident wave, or, upon passing through the array, in a direction opposite to that of mirror scattering. Detailed results of the performed numerical calculations will be reported for some cases of interest.

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Limited applicability of the extended boundary condition method to layered spheroidal scatterers

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The extended boundary condition method is a popular approach to treat the light scattering by particles of simple shape and structure. Within the method the fields are used to be expanded in terms of the spherical wave functions, the expansions are substituted in surface integral equations, and the expansion coefficients are derived from solution of a linear algebraic equation system arisen.

However, numerical studies have demonstrated that the method does not work well for layered particles [1]. In this paper we analytically investigate the applicability of the method to small layered spheroids.

In the case of scatterers very small in comparison with the wavelength the light scattering problem is known to be reduced to an electrostatic one. We apply a direct analog of the method to solve the electrostatic problem for a two-layered spheroidal particle with the confocal layer boundaries. As a result we get the following expression for the particle polarizability:

$$\alpha = \frac{V_1}{4\,\pi}\,\frac{(\varepsilon_1-1)\,[1+(\varepsilon_2-1)\,L_2]+(\varepsilon_2-1)\,[1-(\varepsilon_1-1)(L_1-1)]\,\frac{V_2}{V_1}}{[1+(\varepsilon_1-1)L_1]\,[1+(\varepsilon_2-1)\,L_2]-(\varepsilon_1-1)(\varepsilon_2-1)\,\sum\limits_{l=1}^{\infty}\,I_l^{(1)}I_l^{(2)}}, \tag{1}$$

where ε_1 and V_1 are the permittivity of the mantle and the volume of the particle, ε_2 and V_2 those of the core, L_1, L_2 the known geometric factors of the mantle and core, and $I_l^{(1)}$, $I_l^{(2)}$ some integrals of spherical functions and their derivatives [2].

We analytically prove and confirm by calculations with arbitrary precision that the series in the denominator of Eq. (1) converges under the condition

$$a_1/b_1 < \sqrt{2} + 1, \tag{2}$$

where a_1 and b_1 are the major and minor semiaxes of the particle, and then Eq. (1) becomes equal to the known expression obtained by using the corresponding spheroidal basis [3]. Note that the condition (2) is not related with the position of the foci which would obviously give the condition $a_1/b_1 < \sqrt{2}$, but it can be connected to the inversive spheroid corresponding to image of the focal interval.

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Waves in adiabatic quantum films

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We describe the large time behavior of solutions of the non-stationary one dimensional Schrödinger operator with a potential slowly depending on time. We assume that the spectrum of the stationary Schrödinger operator with the same potential (depending now on the time as on a parameter) has an absolutely continuous component and, possibly, a finite number of eigenvalues. We concentrate on a model problem related to quantum films.

Exact localized astigmatic solutions for Klein–Gordon–Fock and Dirac equations

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We construct exact localized solutions for the Klein–Gordon–Fock equation in form of wave-packets travelling along a straight line. This construction is realized in many-dimensional case, and is based on previous works [1], [2]. The presented solutions have astigmatic behavior. We also discuss similar solutions of the Dirac equation.

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Wave phenomena in the periodic beam

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The models of finite and infinite beam with periodically located inertial inclusions are considered. The propagation of the banding waves, pass and stop bands, power flow asymptotic are analyzed. The location of eigenfrequencies of the corresponding finite system versus pass and stop bands are considered.

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Asymptotic behavior of scattering data and conductivity for small Fermi energy in monolayer graphene

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Electron scattering problem in the monolayer graphene with short range impurities is considered. The main novel element in the suggested model is the band asymmetry of the defect potential in the 2+1-dimensional Dirac equation. This asymmetry appears naturally if the defect violates the symmetry between sublattices. Our goal in the present paper is to take into account a local band asymmetry violation arising due to the defect presence. We analyze the effect of the electron scattering on the electronic transport parameters in the monolayer graphene. The explicit exact formulae obtained for S-matrix for δ-shell and annular well potential cases allowed us to analyze the asymptotic behavior of such scattering data as scattering phases, transport cross section, the transport relaxation time and then conductivity for small values of Fermi energies. The obtained results are in good agreement with experimental data which shows that the considered model is reasonable.

Mobility of a cluster

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Spectrum of an 1D Schrödinger type operator with quasi-random coefficients is a zero measure perfect set, see for instance [1, 2], but if the spectrum in self-similar, then the spectra of the corresponding Liouvillian of two-body Hamiltonian may have an absolutely continuous component, see the mathematical observation in [3] and an extended physical discussion in [4, 5, 6]. We show that for some self-similar Cantor spectra, the spectrum in some few-body sector may have an absolutely continuous component, while the spectra in all sectors with less particles are still singular.

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Oscillation of a punch moving on the free surface of an elastic half space

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The contact problem concerning oscillation of a circular rigid punch, moving uniformly at sub-Rayleigh speed along the surface of an elastic half space, is investigated using a three-dimensional formulation. “Slow” motion of the punch is considered, which implies that the characteristic time for the external loading is much larger than the time interval necessary for shear waves to propagate across the punch. An asymptotic solution for the vertical oscillation of the punch is given. It is shown that the vertical displacement of the punch can approximately be described by the equation of dynamics for a system of one degree of freedom with viscous friction. The dependence of the coefficients for effective viscosity and stiffness, occurring in this equation, on the speed of the punch and Poisson ratio of the half space, is investigated.

Scattering theory for Klein–Gordon equations with non-positive energy

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We study the scattering theory for charged Klein–Gordon equations:

$$\begin{cases} (\partial_t - \mathrm{i}v(x))^2\phi(t,x) + \epsilon^2(x, D_x)\phi(t,x) = 0, \\ \phi(0,x) = f_0, \\ \mathrm{i}^{-1}\partial_t\phi(0,x) = f_1, \end{cases}$$

where:

$$\epsilon^2(x, D_x) = -\sum_{1\le j,k\le n} \left(\partial_{x_j} - \mathrm{i}b_j(x)\right) A^{jk}(x) \left(\partial_{x_k} - \mathrm{i}b_k(x)\right) + m^2(x),$$

describing a Klein–Gordon field minimally coupled to an external electromagnetic field described by the electric potential $v(x)$ and magnetic potential $\vec{b}(x)$. The flow of the Klein–Gordon equation preserves the energy:

$$h[f,f] := \int_{\mathbb{R}^n} \overline{f}_1(x)f_1(x) + \overline{f}_0(x)\epsilon^2(x, D_x)f_0(x) - \overline{f}_0(x)v^2(x)f_0(x)\,\mathrm{d}x.$$

We consider the situation when the energy is not positive. In this case the flow cannot be written as a unitary group on a Hilbert space, and the Klein–Gordon equation may have complex eigenfrequencies.

Using the theory of definitizable operators on Krein spaces and time-dependent methods, we prove the existence and completeness of wave operators, both in the short- and long-range cases. The range of the wave operators are characterized in terms of the spectral theory of the generator, as in the usual Hilbert space case.

The Huygens–Feynman–Fresnel principle and its applications

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The main relationships of both wave and geometrical optics can be derived from the Huygens–Feynman–Fresnel principle [1] which is a combination of the Huygens–Fresnel principle with the Feynman's path integral method [2].

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An impedance boundary condition in elastodynamics and existence of surface waves

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Elastic surface waves are of particular importance in seismology, because they are the most destructive in earthquakes. In a geophysical context, it is usually assumed that the external bounding surfaces are traction-free, which is mathematically expressed by Neumann boundary conditions. Other types of boundary conditions are far less frequent in this area. The present work considers time-harmonic surface waves in an isotropic elastic half-plane with a boundary condition of the impedance type prescribed on its surface (cf. [2]). Some authors that have previously studied impedance-like boundary conditions in geophysics and seismology are Tiersten [5], Bövik [1] and Malischewsky [3, 4].

Given Cartesian coordinates (x_1, x_2), we consider the half-plane $x_2 > 0$ occupied by an isotropic elastic solid. We assume that the surface $x_2 = 0$ is free of normal traction, and the shear traction varies linearly on the tangential displacement u_1 times the angular frequency ω, that is,

$$\sigma_{12} + \omega Z u_1 = 0, \quad \sigma_{22} = 0, \qquad \text{for } x_2 = 0, \tag{1}$$

where σ_{ij} denotes the stress tensor and $Z \in \mathbb{R}$ is an impedance parameter. The associated secular equation (or dispersion relation) for surface waves is given by

$$\left(2s^2 - s_T^2\right)^2 - 4s^2\sqrt{s^2 - s_L^2}\sqrt{s^2 - s_T^2} + \tfrac{Z}{\mu}\, s_T^2\sqrt{s^2 - s_T^2} = 0, \tag{2}$$

where the unknown s is the surface wave slowness (reciprocal of velocity), s_L and s_T stand for the longitudinal and transverse wave slownesses, respectively, and μ is the shear modulus. If $Z = 0$ we retrieve in (1) the usual traction-free boundary condition and (2) becomes the classic secular equation for Rayleigh waves. As (2) only depends on s^2, it suffices to analyse the case $s > 0$. We present some results concerning existence of surface waves, which are summarised as follows:

Proposition 1. *For each impedance $Z \in \mathbb{R}$ the secular equation* (2) *has a unique solution in the range $s > s_T$. It corresponds to the Rayleigh wave slowness and is strictly increasing in Z.*

Proposition 2. *If the impedance Z takes the positive value $Z^* \equiv 2\mu\sqrt{s_T^2/2 - s_L^2}$, then the secular equation* (2) *has one real solution in the range $s_L < s < s_T$. This solution is given by $s^* \equiv s_T/\sqrt{2}$ and corresponds to the slowness of an additional surface wave.*

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Continuous wavelet transform application in diagnostics of piezoelectric wafer active sensors

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Due to increasing complexity of everyday used civil objects such as airplanes, nuclear power plants, bridges and rotor blades of wind turbines, damage detecting technologies are of a great importance. Therefore structural health monitoring (SHM) is a promising branch in worldwide science. One method in this field is to use guided waves actuated and recorded by a certain number of piezoelectric waver active sensors (PWAS). Especially for long term monitoring, sensors might be the weakest link in the SHM system. Failure of actuators might lead to significant problems and evidently the monitoring of actuators themselves is necessary. The aim of the present work is to collect knowledge about the effects of partially debonded PWAS. Therefore an experimental setup with 16 partially debonded actuators is used to investigate the excitation of an aluminum plate. Phenomena accompanying wave excitation by debonded actuators are also analyzed. Collected knowledge is analyzed in order to identify existence, location and shape of a debonded part of the actuator.

Circular piezo wafer active sensor with disk-wrapped electrode are widely used in practical applications therefore they are chosen as a subject of present research [1]. In order to reveal the difference in the wavefield, generated by the bonded and debonded actuators a Laser Doppler Vibrometer setup is used. Sixteen PWASs are glued with various debonding to an aluminum plate of the dimensions 500 mm by 500 mm and a thickness of 2 mm. When a transducer is excited by a Hann-windowed toneburst voltage signal of a certain central frequency a propagating wave is produced and the laser vibrometer measures the velocity of the out of plane motion on the surface of the plate, see more details in [1]. The measurements are made with various central frequencies from 30 kHz to 180 kHz. For a certain sufficiently debonded PWAS some interesting abnormalities are detected for higher frequencies. A sizable increase in the amplitude of the velocity of the motion was observed (up to 300 percent in comparison to perfectly bonded PWAS). It was revealed that the velocities of the motion and carrier frequencies depend on shape of the debonded part of the PWAS. Continuous wavelet transform is applied to the recorded signal in order to compute the carrier frequency of the generated wavefield and to identify the debonded part of the investigated PWAS. The Gabor wavelet is selected as a kernel function [2] due to its waveform is similar to the signal used in the experiment.

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Solution of 3D scattering problems from 2D ones in short waves

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The paper reports on development of a boundary integral equation technique of characterization of scattering by rough three-dimensional (3D) surfaces at a short wavelength of λ. The effect of roughness on the mirror scattering intensity can be rigorously taken into account with the model in which an uneven surface is represented by a grating with a large period of d_i in different perpendicular planes i, which includes an appropriate number of random asperities with a correlation length of ξ_i. The code analyzes the complex structures which, while being multilayer gratings from a mathematical viewpoint, are actually rough surfaces for $d_i >> \xi_i$. If $\xi_i \sim \lambda$ and the number of orders is large, the continuous angular distribution of the energy reflected from randomly rough boundaries can be described by a discrete distribution $\eta(\#)$ in order # of a grating [1]. A study of the scattering intensity starts with obtaining statistical realizations of profile boundaries of the structure to be analyzed, after which one calculates the intensity for each realization, to end with the intensity averaged out over all realizations. By selecting large enough samples, one comes eventually to properly averaged properties of the rough surface; however, this approach does not involve approximations, including averaging by the Monte Carlo method. The more general case of bi-periodic gratings (or 3D surfaces) may be considered in a similar way or by expressing the solution of the 3D Helmholtz equation through solutions of the 2D equation described below, an approach which may be resorted to in some cases [2]. General equivalent rules for determination of the efficiencies of reflected orders of bi-periodic gratings from those calculated for one-periodic gratings can be found, for example, in [3]. The general approach used is based on expansion of the efficiency of a bi-grating with profile boundaries symmetric relative to the horizontal plane in a Taylor series in powers of a boundary profile depth h, with the principal terms of the series retained in the $h < d$ case. Then the efficiencies $e^+_{0,m}$ and $e^+_{0,n}$ of the orders numbered $(0,m)$ and $(n,0)$ propagating in the upper $(+)$ medium for arbitrary linear polarization of light can be defined through the leading (quadratic in h) terms of the expansion as

$$e^+_{0,m} = e^+_{0,1}e^+_{m,2}/R; \quad e^+_{n,0} = e^+_{n,1}e^+_{0,2}/R,$$

where $e^+_{n(m),1(2)}$ are the values of the efficiencies of the corresponding mutually perpendicular one-periodic gratings calculated with the position of the polarization vector left unchanged, and R is the Fresnel reflection coefficient of the grating material. For non-deterministic surface functions some modification of the general approach is required. As follows from a comparison with the results of rigorous calculations performed in [3] and by the present author, the approximate relations (1) give a high-accuracy solution for $\cos\theta_i h_i << d_i$ and $\lambda < d_i$, where θ_i is an incidence angle. In the cases where one minus real part of the refractive index and imaginary part of the material are small, h can be large enough.

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Fox–Li operator, laser theory and Wiener–Hopf theory

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Many problems of diffraction theory can be reduced to the investigation of Wiener–Hopf integral operators over finite intervals. In particular the famous paper (A.G. Fox and T. Li "Resonant modes in a Maser Interferometer. Bell, System Tech. J 40, 1961.") stimulate numerous numerical investigations about behaviour of individual eigenvalues of truncated Wiener–Hopf integral operators over large intervals in the case of strongly oscillating symbols. But there are a few rigorously mathematics results in this field. This reports is devoted to the asymptotic behaviour of individual eigenvalues of truncated Wiener–Hopf integral operators over increasing intervals. The kernel of the operators is complex-symmetric and has a rational Fourier transform. Under additional hypotheses, the main result describes the location of the eigenvalues and provides their asymptotic expansions in terms of the reciprocal of the length of the truncation interval. Also determined is the structure of the eigenfunctions.

Homogenization and "anomalous" dispersion for the wave equation with the fast-oscillating velocity and spatially localized source

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We construct the asymptotic solutions of the wave equation with the velocity having fast-oscillations on the smooth background and the momentary spatially localized source:

$$\frac{\partial^2 u}{\partial t^2} =< \nabla,\, C^2(\tfrac{\Theta(x)}{\epsilon},\, x)\nabla > u, \quad x \in R^2, \quad u|_{t=0} = V(x/\mu), \quad u_t|_{t=0} = 0.$$

Here the $C^2(y,\, x) = C_0^2(x) + a(y,\, x)$ is the velocity, function $\Theta(x) = (\Theta_1,\, \Theta_2)$. Small parameter ϵ is the parameter of oscillations and μ is the parameter characterizing the spatial localization of the source. We assume that $\epsilon \ll \mu$, e.g. $\mu = \epsilon^\kappa$, where $\kappa > 1$. Function V decays at infinity at least like $C/|x|^{2+\kappa}$, $\kappa > 0$. Using the adiabatic approximation in the operator form we provide the averaging and obtain the Boussinesque-type equation with smooth coefficients and with small negative ("anomalous") dispersion. Then we construct the asymptotic solutions of this equation with the help of the modified Maslov operator. They become the combination of the products of the Airy functions and their derivatives in the case when $V(z) = A/(1 + z_1^2/b_1^2 + z_2^2/b_2^2)^2$. The corresponding formulas show that the oscillatory part of the velocity C^2 generate the oscillations in the wave profile running before the main wave (Fig. 1).

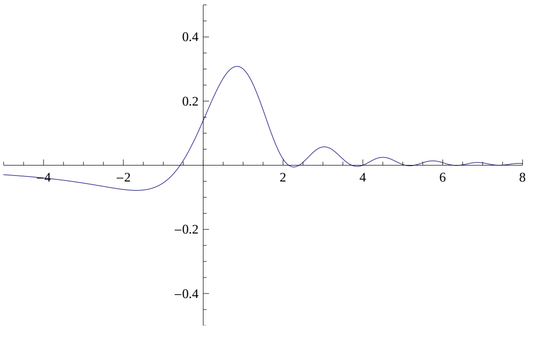

Fig. 1: The wave profile for the "anomalous" dispersion.

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The gene expression model based on Jeffreys type equation

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As a rule a parabolic reaction-diffusion equation is used for modelling of gene expression in the *Drosophila* embryo at early stages of its development:

$$u_t(x,t) = D\, u_{xx}(x,t) - \gamma\, u(x,t) + g(\eta),$$

where $u(x,t)$ denotes the concentration function, γ – degradation coefficient, D – diffusion coefficient and g – synthesis function. A parabolic PDE is well known to provide a non-zero concentration in each point of a system for any $t > 0$ even with exponentially small values [1], [2], [3], i.e., in an embryo a protein propagates infinitely fast, that is called "the diffusion paradox". However in many biological systems the protein motion is quite slow, and its velocity is about few millimeters per minute [4]. We propose a refined model of the transport process based on the Jeffreys type 3^{rd} order equation:

$$\tau\, u_{tt}(x,t) + (1+\gamma\tau)\, u_t(x,t) - \tau D_1\, u_{tt}(x,t) = (D+D_1)\, u_{xx}(x,t) - \gamma\, u(x,t) + g(\eta) + \tau\, g(\eta),$$

where τ is the relaxation time and D_1 is the second diffusion. This equation does not eliminate completely the diffusion paradox, because an exponentially small concentration still exists at infinity, however the main part of mass propagates as a wave smooth front with finite velocity.

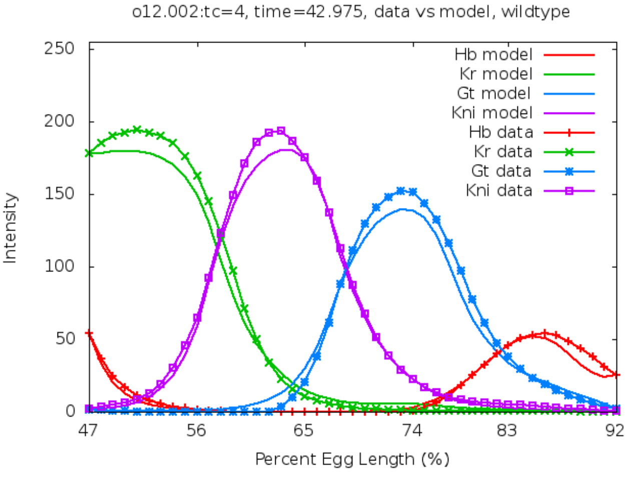

Fig. 1: Numerical solution (simple lines) vs experimental data (marked lines).

We applied the model to experimental data obtained at the early stages of development of the *Drosophila melanogaster* embryo for the gap gene system of 4 genes. The comparison between the numerical solution of the Jeffreys type equations system (solid lines) and the experimental data (marked lines) obtained for different moments of development in wild type embryos is shown in Fig. 1. The model parameters has been obtained by the formal solution fitting to experimental data.

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Nonlinear acoustic waves near the boundary of media with gas bubbles

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The problem of acoustic wave propagation in the media with gas bubbles is under consideration. Due to gas bubbles acoustic waves can change their properties significantly. For example sound speed can decrease by some orders; the strong dispersion appears so the wave propagation differs at low and high frequencies. Gas bubbles in liquid are strongly compressible so acoustic wave propagation is substantially nonlinear. Another important factor which influence on wave properties is the existence of boundaries and interfaces. Also the viscosity of liquids is necessary to take into account, when the interfaces are considered. Boundaries can cause some localizations of acoustic wave that leads to new nonlinear effects and wave profile transformation. Some evolution equations are set up for describing nonlinear acoustic wave and analytical solutions of them are obtained.

Investigation of nanotube vibrations in the liquid

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The vibration of the nanotube cantilever fixed in liquid was investigated. Experimentally and theoretically, this system has been first studied in [1]. In our paper, a model of vibrations of the nanotube, taking into account the friction of the liquid, and the impacts of the liquid molecules is suggested. A similar approach was used in [2] to describe the vibrations nanoplate. We investigated the characteristics of the nanotube vibrations depending on the properties of the nanotube and the liquid.

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Description of electro-mechanical processes by means of Cosserat continuum

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According to the ideas of modern physics Maxwell's equations describe the electromagnetic field which is some kind of abstraction giving a good description of electromagnetic interactions but not having a material carrier. This viewpoint essentially differs from the viewpoint of scientists of the XIX century who consider the electromagnetic field as a special kind of material medium [1]. The famous scientists of the XIX century have proposed a number of mechanical models of this material medium. Numerous attempts to construct models based on the translational degrees of freedom have failed. The models based on the translational degrees of freedom which have been proposed by Maxwell, FitzGerald and Kelvin [1] were not developed at that time. The reason is the fact that in the second half of XIX century the level of development of continuum mechanics made it impossible to solve such problems. The possibility to construct mathematical models of such media appeared much later when in 1909 the brothers Eugene and Francois Cosserat developed methods for describing 3D-continua with rotational degrees of freedom [2]. For a long time the Cosserat approach was lacking for the followers. However, starting with the works by C. Truesdell and J. Eriksen written in the second half of the XX century this approach began to develop intensively. Now many authors use the Cosserat continuum to describe various physical processes and phenomena such as piezoelectricity, ferromagnetism, plasticity, behavior of granular media, etc. At the beginning of XXI century P. A. Zhilin has proposed a new mechanical model of electromagnetic field that is based only on the rotational degrees of freedom [3]. Zhilin's theory describes the electromagnetic field in vacuum and does not consider the interaction between the electromagnetic field and a substance. We propose the mechanical model mathematical description of which can be reduced to the equations describing the electromagnetic field in matter. In the context of proposed model under one simplifying assumption we obtain the Lorentz force and under another simplifying assumption we obtain Maxwell's equations. Therefore we believe that the general equations describing our mechanical model may be of interest for describing electromagnetic phenomena. In order to clarify our idea and method we pay attention to the analogy with the coupled problem of thermoelasticity. If process of heat conduction is studied then inertial terms in the equation of motion can be neglected. If the propagation of acoustic waves is studied then the term in the heat conduction equation which contains temperature Laplacian can be neglected. Thus, in the theory of thermoelasticity we obtain one approximate statement of problem to describe the slow process and another approximate statement of problem to describe the fast process. Something like this we do when we obtain the Lorentz force and Maxwell's equations from the general equations describing the proposed mechanical model. Really, the Lorentz force acts on particles of matter which usually move with velocities much less than the velocity of light whereas Maxwell's equations describe waves propagating with velocity of light. However, if particles of matter are moving with velocities close to the velocity of light then the general equations contain both the Lorentz force and Maxwell's equations can be applied.

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Nonlinear generation of surface acoustic waves at solid-liquid interface

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Nonlinear acoustic phenomena are in focus of current research in physics, promising new methods of study of new materials and non-destructive testing, and promising new alternative instruments that could overcome conventional methods of linear acoustics. For example, papers [1, 2] show interesting results of theoretical and experimental studies of surface acoustic waves (SAW) harmonics in $LiNbO_3$ crystal.

The present experimental work deals with a specific type of non-linear phenomena — contact acoustic nonlinearity (CAN) — which occurs at liquid-crystal interface. The crystal used in the experiments is an YZ-cut $LiNbO_3$ with interdigital transducers (IDT) built in its surface. The IDTs could effectively work at 15, 30 and 45 MHz frequencies. A liquid drop was placed on the surface of the niobate delay line and SAWs were generated by the IDTs. The electric signal had the form of short radio pulses of about 1,5 to 2 μs length with 10 V amplitude. Acetone, water, glycerin and Fairy water solution were used as liquids in the experiments. The SAWs propagating in the substrate are reflected from the triple boundary between the liquid, the crystal and the air. It was shown by study and careful calculations that the amplitude of linear reflection in this case is not more than 1% of the incident wave. The nonlinear reflection was studied at summary and differential frequencies when 15 and 30 MHz harmonic pulses were applied simultaneously at the IDTs. The amplitudes of the nonlinear reflected waves were several times higher than that for the linear reflection. The differential frequency (15 MHz) showed much weaker amplitudes being only about two times lower than the summary one in amplitude. In the case, when the IDT was excited only by the main frequency signal (15 MHz), the generation of the second and third harmonics was clearly observed in the reflected signal. The nonlinear parameters calculated for the different cases of nonlinear generation in the reflections showed more than 1,5 times higher values than that for conventional structural nonlinearity. The mechanism of nonlinear generation is evidently defined by the interaction of SAWs with the liquid drop boundary. This type of nonlinearity could be used for characterization of materials' surface structure and acoustic tomography of surface non-uniformities which are similar in its properties with the contact acoustic nonlinearities.

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Carleman's functions for the polygarmonious functions in unbounded subdomains of even-dimensional Euclidean space

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We consider in this article Carleman's functions, to find integral representation for the polygarmonious functions ($\Delta^n u(y) = 0$) defined in unbounded domain in m-th even Euclidean space which satisfies $2n \geq m$.

Classification and x-ray analysis of molecular structures with certain non-translational symmetries

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The possibility of structure analysis of crystals by x-ray diffraction is due to the translational invariance of both plane waves and crystal lattices. The resonant frequencies reveal the translational structure of the crystal given by its Bravais class. This classification identifies lattices that have the same arithmetic holohedry up to conjugacy in the general linear group over the integers [1].

Recently [2, 3], R. D. James introduced a more general concept of invariant arrangements of atoms or molecules — the so-called objective structures — that include crystal lattices as a special case. The translational structure of the latter is replaced by invariance with respect to a uniformly discrete isometry group. Many structures in nanotechnology and biology are realizations of objective structures: graphene, carbon nanotubes and the buckyball, tails and capsids of certain viruses as well as some secondary and quaternary protein structures.

The classification of objective structures via a suitable generalization of arithmetic holohedry is discussed, as are preliminary ideas regarding the x-ray analysis of such structures.

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Computation of Rayleigh waves in layered anisotropic media

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To find parameters of anisotropic media, which depend on the depth only in the layer close to the surface it may be convenient to use Rayleigh waves, because parameters of the waves depend on media parameters and direction of propagation and we can chose the depth of penetration of the Rayleigh waves close to the inhomogeneous layer. To solve the inverse problem of reconstruction parameters of anisotropic media close to the surface we have to solve direct problem of computation of Rayleigh waves in layered media. The talk is devoted to such computations.

First of all we replace inhomogeneous layer on the pack of homogeneous anisotropic layers. Computations of parameters of the Rayleigh waves in the packs of layers on the homogeneous anisotropic half space are based on ideas of impedance operators for the layered anisotropic media and translation operators for each layer. Geometric parameters of layers and elastic parameters of media as in the layers as in the homogeneous elastic half space are assumed arbitrary. By means translation operators the problem of computation of impedance operator for the layered media is reduced to the construction of the impedance operator for the homogeneous half space, which has been done earlier ([1–3]). Velocity of the Rayleigh waves corresponds to velocity zero eigen value of determinant of the impedance operator and displacement vector on the surface is the eigen vector corresponding to the zero eigen value. In results we obtain velocity of the Rayleigh waves and their polarization vector as function of direction of propagation.

In the talk we present results of computation for several interesting cases.

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Stability of dynamical system under white noise perturbation

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Dynamical system under small stochastic perturbation is considered:

$$d\mathbf{y} = \mathbf{a}(\mathbf{y},t)dt + \mu B(\mathbf{y},t)d\mathbf{w}(t),\ t>0;\quad \mathbf{y}|_{t=0} = \mathbf{x} \in \mathbb{R}^n.$$

Here $\mathbf{a}(\mathbf{y},t)$ is a given vector–function in $\mathbb{R}^n$, $\mathbf{w}(t)\in\mathbb{R}^n$ is a n-dimensional Wiener process, $B(\mathbf{y},t)$ is a given matrix–function $n\times n$.

It is supposed that the point $\mathbf{y}=0$ is a local asymptotic stable equilibrium of the unperturbed system $\dot{\mathbf{y}}=\mathbf{a}(\mathbf{y},t)$, and the matrix of perturbation $B(0,t)\neq 0$. The problem of stability under small perturbation $|\mu|\ll 1$ is analyzed. The main result is as follows: The expectation of $|\mathbf{y}|^2$ along a trajectory $\mathbf{y}=\mathbf{y}_\mu(t,\mathbf{x})$ starting from point $\mathbf{x}$ near equilibrium remains small: $\mathbb{E}|(\mathbf{y}_\mu(t,\mathbf{x})|^2] < M(|\mathbf{x}|^2+\mu^2)$ for a long time interval $0<t<t_0\mu^{-2}$; $(M,t_0=\text{const})$. Both the Lapunov's functions and the method of parabolic equation are applied to prove the random stability. Stability of the autoresonance phenomenon under random perturbations is discussed in the report.

Ostrogradski's high-order derivative formalism and the foundation of quantum mechanics

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Quantum theory describes objects in Hilbert space, i.e. in terms of an infinite number of variables, and thus it gives a more detailed description as compared to classical theory. Thus, the question of quantum-mechanical description being incomplete should be answered so that any theory as a model of physical reality is incomplete following Godel's theorem.

The classical description of physical reality contains an incomparably fewer number of variables. This raises the question: "How the classical description can be completed?" While a possibility of supplementing the quantum mechanical description with additional ("hidden") variables has been debated for long, the question as to how to complete the classical description to make it compatible with the quantum mechanical one has not received a due attention. Classical dynamics of a test particle's motion with higher-order time derivatives of the coordinates was first described in 1850 by Ostrogradski and is known as Ostrogradski's Formalism. In the general case, the Lagrangian takes on the form $L=L(q,\dot{q},\ddot{q},...,\dot{q}^{(n)},...)$.

The classical and quantum theory could be based on the following common axioms:

1. Any reference frame is subject to random external influences. Hence, every reference frame is individual and a transition from one to another reference frame may lead to jump like changes. The notion of the inertial frame in classical mechanics is valid only on the average and, correspondingly, the Galilean relativity is an average notion as well. Then there are many trajectories of a particle corresponding to different reference frames; the Heisenberg uncertainty can be understood as a consequence of the nonexistence of ideal inertial frames; and the Ehrenfest theorem can be seen as a consequence of the inertial frame being an average notion. Correspondingly, *the free body preserves the same order of its time derivative as the constant kinematic characteristics of the class of reference frames*, e.g. in the uniformly accelerating reference frame the free body preserves its acceleration.

2. Using the Taylor expansion

$$a(t\pm\Delta t)=a(t)\pm\dot{a}\Delta t+\frac{1}{2!}\ddot{a}(t)\Delta t^2+...+\frac{(-1)^n}{n!}\dot{a}^{(n)}(t)\Delta t^n+..$$

the function $F=-\frac{\partial L}{\partial q}$ can be expanded as follows:

$$F(q,\dot{q},\ddot{q},\dddot{q}...,\dot{q},^{(k)}...)=ma(q,\dot{q},\ddot{q},\dddot{q}...,\dot{q},^{(k)}...)$$

where $\dot{a}^{(n)}$ denotes n-th time derivative of the acceleration a. *It is the Extended Law of Dynamics in arbitrary reference frames* including the case of the vibration non-inertial reference frames. Correspondingly, the free body preserves the same order of its time derivative like the constant kinematic characteristics of the reference frames. For example, in the uniformly accelerating reference frame the free body preserves its acceleration.

3. The de-Broglie waves $\psi = \psi_0 \exp(-iS/\hbar)$ with the actions functions $S = S(q, \dot{q}, \ddot{q}, ..., \dot{q}^n, ...)$ can be considered as having the gravity-inertial nature following from the fact that every *reference frame is vibrational due to the influence of random gravitational fields and waves* so that every free particle appears to be oscillating.

4. As the action function $S = S(q, \dot{q}, \ddot{q}, ..., \dot{q}^n, ...)$ is a convergent series in high derivatives of q the difference $S(q, \dot{q}, \ddot{q}, ..., \dot{q}^n, ...) - S(q, \dot{q}) = h$ is finite and can be identified with the constant h. Within the presented framework the variables of *the (high order extension of the) phase space do describe the completed dynamics of a particle*, but they cannot be measured because the ideal inertial reference frames do not exist in really. The infinite dimensionality of Hilbert space can also be understood as a consequence of all high order time derivatives being taken into account in the description of the dynamics.

Explicit formulation for the bending edge wave

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This paper is concerned with the explicit formulation for the bending edge wave [1] (see also a recent review [2]), propagating in a semi-infinite elastic Kirchhoff plate $-\infty < x < \infty$, $0 \le y < \infty$ of thickness $2h$, volume density ρ, and bending stiffness D.

The approach is relying on the eigensolution expressed through a single harmonic function [3], and is oriented to extraction of the contribution of the bending edge wave to the general dynamic response of the plate arising from the prescribed non-homogeneous edge boundary conditions.

First, a multi-scale slow time perturbation procedure is developed leading to the two-term asymptotic expansions around the eigensolution. Then, the boundary conditions with the non-zero bending moment $M_0 = M_0(x, t)$ lead to a parabolic beam-like equation at the edge of the plate $y = 0$

$$D\gamma_e^4 \frac{\partial^4 W}{\partial x^4} + 2\rho h \frac{\partial^2 W}{\partial t^2} = Q \frac{\partial^2 M_0}{\partial x^2}, \tag{1}$$

where $W = W(x, y, t)$ is the deflection of the plate, $\gamma_e = \left[(1-\nu)\left(3\nu - 1 + 2\sqrt{2\nu^2 - 2\nu + 1}\right)\right]^{1/4}$ is parameter of the dispersion relation [1], ν the Poisson ratio, and Q material constant given by

$$Q = \frac{\sqrt{1-\gamma_e^4}\left(\nu + \sqrt{1-\gamma_e^4}\right)}{1 - \nu + \sqrt{1-\gamma_e^4}}.$$

The parabolic equation (1) acts as a boundary condition for any of the two elliptic equations

$$\frac{\partial^2 W_j}{\partial y^2} + \lambda_j^2 \frac{\partial^2 W_j}{\partial x^2} = 0, \qquad (j = 1, 2) \tag{2}$$

governing the interior, whereas the deflection is presented through an arbitrary harmonic function W_j decaying as $y \to \infty$ as

$$W(x, y, t) = W_j(x, \lambda_j y, t) - \frac{\nu - \lambda_j^2}{\nu - \lambda_m^2} W_j(x, \lambda_m y, t), \qquad 1 \le j \ne m \le 2, \tag{3}$$

with $\lambda_j = \sqrt{1 + (-1)^j \gamma_e^2}$.

The obtained formulation reveals a dual *parabolic-elliptic* nature of the bending edge wave in contrast to the *hyperbolic-elliptic* one of surface and interfacial waves, see [4] and [5].

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Projection methods for computation of spectral characteristics of weakly guiding optical waveguides

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The original problem on eigenmodes of a weakly guiding step-index optical waveguide with smooth boundary of the cross section domain is considered.

We are looking for surface and leaky eigenmodes which satisfy to the real and complex propagation constants respectively.

The original problem is reduced by the simple layer potential method to a nonlinear spectral problem for the set of weakly singular boundary integral equations.

We approximate the integral operator by collocation method and Galerkin method.

The convergence and quality of these numerical methods are proved by numerical experiments. The collocation method demonstrates better speed of convergence.

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Heart and respiration rate estimation in the radar doppler health monitoring system using of Wigner–Ville distribution method

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The radar Doppler system for unobtrusive and non-contact monitoring of heart and respiration activity from distance is a powerful tool for security, emergency and Telemedicine applications. The back scattered signal to the receiver of this system is an amplitude-frequency (AM-FM) modulated signal which is modulated by the Doppler effect of the heart and respiration chest motion. In this paper, an efficient signal processing method for time-frequency representation (TFR) of the signal and extraction the information from high noisy signal is introduced. This method is based on Wigner-Ville distribution which is used to demodulate the phase of the received signal and extract the vital signs information. The simulation results show that the heart and respiration rates could be obtained with acceptable accuracy from the noisy signal in low SNR levels.

On some problems in application of short-wave theory of diffraction by prolate bodies of revolution

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Nowadays there is a certain interest towards the short-wave diffraction by prolate bodies of revolution, see e.g. [1, 2]. The current report actually represents a continuation of the previous paper [3] and is dedicated to the discussion of several problems, which arise in the method of Leontovich–Fock parabolic equation being applied to such tasks. It is convenient to examine the construction of the diffraction field close to the surface of the body in the orthogonal coordinate system, which is formed by the main curvature lines on the surface, i.e. the meridians (geodesics) and latitudes, together with the external normal to this surface. The oblongness of the body is characterized by the parameter $\Lambda = \rho/\rho_t$, where ρ is the curvature radius along the geodesics (meridians), and ρ_t represents a curvature radius in the transversal direction (latitudes). The large Fock parameter is denoted by $M = (k\rho/2)^{1/3}$, where k is the wave number.

In our report we point out the following problems:

1. In the case of axis-symmetrical diffraction task, the method of parabolic equation turns out to be applicable and leads to relatively simple results, both in the Fock domain and in the area of the creeping waves, provided $\Lambda = M^{2-\varepsilon}$, where $0 < \varepsilon < 2$. However, if $\varepsilon = 0$, the recurrent set of equations in the Fock domain gets singular, including the parabolic equation for the leading term of the asymptotics by itself, and therefore the solvability of this system in terms of smooth functions becomes questionable. In this case, for the ellipsoid of revolution the bigger axis surpasses the smaller one 30 times in length.

2. For the inclined incidence of the oncoming wave, the light-shadow boundary moves close to poles, where the curvature of the prolate bodies is getting bigger and therefore the applicability of the short-wave approach is violated. But for all that, due to the geodesics forming numerous caustics upon the surface of the body of revolution, the complications appear, while selecting the fast oscillation multiplier.

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Paradoxical wavefield in the elastic Green tensor

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Closed-form solution for the time-dependent Green tensor for a homogeneous, isotropic, elastic medium, was first presented in [1]. In case of $\delta(t)$ time-dependence of the source, the wavefield at each point linearly grows with time between the fronts of P and S waves [2]. This result, which still surprises students and researchers, is discussed.

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Error of unidirectional approximation in simulation of intense few-cycle femtosecond pulses propagating in single-mode optical fiber

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We simulate propagation of intense few-cycle femtosecond laser pulses in the single-mode silica fiber using bidirectional propagation approach with initial conditions corresponding to the unidirectional approximation. We describe dispersion with Sellmeier's formula, and nonlinear effects with a simple model of instantaneous cubic nonlinearity. Both dispersion and nonlinearity are nonresonant in the medium transparency range which includes spectra of propagating pulses.

The set of equations for bidirectional fields have the following form:

$$\partial_z G_\pm = \pm ik(\omega)G_\pm \pm \frac{1}{2} ik(\omega)N_\omega(E_+ + E_-), \quad N_\omega(E) = 4\pi F[P_{NL}(E)]/n^2(\omega),$$

where z is propagation coordinate, G_+ and G_- are spectral densities of forward and backward waves, respectively, $k(\omega) = \omega n(\omega)/c$ is the wavenumber, ω is frequency, $N_\omega(E)$ is nonlinear operator in spectral domain, F is Fourier transform, n is linear refractive index. These equations are analogous to the full second-order scalar wave equation and include unidirectional model as a special case $G_- = 0$.

Our numerical simulations show that if the backward wave is initially absent which is default for the unidirectional approach, it is generated due to coupling of waves in the nonlinear term. It consists of two parts, one propagates backward, the other accompanies the forward field and tightly coupled with it. Physically the backward-propagating part can be explained as reflection of input forward pulse from the jump of refractive index induced by nonlinearity.

To explain this behavior of backward wave we derived simplified set of equations, that keep main features of pulse propagation over distances of a few central wavelengths. This set is solved analytically for both forward and backward waves. Solution for the forward wave is simply initial pulse propagating forward with the group velocity. Solution for the backward wave consists of three parts:

$$E_-(z,t) = E_{0-}(t + z/V_g) + \frac{\pi}{n_0 n_g}\left\{ P_{NL}[E_{0+}(t + z/V_g)] - P_{NL}[E_{0+}(t - z/V_g)] \right\},$$

where E_{0-} is initial backward field distribution; $n_0 = n(\omega_0)$; $V_g = [dk(\omega)/d\omega]^{-1} = c/n_g$ is the group velocity and $n_g = n_0 + dn/d\omega$ is the group refractive index, both calculated at $\omega = \omega_0$. The last two terms on the right-hand side are always lost in the unidirectional propagation approach and represent its error.

We derive straightforward analytical estimate of the amplitude of the backward wave compared to the amplitude of the forward wave. Thus, defining the last two terms (in braces) of the solution as E_-^{induced}, we get:

$$\Gamma = \frac{E_-^{\text{induced}}}{E_+} \sim \frac{\pi\chi_3 E_{\max}^2}{3n_0 n_g} = \frac{n_2 E_{\max}^2}{3n_0 n_g} = \frac{2\tilde{n}_2 I}{3n_0 n_g}.$$

This estimate shows that for fused silica the error remains less than 1% for the input intensities up to about 10^{14} W/cm^2. Though for media with stronger nonlinearities it can reach higher values even at lower input intensities. With obtained solution it is obvious to see that reflected part can be eliminated by correct formation of initial backward wave distribution. Our numerical simulations show that exactly the same distribution can be applied for elimination of reflection even in the case of full set of equations.

Inverse spectral theory and Minkowski problem for surface of revolutions

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We solve the inverse spectral problem for rotationally symmetric manifolds, which include the class of surfaces of revolution, by giving an analytic isomorphism from the space of spectral data onto the space of functions describing the radius of rotation. An analogue of the Minkowski problem is also solved. Joint results with Hiroshi Isozaki, Japan.

A mathematical model of electromagnetic wave diffraction on a lattice of the special form: frequency analyze and loss

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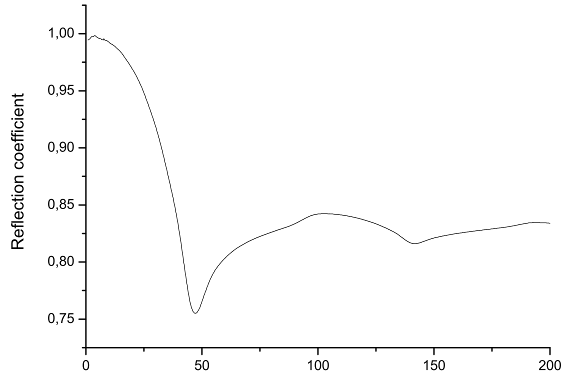
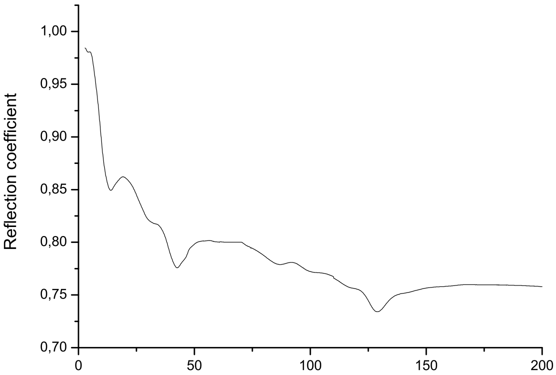
To construct this mathematical model the method represented in [1] and modified in [2] is used. This method solves a two-dimensional diffraction problem for a plane monochromatic electromagnetic wave on a lattice which consists of impedance strips of different widths. As in [2] the mentioned problem is reduced to two boundary integral equations uniformly with respect to polarization, E or H. One equation has a kernel with a logarithmic singularity and the other one is hypersingular.

Two lattices are investigated. Their intersections with a plane has the following form:

1. $\left(-a, \frac{a}{3}\right)$ and nine intervals of $\frac{2a}{30}$ width on $\left(\frac{12a}{30}, \frac{30a}{30}\right)$ separated by gaps of the same width;
2. $\left(-\frac{123a}{123}, \frac{41a}{123}\right)$ and fourth-stage pre-Cantor set on $\left(\frac{42a}{123}, \frac{123a}{123}\right)$.

The following figures show the relation between the reflection coefficient and the product of the wave number denoted as k and a which denotes a half of the lattice width in the case of orthogonally incident E-polarized electromagnetic wave on the lattice. The left figure shows this relation for the first lattice and the right figure shows it for the second one. The following parameters are used: $Z = 10^{-6} - 0,002\mathrm{i}$ is the impedance of the strips, $\varepsilon = 1$ is the permittivity of the medium.





This frequency analyze made on the base of the mentioned mathematical model allows us to estimate the scattering loss.

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Optimal way to derive parabolic equation for bulk-acoustic-wave beams in anisotropic media

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The problem of wave-beam propagation in anisotropic media is of importance for fundamental and applied research. In spite of a long history of study, this problem is still far from its completion. A powerful and advantageous method to describe wave-beam diffraction and focusing is a parabolic-equation formalism. Nevertheless, there are only a few publications which are concerned with the development of this method for bulk acoustic waves in anisotropic solids. In our previous study, an attempt was done to derive the parabolic equation for bulk-acoustic-wave beams in crystals [1]. For that we transform the Green–Christoffel equation into an algebraic equation which is reduced to a quadric form by using the paraxial approximation. Further replacement of the coordinate projections of the wave vector by the derivates with respect to the coordinates gives us a paraxial wave equation. Then simple rotations and scale changes of the coordinates transform the last equation into an equivalent isotropic form from which it is easy to derive the parabolic equation. However, the mentioned replacement is not mathematically rigorous. Besides the wavefunction in the paraxial wave equation is not uniquely defined in this case. The aim of the present study is to develop a more accurate approach to the problem under study. The suggested procedure includes the following steps.

1) The system of equations of motion written in the crystallographic basis is transformed into a rotated coordinates with one of axes parallel to the surface normal of the anisotropic medium.

2) The solutions to the transformed equations are found in the form of three plane bulk acoustic waves propagating along the surface normal and the polarization of these waves is determined.

3) In the equations of motion written in the rotated coordinates related to the normal to the surface of the anisotropic solid, the displacement vector is represented in a new orthonormal basis coinciding with the polarization vectors of the plane waves.

The equations system and its solution in the form of three plane waves obtained as the result of the aforementioned operations have the following properties: a) only one derivative corresponding to the normal to the medium surface is not equal to zero, b) only one component of the displacement vector is not equal to zero, which corresponds to the selected polarization of one of the three plane waves that can propagate in the given direction. The advantage of such a representation of the equations of motion in comparison with alternative forms is the maximum simplification in constructing approximate equations for bulk acoustic wave beams. This is due to the fact that, when the main displacement component of the wave beam is chosen, the other two components as well as the derivatives with respect to the coordinates different from the surface normal are proportional to a small parameter. The described method is used for simulating experiments on anisotropic beam steering and anisotropic diffraction studied in particular in Ref. [2]. There is a good agreement between the calculated and observed results.

It worth to mention that an attempt to find a parabolic equation for acoustic beams in crystals of general anisotropy was previously done in paper [3]. However the equation derived in [3] does not contain all cross derivatives with respect to the coordinates; so it does not describe correctly, in contrast to the current study, the properties of acoustic beams in anisotropic media.

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Modeling of gap gene expression in Drosophila Kruppel mutants

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The segmentation gene network in *Drosophila* embryo determines both the positions and identities of body segments. We have applied the systems-level approach to investigate the regulatory effect of gap gene *Kruppel* (*Kr*) on segmentation gene expression. Careful investigation of acquired large dataset on the expression of gap genes in *Kr* null mutants revealed that the expression levels of these genes are significantly reduced in the second half of cycle 14A. We applied the gene circuit method to explain this biological phenomena [1].

The gene circuit method estimates regulatory parameters that predict a specific network topology by fitting the computed model output to gap gene expression patterns in wild type and in embryos with homozygous null mutation in Kr gene simultaneously using DEEP method [2]. The fitted model correctly reproduces the characteristic features of gap gene expression in *Kr* null mutants. We performed over 200 runs with different initial parameter approximations and control variables. The search space was sampled uniformly for each parameter in the interval defined by biologically relevant limits. Two step procedure was applied to construct the ensemble of parameter sets. On the first stage, the residual mean square (RMS) was checked and the sets with RMS less than 5% of the maximal gene expression value (equals 255 in our data) were accepted for further analysis. Secondly, we inspected the model expression patterns visually. Consequently, the ensemble of 11 parameter sets was obtained that correctly reproduces the dynamics of gene expression in wild type and mutant embryos, in particular the decrease of gap gene expression levels and the anterior shift of gt domain.

We found that the remarkable alteration of gap gene expression patterns in Kr mutants can be explained by the dynamic decrease of activating effect of Cad on a target gene and exclusion of *Kr* gene from the complex network of gap gene interactions, that makes it possible for other interactions, in particular, between hb and gt, to come into effect. The successful modeling of the quantitative aspects of gap gene expression in mutant for the trunk gap gene *Kr* is a significant achievement of this work. This result also clearly indicates that the oversimplified representation of transcriptional regulation in the previous models is one of the reasons for unsuccessful attempts of mutant simulations [3].

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Special solutions of Maxwell equations

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Special solutions being considered hereunder are defined by the fact that the vectors describing electromagnetic field corresponding to them are equal to zero in free space points, i.e. beyond restricted domain V occupied by a substance, at the same time field electromagnetic potentials are different from zero in the whole space. The paper demonstrates that Maxwell equation solutions will be special if current density $\boldsymbol{j}$, polarization $\boldsymbol{P}$ and magnetization $\boldsymbol{J}$, considered as electromagnetic field sources represent gradients of smooth finite functions in V domain under any $t \in (-\infty, \infty)$.

The essence and physical meaning of the special solutions may be explained through consideration of current density $\boldsymbol{j}$. Let V be the restricted domain with smooth boundary S, $\boldsymbol{P} = 0$ and $\boldsymbol{J} = 0$, then Maxwell equations in V domain may be given by:

$$\mathrm{rot}\boldsymbol{H} = \frac{1}{c}\left(\frac{\partial}{\partial t}\boldsymbol{E} + \boldsymbol{j}\right), \quad \mathrm{rot}\boldsymbol{E} = -\frac{1}{c}\frac{\partial}{\partial t}\boldsymbol{H}, \quad \mathrm{div}\boldsymbol{E} = \rho, \quad \mathrm{div}\boldsymbol{H} = 0. \tag{1}$$

Assuming that

$$\boldsymbol{j} = -\frac{\partial}{\partial t}\boldsymbol{E}, \quad x \in V, \quad \boldsymbol{E} = 0, \quad \boldsymbol{j} = 0, \quad x \notin V, \tag{2}$$

it is easy to show that (2) is compatible with (1) only if

$$\boldsymbol{j} = \mathrm{grad}\psi(x,t), \quad x \in V, \quad t \in (-\infty, \infty), \quad \psi(s,t) = 0, \quad s \in S.$$

In this case $\boldsymbol{H} = \boldsymbol{B} = 0$, $\boldsymbol{D} = \boldsymbol{E}$, at that:

$$\boldsymbol{E} = -\mathrm{grad}\int_{-\infty}^{t}\psi\, dt', \quad \Delta\psi(x,t) = \frac{\partial}{\partial t}\rho, \quad \frac{\partial}{\partial n}\psi(s,t) = -\frac{\partial}{\partial t}\sigma, \tag{3}$$

where n – outer normal to S. The absence of the delay in the time between $\boldsymbol{j}$ source values and $\boldsymbol{E}$ field created by it in (2) and (3) is evident from (3). The same result follows from representations of electromagnetic fields through sources obtained in [1]. Again, [1] implies also dissimilarity of electromagnetic potentials from zero within the whole space. Special solutions for $\boldsymbol{P}$ and $\boldsymbol{J}$ are developed in similar way.

Consideration of equation (1) solution in [2] for AC flowing in infinitely long cylindrical conductor of R radius under the effect of Ohm law $\boldsymbol{j} = \gamma\boldsymbol{E}$ and $\omega \ll \gamma$ reveals that volumetric density of Lorentz force $\boldsymbol{f}$ represents potential vector identical in each conductor section and directed to its axis along radius:

$$\boldsymbol{f} = -\mathrm{grad}\frac{1}{c}[\boldsymbol{j}, \boldsymbol{H}] = -\mathrm{grad}\ \mathrm{F}(r,t) = -\boldsymbol{e}_r {F_r}'(r,t) = n_0\boldsymbol{e}_0\boldsymbol{E}'(r,t),$$

where $\boldsymbol{j}$ and $\boldsymbol{H}$ are defined by general solution of equations (1) in this case, n_0 – density of conductivity electrons, e_0 – their charge. The last equality in (4) shows, that the effect of $\boldsymbol{f}$ force on conductivity electrons being current components is equivalent to the effect of the potential electrical field $\boldsymbol{E}'$. The effect of such force $\boldsymbol{f}$ on moving electrons makes them concentrating on cylindrical axis. In fact reverse process takes place-current concentration nearby cylindrical surface. It means that Lorentz force effect is ample compensated by some unaccounted process. This compensation may be fulfilled by the electrical field $\boldsymbol{E}_* = -\boldsymbol{E}'$ created by the electrical charge density $\rho_* = \mathrm{div}\boldsymbol{E}_*$. The variable part of this charge density creates the electrical current flowing along radius with density:

$$\boldsymbol{j}_* = -\mathrm{grad}\frac{\partial}{\partial t}\frac{1}{n_0 e_0}F(r,t) = -\frac{\partial}{\partial t}\boldsymbol{E}_* = \frac{\partial}{\partial t}\boldsymbol{E}'.$$

Thus current density $\boldsymbol{j}_*$ represents special solution of equation (1) for the considered case.

It should be noted that within static limit for general case the number of alternate special solutions increases. There are solutions in form of completely solenoidal vector functions – for $\boldsymbol{P}$ and $\boldsymbol{J}$, and in form of rot from the finite solenoidal vector functions – for $\boldsymbol{j}$ ([3]).

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Numerical algrorithm of 1D dusty void model formation

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In present work we consider a one-dimensional model of dusty void formation according to from numerical methods point of view. Such kind investigation is related to the problem of modelling a quasi-periodic structure of electron clusters both dusty [1] and electron–ion plasma [2] fluid.

We consider the following system of governing equations in vector form:

$$\frac{\partial \mathbf{u}(x,t)}{\partial t} + \frac{\partial F(\mathbf{u}(x,t))}{\partial t} = \mathbf{f}(x,t), \quad \mathbf{u}(x,t) = \begin{pmatrix} n_d \\ v_d \end{pmatrix}, \quad \mathbf{f}(x,t) = \begin{pmatrix} 0 \\ (a/(b+|v_i|^3)-1)E - \alpha_0 v_d \end{pmatrix}, \tag{1}$$

$$\frac{\partial \mathbf{w}(x,t)}{\partial x} = \mathbf{g}, \quad \mathbf{w}(x,t) = \begin{pmatrix} n_e \\ E \end{pmatrix}, \quad \mathbf{g}(x,t) = \begin{pmatrix} -n_e E/\tau_i \\ 1 - n_d - n_e \end{pmatrix} \tag{2}$$

in the rectangle $x \in [0, L]$, $t \in [0, T]$ under the initial-boundary conditions

$$\mathbf{u}(x,0) = \begin{pmatrix} 0 \\ 5\cdot 10^{-4} \end{pmatrix}, \quad \mathbf{w}(0,t) = \begin{pmatrix} \alpha_1 \\ 0 \end{pmatrix}, \quad \mathbf{w}(L,t) = \begin{pmatrix} 0 \\ \beta_1 \end{pmatrix}, \quad \frac{\partial n_d(L,t)}{\partial x} = 0. \tag{3}$$

The algorithm of integrating of (1)–(3) is the following:

- Computation of the initial–value problem (1), (3) by one of the explicit first order schemes [3], for example Lax–Friedrichs scheme (the simplest one) on time step of integration $t^n \ \Rightarrow\ \mathbf{u}^\star$ is known.
- Computation of the boundary–value problem (2), (3) by tridiagonal block algorithm on the same time step of integration $t^n \ \Rightarrow\ \mathbf{w}$ is known.
- Recomputation of (1), (3) on the next time step of integration t^{n+1} with respect to $\mathbf{u}^\star$, $\mathbf{w}$.

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The new modified kernels and weight functions in the generalized Kravchenko–Kotelnikov sampling theorem

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The report consists of two parts. In the first part the new designs of kernels of the generalized cardinal series [1–6] are suggested and proved. This approach providing high interpolation properties and time–and–frequency localization. They are obtained by means of modification of Whittaker–Kotelnikov–Shannon function [1,2], and also family of the atomic functions (AF) [3–5]. Suggested modifications are focused on reduction of the function effective support at quality of frequency properties and reduction of an recovery error of sampled information signals. In the second part the new class of the weight functions (WF) for improvement of convergence of the generalized cardinal series is constructed. Inclusion of these functions improves restoration of signals at insufficient number of samples or in oversampling. WF are entered directly into a cardinal series and allow to eliminate the negative phenomena of transformation, and also to provide necessary time–and–frequency localization of an input and restored signal. The generalized cardinal series is written so $f(x) = \sum_{k=0}^{N-1} f_D[k]\, w(x)\, \varphi(x - \Delta k)$, where $f_D[k] = f(\Delta k)$, $k = 0,..,N-1$ is sampled signal $f(x)$ on an interval $[0,T]$, Δ is sampling step ($\Delta = T(N-1)$), $w(x)$ is the WF, $\varphi(x)$ is kernel function. If the restored signal is displaced on frequency, it is necessary to carry out wavelet-modulation of weight function $\tilde{w}(x) = w(x)\exp(i\eta x)$, where η is the parameter of modulation close to the central frequency of the restored information signal. Let us consider the following kernel functions: $\varphi_1(x,b) = \frac{\sin(\omega_c x)}{\sinh(b\omega_c x)}$, where $\omega_c = \frac{\pi}{\Delta}$ is the cutoff frequency, b is the parameter regulating width of the effective support of $\varphi_1(x,b)$, $\varphi_2(x,a,M) = \prod_{k=1}^{M} \operatorname{sinc}\left(\frac{\omega_c x}{a^k}\right)$ is Kravchenko–Kotelnikov kernel. We analyze these functions by the following [4] physical characteristics: relative position of amplitude frequency response (AFR) first zero γ_2, AFR width on level –3 dB γ_3, relative AFR width on level –6 dB γ_4, coherent amplification γ_7, maximum level of the side lobes (in dB) γ_9.

Behavior of kernel functions for the support $x \in [-1,1]$, and also their normalized AFR in Fig. 1 are shown, and their main physical characteristics agrees [4] in the Table 1. We used such functions as WFs

$$w_1(x,m) = \left[\operatorname{sinc}\left(\frac{x}{\tau}\right)\right]^m, \quad w_2(x;\delta) = \sin\left(2\pi\delta\sqrt{\left(\frac{x}{\tau}\right)^2 - 1}\right) \Big/ \left(\sinh(2\pi\delta)\sqrt{\left(\frac{x}{\tau}\right)^2 - 1}\right),$$

$$w_3(x,N) = \operatorname{fup}_N\left(\frac{N+2}{2\tau}x\right),$$

where the support is $x \in [-\tau,\tau]$. Their main physical characteristics in Table 2 are presented.

Table 1: Main physical characteristics of cardinal series kernels ($\Delta = 0.2$, $\omega_c = 5\pi$).

	γ_3	γ_4/γ_3	γ_2/γ_3	γ_9	J
Whittaker–Kotelnikov–Shannon kernel					
–	14,65	1,05	1,20	-22,7	0,241
$\varphi_1(x,b)$					
b =0.15	14,65	1,06	1,28	-33,3	0,220
b =0.20	14,50	1,07	1,39	-39,3	0,237
b =0.25	14,50	1,07	1,68	-50,1	0,282
Kravchenko–Kotelnikov kernel $\varphi_2(x,a,M)$					
$M=2$, a =5	14,50	1,07	1,37	-38,8	0,233
$M=2$, a =6	14,65	1,06	1,31	-35,9	0,220
$M=2$, a =7	14,65	1,06	1,28	-32,2	0,224
$M=2$, a =8	14,65	1,06	1,26	-29,6	0,228

Table 2: Main physical characteristics of weight functions.

	γ_3	γ_4/γ_3	γ_2/γ_3	γ_7	γ_9	AC	γ_{10}	γ_{11}
Weight function $w_1(x,m)$								
m =1	1,94	1,38	2,62	0,49	-26,4	0,54	1,89	3,03
m =2	2,39	1,37	3,06	0,50	-39,6	0,51	1,22	3,34
m =3	2,84	1,37	3,42	0,47	-53,4	0,50	0,89	3,72
Weight function $w_2(x,\delta)$								
δ =2	2,84	1,47	4,53	0,51	-97,1	0,50	0,82	3,86
Weight function $w_3(x,N)$								
N =0	2,39	1,37	2,62	0,50	-23,3	0,54	1,21	3,30
N =1	2,69	1,44	3,50	0,48	-37,2	0,51	0,93	3,64
N =2	2,99	1,45	4,20	0,50	-50,8	0,50	0,75	3,96

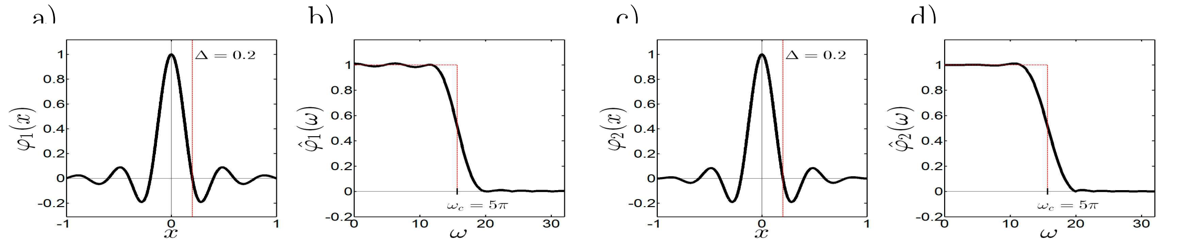

Fig. 1: Kernel functions $\varphi_1(x)$ (a), $\varphi_2(x)$ (c) on $x \in [-1, 1]$, and their normalized AFC (b), (d).

Conclusions. Numerical experiment and the analysis of results showed advantages of use of new kernels and WF in the generalized cardinal series. The suggested approach can be applied in various problems of digital signal and image processing, wavelet 1D and 2D analysys. This work was supported by Russian Foundation for Basic Research (Project No. 12-02-90425).

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Weight functions based on the convolutions of atomic functions

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In the report new weight functions based on convolutions of atomic functions [1] are presented. Weight functions (windows) are widely used in digital signal processing. To characterize quality of weight function several numerical characteristics were introduced. Among them there are coherent gain and highest side-lobe level [2, 3]. From these characteristics worst case process lost characteristic was constructed. Good window must have low side-lobes and high coherent gain. According to uncertainty principle these requirements are conflicting.

Atomic functions are infinite convolutions of rectangular pulses, then their side-lobes decreasing very fast [3]. According to the properties Fourier transform convolution in spatial domain corresponds to the multiplication in frequency domain [4]. Then if given function has some side-lobe level, it will double after convolution of function with itself and multiplied by n after n-time convolution. Then, to obtain window with low side-lobes one can take convolutions of some functions. Note, that there are effective algorithms for computation of convolutions of atomic functions then computation of multiple convolution of atomic functions is very convenient [5]. In this report windows constructed from atomic functions $\mathrm{ch}_{a,n}$ and fup_n are discussed.

Due to uncertainty principle coherent gain for such window will be too low. To increase coherent gain we will cut the window. After this operation spectrum of the window will be convoluted with sinc(ω) spectrum of rectangular pulse and side-lobes will increase, but if their level was very low, it will state acceptable. To smooth discontinuity at the ends of segment and reduce increase of side-lobes additional window can be used. For this purpose window with high coherent gain will be useful.

This approach allows to obtain windows with good properties confirmed by the numerical experiment. The degree of cut of the function determine the relation between side-lobes and coherent gain. Usually the best windows obtained with the cut near to the effective support of function.

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Homogenization of high order elliptic systems with periodic coefficients

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The report focuses on homogenization problem for high-order elliptic systems with periodic coefficients. The method is generalization of the operator-theoretic approach for second-order operators suggested by M. Birman and T. Suslina [BSu1-2] and developed by N. Veniaminov [V] for some class of high-order systems.

In $L_2(\mathbb{R}^d;\mathbb{C}^n)$ we consider an elliptic operator of order $2p$, admitting a factorization of the form

$$A_\varepsilon = b(\mathbf{D})^* g(\mathbf{x}/\varepsilon)\, b(\mathbf{D}).$$

Here $\varepsilon > 0$ and g is a positive definite bounded $(m \times m)$ matrix-valued function periodic with respect to some lattice in $\mathbb{R}^d$; $b(\mathbf{D})$ is an $(m \times n)$ matrix homogeneous differential operator of order p. It is assumed that $m \geqslant n$. The symbol $b(\boldsymbol{\xi})$ is subjected to the following condition:

$$\operatorname{rank} b(\boldsymbol{\xi}) = n, \quad \mathbf{0} \neq \boldsymbol{\xi} \in \mathbb{R}^d.$$

We study the behavior of the resolvent $(A_\varepsilon + I)^{-1}$ for small ε. It turns out that $(A_\varepsilon + I)^{-1}$ can be approximated in the norms of $\mathbf{B}(L_2)$ and $\mathbf{B}(L_2; H^p)$:

$$\left\| (A_\varepsilon + I)^{-1} - \left(A^0 + I\right)^{-1} \right\|_{\mathbf{B}(L_2)} \leqslant C\varepsilon, \tag{1}$$

$$\left\| (A_\varepsilon + I)^{-1} - \left(A^0 + I\right)^{-1} - \varepsilon^p \mathcal{K}(\varepsilon) \right\|_{\mathbf{B}(L_2;H^p)} \leqslant \widetilde{C}\varepsilon. \tag{2}$$

Here

$$A^0 = b(\mathbf{D})^* g^0 b(\mathbf{D})$$

is the so-called *effective operator* with constant matrix g^0; the operator $\mathcal{K}(\varepsilon)$ is the corrector. Estimates (1) and (2) are order-sharp, the constants C and $\widetilde{C}$ are well controlled in terms of the problem data.

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On high spots of the fundamental sloshing eigenfunctions in axially symmetric domains

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We investigate the classical eigenvalue problem that arises in hydrodynamics and is referred to as the sloshing problem. It describes free liquid oscillations in a liquid container $W \subset \mathbb{R}^3$. The Cartesian coordinates (x, y, z) are chosen so that the mean free surface of the liquid F lies in the (x, z)-plane and the y-axis is directed upwards. We study the case when W is an axially symmetric, convex, bounded domain such that $W \subset F \times (-\infty, 0)$. Our first result states that the fundamental eigenvalue has multiplicity 2 and for each fundamental eigenfunction φ there is a change of x, z coordinates by a rotation around y-axis so that φ is odd in x-variable.

The second result of the paper gives the following monotonicity property of the fundamental eigenfunction φ. If φ is odd in x-variable then it is strictly monotonic in x-variable. This property has the following hydrodynamical meaning. If liquid oscillates freely with the fundamental frequency according to φ then the free surface elevation of the liquid is increasing along each line parallel to the x-axis during one half-period of time and decreasing during the other half-period. The proof of the second result is based on the method developed by D. Jerison and N. Nadirashvili for the hot spots problem for the Neumann Laplacian.

The talk is based on my paper with M. Kwaśnicki "On high spots of the fundamental sloshing eigenfunctions in axially symmetric domains", Proc. London Math. Soc. 105 (2012), 921–952.

Characterization of binary gratings using scatterometry

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We present a nondestructive technique for the characterization of transmission gratings using scatterometry. Obtained experimental results compared with SEM data are found very satisfactory.

Scatterometry is a well-known technique widely used for studying structures of the order of wavelength of light. Here, we studied binary transmission gratings fabricated on glass substrate. The period of the gratings was known beforehand but parameters like fill factor (c), height (h), and sidewall angle (α) were unknown. For our experiments, we used a homemade scatterometric set-up. He-Ne laser with wavelength 633 nm (<10mW) was used with a standard photodiode sensor as a detector. Using TE-polarized input light, the angle of incidence was varied from 0 to 30° with a step of 5°, and diffraction efficiencies of -2, -1, 0 and $+1$ orders were recorded. Experiment was then repeated for the TM polarized input. The resulting efficiency data was then imported into our rigorous modeling tool, based on the Fourier Model Method (FMM), which was used to fit the unknown parameters. An SEM image of one of the grating samples, with period 1 μm, is presented below. For this particular sample, our scatterometry-based fitting gave values $c = 395$ nm, $h = 990$ nm, and $\alpha = 87.4°$, which are very close to data obtained from SEM image, i.e., $c = 400.7$ nm, $h = 1030$ nm, and $\alpha = 87.4°$.

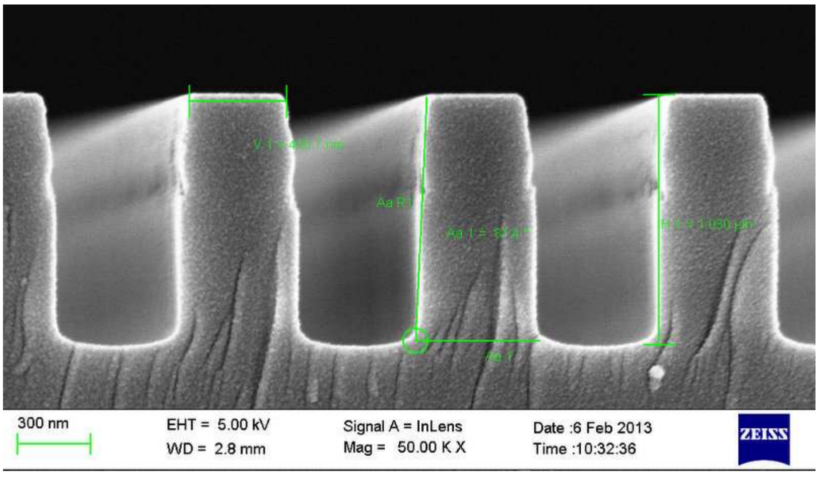

Fig. 1: SEM image of a sample transmission grating with period 1 μm.

Steady water waves with vorticity: spatial Hamiltonian structure

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Spatial dynamical systems are obtained for two-dimensional steady gravity waves with vorticity on water of finite depth. Some of these systems have Hamiltonian structure and Hamiltonian in these cases is the flow-force invariant.

The presented material is based on a joint work with Vladimir Kozlov, Linköping University, Sweden.

The application of the method of continued boundary conditions to the problem of wave diffraction on an impedance screen

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The method of continued boundary conditions (MCBC), offered in work [1], has been subsequently applied to the solution of a wide class of problems of the theory of diffraction, and high efficiency of the method has been shown in all cases [2, 3]. In the present work the scattering on a thin screen as the body of revolution with the variable impedance is considered. Both the stationary and nonstationary problems are examined in the paper. The nonstationary problem is reduced to the stationary one by using the Fourier integral.

Consider the mirror antenna as the thin screen of revolution. Below we formulate the stationary boundary problem. We assume that the equations of the contour of the cross section of the body are $\rho = \rho_s(s)$, $z = z_s(s)$, where $s \in [0, s_2]$. In the central part of the screen (that is for $s \in [0, s_1]$) the

Dirichlet condition is fulfilled and on the rest part of the surface of the screen (for $s \in [s_1, s_2]$) the following conditions are valid

$$u_+ - u_- = 0,$$
$$u = \frac{W}{k}\left(\frac{\partial u_+}{\partial n} - \frac{\partial u_-}{\partial n}\right). \tag{1}$$

Here $u = u^0 + u^1$ where u^0 is the primary field and u^1 is the secondary or scattered field, k is the wave number in the surrounded space, $\partial/\partial n$ means the derivative on the normal to the surface of the antenna directed to the source of the field. The values $u_+,\ u_-$ in the formula (1) are the meaning of the wave field at the two sides of the screen. That is we have the jump of the normal derivative on the surface of the screen. Note that the value W in the formula (1) is the function of coordinates.

We assume that the screen is irradiated by the field produced by the point source located on the axis of symmetry of the body. In accordance with MCBC we assume that the condition (1) is valid on the surface S_δ which is located at the little distance from the surface of the screen. Thus the following integral equation can be derived

$$W(\vec{r})j(\vec{r}) + \frac{k^2}{4\pi}\int_S G(\vec{r},\vec{r}')j(\vec{r}')d\sigma' = u^0(\vec{r}), \quad \vec{r} \in S_\delta \tag{2}$$

where $G(\vec{r},\vec{r}') = \frac{\exp(-ikR)}{kR}$, $R = |\vec{r} - \vec{r}'|$, $j(\vec{r}')$ is the unknown function proportional to the jump of the normal derivative. The equation (2) is solved numerically using the collocation technique.

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The T-matrix method on the basis of modified null-field method

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The T-matrix method, proposed by P. C. Waterman [1], belongs to the most actively used methods of solving wave diffraction and scattering problems [2]. Its popularity is caused mainly by simple relation, connecting coefficients of some spherical or cylindrical basis expansion of incident and reflected waves, interacting with the scatterer. However, as it was recently shown in our papers [3, 4], the T-matrix method is valid only for so called Rayleigh scatterers [5], which have all diffracted field analytical continuation beyond area of original definition singularities contained inside of a sphere (circle), entirely inscribed in the scatterer. The class of such geometries is quite limited.

In the present work we propose the approach, in which the null field condition, underlying T-matrix method, is satisfied on some surface (or curve), obtained by analytical deformation of the scatterer boundary, which incorporate all wave field analytical continuation singularities, rather than on a sphere (or a circle in two-dimensional case). As a result, we obtain the T-matrix relation between scattered wave coefficients of spherical (cylindrical) basis expansion and values of incident wave at discrete points of surface (curve). As opposed to traditional T-matrix method, the proposed modification is applicable to solving diffraction problems on scatterers with practically any geometry.

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Scattering of waves by periodic surface: modified null-field method

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The modified null-field method (MNFM), offered in [1], has been successfully applied by a number of authors to solve the problem of scattering (diffraction) of waves by compact bodies (three-dimensional problem) or infinite cylinders with a limited cross-section (two-dimensional problem) (see, e.g., [2] and references therein).

Within the framework of the MNFM the general null-field equation considered on the certain (so-called "auxiliary") surface (contour) inside scatter. The auxiliary surface should construct by the analytical deformation of the boundary of body. Thereto the auxiliary surface should cove all the singularities of analytical continuation of scattered wave field. This technique provides simplicity, effectiveness and stability of the MNFM [1, 2].

In this paper we present application the MNFM to investigation the two-dimensional problem of electromagnetic scattering by a perfectly conducting (the Dirichlet problem) periodic surface (grating) with a unlimited cross-section. The periodic surface examples include

a) a cosinusoidal surface: $y = h(x) = a\cos(px), a > 0, p = 2\pi/l, x \in \mathbf{R}$, where l is the period;

b) a cycloidal surface: $x = a(t + \tau\cos t), y = a\tau\sin t, a > 0, 0 < \tau \le 1, t \in \mathbf{R}$.

In the complex plane the coordinates of the singular points we can write as

a) $\zeta_0 = ql - ia\left(\dfrac{1}{ap}\ln\dfrac{1+\sqrt{1+a^2p^2}}{ap} - \dfrac{\sqrt{1+a^2p^2}}{ap}\right), q = 0, \pm1, \pm2, ...;$

b) $\zeta_0 = a\left(\dfrac{\pi}{2} + 2\pi q\right) + ia(1 + \ln\tau), q = 0, \pm1, \pm2, ...$

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The perturbation action optimization of the powerful electromagnetic wave in the ionospheric plasma

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In the experiments on the effects of powerful electromagnetic radiation to the ionosphere plasma F-region, ion acoustic perturbations of the plasma concentration arise. Previously, we analyzed such disturbances, caused by short pulse of a standing electromagnetic wave [1]. The expression for the plasma concentration perturbation $n(z,t)$, similar to the Green function and can be used in deducing the solution, caused by the long time powerful wave action. Thus, you can get for the perturbations of the plasma concentration far from the level of reflection of a powerful wave $k(z)\cdot z \gg 1$:

$$n(z,t) = C\cdot\int_0^\tau \frac{\sin((t-t')\sqrt{(2k(z)\cdot V)^2-\gamma^2})}{\sqrt{(2k(z)\cdot V)^2-\gamma^2}}\cdot\exp(-\gamma\cdot(t-t'))\cdot\sin\left(\frac{4}{3}k(z)\cdot z+\varphi(t')\right)\cdot dt'.$$

Here $k(z)$ is the wave number of a powerful wave at the level of z, V is the velocity of the ion sound waves, and γ is its damping decrement, τ is the duration of perturbation wave impulse (we assume $t > \tau$). Upon deduction of this expression the linear dependence was used of the dielectric permittivity of the coordinates z, which corresponds to the Airy function for powerful wave electric field. Disturbance of phase $\varphi(t')$ ($\varphi(0) = 0$) takes into account the distortion of the profile of plasma near the level of reflection of an electromagnetic wave (at $z = 0$).

In this expression integrand changes sign with time because of the multiplier $\sin((t-t')\cdot\Omega(z))$, where $\Omega = \sqrt{(2k(z)\cdot V)^2-\gamma^2}$ is the frequency of forced ion-sound waves at the level of z. It is clear that the continuous long-term action of a powerful wave inefficient excites ion-sound of perturbations. Under the conditions $\gamma << \Omega$, $\varphi = 0$ it is easy to obtain the expression:

$$n(z,t) \approx const\cdot\exp(-\gamma t)\cdot[\exp(\gamma\tau)\cdot\cos(\Omega\cdot(t-\tau)) - \cos(\Omega\cdot t)]$$

from which follows, that the amplitude of perturbations in the case $\tau = \pi/\Omega$ excess of the amplitude in the case (when $\tau = 2\pi/\Omega$) of prolonged action in $\frac{\exp(\gamma\tau)+1}{\exp(\gamma\tau)-1} \simeq \frac{2\Omega}{\gamma} >> 1$ times.

So, to increase the amplitude of the perturbations in the some height z, you can instead of a continuous wave the use of pulse action with equal duration of pulse and pause $\Delta t = \pi/\Omega(z)$.

The message will also discuss the limitations, which makes the case $\varphi(t) \neq 0$ on the above simple considerations. Phase perturbations was obtained by the use of numerical integration of the equations for the electromagnetic field and deformation of plasma concentration profile because of the ion-acoustic waves in a neighborhood of the level of reflection $z \approx 0$, where asymptotic methods of analysis are inapplicable. Some analitical results on the quasistatic plasma profile deformation by the electromagnetic field published in [2]. But in the case of interest quasistatic approximation takes no place.

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New boundary integral equation methods for optical waveguides with corners

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Finding guided or leaky modes for optical waveguides is a classical problem that has been considered by many authors. For waveguides of current interest, including silicon waveguides, plasmonic waveguides and photonic crystal fibers, a rigorous analysis based on the full Maxwell's equations is essential. Existing numerical methods such as the finite element method have limitations in accuracy and efficiency due to field singularities at waveguide corners and/or complicated microstructures. Boundary integral equation (BIE) methods have also been used to analyze optical waveguides [1, 2]. For waveguides with smooth interfaces (discontinuities of the refractive index profile), full-vectorial BIE methods usually solve four functions on the interfaces. In a recent work [3], we developed a more efficient BIE method that solve only two unknown functions on the interfaces, but it is still limited by field singularities at waveguide corners. In another work [4], we developed a high order BIE method for waveguides with corners, but it requires solving four unknown functions on the interfaces. In this paper, we present two new high order BIE methods for waveguides with corners, solving two or three unknown functions on the interfaces, respectively. The required number of operations of these new methods is roughly 12.5% or 42.2% of the four-function method [4]. The two-function method relies on a new high order scheme for discretizing a related hypersingular boundary integral operator. The three-function method is simpler to implement.

For an optical waveguide given by a piecewise constant refractive index profile $n = n(x, y)$, where the waveguide axis is parallel to the z-axis and the discontinuities of n are located on the interface Γ, a guided mode is a solution of Maxwell's equations where the z dependence is separated as $e^{i\beta z}$ and the electromagnetic field decays to zero as $r = \sqrt{x^2 + y^2} \to \infty$. Our BIE methods are given in the following form

$$F(\beta)\boldsymbol{\phi} = 0 \quad \text{on} \quad \Gamma,$$

where $F(\beta)$ is a matrix with operator entries which can be calculated by solving BIEs. In our two-function method, $\boldsymbol{\phi}$ is the a vector for the x and y components of the megnetic field $\boldsymbol{H}$. In the three-function method, $\boldsymbol{\phi}$ is a vector for the normal derivative of the normal component of $\boldsymbol{H}$ (which is continuous across Γ), and the one-side limits of normal derivative of the tangential component of $\boldsymbol{H}$ (which is not continuous across Γ). If Γ is discretized by N points, F is approximated by a $(2N) \times (2N)$ or $(3N) \times (3N)$ matrix for these two methods, respectively. The value of β is determined from the condition that $F(\beta)$ is singular. To find F, we solve BIEs using a Nyström method with kernel-splitting and graded mesh techniques. The two-function method requires a new kernel-splitting technique for a hypersinbgular integral operator, since the original method developed by Kress [5] cannot be used with a graded mesh technique for waveguides with corners.

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The problem of surface electromagnetic TE wave propagation in an inhomogeneous plane layer dielectric waveguide

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The problem of surface electromagnetic TE wave propagation in an inhomogeneous plane layer dielectric waveguide is considered. The permittivity inside the layer is described by Kerr law. Inhomogeneity of the waveguide is modeled by different functions. The problem is reduced to a nonlinear integral equation with the Green function kernel. The existence of surface TE waves is proved. Two methods are proposed for the numerical solution of the problem: an iterative algorithm (with proven convergence), and a method based on the solution of the auxiliary Cauchy problem (false position method). The existence of the roots of the dispersion equation — the waveguide propagation constants — is proved. Linear and nonlinear cases are compared. Numerical results are presented.

Buldyrev-type interference head wave in diffraction by inhomogeneous halfspace

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We consider an interface of two halfspaces, one of which is homogeneous and the other is vertically inhomogeneous. The square of wave-number inside inhomogeneous region is supposed to decrease linearly with distance from the boundary. High-frequency point source is acting in the homogeneous medium. The goal is derivation of the asymptotic of a whispering gallery waves and of Buldyrev-type interference head wave [1] as $k \to \infty$, where k is the wave-number in a homogenous halfspace.

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Application of the generalized variational principle for description of sound propagation in non newtonian fluids

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The generalized variational principle (GVP) for dissipative continuum mechanics [1–3] combines Hamilton's variational principle for dissipationless mechanics with Onsager's variational principle for dissipative thermodynamical systems. It was shown that the motion equations of dissipative hydrodynamics can be derived on the basis of GVP. The shear and bulk viscosities can be introduced into equations of dissipative hydrodynamics by using of the Mandelshtam–Leontovich theory of internal parameters [1, 3]. This approach generalizes Navier–Stokes equation taking into account viscosity relaxation phenomenon. It was shown [4–5] that the internal parameter responsible for shear viscosity can be interpreted as a consequence of relaxation of angular momentum of material points constituting mechanical continuum. The rotational degree of freedom appears as independent variable additionally to the mean mass displacement field. For the dissipationless case this approach

leads to the well-known Cosserat continuum. When dissipation prevails over inertion this approach describes local relaxation of angular momentum and corresponds to the sense of internal parameter.

In the report GVP is applied for description of sound propagation in non Newtonian fluids.

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Particle storage in a nanolayer: Hartree–Fock approximation

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The system of interacting electrons placed into a two dimensional deformed nanolayer is considered. The interaction is described by δ-potential. The discrete spectrum of the system is studied. To treat the multi-particle problem the Hartree–Fock approach and the finite element method are engaged. Comparison of the storage efficiencies in cases of different geometries, number of particles and total spin is made.

Homogenization in the Sobolev class $H^1(\mathbb{R}^d)$ of the Cauchy problem for a parabolic equation with rapidly oscillating coefficients

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The talk is devoted to homogenization of periodic differential operators. We consider a matrix elliptic second order differential operator B_ε, $0 < \varepsilon \leqslant 1$, acting in the space $L_2(\mathbb{R}^d, \mathbb{C}^n)$. The operator B_ε is positive definite. The principal part of B_ε is given in a factorized form $b(\mathbf{D})^* g(\varepsilon^{-1}\mathbf{x}) b(\mathbf{D})$, where g is a periodic, bounded and positive definite matrix-valued function. Next, $b(\mathbf{D})$ is a matrix first order operator; its symbol has maximal rank. The operator B_ε also includes zero and first order terms with unbounded coefficients. The coefficients are periodic with respect to the lattice $\varepsilon\Gamma$.

We study homogenization in the small period limit for a periodic parabolic Cauchy problem:

$$\partial_t \mathbf{u}_\varepsilon(\mathbf{x},t) = -B_\varepsilon \mathbf{u}_\varepsilon(\mathbf{x},t); \quad \mathbf{u}_\varepsilon(\mathbf{x},0) = \boldsymbol{\phi}(\mathbf{x}). \tag{1}$$

Here $\mathbf{u}_\varepsilon(\mathbf{x},t)$ is a $\mathbb{C}^n$-valued function of $\mathbf{x} \in \mathbb{R}^d$ and $t \geqslant 0$. Suppose that $\boldsymbol{\phi} \in L_2(\mathbb{R}^d;\mathbb{C}^n)$. Our goal is to approximate the solution $\mathbf{u}_\varepsilon$ in terms of the solution of the "homogenized" problem:

$$\partial_t \mathbf{u}_0(\mathbf{x},t) = -B^0 \mathbf{u}_0(\mathbf{x},t); \quad \mathbf{u}_0(\mathbf{x},0) = \boldsymbol{\phi}(\mathbf{x}).$$

Here B^0 is the so-called effective operator with constant coefficients.

The problem reduces to the study of the operator exponential $\exp(-B_\varepsilon t)$ for small ε. We find an approximation for $\exp(-B_\varepsilon t)$ in the operator norm. Results of this type are called "operator error estimates" in homogenization theory. This approach allows us to approximate the solution $\mathbf{u}_\varepsilon$ of the problem (1).

Theorem. *Under the above assumptions, we have the following sharp order estimates*

$$\|\mathbf{u}_\varepsilon(\cdot,t) - \mathbf{u}_0(\cdot,t)\|_{L_2(\mathbb{R}^d;\mathbb{C}^n)} \leqslant C_1\varepsilon(\varepsilon^2+t)^{-1/2}\exp(-C_2 t)\|\boldsymbol{\phi}\|_{L_2(\mathbb{R}^d;\mathbb{C}^n)}, \quad t>0;$$
$$\|\mathbf{u}_\varepsilon(\cdot,t) - \mathbf{u}_0(\cdot,t) - \varepsilon\mathbf{v}_\varepsilon(\cdot,t)\|_{H^1(\mathbb{R}^d;\mathbb{C}^n)} \leqslant C_3\varepsilon t^{-1}\exp(-C_2 t)\|\boldsymbol{\phi}\|_{L_2(\mathbb{R}^d;\mathbb{C}^n)}, \quad 0<\varepsilon\leqslant t^{1/2}.$$

Here $\mathbf{v}_\varepsilon$ is the corrector. It contains rapidly oscillating factors and so depends on ε. The constants C_1, C_2, C_3 are well controlled in terms of the problem data.

The method of investigation is based on the abstract operator theory approach for selfadjoint operator families. This approach was developed for elliptic problems in [1, 3]. We apply the scaling transformation, the Floquet–Bloch theory and the analytic perturbation theory. It turns out that the homogenization procedure is determined by the spectral characteristics of the periodic operator near the bottom of its spectrum.

The homogenization problem for parabolic equations has been studied by this method in [2, 4] in the absence of lower order terms. We study a more general problem in the presence of lower order terms.

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Semiclassical asymptotics and density of states for 2D Schrodinger and Dirac equations with a tip-like potential

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Our study is related with the question: how does a tip of a scanning tunneling microscope affect what it measures. Mathematical problem consists in constructing explicit semiclassical asymptotic formulas for generalized eigenfunctions of 2D Schrodinger and Dirac equations with axisymmetric potential that simulates a potential on a surface induced by a charged tip. Then density of states for these eigenfunctions below the tip is to be found. It is important that to obtain essential physical results one requires not only the leading term of the asymptotics but also corrections to it, which makes the problem not so trivial. To solve the problem we use ideas of V. M. Babich, Yu. A. Kravtsov and recently proposed new representation of Maslov canonical operator near focal points.

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On evaluation of Green function for 3D ship wave problem

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The problem of steady forward motion of bodies in an open ocean is considered in the framework of the linear surface wave theory; most attention is paid to the Green function and methods of its numerical evaluation. The subject has got rather intensive development (see e.g. [1, 2] and references therein), directed to finding representations of the Green function and its derivatives, to elaborating accurate and fast computation techniques.

One of the problems in numerical evaluation is due to contribution of the integral (containing two differently oscillating factors) $\int_0^\infty e^{y(1+t^2)+i(x+zt)\sqrt{1+t^2}}\,dt$, where $x, y < 0$, $z \in \mathbb{R}$. A review and discussion of known approaches to approximate the integral and its derivatives can be found in [2]. In this work we suggest two alternative methods.

First of the methods is based on Levin's idea [3] allowing us to evaluate the above integral by finding one particular non-oscillating solution of an ordinary differential equation. The second method involves application of the steepest descent approach and Clenshaw–Curtis quadrature [4]. The methods are tested and compared for a wide variety of parameters, and shown to constitute a numerical scheme, being fast and robust for all moderate values of (x, y, z) except a small vicinity of the source's track $(x, 0, 0)$ (where Green's function is known to behave irregularly, see [5]).

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Discrete mathematical model of TM wave diffraction on pre-Cantor impedance strips on a shielded dielectric layer

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Consider a TM plane wave of unitary amplitude falling from infinity onto the top of a diffraction structure at an angle α:

$$u_{inc}^N(y,z) = H_x(y,z) = e^{ik(y\sin\alpha - z\cos\alpha)}. \tag{1}$$

Here α is an angle between positive direction of the Z axis and the direction of propagation of a plane monochromatic wave, and k is the absolute value of a wave vector (Fig. 1).

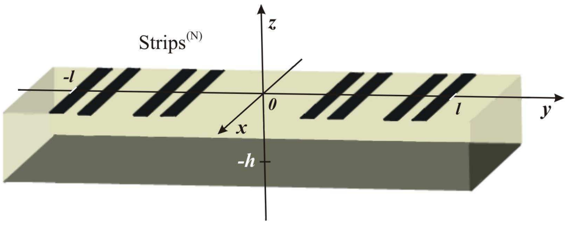

Fig. 1: Schematic of the considered diffraction structure.

In this paper we consider a TM case where the unique independent component of the magnetic field $H_x = u(y,z)$ satisfies two-dimensional Helmholtz equation above the flat screen reflector outside of the strips:

$$\Delta H_x(y,z) + k^2 H_x(y,z) = 0, \quad k = \frac{\omega}{c}. \tag{2}$$

It is also required to satisfy impedance boundary conditions of Shchukin–Leontovich on the strips and the shield, Sommerfeld radiation conditions and Meixner condition on the edges of strips.

Components of electromagnetic field vectors $\overrightarrow{E}(y,z)$ and $\overrightarrow{H}(y,z)$ are represented by $H_x(y,z)$ in the following form: $E_x(y,z) = H_y(y,z) = H_z(y,z) = 0$,

$$E_y(y,z) = -\frac{1}{i\omega\varepsilon}\frac{\partial}{\partial z}H_x(y,z), \quad E_z(y,z) = \frac{1}{i\omega\varepsilon}\frac{\partial}{\partial y}H_x(y,z). \tag{3}$$

As shown in the monograph [1] the boundary-value problem considering all conditions is reduced to system of singular integral equations of the first kind with the supplementary conditions and Volterra integral equations of the second kind. The solution of these integral equations is based on the method of parametric representation of integral operators.

The overall aim of this work is to construct a discrete mathematical model of boundary integral equations with the help of an efficient numerical discrete singularities method [1], [2].

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Laser diffractometry and evaluation of statistical characteristics of inhomogeneous ensembles of red blood cells

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We consider laser beam diffraction by an ensemble of red blood cells, deformed by a shear stress in the laser diffractometer. We propose an algorithm, which allows for determining the dispersion of the erythrocyte shape parameter and the asymmetry of the distribution in shapes for the erythrocytes, deformed in a shear flow in vitro, using the laser diffractometry technique.

An important parameter of red blood cells is its deformability, defined as a measure of cells ability to change their shapes in response to an external force. The deformability of blood cells is one of the main factors which determine the effective viscosity of blood. Also it determines the ability of the cells to move through the smallest vessels — capillaries. There are a number of techniques to measure the deformability, but usually only the deformability of individual cells or mean deformability of the cells in a blood sample can be evaluated. Meanwhile different cells even of a healthy person, and all the more so, the cells of a sick patient, possess different ability to deform. It provides a basis to consider the deformability as a statistical characteristic of the red blood cells population and to describe it by such notions as the distribution function, mean value, and variance.

Laser diffractometry (ektacytometry) is one of the technique to measure the red blood cells deformability. The method is based on the observation and analysis of diffraction patterns which arise when a laser beam is scattered by the diluted suspension of red blood cells. Usually conditions of single scattering and small angle scattering are realized. In one of the experimental realization of the ektacytometer the suspension is placed in a gap between the walls of two transparent coaxial cups, one of which is fixed and another can rotate with a prescribed angular velocity (so called Couette cell). The rotation of the movable cup causes fluid to flow and leads to appearance of a shear stress which deforms the red blood cells. To detect the deformation, the suspension is illuminated by a laser beam. Then the diffraction pattern arises which brings information about shapes of the particles under investigation.

In this paper we consider a problem of how to evaluate statistical moments of the red blood cells distribution in shapes using laser diffractometry. To solve the problem, we calculate the laser beam diffraction by an inhomogeneous ensemble of transparent elliptical discs, modeling red blood cells in the ektacytometer. In the anomalous diffraction approximation the problem is reduced to calculation of the Fraunhofer diffraction by an elliptical opening with definite square and orientation but with a random excentricity (elongation). We calculate spatial distribution of the scattered light intensity at the observation screen in the vicinity of the boarder of central diffraction maximum. Basing on the analytical expressions obtained, we have developed a number of algorithms which enable one to evaluate variance of the erythrocytes shape parameter, as well as to estimate the asymmetry of the particles distribution in shapes. As an example, we have considered laser beam scattering by a bimodal ensemble of particles. Comparison of the calculation results with available experimental data let us to conclude that the developed algorithms allow to obtain reliable data.

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The dispersion functions of quasi-2D periodic structures via Dirichlet-to-Neumann map

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The question on dispersion or conductance of quasi-2D periodic structures is important in studying of superconductivity in hand-made materials like quasi-periodic B-Si-B sandwich and in low-threshold field emission from cathodes covered with the quasi-2D carbon nano-structures. The B-Si-B sandwich reveals surprising properties, including high-temperature superconductivity at 145 K, see [1] which needs deeper understanding. Similarly, the low-threshold field emission from cathodes covered with quasi-2D periodic carbon nano-structures, see the references in [3], needs refurbishing of the traditional explanation of the low-threshold phenomenon based on presence of micro-protrusions on the surface of the cathode, which play a role of the field enhancement factor. This explanation does not work in our experiments , because surface of the carbon flakes is very smooth.

In this paper we suggest fitted models for quasi- 2D periodic lattices and sandwiches and calculate typical dispersion function of the objects based on rational approximations of the corresponding Dirichlet-to-Neumann maps (DN-maps) of the period. The rational approximations are interpreted as DN-maps of the corresponding fitted solvable models, which reveal some interesting physical properties like Landau–Zener enhancing of the BCS gap which may imply the high temperature superconductivity phenomenon, see [5, 2]. Based on a fitted solvable model of the space-charge region/ vacuum interface, we also analyse the low-threshold field emission from cathodes covered with the quasi-2D carbon nano-structures. In our model the role of the field enhancing factor is played by the small effective mass m_e of electron in the carbon structure, see [4]. The formula in [4] for the transmission coefficient was derived under assumption that the spectrum of the 1D size-quantization model is diccrete , while in 2D case it is continuous and consists of a sequence of flat spectral bands, with nontrivial dispersion $E = E(p)$ and the density of states $1/|dE/dp|$. Based on the 2D size quantization model with continuous spectrum, we suggest an algorithm of calculation of the transmission of electrons from the space-charge region into vacuum.

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Asymptotic solution for the problem of acoustic waves propagation in a penetrable truncated wedge

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We consider time-harmonic acoustic waves propagation in a wedge-shaped waveguide in shallow water. The wedge is formed by the sea surface (plane $z = 0$, hereafter z is depth, x, y are horizontal coordinates) and the water–bottom interface $z = h(x, y)$, where $h(x, y) = h_0 + h_1(y)$,

$$h_1(y) = \begin{cases} -\zeta L, & \text{for } \ y < -L\,; \\ \zeta y, & \text{for } \ -L \le y \le L\,; \\ \zeta L, & \text{for } \ y > L\,. \end{cases}$$

Sound speed and density in the water layer (i.e. for $0 \le z \le h(x, y)$) are c_w and ρ_w, respective parameters in the bottom are c_b and ρ_b (we assume that $c_w < c_b$). The time-harmonic point source of frequency f is located at $x = 0$, $y = 0$, $z = z_s$. The asymptotic solution is sought along the acoustic track $y = 0$ directed across the bottom slope for the case of small ζ.

To solve the problem, we apply the method of mode parabolic equations [1, 2]. In our case, i.e. for small zeta, when L is sufficiently large, the solution of mode parabolic equation for piecewise linear $h(y)$ may be accurately approximated by the solution of the same equation with the potential replaced by a linear function in y. Such Schrödinger-type equation with linear potential may be solved analytically. Thus we obtain a closed formula for acoustic pressure along the track $y = 0$ in a wedge-shaped waveguide which writes as

$$p(x, y, z) = \sum_{j=1}^{N_m} A_j(x, y)\phi_j(z)\mathrm{e}^{\mathrm{i}k_j x}\,,$$

where k_j and $\phi_j(z)$ are the wavenumbers and mode functions of the discrete spectrum of standard acoustic spectral problem [3] for the reference waveguide (with $h_1 = 0$) and A_j are determined by

$$A_j(x,y) = \mathrm{e}^{i\left(\frac{\alpha x}{2k_j} + \frac{\beta y x}{2k_j} - \frac{x^3\beta^2}{24k_j^3}\right)} \sqrt{\frac{\sigma^2}{\sigma^2 + \frac{2ix}{k_j}}} \exp\left(-\frac{\left(y - \frac{x^2\beta}{4k_j^2}\right)^2}{\sigma^2 + \frac{2ix}{k_j}}\right).$$

Here we introduced the following notations: $\sigma = 1/k_j$, and

$$\beta = \zeta\left(\phi_j^2(h_0)(\gamma_w\kappa_w^2 - \gamma_b\kappa_b^2 + k_j^2(\gamma_b - \gamma_w)) - \gamma_w^2\phi_{jz-}^2(h_0)(\rho_b - \rho_w)\right), \quad \alpha_0 = \nu_b\gamma_b\int_{h_0}^{\infty}\phi_j^2 dz\,.$$

$(\gamma_w = 1/\rho_w,\ \gamma_b = 1/\rho_b)$.

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Instability of nanocluster shape

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The process of nanocluster growth due to agglomeration of nanoparticles is described. The perturbation of cluster surface is expanded in series of Legendre polynomials. The coefficients of expansion change over time in dependence on index n.

There is an area of indexes n at certain ratios between parameters of system where the loss of stability of a nanocluster surface happens. We investigated various modes at which the surface of nanocluster loses the stability and described dynamics of instability development of surface.

If the unperturbed velocity of cluster radius increasing is greater than some critical value, coefficients of expansion correspond to unstable harmonics and speed of instability development increases with index growth. That corresponds to formation of fractal structure of a cluster surface.

Radiation from a convex equiphase surface: Malyuzhinets' parabolic equation

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The problem of electromagnetic wave or sound radiation by a curved antenna conformal with the craft surface presents a clear technical interest. In order to obtain an approximate analytical solution we consider a simplified 2D problem: harmonic wave field produced by equiphase excitation of a convex body surface. The excitation amplitude may vary slowly from point to point, resulting in diffraction effects that determine the far-field radiation pattern.

Considering the emitting "surface" (curve) S_0 as an initial wave front we introduce ray coordinates (ξ, s) where ξ is distance along the normal vector to S_0 and s is arc length ofS_0.

In an early Malyuzhinets' paper [1] a bold generalization of the Leontovich–Fock parabolic equation (PE) [2] was proposed. In our problem, Malyuzhinets PE in ray coordinates ("transversal diffusion equation") governing the slowly varying complex wave amplitude $u(\xi, s)$ of the emitted harmonic wave $E = u\, e^{i(k\xi - \omega t)}$ has the following form:

$$ik\left(2\frac{\partial u}{\partial \xi} + \frac{u}{h_s}\frac{\partial h_s}{\partial \xi}\right) + \frac{1}{h_s}\frac{\partial}{\partial s}\left(h_s\frac{\partial u}{\partial s}\right) = 0, \qquad (1)$$

where $h_\xi = 1,\ h_s = 1 + \frac{\xi}{\rho(s)}$ are Lame coefficients, $\rho(s)$ being the curvature radius of the initial wave front S_0. Equation (1) has a marching type and admits a correct initial value problem formulation: $u(0, s) = u_0(s)$.

In this work we show that a series of variable substitutions identically reduces Eq. (1) to a standard Leontovich PE

$$2i\frac{\partial w}{\partial \tau} + \frac{\partial^2 w}{\partial \psi^2} = 0, \quad w(0, \psi) = w_0(\psi) \qquad (2)$$

in a finite rectangular domain: $0 < \psi < \gamma(L)$, $\ 0 < \tau < \frac{\gamma(L)}{kL}$. Here $\gamma(s) = \int_0^s \frac{ds}{\rho(s)}$, L being the full arc length of S_0. For a closed wave front S_0 ("omnidirectional antenna") L is its perimeter and we have a periodic initial value $w_0(\psi)$. An open curve S_0 corresponds to a finite antenna aperture, which presumes additional edge diffraction effects. Within PE approximation, they can be correctly taken into account by imposing transparent boundary conditions [3] on the lateral sides of the computational domain. In both cases, the problem (2) can be solved analytically by separation of variables or numerically, using finite differences.

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Suppression of the scheme dispersion for coupled nonlinear equations

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The choice of the most efficient numerical scheme for capturing the shock waves propagation requires sharp resolutions but without oscillations. Often these schemes generate oscillations on the shock profile, in particular, studying the slowly moving shocks of the isothermal Euler equations,

$$\rho_t + (\rho\ u)_x \ = \ 0, \qquad (1)$$

$$(\rho\ u)_t + (\rho\ u^2 + a^2\ \rho)_x \ = \ 0. \qquad (2)$$

written for the velocity, u, and density, ρ. An addition of artificial viscosity helps to reduce numerical oscillations but simultaneously affects the slope or the steepness of the shock. One can decrease the influence of the bad factors by varying time and space steps and modifying the method of discretization. A possibility to know how to do it is the application of the method of differential approximation [1].

In this paper, we examine additional nonlinear terms that may be added in the scheme to suppress oscillations and keep the steepness of the shock wave. The second order Lax–Wendroff scheme will be used for discretization of Eqs. (1), (2). The differential approximation in the form of the coupled nonlinear partial differential equations will be obtained for the scheme and an influence of artificial viscosity and additional nonlinear terms will be studied using exact solutions of the differential approximation. The form of the nonlinear terms will be defined when a smooth exact shock wave solution of the modified differential approximation exists.

Previously, we used the technique for a single nonlinear advection equation [2].

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On absolute continuity of spectrum of the periodic Maxwell operator in a cylinder.

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Let us consider the Maxwell operator in a cylinder $\Pi = U \times \mathbb{R}$, where U is a bounded domain in the plane. The Maxwell operator acts by the formula

$$M \begin{pmatrix} E \\ H \end{pmatrix} = \begin{pmatrix} i\varepsilon^{-1} \operatorname{rot} H \\ -i\mu^{-1} \operatorname{rot} E \end{pmatrix},$$

where divergence-free requirement

$$\operatorname{div}(\varepsilon E) = \operatorname{div}(\mu H) = 0$$

and boundary conditions of perfect conductivity

$$E_\tau|_{\partial\Pi} = 0, \, (\mu H)_\nu|_{\partial\Pi} = 0$$

are imposed on vector functions E and H.

We assume that coefficients ε and μ are scalar functions, periodic along the axis of the cylinder, satisfying

1. $0 < c_0 \le \varepsilon, \mu \le c_1 < \infty$,
2. $\varepsilon, \mu \in W^2_{3/2}(\Pi \cap B_R, \mathbb{R}), \forall R < \infty$.

We prove, that the spectrum of such operator is absolutely continuous in cases, when the cross-section U of the cylinder is a rectangle or a circle.

Earlier, the results of absolute continuity of spectrum of the periodic Maxwell operator were discovered in cases of the whole space $\Pi = \mathbb{R}^3$ and the layer $\Pi = [0, a] \times \mathbb{R}^2$ (see [1, 2]).

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Influence dispersion in mathematical models of mechanics

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The mathematical modeling of different phenomenons is based on solution of the two problems: physical that is creation of adequate model and mathematical that is formulation of the problem and the development solution method. For continuous mechanics we have two effects: dissipation and nonlinearity but dispersion as I see it plays important role. Nonlinearity changes form of the perturbations, dissipation decreases amplitude, dispersion leads to become blur. The Navier–Stokes equations include dissipation and nonlinearity, they are without dispersion. Dispersion is present at the Korteweg-de Vries and Korteweg-de Vries – Burgers equations. In the classical Newton mechanics we have four conservation laws: masses, liner momentum, energy, angular momentum. In continuous mechanics we use only three first laws and as a result we lose dispersion. Last law degenerates in symmetric tensor P that breaks disturbance of continuous medium. The kinetic theory does not save the situation. The law of angular momentum does not implement in the Boltzmann equation. This follows from at present for continuous mechanics formulation of equilibrium force conditions are used. These give us symmetric pressure tensor and disturbance of continuous medium. The purpose of this work is development of more full mathematical model for continuous mechanics and rarefied gas. It is devoted to the influence of consideration an angular momentum variation in an elementary volume and cross flux on its sides. Some asymptotical methods are verified. Taking into account the angular momentum law nonsymmetrical stress tensor is received. The method for calculation of nonsymmetrical part is suggested. For rarefied gas the second term in collision integral of the Boltzmann equation is taken into account to calculate the self-diffusion and thermo-diffusion.

We discussed the problems that can be appearing for consideration the angular momentum variation in an elementary volume near the surface and into boundary layer. Conditions of the existence A. N. Kolmogorov inertia interval is established. Resolved the G. Hilbert paradox in the solution of the Boltzmann equation by the Chapman–Enskog method. In general case (nonstationary) the collision integral can be written in form suggested by B. Aleexeev. Constants are defined by the species of concrete potential. Always in spite of that concrete time for two molecules we will be exploit average time which is proportional to inverse value of frequency. For numerical solution can be used the average values of collision integral for concrete velocity. Classical theory predicts the existence of the second viscosity but usually we assumed that it should be take into account for molecules with inner degree of free or for dense gas. The modified kinetic theory gives the second viscosity for molecules without structure. The order of the new equations (for the density and for the linear momentum, energy) is more than in classical case. If we deal with continual medium the external boundary condition for boundary layer can be determined as the value of rotor velocity or as of value normal velocity. That is for the vertical velocity. For the longitudinal velocity it is need to put friction. In turbulence layer we need to set a friction too. For the rarefied gas the boundary conditions would be included the value gas flow besides the classical boundary conditions. The results of numerical and analytical studies of certain problems of the boundary layer, the interaction of gas incident flow with the crystal surface, the simplest problems of elasticity theory are discussed.

Square integrable solutions of spheroidal Coulomb equation of the imaginary variable

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The boundary-value problem with homogeneous boundary conditions

$$\begin{aligned} &\frac{d}{d\xi}(\xi^2+1)\frac{d}{d\xi}X_{mk}(\xi;R) - \left[\lambda + p^2(\xi^2+1) - a\xi - \frac{m^2}{\xi^2+1}\right] X_{mk}(\xi;R) = 0\,, \\ &|X_{mk}(\xi;R)| \xrightarrow[\xi\to\pm\infty]{} 0, \quad \xi \in (-\infty,\infty)\,, \end{aligned} \tag{1}$$

with additional condition $X_{mk}(\xi;R) \in \mathcal{L}_2(-\infty,\infty)$ is considered. After a change of variable $\xi \to \imath z$ we have a boundary problem for the Coulomb spheroidal equation [1] on the imaginary axis. It is clear that the special points $z_1 = -1$ ($\xi_1 = -\imath$) and $z_2 = +1$ ($\xi_2 = +\imath$) are "on different sides" of the Im(z), and so any attempt to present $X_{mk}(\xi;R)$ in the form of series faces the problem of the circle of convergence. It is not possible to use the standard technique here, such as the transformation of Jaffe [2] or Jaffe–Lay [3], and therefore it is necessary to look for a more general method including the standard methods as special cases. In this report we propose a generalized Jaffe transformation and use it to obtain the convergent series representation for the eigenfunctions. We show that the coefficients are related to each other via recurrent relations of Poincare–Perron type. Some possible physical applications are discussed.

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Time-frequency integrals and quaternionic analysis in problems of wave propagation in chiral media

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We obtain a representation of the fields of moving sources in chiral media of the form of double time-frequency oscillating integrals applying quaterninic analysis methods. Some additional assumptions concerning the source allow us to introduce a large dimensionless parameter $\lambda > 0$ which characterizes simultaneously the slowness of variations of the amplitude and of the velocity of the source. Applying to the integral representation of the field the two-dimensional stationary phase method we obtain asymptotic formulas for the electromagnetic field for large $\lambda > 0$, and efficient formulas for the frequency and the time Doppler effects in dispersive chiral media. As an application of the proposed method we consider the Vavilov–Cherenkov radiation in chiral dispersive media.

Semiclassical theory of potential scattering for massless Dirac fermions

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The dynamics of massless Dirac fermions was recently revisited due to the discovery of graphene, whose charge carriers are exactly these particles. Although it was known that tunneling phenomena for Dirac fermions are substantially different from those for Schrödinger particles, a complete mathematical description was still missing. In our work [1] we study scattering of two-dimensional massless Dirac fermions by a one-dimensional potential within the Wentzel–Kramers–Brillouin (WKB) approximation [2, 3].

Depending on the energy of the incoming particle and its angle of incidence, we differentiate *(i)* the regime of Klein tunneling, i.e. the regime when the scattering of electrons is mediated by hole states supported by the barrier, *(ii)* the above-barrier scattering regime and *(iii)* the conventional tunneling regime. To find the reflection and transmission coefficients in each of these regimes, we adapted the WKB approximation [2, 3]. To describe near-normal incidence on a potential barrier, when the turning points are nearly degenerate, we reduced the initial problem to a non-conventional comparison (model) equation [4]. Using its solution, we obtained the reflection and transmission coefficients for near-normal incidence, which is crucial for physical applications.

Another important physical problem is provided by a monotonously increasing potential. Such a potential simulates an electronic junction, widely used in semiconductor physics. Surprisingly, we found that the semiclassical analysis of the Stokes diagram for this case (see Fig. 1) is closely related to the analysis of Klein tunneling. Therefore above-barrier scattering for such a junction can be treated as "virtual Klein tunneling" in the complex plane. The predictions provided by our analytic expressions show good agreement with numerical calculations [1] for all considered cases.

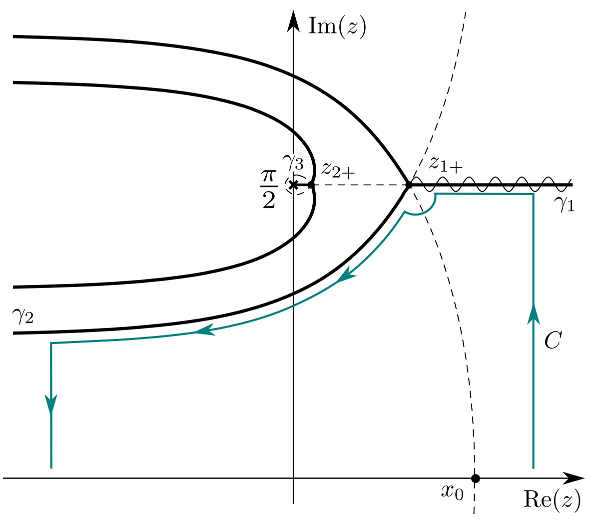

Fig. 1: The Stokes diagram for above-barrier scattering for a monotonously increasing potential. The solid lines are the Stokes lines, where we use the terminology from [2], and the cross denotes a pole of the effective potential.

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The theory of near-field microwave microscopy of plain-layered media: application for semiconducting films characterization

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We have developed the electromagnetic theory of near-field (NF) microwave microscopy of arbitrary plain-layered medium. The proposed theory consider the probe of NF microscope (NFM) as an electrically small antenna posed above the half space with one-dimensional profile of permittivity $\varepsilon(z)$ — piece-wise or continuous. We obtained the antenna input impedance Z_a using the ordinary scheme for calculation of plain wave propagating characteristics which was generalized to determine the antenna quasi-stationary field distribution. The considered NFM consists of the microwave resonator which includes the impedance Z_a as a load. The NFM frequency response was calculated via the resonators equivalent circuit. Its resonance frequency f_0 and unloaded quality factor Q_0 are sensitive to the electromagnetic and geometry properties of the sample under test. Therefore, we have measured and calculated parameters f_0, Q_0 to determine characteristics of the investigated samples. We have applied the above theory to solve the problem of nondestructive quantitative determination of sheet resistance R_{sh} of semiconducting films on dielectric substrate. The investigated samples were n-GaN films (thickness 0.25–2.5 μm, resistance 0.03–15 kΩ) grown on the sapphire substrate. The fitting parameters of the theoretical model were sought using the universal set of calibrating standards, specifically, bulk homogeneous silicon slabs varying in their conductivity. Experimental investigations were assisted by a 3 GHz resonance probe with an aperture of about 1 mm. The accuracy of the NFM theory proved sufficient for determination of R_{sh}, if the fitting parameters of our model are specified so as to fit the measured value of R_{sh}. To estimate the method accuracy we compared the values of R_{sh} measured by microwave NF and direct current techniques. The last one is usual Van-der-Pauw method — contact and destructive. We obtained the root-mean-square deviation 20% for $R_{\mathrm{sh}} < 4$ kΩ.

Hyperbolic Hamiltonian flows and the semi-classical Poincaré map

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We consider a h-Pseudo Differential Operator $H(x, hD_x)$ of principal type on $L^2(\mathbf{R}^d)$ with analytic coefficients, and whose symbol H_0 has a closed, isolated orbit γ in energy shell $H_0^{-1}(E)$. The main tool for describing the classical dynamics in a neighborhood of γ is Poincaré (or first return) map $\mathcal{P}(E)$ acting on a transverse section Σ to γ. The microlocal kernel of $H(x, hD_x)$ near γ (including complex values of E) reduces essentially to the microlocal kernel of id $-\mathcal{M}(E)$, where $\mathcal{M}(E)$ (the monodromy operator) is a Fourier Integral Operator whose canonical relation is associated to $\mathcal{P}(E)$. We assume this map is hyperbolic, i.e. $d\mathcal{P}(E)$ has no eigenvalue of modulus 1. Then, in a special action-angle coordinate system (φ, ι) on Σ, $\mathcal{P}(E)$ is the time-one map $\exp X_q$ of an Hamiltonian q depending on action variables ι only. Further, we can take $\mathcal{M}(E)$ to its so-called Sternberg normal form $\mathrm{Op}^w(e^{iF(\iota;h)/h})$, whose symbol $F(\iota; h) = F_0(\iota)+hF_1(\iota)+\cdots$ has principal part $F_0(\iota) = q(\iota)+\mathrm{const}$ When the eigenvalues of $d\mathcal{P}(E)$ are rationally independent, and under some suitable assumptions on the Hamilton flow for H_0 at infinity, this allows to construct Bohr–Sommerfeld quantization rules for all resonances of $H(x, hD_x)$ near E in a complex window of the form $[E-\varepsilon_0, E+\varepsilon_0]-i]0, Ch^\delta]$, with $0 < \delta < 1$. When $d = 2$, we can take $\delta = 0$. We investigate next the case when Σ contains a non trivial center-manifold. Such dynamical systems arise for special motions in the 2+1 body problem. This is a joint work with PhD students H. Fadhlaoui and H. Louati.

Asymptotic solution of the phase field system in the case of the high thermal conductivity and the small coefficient of the velocity of the free boundary

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We consider the phase field system [1]

$$\frac{\partial\theta}{\partial t} - k\triangle\theta = -\frac{1}{2}\frac{\partial u}{\partial t}, \tag{1}$$

$$\varepsilon\alpha\frac{\partial u}{\partial t} - \varepsilon\beta\triangle u = \frac{1}{\varepsilon}\left(u-u^3\right) + \chi\theta\left(1-u^2\right) \tag{2}$$

in the spherically symmetric region. Here θ is the approximation of the temperature and u is the order function, ε is the small parameter (parameter of the regularization), α, β, χ are some constants. If we formally assume $\varepsilon \to 0$ in system (1), (2) then we obtain the limit Stefan–Gibbs–Thomson problem

$$\frac{\partial\bar\theta}{\partial t} - k\triangle\theta = -\frac{1}{2}\frac{\partial u}{\partial t},$$

$$k\left[\frac{\partial\bar\theta}{\partial t}\right]\Big|_{\Gamma(t)} = \gamma\varphi(t)\varphi'(t),$$

$$\bar\theta\big|_{\Gamma(t)} = \alpha\varphi'(t) + \frac{2\beta}{\varphi(t)},$$

where $\Gamma(t)$ is the free boundary, $\bar\theta$ is the temperature.

Using the weak asymptotics method [2], [3] we construct the asymptotic solution of the phase field system (1), (2) in the case of the high thermal conductivity ($k \to \infty$) and the small coefficient of the velocity ($\alpha \to 0$) and we numerically analyze the behaviors of the limit Stefan–Gibbs–Thomson problem in the case considered.

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A model of diffusion, based on the equation of the Jeffreys type

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The classic diffusion equation

$$\frac{\partial u}{\partial t} - D\Delta u = f,$$

where D is the diffusion coefficient and f is the source term, is widely used for approximate macroscopic description of diffusion (dispersion of species). The diffusion equation results from the continuity equation and Fick's law. However, Fick's law does not take into account the mass (inertia) of moving particles (molecules), and, therefore, the diffusion equation gives an appropriate and accurate model for diffusion phenomena in weakly inhomogeneous media and/or for slow processes, i. e., when relaxation time is short compared to the characteristic time scale. Otherwise the description of diffusion phenomena by the diffusion equation may fail [1].

Many biological media, e. g., cellular cytoplasm, are strongly inhomogeneous and highly viscoelastic (non-Newtonian), therefore, diffusion in them is not Fickian and its description by the diffusion equation is questionable [2, 3].

The constitutive relation of the Jeffreys type was proposed for description of transport in rheological (non-Newtonian) media [4, 5, 6]. This relation generalizes Fick's law, it takes into account the nonlocality of transport both in time and space (time delay and inertia of molecules). The relation of the Jeffreys type together with the continuity equation leads to the partial differential equation of the third order

$$\tau\frac{\partial^2 u}{\partial t^2} + \left(1 - \tau\frac{\partial f}{\partial u}\right)\frac{\partial u}{\partial t} - \tau D_1\frac{\partial \Delta u}{\partial t} - (D_1 + D_2)\,\Delta u = f + \tau\frac{\partial f}{\partial t},$$

where τ is the relaxation time. We call this the equation of the Jeffreys type. If $\tau = 0$ the equation of the Jeffreys type is reduced to the diffusion equation with $D = D_1 + D_2$.

In this paper we consider the one-dimensional model of diffusion, based on the equation of the Jeffreys type. We study properties of the model. We show that the model has unusual properties.

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Discrete Schrödinger operators on periodic graphs

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We consider Schrödinger operators with periodic potentials on periodic discrete graphs. We show that the spectrum of the Schrödinger operator consists of an absolutely continuous part (which is a union of finite number of spectral bands, i.e., open intervals) plus finite number of flat bands, i.e., eigenvalues of infinite multiplicity. The following results are obtained:

1) estimates of the Lebesgue measure of the spectrum in terms of geometric parameters of the graph,

2) detailed analysis of all bands for specific periodic graphs, including the face-centered cubic lattice,

3) the existence and positions of the flat bands for specific graphs; in particular, for any integer N

there exists a graph, such that the corresponding Laplacian has two spectral bands and N flat bands between the bands,

4) stability estimates of bands and gaps in terms of potentials.

The proof is based on the Floquet theory and the precise representation of fiber Schrödinger operators, constructed in the paper.

On homogenization for periodic elliptic second order differential operators in an infinite rectangular cylinder

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The talk is devoted to homogenization for the periodic elliptic operator in $L_2(\Xi)$, $\Xi = \mathbb{R}^{d_1} \times \Omega_2$ and Ω_2 being a cell in $\mathbb{R}^{d_2}$, that is defined by the differential expression

$$\begin{aligned} \mathcal{B}_\lambda^\varepsilon = \sum_{j=1}^{2} \big(\mathbf{D}_j^* g_j(\mathbf{x}_1/\varepsilon,\, \mathbf{x}_2)\,\mathbf{D}_j + \mathbf{h}_j^*(\mathbf{x}_1/\varepsilon,\, \mathbf{x}_2)\,\mathbf{D}_j + \mathbf{D}_j^*\mathbf{h}_j(\mathbf{x}_1/\varepsilon,\, \mathbf{x}_2)\big) \\ + Q(\mathbf{x}_1/\varepsilon,\, \mathbf{x}_2) + \lambda Q_*(\mathbf{x}_1/\varepsilon,\, \mathbf{x}_2) \end{aligned} \tag{1}$$

with periodic boundary conditions on $\partial\Xi$; here $\varepsilon > 0$ and $\mathbf{D}_j = -i\nabla_{\mathbf{x}_j}$, $j = 1, 2$, $\mathbf{x}_1 \in \mathbb{R}^{d_1}$, $\mathbf{x}_2 \in \Omega_2$. All the coefficients are supposed to be periodic in the first variable with respect to some lattice in $\mathbb{R}^{d_1}$ and smooth in some sense in the second one.

Sharp-order approximations for the inverse of $\mathcal{B}_\lambda^\varepsilon$ in the norms of $\mathbf{B}(L_2(\Xi))$ and $\mathbf{B}(L_2(\Xi),\, H^1(\Xi))$ are obtained, with error terms being $O(\varepsilon)$:

$$\left\| \left(\mathcal{B}_\lambda^\varepsilon\right)^{-1} - \left(\mathcal{B}_\lambda^0\right)^{-1} \right\|_{\mathbf{B}(L_2(\Xi))} \le C\varepsilon, \tag{2}$$

$$\left\| \left(\mathcal{B}_\lambda^\varepsilon\right)^{-1} - \left(\mathcal{B}_\lambda^0\right)^{-1} - \varepsilon\mathcal{K}_\lambda^\varepsilon \right\|_{\mathbf{B}(L_2(\Xi),\, H^1(\Xi))} \le \widetilde{C}\varepsilon. \tag{3}$$

Here $\mathcal{B}_\lambda^0$ stands for the effective operator for $\mathcal{B}_\lambda^\varepsilon$; it has a similar form to (1), but its coefficients depend on the nonperiodic variable $\mathbf{x}_2$ only. The operator $\mathcal{K}_\lambda^\varepsilon$ is called the corrector; it is of zeroth order with respect to ε, but includes rapidly oscillating factors.

The inequalities (2) and (3) for the case $d_1 = d_2 = 1$ have been established in [2]. The work presented herein continues the investigation of that paper; the method of proving the estimates is based upon the operator-theoretic (spectral) approach introduced in the article [1] and is further development of the scheme suggested in [3].

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Diffraction of high-frequency grazing wave on grating with screens of different height

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A 2D problem of diffraction by an aperture line is studied. The line is formed by absorptive screens (Fig. 1). This problem can be interpreted as a reduction of a waveguide problem [1]. It corresponds to the Fabry–Perot resonator with displaced mirrors (Fig. 2). The incident wave is assumed to have wavelength short comparatively to the scale of the aperture line, and the incidence angle is small, i.e. the incident wave propagates almost parallel to the edge of the aperture line. We assume that the scattering occurs mainly under small angles and use the parabolic approximation to describe the wave process. A recently developed approach [2, 3] based on the embedding formula and the "spectral" equation for the directivity of an edge Green's function is applied to the problem. We prove the embedding formula for this problem, which express reflection coefficients through the directivity of an edge Green's function. Then we introduce spectral equation, which is an ordinary differential equation with unknown coefficient but with known boundary conditions. To determine this coefficient we construct Ordered Exponential (OE) equation. Then we solve OE-equation numerically.

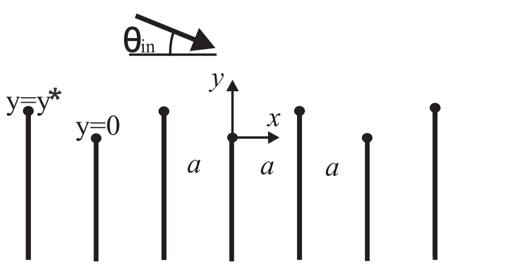

Fig. 1: Geometry of the array of scatterers.

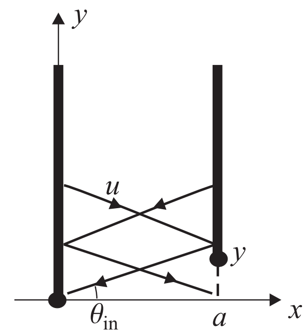

Fig. 2: Fabry–Perot resonator geometry.

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Homogenization of nonlinear Robin boundary conditions for cavities and associated spectral problem

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Let u_ε be a solution of the Poisson equation in a domain periodically perforated along manifold $\gamma = \Omega \cap \{x_1 = 0\}$ with nonlinear Robin type boundary condition on the perforations (the flux here being $O(\varepsilon^{-\kappa})\sigma(x, u_\varepsilon)$), Ω is a domain of $\mathbb{R}^n$, $n \geq 3$, the small parameter ε that we shall make to go to zero, denotes the period, and the size of each cavity is $O(\varepsilon^\alpha)$ with $\alpha > 1$. The function σ involving the nonlinear process is a $C^1(\overline{\Omega} \times \mathbb{R})$ function and the parameter $\kappa \in \mathbb{R}$. Depending on the values of α and κ the effective conditions on γ are obtained. We provide a *critical relation* between both parameters which implies a different average of the process on γ ranging from linear to nonlinear. For each fixed κ a critical size of the cavities which depends on n is found. As $\varepsilon \to 0$, we show the convergence of u_ε in the weak topology of H^1. All this allows us to derive convergence for the eigenelements of the associated spectral problems in the case of σ is a linear function.

Discrete spectrum of periodic Schrödinger operator with non-constant metric in the case of non-negative perturbations

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Let A be an elliptic periodic self-adjoint operator of the second order in $L_2(\mathbb{R}^d)$, $d \geq 1$, and let V be the multiplication by the function $V(x) \geq 0$ which tends (in an appropriate sense) to zero as $|x| \to +\infty$. Let (α, β) be an *inner* gap in the spectrum of A and $\lambda \in [\alpha, \beta]$ be a fixed number. The spectrum of the operator $B(t) := A + tV$, $t > 0$ inside the gap (α, β) is discrete. Denote by $N(\lambda, \tau)$ the number of eigenvalues of the operator $B(t)$ that have passed the point λ as t increased from 0 to τ. The asymptotics of $N(\lambda, \tau)$ is obtained for $\tau \to +\infty$ in the case when the perturbation $V(x)$ has power-like asymptotics at infinity, $V(x) \sim \omega(x/|x|)|x|^{-\rho}$, $|x| \to +\infty$, $\rho > 0$.

The main result can be represented in the following form: $N(\lambda, \tau) \sim \Gamma_\rho(\lambda)\tau^{d/\rho}$, $\tau \to +\infty$. Here the coefficient $\Gamma_\rho(\lambda)$ is computed in terms of *the zone functions* of the operator A. Under certain conditions, this asymptotics holds on the left edge of the gap as well, $\lambda = \alpha$. We impose no additional restrictions on the smoothness of the coefficients of the operator A.

The verification of the main result is based on analysis of the asymptotics of singular numbers of certain integral operators. Down this route, we employ different generalizations of the Cwikel estimate. The derived asymptotics is non-local with respect to energy, its order is different from the "standard" $\tau^{d/2}$. The Weyl nature of the asymptotics reveals itself if the roles of coordinates and quasi-impulses are switched.

On processing and recognition of radionavigation signals in control of orbital complex

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In detection, recognition and treatment of radio navigation signals from orbital complexes a problem of signal distortion [1, 2] arises. The distortion may complicate separation of signals and distort their important characteristics. Moreover, the effect of blinking may lead to the disappearance of the signal [3]. In the paper an approach to reduction of computing noise is described. Also, method for comparing of signal processing algorithms is proposed. This method allows to identify parameters that determine the stability of the signal processing algorithms and to quantify characteristics of stability of algorithms.

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Permittivity determination of thin multi-sectional diaphragm in a rectangular waveguide

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In this paper we consider the inverse problem of the permittivity determination of thin multi-sectional diaphragm in a rectangular waveguide. We perform a detailed analysis for cases of one-, two- and three-sectional thin diaphragms. Numerical results are presented.

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Problem of electromagnetic TE wave propagation in a inhomogeneous nonlinear two layered dielectric waveguide

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Problem of electromagnetic TE wave propagation in a inhomogeneous nonlinear two layered dielectric waveguide is considered. The waveguide has circular cross-section. The permittivity in the first (inner) layer of the waveguide and in outer space are constants. The permittivity in the second (exterior) layer depends on radius of the waveguide (inhomogeneity) and on modulus of the electric field intensity by the Kerr law. The problem is to determine propagation constants of the waves, which propagate along the boundary of the waveguide. Physical problem is reduced to a nonlinear eigenvalue problem. Numerical results for a few inhomogeneities are presented. Comparison with a linear inhomogeneous waveguide is given.

Diffraction of an elastic wave by the jump inhomogeneity in the elastic layer

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An elastic layer of finite thickness represents a plane elastic waveguide. The problem of a waveguide joint, which is a the particular case of the general wave diffraction problem, is the highly topical problem of the wave propagation and diffraction theory. In this study, investigated is the two-dimensional diffraction problem of elastic wave by the jump inhomogeneity of elastic parameters of the medium in the plane elastic waveguide having rigidly fixed sides.

The problem is considered in the cartesian reference system. We take axes in such a way that the line $x = 0$ divides the waveguide into two sections, each of which is filled with different elastic materials. We assume that media filling in the plane elastic waveguide are linear, homogenous and isotropic.

Let us consider an elastic wave falling onto the waveguide inhomogeneity in case $x = 0$. Desired is the field occuring under the wave deformation, reflection and refraction. The mathematical statement of the problem of diffraction of elastic wave by the interface within the plane waveguide having fixed sides is described as following. Sought is the solution of the Lame equation system in case $x \in (-\infty, +\infty)$ and $y \in (0, h)$:

$$\begin{aligned} &(\lambda + 2\mu)\frac{\partial^2 u_x}{\partial x^2} + \mu\frac{\partial^2 u_x}{\partial y^2} + \rho\omega^2 u_x + (\lambda + \mu)\frac{\partial^2 u_y}{\partial x \partial y} = 0, \\ &(\lambda + \mu)\frac{\partial^2 u_x}{\partial x \partial y} + \mu\frac{\partial^2 u_y}{\partial x^2} + (\lambda + 2\mu)\frac{\partial^2 u_y}{\partial y^2} + \rho\omega^2 u_y = 0, \end{aligned} \tag{1}$$

which turns to zero in case $y = 0$ and $y = h$ (on the waveguide sides), and the solution is to be continuous at the interface and satisfy the radiation conditions. We consider that the elastic modula λ, μ and ρ are piecewise constant functions and jumps can appear in case $x = 0$.

All oscillations of the homogenous elastic layer, resulting from solution of (1) with constant coefficients, can be represented in the form of expansion into eigenwave series [1]. The finite number of eigenwaves corresponds to the real value of eigen propagation constants, and the denumerable number of the waves corresponds to imaginary values of the constants [2]. The problem of the elastic wave diffraction by the vertical interface in the plane elastic waveguide reduces to the solution of the system of equations.

Let us represent the results of the numerical experiments for different fillers of elastic waveguides, which corresponds to geological strata.

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Discrete diffraction of light beams in two-dimensional sinuous structures

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In this paper we report on the investigation of discrete diffraction phenomenon in linear and nonlinear periodic snaking structures. We provide numerical simulation of light propagation in two models of medium: model of periodically-curved arrays of optical waveguides [1] and model of continuum with a periodic modulation of the refractive index both in transverse and longitudinal direction [2, 3]. Refractive index distribution in our continuum model is determined by following equation:

$$n(x, z) = n_0 + \Delta n \cos\left(\frac{2\pi}{L_1}\left(x + b\sin\frac{2\pi}{L_2}z\right)\right), \tag{1}$$

where z is the paraxial propagation distance, L_1 and L_2 — periods of transverse and longitudinal modulation respectively, b — amplitude of longitudinal modulation of refractive index n.

Changing various parameters of medium and light beam we can observe various effects. In our numerical experiments we demonstrate reduction (Fig. 1) and even complete suppression of discrete diffraction. We identify the influence of medium inhomogeneity (Fig. 2), frequency of the radiation and the angle of incidence of the light beam to its diffraction in this medium.

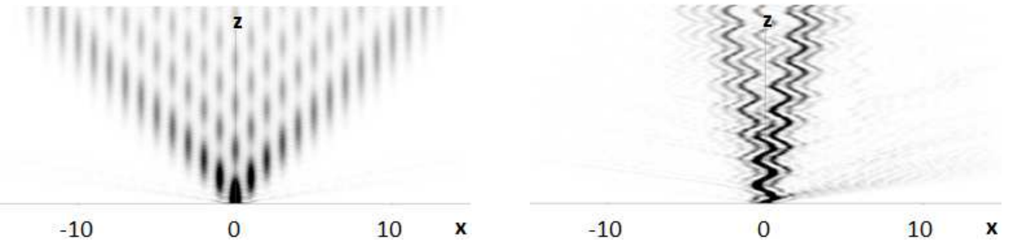

Fig. 1: The intensity distribution in the medium with (right) and without (left) periodic axial modulation of the refractive index.

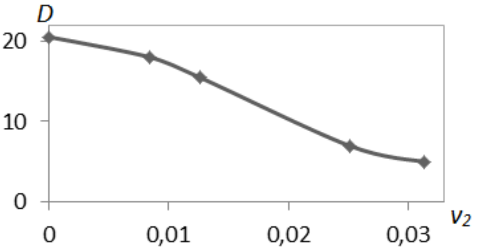

Fig. 2: Beam width D dependence on frequency ν_2 of longitudinal modulation.

Similar analysis is performed for nonlinear media, where only nonlinear electric susceptibility coefficient has spatial modulation as shown in Eq. (1). So discrete diffraction phenomenon appears only under the influence of powerful pump wave.

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Homogenization of the Neumann problem for an elliptic system with periodic coefficients

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Let $\mathcal{O} \subset \mathbb{R}^d$ be a bounded domain of class $C^{1,1}$. We consider the Neumann problem for an elliptic system:

$$b(\mathbf{D})^* g(\mathbf{x}/\varepsilon) b(\mathbf{D})\mathbf{u}_\varepsilon + \lambda \mathbf{u}_\varepsilon = \mathbf{F} \text{ in } \mathcal{O}, \quad \partial_\nu^\varepsilon \mathbf{u}_\varepsilon|_{\partial\mathcal{O}} = 0, \tag{1}$$

with $\mathbf{F} \in L_2(\mathcal{O}; \mathbb{C}^n)$. Here an $(m \times m)$-matrix-valued function $g(\mathbf{x})$ is bounded, uniformly positive definite and periodic with respect to some lattice Γ. The elementary cell of Γ is denoted by Ω. Next, $b(\mathbf{D}) = \sum_{j=1}^d b_j D_j$ is an $(m \times n)$-matrix first order differential operator (b_j are constant matrices). It is assumed that $m \geq n$ and the symbol $b(\boldsymbol{\xi}) = \sum_{j=1}^d b_j \xi_j$ has maximal rank: $\operatorname{rank} b(\boldsymbol{\xi}) = n$ for $0 \neq \boldsymbol{\xi} \in \mathbb{C}^d$. The symbol ∂_ν^ε stands for the conormal derivative: $\partial_\nu^\varepsilon \mathbf{u}_\varepsilon = b(\nu(\mathbf{x}))^* g(\mathbf{x}/\varepsilon) b(\nabla)\mathbf{u}_\varepsilon(\mathbf{x})$, where $\nu(\mathbf{x})$ is the unit normal vector to $\partial\mathcal{O}$ at the point $\mathbf{x} \in \partial\mathcal{O}$.

The problem (1) is understood in the weak sense; λ is either sufficiently large or $\lambda = 0$ (in the latter case we assume that $\int_{\mathcal{O}} \mathbf{F}(\mathbf{x})\, d\mathbf{x} = 0$ and the solution is subject to the same condition).

It turns out that $\mathbf{u}_\varepsilon$ converges in $L_2(\mathcal{O};\mathbb{C}^n)$ to $\mathbf{u}_0$, as $\varepsilon \to 0$. Here $\mathbf{u}_0$ is the solution of the "homogenized" Neumann problem

$$b(\mathbf{D})^* g^0 b(\mathbf{D})\mathbf{u}_0 + \lambda \mathbf{u}_0 = \mathbf{F} \text{ in } \mathcal{O}, \quad \partial_\nu^0 \mathbf{u}_0|_{\partial\mathcal{O}} = 0.$$

The effective matrix g^0 is a constant positive $(m \times m)$-matrix defined as follows. Denote by $\Lambda(\mathbf{x})$ the $(n \times m)$-matrix-valued periodic solution of the equation $b(\mathbf{D})^* g(\mathbf{x})(b(\mathbf{D})\Lambda(\mathbf{x}) + \mathbf{1}_m) = 0$ such that $\int_\Omega \Lambda(\mathbf{x})\, d\mathbf{x} = 0$. Then $g^0 = |\Omega|^{-1} \int_\Omega g(\mathbf{x})(b(\mathbf{D})\Lambda(\mathbf{x}) + \mathbf{1}_m)\, d\mathbf{x}$.

Theorem 1. [1] *We have the following sharp order error estimate*:

$$\|\mathbf{u}_\varepsilon - \mathbf{u}_0\|_{L_2(\mathcal{O};\mathbb{C}^n)} \le C\varepsilon \|\mathbf{F}\|_{L_2(\mathcal{O};\mathbb{C}^n)}. \tag{2}$$

Now we give approximation of $\mathbf{u}_\varepsilon$ in the Sobolev space $H^1(\mathcal{O};\mathbb{C}^n)$. For this, the first order corrector must be taken into account.

Theorem 2. [1] 1) *Denote* $\Lambda^\varepsilon(\mathbf{x}) = \Lambda(\varepsilon^{-1}\mathbf{x})$. *If* $\Lambda \in L_\infty$, *then*

$$\|\mathbf{u}_\varepsilon - \mathbf{u}_0 - \varepsilon \Lambda^\varepsilon b(\mathbf{D})\mathbf{u}_0\|_{H^1(\mathcal{O};\mathbb{C}^n)} \le C\varepsilon^{1/2} \|\mathbf{F}\|_{L_2(\mathcal{O};\mathbb{C}^n)}, \tag{3}$$

2) *In the general case, we have*

$$\|\mathbf{u}_\varepsilon - \mathbf{u}_0 - \varepsilon \Lambda^\varepsilon b(\mathbf{D})(S_\varepsilon \widetilde{\mathbf{u}}_0)\|_{H^1(\mathcal{O};\mathbb{C}^n)} \le C\varepsilon^{1/2} \|\mathbf{F}\|_{L_2(\mathcal{O};\mathbb{C}^n)}. \tag{4}$$

Here $\widetilde{\mathbf{u}}_0 = P_{\mathcal{O}}\mathbf{u}_0$ *and* $P_{\mathcal{O}} : H^2(\mathcal{O};\mathbb{C}^n) \to H^2(\mathbb{R}^d;\mathbb{C}^n)$ *is a continuous extension operator,* S_ε *is the Steklov smoothing operator defined by* $(S_\varepsilon \mathbf{u})(\mathbf{x}) = |\Omega|^{-1} \int_\Omega \mathbf{u}(\mathbf{x} - \varepsilon\mathbf{z})\, d\mathbf{z}$.

We use the results of M. Birman and T. Suslina for homogenization problem in $\mathbb{R}^d$: the analogs of estimates (3), (4) in $\mathbb{R}^d$ are of sharp order ε. The problem is reduced to estimating the "boundary layer corrector" $\mathbf{w}_\varepsilon$, which is the solution of the homogeneous equation $b(\mathbf{D})^* g(\mathbf{x}/\varepsilon) b(\mathbf{D})\mathbf{w}_\varepsilon + \lambda \mathbf{w}_\varepsilon = 0$ with appropriate boundary condition. We show that the norm of $\mathbf{w}_\varepsilon$ in H^1 satisfies estimate of order $\varepsilon^{1/2}$. This leads to (3), (4). At the same time, the norm of $\mathbf{w}_\varepsilon$ in L_2 is of order ε, this allows us to prove sharp order estimate (2).

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Non-stationary 'complex source' wavefields in real space

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Complexified spherical waves attracted earlier much attention, e. g. [1], as exact solutions showing high localization. They are obtainable from the non-stationary spherical waves $u = f(R-ct)/R$, $R = \sqrt{x^2 + y^2 + z^2}$ (where waveform f is an arbitrary function) by the 'complex shift of the source' $z \to z - ia$, $a > 0$. The distance R is thus replaced by a multi-valued function $R_* = \sqrt{x^2 + y^2 + (z - ia)^2}$ and a branch cut must be described. Now, u has a jump in the real 3D space and satisfies the inhomogeneous wave equation $u_{xx} + u_{yy} + u_{zz} - c^{-2}u_{tt} = F$ with a certain source function F. We are interested in description of such sources for different choices of the cut and in asymptotic behavior of u. We present waveforms f, showing packetlike Gaussian localization.

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A ray mode parabolic equation for shallow water acoustics propagation problems

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We consider the propagation of time-harmonic sound in the three-dimensional waveguide $\Omega = \{(x, y, z)|0 \le x \le \infty, -\infty \le y \le \infty, -H \le z \le 0\}$ (z-axis is directed upward), described by the acoustic Helmholtz equation

$$(\gamma P_x)_x + (\gamma P_y)_y + (\gamma P_z)_z + \gamma\kappa n^2 P = 0\,, \tag{1}$$

where $\gamma = 1/\rho$, $\rho = \rho(x, y, z)$ is the density, κ is the reference wave-number and $n = n(x, y, z)$ is the index of refraction. We assume the appropriate radiation conditions at infinity in x, y plane, the pressure-release boundary condition at $P = 0$ $z = 0$ and rigid boundary condition $\partial u/\partial z = 0$ at $z = -H$. At $x = 0$ we impose the Dirichlet boundary condition $P = g(z, y)$ modeling the sound source located outside Ω.

We apply the method of multiple-scale expansions [1] to eq. (1) in ray scaling with the slow variables $X = \epsilon x$, $Y = \epsilon y$ and the fast variables $\eta = (1/\epsilon)\Theta(X, Y)$, $\xi = (1/\epsilon^{1/2})\Psi(X, Y)$. The expansions of dependent variables and parameters are as follows

$$n^2 = n_0^2(X, z) + \epsilon\nu(X, Y, z)\,, \quad \gamma = \gamma_0(X, z) + \epsilon\gamma_1(X, Y, z)\,, \quad P = P_0(X, Y, \eta, \xi) + \epsilon P_1(X, Y\eta, \xi) + \ldots\,.$$

At $O(\epsilon^0)$ we obtain that $P_0 = \exp((\mathrm{i}/\epsilon)\Theta)A\phi$, where

$$(\gamma_0\phi_z)_z + \gamma_0\kappa n_0^2\phi = k^2\gamma_0\phi\,, \quad \phi(0) = 0\,, \quad \left.\frac{\partial\phi}{\partial z}\right|_{z=-H} = 0\,,$$

and

$$(\Theta_X)^2 + (\Theta_Y)^2 = k^2\,. \tag{2}$$

At $O(\epsilon^{1/2})$ we obtain

$$\Theta_X\Psi_X + \Theta_Y\Psi_Y = 0\,.$$

As a solvability condition of the $O(\epsilon)$-problem, we obtain the equation

$$2\mathrm{i}\left[\Theta_X A_X + \Theta_Y A_Y\right] + \mathrm{i}\left[\Theta_{XX} + \Theta_{YY}\right]A + \left[(\Psi_X)^2 + (\Psi_Y)^2\right]A_{\xi\xi} + \alpha A = 0\,, \tag{3}$$

where the potential α depends on the medium parameters.

Considering eq. (2) as the Hamilton–Jacobi equation, we choose some ray. In normalized ray coordinates (s, ξ) eq. (3) can be written as

$$2\mathrm{i}kA_s + \mathrm{i}DA + A_{\xi\xi} + \alpha A = 0\,, \tag{4}$$

where $D = \frac{1}{J}\frac{d}{ds}(Jk)$ and

$$J = \det \begin{pmatrix} x_s & x_\xi \\ y_s & y_\xi \end{pmatrix} .$$

Eq. (4) is solved numerically in some typical environments (wedge et cetera). Some beam solutions of this equation are also obtained.

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Singular statistics revised

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We analyze [1] the 'singular statistics' of pseudointegrable Šeba billiards, i.e. billiards perturbed by zero-range perturbations. The Šeba billiard [2] is often used as an example of a quantum chaotic system whose classical counterpart is almost integrable. The conclusion that the Šeba billiard should be considered as a chaotic one is based on the statement [2, 3] that it manifests 'semi-Poissonian'-like statistics and the level repulsion. We have shown that the computation of a spectrum of the Šeba billiard can be reduced to a summation of exponentially convergent series. This provides an efficient scheme to compute excited eigenvalues of the problem. Increasing the number of resonances, taken for the statistics, we observed a transition from 'semi-Poissonian'-like to Poissonian one. This observation is in agreement with the argument that a classical particle does not feel a point perturbation.

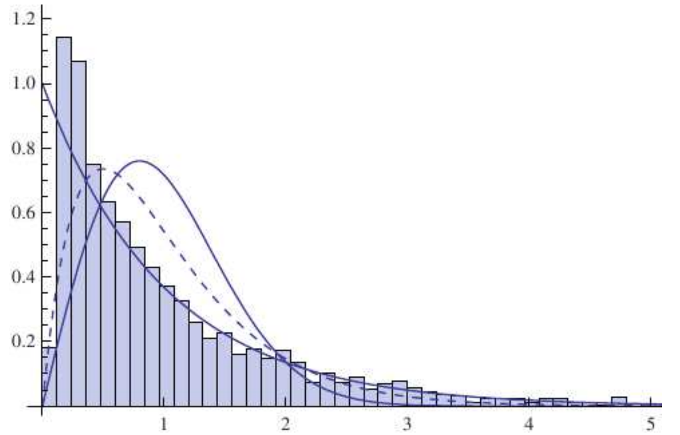

Fig. 1: The level spacings statistics (histogram) for the resonances 25 000 – 27 000 for the Šeba billiard with side ratio $\sqrt{5} - 1$. Lines show Poissonian, semi-Poissonian (dashed line) and GOE (Gaussian) distributions.

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Effect of particle size distribution on the parameters of the diffraction pattern obtained by laser diffractometry technique

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Laser diffractometry is a technique commonly used to characterize the ensembles of biological particles, e.g., live cells in glass smears or fluid suspensions. In the latter case, the suspension can be either stationary, in this case the cells are randomly located and oriented due to diffusion, or dynamic, in this case the cells may be deformed by shear stress and fully or partially oriented along the fluid flow direction. Particular behavior of the particles in shear flow depends on their sizes and shapes as well as on the viscoelastic properties of their membranes and intracellular cytosol. In most cases, the cell suspensions are not uniform, i.e. they can be characterized by statistical distributions of their major properties. Illumination of the suspension under study with a laser beam yields a diffraction pattern in the far field. Typically the sizes of biological particles/live cells are around an order of magnitude higher than the laser wavelength, which implies that the diffraction pattern is limited to low scattering angles. Due to random location of the particles within the illuminated area the superposition of partial waves scattered by the particles produces a diffraction pattern symmetrical relative to the zero angle, i.e. the central maximum. The information about the statistical distributions of the particle ensemble properties is incorporated, in particular, in the shapes and locations of the lower order diffraction maxima and minima, and their visibilities.

In this work, we explore the information content in the diffraction pattern typically obtained from a dilute suspension of red blood cells (RBC) in ektacytometry [1]. In the condition of shear flow the RBC are shear deformed, which implies that the shape of the diffraction pattern resembles the average shape of the cells. However the distribution in sizes and in shapes of the particles reduces the visibility of the diffraction extrema. Considering that RBC become deformed only after the shear stress exceeds a certain level (shear yield) and that, in addition, RBC are often modeled by equivolume spheres, in this work, we estimate the contrast (visibility) of the diffraction pattern as a function of relative variation of particle sizes. In our previous work [2], the particles were modeled by homogeneous flat disks oriented perpendicular to the laser beam. We showed that this dependency is monotonic in the case that the relative spread of particle sizes is below 0.17. In this work, we calculate a similar dependency assuming the particles to be spherical in shape. We analyze such dependencies obtained numerically for different size distributions of the optically soft spheres.

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Generation of superluminal sources for localized waves: a realizable approach

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Searching for realistic scenarios of localized wave generation is a modern, rapidly growing interdisciplinary area in the intersection of radiophysics, optics and acoustics [1, 2]. One promising direction of this activity is the investigation of waves belonging to the superluminal localized solutions (SLSs), the most interesting of which being the X-shaped waves [3]. Several methods of the experimental generation of SLSs, relying on suitable *interference* of ordinary-speed waves, have been discussed in the literature (see, e.g., [1] and references therein). In contrast to this ansatz, in 2004 Recami et al. [4] investigated *source-related generation* schemes and showed that an everlasting point charge (tachyon) at rectilinear motion with a constant superluminal speed is a source of a "true X-wave". In 2011 this approach was extended to launching similar localized waves by transient superluminal currents of finite support in space-time, within the framework of a causal model involving an initial value problem for the z-component of the vector potential **A** [5].

In order to support the physical feasibility of such model, the report discusses how to induce transient superluminal currents without violating traditional special relativity, recurring to neither tachyons nor other phenomena lying outside the scope of well-established today's experimental physics.

It is well known and universally accepted that a shadow can move with superluminal velocity. Notably, a shadow boundary is a frontier between light and darkness. If light can generate some particles (e.g., through ionization), the shadow boundary may be treated as a (possibly, superluminal) front of a pulse generating these particles. For example, when a pulse of ionizing radiation with a locally plane wavefront obliquely incidents a surface of some easy-to-ionize material, the region where ionization occurs propagates along the surface with a superluminal speed. As the velocity distribution of the photoelectrons is anisotropic (with a maximum in the direction of photon propagation), this leads to the development of a macroscopic current pulse whose fronts move with the same superluminal velocity. Here the resulting "superluminal sources" are quite ordinary macroscopic currents formed in regions whose boundaries move with superluminal speed due to specially chosen geometry of current generation. A concrete scheme based on this concept will be presented, together with brief analysis of the space-time structure of the electromagnetic pulse accompanying the macroscopic current development.

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Electromagnetic fields formed by ionization of the gas environment by hard nuclear radiation

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A method is proposed to calculate the space-time distributions of the electromagnetic fields formed by passing beams of gamma and charged particles through a matter, taking into account the ranges, the angular and velocities distributions of separate electrons, generated in the process of ionization.

Algorithm and the calculations are fulfilled in assumption that all process may be described as the effect of two constituents. The first component answers for the interaction of particles and the atoms of a matter. The calculations are carried out with the use of methods of quantum electrodynamics, in practice it is the use of computer packages such as GEANT [1]. The second one is connected with the movement of charged particles along the broken trajectories between the points of interactions with the atoms. The calculations are conducted by the methods of classical electrodynamics for delta pulse currents [2] by applying the principle of superposition.

Results. The algorithm is proposed for calculations the space-time distributions of the fields generated by interaction particle and matter which encompasses two mechanisms. It is shown that at velocities are in close to light one interference may influence on the distributions of fields.

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Analytical heuristic solution for the problem of elastic wave diffraction by a polygonal flat 3D scatterer

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Analytical formulas for solving the problem of elastic wave diffraction by 3D flat polygonal scatterer are obtained with use of heuristic approach. Firstly a problem of electromagnetic diffraction by a scatterer of similar shape has been solved by known methods [1–5]. Then diffraction coefficients for electromagnetic wave have been replaced by diffraction coefficients for elastic wave. Comparison between analytical formulas calculation results and numerical calculation results for the case of symmetrical relative position of the scatterer, source and receiver of elastic wave has shown good agreement. In case of necessity a generalization of obtained analytical formulas can be done for the case of non-symmetrical relative position of the scatterer, source and receiver of elastic wave, and for the case of more complicated scatterer shape. Results of this work can be applied in solving a geophysics problem on seismic waves diffraction, ultrasonic flaw detection, etc., and also for investigation of wave fields with physical nature different from this of electromagnetic and elastic waves.

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Electrical tuning of planar thin film ferrite-ferroelectric resonator

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Tuneable microwave (MW) resonators are one of the most demanded elements used in the tunable filters and generators. In order to design such resonators, materials with tunable properties could be used, for example, ferrites, ferroelectrics and piezoelectrics. Nowadays a lot of works devotes to the investigation of the MW applications of th multiferroics. One way to create an artificial multiferroics is the use of ferrite-ferroelectric layered structure. It was shown [1, 2] that resonators based on the layered ferrite-ferroelectric structures can be tuned by external electric and magnetic fields. Later it was shown [3] that it is possible to create resonator based on the thin ferrite and ferroelectric films by using the slot line waveguide structure. In the present work we have designed and fabricated the planar thin film multiferroic resonator and measured the electric tuning range.

Fabrication process consists of the four stages. At the first stage rectangular ferrite film resonator was created. This resonator was formed by the single crystal yttrium iron garnet (YIG) film with in plane dimensions 3x3 mm^2 and the gadolinium gallium garnet substrate with in plane dimensions 20x10 mm^2. At the next stage ferroelectric film (barium strontium titanate $Ba_{0.5}Sr_{0.5}TiO_3$) was grown by the RF magnetron sputtering on the surface of YIG-resonator. The thickness of the film was 5 μm. At the final stage copper slot line electrodes were created by the thermal evaporation method on the ferroelectric film surface. These electrodes formed planar waveguide structure and were tuning electrodes for applying bias voltage simultaneously. The slot line gap width was 150 μm.

MW resonator was excited by the coaxial cable and placed in the external magnetic field that was directed perpendicular to the slot line and parallel to the resonator surface. Magnetic field strength was 900 Oe. Resonance frequency at zero bias voltage was 4.35 GHz. It was found that change of the bias voltage leads to the change of the resonance frequency. The maximum electric tuning range was 30 MHz at the voltage 150 V that is equal to the electric field strength 1 V/μm.

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Non-stationary waves with a complex eikonal near the boundary of an elastic medium

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The work investigates the kinematic and dynamic characteristics of inhomogeneous elastic waves (with a complex phase) that arise near either a boundary surface or any internal interfaces of an elastic medium.

We consider the medium to be anisotropic and inhomogeneous, and the boundary surface smooth and free of stress. Here, we are dealing with a boundary problem for the equation of motion in an elastic medium, where the non-stress boundary conditions take into account the total contribution of all types of waves (including inhomogeneous) arising from the interaction of the incident waves with the medium's boundary.

The kinematics of these waves is defined by the expressions for their complex phases obtained (locally) at points on the boundary surface in accordance with the law of wave reflection. The analytical formulas for the displacement of waves with a complex phase, expressed in space-time as asymptotic ray series, generalize the results previously obtained in [1, 2] for the case of monochromatic waves in homogeneous elastic media with a flat boundary and interfaces.

The asymptotic formula (in the main asymptotic approximation) for the energy flux carried by an inhomogeneous wave, describes it as a surface one with an amplitude exponentially decaying with depth. The group velocity of this wave is found as the superposition of a group of quasi-stationary waves with similar frequencies near the boundary surface. This result is obtained by using both the Fourier transform and theory of functions of several complex variables.

The amplitudes of the inhomogeneous waves at boundary points are found from the boundary conditions with a given displacement of the incident waves. The ray method is then used to compute the wave amplitudes at points within the elastic medium away from its boundary.

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Instability of electromagnetic waves supported by the composite cylinder with dielectric covering

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The study is made of the interaction of the surface waves guided by infinitely long anisotropic cylinder with dielectric covering. The cylinder is embedded in a uniform background medium and aligned with an anisotropic axis which parallel to the z axis of a cylindrical coordinate system. The composite single-axis medium inside the cylinder is described by the permittivity and permeability tensors $\hat{\varepsilon}$ and $\hat{\mu}$ with zero off-diagonal elements. The main attention is given to the parametric instability of the surface waves guided by the cylinder in the presence of the external time-harmonic magnetic field. The intense magnetic field may effect on the medium properties and as a result the components of the permeability tensors depend on the amplitude of magnetic field. The parametric instability has been developed if the space-time conditions between an external magnetic field and surface waves take place. The equations for the amplitude of the surface waves can be obtained from the Maxwell's equations in the approximation of a week nonlinearity. The expressions of the parametric instability increment are given.

We analyzed of the dispersion characteristics of the waves of TE and TM types as well the nonlinear interaction of the waves for the case of isotropic composite medium by numerically. For the monochromatic field, permittivity and permeability constants of the medium can be written in the form $\varepsilon = 1 - (\omega_p/\omega)^2$ and $\mu = 1 - F\omega^2/(\omega^2 - \omega_m^2)$ where parameters F, ω_p, ω_m are determined by the technological properties of elementary cells of composite materials [1]. The results of the computations will be reported.

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The program code for FDTD modelling of very large size problems

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We offer `CFmaxwell` (http://cfmaxwell.com) — an open-source program code for 3D Finite-Difference Time-Domain (FDTD) modelling of up-to-date artificial optical and electromagnetic devices and materials such as photonic crystals, metamaterials, antennas etc. There are many other FDTD libraries and solvers, but in general they aimed on relatively small problems (up to billion Yee cells in computational grid). So it is impossible to use them for the full significant exploring of electromagnetic processes in structures with the scale of characteristic sizes by an order of magnitude.

The code is based on the Local-Recursive nonLocal-Asynchronous Algorithms (LRnLA) [1], which makes possible to reach peak rate of program's effectiveness in problems with very large computational grid size (more than 10^{12} Yee cells). "Effective" algorithm means such one, that has real rate coming up to theoretical. In the work the implementation of such algorithm is offered for Maxwell's equations' modeling. `CFmaxwell` is primarily designed for realistic fully 3D modelling and for problems in which the characteristic dimensions should vary by many times (for example, the size of one cell of photonic crystal's lattice and the linear physical size of the sample).

One may set different boundary conditions, including PML. Also following models for material equations have been realized: simple linear undispersion materials, Drude dispersion model with several terms, anisotropic media. In report it will be shown the effectiveness of `CFmaxwell` in comparison with other FDTD codes at different parameters and grid sizes.

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Near field influence on Green's function retrieval in seismic interferometry (wave equation case)

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Seismic interferometry is a modern branch of seismology. Its main objective is to reduce near surface experimental responses to points convenient for a needed horizon observation. Thus, it allows not to concern complex near surface overburden. Usually such speculations are based on the following identity (e.g. [1]):

$$I \equiv 2i\,\mathrm{Im}G(\mathbf{x_B},\mathbf{x_A}) = \int_S \left[G^*(\mathbf{x_B},\mathbf{x}) \frac{\partial G(\mathbf{x_A},\mathbf{x})}{\partial \mathbf{n}} - G(\mathbf{x_A},\mathbf{x}) \frac{\partial G^*(\mathbf{x_B},\mathbf{x})}{\partial \mathbf{n}} \right] dS. \tag{1}$$

This identity for a Helmholtz equation causal Green's function in principle allows to get the so-called virtual source [2] response by crosscorrelating observations in the time domain and integrating over some closed surface. But in practice there is no easy way to measure normal derivatives and the right part of (1) should be simplified. One can reduce it to a single crosscorrelation product in the "high frequency regime" [3] with some extra assumptions about the medium. We emphasize that such a high frequency approach already contains an error due to sources-receivers closeness and acts as the far field approach at the same time. If receivers $\mathbf{x_A}$, $\mathbf{x_B}$ are located rather far from the surface with sources S, then

$$I \approx I_F \equiv \int_S G^*(\mathbf{x_B},\mathbf{x})G(\mathbf{x_A},\mathbf{x})W_F(\mathbf{x},\mathbf{x_A},\mathbf{x_B})dS, \qquad \text{where}$$
$$W_F(\mathbf{x},\mathbf{x_A},\mathbf{x_B}) \equiv -i\frac{\omega}{v(\mathbf{x})}(|\cos\alpha_A(\mathbf{x})| + |\cos\alpha_B(\mathbf{x})|) \tag{2}$$

is weight function associated with incidence angles.

In this paper we examine the accuracy of approach (2). We show that more accurate approximation in near field can be obtained leaving the next term in inverse frequency by ray series substitution. This correction includes only zero order ray series terms:

$$I \approx I_F + I_N, \quad I_N = \int_S G_0^*(\mathbf{x_B},\mathbf{x})G_0(\mathbf{x_A},\mathbf{x})W_N(\mathbf{x},\mathbf{x_A},\mathbf{x_B})dS, \qquad \text{where}$$
$$W_N(\mathbf{x},\mathbf{x_A},\mathbf{x_B}) \equiv \frac{\partial}{\partial\mathbf{n}}\,\mathrm{Ln}\,\frac{A_0(\mathbf{x},\mathbf{x_A})}{A_0^*(\mathbf{x},\mathbf{x_B})}$$

is weight function associated with zero order ray series aplitudes.

The influence of this correction as closeness of receivers to the surface S function and also as function of some geometry parameters is examined for several simple medium models and surfaces. Appropriate integrals are estimated with stationary phase method and numerical integration.

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Meta-session

Electric and magnetic resonances of metasurfaces: impact of randomization

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We theoretically, numerically and experimentally study the arbitrary incidence of plane waves to periodic, aperiodic and fully amorphous grids of plasmonic inclusions which exhibit two resonances – the electric-mode and magnetic-mode ones. We study the impact of randomization and peculiarities of these resonances. The randomness in the particle positioning influences equally on the scattering loss from both electric and magnetic dipoles, however, the observed impact is very different. The theory is illustrated by numerical examples of paired nanopatches of Ag and Au and confirmed by collaborative experimental investigations.

Introduction. Traditionally, bulk metamaterials and their planar analogues called metasurfaces – optically dense arrays of small individual resonant inclusions are realized as periodical arrays. However, recently, random or amorphous metamaterials start to attract attention. This is due to novel technological possibilities to manufacture amorphous structures cheaply and on a large scale, using advanced self-assembly techniques. In addition, undesirable effects of strong spatial dispersion can be in some cases suppressed by disorder. We have developed a simple model which explains the electromagnetic effects in transition from regular to random states of resonant optically dense arrays of inclusions as well as numerical and experimental results for scattering and extinction versus the angle and in transition to the amorphous state.

Theoretical, numerical and experimental study. We consider a grid of silver and gold pairs of nanopatches in 3 states: periodic, aperiodic and fully amorphous. In all 3 states the averaged distance between nanopairs is kept the same. The whole configuration is shown in Fig. 1. The nanoslab thickness in our study was within $g = 45-50$ nm while the patch dimensions are $165 \times 165 \times 30$ nm with a lattice period $a = 500-512$ nm. In Fig. 1(a) the referential periodic structure is shown. The randomization is achieved by introducing a positional disorder in a deterministic fashion. The disorder parameter D is proportional to the maximal random displacement of every single cut-plate pair from its original position: $D = 0$ for a periodic grid, $D = 0.4$ for an aperiodic one (adjacent nanopatches do not touch one another), and $D = 1000$ for a fully amorphous array with clusters and sparse domains. The proposed structure was modeled as an effective sheet of surface susceptibilities (Fig. 1(b)). We have derived formulas to predict R, T and E for any θ through these parameters measured or simulated for two values of θ. We have derived conditions under which randomizing the particle positions gives either negligible or strong effects on the reflectance R, transmittance T and extinction (absorbance) E. We have done extended numerical studies versus the wavelength λ, incidence angle θ for all 3 cases $D = 0,\ 0.4,\ 1000$. Angle-resolved measurements of the experimental samples were performed within the wavelength range from 500 to 1300 nm. The transmittance was measured with a home-made white-light spectrometer setup consisting of a tungsten light source and an optical spectrum analyzer. The reflectance measurements were carried out on a motorized two-circle goniometer in a $\theta - 2\theta$ configuration, where a Xenon arc-source filtered by a monochromator was used for illumination. Theoretical predictions, simulations and measurements are in a good agreement.

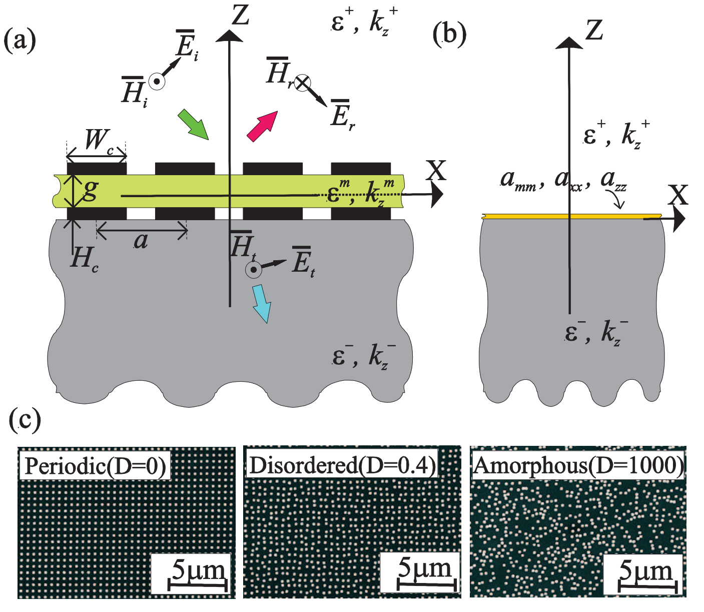

Fig. 1: (Color online) (a) Geometry of the periodic array of cut-plate pairs (side view); $a = 510$ nm, $W_c = 165$ nm, $H_c = 30$ nm, $g = 50$ nm, $\varepsilon^+ = 1$, $\varepsilon^m = 2.96$, $\varepsilon^- = 2.25$. Note the symmetry along the X-direction is identical to the symmetry in Y-direction, thus the latter is omitted for the sake of brevity. (b) Effective sheet model of the structure shown in (a), Notice that the sheet thickness tends to zero in our model. (c) shows three normal view scanning electron microscope images of the fabricated metasurfaces composed of cut-plate pairs. The three corresponding disorder parameters $D = 0$, $D = 0.4$ and $D = 1000$ reflect the transition from the periodic to the amorphous state.

Conclusion. We have revealed following peculiarities of electric and magnetic modes analyzing the frequency dependencies of the extinction coefficient E and reflectance R:

1. Electric resonance of nearly regular arrays:

- For small angles of incidence this resonance corresponds to the local minimum of E and strong maximum of R. There is a weak maximum of E linked to the electric mode. It is red-shifted with respect to the electric resonance.
- For oblique incidence the minimum of E strongly sharpens versus θ and the strong maximum of R keeps (both shift to the blue versus θ). The corresponding maximum of E weakens versus θ and disappears within $\theta = 20 - 30°$ where the Wood anomaly arises.

2. Electric resonance of an amorphous array:

- Randomization suppresses the minimum of E whereas the maximum of R keeps.
- The red-shifted maximum of E at the normal incidence and for small values of θ keeps. Integral losses over the band of the electric resonance dramatically increase at all incidence angles compared to the periodic case.

3. Magnetic resonance.

- No minimum of E in the resonance band, a weak maximum of R coincides with the strong maximum of E
- This resonance does not depend on θ and does not feel the randomization.

We found that the magnetic resonance is practically not affected by randomness due to much higher absorption losses in that mode. The electric resonance corresponds to much smaller dissipative losses and is dramatically affected by the randomization which brings the dominating scattering losses. The Wood anomaly and the electric resonance of reflectance do not survive the randomization.

Deformation of the mollow triplet by influence of a plasmonic nanoparticle

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We study the shape of the fluorescent emission spectrum of two level system (TLS) placed near a plasmonic nanoparticle (PNP). The spectrum of an isolated TLS in the strong external field was calculated in [1, 2] and observed in [3, 4]. Due to modulation of the fluorescent transition of the TLS by the Rabi oscillation in the external field, the triplet-like response, the so-called Mollow triplet, arises. The influence of a PNP on the resonant fluorescent spectrum of TLS was considered in [5] where the variation of the density of states (the Purcell factor) was calculated. Such an approach does not take into account the possible phase difference between oscillations of the dipole moments of TLS and PNP.

In this communication we have shown that the interference of the oscillations can significantly change the spectrum. Moving the TLS closer to PNP firstly results in asymmetry of the spectrum due to the Fano resonance of the oscillation, then in disappearance of the satellite peaks, and finally in the formation of a single Lorentz line.

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Terahertz wave manipulation with metamaterials based on metal and graphene

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The terahertz (THz) technology provides with exciting possibilities for spectroscopy, food quality control, defense, communication and biomedical imaging [1]. Being relatively young (the massive exploitation of the THz range began only in the beginning of 1990-ies), the THz science demands for active and passive materials and devices. Metamaterials, metal-dielectric artificial composites, propose wide possibilities for achieving unconventional electromagnetic properties, not found in nature. Moreover, metamaterials constructed of graphene, a monolayer of carbon atoms, allow for tunable response.

In this presentation we overview our results on theory, fabrication and characterization of metal and graphene based metamaterials for the THz range. We show that the multiple layers of structured graphene can form a hyperbolic dispersion medium lens able to resolve the subwavelength features [2]. We analyze the limitations and demonstrate numerically and experimentally the chiral and non-chiral thin-film metamaterial based polarization converters [3–5] and graphene total absorbers for THz radiation [6].

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Modeling of electromagnetic wave absorption by powder composite

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Dependences of absorption and power loss density of microwaves in the periodically distributed spherical conductive microparticles on the frequency of the incident radiation and the size of the particles are obtained and investigated. Absorption is increasing at decreasing of the size of particles.

Features of the interaction of electromagnetic radiation with a composite powder of conductive particles are of interest from the practical point of view. This is due to the possibility of efficient heating of metal powders under the influence of electromagnetic waves. There are the theoretical and experimental fundamentals of microwave heating of metals by electric and magnetic fields in [1, 2]. The heating curves of electromagnetic radiation heating of copper powder for various particle sizes and porosity are obtained in [3].

The aim of this research is to study the features of absorption of electromagnetic wave in the volume of the ordered structure consisting of conducting spherical particles of specified sizes, not in contact with each other when falling on them electromagnetic wave with specified frequencies. This study allows to describe and classify features of electromagnetic wave heating of various metal powders and to predict the performance, in which it will be effective heating of metal powders by electromagnetic radiation. Layers of conductive spherical particles as the shell, and without it are accepted in our work as the most common model of powder metals.

This study was supported by the Russian Foundation for Basic Research and the Federal Target Program 'Scientific and Teaching Personnel of Innovative Russia' for 2009–2013 of the Ministry of Education and Science of the Russian Federation.

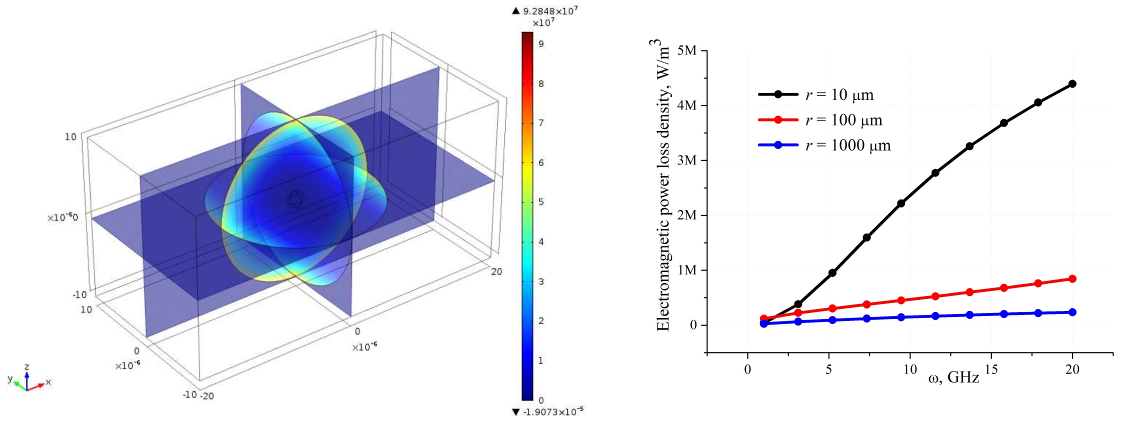

Fig. 1: Electromagnetic power loss density for structure with different sizes of spherical conductive particles (r).

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Giant enhancement of terahertz emission from highly anisotropic metamaterials

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A promising way of fabrication of modern sources of terahertz radiaton is based on separation of light-induced nonequilibrium charge carriers in surface band bending. Increasing surface area can be achieved by synthesis of nanoporous matrices in semiconductors.

We have ivestigated the emission of terahertz radiation from semiconductor nanoporous matrices based on GaP. Nanoporous GaP matrices were made by anodic electrochemical etching [1]. Terahertz raditon was excited by short light pulses (less 150 fs) laser radiation ($\lambda = 515$ nm). We found a giant (by 4–5 orders of magnitude) increased emission of terahertz radiation from the nanoporous matrix based on GaP compared to bulk GaP.

An intensification of emission for both polarizations for frequencies from 0.45 to 1.5 THz compared to the intensity of emission of terahertz radiation from bulk GaP was observed. For s-polarization, the emission intensity was an order of magnitude smaller, while for the bulk GaP it was absent.

The achieved emission intensity of terahertz radiation from porous matrix GaP and nanocomposites was only an order of magnitude less than the intensity of narrow-band emission from the best materials (p-InAs [2]), see Fig. 1. The proposed approach can serve as the basis for the formation of new sources of high-power terahertz radiation. An additional improvment by filling pores with metal will be discussed.

This work has been supported by the RFBR (project 12-02-31439-mol-a) and by the Ministry of Education and Science of the Russian Federation (project 14.132.21.1403).

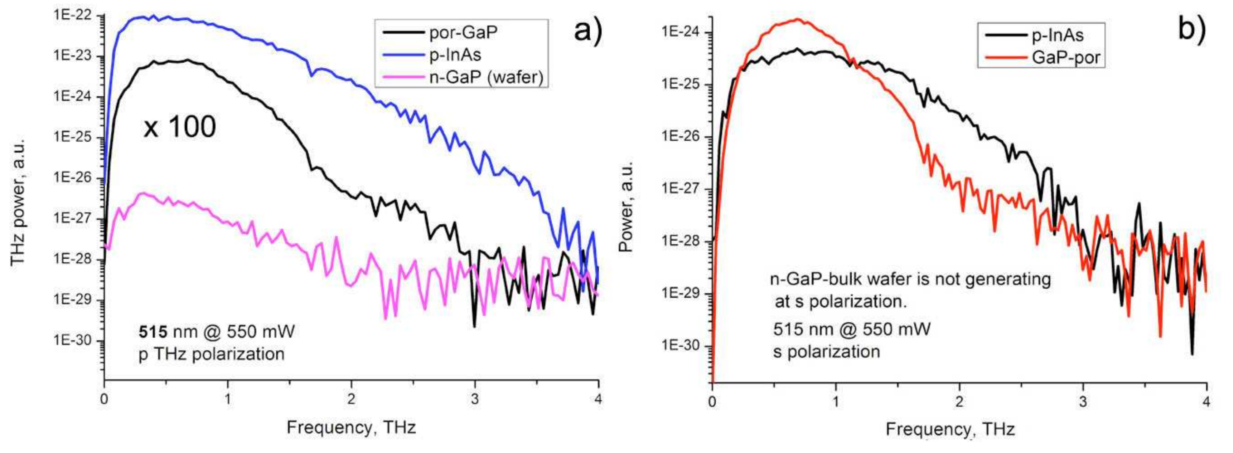

Fig. 1: Emission spectrum of THz radiation of GaP semiconductor nanoporous matrices for p-and s-polarization (*a* and *b* respectively) in comparison with the bulk GaP and p-InAs.

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Unusual properties of hyperbolic media

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Electromagnetic metamaterials, artificial media created by structuring on the subwavelength scale, became a paradigm for engineering electromagnetic space and controlling propagation of light and waves. Such materials demonstrate many unusual properties not available in nature, and they can be employed for achieving novel functionalities of the emerging metadevices operating with light and waves. In the talk, we review one of the most unusual classes of such materials that possess hyperbolic dispersion described by the electric tensors with the components of the opposite sign. Distinct properties of such meta-anisotropic materials include giant enhancement of spontaneous emission, diverging density of states, peculiar properties of negative refraction, and superlensing effects. Different realizations of hyperbolic media based on use of arrays of metallic nanorods [1], layered metal-dielectric nanostructures and even multilayered graphene [3] are discussed.

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Optical antennas for manipulating light-matter interaction at the nanoscale

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In this talk, I will describe our recent advances on optical antennas. In a first part, I will describe an analytical model that describes the directivity of a metallic particle coupled with a single dipolar emitter and will evidence the important role of the distance between the emitter and the particle on the direction of emission [1, 2]. In a second part, I will describe a collaborative work carried out with Institut Langevin in Paris on the fabrication of fast single photon sources by coupling a single organic emitter to one or two gold particles with DNA strands [3, 4]. The calculation of the electric local density of states is able to predict the distribution of fluorescent emission lifetimes with respect to both (i) emitter direction and (ii) emitter/particle distance. The third and last part of the talk will be dedicated to the control of the electric and magnetic LDOS with dielectric Mie resonators.We will see how dielectric resonators can promote either the magnetic or the electric emission rate. The analytical expressions of the electric and magnetic LDOS explain why in transverse coupling an electric dipole emission can be controlled with magnetic modes (and vice versa), while in longitudinal coupling magnetic (electric) transitions rates can be controlled with magnetic (electric) modes only [5]. Also, Mie resonators can be used to control the direction of emission by extending the so-called kerker's conditions to the case of near field excitation [6].

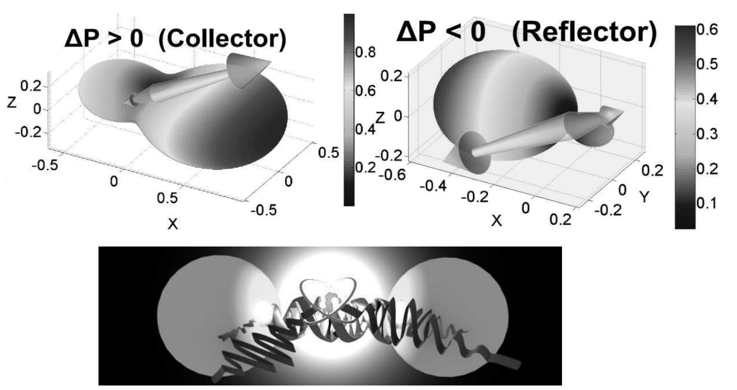

Fig. 1: (above) Forward or backward scattering of light when a metallic particle is coupled with an electric dipolar emitter. (below) Bioinspired antenna fabricated with two gold particles linked a double DNA strand. A single fluorescent molecule (ATTO647) is grafted to one DNA strand, at the center of the nanogap.

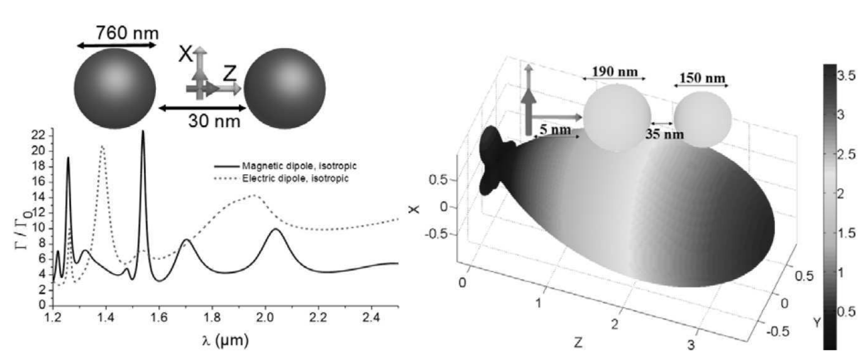

Fig. 2: (left) Isotropic averaged decay rates when an ED (dashed red line) or MD (solid black line) is placed in the centre of a Si dimer in a n=1.45 host medium (inset: schematic). (right) Radiation diagram at 570 nm of an ED transversely coupled to two GaP spheres in a n=1.45 host medium (inset: schematic).

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Effective dynamic permittivity and permeability of composite media and metamaterials

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Direct numerical finite element method (FEM) for solving differential equations of Maxwell determined the effective permittivity and permeability of the composite medium as orderly arranged conductive spherical particles and long cylinders. Shows the dependence of the effective permeability of the frequency of the alternating electromagnetic field, dielectric and magnetic parameters, conductivity, geometry and concentrations of the individual components.

In this paper to analyze the effectiveness of the dynamic dielectric permittivity and magnetic permeability of heterogeneous systems is given an external alternating magnetic field. Direct numerical finite element method is the solution of differential equations of Maxwell. The grid system is set so that the size of the cells was much less than the size of the inhomogeneities, the length of the electromagnetic wave and the thickness of the skin layer. Constructed distribution of the field inside the system. From this distribution, calculated wave impedance at the boundaries and determined the effective permeability of the system. This method allows to take into account the mutual polarization of the irregularities of the distance between them and the frequency of the electromagnetic field.

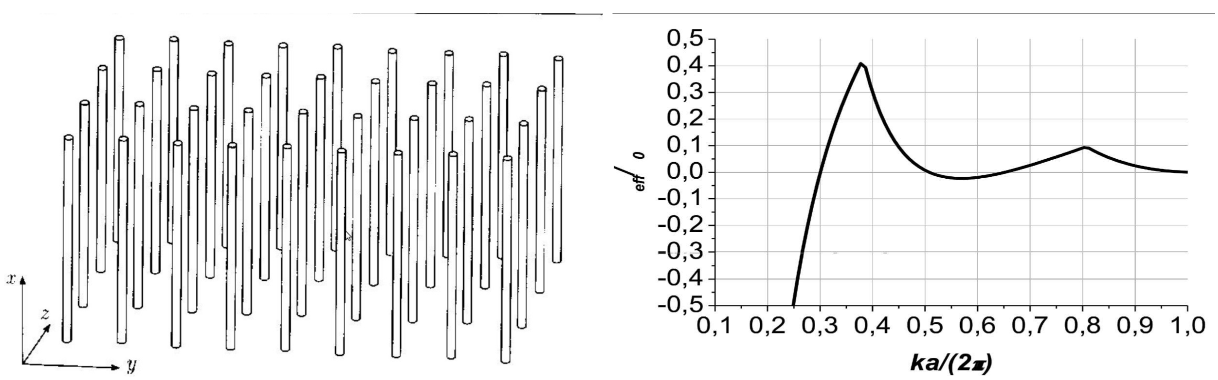

Fig. 1: The model of the structure in the form of ordered thin wires arranged in a dielectric matrix ε_0 (left). The effective permeability of the medium (right) from $ka/2\pi = \sqrt{\varepsilon_0}\nu a/c$. Here ν is frequency of the electromagnetic field, a is the lattice constant, ε_0 is permeability matrix.

On the resulting fig. 1 shows the alternation of “transparency zones” (effective permeability is positive) and “forbidden zones” (effective permittivity is negative).

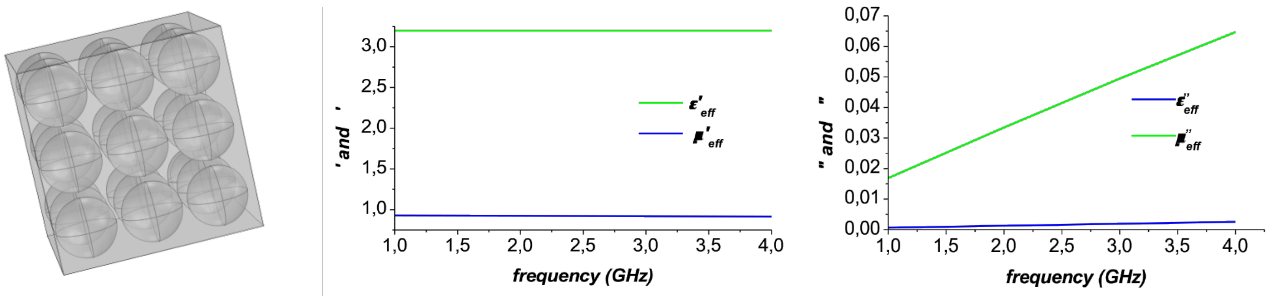

Fig. 2: The frequency dependence of the effective permeability and permittivity for structure of spherical conductive particles. Radius: 10 μm, volume fraction: -0.3, conductivity: -10^6 S/m, permeability of spherical conductive particles: 1.

This study was supported by the Russian Foundation for Basic Research and the Federal Target Program 'Scientific and Teaching Personnel of Innovative Russia' for 2009–2013 of the Ministry of Education and Science of the Russian Federation.

Tunable metamaterials using superconducting circuits with Josephson junctions

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One of the main limitations of many metamaterial designs is the restriction of their usability to a narrow frequency band. By using resonant elements in order to achieve the desired effects (such as a negative magnetic permeability μ_r) the "meta-atoms" are usually made for a fixed operation frequency range by design. Superconductors in general and Josephson junctions in particular are ideal constituents of tunable meta-atoms due to the strong dependence of their inductance on external parameters such as temperature and magnetic field. In this work, we experimentally demonstrate tunability via magnetic field by replacing the SRR by a superconducting quantum interference device (SQUID). This approach has previously been suggested in the theoretical work by Lazarides and Tsironis [1] and also further investigated by Gabitov and Maimistov [2].

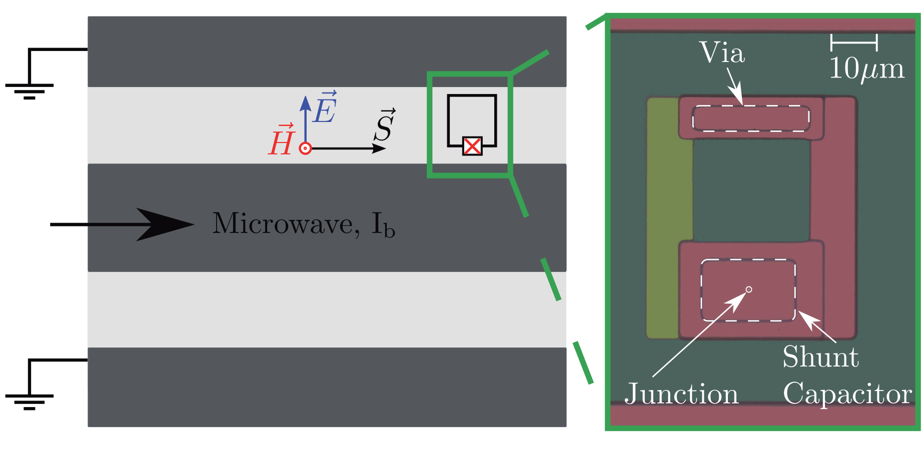

Fig. 1: Sketch of a single rf-SQUID embedded into a coplanar transmission line. The darker areas represent the Nb electrodes. The inset on the right shows an optical microscope image of a single rf-SQUID.

We will present experiments with superconducting microwave-range metamaterials containing SQUIDs. Like the split ring resonators, these elements can be seen as LC-resonators that couple to the magnetic component of the incoming wave. The advantage of superconducting thin-film metamaterials is that, due to the tunable intrinsic inductance of the Josephson junction, the resonance frequency of the SQUID can be changed by applying an external dc magnetic field. We will present experimental results that demonstrate the tunability of the resonance frequency of these devices [3]. Using same approach, our work in progress involves tunable left-handed transmission lines as well as semi-conventional tunable split-ring resonators.

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Metal nanoisland films for plasmonics

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The excitation of surface plasmon polaritons in random media [1] like sputtered glass metal nanocomposites (GMN) and metal island films (MIF) has recently been demonstrated. To apply these media in plasmonics their 2D structuring is necessary. GMN structuring can be performed with electron beam lithography (EBL) while the use of EBL for MIF processing is doubtful for they are too affectional. At the same time MIF are of special interest because the distance between the metal islands can lay within unavailable with EBL several nanometers scale to provide extra high local electric fields. We present a novel technique of MIF formation and 2D structuring, which is based on metal out-diffusion from glass substrate and glass poling. These allow precise control of MIF properties and MIF patterning.

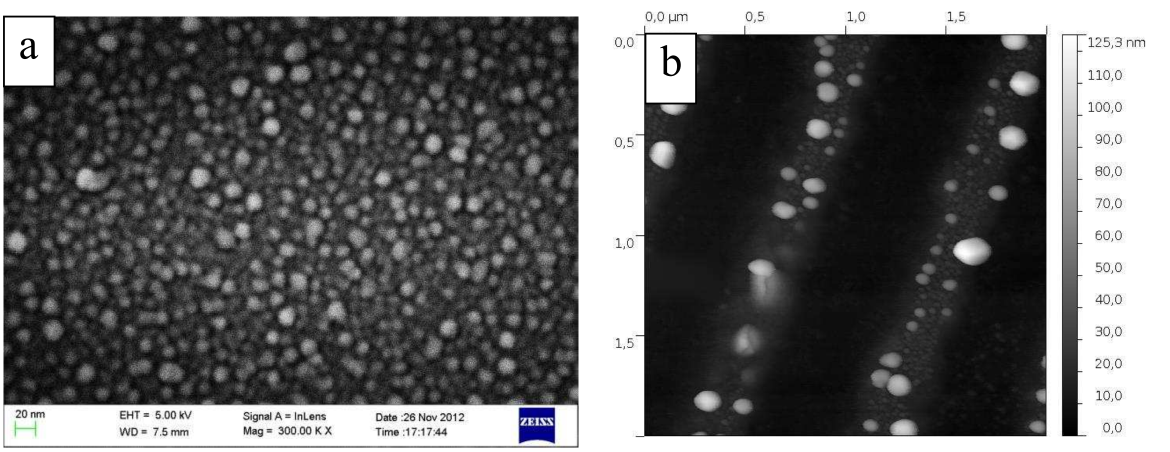

Fig. 1: (a) SEM image of metal nanoisland film formed in 10 min at 150°C processing of soda-lime glass substrate exchanged in $Ag_{0.5}Na_{0.95}NO_3$ melt at 325°C for 20 min. (b) AFM image of nanoisland film stripes formed with grating electrode.

Silver MIF were formed in the course of anneal of silver-enriched glass substrates in reducing hydrogen atmosphere. Varying the conditions of ion exchange doping the glasses with silver and hydrogen processing allowed to govern silver island size and concentration (Fig. 1(a)). The increase in the concentration of our semi-spherical islands allowed to go from and subnanometer distance between the islands to percolated and solid silver films.

Thermal poling of ion-exchanged glass substrates with structured electrode was used for 2D patterning of silver ions distribution beneath the glass surface. Silver ions shifted down from the glass surface in poled region of the glass substrate were not reduced in hydrogen processing due to the limited depth of hydrogen thermal diffusion. Thus neutral silver was formed only in unpoled area of the substrate and, respectively, metal island film was poled only there. The strips of silver islands were successfully formed in performed experiments (Fig. 1(b)). According to preliminary data the patterning within hundreds of nanometers scale, that is comparable with EBL patterning, is possible. It is worth to note that the same technique is applicable for bulk GMN nanopatterning [2].

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Qualitative models in nanophotonics: scientific and educational aspects

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> "It is necessary to reconstruct the textbooks and change some pedagogical methods in lecturing of electrodynamics... It turned out to be necessarily to elaborate the electrodynamics from fundamental principles taking into account possible magnetic effects", Prof. V.G. Veselago

Recent technological advancements allowing creation of nanoobjects/metamaterials in the optical domain has forced the revisiting of basics electrodynamic principles and assumptions. A large amount of new experimental and theoretical data has to be structured within the framework of a new unified approach, in order to distinguish the really fundamental knowledge from various applications and particular cases. Experience has shown that a unified approach appears to be extremely important for educational courses in the area of nanophotonics/optical metamaterials. In particular it allows us to present a self-consistent physical picture, which in turn minimizes the amount of educational material to be memorized to the crucial physics. The significance of this conceptual approach in fundamental education becomes extremely important in view of the current problems of market economy.

Hyperbolic metamaterials with topological transitions

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Hyperbolic metamaterials, being a particular class of indefinite media [1], are described by the electric or/and magnetic tensors with the components of the opposite sign. Due to the hyperbolic isofrequency contours in the wave-vector space, such structures exhibit a number of unusual properties. First, waves at their boundaries may exhibit negative refraction, similarly to the case of double-negative metamaterials. Second, they have a diverging density of photonic states that allows to enhance the strength of light-matter coupling [2]. This makes a concept of hyperbolic media very promising for tailoring broad-band light-matter interaction, nanophotonics applications, including single-photon generation, sensing, and photovoltaics. In our earlier work [3], we have proposed the realization of hyperbolic metamaterials with artificial two-dimensional transmission-line (TL) metamaterials. We have demonstrated experimentally that the emission pattern of a current source, being a fingerprint of hyperbolic media, has a pronounced cross form. In this contribution, we propose a novel design of TL metamaterials with topological transitions [4]. We have utilized the unit cell with parallel tank of inductance and capacitance where the admittance sign reverses at the resonance frequency [Fig. 1(a)]. The resonance frequency is chosen to be 160 kHz. Numerically simulated voltage distribution, excited in the structure, is obtained by solving the Kirchhoff equation, and it is shown in Figs. 1(b, c). At the frequencies below the resonance, the voltage distribution of the metamaterial structure has an elliptic shape while at the frequencies above the resonance it becomes hyperbolic. Thus, by choosing the operating frequency, we observe a transition from the elliptic to hyperbolic metamaterial regimes.

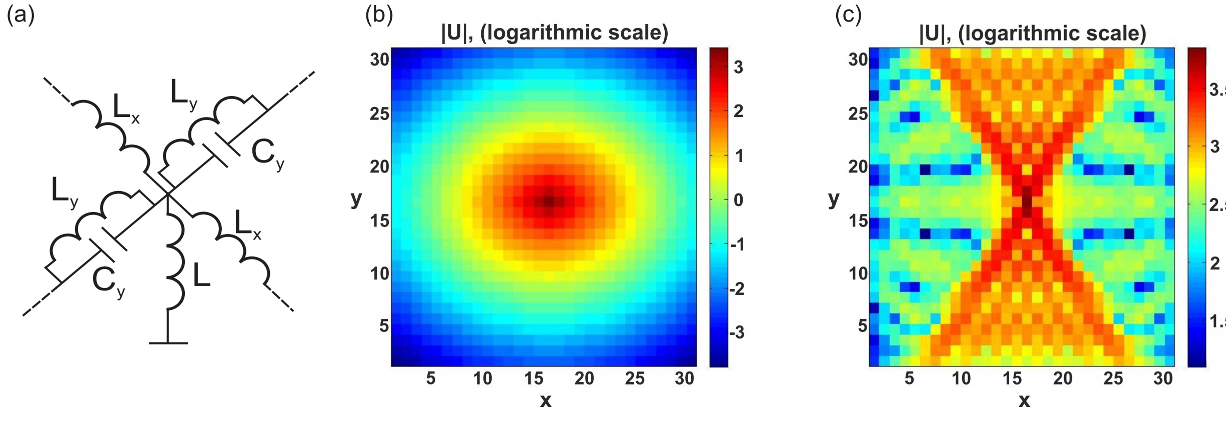

Fig. 1: (a) Unit cell of our two-dimensional TL metamaterial structure supporting topological transitions. Simulated voltage distribution of the two-dimensional TLs metamaterial structure composed of 31×31 unit cells: the voltage magnitude in logarithmic scale at the frequency: (b) 100 kHz and (c) 250 kHz.

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Resonant properties of a layer of bi-isotropic composite material

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Most research works, devoted to the problem of bi-isotropic chiral media investigation, as a rule, present the results for the case of arbitrary electrodynamic parameters, such as dielectric permittivity and magnetic permeability, chirality factor. But chiral media in the form of composites, containing conductive inclusions (few-coil helices, omega-particles, combinations of a helix coil with attached straight conductors etc.) possess resonant properties in a certain frequency range. The conducted research shows that, for instance, the frequency dependence of the reflection coefficient of an electromagnetic wave (EMW), reflected from a layer of a composite on metallic surface, has a resonance character, and the bandwidth of the resonance curve is rather narrow [1–2].

Current paper presents the method and investigation results that aim to broaden the resonance bandwidth of a similar composite using a two-component (and in general case, a multi-component) mixture of particle of identical geometry. At this the component concentration is identical, and geometrical dimensions proportionally increase. The solution of this task is based on assumption, that the composite is formed by conductive fibers that consist of coupled conductive particles chaotically dispersed in a dielectric matrix. Following the method, described in [2–3], the task of electromagnetic wave scattering on such a particle is solved, it polarizability coefficients are defined. Then, using Maxwell–Garnett method, the effective electrodynamic parameters of a composite are defined: dielectric permittivity and magnetic permeability, chirality factor. Taking into account obtained material parameters of a composite the investigation of interaction between electromagnetic waves of E- and H-polarization and a flat layer of a composite is conducted as shown in [4–5].

The article presents the investigation of bi-isotropic composite material that represents a dielectric matrix, containing coupled particles in the form of two identical few-coil helices, located on some distance from each other. The calculation of polarizability factor was carried out for such particles, material parameters of a composite and the coefficient of EMW reflection by a layer of a composite on a metallic backing, transmission coefficient for a layer of such a composite. The influence of mutual orientation of few-coil helices and the distance between them on the values of material parameters, reflection coefficients and transmission coefficients was investigated. As the outcomeof numerical experiments, it was ascertained that within the limits of frequency range of the first resonance the calculation results almost do not depend on mutual orientation of helices, when the distance between the helices does not exceed half of the wavelength of electromagnetic field.

Obtained results allow recommending the proposed method to analyze more complicated systems. In particular, the analysis of interaction between EMW and a layer of a composite material, filled with coupled particles in the form of two few-coil helices, when the size of one of them is **K** times bigger than the other, is conducted.

It was established that at some definite value of scaling of coefficient **K** it is possible to broaden the resonance bandwidth for reflection coefficient and transmission coefficient in comparison to a composite made of single particles. Further increase of coefficient **K** leads to appearance of two resonances in frequency dependency of reflection and transmission coefficients.

Thus, a methodology for calculating material parameters of a composite, consisting of thin-wire inclusions of different size, is proposed. It uses the method of integral equations to analyze diffraction of electromagnetic wave on wire chiral scatterer. A possibility to broaden resonance bandwidth of multi-layer chiral structures at the expense of using a mixture of chiral elements with different geometric dimensions is proposed.

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Practical antenna application of extremely anisotropic materials: reality or fiction?

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The paper presents an insight into recent authors' attempts to develop practical antenna devices based on extremely anisotropic materials. It summarises theoretical, numerical and experimental results which have been obtained during the last 3–4 years of research. in introduction, the paper describes basic concepts of wire media and sub-wavelength imaging, which is achievable through a proper utilization of wire media [1]. Also, an attention is paid to application of wire media in dielectrics. The following sections explain how wire media can be used in antenna applications. First of all, an antenna imaging is mentioned where the antenna image can be simply transferred from one interface of the wire medium into another without a loss of information about its near-filed. However, those experiments have been done in free-space and, despite amasing results [2], are pretty useless for antenna business. More attractive results have been obtained with magnifying media [3]. It allows achieving unusual antenna field distribution, non possible otherwise, for example, for dipole or horn antennas [4]. Another area of research concerning wire media application is its utilization as antenna radomes. The main idea was to retrieve near-filed distribution of the antenna placed within a cavity, even metal one. the idea had been successfully proven [5]. However, not all antennas can be "resurrected" as it was shown [6]. In the presented paper, the authors try to show which problems can be anticipated when attempting applying wire media for antenna application. As an example a printed Yagi–Uda antenna has been designed and thoroughly numerically and experimentally investigated in terms of radiation patters, near-filed distribution, etc. General conclusions and discussion finalise the paper, pointing several important limitation of wire media applications in terms of performance and not less important manufacturing issues.

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Photo-induced nonlinear-optical response of dielectric nanoparticles

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The comprehensive study of optical properties of Al_2O_3, SiO_2, TiO_2, and ZnO nanoparticles in weak optical fields is presented. We found that nanoparticles have unique optical nonlinearity: The values of the nonlinear portion of the refractive index and absorption coefficient for nanoparticles increase to maxima, and then decrease to zero when the radiation intensity changes from 1 to 500 W/cm^2. We estimated electron energy structures of nanoparticles and experimentally determined that such nonlinearity is directly related to peculiarities of the energy structure. Using the proposed theory of optical properties of dielectric nano-objects in weak optical fields, we obtained a good fitting of theoretical results to the experimental ones (e.g. absorbance spectra of nanoparticles; the dependence of nanoparticle refractive index and absorption coefficient on radiation intensity).

Optical resonant properties of Si nanoparticles

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Strong resonant light scattering by Si nanoparticles is experimentally and theoretically demonstrated, revealing pronounced resonances associated with the excitation of magnetic and electric modes in these nanoparticles. In contrast to localized surface plasmon resonances of metal nanoparticles, induced by the light electric field, Si nanoparticles can resonantly interact with both electric and magnetic components [1–4]. In the last case one can speak about optical magnetism in the visible range. This unique property is interesting from both fundamental and applied points of view. Si nanoparticles can be applied as nanoantennas in different complex systems for sensing magnetic fields at optical frequencies. On the basis of Si nanoparticles, novel optical metamaterials with a strong magnetic response can be realized. Moreover, accumulation of magnetic energy at the resonant conditions can be exploited for realization of local magnetic-field enhancements and for the design of novel nanoscale optical devices. Here we present our recent results on investigations of resonant properties of single Si nanoparticles with different sizes and shapes, and of Si nanoparticle arrays. Possible applications of resonant Si nanoparticles are discussed.

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Quantum plasmonics

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Surface plasmon nanolasers, also known as SPASERs attract much attention in recent years due to the numerous potential applications in the plasmonics. In this work, we consider the metal horseshoe nanoresonator filled with the active dielectric medium. The size of the considered resonator is much less than the wavelength. On this scale, the plasmon field inside the nanoresonator behaves as a quantum object. The plasmon quantum field is subject of the random external force because of the quantum-mechanical process of the photon absorption in the active medium. Due to the small size of the resonator, the coupling between the plasmon field and an atom of the active medium is anomalous strong. The interaction cannot be considered as just emission and absorption processes. We develop the quantum dynamics of the plasmon field coupled with the active medium. Thus, we consider how the quantum fluctuations influence the process of the luminescence and find how the lasing threshold is changed. The coherence of the light emitted by the plasmon laser is also considered. We predict that the light beams, radiated from non-interacting plasmon nanolasers, could not give the interference pattern. The quantum properties of the nanolaser are important for many applications, such as transmitting and processing optical signals on a scale much smaller than the wavelength.

Features of terahertz emission interaction with "probe-object" system in terahertz apertureless near-field microscope

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In this report, we give a brief review of our experimental and theoretical study on effects occurring in terahertz apertureless near-field microscope while the terahertz excitation pulse interacts with the metal probe and the object underneath. We show that the amplitude and the spectrum of the pulse after this interaction is determined by some parameters including the shape of the probe, the amplitude of its modulation, and the local dielectric permittivity of the probe and the object under study. Taking into account the inhomogeneous excitation of the probe by the terahertz pulse, we show that this excitation results in diffraction edge waves scattering from the tip of the probe and from the transition region, which serves as a boundary between the illuminated part of the probe and the shadow region. These edge waves form the angular and the frequency spectra of the terahertz pulse registered in the far field. We also show that terahertz apertureless near-field microscope is capable of acquiring some fundamental parameters of semiconductors, such as charge carriers' density, with 100 nm lateral resolution.

Femtosecond nanoplasmonics in metamaterials

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Polarization is a property of electromagnetic waves which describes the time-averaged trajectory of the electric field vector at a given point of space. It is a commonplace that the timescale over which the polarization is averaged is usually much greater than the period of a single electromagnetic field oscillation. However, recent works demonstrated that the polarization state could be switched on the sub-picosecond scale by means of elementary excitations in quantum-sized media at subzero temperatures. A convenient system for observation of sub-picosecond-lifetime elementary excitations at room temperatures is a modulated surface of a noble metal film where surface plasmon-polaritons (SPPs) are excited. Proved by numerous ultrafast experiments involving femtosecond laser pulse sources the mean lifetime of a resonantly excited SPP is found to be varying from tens to hundreds of femtoseconds which is mainly defined by radiative losses of the SPPs. In this contribution we use resonantly excited surface plasmons to control the state of polarization (SoP) inside a single sub-picosecond telecom laser pulse reflected from a plasmonic nanograting with enormous spectrally dependent optical anisotropy[1]. Plasmon-induced birefringence and dichroism with a Fano-type spectral shape cause a pronounced shift of the polarization state inside a single femtosecond telecom laser pulse at the maximal rate of 13 ps^{-1} in the Stokes vector space. Time-dependent non-zero depolarization is found which indicates the sub-130 fs polarization change inside the pulse. We support the experimental data with an analytic model which predicts the four-fold enhancement of polarization conversion which makes plasmonic crystals a perspective media for ultra- fast polarization control. Temporal modification of femtosecond pulses upon the resonant excitation of surface plasmon–polaritons is also studied in one-dimensional metallic nanogratings by femtosecond cross-correlation spectroscopy when the laser pulse duration is comparable with the SPP relaxation time. Modification reveals itself in the pulse duration changes and the pulse shift relative to the unperturbed pulse and manifests itself in the maximum shift and the width changes of the second-order cross-correlation function. Spectral behavior of the pulse shape changes is governed by the femtosecond SPP relaxation dynamics described by the Fano-type resonance. Both a decrease and an increase in the reflected pulse duration are found. Leading and delaying of the pulse reflected from the sample relative to the unperturbed pulse are found to be up to 24 fs and 43 fs, respectively, for the pulse width 200 fs and SPP time decay about 90 fs. The technique of time-resolved Stokes parameters measurements allows one to measure the evolution of all the Stokes vector components of the beam reflected from the sample in only two delay line scans. Femtosecond-scale magnetic field-controlled shaping of 200fs laser pulses reflected from a one-dimensional magnetoplasmonic crystal is experimentally demonstrated. Magnetic field-induced modification of the pulse shape is revealed by measuring the second-order intensity correlation function (CF) of femtosecond pulses reflected from the sample. The sign of the magnetic contribution to the CF is reversed within the pulse. Such temporal shaping of the pulses is attributed to modification of the Fano-type surface plasmon spectral response function under magnetization of the sample in the Voigt configuration[2].

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Quantum theory of the plasmonic nanolaser

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The modern story of the utilization of the nanoparticle-living plasmons started a decade ago, when the possibility of the enhancement of surface plasmons in the nanoparticles (NPs) by optical gain in dielectric medium was predicted theoretically [1] and demonstrated in the experiments [2]. It has been realized, that extremely tight field confinement in surface plasmon modes can be used to achieve the strong coupling regime with the emitters. The metal NPs became a promising platform for implementation of the great variety of effects known in quantum optics and cavity QED. In parallel, papers [3, 4] claimed the lasing on the individual NPs, and a lot of work focused on the realization of smaller and faster sources of light.

A metal NP adjacent to the active medium appeared as a basic unit cell of many applications of nanoplasmonics, such as metamaterials, nanosensing, optical logic, etc. Though, theoretical description of this object (which essentially constitutes a nanolaser, alternatively known as spaser) trails far behind the experimental progress. Recently, we have shown that the semiclassical limit of quantum theory (see, e.g. [5]), which is eventually equivalent to the classical Maxwell–Bloch approach [6], is unadaptable for the nanolaser. It corresponds to the so-called thermodynamic limit of the laser [7], which is not the case, since in the nano-sized resonant cavity most of the spontaneous radiation of the emitters goes directly to the laser mode. In another words, the precondition of the classical theory is that the number of quanta in the laser mode is macroscopically large. We show that in real plasmonic nanoresonators this value is limited from above by strict thermal and strength limitations. Actually, the nanolaser in CW regime is not expected to support mode than several plasmonic quanta. That is, the nanolaser is truly a nonlinear noise device, where fluctuations play a crucial role.

Here, for the first time, we make an attempt to describe the plasmonic nanolaser in the formalism of the density matrix. We use a low-loss approximation, which holds for the plasmonic nanolasers with cavities starting from the FOM of 10^2, which is the case for many realizations. This allows one to employ the maser approximation and solve the master equation analytically. We use this solution to study the statistical and coherence properties of the plasmonic radiation. Besides the plasmonic nanolasers, our theory can be applied to the wide range of the high-Q microcavity lasers.

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Nonlinear phenomena in transitional metamaterials

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The unusual properties of metamaterials are most manifest as boundary phenomena. These phenomena include the multi-wave interactions, when the frequencies of the interacting waves correspond to the frequency domains with different signs of the refractive index. The role of the boundary in the frequency domain plays a transition frequency corresponding to the change of sign of the refractive index. An example of boundary phenomena in the spatial domain is the propagation of electromagnetic wave through a transition layer in which the refractive index continuously changes sign. Resonant field enhancement in the neighborhood of the "zero point" is an example of the unusual behavior of the electromagnetic field in the transitional metamaterials. This enhancement creates preconditions for nonlinear media response at moderate intensity values of the incident electromagnetic field. We studied the second-harmonic generation in the transition layer for both cases of continuous field and electromagnetic pulses. It is shown theoretically that the phenomenon of second harmonic generation in transition layers occurs at low intensities of the fundamental harmonic. In traditional dielectrics the second harmonic at such intensities is not noticeable. The intensity of the generated second harmonic intensity is comparable to the fundamental harmonic.

Quantum opto-mechanical phenomena on the nano-scale

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Optical forces are of significant interest from both fundamental and applicative point of view and already proven to be important in various fields of science, from astronomy to biology. The term of radiation pressure, i.e., force acting on uncharged body situated in electromagnetic field, was introduced by Maxwell in 1871, but the real boost of practical applications was initiated with the discovery of optical tweezers by Ashkin in 1970 [i]. One of the very promising and already tested approaches for optical force enhancement relies on the increase of the field gradient with the help of so-called plasmonic nanostructures. Noble metals with negative permittivity at optical and infra-red wavelengths (plasmonic metals) can support surface plasmon modes with the deep subwavelength localisation of the electromagnetic energy, overcoming the conventional diffraction limit and creating strong field gradients.

In this contribution we will give a review on recent achievements on optical forces, involving nanophotonics structures. In particular self-induced quantum opto-mechanical forces, enhanced in plasmonic metamaterials [ii], and novel approaches for optical signals modulation [iii] will be discussed.

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Application of atomic layer deposition in nanophotonics

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In this talk we review our recent results on employing the Atomic Layer Deposition (ALD) in fabrication of nanophotonic devices. The ALD is a unique thin film deposition method providing several unparalleled advantages: atomic level control of film composition and thickness, perfect step coverage, and large-area uniformity. We describe several devices utilizing the advantages of the ALD in connection with Si-nanophotonics and with replicated polymer based resonance waveguide gratings.

Si-nanowaveguides are gaining significant importance and they are expected to play a crucial role in the new generation of photonic devices, in particular for optical telecommunications. Optical telecommunications will continue to revolutionize our society, e. g., by further increasing the possibilities of the Internet. However, the functionality of silicon nanophotonic devices can be dramatically increased by taking advantage of thin film materials that have superior optical properties compared to silicon. For example, integration of silicon nanowaveguides with materials featuring very large third-order nonlinearity would be attractive. In our research we investigate the use of thin films grown by ALD in nonlinear silicon nanophotonics. Here we present several new structures based on filling the so-called slotted Si-nanowaveguides with ALD-grown materials.

In addition, we present our recent results of polymer based Guided-Mode Resonance Filters (GMRFs), e. g., for Surface Enhanced Raman Scattering (SERS). In our approach the GMRFs are fabricated by combining replicated polymer materials and ALD-grown thin films. GMRFs are diffraction gratings with spectrally narrow reflectance peaks due to resonance anomalies, i. e., coupling of incident light into a semi-guided mode in a corrugated waveguide. A unique feature in our replicated gratings is the use of ALD in producing uniform and conformal high-index films on the corrugated grating structures. Our approach of combining replication techniques with the ALD can lead to low-cost and sensitive sensor devices, e. g., for medical diagnostics.

Plasmons and magnetoplasmons in single-, double- and multilayer graphene structures

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Graphene, two-dimensional lattice of carbon atoms, exhibits a wide range of unique electronic and optical properties [1]. It was shown theoretically and demonstrated experimentally [2] that a specific type of localized waves, surface plasmon polaritons, can propagate along a single layer of graphene or its bilayer. It was shown that both TM and TE polarized plasmons can exist in graphene [3], and their dispersion properties can be changed by applying an external gate voltage to the graphene sheet which allows to construct effective two-dimensional metamaterial structures based on graphene [4].

We study optical properties and eigenmode dispersion of different graphene based structures shown in fig. 1.

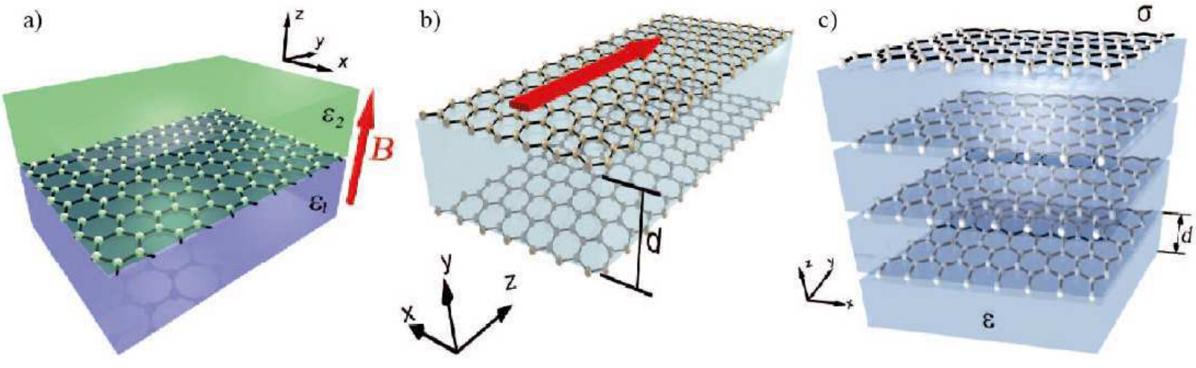

Fig. 1: Geometries of the considered structures
a) semi-infinite dielectric medium covered by a layer of graphene in the presence of a strong external magnetic field [5]
b) double graphene layer structure with a dielectric medium between lists of graphene [6]
c) metamaterials for THz frequencies based on multilayer graphene structures [7].

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Non-absorbing metamaterial film with dispersion of effective refractive index

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The present work studies the effective refractive index in a composite film. It is a dielectric film with an embedded lattice composed of orderly distributed nanosize cavities of the same size. Such a structure was proposed as an antireflection coating in [1]. General approach for determination of the composite film effective refractive index is developed, for example, in [2]. This method allows to define physical parameters of a medium through so-called S-parameters, which are the components of scattering matrix. The effective refractive index is related to S-parameters as follows [2]

$$n_{\mathit{eff}} = \frac{1}{\mathbf{k}_0 \mathbf{\Delta}} \arccos \left[\frac{1}{2S_{21}} \left(1 - S_{11}^2 + S_{21}^2\right) \right].$$

In this work, we perform numerical calculations of S-parameters employing the finite elements method software Comsol Multiphysics. The figure shows a dispersion dependence of the effective refractive index of a composite film that is a dielectric matrix with imbedded nanosize cavities. One can see that the effective refractive index of the system exhibits the positive dispersion. This refractive index dispersion is specified by a character of light scattering by nanosize objects and cooperative effects in a quasicrystal associated with wavelength/interparticle distance relation. For comparison, Figure shows the dependence obtained from the effective medium theory of Maxwell–Garnett (with the same filling factor as the ordered layer has) corresponding to a composite film with chaotic distribution of nanosize cavities. In this case, the refractive index possesses no frequency dispersion. In long-wave limit, the effective refractive index of a nanostructure tends to the value predicted by the Maxwell–Garnett theory. Note, the system is non-absorbing. The obtained result does not contradict the Kramers–Kronig relation that originates from the causality principle and requires non-zero imaginary part of the dielectric permittivity in case of real part dispersion. In this case, the effective dielectric permittivity is not a "response function" of a system ([3]) but an abstract quantity calculated from the analyses of transmission spectra and total system reflection. This quantity can not be used to determine the displacement vector at each point in the film's volume.

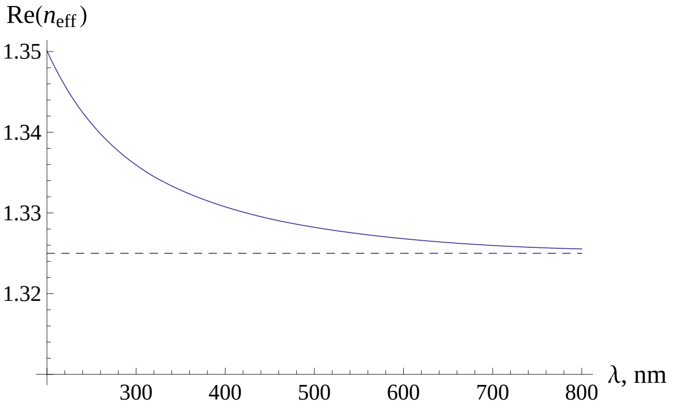

Fig. 1: Dispersion of effective refractive index. Refractive index of dielectric matrix is equal to 1.5. Refractive index of dielectric matrix is equal to unity, composite film thickness — 100 nm. Dashed line is the calculation by Maxwell–Garnett formula.

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Reduced absorption of light by metallic intra-cavity contacts: Tamm plasmon based laser mode engineering

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It was widely accepted that embedding of metallic layers into optoelectronic structures is detrimental to lasing due absorption in metal. However, recently macroscopic optical coherence and lasing was observed in microcavities with intra-cavity single metallic layer. Here we propose the design of the of microcavity-type structure with two intra-cavity metallic layers which could serve as contacts for electrical pumping. The design of optical modes based on utilizing peculiarities of Tamm plasmon provides vanishing absorption due to fixing of the node of electric field of optical mode to metallic layers. Proposed design can be used for fabrication of vertical cavity lasers with intra-cavity metallic contacts.

Recently, a novel type of localized mode of the electromagnetic field (Tamm plasmon) were predicted theoretically [1] and subsequently demonstrated experimentally [2]. Tamm plasmons (TP) are localized at the interface of a specially designed Bragg reflector and a metal and are the analogues to Fabry–Perot cavity modes. Despite metallic mirrors being the most commonly used type of light reflectors in normal life, metallic optical components have not found wide use in optoelectronics. The main obstacle preventing the application of metallic mirrors in optoelectronics is the optical loss and heating of the metal due to optical absorption which leads to a catastrophic degradation of the mirrors and surrounding materials. A peculiarity of TP localized on thin metal films of sub-wavelength thickness embedded into a microcavity is the occurrence of a node of electric field at the metal. Thus, absorption in such structure is substantially reduced, and makes possible the experimental observation of macroscopic optical coherence [3] in a microcavity with single metallic layer. For an implementation of the efficient electrical pumping of a microcavity, the two metallic layers should be inserted into the structure.

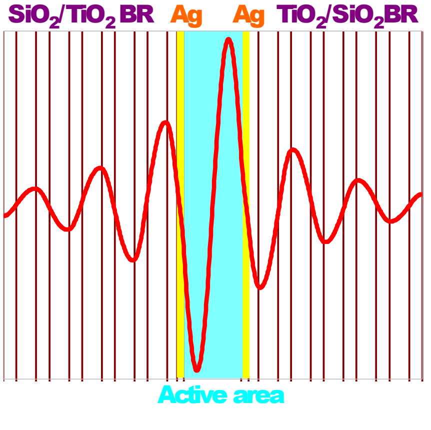

Fig. 1: Design of the structure: two silver layer of the thickness 40 nm are placed are the boundaries between Bragg mirrors and cavity forming a microcavity structure. Solid line shows the profile of electric field eigenmode with minimal decay.

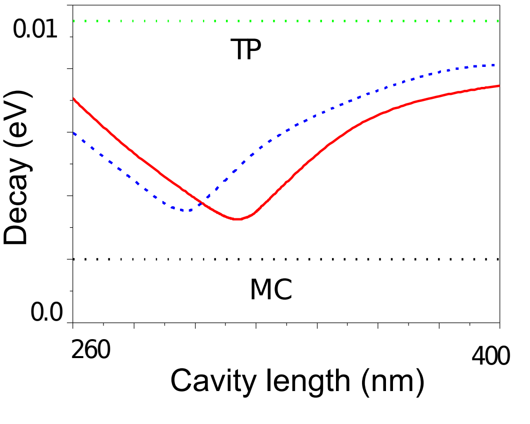

Fig. 2: Dependence of decay of the hybrid mode of the microcavity with minimal decay as a function of cavity length for the structure shown in figure 1 (solid line), and for the structure, where additional silver layer of the thickness 20 nm is placed at the centre of the cavity. Dotted lines show the decay for TP mode and for the mode of microcavity without metal layers.

The structure used for modelling is shown in figure 1. Active area made of optically active organic material DCM+Alq3, is sandwiched between two silver layers and surrounded by two SiO_2/TiO_2 Bragg reflectors. On each interface between silver layer and Bragg reflectors, TP can be localized. Another Fabry–Perot can be localized between two silver layer. When the frequencies the three modes are matched, three hybrid modes appears, the energy splitting of these three modes is defined by a thickness of intra-cavity metallic layers. Each mode have different field distribution distribution. The mode of interest, which will potentially support lasing, should possess the following properties: i) minimized overlapping with metallic layers (to reduce absorption); ii) maximized overlapping with active area (to increase amplification of light). We can vary the field profile of eigen-mode by tuning the phases gained by light propagating through various layers.

Figure 2 shows the depenencies of decay of hybrid mode with localized within an active area of the structure on the distance between metallic layers. It can be seen, that there is optimal length of active area, when the value of the mode decay is minimized. Note, that for hybrid eigen-mode of the structure with metallic elements, decay in increased only by 50% in respect to the mode of microcavity with zero absorption. To explain this phenomenon, one should analyse the profile of such optimized mode shown in figure 1.

It can be seen, the electric field of optimized mode have the node at the centre of active area and on metallic layers. Since absorbing region is placed near at the place of a node, absorption is reduced. It should noted, that for two other eigen-mode field is not localized upon based active area.

Insertion of metallic layers into the nodes of the electric field of an eigen-mode can be used for construction of the structures, where not with just two, but with many metallic layers in the active area of the laser. Dashed lines in figure three shows the dependence of resonant energy and decay of optimal eigen mode for the structure, similar to those shown in figure1, but with an additional silver layer placed at the centre of the cavity, where the node of the electric field occurs. It can be seen that the additional layer does lead to a substantial increase of absorption in the structure.

Since the metallic layers in such structure are adjacent to the active area, there the ohmic resistance of the structure will be reduced, and the wall-plug efficiency of the electrically pumped laser will be increased. Also, contacts of this type can provide uniform distribution of pumping current over the active area of a large lateral size.

In summary, we have demonstrated possibility of lasing from microcavity with two metallic intracavity layers which can serve as a contacts. It was demonstrated, that eigen-mode in such structures can be engineered in the way, which provide nodes of electric field upon metallic layer, and absorption in such structure is substantially reduced.

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Competing nonlinearities with metamaterials

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Metamaterials as a novel class of electromagnetic media offers exceptional opportunities for creating artificial materials with a wide range of macroscopic parameters through appropriate arrangement of their subwavelength elements [1]. Moreover, it has been recognized that the concept of metamaterials allows engineering tunable, active and nonlinear response as well [2].

Recently we have proposed a novel design of light-tunable split-ring resonator (SRR) [3] which is a prototype of a meta-atom for tunable microwave metamaterials. In such metamaterials the local electromagnetic properties can be changed arbitrarily by illuminating the sample with required patterns of light [4].

In this work we focus on nonlinear properties of light-tunable SRRs. The schematic of the light-tunable SRR structure is shown in Fig. 1(a). The design is based on the standard side-coupled SRR formed by two broken rings of a metal strip (copper) printed on a dielectric substrate (FR4 fiberglass, $\varepsilon_r \approx 4.4$). To achieve tunability, we introduce an additional gap in the outer ring of the SRR in which we solder a varactor diode. The bias voltage for the varactor diode is produced by a photodiode that operates in the photovoltaic mode. In this design we use two photodiodes connected in series to provide higher value of the biasing voltage. To prevent shortening of the varactor diode by the large parasitic capacitance of the photodiodes, we use chip inductors connected in series with the photodiodes. An efficient isolation is achieved at the self-resonant frequency of the inductors that is chosen to be close to the operational frequency of the structure. As the voltage produced by the photodiodes depends on the intensity of the incident light, the capacitance of the varactor diode changes with the light intensity. In turn, the resonant frequency of the SRR depends on the total loading capacitance of the entire loop, to which the capacitance of the varactor diode contributes. In this way, the resonant frequency of the SRR becomes dependent on the light intensity.

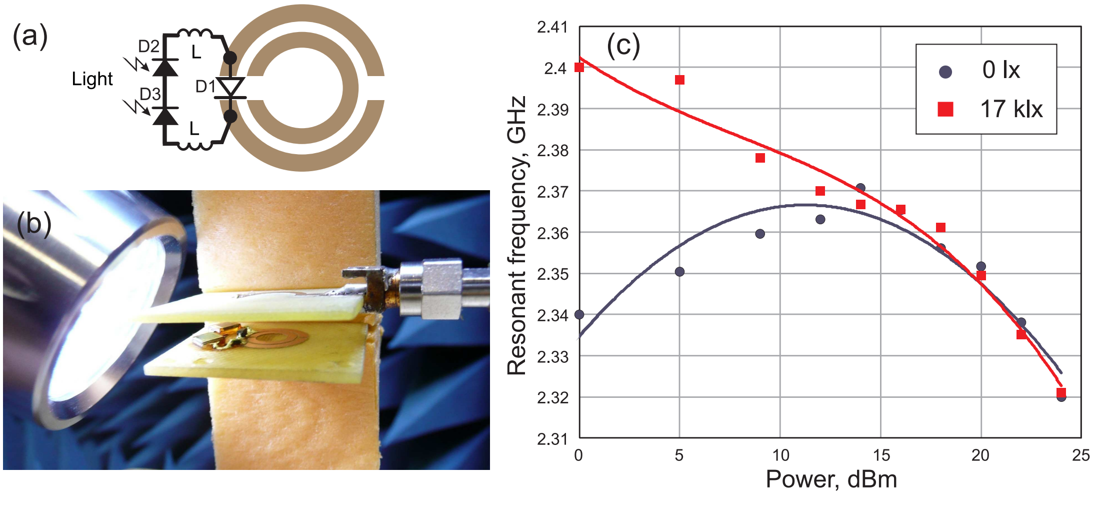

Fig. 1: (a) Schematic of the light-tunable SRR and attached electronic components. (b) Photograph of the SRR sample (bottom dielectric board) excited by a loop antenna (top dielectric board) and illuminated by a light source. (c) Measured shift of the SRR resonant frequency as a function of input power, for two values of light source intensity. Measured data are shown by markers, whereas the lines are guide for eye.

We investigate nonlinear properties of the two light-tunable SRR designs: i) the photodiodes are mounted to ensure constant reverse voltage on the varactor diode [5]; ii) the orientation of the photodiodes is changed to provide the constant direct voltage on the varactor diode [6]. To characterise the SRR resonance as a function of the incident signal power we place a loop antenna above the SRR sample, so that the incident magnetic field is normal to the SRR (see Fig. 1(b)). The loop antenna is excited by an Agilent PNA E8362C vector network analyzer. To amplify the microwave signal we use a 1W power amplifier. The power of the incident signal changes from 0 dBm up to 24 dBm. To ensure that the nonlinear response of the amplifier does not contribute to the measurements, we performed power calibration for each individual power level. A directional coupler is used to sample the output from the amplifier, so that the power calibration can be performed in situ, and to provide a reference level. With the aid of the directional coupler we measure the reflection coefficient of the loop antenna. The resonant frequency is determined from the minimum of the reflection coefficient. The structure is studied under the light intensities illuminating photodiodes of 0 lx (dark state) and 17 klx (bright state).

In the case when the reverse voltage is applied to the varactor diode we demonstrate that the effective nonlinearity can change its sign as we increase the intensity of a microwave signal (see Fig. 1(c)). We show that such nonmonotonic nonlinear response corresponding to the competing nonlinearities is caused by effective switching between the two nonlinear regimes depending on light illumination. In the case when the forward voltage is applied to the varactor diode the nonlinear response can be tuned by illuminating the photodiode with an external light source. For this SRR configuration we have also found a bistability region which appears at large amplitudes of the incident signal.

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Enhancement of the luminescence of the quantum dot layer hybridized with high-Q all-dielectric metamaterial

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Double periodic planar structures with periods smaller than the wavelength are basis of the optical metamaterials. Today huge interest are devoted to the trapped mode resonances in such structures [1, 2, 3, 4]. The trapped mode resonance may be excited in the double periodic planar array with two or more metal elements in periodic cell and it is caused by arising in the metal elements of anti-phased currents. The anti-phased current mode has not electromagnetic coupling with plane wave in free space for the symmetry metal elements case, so this mode could not excited by plane wave in the structure and has infinite Q-factor in non-dissipative structure [1]. These properties of anti-phased current mode explain both its name and strong dependence of radiation losses level on asymmetry factor of the structure. The asymmetry of metal elements provides the electromagnetic coupling between anti-phased current mode and incident plane wave in free space and substrates and as result in structures with two asymmetry metal elements in periodic cell may be excited trapped mode resonance. The level of the radiation losses of trapped mode resonance is strongly depended on asymmetry factor. Unfortunately a huge energy dissipation inherent in metals in infrared extremely limits the Q-factor of the trapped mode resonance. As results although the Q-factor of trapped mode resonances are an order magnitude higher than the Q-factor of regular dimensional resonance in plasmon structures, but it does not exceed a few dozen.

The trapped mode resonance is accompanied with the strong local fields. This intensive local field allows providing the strong coupling between array in the trapped mode regime and the substrate from gained or nonlinear material. For example, it was shown the enhancement of luminescence of quantum dot layer hybridized with periodic array in trapped mode regime [5].

Recently existence of trapped mode resonance in all-dielectric double periodic structures was shown in [6]. Low dissipation level inherent in a dielectric provides the trapped mode with high-Q factor, which is several hundred in infrared for the structure with two silicon bars in periodic cell. In presentation the diffraction approach for calculating of the photoluminescence of the quantum dot layer will be proposed and tested on example of the quantum dot layer hybridized with plasmon array in trapped mode regime, which was considered in [5]. Moreover the results of calculation of the luminescence enhancement in the quantum dot layer hybridized with all-dielectric array in trapped mode regime by diffraction approach will be presented.

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Nanostructured plasmonic metafilms: doing the visible work

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Here, we focus at the theory and design of the experimental samples of novel optical devices made of nanostructured thin metal films, including for example a computer-generated hologram for a visible wavelength and an ultra-thin planar lens.

Plasmonic metasurfaces — artificial ultra-thin plasmonic structures for optical components with improved or completely new functionalities — open a viable way of controlling light on a subwavelength scale. Plasmonic metasurfaces or metafilms (see [1] and references therein) consisting of nano-antennas have been used to achieve many unparalleled applications such as bending the light abnormally [2] in a fairly broad range of wavelengths, [3] generating an optical vortex beam, [2, 4] coupling between propagating waves and surface waves, [5] creating a macroscopic near-infrared lens. [6]

We design and test a computer-generated phase hologram recorded in a plasmonic metasurface by perforating complimentary nano-antennas in a 30-nm-thick gold film. [7] The hologram creates discrete amplitude and phase shifts and hence reconstructs a desired wavefront of the image. This ultra-thin, compact meta-hologram works at a visible wavelength producing a high-resolution, low-noise holographic image.

We also demonstrate compact planar lenses created by such metasurfaces. [8] The lenses work at entire visible spectral range and have very strong focusing ability. Indeed, if the virtual object is chosen to be a point source in the design, the meta-hologram behaves as an optical lens. We have obtained both measured and simulated results for such metasurfaces lenses designed with different focal lengths at 616 nm. We have observed both experimentally and with a full-wave model that the lenses also work at 531 nm and 476 nm.

Our numerical and experimental results have tested a viable route for producing ultra-thin complex photonic devices for the visible from nanostructured plasmonic metasurfaces. We fabricate the proof-of-concept samples built on a Babinet-inverted nanoantenna design and generated high-resolution, low-noise holographic images at a visible wavelength. We applied similar technique to build metasurface lenses, which work at entire visible spectral range and exhibit very strong both focusing and chromatic aberration, which is typical for diffraction optics.

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Control of waves with metamaterials: from microwaves to optics

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Over the last decade metamaterials became a paradigm for engineering electromagnetic space and controlling propagation of waves. This rapid progress has shifted the research agenda towards the development of novel metamaterial applications and metadevices [1] with tunable, switchable, and nonlinear functionalities. This talk will review our recent theoretical and experimental progress in the control of electromagnetic waves with metamaterials and plasmonic structures. In particular, we will discuss several effects associated with the Fano resonance, subwavelength focusing and shaping of light in plasmonic waveguides, generation of Airy plasmons, and control of electromagnetic waves with metamaterials. We will also mention other research activities at Nonlinear Physics Center in Canberra and Metamaterial Laboratory in St. Petersburg supported by the megagrant of the Ministry of Education and Science of the Russian Federation.

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Energy sinks in optics of metamaterials

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Due to energy conservation law the energy emitted by one body (source) will be always absorbed by other bodies (sinks). Usually people do not worry about sinks because far fields are of primary interest. However in nano-optics and especially in nano-optics of metamaterials the role of sinks becomes very important. In fact even in usual media (vacuum) absorbing point dipole drastically disturbs the field distribution and can be neglected by no means (see Fig. 1).

In the case of closed cavities and metamaterials the situation becomes more complicated because in the stationary lossless regime strong spatial singularities arise there. From our point of view these singularities indicate that sinks (absorbers, detectors) should be explicitly incorporated in description of wave phenomena in metamaterial [1–4]. Our approach is not to try to avoid the appearance of new singularities by overdamping or changing geometry, but to include these singularities into wave propagation theory. So we suggest to put real sources and sinks (absorbers) in the places of possible singularities and to see what will happen. This expanded description results in new systems which turn out to be full of new physical sense. Moreover, such systems can serve as a base for a development of new applications of nano-optics. We have analyzed several negative refraction geometries (slab, sphere, wedge etc.) and closed elliptical cavities where singularities appear in formal solution of Maxwell equations. In all cases we found that it is possible to attribute real physical meaning to them. As an example of possible application of our paradigm the scheme of a perfect nano-absorber is shown in Fig. 2. It is proved by full scale simulations that this system can absorb 100% of wide angle incoming energy in deep subwavelength region. Other applications of our paradigm (e.g. preparation of entangled states of atoms for quantum computers) will be also discussed.

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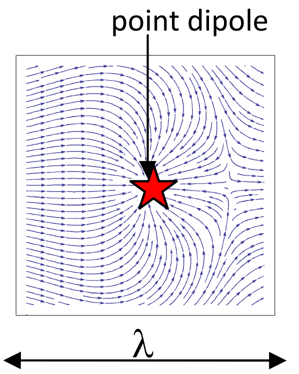

Fig. 1: Resonant light absorption by point dipole.

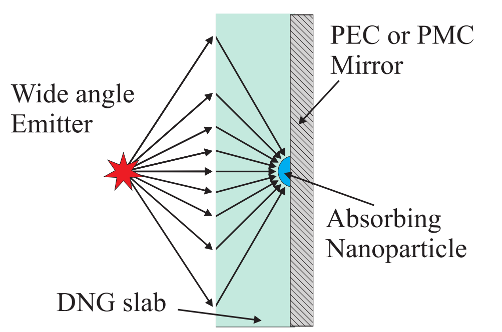

Fig. 2: Schematic diagram of a perfect nanoabsorber based on our paradigm.

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Radiative and non-radiative channels of molecule fluorescence near hyperbolic metamaterials

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Now hyperbolic metamaterials (HMM) are considered as promising media to enhance decay rate of simple quantum systems. Due to huge local density of photon states the decay rate of molecule near surface of HMM tends to infinity as

$$\Gamma^{\text{rad}} \propto \frac{2\sqrt{|\varepsilon_x|\,\varepsilon_z}}{d^3\,(1+|\varepsilon_x|\,\varepsilon_z)} \tag{1}$$

where $d \to 0$ is distance between molecule and HMM surface and $\varepsilon_x = \varepsilon_y, \varepsilon_z$ are lossless components of permittivity tensor.

However if we take into account losses in HMM it is easy to show that non-radiative (Joule) losses also tend to infinity when molecule gets closer to surface of HMM

$$\Gamma^{\text{nonrad}} \sim \frac{Im\varepsilon}{d^3} \tag{2}$$

In this work we will present results of investigation of relations between (1) and (2) for different materials, molecule positions and orientations of its dipole momentum, and find conditions where nonradiative rate is small in comparison with radiative one.

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Spin wave chaos in resonant rings based on metalized ferrite films

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This work reports generation of chaotic signal in a feedback active ring structure based on metalized ferrite film. It was experimentally observed that the active ring can generate dynamical chaos, with frequency bandwidth in a range of 1,5 GHz. In contrast to previous papers devoted to broadband self-generation of dynamical chaos through three and four wave parametric processes [1,2], this work was done in condition when spin system nonlinearity is caused by four-wave parametric processes only. For this condition the use of the metalized ferrite film allows to increase significantly spin wave excitation bandwidth [3] and therefore to obtain the broadband chaotic signal generation. Note that in previous experiments with not screened ferrite films the chaotic signal bandwidth did not exceed 200 MHz [4,5].

The active ring consisted of broadband microwave amplifier, adjustable attenuator, directional coupler and nonlinear phase shifter. Microwave amplifier compensated losses appearing in the ring. Adjustable attenuator was used for ring gain coefficient control. Microwave signal was detected by directional coupler and transmitted to oscilloscope and spectrum analyzer for temporal and frequency analysis. Nonlinear phase shifter was constructed on magnetic yttrium iron garnet (YIG) film 65 μm-thick, 40 mm-long and 2 mm-width. Saturation magnetization $4\pi M_s$ was 1750 Gs. For excitation and detection of spin-waves we used slotline antennas 8 mm long and 50 μm wide. Distance between antennas was 10 mm. Both antennas were fabricated on one substrate. The YIG film was placed on this structure. Thus one surface of the film was metalized. This led to increase of spin wave group velocity and broadening of its excitation band. A static magnetic field was applied in the film plane and perpendicular to spin wave propagation direction. Experimental research was done for several values of bias magnetic fields $H_1 = 1462$ Oe, $H_2 = 1626$ Oe, $H_3 = 1717$ Oe. For these field values three-wave processes were forbidden by conservation laws.

The experiments demonstrated that with increasing of the gain coefficient for all values of H the auto-generation regime was changing from monochromatic generation to generation of periodic sequence of solitonic pulses, and then to broadband chaotic signal generation. In stationary regime, self-generation of dark and bright envelope solitons were observed. Frequency band of dynamical chaos depends on bias field value and varies in a range of 1–1.5 GHz. Numerically calculated characteristic of fractal dimension vs. ring gain demonstrates increase of fractal dimension for all values of bias magnetic field till certain saturation level. Slope of these characteristics and saturation level depends on magnetic field value.

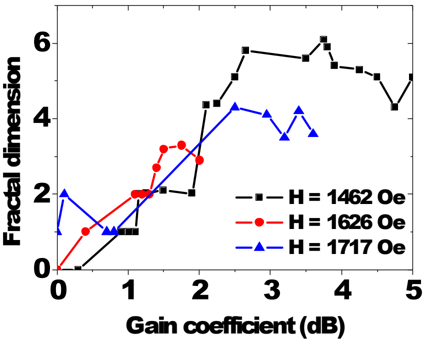

Fig. 1: Fractal dimension vs. ring gain coefficient characteristic.

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Superdirective magnetic nanoantennas with effect of light steering

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We introduce a novel concept of superdirective nanoantennas based on the generation of higher-order optically-induced magnetic multipoles. Such an all-dielectric nanoantenna can be realized as an optically small spherical dielectric nanoparticle with a notch excited by a point source located in the notch. We also confirm the predicted superdirectivity effect experimentally through scaling to the microwave frequency range.

Electrically small radiating systems whose directivity exceeds significantly that of a dipole are usually called superdirective [1]. Superdirectivity is an important property of radio-frequency antennas employed for space communications and radioastronomy, and it can be achieved in antenna arrays in a narrow frequency range and for a sophisticated system of phase shifters [1]. Here we reveal a way for achieving superdirectivity of nanoantennas with a subwavelength (maximum size 0.4–0.5 λ) radiating system without using complex nanoantenna arrays. First, we demonstrate possibility to create a superdirective optically small nanoantenna that does not require metamaterial. We consider one semiconductor nanoparticle with the permittivity $\mathrm{Re}\,\varepsilon = 15 - 16$ radiated by light at wavelength λ (for $\lambda = 440$–460 nm this corresponds to a nanoparticle made of crystalline silicon [2]) and the radius $R_\mathrm{S} = 90$ nm being almost five times smaller than λ. For a perfect sphere, lower-order multipoles for both electric and magnetic fields are excited while the contribution of higher-order modes is negligible [3]. However, making a small notch in the spherical particle breaks the symmetry allowing the excitation of higher-order multipole moments of the sphere. This is achieved by placing a nanoemitter (e.g. a quantum dot) within a small notch created on the sphere surface, as shown in Fig. 1A. The notch in our example has the shape of a hemisphere with a radius $R_\mathrm{n} \ll R_\mathrm{S}$. The emitter can be modeled as a point-like dipole and it is shown in the figure by a red arrow. It turns out that such a small modification of the sphere would allow the efficient excitation of higher-order spherical multipole modes [4].

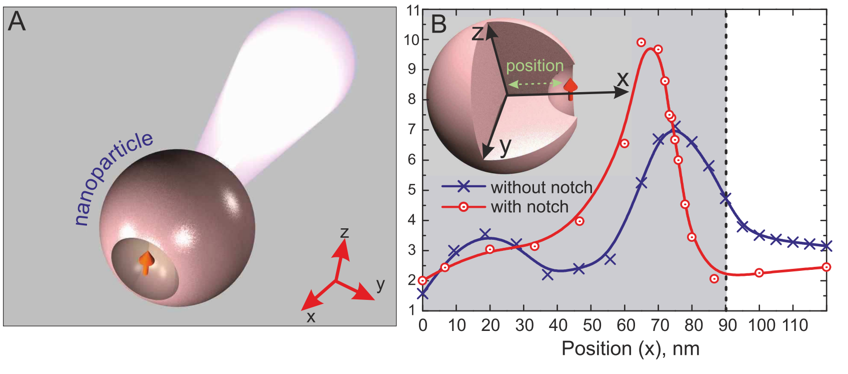

Fig. 1: (**A**) Geometry of the notched all-dielectric nanoantenna. (**B**) Maximum of directivity depending on the position of the dipole ($\lambda = 455$ nm) in the case of a sphere with and without notch, respectively.

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Anisotropic diamagnetic metamaterial based on closed rings

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Metamaterials are new materials with unusual properties which enable useful devices through structural engineering of appropriate constitutive elements ("meta-atoms"). The advantage of metamaterial is that we can choose internal of structure "meta-atoms" and their arrangement depending on a particular purpose. For example, metamaterials based on split-ring resonators offer a resonant magnetic response, suitable for achieving negative effective permeability [1]. Importantly, properties of metamaterials are not only determined individual forms of "meta-atoms" but also the lattices structure.

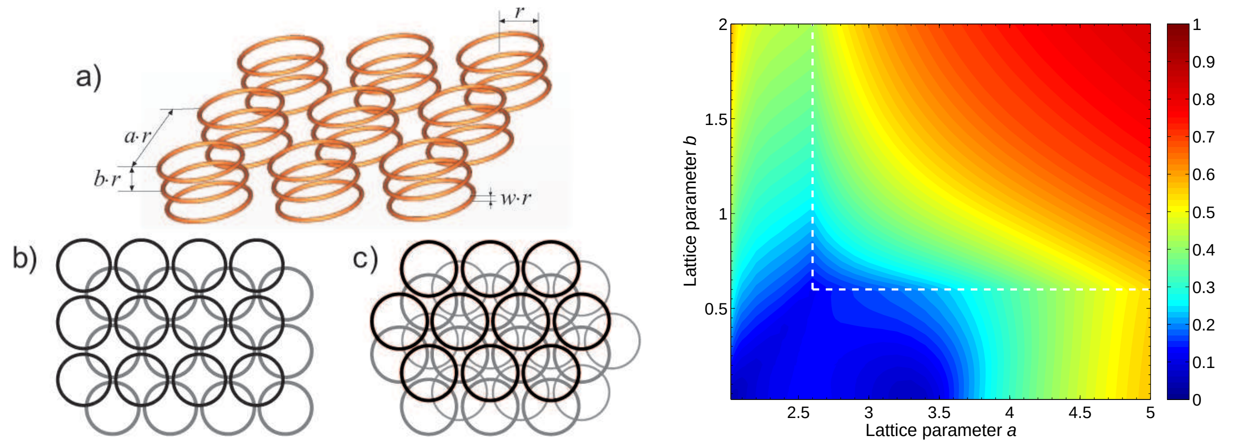

Fig. 1: Left. Various types of lattice arrangements. (a) simple tetragonal arrangement in the plane (b) tetragonal lattice with each layer shifted by one half of the lattice constant, (c) hexagonal lattice with each layer displaced by one half of the lattice constant. Right. Minimal values of the effective permeability (as indicated with the colour bar) in shifted hexagonal lattices and variable w depending on a and b as long as $w < 0.3$, and fixed $w = 0.3$ for $a > 2.6$, $b > 0.6$; the transition between the variable and fixed w is shown with white dashed lines.

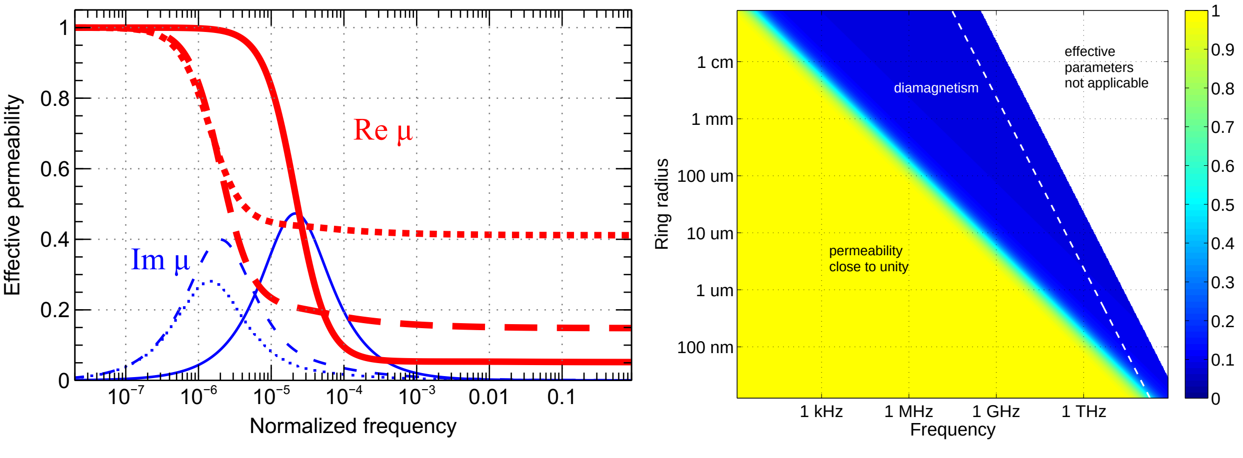

Fig. 2: Left. Effective permeability for different lattices: normal tetragonal lattice with $a = 2.5$, $b = 0.5$, $w = 0.2$ (dotted curves), shifted tetragonal lattice with the same parameters (dashed curves), and shifted hexagonal lattice with $a = 2.02$, $b = 0.02$, $w = 0.01$ (solid curves). Real part is shown with red thick lines and imaginary part with blue thin lines. Right.Real part of the effective permeability (the scale is shown with the colour bar) observed at various frequencies in metamaterials with different ring radius. The white dashed line indicates a reliability threshold taken as $\omega_0/10$, so only the data to the left from this line can be used in practice.

In our contribution we demonstrate that, using metamaterials, we can achieve a strong diamagnetic response which does not exist in nature. To obtain this effect we used a suitable mutual arrangement of closed conductive rings in a lattice (see Fig. 1(left)). To analyse the effective parameters of these lattices we developed a method, similar to that of Ref. [2]. We derive an analytical expression for the effective permeability:

$$\mu = 1 - \left[\frac{ba^2}{\gamma \pi^2} \left(\frac{L_\Sigma}{\mu_0 r} - \frac{1}{\zeta} \frac{J_0(\zeta)}{J_1(\zeta)} \right) + \frac{1}{3} \right]^{-1} \tag{1}$$

where γ is a geometrical factor, $L_\Sigma = L_\mathrm{e} + \mu_0 r \Sigma$ is the total inductance (including mutual interactions), and zero (J_0) and first (J_1) order Bessel functions are taken with $\zeta = (\mathrm{i} + 1) w r \sqrt{\mu_0 \omega / 2\rho}$, where w is the ratio of the radius of the wire of the wire used to make the rings, to the ring radius r, and ρ is resistivity.

We present an overall parametric analysis of the influence of lattice constants and types, as well as the geometrical characteristics of the rings, on the minimum achievable magnitudes of the real part of the permeability (see Fig. 1(right)). We discovered that minimum value of permeability is achieved in shifted hexagonal lattice with dense packing of rings.

In our research [3] we found that the metamaterial of closed conductive rings can provide strong diamagnetic response and that the permeability as low as 0.05 can be achieve by using optimal parameters (Fig. 2(left)). Our theoretical analysis is valid in wide frequency range and predicts an ultra-broadband diamagnetism, as shown in Fig. 2(right). We expect that such diamagnetic response is appealing for applications in magnetic screening, cloaking or even levitation.

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Performance of FDTD method CPU implementations for simulation of electromagnetic processes

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We analyze the performance of finite-difference time-domain (FDTD) method implementations for 2D and 3D problems. Implementations in Fortran, C and C++ (with Blitz++ library) languages and performance tests on several hardware setups (AMD, Intel i5, Intel Xeon) are considered. The performance of implementations using traditional FDTD algorithm for the largest size of test problem is limited by the bandwidth of computer random-accessed memory (RAM). Our implementations are compared with a commercial simulation software package Lumerical FDTD Solutions and an open source project Meep.

Obtained results showed that C implementation is several percent faster than Fortran for the majority of problem spatial task sizes, and C++ implementation is much slower on small spatial sizes, but rapidly improves performance with spatial size growth. For single core computations our implementations proved to be $1.6-1.7$ times faster than Meep, whereas Lumerical FDTD Solutions shows similar performance for maximum spatial size and $1.7-2.2$ times lower performance for smaller sizes. In the parallel four cores regime Lumerical FDTD Solutions shows 1.1 times faster stepping time for maximum spatial size and 3.3 slower for minimum size.

Performance of implementations for maximum problem spatial size changes in several times for different setups, whereas the arithmetical performance of tested processors differs a lot less. This occurs because of the limitation by RAM bandwidth. Calculations and tests showed that our best implementation uses up to 90% of bandwidth limitation, which means that further improvement of source code is ineffective. It is only possible to improve performance using setups with better RAM bandwidth.

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Dynamics of quantum emitters in structured enviroment

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We propose an efficient numerical method to study the emission dynamics of a few two-level quantum systems placed in the vicinity of photonic and plasmonic nano-structures. All parameters of an inhomogeneous environment relevant for the evolution of the quantum systems are appropriately included via the classical Green's function calculated with three-dimensional finite element or finite-differences time-domain methods.

The proposed technique includes naturally weak and strong coupling regimes. In weak coupling regime obtained results are in full correspondence with markovian approximation. Numerical solution for the emission dynamics of a two-level system near a gold nano-cylinder demonstrates excellent agreement with the available analytical solution in the limit of the Lorenzian-shaped local density of states in strong coupling regime.

We show numerically that the strong coupling regime can be realized for realistic strength of the dipole moment for a quantum emitter in the gap of a gold bowtie nano-antenna. Also we explore the joint emitters dynamics embedded in photonic crystall membrane and estimate the field-emitter bound state.

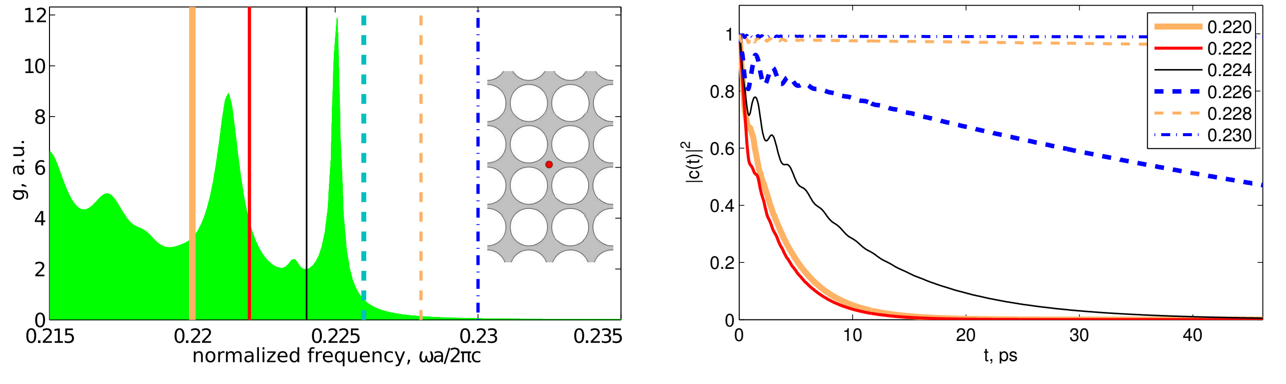

Fig. 1: Left: Two dimensional photonic crystall spectrum. Right: Single two level atom excited state dynamics in two dimensional photonic crystall on different frequencies.

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Modeling of acoustic processes in structurally nonuniform mediums by the method of minimal autonomous blocks

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The wide class of applied problems (communication, a sound and ultrasonic location, a tomography, information security, seismology etc.) is connected with research of acoustic processes in nonuniform mediums. The special attention is given to research of acoustic characteristics of composites, metamaterials, acoustic crystals (functional analogues of photon crystals).

The complex approach to the analysis of acoustic characteristics of structurally nonuniform mediums, based on a method of the minimal autonomous blocks (MAB) is considered. This method is successfully used for the solving of electrodynamic problems similar on statement [1–3].

The investigated area is divided into system of blocks in the form of rectangular parallelepipeds. The sizes and materials of blocks can be various. Acoustic characteristics of blocks are described by scattering matrixes in relation to the waves propagating in virtual waveguides, connected to sides of blocks. For satisfaction of boundary conditions between blocks with various materials additional blocks are used. Scattering matrixes of all types of blocks are used in frameworks the recomposition, the iterative and combined algorithms.

Features of realization of models of various types of sources, and absorbing boundary conditions on external boundaries of investigated area are considered.

For increase of computing efficiency of modeling of systems which structure includes composites and metamaterials, it is offered to use average scattering matrixes which are alternative to the effective material parametres characterizing acoustic parameters of nonuniform materials.

Efficiency and universality of the proposed approach is illustrated on the solving of the following types of problems: propagation of acoustic waves in buildings and constructions; interaction of plane acoustic waves with periodic array from any on structure and material objects; interaction of acoustic waves with composites, metamaterials, acoustic crystals.

Possibility of use of the MAB method for the joint analysis of acoustic and electrodynamic processes in nonuniform mediums is considered.

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Optical second harmonic generation in G-shaped metamaterials

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Planar chiral metamaterials attract much attention as perspective materials for the control over the main properties of light that interacts with them. G-shaped nanostructures have been shown to reveal a strong optical anisotropy, as well as a strong influence of the spatial light localization on their optical response. In this paper, we study experimentally the effect of circular dichroism (CD) in the linear and second-order nonlinear-optical output of G-shaped metamaterials.

The samples under study are regular planar arrays of gold G (and mirror-G)-shaped elements that are made by the electron beam lithography from a 25 nm thick Au film deposited onto $Si(100)/SiO_2$ substrate. The lateral size of G-elements is about 1 μm, the distance between them being 0.4 μm. The elementary cell of a 2D lattice of the metamaterial consists of four elements rotated by 90° with respect to each other, so that they form an in-plane symmetrical and chiral planar structure.

Optical second harmonic generation (SHG) in G-shaped metamaterials is studies when using an output of a Ti-sapphire laser at the wavelength of 800 nm and pulse duration of 100 fs. All the combinations of the pump and SHG polarizations (linear and circular) were studied and all the Stokes parameters are determined.

We show that a strong CD effects is observed in second harmonic generated by G-shaped structures, which consists in different SHG intensity when the sample is excited by left- or right-circularly polarized fundamental radiation. It should be underlined that the sign of the CD effect is changed for the case of a mirror-symmetrical G-shaped structures. Moreover, it turns out that the direction of the SHG polarization plane rotation is the opposite for different G-enantiomers. At the same time, CD effect in the linear optical response was shown to be at least two orders of magnitude smaller.

We also demonstrate that the CD effect in the SHG response is strongly dependent on the geometry of the nonlinear interaction, namely it decreases with increasing angle of incidence of the pump radiation. Our model calculations have shown that this is due to modification of the optical field localization within the metamaterial under different experimental conditions.

Electromagnetic quantum friction on monomolecular layers

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The electromagnetic quantum friction is a phenomenon in which the kinetic energy of bodies in relative motion is nonreversibly converted into the energy of fluctuating electromagnetic field surrounding the bodies. In this work, we consider a pair of parallel material layers (modeled as planar arrays of electric dipoles) which slide one with respect to another with a nonrelativistic velocity $v \ll c$ (Fig. 1, left). On the nanoscale, these layers can be realized, for example, as graphene sheets, whilst on the macroscopic scale the same effect can be achieved with metasurfaces formed by dipolar inclusions.

Using a Hamiltonian quantum-mechanical model we show that electrostatic interaction between the sliding arrays may lead to generation of surface excitations on the two arrays, which can be understood as surface plasmon-polaritons. These boson-type quasiparticles are born in pairs in the two layers, such that the wave momentum associated with each of them is negative with respect to the kinetic momentum of the layer. Because the energy that is required for transitions that result in quasiparticle generation is borrowed from the kinetic energy of the slabs, there appears an effective force that tends to slow down the slabs — the electromagnetic quantum friction force (Fig. 1, right).

Unlike previous models [1], here we consider the quantum friction force acting on arrays of oscillators with a controllable dispersion, which allows us to prove that the quantum friction vanishes in arrays of nondispersive oscillators. We also show that a similar effect can be obtained with a semi-classical model that considers the system of two sliding arrays as a pair of parametrically coupled waveguides. Such coupling results in parametric amplification and generation of the polarization waves in the two arrays, which gives rise to a friction force.

In this presentation we will also outline possible applications of the effect of electromagnetic friction on the micro- and macroscale. Further details can be found in [2].

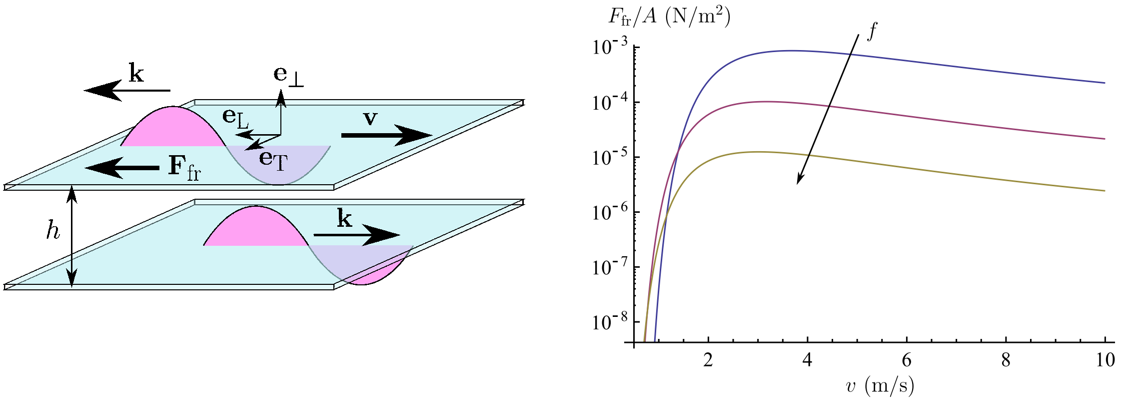

Fig. 1: Left: Two thin material layers in relative motion. Right: Friction force F_{fr} (per unit of area) as a function of the relative velocity v for two nanoscale arrays separated by 3 nm, with the dispersion parameter $m_{\mathrm{eff}}\Omega^2 = f \times 9.10938$ kg·s^{-2}, where $f = 0.3$, 1, and 3.

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Magnetic light: Optical magnetism of dielectric nanoparticles

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Dielectric nanoparticles were predicted to exhibit strong magnetic resonances in the visible [1]. A magnetic resonance originates from the excitation of a particular electromagnetic mode inside the nanoparticle with a circular displacement current of the electric field. This mode is excited when the wavelength of light is comparable to the particle's diameter. It has an antiparallel polarization of the electric field at the opposite sides of the particle while the magnetic field is oscillating up and down in the middle [see Fig. 1(a)]. Recently, this fundamental phenomenon of strong magnetic resonances was experimentally observed throughout the whole visible spectral range from blue to infra-red for silicon nanoparticles with sizes ranging from 100 to 270 nm [2]. Similar results in red and infra-red spectral region for Si particles ranging from 200 to 265 nm have also been published in [3].

Interaction of magnetic and electric dipoles may lead to entirely new scattering properties [4]. In particular, an interference between two optically induced dipole resonances results in azimuthally symmetric unidirectional scattering, that can be realized in layered nanoparticles with metal cores and dielectric shells [5]. A superposition of electric and magnetic resonances of a single core-shell nanoparticle may result in the suppression of the backward scattering and unidirectional emission by a single subwavelength element [5]. The directivity can be further enhanced by forming a chain of such nanoparticles. Together with low losses of dielectric materials, this property suggests a novel principle of optical nanoantennas made of dielectric nanoparticles. Such all-dielectric nanoantennas exhibit much higher radiation efficiency than their plasmonic counterparts allowing more compact designs.

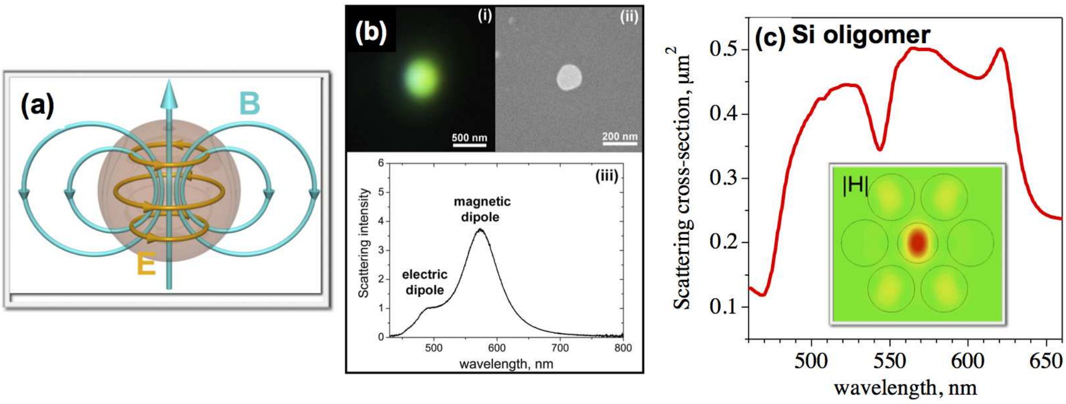

Fig. 1: Optically induced magnetic response of high-refractive index dielectric nanoparticles: (a) Schematic of electric (yellow) and magnetic (blue) field distributions inside a high-refractive index dielectric nanoparticle at the magnetic resonance. (b) Experimental dark-field scattering spectra exhibiting very strong magnetic dipole (md) resonance. (c) Scattering cross-section of a Si heptamer exhibiting strong Fano resonance.

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Plasmonic nanocomposites for functional optical layers and surfaces

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Metamaterials with extraordinary electromagnetic properties can find a variety of applications, in particular for manufacturing superlenses, nonreflecting (absorptive) materials, as well as controlling the optical beam intensity and propagation direction, etc. In this work, a more detailed investigation of optical properties of a matrix metal-dielectric medium with metal inclusions is performed, and the possibility to realize plasmonic structures with beneficial effects in the visible region is considered. The possibility of using plasmonic nanocomposite with nonspherical metal inclusions as functional optical layers and surfaces is considered.

The dependence of optical properties of the plasmonic nanocomposite with metal nanoparticles randomly distributed over the whole matrix volume on the geometric (shape and concentration of inclusions) and material (permittivities of the matrix and metal nanoparticles) parameters are calculated within the effective-medium approximation. Specifically, we apply the Maxwell–Garnett model, whose results for matrices with a moderate content of spheroidal inclusions are in fairly good agreement with the results of exact electrodynamic calculation. The data obtained with the help of Maxwell–Garnett effective medium model show that a heterogeneous medium with plasmonic impurities, such as silver nanoparticles with a concentration of about 10^{21} per m^3, is an interesting object of research with many perspectives for applications. Our results show that such plasmonic medium can be used as a low-loss anti-reflection coating, weakly reflecting light-absorbing filter, polarizing beam splitter with high performance in transmission and reflection.

Nanocomposite coatings and layers are considered also to be formed by one, two, or more monolayers of uniformly oriented metal nanoparticles suspended in a transparent media. Monolayers are oriented parallel to the substrate boundary. In order to calculate the coefficients of the direct light transmission and specular reflection for a stack made of monolayers, we combine the quasi-crystalline approximation applied for calculations of the transmission properties of individual monolayers, with the transfer-matrix technique used for subsequent calculations of the transmission properties of multilayer structures. Our results show that the optical characteristics of the antireflection coating and the polarizing beam splitter can be improved using discontinuous plasmonic nanocomposite.

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Vavilov–Cherenkov radiation in materials in millimetre wavelength region

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We investigated radiation characteristics of composite metamaterial, which is constructed from thin copper wires and split ring resonators on a dielectric plate. The metamaterial target was designed by a set of such plates. We have found the optimal target parameters from measurements of the single plate phase delay. Using the target designed we had investigated the angular, spectral and orientation dependences of the refracted radiation using source of mm-radiation [1] as well as the radiation generated by electrons with enrgy 6 MeV. The measurements were performed in millimeter wavelength region. The measured dependences demonstrate the existence of radiation characterized by negative refraction index. For the same target we investigated characteristics of Vavilov–Cherenkov (V-Ch) radiation and observed it in the backward semi-sphere. Comparison of obtained results with characteristics of conventional V-Ch radiation from a target with positive refractive index (Teflon) [2] have been performed.

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Wave properties of asymmetric hyperbolic media

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Now we observe growing interest to hyperbolic media (media whose wave properties are characterized by unclosed isofrequency contours in space of wave vectors). One of the most remarkable properties of hyperbolic media is a very high electromagnetic density of modes (DOM) that results in a strong enhancement of interaction between electromagnetic field and matter. All nonlinear processes in hyperbolic media, such as spontaneous emission [1], Purcell effect [2], etc., are strongly enhanced in hyperbolic media (HM). From electromagnetic point of view listed above effects are caused by a very wide spatial spectrum of propagating waves which would be evanescent ones in usual media.

In our work we present a concept of *asymmetric hyperbolic media* (AHM) and demonstrate new opportunities offered by such media for light absorption, high-directive thermal emission and control of the spontaneous emission. Under the asymmetry we understand different normal components of the wave vector $k_z^{(1,2)}$ for waves, propagating upward and downward with respect to interfaces in hyperbolic media with tilted anisotropy axes (see Fig. 1). This effect takes place if the tangent component k_x of the wave vector is fixed (for example, by an incident wave). Components of the permittivity tensor have opposite signs for HM. A special of interest is if they have close to equal absolute values, $\epsilon_{x'x'} \simeq \epsilon_{z'z'}$. In this case diagonal components of the permittivity tensor become close to zero in the coordinate system (xyz), associated with interfaces. It leads to such a remarkable property as a very large either real or imaginary part of k_z, namely, $\left|k_z^{(1)}\right| \to \infty$ if parameter of losses goes to zero. It results in the total absorption of light by optically ultra-thin layer of AHM [3].

The second important property of AHM appears as ability of modes with a very high DOM to be radiated from AHM without internal total reflection, as takes place in conventional HM. We demonstrate that AHM can produce high-directive and quite strongly coherent thermal emission in far zone that opens a door for creation of simple and cheap light sources in the near- and mid-infrared ranges.

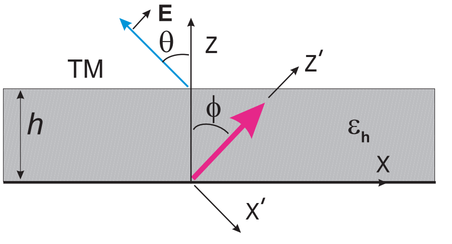

Fig. 1: Schematic view of a hyperbolic medium slab with the tilted optical axis.

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Estimation of quantum storage effects in logic gate operation inside photonic crystals

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An atom embedded in photonic crystals is a promising candidate tool for storing quantum information for quantum computation, where atomic decay is suppressed due to the presence of a photonic band gap (PBG), and a nonzero steady-state atomic population is formed in an excited state. This leads to the storage of quantum information [1]. In addition to such storage, the processing of quantum information through quantum logic gate operation is necessary to carry out quantum computation [2]. This requires population transfer between the excited states of the embedded atom [3].

In this work, by calculating the time-averaged probabilities of the excited states of the embedded atom showing several quantum states including a nonzero steady-state atomic population and population transfer, we estimate the quantum storage effect in logic gate operation inside photonic crystals. Here, the atom has a three-level energy configuration, where a control laser is used for coupling the two upper levels $|1\rangle$ and $|2\rangle$. At the initial time of this system, an ultra-short laser pulse is used for pumping the atom as $|\psi(0)\rangle = \cos(\theta/2)|1\rangle + e^{i\phi}\sin(\theta/2)|2\rangle$. The initial ratio θ determines the final state of the atomic state, such as a steady-state population case, a population transfer case or their intermediate cases.

Figure 1 shows the time-averaged probabilities (namely, the probabilities of finding the atom on the excited states for a long-time limit), using a polar coordinate as a function of the initial ratio θ. Figure 1(a) shows a narrow-PBG case, where the probabilities are given by an ellipse. In the cases of the initial ratio giving the major and minor axes, the final atomic states show the steady-state population and the population transfer, respectively. In the latter case, the quantum storage effect is further suppressed by increasing the control laser strength Ω. However, in the wide-PBG case, as shown Fig. 1(b), we find that atomic decay is suppressed at several Ω. These results show that, for the population transfer necessary for the quantum logic gate operation, the quantum storage effect may be suppressed by several factors; however, this suppression is minimized in a wide-PBG case. The demonstration of optimizing the optical parameters is considered to be an issue for the future studies.

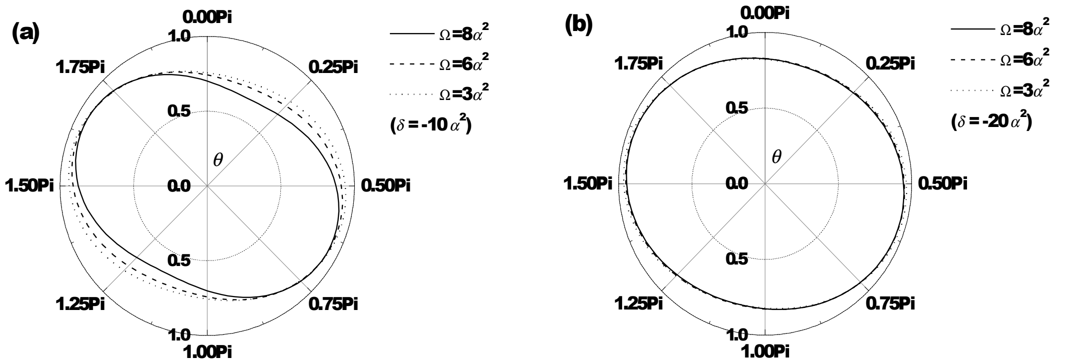

Fig. 1: Time-averaged probabilities: (a) narrow-PBG case, (b) wide-PBG case.

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Dynamical metamorphoses in arrays of nonlinear plasmonic nanoparticles

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Nonlinear plasmonic nanostructures — such as gold or silver nanospheres, nanorods, and their assemblies — are becoming increasingly important in the development of the emerging field of nanophotonics [1]. Due to boosting the intrinsic metallic nonlinearity through the resonant plasmon field enhancement along with the deep subwavelength localization of surface plasmons, they enable scaling down the size and required optical powers for nonlinear components, which is important for fully functional nanophotonic circuitry. Identification of novel nonlinear phenomena in plasmonic system offer intriguing prospects for further revolutionizing modern photonics [2, 3, 4, 5, 6].

Here we present a comprehensive study of MI and bistability in subwavelength nonlinear systems for 1D and 2D arrays of optically driven metallic nanoparticles with a Kerr-like nonlinear response, as shown in Figs. 1 (a) and (b). In particular, we demonstrate that a bistable nonlinear response of each nanoparticle in the array can lead to the formation of novel types of nonlinear localized modes — plasmonic kinks (which describe switching waves connecting two different states of polarization of metallic nanoparticles) and solitons (see Figs. 1 (c) and (d)).

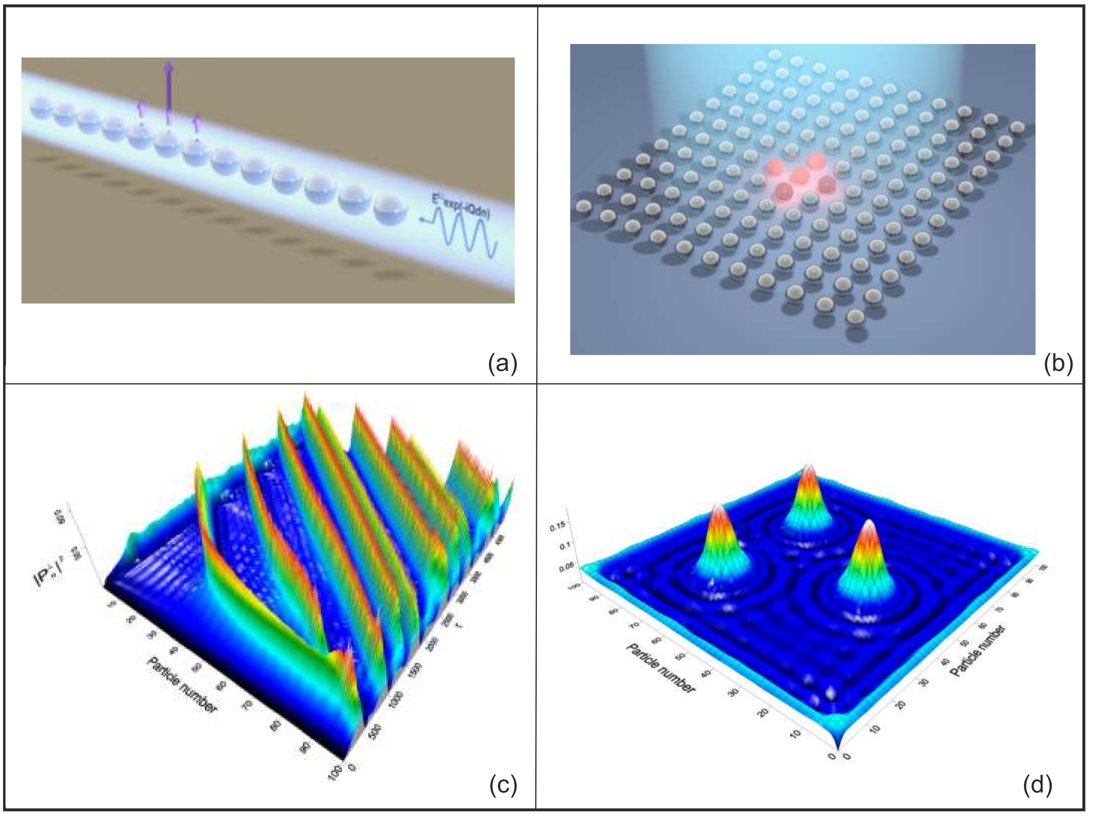

Fig. 1: Artistic views of (a) a chain and (b) a square lattice of metallic nanoparticles illuminated by laser beams. Reddish particles indicate a discrete plasmon soliton. Panels (c) and (d) present examples of 1D and 2D solitons.

We also show that modulational instability in such nanostructures can lead to generation of long-lived standing and moving nonlinear localized modes in the form of oscillons. We reveal that a wide variety of scenarios of MI development allows mode transformation from one type to another, changing movement direction and a velocity of drifting oscillons and solitons as well as formation of stable domain walls connecting not only different stationary states but also the states with different types of nonlinear dynamics (e.g., chaotic-like and regular).

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Effect of losses in nonlocal metal-dielectric multilayers

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Optical metamaterials formed by metal-dielectric multilayers have striking electromagnetic properties that have been found last decade, including negative refraction without use of magnetoactive constitutive materials, epsilon-near-zero and epsilon-very-large behavior, and strong nonlocal optical response [1]. Multilayered metal-dielectric metamaterials are also able to realize so-called hyperbolic media [2] possessing ultra-high values for the photonic density of states and, consequently, the Purcell factor. It is even become possible to study cosmology with such metamaterials and mimic black holes.

However, there is a question of how these phenomena behave in real structures where realistic losses should be taken into account, hence we present our study on complex modes of the multilayered metamaterial and clarify the effect of losses. Varying losses from weak to realistic ones, we analyze band structure of the metamaterial and clarify effect of losses on its intrinsic electromagnetic properties. The structure operates in a regime with infinite numbers of eigenmodes, whereas we analyze dominant ones.

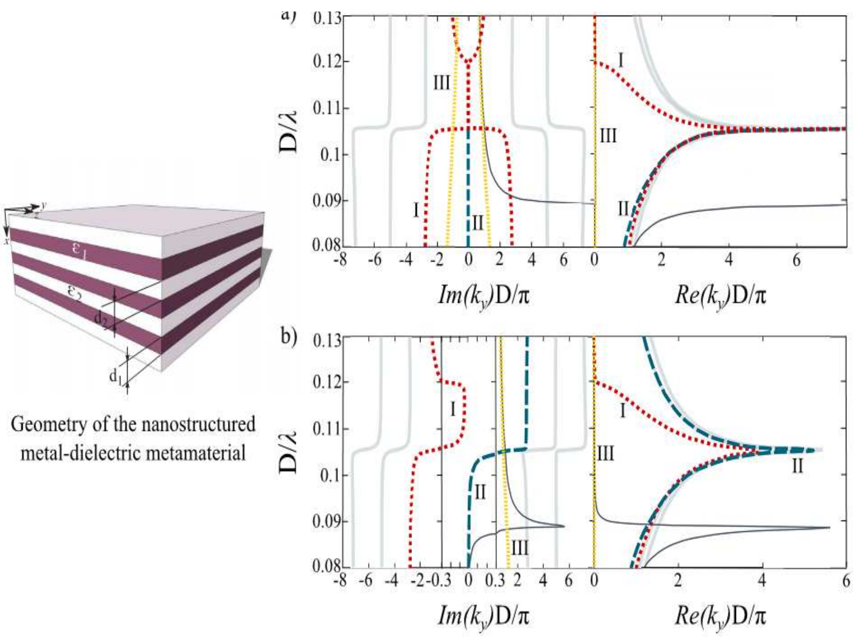

Fig. 1: Band structure of the multilayer with $d_1 < d_2$ in the absence of losses (a) and with the losses (b). Imaginary parts of the modes are on the left, real parts are on the right. Low-loss modes are marked with colors, gray curves correspond to high-order modes.

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Studies of magnetically soft ferromagnetic wires and tuneable composite materials containing wire inclusions

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Studies of magnetic properties of glass-coated microwires attracted considerable attention mainly owing to their good soft magnetic properties and giant magneto-impedance, GMI, effect and their potential applications in sensors and multifunctional composites [1–3]. GMI effect, consisting of large sensitivity of the impedance of magnetically soft conductor on applied magnetic field, attracted great attention [1, 2] due to excellent magnetic field sensitivity. One of the recent tendencies related with development of industrial applications is the miniaturization of the magnetic sensors. Therefore soft magnetic wires with reduced dimensionality, such as glass-coated microwires with thin diameters (between 1–40 μm) recently gained much attention [1, 2]. Certain progress on improvement of soft magnetic properties and achievement of high GMI effect of glass coated microwires has been reported at the laboratory level [2]. Cylindrical shape and high circumferential permeability observed in Co-rich amorphous wires with vanishing magnetostriction constant are quite favourable for achievement of high GMI effect [1, 2].On the other hand the composites with embedded arrays of metallic wires may demonstrate a strong dispersion of the effective permittivity ε_{ef} in the microwave range. As mentioned above the impedance of magnetically soft microwires can be very sensitive to external stimuli as magnetic field and mechanical load. A possibility to control or monitor the electromagnetic parameters (and therefore scattering and absorption) of composite metamaterials is of great interest for different application such as remote non-destructive testing, remote stress and temperature monitoring, microwave tunable coatings and absorbers. The dispersion of the effective permittivity ε_{ef} is determined by the geometry of wire medium and the frequency dependence of wire impedance.

We present our results on tailoring of soft magnetic properties and GMI effect in thin microwires paying special attention to achievement of low hysteretic high GMI effect in extended frequency range (till 4 GHz). The magnetic anisotropy of Co and Fe-rich amorphous microwires is determined mostly by magnetoelastic contribution and can be tailored by stress or magnetic field annealing. Varying the time and the temperature of such stress annealing we are able to tailor both magnetic properties and GMI. Additionally, magnetic response of linear microwires arrays and GMI effect of the system containing few microwires can be tailored through the magnetostatic interaction between the microwires.

We present also our results on studies of composite materials consisting of soft magnetic wires embedded in a polymeric matrix. The combination of two effects, namely, a strong dispersion of the effective permittivity in metallic wire composites (resonance or plasmonic type) and giant magnetoimpedance effect in wires results in unusual behavior of the developed metamaterials that is dependence of an effective dielectric response on the wire magnetization which can be changed with different external stimuli.

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Three-dimensional reconstruction of electromagnetic field distribution in the vicinity of subwavelength hole in thin metal film

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Light manipulation at a subwavelength scale is desirable for various applications in nanophotonics. This can be achieved by fabrication of nanostructures that allow to excite and guide surface plasmon polaritons (SPPs) localized in the vicinity metal surfaces. One of the simplest structures allowing to excite a SPP is an isolated nanoscale hole in metal film [1, 2]. On the other hand, the scanning near field optical microscope (SNOM) which allows to obtain information about the structure of the electromagnetic (EM) field near the object under study plays important role in contemporary science [3]. Metal-coated probe with a subwavelength hole in it is a critical part of this microscope. Therefore, the problem of interaction of light with subwavelenght aperture in metal film appears rather important.

The aim of this study is to perform experimentally the three-dimensional reconstruction of EM field distribution in the vicinity of a subwavelenght aperture in thin metal film. Nanoscale holes in metal films have been intensively studied both theoretically and experimentally. However, to our knowledge, such an experimental EM field reconstruction reported in this work was performed for the first time.

An isolated hole with the diameter of order of 200 nm was drilled by means of focused ion beam in 75-nm-thick silver films sputtered onto a glass substrate. The sample was excited from the bottom by a focused laser beam ($\lambda = 532$ nm). The intensity of EM field collected with the Al-coated fiber probe from the top was measured by photomultiplier tube.

The scanning was performed both in contact and plane scan modes. The latter regime allowed us to perform scanning within a set of planes on different elevations over the sample surface. After processing the experimental data, we performed the three-dimensional reconstruction of EM field intensity distribution near the subwavelength hole.

The numerical simulations performed CST Microwave Studio reproduced and allowed us to interpret all the features observed experimentally. First, because of the small thickness of the metal film, the incident light was partially transmitted through the sample. The interference of the incident beam with the waves scattered by the hole induced spacial modulation of the intensity of the optical signal and provided information about the phases of the scattered waves. Second, the diffraction of EM waves from the aperture was observed. Third, the SPP excited by the hole was observed experimentally. It was strongly localized near the metal surface and propagated primarily along the direction of polarization of the incident beam.

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Near infrared hyperbolic metamaterial with thin gold layers

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Hyperbolic media are uniaxial and demonstrate different signs of dielectric constant. They have a hyperbolically shaped equifrequency contour for one of the two principle polarisations. This hyperbolic shape can be used for focusing applications. But more important is the density of states divergence in such materials [1], which leads to enhanced spontaneous emission and thermal radiation. Hyperbolic media are realised as layer or wire metamaterials [2] combining metallic and dielectric materials. For common configurations the transition to hyperbolic behaviour is obtained already at visible wavelengths. The region of near infrared is difficult to access due to strong increase of negative dielectric constant in metals. We propose here a concept to shift the hyperbolic transition to 1.7 μm wavelength. This is achieved by using very thin gold layers of 7 nm in combination with relatively thick silicon layers of 43 nm. The silicon is chosen due to its large dielectric constant of approx. 12.25, which allows compensating the strong negative dielectric constant of the metal. Gold was taken instead of typical silver to achieve very thin films. It is reported that silver layer thickness is difficult to reduce below 20 nm [3].

Our metamaterial was produced using magnetron sputtering on silicon substrate. 13 layers of gold were deposited, with full thickness of metamaterial equal to 600 nm. Broadband angle dependent reflection characteristics for s- and p-polarisation were measured with FTIR spectroscopy. This data were fitted with transfer matrix method to find out the properties of gold in very thin films. Collision frequency was chosen as a fitting parameter. The obtained collision frequency is 10 times larger than the one reported for bulk gold [4]. This influences not only the imaginary part of dielectric constant but also the real part, which becomes less negative. The dielectric constant of the gold in thin films was then used to obtain effective parameters of the metamaterial (see Fig. 1). As can be seen in Fig. 1 the hyperbolic transition for gold films takes place even at longer wavelengths compared to what one would expect with bulk gold properties. The losses of the metamaterial are comparatively much higher. At the same time the figure of merit is still higher than expected for hyperbolic metamaterials with alternative NIR plasmonic materials as for example TiN [5].

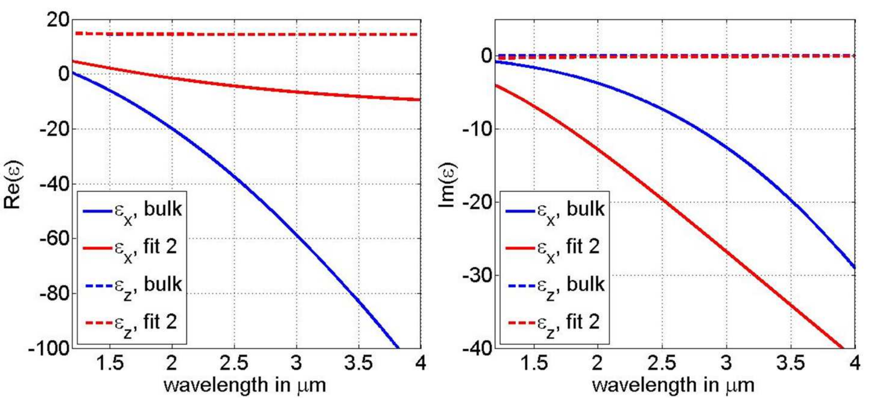

Fig. 1: Left: Real part of the metamaterial dielectric constant for polarisation parallel (x) and orthogonal (z) to layers. Right: Imaginary part of the metamaterial dielectric constant. The data are presented for the metamaterial with bulk gold properties (blue) and properties of real thin film gold (red).

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Purcell effect for electric and magnetic dipole emission in wire medium

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Wire metamaterials are composed of arrays of optically thin metallic rods embedded in a dielectric matrix [1]. Experimental realizations of such structures span from microwaves to optics, and they are very promising for a number of applications including the subwalength transmission of images, negative refraction phenomena, superlensing, and biosensing applications.

Here, we study theoretically the enhancement of spontaneous emission in wire metamaterials. We analyze the dependence of the Purcell factor on the wire dielectric constant for both electric and magnetic dipole sources and find an optimal value of the dielectric constant for maximizing the Purcell factor for the electric dipole. We obtain analytical expressions for the Purcell factor and also provide estimates for the Purcell factor in realistic structures operating in microwave, terahertz and optical spectral ranges [2]. Effect of source polarization is analyzed. Finite size effects in the wire medium are also discussed and it is demonstrated, that the large Purcell factor may be attained for wire arrays with the sizes in order of the light wavelength.

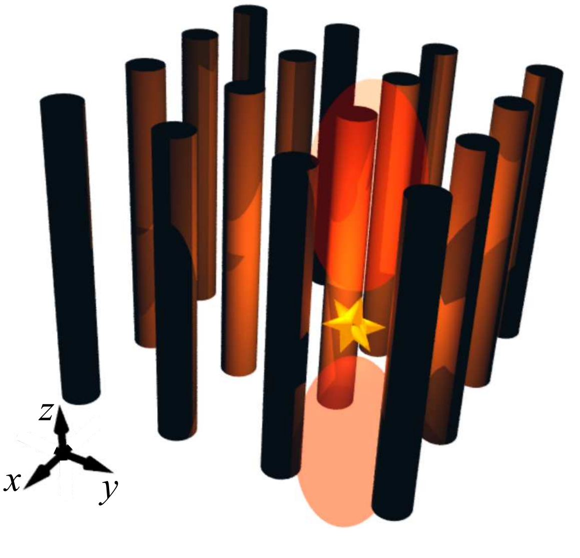

Fig. 1: Schematic illustration of wire medium with embedded light source.

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Cloud computing tools for modeling optical metamaterials and tunable graphene-based photonic devices

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In this paper we review our recent progress in developing online simulation tools and material numerical models for nanostructured optical metamaterials, and plasmonic devices. The emphasis is made on electrically tunable graphene-based devices.

Update: simulation tools at nanoHUB.org. First, we focus on new and updated software tools freely available at nanoHUB.org as cloud computing services. We have worked through the most important features of the software for simulating and optimizing nanostructured devices. We developed the common GUI platform, which is used now for all new generation tools [1]. The new features include cache-based algorithms, cloud storage of previous calculations, quick search-as-you-type access to the database of optical material properties, adjustable two-parameter sweeps for custom optimizations. Two new tools are presented: (i) PhotonicsPOS — frequency-domain tool for simulating spherical core-multishell structures located above a multilayer substrate and (ii) PhotonicsGain-0D [2] — time-domain tool for simulating the local response of a multi-level gain media, the population kinetics at all levels, and the dependence of probe amplification on pumping parameters. Gain media is of interest in plasmon laser applications and active metamaterials.

Time-domain modeling of graphene-based devices. Second, we present a Drude-Continuous Critical Points (DCCP) model [3] for time-domain modeling (e.g. modeling with FDTD, FVTD, FETD techniques) of tunable graphene-based devices. Accurate yet physical and computationally efficient model of multi-parametric dielectric function is the core challenge in modeling the time-domain response of a tunable single layer graphene (SLG). The dielectric function of SLG, already validated by a number of experiments [4] can be derived using the random phase approximation as a sum of integral expressions for interband and intraband contributions [5]. The final integrals depend on frequency, temperature and chemical potential (electrically controlled by applied voltage), so that the numerical approximation has to be accurate within the joint parametric space for all three ranges of interest. Another challenge is to incorporate the dielectric function into a time-domain numerical model with the minimal computational overhead [3, 6]. Application examples include electrically controlled metasurfaces with SGL [4] and recently proposed transparent conductive electrodes with SLG cover [7].

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Spectrum of surface plasmons excited by spontaneous transitions in quantum dot

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In last decade, quantum plasmonics, which studies quantum phenomena of electrodynamics of plasmonics nanostructures, has been highly developed. Surface plasmon amplification by stimulated emission of radiation, which was predicted in [1, 2] and experimentally observed in [3] is one of the most intriguing phenomenon, in which quantum properties are exhibited most clearly.

In the present communication we consider the spectrum of quantum fluctuations of near fields of a plasmonic nanoparticle (PNP). It is supposed that the fields are resonantly excited by spontaneous transition of quantum emmiter placed near the PNP. The pumping of the quantum emmiter is assumed to be below the threshold of spasing. We show that under this condition, the shape of the line differs from the one predicted by the Shawlow–Tauns formula. In contrast to the Shawlow–Tauns approach, the developed approach is valid in the regime of small number of excited plasmons.

Our calculations predict a broad line, which corresponds to the plasmonic resonance, with narrow peak corresponding to the quantum emmiter transition. Taking into account spontaneous radiation allows to describe thresholdless transition to spasing.

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Thin composite layers for arbitrary transformations of plane electromagnetic waves

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In this review presentation we will discuss approaches to realizing thin composite layers (sheets) for arbitrary control over reflection and transmission coefficients for plane waves. Among usually desired functionalities there are full absorption, artificial magnetic wall, polarization transformations, and so on. We are interested in understanding how to find and realize such micro- (nano-) structures of the sheet material, that the desired response for exciting plane waves would be achieved.

Our starting point is that to reach this goal, we will need to be able to synthesize materials with desired electromagnetic properties, and it is expected that for many of the thought functionalities it will be not possible to find needed materials among the natural materials. Thus, we will need to make use of the concept of *metamaterials* as artificial electromagnetic (multi-)functional materials engineered to satisfy prescribed requirements. They can have new or rare electromagnetic properties as compared to what can be found in nature.

Our other starting point is that it is desirable to have as thin functional layers as possible, and we will target ultimately thin designs. Thinking about the limits for reducing thickness, we realize that the thinnest possible layer of a composite material is the layer which contains only one single layer of molecules (or "artificial molecules" in the realm of metamaterials). Thus, the realizations which we will consider will contain only one layer of polarizable particles, and we will deal with effectively two-dimensional "metasurfaces" rather than volumetric metamaterials.

The next issue to think about is what kind of particles we should use to realize the desired response. It is clear that for small particles the dominating polarization effects will be in form of induced electric and magnetic moments. Of course, for complex-shaped particles the spatial field distribution in the vicinity of the particle can be quite complicated, and the dipole moment model is not appropriate if we would need to model this distribution, but, on the other hand, the plane-wave reflection coefficients depend only on the surface-averaged electric and magnetic current densities, which, in turn, depend only on the lowest-order (dipole) moments of the currents induced on the inclusions. Thus, for our purpose it is appropriate to model the particles as electric and (possibly) magnetic moments induced by the incident fields. Higher-order moments of the induced currents contribute only indirectly: basically, via the influence of the reactive near fields on the dipole polarizabilities.

For applications, it is always desirable to come up with as simple realization of the device as possible. Thus, the question arises, if we can fully control reflection and transmission properties of an array of small particles assuming that they are only electrically polarizable (magnetic moments of the particles are zero). The answer is "no", and it follows from the fact that a planar array of electric dipoles radiates symmetrically towards both directions away from the plane. The same is true for arrays of only magnetic dipoles. On the other hand, it is possible to prove that an array of *both* electric and magnetic dipoles can be designed so that the reflection and transmission can be fully controlled (within the fundamental limitations of causality, energy conservation, and similar constrains).

Following the above observations, we can formulate a general framework for design and optimization of ultimately thin layers for arbitrary control of reflection and transmission coefficients. We will derive formulas to connect the reflection and transmission coefficients with the polarizabilities of dipole particles. This will allow us to find out what polarizabilities are needed to realize the desired operation of the sheet. Next, knowing the theory of dipole scatterers, we can determine what geometrical symmetry and physical properties of the particles we will need (chiral or non-chiral, reciprocal or non-reciprocal, and so on). Finally, knowing the particle type, we can find the topology and dimensions of the inclusions, and complete the synthesis task.

In the talk, we will present the needed analytical derivations (general formulas for reflection and transmission coefficients, and so on) and discuss our recent results on ultimately thin perfect absorbers, twist polarizers, one-way transparent sheets, ultimately thin high-impedance surfaces, and other devices. Some of these results can be found in our preprints [1, 2, 3, 4].

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Towards all-dielectric metamaterials: Cascades of Fano resonances in the Mie scattering by dielectric rods

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High-index dielectric rods are considered as building blocks of all-dielectric metamaterials. For a deeper understanding of the wave propagation in such structures, here we study the scattering properties of a single infinite dielectric rod by varying its refractive index from low to high values. We reveal new features in the spectra of the Mie scattering by high-index dielectric rods. Figure 1 shows the numerically calculated spectra of the Lorenz–Mie coefficient for the scattered TE_0 cylindrical wave by a single dielectric circular rod. We observe a cascade of resonances characteristic for Mie scattering with responses affected by the interference between the incident and scattered waves and explain the specific features of the spectra by the physics of Fano resonances.

To provide justification of our interpretation and to extract the Fano asymmetry parameter q, we fit the calculated spectra with the Fano formula $I = A(\Omega+q)^2/(\Omega^2+1)$, where $\Omega = (\omega-\omega_0)/(\gamma/2)$ is the dimensionless frequency, ω_0 is a central frequency, and γ is the width of the $TE_{0,n}$ band. The insert shows an example of the dependence of the Fano parameter q on the frequency ω for $TE_{0,n}$ at $\varepsilon = 50$. The dependence is of the same cotangent-type as one recently demonstrated in Ref. [1] for a disordered photonic structure.

This work was supported by the Ministry of Education and Science of the Russian Federation.

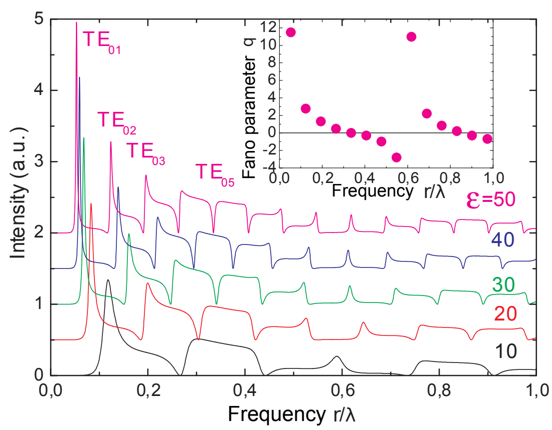

Fig. 1: The Lorenz–Mie coefficient spectra of a single dielectric circular rod with the radius r and for different values of dielectric permittivity ε. Insert: Dependence of the Fano parameter q on the dimensionless frequency r/λ.

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Complex eigenmodes of an infinite chain of dielectric nanoparticles

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Chains of metallic nanoparticles were extensively studied in the last decade due to their ability to support subwavelength localized waves through surface plasmon polaritons (SPP) [1–2]. Besides numerical simulations and experimental measurements dispersion relations of such chains were obtained theoretically, using an analytical model of coupled electric dipoles. All studies indicate the main problem of such systems — severe losses in metal at optical frequencies and thus low propagation constant of guided modes.

One of the possible approaches to address this issue is to use dielectric nanoparticles with high refractive index instead of metal. Silicon nanoparticles with refractive index $n \approx 4$ and linear size about 100–200 nm in addition to electric dipole resonance have magnetic dipole resonance in optical frequency range. It was demonstrated theoretically [3] and experimentally [4]. Moreover, losses in silicon are about one order less than losses in metal.

Here we study the dispersion properties of an infinite one-dimensional array of silicon nanoparticles, which have their electric and magnetic resonances at two different frequencies, that can be adjusted by stretching or squeezing the particles. Thus different dispersion relations can be achieved due to the potential shift of the resonances just by tailoring the geometry of the particles. To obtain the dispersion equation we use a common coupled dipole model with electric and magnetic polarizabilities provided by the Mie theory [2].

An example of the dispersion diagram $k_h a(\beta a/\pi)$ (where k_h is a host wavenumber and β is a complex wavenumber of the transverse-polarized eigenmode) for the chain of silicon nanospheres is shown in Fig. 1.

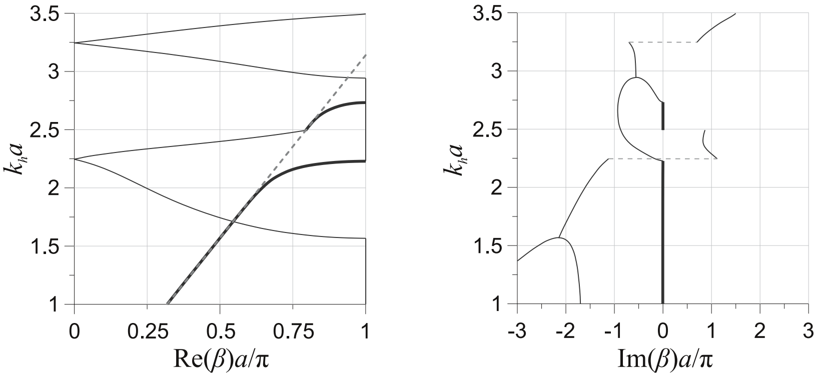

Fig. 1: Dispersion diagram for the chain of silicon nanospheres with $\varepsilon = 16$, $\mu = 1$ and $a/R = 3$, where a is the spacing. Thick solid lines show real solutions of the dispersion equation, thin solid lines are the complex modes.

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Electromagnetic waves in artificial helically structured systems with optimum parameters

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It is shown in the presentation, that optimum parameters of a helix can be obtained, at which the equally significant electric dipole moment and magnetic moment arise in helix at resonant influence of an electromagnetic field. For a helix with optimum parameters all three polarizabilities are equal: dielectric, magnetic and chiral.

In conditions of the main resonance optimum parameters of a helix are determined by a pitch angle and number of coils of helix and do not depend on wavelength of an electromagnetic field.

Optimum parameters do not change at interaction of helices, they are identical both to a single helix, and for metamaterials with anyone concentration of helical elements, including the highest values of concentration.

Helices with optimum parameters can be used in various devices, for example converters of the polarization, low reflecting coverings, the structures providing a flow of objects by electromagnetic waves, artificial media with a negative refraction index.

Helices with optimum parameters exist in the nature, in particular, molecule of DNA has the optimum form for soft X-ray radiation.

Nonlinear response of torsional metamaterials

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Metamaterials provide novel approaches to harness electromagnetic waves, and it is particularly important that we can also control metamaterial properties by light. Recently, a new type of nonlinear metamaterials – magnetoelastic metamaterials shed light on the possibility of modifying the properties of metamaterials directly by light [2]. This approach allows us to achieve a rich variety of nonlinear effects via coupling of electromagnetic (EM) and elastic properties within the structure.

Here, we propose a new design to achieve strong EM-elastic coupling by using the rotation within chiral meta-atoms [3]. Our new design has high sensitivity to electromagnetic wave power, and the elastic and electromagnetic properties can be independently designed to optimise the response. The tunability of nonlinear response in such structures is confirmed with a microwave experiment.

The schematic view and operation principle of such metamaterials are shown in Fig. 1. Each element of metamaterial is a dimer composed of two split-ring resonators with their centres connected elastically. When the structure is subject to an external field, the mutual electromagnetic (EM) torque between the two rings changes the twist angle. The rings reach a new equilibrium state, where the EM torque M_{EM} and the restoring torque M_R from the wire are balanced. One of the prominent advantages of this design is its ultra-sensitive elastic feedback, since the effective lever arm of M_{EM} can be much larger than M_R. This enables us to achieve an ultra-large nonlinear resonance shift even at moderate pump power levels [Fig. 1 (c)]. To confirm the predicted effects we perform a pump-probe microwave experiment. The predicted evolution from bistable to monotonic nonlinearity is clearly observed when the pump frequency approaches to the initial resonance.

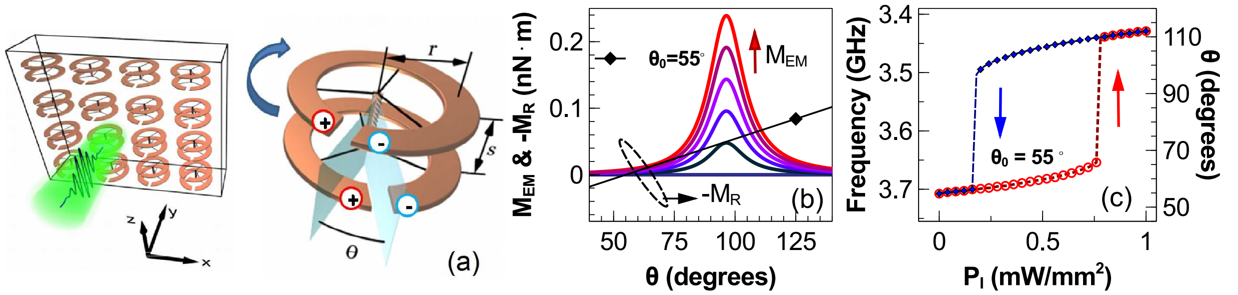

Fig. 1: (a) Conceptual layout of a torsional metamaterial and its rotational "meta-atom". (b) Twist angle dependency of EM torque M_{EM} for a given pump frequency, which is linearly proportional to the incident power. The restoring torque M_R is approximated by Hook's law. (c) The bistable response of resonant frequency and the corresponding twist angles.

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New approach to nonlinear modulation of light

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Currently, interest is on the rise for the development of different ways for modulation of light intensity on nano-scales. Interaction between light beams at different frequencies demands an object, mediating the process. Most commonly, nonlinear materials are used for this purpose, but having naturally small nonlinear susceptibilities, they demand high light intensities which put certain limitations on devices performances. Here we investigate an approach for nonlinear optical interactions, mediated by nanoelectromechanical systems.

Modulation of optical signals at the nano-scale is challenging due to several reasons. First, the classical limit of diffraction imposes the usage of noble metals (plasmonic) structures in order to achieve deep nano-scale confinement, which however results in high propagation losses and put certain limits on overall length, available for devices implementations. Either phase or amplitude modulation of signals requires sufficient propagation lengths. Both interference and attenuation effects demanded stronger light matter interactions in order to reduce the dimensions. While various optical functionalities were demonstrated on the plasmonics platform — [1] for the very partial list on the subject, the approach of signal modulation with nano-mechanical phenomenon was not considered. Here we propose to implement the ancient approach of "Bonfire and Beacon chains" blocking/unblocking the propagation of signals with nano-shutter, or resonant scatterer moving in and out of the signal beam.

In this report we discussed new concept of all-optical NEMS modulator. The layout of the structure is depicted on Fig. 1 — nanoparticle in the v-grove plasmonic waveguide is driven into oscillatory motion by pump beam (red). The particle moves in and out the area of high intensity of the fundamental mode (green) and scatters the signal, modulating its intensity. While many other realizations are possible, the usage of plasmonic v-groove waveguide is potentially promising as it supports moderate subwavelength mode confinement and the same time reasonable propagation distances of dozens of microns [2], sufficient for devices realizations. Moreover, being tapered structure in the direction perpendicular to the mode propagation, it supports subwavelength light focusing at its bottom while being illuminated from above [3], which is demanded in order to obtain pulling forces [4, 5]. As a result, we obtained transmission variation of 10% per single particle. Moderate intensities, efficient interactions and deep subwavelngth devices dimensions make nanoelectromechanical systems to be of a primary interest for future opto-electronic technology.

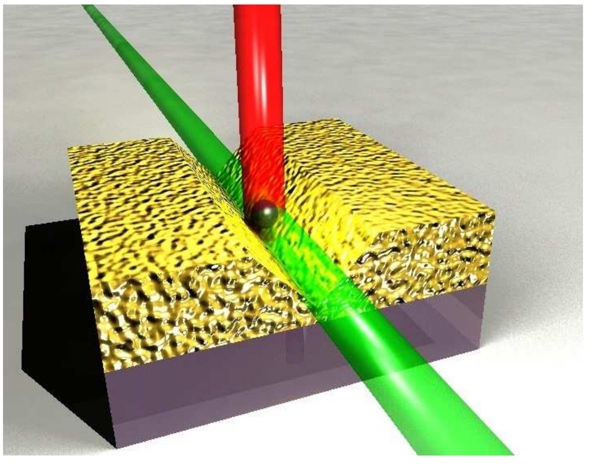

Fig. 1: Figure 1. Schematics of all-optical NEMS modulator — nanoparticle in the v-grove waveguide is driven into oscillatory motion by pump beam (red). The particle moves in and out the area of high intensity of the fundamental mode (green) and scatters the signal, modulating its intensity.

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Control of near field localization in silver nanotrimers.

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Nanoantennas keep promises for a variety of applications in nanophotonics with advantages of enhanced light–matter interaction [1, 2]. For many of them, however, an active control over the subwavelength light localization near nanoantennas is required. Dynamic switching of the field enhancement has been demonstrated by means of external beam shaping in a number of previous works [3, 4]. Also tunable localization of light through spatial-phase modulation of the incident beam has been considered for a linear trimmer composed of three identical gold nanoparticles [5].

In this Report we focus on further investigation of near field optical switching in trimmer metallic clusters. In particular, we study three silver nanoparticles with the 40-nm diameter and separated by the distance of 5 nm, as shown in Fig. 1 (a). We show that the variation in orientation and frequency of the incident plane wave may control the balance between excitation of symmetric and asymmetric eigenmodes. As a consequence, one may achieve a strong increase of the near field (hot spot) in one nanogap of the trimmer, keeping the comparatively weak field (cold spot) in the other. Moreover, mutual placement of cold and hot spots can be easily tuned by the proper choice of frequency, as shown in Fig. 1 (b). We also extend this concept of switching on the trimers containing nanoparticles with different shape such as cylindrical and spherical.

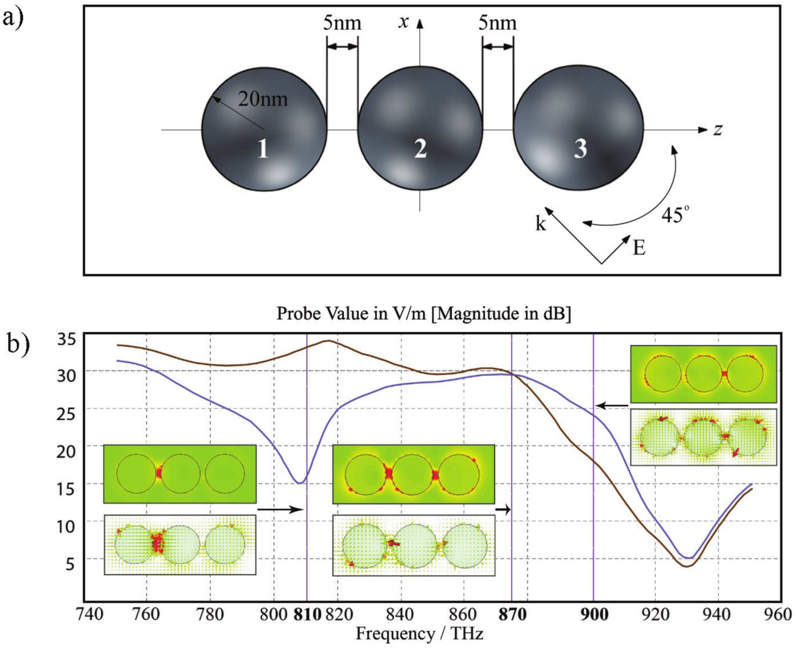

Fig. 1: (a) Schematics showing the structure, parameters, and direction of an incident plane wave. (b) Electric field in the gaps between the first and second spheres (purple line), the second and third spheres (brown line). Insets demonstrate the electric field distribution at 810, 870, and 900 THz.

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Circuit model in design of transparent electrodes based on metallic fishnet metamaterials

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Transparent conducting electrodes (TCEs) are expected to have good electrical conductivity together with high optical transmission. In recent research efforts, graphene films, carbon nano-tube networks, and unpatterned thin metal films have been examined as potential replacements for the conventional Indium Tin Oxide (ITO) electrodes, which are not desirable because of their scarcity, high cost, and brittle nature [1]. Here, we report a new scheme to design an optically transparent metallic mesh on a silicon substrate, and thus, we obtain an almost completely transparent electrode at a desired frequency. The proposed architecture is made of a metallic fishnet mesh. The structure is filled with Silica (SiO_2) and is placed on a semi-infinite Si substrate. It is schematically depicted in Fig. 1(a).

We propose a circuit model for this structure, and then employ the standard binomial transformer to find the appropriate values for the dimension of the structure to achieve transparency at the desired frequency. The proposed circuit model is shown in Fig. 1(b). In the proposed model, very similar to the circuit model presented in [2], the substrate and the cover region of the structure are represented by scalar wave impedances denoted by Z_3 and Z_1, respectively. Appropriate surface impedances, Z_g^1 and Z_g^3 and a transmission line are also proposed to model the interfaces of different layers and the metallic mesh, respectively. Fig. 1(c) shows the reflection of the designed structure computed by the proposed circuit model (blue solid line) compared with the full-wave simulation (red dashed-dotted line).

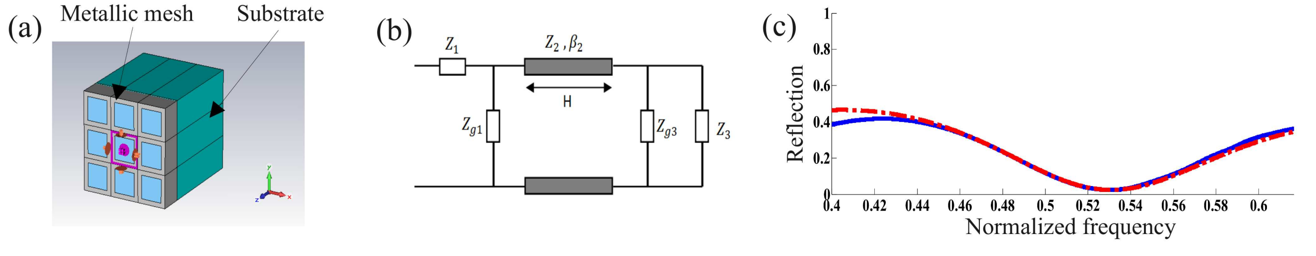

Fig. 1: (a) The proposed structure, (b) The proposed circuit model, and (c) Simulation results, CST (solid curve), and circuit model (dash-dotted curve).

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Multiple Bragg diffraction in synthetic opals: spectral and spatial dispersion

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Synthetic opals demonstrate bandgap properties of the optical spectra in the visible range due to a typical size of their constitutive SiO_2 particles of some hundreds of nanometers. This provides a unique opportunity to study photonic properties not only by traditional methods such as the study of transmission or reflection spectra with a spectrometer, but also directly, by observing diffraction patterns on a screen placed behind or near the sample. If the Laue condition holds simultaneously for two systems of planes with different Miller indices, the optical spectra exhibit special effects caused by the so-called multiple Bragg diffraction of light.

Here, our aim is to study multiple Bragg diffraction from opal films using new experimental setup with a holder formed from two quartz hemispheres. As a result, we overcome the problem of the total internal reflection in an opal film or in a substrate; so that the angle-resolved and frequency-resolved diffraction patterns have been successfully investigated in various scattering geometries including nontrivial ones with the patterns from the $\{\bar{1}11\}$ family of planes. Figure 1(a) presents an example of a complete set of transmission and diffraction experimental data for a given angle of incidence.

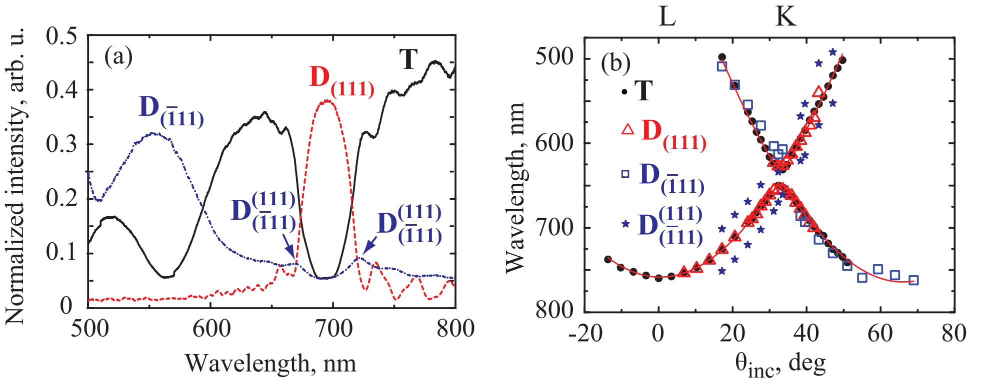

Fig. 1: (a) Transmission (T) and two diffraction ($D_{(111)}$,$D_{(\bar{1}11)}$) spectra of an opal film in air at $\theta_{inc} = 21°$. The $D_{(111)}$ spectrum is collected in the spatial region of $\theta^{(111)}_{diff}$= -21°, and the $D_{(\bar{1}11)}$ — in the region of $\theta^{(\bar{1}11)}_{diff} = 45°$. (b) Experimental and calculated positions of (111) and $(\bar{1}11)$ bands in the T and D spectra.

Here we present the diffraction spectrum $D_{(\bar{1}11)}$ that contains pronounced background superimposed by three bands: strong $(\bar{1}11)$ diffraction band and two weak features $D^{(111)}_{(\bar{1}11)}$. The dissimilarity in permittivity of SiO_2 particles caused the background that can be described as Mie scattering with more intense forward lobe. The Fano-type (111) band [1] perturbs a rather flat background at any scattering angle including $\theta^{(\bar{1}11)}_{diff}$=45° by adding two diffraction features $D^{(111)}_{(\bar{1}11)}$.

In the region of multiple Bragg diffraction (the K point of the fcc Brillouin zone), the anti-crossing effect is observed for dispersion curves both in the T and D spectra [see Fig. 1(b)].

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Nanoscale patterning of metal nanoparticles distribution in glasses

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Nowadays, plasmonic materials and structures are the subject of wide-scale studies. In addition to metals, new materials like wide bandgap semiconductors and glass-metal nanocomposites (GMN)[1], that are glasses embedded with metal nanoparticles, have recently been implemented in plasmonics. Since the dielectric function and, consequently, the propagation of surface plasmon polariton modes in the latter materials can be controlled by varying the volume fraction, size and type of metal inclusions[1, 2], the flexibility of GMN makes them attractive for plasmonics.

Recently, the usability of electric field imprinting process for nanostructuring of GMN has been shown[3]. Electric field imprinting (EFI) of GMN is based on electric field assisted dissolution[4, 5, 6] (EFAD) of nanoparticles in glass matrix at elevated temperature. This allows to control their spatial distribution via applying DC voltage to the GMN using a structured electrode (stamp). The imprinting enables multiple replication of the stamp image to GMN[4], that is, mass fabrication of GMN structures.

This paper is focused on the characterization of the spatial resolution of GMN EFI using atomic force microscopy (AFM) and scanning near-field optical microscopy (SNOM).

For this purpose, a silver-based GMN sample was prepared in a plate of commercial 1 mm thick soda-lime glass using silver-to-sodium ion exchange followed by hydrogen-assisted reduction of silver ions and metal clustering as it was reported elsewhere [6]. Optical spectrum of the GMN exhibits strong surface plasmon resonance (SPR) absorption centered at 415 nm, and the SPR peak drops after the electric field imprinting (see Fig. 1). To find the linewidth achievable in the EFI, a profiled glassy carbon stamp with the set of 350 nm deep grooves of different width was fabricated with EBL. The stamp was used as the anode in the EFI of both the GMN sample and the plate of virgin glass. The imprinting resulted in the dissolution of silver nanoparticles everywhere except the regions beneath the stamp grooves, that is in the formation of GMN strips (see the inset in Fig. 1). In the virgin glass the imprinting resulted in poling of the glass [5] except the strips beneath the stamp grooves.

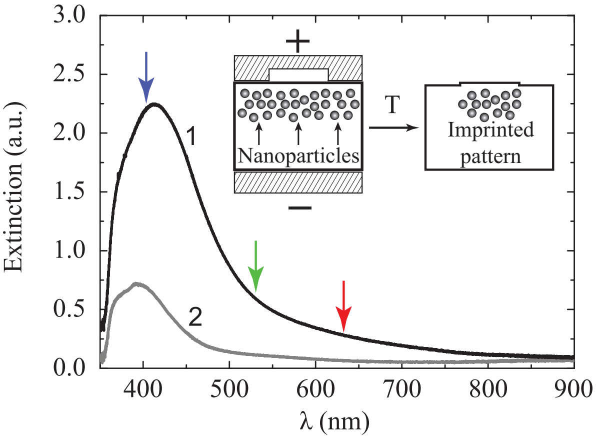

Fig. 1: Absorption spectra of the GMN before (1) and after (2) the imprinting; the wavelengths of lasers used in the near-field experiments are marked with arrows. The process of imprinting is schematically illustrated in the inset.

The obtained samples were studied using AFM and SNOM techniques. The AFM measurements showed that the samle surface of both GMN and virgin glass contains humps replicating the profile of the used stamp. The surface profiling is caused by the relaxation of volume defects generated in the glass matrix after the evacuation of alkali ions from subanodic region towards cathode in the course of EFAD [4, 5]. The subsidence process is suppressed under the stamp grooves where neither alkali evacuation nor nanoparticles dissolution occurs.

To characterize the nanoparticles distribution we resorted to near-field optical microscopy using the aperture-type SNOM (AIST-NT CombiScope Scanning Probe Microscope with optical fiber probe) in transmission mode (the sample was excited through the objective, and scattered light was collected with fiber probe). Three excitation laser wavelengths were used: 633 (red), 532 (green), and 405 nm (violet). In contrary to the virgin glass sample, the optical transmission of GMN strips showed significant dependence on the excitation wavelength. In contrast to relatively low modulation of optical signal at 633 and 532 nm wavelengths, the transverse scan of the intensity profile at 405 nm contained sharp dips corresponding to the silver nanoparticles surface plasmon resonance absorption in the imprinted strips. However, the signal from the narrowest strip (100 nm) was smeared after the averaging, and thus we are not able to claim the imprinting of this strip.

To interpret the obtained experimental results, the numerical modelling was carried out using COMSOL Multiphysics® package. The Maxwell Garnett effective medium approach with filling factor $f = 0.01$ was used for modeling of GMN optical parameters. Numerical simulations of near-field signal were performed under the assumption that the nanoparticles concentration is equal in all the strips and showed good agreement with our experimental data. Finally, this study proved that glass-metal nanocomposite elements with linewidth down to at least 150 nm can be fabricated with electric field imprinting technique.

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Improving sensitivity of magnetic resonance imaging with highly anisotropic metamaterials

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The amplification of evanescent spatial harmonics inside of a wire metamaterial (WM) with length divisible by half-wavelength has been theoretically predicted in [1]. The physics of the phenomenon is drastically different as compared to the amplification observed for slabs of left-handed media [2]. The amplification of evanescent waves in the slab of left-handed material appears due to resonant excitation of the surface plasmons at the interfaces of the slab. This effect is extremely sensitive to the losses in the left-handed material. The amplification in wire metamaterial sample happens because of resonant excitation of the standing waves with high spatial frequencies inside of the slab, that is why this effect is not sensitive to losses in the metamaterial. The enhancement effect may be used for further improvement of magnetic resonance imaging (MRI) systems [3] and for creation of subwavelength imaging devices of new generation [4].

We report the first experimental verification of evanescent waves enhancement inside wire metamaterial and consider the possibility to improve MRI image quality through enhancement of the amplitude of radio frequency signal at the detecting coil due to the presence of the wire metamaterial.

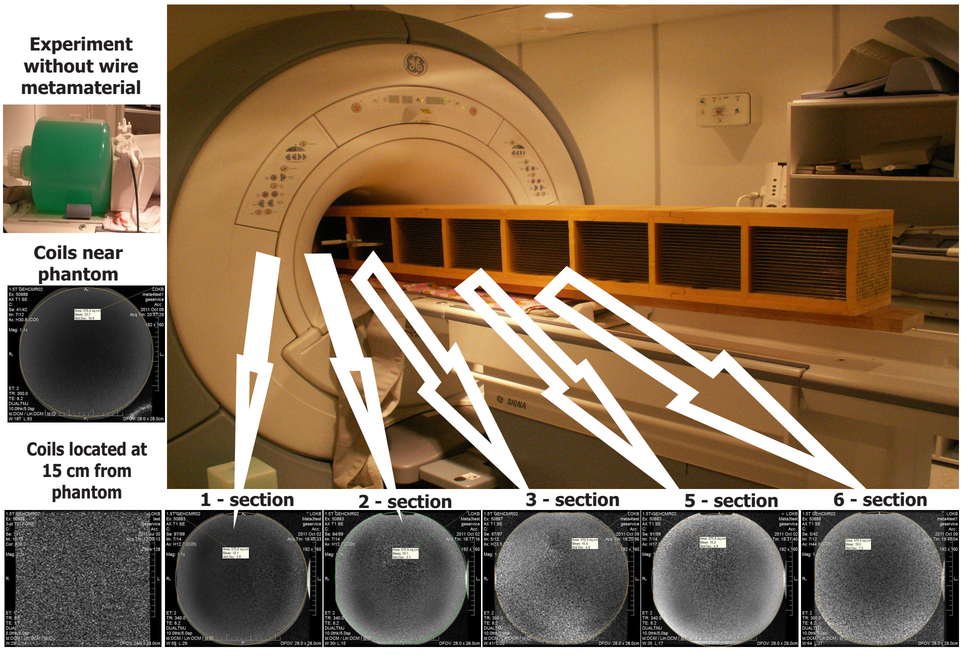

Fig. 1: MR images of phantom obtained when surface coils were located in different sections of wire metamaterial.

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Coupled-mode theory for nonlinear metamaterials with loss and gain

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Artificial periodic structures including metamaterials offer unique possibilities to control both linear light propagation and nonlinear optical interactions. In particular, nonlinear optical interactions can be enhanced due to a strong field concentration at metal interfaces [1] and new regimes of nonlinear wave mixing between forward and backward waves can be efficiently realized in negative-index metamaterials [2, 3].

A powerful analytical method for simulating nonlinear wave propagation in periodic waveguides and metamaterials is based on the derivation of coupled-mode equations for the amplitudes of linear eigenmodes, where the amplitudes can change due to optical nonlinearity. In this work, we formulate a rigorous systematic procedure for obtaining coupled-mode equations, which does not rely on homogenization. We derive explicit expressions for the coupling coefficients in the form of overlap integrals involving forward and backward Bloch modes. Our approach is based on the Lorentz reciprocity theorem, and it is applicable for periodic structures with loss, gain, and nonlinearity.

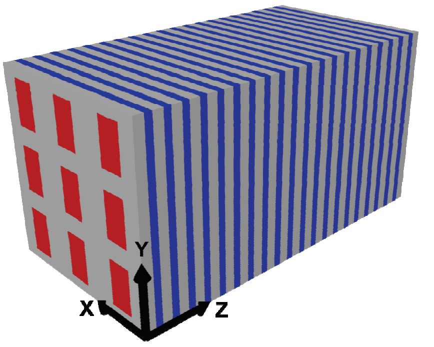

Fig. 1: Schematic of a fishnet metamaterial made of 45 nm thick silver layers (grey) separated by 30 nm thick silicon layers (blue). Arrays of holes with lattice constant 270 nm and dimensions 110×190 nm are filled with chalcogenide glass (red).

As an example, we consider a fishnet metamaterial as shown in Fig. 1. Whereas a metal nonlinearity can play an important role, in order to illustrate the effect of loss on effective nonlinear response, we analyze the nonlinear polarization due to a nonlinear dielectric embedded between the metal. We choose the nonlinear dielectric susceptibility to be purely real, however we find that in fishnet structure, the effective nonlinear coefficient becomes complex, where the imaginary part is of the order of 10^{-1} of the real part. Therefore, nonlinearity would generally modify both the linear and nonlinear parts of the refractive index. Importantly, our method can also be used to describe multi-wavelength interactions at different frequencies, such as in the process of harmonic generation and four-wave mixing.

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Nonlinear interactions in media with managed dispersion and diffraction

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The pulse interaction in quadratic media with dispersion management are considered. Extremely short electromagnetic pulses, or few-cycle pulses containing a small number of field oscillations or even half an optical cycle are of great interest in optics. The rapid development of pulse compression methods has made the experimental production of such few-cycle, even single-cycle, pulses feasible. They attract a great deal of attention as perspective instruments in photonics, telecommunication systems, ultrafast processes spectroscopy and medicine. The interest to these pulses is stimulated by a possibility of a drastic increase in the rates of data transmission and processing. For this reason, much attention is being paid to their propagation in vacuum, linear and nonlinear media, and optical systems.

One of the main problems that arise at few-cycle pulses application is the strong influence of dispersive effects distorting pulse profile and decreasing interaction efficiency. For example velocity mismatch limits interaction efficiency and leads to splitting of generated waves into two sub- pulses. Such effect can be suppressed in the medium with periodic modulation of quadratic nonlinearity. Dispersive spreading can be reduced in layered medium with alternating third-order dispersion coefficients. We show that dispersive spreading and walk-off also can be effectively suppressed in layered medium with simultaneously alternating velocity mismatch and third-order dispersion coefficient that enables to increase nonlinear interaction length for extremely short pulses. Such kind of dispersion management can be applied for effective realization of few-cycle pulses nonlinear interactions. So, quadratic photonic crystals with managed dispersion are perspective instruments for few-cycle pulses nonlinear optics.

Another problem is related to the suppression of diffraction spreading. We propose to use a layered metamaterial in which the sign of diffraction is reversed from layer to layer. Each layer of the metamaterial becomes a focusing lens that provides a quasi waveguide propagation of the pump beam. The dynamics of the second harmonic generation and frequency mixing in quasi-waveguides for the pump and harmonics is discussed. The analytical theory is accompanied with the numerical simulation results.

GMI effect in nanocrystaline magnetic wires

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Recent trends in the development of magnetic sensors are focused on the miniaturization of their size, improvement of their features and on finding of new operating principles based on fundamental studies of new materials. Among new magnetic materials a family of thin wire with reduced dimensions recently gained considerable attention [1, 2]. Particularly, studies of giant magneto-impedance (GMI) effect of thin microwires attracted considerable interest.

One of the peculiarities of the fabrication technique of glass-coated microwires is that it involves the simultaneous solidification of composite microwire consisting of ferromagnetic nucleus surrounded by glass coating. Quite different thermal expansion coefficients of the glass and the metallic alloys introduce considerable internal stresses inside the ferromagnetic nucleus during simultaneous fast solidification of the composite microwire [2]. Strength of these internal stresses depends on ρ-ratio defined as the $\rho = d/D$, where d is the metallic nucleus diameter and D — total microwire diameter.

We present the results on tailoring of soft magnetic properties and GMI effect in thin microwires with nanocrystaline structure in extended frequency range (till 4 GHz). The nanocrystalline structure has been observed generally in annealed FeSiBCuNb microwires, although even some as-prepared samples present nanocrystalline structure. Both GMI ratio and hysteresis loops of Fe-rich amorphous and nanocrystalline microwires exhibit strong sensitivity to the internal stresses related with the ratio, ρ, of the metallic nucleus diameter to the total microwire diameter. The hysteresis loops exhibit low coercivity (generally below 100 A/m).

Although GMI effect in as-prepared amorphous Fe-rich microwires is rather small, after annealing we observed increasing of the GMI effect. Enhancement of the $\Delta Z/Z$ ratio is related with magnetic softening of studied microwires after annealing.

Observation of GMI effect in as-prepared $Fe_{73.8}Cu_1Nb_{3.1}Si_{13}B_{9.1}$ microwires must be attributed to lower magnetostriction constant in partially crystalline samples containing nanocrystallites observed in some of studied samples.

We measured GMI effect in Finemet-type microwires with different ρ-ratio. Although generally GMI is rather small, the samples with higher ρ-ratio (lower internal stresses) generally present higher $\Delta Z/Z_{\max}$ values.

Generally, measurements of the GMI effect in nanocrystalline microwires involve preparation of the electrical contacts of rather brittle nanocrystalline samples obtained by recrystallization of amorphous samples after annealing. Therefore preparation of less brittle nanocystalline microwires directly by rapid quenching of composite microwire from the melt might be useful from the point of view of applications. The work on optimization of the GMI ratio during nanocrystallization is in progress.

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Sub-diffraction imaging in the THz using wire array metamaterial fibres

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We will present recent progress in drawn metamaterials [1]. In particular we demonstrate sub-diffraction THz imaging with up to $\lambda/13$ resolution through a wire array metamaterial [2] fibre several wavelengths long, with wire arrays up to three orders of magnitude smaller than recent experiments at microwave frequencies [2, 3]. The fibre is fabricated using fibre drawing methods [1]: individual macroscopic rods of cyclo-olefin polymer with low dielectric losses in the THz spectrum are filled with indium. These rods are then drawn using a method similar to the Taylor wire process [4], at temperatures at which the indium is liquid and the polymer is viscous. The resulting indium-cored fibre are stacked into a hexagonal array, then drawn again to fibre, maintaining the cross sectional geometry of the initial preform. Our metamaterial fibre contains hundreds of indium wires of roughly 10 μm diameter, in a hexagonal array with 50 μm spacing, with the entire wire array cross section forming a 1 mm diameter cylinder. The fibres are cleaved and polished to various lengths, and used to image apertures using a THz time domain spectroscopy near-field imaging setup. The figure shows the measured x-polarized image of a circular 200 μm aperture after propagation through a 3.4 mm long fibre, for a range of frequencies corresponding to wavelengths from 2.7 mm to 285 μm. Good imaging is obtained at Fabry–Perot resonances spanning well over one octave in frequencies, with imaging limited by bulk resonances of the finite length wire array in between [3]. Our experimental results, in good agreement with full vector finite element simulations, open up the possibility of manipulating THz fields at the sub-wavelength scale [2,3] with devices that are readily mass-produced.

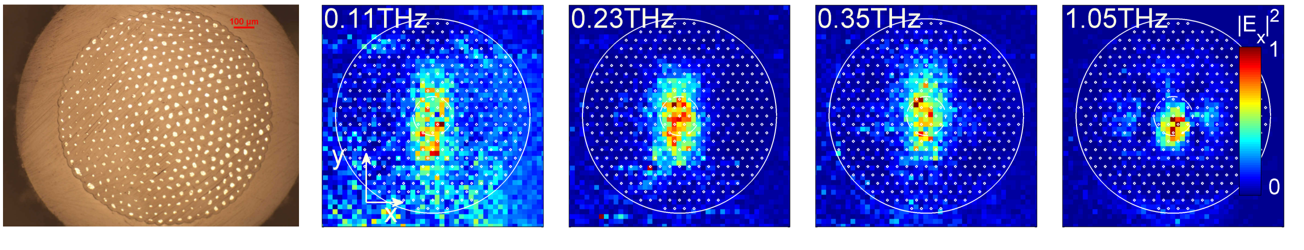

Fig. 1: Left, cross section of the metamaterial fibre; right: near-field images of a 200 μm circular aperture after propagation through a 3.4 mm metamaterial fibre, at several frequencies.

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Nonlinear spin wave processes in ferrite films, magnonic crystals, and multiferroics

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For many years, investigation of nonlinear dynamics attracts a great research interest. Such fascinating nonlinear wave phenomena as solitons and self-modulation instability have received perhaps the most attention. They were studied in diverse physical systems such as waves on the surface of water [1, 5], ion-acoustic waves in plasma [1], electromagnetic waves in nonlinear transmission lines [2], light waves in optical fibers [3], microwave spin waves in magnetic films [4], matter waves in Bose–Einstein condensates [6], and others. In solid state media, such as optical fibers or magnetic films, the solitons represent a solitary wave of envelope of a carrier wave. Magnetic films are excellent objects for experiments on nonlinear wave phenomena owing to their rich nonlinear and dispersive spin-wave properties. In the present talk recent advances in this field will be discussed. The first part of the talk is devoted to chaotic solitons recently discovered in yttrium iron garnet (YIG) films. The chaotic solitons are formed from the stationary soliton train through its self-modulation instability (SMI), which we name as the secondary SMI with respect to the primary SMI of the initially monochromatic carrier spin waves. After the chain of instabilities, the nonlinear waves attain the form of a chaotic soliton sequence or, in the other words, they become a chaotic soliton train. The envelope solitons in the sequence have chaotically varied amplitude and period. The second part of the talk is devoted to gap solitons in magnonic crystals. In this part an influence of the finite-size effects and the dynamic magnetic losses on formation of bright and dark solitons via the induced and the coupled modulation instabilities is considered. Self-generation of spin-electromagnetic wave solitons and chaos in active ring systems based on the ferrite-ferroelectric layered structures (multiferroics) is discussed in the last part of the talk. Possible controllability of the parameters of the spin wave envelope solitons and chaotic waves due to variation of the dielectric permittivity of the ferroelectric is demonstrated.

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Secondary self-modulation instability of microwave spin waves in ferromagnetic films

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Modulation instability is a nonlinear wave phenomenon which has been attracting noticeable research interest. This phenomenon was observed and studied in diverse physical systems such as waves on the surface of water, ionacoustic waves in plasma, light waves in optical fibers, microwave spin waves in magnetic films, and others [1–3].

This work reports the experimental observation of the secondary self-modulation instability (SMI) of the spin waves propagating in ferromagnetic films. The specific experiments were carried out with 2.2-μm-thick, 1.5-mm-wide, and 40-mm-long yttrium iron garnet (YIG) film waveguide. The film was epitaxially grown on 500-μm-thick gadolinium gallium garnet substrate. The spin waves in the YIG film waveguide were excited and detected by microstrip transducers separated by a distance of 1.8 mm. The bias magnetic field of 4058 Oe was directed perpendicular to the YIG film plane. The YIG film had asymmetrically pinned surface spins, a ferromagnetic resonance line-width of 0.4 Oe at 6 GHz, and a saturation magnetization 1750 G. A distinguished feature of the film is a strong frequency dispersion of the spin wave group velocity and consequently the so called "dipole gaps" are distinctive for the spin wave transmission [3].

During the measurements we excited an input monochromatic spin wave at the carrier frequencies in the vicinity of the lowest-frequency dipole gap. Then we gradually increased the input power Pin and observed the output waveforms. The SMI of the carrier spin wave developed for small values of the input power in the frequency range of 6487–6492 MHz. At a particular frequency of 6490.5 MHz the development of the SMI resulted in the formation of the stationary soliton train [3]. Correspondingly, in the frequency domain we observed sidebands around the input carrier harmonic. The primary SMI emerged at $\mathrm{P}_{in1} = 1.36$ mW and had a frequency of 2.15 MHz. A further increase in the P_{in} led to after-modulation of the amplitude of the soliton train or, in other words, in the self-modulation instability of the soliton train. We refer to it as the secondary SMI because it appears to be an "envelope of the envelope" of the initially excited monochromatic carrier spin wave. In the frequency domain we observed new small sidebands around each sideband created by the primary SMI. The secondary SMI emerged at $\mathrm{P}_{in2} = 2.02$ mW and had a modulation frequency of about 100 kHz. For P_{in} of more than 2.48 mW a destruction of the periodic after-modulation of the soliton train took place and the variations in the amplitude of the carrier solitons became chaotic.

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Spontaneous radiation of a two-level system placed near metal nanosphere with $\varepsilon = -1$

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We study radiation of a two-level system (TLS) placed near a spherical metal nanoparticle. We assume that the TLS transition frequency is the one which metal dielectric permittivity is $\varepsilon = -1$. It is known that if the transition frequency coincides with the frequency of the plasmon resonance and Joule losses are negligible, then the Rabi oscillations arise in the system [1]: the TLS relaxes to the ground state and causes radiationless excitation of a surface plasmon at the nanoparticle (isolated mode). In turn, the plasmon field excites the TLS. On the other hand, TLS radiation into the continuum of modes (e.g. into the free space) should cause the exponential decrease of the probability of the TLS being in the excited state [2].

The frequency at which dielectric permittivity of metal $\varepsilon = -1$ is the condensation point for multipole plasmon resonances (modes) of a spherical particle [3]. We find that an intermediate behavior can be observed. Part of the energy released by the TLS is used to excite infinite number of modes. The probability of the TLS to be at an exited state sharply decreases (spontaneous emission). When the stationary distribution of excited modes is established, the exponential decrease is replaced by Rabi oscillations which specter includes all frequencies. We show that when the distance between the TLS and the nanoparticle decreases, the residual probability of the TLS excitation decreases. In other words, the process of the spontaneous emission is enhanced. At the same time, the relative amplitude of Rabi oscillations grows. There is a characteristic distance at which the probability of excitation stops decreasing and the amplitude of the Rabi oscillations reaches 100% of this value. At shoter distances, the frequency of the Rabi oscillations is determined by the frequency of the multipole with the greatest excitation. As a result of the distance decrease, the order of the multipole with the greatest excitation increases.

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Radiation of charged particle bunches moving along boundary of a wire metamaterial

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Wire metamaterial represents a periodical volume structure of long parallel wires. If wavelengths of the electromagnetic radiation are much greater than the structure period, such material can be described as a medium which possesses both frequency and spatial dispersions. Previously, the field of a point charge and a small bunch flying in unbounded metamaterial perpendicularly to the wires was considered [1, 2]. It was shown that the radiation concentrates in a small vicinity of the determined lines behind the bunch and the Pointing vector is directed along the wires. Now we consider such a problem for the charge moving along the boundary of the wire metamaterial perpendicularly to the wires. Note that the case of linear array of charges, moving in perpendicular to itself direction was considered in [3]. We study, firstly, the case of point charge and, further, the case of finite bunch.

For the problem with a point charge, analytical and computational investigations were carried out. In particular, it was shown that in the case of ultra-relativistic motion the radiation field can be presented as single integral over frequency. The radiation is non-divergent as well as in the problem with unbounded medium. As opposed to this problem, the radiation field concentrates in the vicinity of certain plane (not line), and it is asymmetric relatively to these plane. If the distance from the charge trajectory to the metamaterial tends to zero, then the field has the same singularity as in the case of unbounded medium. On base of analytical results for the point charge we developed an algorithm of computation of fields of bunches with finite length. Typical examples of bunches' fields are presented. They show that the radiation in metamaterial can be applied for diagnostics of charged particle bunches.

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Broadband μ-near-zero metamaterial

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We fabricate and investigate a metamaterial composed as a square lattice of nonmagnetic metallic cubes. We present numerical simulations of this metamaterial and confirm a possibility to obtain extremely low permeability values. We show that the practical assembly is quite sensitive to fabrication tolerances, and demonstrate that permeability of about $\mu = 0.15$ can be achieved experimentally.

Nowdays, diamagnetism of artificial media attracts more and more attention. For example, diamagnetic properties of the metamaterials were discussed recently in a system of split-ring resonators above the resonance [1, 2], and also diamagnetic response at "zero" frequency using superconductors was suggested [3].

Here we provide a practical evaluation of an isotropic artificial diamagnetic, assessing the lowest magnitudes, likely to be feasible in practice. We present the results of our numerical simulations and experimental results for isotropic metamaterials composed of a dense stack of metallic cubes.

We consider the cubes of size of $b = 1.5$ mm separated by gaps of width $z = 0.13$ mm, so that the lattice constant of the structure is $a = b + z = 1.63$ mm. We have calculated the estimated permeability of an infinitely large lattice with these dimensions using the equations from Refs. [3, 4] and have obtained $\mu = 0.15$.

A common way to extract metamaterial effective parameters is to measure the complex S-parameters in a waveguide followed by the Nicholson–Ross–Weir (NRW) retrieval technique [5, 6, 7]. We performed CST simulations to estimate S-parameters numerically. A unit cell of metamaterial was surrounded by the walls with periodic boundary conditions in the direction transverse to the wave propagation. The metamaterial samples containing 1, 2 and 4 layers of cubes have been studied. The effective parameters were extracted by the NRW technique.

We have fabricated several metamaterial samples consisting of 1, 2 and 4 layers. The cubes of size of 1.6 mm were made from aluminium. In order to provide a structural stability and a thin gap between the cubes in the range of 0.1–0.2 mm, the glue was used.

The sample with one layer of cubes was investigated first. The material parameters were extracted from the measured S-parameters. The mean value of the refractive index yields n about 3 for the real part, and about 0.5 for the imaginary part within the frequency range of 8.5–12 GHz. The permittivity then was found to vary from 60 to 50 with the imaginary part around 30. The high value of the imaginary part can be explained by strong dissipation in the glue as well as due to the sample roughness. The obtained permeability to have a real part about 0.15 in with the imaginary part close to 0.01 [Fig. 1(a)].

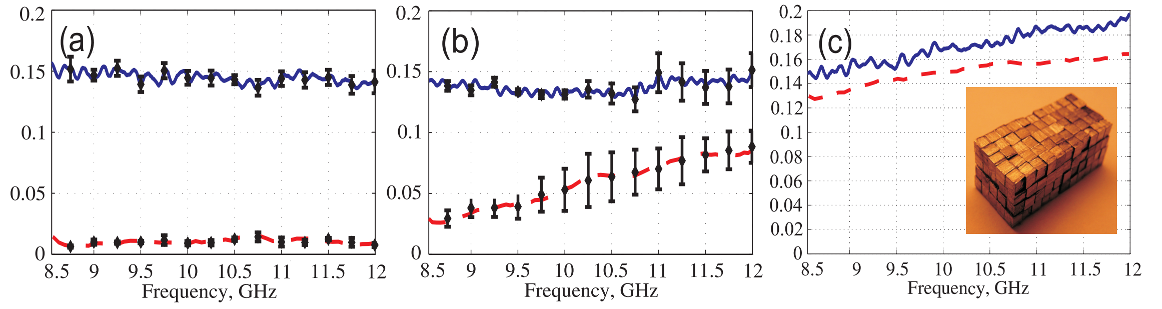

Fig. 1: The permeability extracted from the measured data for (a) 1-layer, (b) 2-layer and (c) 4-layer structure. The solid line shows the real part, the dashed line shows the imaginary part of the permeability. The inset shows the photograph of the 4-layer sample.

The evaluated values of permeability of the two-layers is shown in [Fig. 1(b)]. The refractive index data is matching with the results for a single-layer structure with a higher value of imaginary part lying within 0.5–1.0 region. The mean value of permittivity decreases from 60 to 10 compared to single-layer structure with the increase in imaginary part up to 50.

Finally, the fabricated 4-layer metamaterial had shown the results comparable with the samples consisting of one and two layers. Extracted real part of permeability was about 0.15.

We have demonstrated, both experimentally and numerically, that a sample of densely packed cubes can have quite low values of the effective permeability. For the described setup, we predict that the effective permeability down to $\mu = 0.15$ is feasible. We hope that our results are helpful for further development of artificial diamagnetics, and are promising for magnetic levitation and μ-near-zero metamaterials.

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Magnetism in optical domain using metal-dielectric multilayers

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Multilayered metal-dielectric metamaterials are known to have unusual electromagnetic properties being promising for many applications especially in the optical frequency range. Optical multilayered metal-dielectric nanostructures comprising the periodic arrays of alternating dielectric and metal layers with thicknesses of several tens of nanometers, are known to have striking electromagnetic properties.

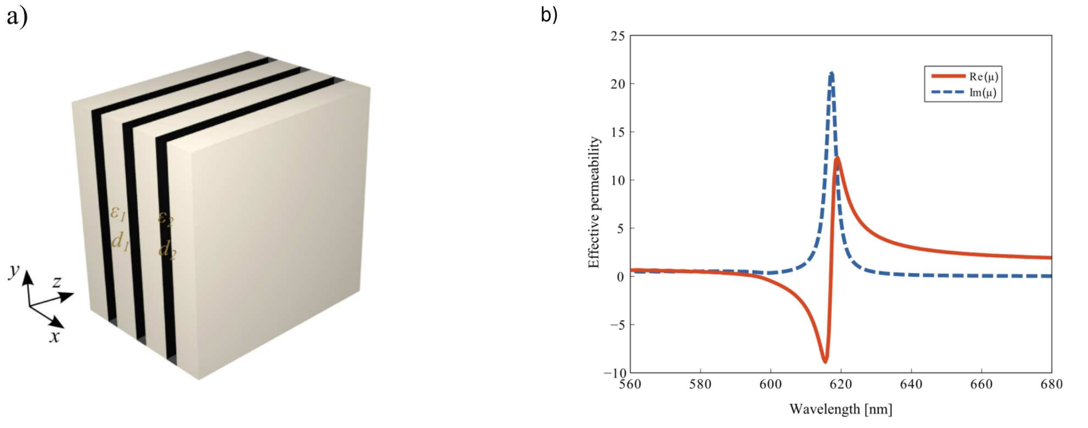

Fig. 1: (a) Schematic view of the multilayer. (b) Extracted effective permeabilty of the metamaterial formed by layers with equal thicknesses.

In this work we show another fascinating property demonstrating magnetic response of multlayered metamaterials in optical domain. Magnetic permeability is obtained numerically using a method of extraction of material parameters of layered structures [1,2] where at first reflection and transmission coefficients should be calculated and then material parameters are extracted. We analyze a role of geometric parameters of the structure on the discovered optical magnetic resonance shown in the figure. Also, limitations of the extraction procedure are discussed.

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Circuit model for periodic array of metallic slits with multiple propagating diffracted orders

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Metallic gratings have a great potential for creation of high-impedance surfaces, effective plasmonic behavior, and high index of refraction. In [1], a circuit model is proposed for one dimensional periodic array of slits perforated in a metallic slab (see Fig. 1.a). In this model, the grating region for TM polarized waves is modeled by a transmission line terminated to a capacitor modeling the effect of the evanescent diffracted orders. However, the model is valid only for the sub-wavelength regime, i.e. when the working frequency is low enough to ensure that there is only one propagating diffracted order outside the grating. In this paper, the circuit model is extended to include higher diffracted orders when they are propagating. This is achieved by relating the capacitance presented in [1] to the artificial surface conductivity concept as introduced in [2]. The surface conductivity is the infinite summation of partial surface conductivities, each one representing a specific diffracted order. The partial surface conductivity is imaginary when its corresponding diffracted order is below cut-off.

It becomes real when its corresponding diffracted order becomes propagating. Real and imaginary partial surface conductivities in the proposed model represent resistors and capacitors, respectively. It is shown that the power lost to each resistor in the proposed model is the power transmitted to its corresponding diffracted order. All the parameters of the proposed circuit model are given by closed form expressions. The proposed model is verified by full wave simulations.

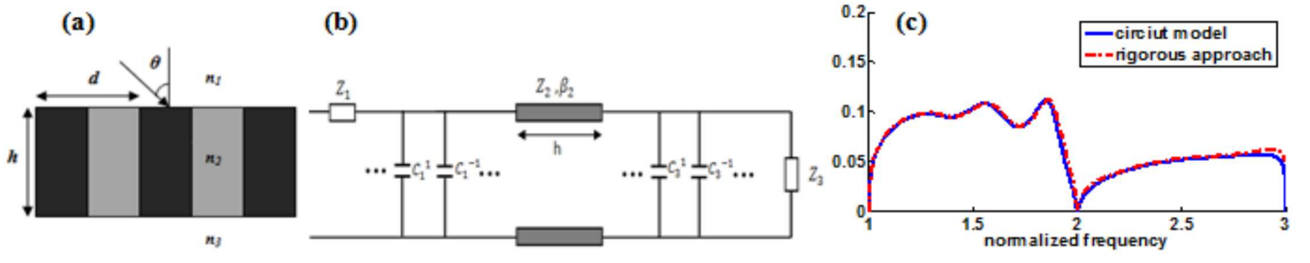

Fig. 1: (a) The geometry of the structure, (b) The proposed circuit model, (c) diffraction efficiency of the first transmitted order.

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Fano resonance in bimetallic dimers and trimers

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Fano resonance (FR) is a type of resonant scattering phenomenon that gives rise to an asymmetric line-shape. Investigation of this phenomena has become popular in recent few years due to a large number of potential applications such as chemical or biological sensors, filters, lasing, switching, nonlinear and slow-light devices and electro-optics, etc [1, 2]. Special attention has been payed to Fano resonance generation in plasmonic nanostructures, where FR occurs via the spectral overlap of broad superradiant(see fig.1 top inset) and sharp subradiant (see fig.1 bottom inset) resonance modes, which are typically characteristics of dipolar and high-order modes, respectively. However, predominant efforts have been focused on FR generation and tuning only through the variation in the geometry of nanostructures so far. Therefore, identification of novel physical mechanisms leading to FR offers promising possibilities for further advances in this field.

In this report, we consider bimetallic plasmonic clusters where FR is caused by different material composition of nanoparticles. Within the point-dipole approximation, we analyze the extinction cross-sections for dimers and trimers containing silver, gold, and copper nanoparticles and find conditions for FR generation. Extinction cross-section of bimetallic dimer composed of silver and gold nanoparticle with transverse polarization of external field is showed in fig.1. We demonstrate unique features of FRs in such structures associated with tunable placements of field cold and hot spots along gaps between nanoparticles.

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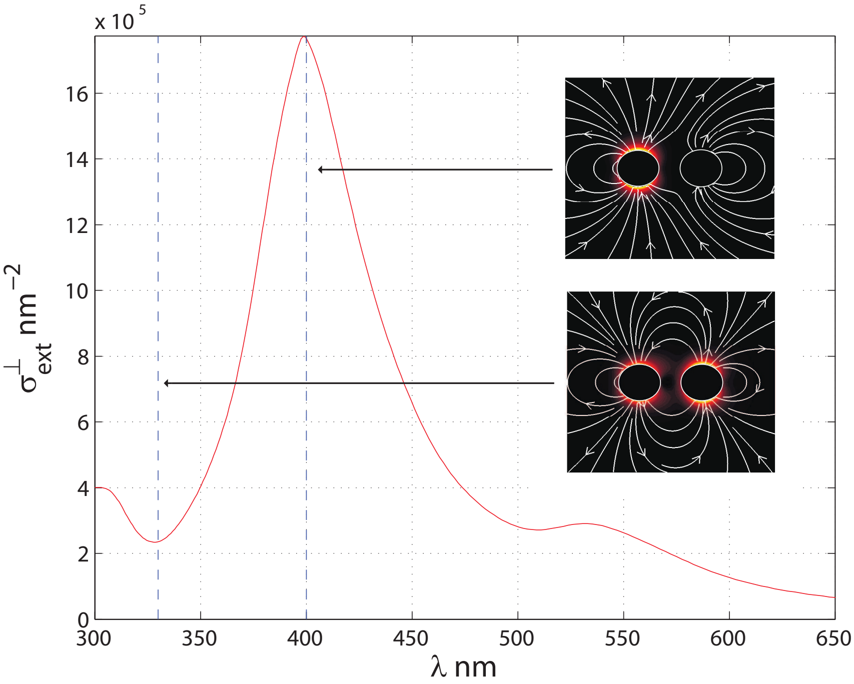

Fig. 1: Extinction cross-section of bimetallic nanodimer composed of the gold and silver. Field distribution within the bimetallic dimer at 400 nm and at 330 nm top inset and bottom inset, respectively. Polarization incident field is transverse the axis dimer. Size of particles is 30 nm, the gap between center of the particles is 90 nm.

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Liquid meta-crystals in ac/dc electromagnetic fields: theoretical treatment

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This report is devoted to the discussion of some linear and nonlinear optical properties of new type of metamaterial which we call *liquid meta-crystals* (LMC) in both ac and dc electromagnetic fields. We consider an array of identical metallic nanoparticles (meta-atoms) having the form of elongated dumbbells suspended in viscous liquid. The action of dc (controlling) electric field orients the particles along the field direction that makes LMC anisotropic medium for electromagnetic radiation. The axis of anisotropy can be re-oriented and this type of tunability resembles that of liquid crystals in nematic phase. The artificial meta-atoms can be designed as a classical oscillators to operate at different frequencies and to enhance their tunability. Furthermore, our meta-atoms can be re-oriented in response to the electromagnetic field, that suggests the strong nonlinear properties which LMC may exhibit.

As a particular example, we choose the particle geometry so that the frequency of fundamental eigenmode lies within terahertz domain. For the description of the wave interaction with such resonant artificial medium we use the dynamical mechanical equations for meta-atoms, which at the same time are the electric dipoles induced by external ac/dc electric field, along with Maxwell's equations written for corresponding effective medium. We study both linear and nonlinear interaction of electromagnetic waves with LMC. More specifically, we consider a linear wave propagation in this strongly anisotropic medium and show the possibilities of effective control of transmission and reflection via changing of dc electric field direction. In the intensive electromagnetic field the interaction of electromagnetic wave with LMC may become strongly nonlinear because of self-consistent re-orientation of dipoles in ac electric field. The characteristic ac field tension of nonlinearity is just the tension of dc electric field and in this sense the nonlinear effects may be giant or even colossal. To illustrate this fact we consider nonlinear transmission of THz electromagnetic wave through a thin LMC layer and show that nonlinear effects strongly depend not only on the field intensity but also on incident wave polarization and frequency detuning from resonance value. Apart from the study of volume wave propagation we consider linear surface waves supported by air/LMC interface and thin film of polarized LMC. We show that air/LMC interface supported the surface waves within quite narrow gap of the frequencies as well as wavenumbers which collapses as far as the angle of LMC anisotropy axis tends to perpendicular to the interface direction. In the case of thin film we demonstrate that TE-polarized surface waves exist at the frequencies below resonant and exhibit closed isofrequency contour (effective elliptic medium), whereas TM-polarized surface waves have the frequency greater than resonant and, in turn, exhibit unclosed isofrequency contour (effective hyperbolic medium).

Perfect non-magnetic invilibility cloak

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Cloaking, or an electromagnetic concealment, attracts a lot of attention recently. Transformational invariance of Maxwell's equations is at the heart of the method of the cloaking design. Simple formulas of transformational optics (TO) give a recipe of mapping of free space and the corresponding vacuum fields onto inhomogeneous medium with anisotropic material parameters. Topological invariance of virtual (vacuum) and physical space ensures zero scattering of electromagnetic field. The benefit of the method is a potential to create a perfect masking shell regardless of the cloaked object size and form. However, the perfect cloaking based on TO requires singular material parameters and also symmetric dielectric and magnetic properties, $\hat{\mu} = \hat{\epsilon}$. As a result, in the most important optical applications it is actually impossible to apply the TO approach because of magnetic degeneration, $\mu \to 1$, of all the media with the frequency growth. To overcome this problem a method was proposed to create a simplified (not perfect) cloaking shell using only dielectric materials [1]. It implies taking the refraction index just the same as in TO design but keeping magnetic permeability to be 1, i.e. creating a shell with $\hat{\epsilon}_{shell} = \hat{\epsilon}_{TO}\hat{\mu}_{TO}$, $\hat{\mu}_{shell} = 1$. This simplified cloaking design is apparently based on geometro-optical approach. Numerical simulations confirmed the substantial reduction of the scattering cross-section of the object covered by such a shell. However, the quality of concealment is aloud worse compared with the perfect TO cloak.

In this work we introduce a designing method for *perfect non-magnetic invisibility cloak*. The method can be applied for 2D scattering of the TM-polarized electromagnetic radiation by cylindrical object. The procedure performs a mapping of the virtual vacuum (r, ϕ) onto deformed (physical) space (R, ϕ) by means of an (arbitrary) auxiliary function and obtains the corresponding radial structure of $\hat{\epsilon}_{shell}(R)$ which provides non-scattering concealment volume in non-magnetic medium.

An extra freedom in mapping formulas favors the practical implementation of the proposed design. In particular, in contrast to TO, it does not necessarily lead to a singularity of ϵ on the inner boundary of the cloak. We show that using the anisotropic material with diagonal dielectric permittivity tensor $\hat{\epsilon}_{shell} = (\epsilon_{rr}(R), \epsilon_{\phi\phi}(R))$ (the components $\epsilon_{rr}, \epsilon_{\phi\phi}$ bounded between 0 and ϵ_{max}) and $\mu = 1$ makes it possible to eliminate the scattering from the cylindrical object completely. It is shown that a cylindrical layered shell with the layers of isotropic $\epsilon_1 = \epsilon_{max}$ and $1 > \epsilon_2(R) \geq 0$ gives the required tensor permittivity with the help of changing the relative thickness of the layers (1) and (2). The results make an important step towards the application of the invisibility cloaks for practical devices.

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Cascaded interaction of laser pulsed beams in quadratic nonlinear media

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In this paper we investigate interaction of three pulsed laser beams in quadratic nonlinear media [1, 2]. Pulsed beams are the waves limited in time (along the direction of propagation) and space (in the transverse direction) [3].

We use wave theory to investigate nonstationary effects. The equations for slowly varying amplitudes of pulsed beams (taking into account group velocity dispersion, quasi-optical diffraction) are of the form:

$$\frac{\partial A_1}{\partial z} + i[D_{1x}\frac{\partial^2 A_1}{\partial x^2} + D_{1\tau}\frac{\partial^2 A_1}{\partial \tau^2}] = -i\gamma_1 A_3 A_2^*, \tag{1}$$

$$\frac{\partial A_2}{\partial z} + i[D_{2x}\frac{\partial^2 A_2}{\partial x^2} + D_{2\tau}\frac{\partial^2 A_2}{\partial \tau^2}] + \nu_2\frac{\partial A_2}{\partial \tau} + \alpha\frac{\partial A_2}{\partial x} = -i\gamma_2 A_3 A_1^*, \tag{2}$$

$$\frac{\partial A_3}{\partial z} + i[D_{3x}\frac{\partial^2 A_3}{\partial x^2} + D_{3\tau}\frac{\partial^2 A_3}{\partial \tau^2}] + \nu_3\frac{\partial A_3}{\partial \tau} = i\Delta k A_3 - i\gamma_3 A_1 A_2. \tag{3}$$

The wave frequencies obey $\omega_1 + \omega_2 - \omega_3 = 0$, and the wave vectors are mismatched $\Delta k = k_1 + k_2 - k_3$.

When the wave mismatch is sufficiently large, the parametric interaction takes form of the cascade process [4, 5]. As the signal is weak ($A_2 << A_1$) we can identify the main features of pulsed beams nonlinear interaction using only the equation for amplitude of the signal:

$$\frac{\partial A_2}{\partial z} + \alpha\frac{\partial A_2}{\partial x} + \nu\frac{\partial A_2}{\partial \tau} + iD_x\frac{\partial^2 A_2}{\partial x^2} + iD_\tau\frac{\partial^2 A_2}{\partial \tau^2} = ik_2 n_{nl2}(x,\tau)A_2, \tag{4}$$

here $\tau = t - \frac{z}{u_1}$, $\alpha = \angle(k_2, z)$, $\nu = u_2^{-1} - u_1^{-1}$, $D_x = \frac{1}{2k_2}$, $D_\tau = -\frac{1}{2}\frac{\partial^2 k}{\partial \omega^2}$. Equation (4) describes the propagation of a pulse in a medium with refractive index inhomogeneity

$$n_{nl2} = n_2 \left|A_1(\tau, x, z)\right|^2. \tag{5}$$

We considered the signal transmission through the pump pulsed beam, single and double total reflection, signal wrapping of the reference pulsed beam by numerical simulation of three-wave parametric interaction. We have focused our study on the signal total reflection from the pump (Fig. 1).

If the collision is not centered (detuning in time and in space are introduced), we introduce the minimum distance (R_{min}) between the centers of the Gaussian pulsed beams for the signal and the pump to estimate the “transparency” of the inhomogeneity expressed by the formula:

$$R_{min}^2 = \frac{(\tau_0\alpha - d\nu)^2}{\alpha^2 T^2 + \nu^2 a^2}. \tag{6}$$

Pulsed beams converge to the point $R_{min} = 0$ if length of the group delay is equal the length at which the beams intersect in space:

$$\tau_0/\nu = d/\alpha. \tag{7}$$

Using the geometrical optics method we derived equations for the signal trajectories $x = x_s(z)$, $\tau = \tau_s(z)$. For the initial conditions corresponding to a collision at a small angle signal Gaussian pulsed beam with not changing its shape over time pump, trajectories were obtained for different values of detuning the velocity and intensity of the fundamental wave (Fig. 2). The results which gives the geometrical optics theory are compared with the wave theory simulation.

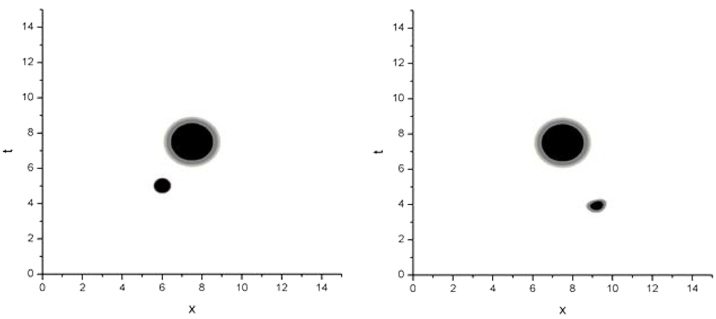

Fig. 1: Single signal reflection from the reference beam pulse time.

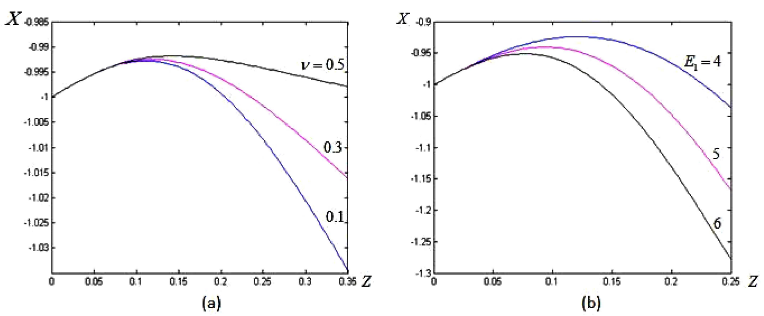

Fig. 2: The signal path in the case of total reflection for different detuning velocity (a) and the intensity of the fundamental wave (b).

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Pseudo-Hermiticity in optics

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In Ref. [1] it was shown that a large class of non-Hermitian Hamiltonians can have real eigenvalues. Such Hamiltonians are called PT-symmetric. PT-symmetric Hamiltonians may also have complex eigenvalues. Moreover, in these systems, spontaneous phase transition from real to complex eigenvalues can be observed. The similarity of the equations of quantum mechanics and electrodynamics has stimulated studies of PT-symmetric optical systems [2, 3].

Near the phase transition point it is possible to create optical devices with unusual properties, such as systems consisting of alternating amplifying and absorbing layers, which are completely invisible in the finite frequency range from one side and visible from the other side [4, 5]. However, in order to use such devices, it is necessary that PT symmetry would exist over a frequency range. We show that because of the medium dispersion, PT symmetry can exist at isolated frequencies, but not in a range of frequencies.

Spontaneous phase transition in the spectrum of eigenstates can be observed in a wide variety of systems [6], called pseudo-Hermitian, which do not possess PT symmetry. We suggest examples of pseudo-Hermitian optical systems. We analyze their optical properties and show that unlike PT symmetric systems, in the proposed pseudo-Hermitian optical systems phase transition may occur with the frequency variation.

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