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DAYS ON DIFFRACTION 2014

ABSTRACTS

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FOREWORD

"Days on Diffraction" is an annual conference taking place in May–June in St. Petersburg since 1968. The present event is organized by St. Petersburg State University, St. Petersburg Department of the Steklov Mathematical Institute, the Euler International Mathematical Institute and the ITMO University.

The abstracts of 192 talks to be presented at oral and poster sessions during 5 days of the Conference form the contents of this booklet. The author index is located on the last pages.

Full-length texts of selected talks will be published in the Conference Proceedings. They must be prepared in LaTeX format and sent not later than 15 June 2014 to `diffraction14@gmail.com`. Format file and instructions can be found at `http://www.pdmi.ras.ru/~dd/proceedings.php`. The final judgement on accepting the paper for the Proceedings will be made by the Organizing Committee after peer reviewing.

As always, it is our pleasure to see in St. Petersburg active researchers in the field of Diffraction Theory from all over the world.

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Complex Ince–Gaussian beams

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Ince–Gaussian (IG) modes is a third family of structurally stable solutions of the paraxial equation, separated in an orthogonal coordinates in the plane [1, 2]. Two others are Hermite–Gaussian modes and Laguerre–Gaussian mode families. We propose a new family of complex IG modes, which is rather different from helical IG modes [3], and investigate some of its properties.

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Nonlinear theory of localized and periodic waves in crystal media with a complex lattice

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The nonlinear model of deformation of crystal solid bodies with a complex lattice has been offered in works [1], [2]. Shift of the center of inertia of atoms of an elementary lattice is described by a vector $\mathbf{U}(x, y, z, t)$ (acoustic mode), and relative shift of atoms is described by a vector $\mathbf{u}(x, y, z, t)$ (optical mode). The shifts of sublattices can be arbitrary large magnitude unlike the classical theory [3]. The nonlinear equations of movement predict formation of defects, phase transformations, formation of a superlattice (fragmentation) and other physical processes which arise in the field of intensive external loadings. Such processes are not described by the linear classical theory of elasticity. Therefore development of numerical and analytical methods of the solution of the equations of the nonlinear theory is necessary.

Wave processes in thin membranes are considered. The equation of motion, defining $\mathbf{U}(x, y, t)$ for plane deformation, have a form of the standard equations of mechanics of continuous media. The equations for optical mode $\mathbf{u}(x, y, t)$ represent system of two coupled nonlinear equations similar to Klein–Fock–Gordon type equations

$$\mu\frac{\partial^2 u_i}{\partial t^2} + P\frac{\partial \Phi(u_R)}{\partial u_i} = k_1 \Delta u_i + k_2 \frac{\partial}{\partial x_i}\operatorname{div}\mathbf{u}, \qquad (i = x, y), \tag{1}$$

$$P = \frac{u_i}{u_R}\left[P_1(x, y, t) + P_2(x, y, t)\Phi(u_R)\right], \tag{2}$$

$$\Phi(u_R) = 1 - \cos u_R + \delta(1 - \cos 2u_R), \qquad u_R = \sqrt{u_x^2 + u_y^2}. \tag{3}$$

Here μ is the specified density of couple of atoms, k_1, k_2 are microelasticity coefficients, $P_1(x, y, t)$ and $P_2(x, y, t)$ are the functions which are unambiguously determined by a stress tensor, $\Phi(u_R)$ is the energy of interaction of sublattices.

For the case ($k_1 = k_2$, $P_1, P_2 = \text{const}$) functionally invariant solutions of the system of Eqs. (1) are found. Exact analytical solutions are expressed through two arbitrary functions depending on one ansatz $\alpha(x, y, t)$. For potential $\Phi(u_R) = 1 - \cos u_R$ the solution is given by inversion of elliptic integral, and for $\Phi(u_R) = 1 - \cos u_R + \delta(1 - \cos 2u_R)$ — by inversion of hyperelliptic integral. For the case $\delta = 0$ the particular solutions are found and the dependence of nature of perturbations (periodic waves, localized waves like a kink, a soliton, etc.) from the parameters of model is established. Phase portraits of the equations of optical mode are constructed for the cases $\delta = 0$ and $\delta \neq 0$. Research of phase portraits allowed to establish dependence of dynamic structure of the crystal medium from a type of potential $\Phi(u_R)$.

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Diffraction of localized shear wave on the edge of semi-infinite crack in composite elastic space

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The problem of Love localized shear plane wave diffraction, incident from infinity in elastic piecewise-homogeneous space, weakened by a semi-infinite crack parallel to non-homogeneity line, is considered. Diffractional wave field determination is reduced to a problem of Riemann type of analytic functions theory on the real line with right-hand side of generalized function $\delta(x)$. Solution of functional equation, obtained in generalized functions, allows us to obtain the distribution of wave field in any region of the elastic space. Asymptotic formulas allowing determine wave field behavior in far regions are also obtained.

Let us consider a piecewise-homogeneous elastic space, consisting of two different semi-spaces occupying domains $\Omega_1(|x| < \infty, y > 0, |z| < \infty)$ and $\Omega_2(|x| < \infty, y < 0, |z| < \infty)$ in Cartesian coordinate system $Oxyz$. Semi-space Ω_2 is weakened by a semi-infinite through crack of longitudinal shear, occupying domain $\Omega_0(x < 0, y = -h, |z| < \infty)$. Semi-spaces contact by $y = 0$ plane and are in full contact conditions. In $\Omega_{12}(x < 0, y > 0)$ quadrant

$$u_z^{(\infty)}(x, y, t) = w_{\mathcal{L}}(x, y)e^{-i\omega t},$$

with amplitude

$$w_{\mathcal{L}}(x, y) = A_m^{(\mathcal{L})} \exp\left(i\sigma_m x - \sqrt{\sigma_{m1}^2 - \kappa_1^2}\, y\right).$$

Here $\kappa_1 = c_1\omega$ is the wave number, $c_1 = \sqrt{\mu_1/\rho_1}$ is the shear plane wave velocity, μ_1, ρ_1 are the shear modulus and density of medium in Ω_1 domain, ω is vibrations frequency, t is the time, σ_{m1} is the wave number of localized wave, i.e. the positive root of Love function

$$\mathcal{L}_1(\sigma) = \mu_1\gamma_1 \cosh(\gamma_2 h) + \mu_1\gamma_2 \sinh(\gamma_2 h),\ \ \gamma_j = \sqrt{\sigma^2 - \kappa_j^2}\ (j = 1, 2),\ \ \kappa_1 < \sigma_{m1} < \kappa_2.$$

Diffracted wave field determination in composite space under assumption, that it is in antiplane deformation conditions, is reduced to a problem of Riemann type of analytic functions theory on the real line. Solution of resolving functional equation is obtained by factorization technique. Explicit expressions for wave fields determination in all regions of problem under consideration are obtained. Questions concerning with surface waves emergence and asymptotic formulas determination, describing wave fields behavior in far regions are also investigated.

Numerical study of a model scattering problem

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A new approach to estimating scattering functionals is under consideration. We study numerically scattering of a plane wave on a conducting body. The method is based on Monte Carlo simulation of the induced currents on the body surface. We discuss the convergence and the accuracy of the proposed method and verify our estimates in numerical experiments.

We consider also a problem of accurate numerical modeling of resonant scattering on conducting bodies. A few model problems are solved numericaly by three different well-known methods such as finite element method, method of moments, and fast multipole method. The results are compared with the Mie series for the unit conducting sphere. The estimates by the proposed Monte Carlo simulation are verified in the numerical tests against the results of the above methods.

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GPU-based calculations in electromagnetic wave diffraction problems

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In their papers Z.S. Agranovich, V.A. Marchenko, V.P. Shestopalov, V.G. Sologub proposed to solve the diffraction problem of electromagnetic waves by the method of solution of the Riemann–Hilbert boundary-value problem. By this method diffraction problem is reduced to system of functional equations. The solution of this system can be found by the expansion of the required function into Fourier series. Unknown coefficients of this function satisfy an infinite set of linear algebraic equations

$$Ax = f. \tag{1}$$

In their papers N.B. Pleshchinskii, I.E. Pleshchinskaya, D.N. Tumakov proposed to solve the diffraction problem by the method of solution of the over-determined Cauchy problem. The electromagnetic wave diffraction problem on a thin conducting screen is equivalent to a regular infinite set of linear algebraic equations (1) relative to unknown coefficients of expansion into a Fourier series. The infinite set of equations (1) can be solved by the reduction method with any degree of accuracy.

Generally, especially in three-dimensional waveguide structures, calculation of matrix elements $a_{k,j}$ and elements f_k in system (1) occupies most of time. These elements are data-independent of each other. So, they can be calculated in parallel mode. Solution of system (1) takes much less time. It can be found in serial mode by method of Gaussian elimination, for example. Parallel algorithm using CUDA is following:

1. copy input datas from host memory to device memory;
2. invoke kernel, executed on GPU, for calculation of matrix elements $a_{k,j}$;
3. invoke kernel, executed on GPU, for calculation of elements f_k;
4. copy matrix elements $a_{k,j}$ and elements f_k from device memory to host memory;
5. solve the system (1) in serial mode on CPU.

Programs were launched on CPU Intel Core i5 2.3GHz and GPU NVidia GeForce GT 525M.

Running time of sequential program and parallel program for solving diffraction problem on metallic screen in rectangular waveguide is shown in the table, where N is number of coefficients in expansion of the required function into the series of eigenwaves.

N	4	8	12	16	20	24
Running time of sequential program (ms)	3634	$5.5 * 10^4$	$2.8 * 10^5$	10^6	$2 * 10^6$	$4 * 10^6$
Running time of parallel program (ms)	731	1800	2506	4890	4225	4276
Speedup	4.9	30.5	111.7	204.4	473.3	935.4

Diffraction by an elliptic cylinder with a strongly elongated cross-section

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We consider diffraction of a stationary plane wave incident on an elongated elliptic cylinder and construct the high-frequency asymptotics of the diffracted field under the assumption that the semiaxes: minor a and major b of the ellipse in the cross-section of the cylinder and the wave number k form the parameter

$$\chi \equiv ka^2/b$$

on the order of unity, while kb is asymptotically large. The angle ϑ between the direction of incidence and the major axis of the ellipse is considered small, such that $\beta = \sqrt{kb}\vartheta$ also is a quantity on the order of unity. The surface is considered to be ideal, i.e. described by the Dirichlet or the Neumann boundary conditions.

The problem is decomposed into even and odd parts with respect to the major axis of the ellipse. The asymptotics of the field in a boundary-layer near the surface is represented as the sum of two wave processes. One is formed by waves that travel along the surface in the direction of the incident wave, the other is formed by waves that circumvent the shaded tip of the ellipse and travel in the backward direction. The interference of these two wave processes forms the field with an oscillating amplitude. Comparison with the induced currents computed with the help of pdetool in Matlab shows that the obtained asymptotic representation provides a sufficiently good approximation starting with $kb \approx 3$.

Another asymptotics is obtained for the far field in the forward directions. This asymptotics demonstrates the property of uniformity with respect of the rate of elongation χ and allows the transformation of the far field amplitude to be seen from the case of the circular cylinder, when it does not depend on the angle of incidence ϑ, to the case of the strip, when it is symmetric with respect to the major axis of the ellipse.

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Eigen-modes of the linearised problems at the resonant scattering and generation of oscillations for a nonlinear layer

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Nonlinear dielectrics with controllable permittivity are subject of intense studies and begin to find broad applications in device technology and electronics. In the range of resonant frequencies, the problem of scattering and generation of oscillations by the excitation of a nonlinear cubically polarisable layer was studied. We restricted our investigations to the third harmonic generation [1–3].

The approximate solution of the self-consistent nonlinear problems was obtained by solving linear problems with an induced nonlinear dielectric permeability by means of an iterative method. The analytical continuation of these linear problems into the region of complex values of the frequency parameter allowed us to switch to the analysis of spectral problems. We were able to show the characteristic dynamical behaviour of the relative Q-factor of the eigen-modes and the energy of the higher harmonics generated by nonlinear layers.

Numerical results for the problem of the third-harmonic generation by resonant scattering of nonlinear waves on layers having either decanalising or canalising energy-dissipation properties were investigated. In particular, in the numerical investigation of a nonlinear decanalising single-layered structure the effect of type conversion of the generated field in the region of higher-harmonics generation could be observed. Within the framework of the self-consistent formulation of the problem it was demonstrated that the induced imaginary part of the dielectric permittivity, which is determined by the nonlinear part of the polarisation, characterises the energy-loss in a nonlinear medium spent for the generation of the electromagnetic field at the third harmonic.

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Multidimensional tunneling between potential wells at non degenerate minima

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We consider tunneling between 2 symmetric wells for a d dimensional semi-classical Schrödinger operator with potential V at an energy level E close to the minimum $E_0 = 0$ of V. According to the common wisdom, splitting of eigenvalues $E_j(h)$ near E takes generally the form

$$\Delta E_j(h) \sim A(h)e^{-S(E(h))/h}, \quad h \to 0$$

where $S(E(h))$ is an action that measures the life-span of the particle in the classically forbidden region, and $A_j(h)$ an amplitude. This formula holds true in the one dimensional case, even for excited states. In the multidimensional case, it holds true for the ground state, but not necessarily for excited states, depending in particular on integrability properties of the underlying classical dynamics. We give a precise meaning to this formula for semi-excited states, i.e. with energies near the (quadratic) mimimum of V, in two cases: (1) excitations of the lowest frequency ω_1 in the harmonic oscillator approximation of V; (2) more general semi-excited states in dimension 2, i.e. with energies $E \leq Ch^\delta$, provided frequencies of the harmonic oscillator are simply irrational or Diophantine.

Case (1) resorts partially to the one dimensional problem: $S(E(h))$ identifies with Agmon distance between the wells, and we can also obtain a first order asymptotics for $A(h)$ of the form

$$\Delta E_j(h) = b_j \frac{\omega_1 h}{\pi} e^{S(E(h))/h}(1 + o(1)), \quad h \to 0$$

where b_j is a geometric constant, $b_j \to 1$ as $j \to \infty$.

In case (2) $S(E_j(h))$ identifies again with Agmon distance between the wells for Agmon metric, but the structure of $A_j(h)$ may be lost, because tunneling involves in a complicated way the analytic continuation of quasi-modes in the classically forbidden region.

On some of the peculiarities of propagation of an elastic wave through a gradient transversely isotropic layer

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Peculiarities of propagation of longitudinal waves through inhomogeneous transversely isotropic layers with gradient-like distribution of density and elastic parameters are of interest for modeling elastic wave propagation in the real media. In particular, problems of reflection and propagation of longitudinal waves through heterogeneous alloys, composite materials and spatially confined porous structures were under investigation by a number of researchers in the past.

In this study we investigate the problem of diffraction of an elastic wave by the inhomogeneous transversely isotropic layer with a continuous distribution of elastic parameters. Peculiarities of propagation of the plane wave through the gradient isotropic layer were already considered by these authors in the past in [1]. This study represents continuation of investigations that started in [2]. Our goal is to detail characteristic features of frequency-response characteristics of the elastic wave diffraction by transversely isotropic layers.

Differential equations for describing the diffraction problem are considered separately for half-planes and for the layer. The elastic parameters in the layer are defined through the elasticity tensor. Problems in the half-planes are overdetermined, which allow establishing a connection between traces of the required functions at media interfaces. Thus, the original problem reduces to the boundary value problem for the system of partial differential equations with boundary conditions of the third type. The Fourier transformation is applied with respect to the variable for which homogeneity of the problem is preserved. The obtained boundary value problem for the system of ordinary differential equations is solved using the grid method.

Results of numerical calculations are presented for realistic and "synthesised" geologic environments. Characteristic extrema in transmittance ratios of the elastic wave are determined. Dependencies of frequency-response characteristics of energy of the passed wave on velocities of the elastic waves in the layer are presented.

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Isomonodromic deformations of Fuchsian systems and symplectic geometry of space of matrices

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It is natural to consider Fuchsian system $dY = \omega Y$ up to a linear change of unknown Y in the Isomonodromic Deformations Problem. It leads us to the investigation of geometry of the *a symplectic quotient of the product of (co)adjoint orbits of* $\mathrm{GL}(N)$ that is an algebraic symplectic space. I will talk about the birational Darboux coordinates on the space, the connection between isomonodromic problem and the Painlevé equations.

The main points are the following. The constant monodromy is equivalent to the single-valuedness of the fundamental solution $Y = Y(z, z_i)$, where z_i are the positions of the singular points, considered as parameters of deformation. The suggestion of the similarity of the behaviour with respect to z and with respect to z_i at the singularity: $Y(z, z_i) = \tilde{Y}(z - z_i)(\mathrm{I} + o(1)), z \sim z_i$ implies compatible evolution equations on A_i. A projection of the equations on the symplectic-quotient space in 2×2 case gives Painlevé VI equation.

I will show how the symplectic structure arise and work in this problem, how the elementary geometrical construction makes possible to write down the equations and investigate their symmetries.

Eigenvalue asymptotics and trace formulas for fourth order operator on the unit interval

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We consider the fourth order operator on the unit interval with the Dirichlet type boundary conditions. It is well known that such operators arise in many physical models. One of the new application is the wave propagation in the layered spherically symmetric materials (for example, radial wave crystals). It is described by the partial differential equation with coefficients, depending on radius and not depending on corners. Separation of variables in such equation in some approximation leads to the system of the second and fourth order equations. Special case of the fourth order operator is the Euler–Bernoulli operator which describes the bending vibrations of thin beams and plates.

We determine few trace formulas for such operators. Moreover, we determine the sharp eigenvalue asymptotics at high energy. These asymptotics are expressed in terms of the Fourier coefficients of all coefficients of the operator. The asymptotics are new and they are the basis for the proof of the trace formulas.

Electron multichannel scattering at narrows of quantum waveguides

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In the recent times, the study and design of electronic components, based on ballistic electron transport in nanowires, show steep increase of interest. The components can be field effect transistors [1], resonant tunneling diodes [2], lasers [3], cubits [4], etc. Quantum resonators in nanowires (quantum waveguides) can be made by applying of external potential to chosen parts of the waveguide [2]. But there exist much easier way to make quantum resonators by forming two or several narrows in the waveguide. Here we analyze elastic scattering of electron wave at a narrow in a quantum waveguide in the case where transitions with a change of quantum number are possible (multichannel scattering).

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Fano resonances and determination of the resonance parameters from transition coefficient curves

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Ballistic electron multichannel scattering in a waveguide with two narrows is discussed. The waveguide narrows serve as effective potential barriers and make the domain between them a resonator. Having enough energy, incident electron wave scatters at the first narrow into several (allowed) states with different quantum numbers, the total electron energy being constant. One or several of the states may have resonant energies while the other states pass (and are reflected) without resonance. This causes specific bendings of the transition coefficient curve. We suggest a simple model explaining the emergence of the Fano resonances in the considered case and propose a method for determination of the resonance parameters.

Inverse dynamical problem for the 1-d Dirac system

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We consider the inverse dynamical problem for the one-dimensional Dirac system. We give a optimal-in-time procedure of the reconstruction of the matrix-valued potential from the given operator of reaction. We also answer the question on the characterization of the inverse data: i.e.

we describe the set of operators which are reaction operators for 1-d Dirac system with some locally summable potential.

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Transfer matrix of the sixth order

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Method of the transfer matrix is used to describe the six-wave diffraction in layered media. Transfer matrix $T(z) = \|t_{ij}(z)\|_1^6$ expresses the wave field components $\varphi_i(z)$ at depth z stratified medium through the field components on the surface $z = 0 : \varphi_i(z) = t_{ij}(z)\varphi_j(0)$. The system of differential equations

$$\frac{d\varphi_i(z)}{dz} = w_{ij}\varphi_j(z), \quad \varphi_j(0) = \varphi_{j0}, \quad i, j = 1, ..., 6, \tag{1}$$

that determine the 6-order transfer matrix T is solved by the method of symmetric polynomials [1]. The algorithm for numerical solution of problem (1), which uses symmetric polynomials and scaling the matrix $W \equiv \|w_{ij}\|_1^6$, is presented.

This method is applied to the elastic waves in crystals. Influence of layer thickness and frequency of the wave on the scaling parameter is investigated.

Analytic solutions describing the transfer of elastic stresses in the crystalline layers of the cubic, hexagonal and orthorhombic systems are obtained.

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A waveguide problem in aeroelasticity

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This paper extends previous works [1, 2] in the area of elastic waveguides by including the forces due to aerodynamics effects when there exists a transverse flow over a long elastic structure. An infinitely long waveguide is considered analytically. Such simplification does not account for reflections at the ends for a real structure such as an aeroplane wing or a long hanging cable under aerodynamic excitation; however it provides interesting closed form results and generic understanding of the phenomenon in addition to approximations to real aeroelastic problems. The model under consideration analytically examines the spatio-temporal stability of such waveguides. Simple structural model which includes bending-torsion coupling and aerodynamics which provides coupling of the equations of motion additionally are analytically solved to obtain the dispersion relations and the stability information there from. The dependence on various structural and aerodynamic parameters is explicitly brought out. Stability regions in the parameter space are obtained. Illustrative examples are provided.

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Scattering of electromagnetic waves in a plane channel with sharp corners

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The model plane scattering problem in the strip region with sharp ledge is considered. The problem is governed by the Helmholtz equation together with the Neumann condition on the boundary and the radiation condition in infinity. A lot of methods are used for this classical problem; nevertheless, it is very difficult to find suitable numerical method, since the solution is singular at the sharp corners.

To solve such problem effectively, the method of discrete sources (MDS) seems to be one of the most promising one. In turn, effectiveness of the MDS depends essentially on the way of source placement. Moreover, the situation becomes dramatically ill for domains with sharp corners.

The original ideas for the way of source allocation for sharp-pointed domains are suggested. An algorithm, permitting us to find the optimal source placement, based on the singular value decomposition technique, is presented. Numerical experiments illustrates our investigations.

On dimensions of oscillator algebras

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We consider a generalized oscillator algebra connected with arbitrary system of orthogonal polynomials on the real line. We discuss the question: under which conditions such oscillator algebra is finite-dimensional. The answer to this question was given in a recent work of G. Honnouvo and K. Thirulogasanthar only for oscillators related to polynomials which orthogonal with respect to a symmetric measure on the real axis. In our talk we extend the results of the cited above work to the case of arbitrary orthogonal polynomials on the real axis. Besides, we give some consideration of the oscillator algebra associated with Chebyshev–Koornwinder polynomials in two variables.

Internal gravity waves dynamics in stratified medium with variable depth: exact solutions and asymptotic representations

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Wave dynamics of stratified medium (ocean, atmosphere) is highly dependent on bottom topography. The exact analytical solution is obtained only if the water distribution density and bottom shape described by some model functions. When the characteristics of the medium and the boundaries are arbitrary and can be built only numerical solutions of such problems. However, numerical solutions are not qualitatively analyze the characteristics of the wave of the fields. The need for a qualitative analysis of the far field of internal waves arise in the study of internal waves remote methods by means of aerospace-parameter radar. Then the description and analysis of wave dynamics can be made only on the basis of the asymptotic models. In this paper uniform asymptotic forms of the far field of internal gravity waves which propagate in stratified medium with a smoothly varying bottom are constructed. The solution is proposed in terms of wave modes, propagating independently in the adiabatic approximation, and described as a power series of a small parameter characterizing the stratified medium. The effect of the space frequency “blockage” of the wave fields

is characteristic of the real oceanic shelf. Depending on the frequency characteristics of the wave field and bottom topography, far internal gravity waves are either localized in some limited spatial domain (captured waves) or propagated over long distance (progressive waves). The spatial domain, where progressive waves propagate depends on ocean stratification and bottom topography. Using asymptotic methods, one can consider a wide class of interesting physical problems, including problems concerning the propagation of non-harmonic wave packets of internal gravity waves in diverse non-homogeneous stratified media under the assumption that the modification of the parameters of a vertically stratified medium are slow in the horizontal direction. The specific form of the wave packet can be finally expressed by using some special functions, slay, in terms of oscillating exponentials, Airy function, Fresnel integral, Pearcey-type integral, etc. The above approaches are quite general and, in principle, enable one to solve a broad spectrum of problems from the mathematical point of view; however, the problem of their practical applications and, in particular, of the visualization of the corresponding asymptotic formulas based on the Maslov canonical operator is still far from completion, and in some specific problems to find the asymptotic behavior whose computer realization using software of Mathematica type is rather simple. The results presented in the paper have been obtained by research performed under projects supported by the Russian Foundation for Basic Research (No. 14-01-00071, 14-01-00466, 14-08-00701).

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Comparison of different current-based hybrid methods for analysis of electromagnetic waves diffraction by finite thickness large scatterers

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This paper suggests analyzing finite thickness electrically large scatterers as complete bodies considering currents on both sides of a scatterer. Despite increased computational complexity, it opens new possibilities for building methods of electromagnetic analysis for scatterers which lead to accuracy increase. The best way of ensuring accuracy is to use precise electromagnetic methods based on integral equations (IE) [1]. However, precise electromagnetic analysis of large scatterers is practically impossible due to lack of computational power. The most reasonable way of solving this problem is using hybrid methods [1, 2]. The essence of this approach is to use both computationally expensive precise methods and computationally inexpensive approximate methods for reaching the most profitable combination of accuracy vs. computational complexity.

Based on of approximate method used, hybrid methods are divided into two types: current-based and field-based [2]. In this paper to find the surface current density the current-based methods combining IE and physical optics are used [1, 3]. Traditionally, finite thickness scatterers are being modeled without considering their thickness [1, 3, 4]. Therefore, most current-based hybrid methods are based on electric field integral equation [3, 4]. However, there is the possibility of building hybrid methods based on novel magnetic field integral equation [5, 6]. Each approach has its advantages and disadvantages, as well as its field of application.

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Homogenisation of elastic composite plates in the non-linear bending regime

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We consider a problem of homogenisation (i.e. finding an effective limit description) of a thin elastic periodic composite plate in the bending regime as both parameters — thickness of the plate h and period of the composite microstructure ε — go to zero. The plate in the reference configuration occupies thin domain $\Omega_h := \omega \times [-h/2, h/2]$, where $\omega \subset \mathbb{R}^2$, $h << 1$. Our setting is fully non-linear, the elastic energy of the deformation $u \in H^1(\Omega_h)$ is given by

$$\int_{\Omega_h} W(\varepsilon^{-1}x, \nabla u)dx,$$

where $W(y, \xi)$ is the stored elastic energy function periodic with respect to the in-plane variable $y \in \mathbb{R}^2$. In the non-linear bending regime (which allows displacements of order one, not to be confused with the Föppl – von Karman bending theory) the elastic energy is of order h^3. The rigorous derivation of non-linear bending plate theory (in the homogeneous case) in [1] inspired the resent interest in this area. In particular, in [2] the authors consider the described problem in the regimes $\varepsilon << h$ and $\varepsilon \sim h$. In our work we study the regime when the thickness of the plate is asymptotically smaller that the period of the composite microstructure ($\varepsilon >> h$). The limit homogenised elastic functional (after the appropriate rescaling) in all regimes is given by the formula

$$\int_{\omega} Q_{hom}(\mathrm{II}(x_1, x_2))dx_1 dx_2,$$

defined on the second fundamental form II of (roughly speaking) the mid-surface of the bending deformation u. However, the formula for the quadratic form $Q_{hom}(\mathrm{II})$ is different for different regimes. In particular, in the regime when $h << \varepsilon^2$ we have obtained very interesting and somewhat surprising result implying that the homogenised stored elastic energy function $Q_{hom}(\mathrm{II})$ is a discontinuous function of its argument II.

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Resolvent estimates for high-contrast elliptic problems with periodic coefficients

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I discuss the asymptotic behaviour of the resolvents $(\mathcal{A}^\varepsilon + I)^{-1}$ of elliptic second-order differential operators $\mathcal{A}^\varepsilon$ in $\mathbb{R}^d$ with periodic rapidly oscillating coefficients, as the period ε goes to zero. The class of operators covered by the discussion includes both the "classical" case of uniformly elliptic families (where the ellipticity constant does not depend on ε) and the "double-porosity" case of coefficients that take contrasting values of order one and of order ε^2 in different parts of the period cell. I describe a construction for the leading order term of the "operator asymptotics" of $(\mathcal{A}^\varepsilon + I)^{-1}$ in the sense of operator-norm convergence and prove order $O(\varepsilon)$ remainder estimates. This is joint work with Shane Cooper.

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The study of the asymptotic behavior of scattering coefficients in the modified methods of discrete sources and null field

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The modified methods of discrete sources (MMDS) and null field (MMNF) are among the most efficient tools for solving problems of diffraction and scattering of waves [1]. Let us emphasize again that we are talking about the modifications of these methods, since only these options allow you to create the correct and efficient numerical algorithms. The mentioned modification significantly based on a priori information on the analytical properties of the wave field [1].

The study of the asymptotic behavior (on serial number) of scattering coefficients in the MMDS and MMNF was performed. As is well known, the asymptotics is closely connected with the geometry of the set of singularities of diffraction field analytical continuation in the unphysical area (inside of the scatterer). Calculations were performed using both methods. It is shown that, despite the high accuracy of calculations by both methods (that is confirmed by comparison of their results, checking the optical theorem), the asymptotics of scattering coefficients is correct only when using MMDS, while MMNF gives incorrect results.

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Nonstationary solutions of a generalized Korteweg–de Vries–Burgers equation

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Nonstationary solutions of the Cauchy problem are found for a model equation that includes complicated nonlinearity, dispersion, and dissipation terms and can describe the propagation of nonlinear longitudinal waves in rods. Earlier, within this model, complex behavior of traveling waves has been revealed; it can be regarded as discontinuity structures in solutions of the same equation that ignores dissipation and dispersion. As a result, for standard self-similar problems whose solutions are constructed from a sequence of Riemann waves and shock waves with stationary structure, these solutions become multivalued. The interaction of counterpropagating (or copropagating) nonlinear waves is studied in the case when the corresponding self-similar problems on the collision of discontinuities have a nonunique solution. In addition, situations are considered when the interaction of waves for large times gives rise to asymptotics containing discontinuities with nonstationary periodic oscillating structure.

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The new laws of the Rayleigh scattering

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The new laws of the Rayleigh scattering are obtained. These laws are violation of the Rayleigh law of scattering.

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Connection between violation of the Rayleigh law of scattering and the resonance scattering

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Resonance scattering of the Rayleigh surface acoustic wave on a near-surface inhomogeneity of isotropic solid is considered in details. Inhomogeneity is statistical in a plane parallel to the surface and deterministic in the direction perpendicular to the free surface. Resonance scattering is the regime of scattering when the wavelength of incident wave and the character size of inhomogeneity are of the same order in magnitude, contrary to the Rayleigh scattering, when the wavelength of incident wave is much greater then the character size of inhomogeneity. Resonance scattering is the elastic analogue of the Raman [1, 2] scattering regime when "eigenwavelength" of inhomogeneity, that is its character size, defines the maximum scattering. A strong influence of the Rayleigh law of scattering violation on a form of the resonance scattering spectrum is obtained and investigated theoretically. Violation of resonance law of scattering about maximum scattering in resonance limit, when the wavelength of incident wave and character size of inhomogeneity are of the same order in magnitude, is obtained.

It is obtained that violation of the Rayleigh law of scattering about proportionality of scattering coefficient and of angular distribution of scattering to the fifth power of frequency in the Rayleigh limit gives rise to possibility of the angular distribution of scattering oscillations in dependance on frequency in the Rayleigh limit and to possibility of appearance of angular distribution of scattering arbitrary number of zeroes in angle of scattering in this limit. That is violation of the Rayleigh law about isotropy of angular distribution of scattering in the Rayleigh limit can take place.

These violations of the Rayleigh laws of scattering and of resonance law of scattering obtained in the present work are caused by a strong modulation of scattering by the form of inhomogeneity, in particular by the form of the correlation function of inhomogeneity approximated by the sum of Gaussian exponents. This strong modulation of the Rayleigh and resonance scattering is obtained and investigated in the present work.

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Nonlinear wave interaction processes ruled by 1 + 1 quasilinear hyperbolic systems

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Within the theoretical framework of nonlinear wave interactions recent research interest was focused on developing suitable reduction approaches for quasi-linear hyperbolic systems of first order PDEs in order to determine exact solutions whose behavior along different families of characteristic curves permits an accurate and analytical description of the wave dynamics which underlies the resulting interaction processes. In the latter context a prominent role is certainly played by strictly hyperbolic systems involving two dependent and two independent variables (2×2). As well known these mathematical models can be recast into a form which expresses the evolution of a privileged set of field variables, the Riemann variables, along the related characteristic curves. Moreover in the homogeneous (non dissipative) case the quasi-linear field system can be reduced to linear form through the classical hodograph transformation even though the resulting governing pair of equations cannot be directly integrated in a closed form. Therefore 2×2 homogeneous or nonhomogeneous models represented a prototype for determining classes of governing systems whose canonical structure allows for exact solutions which also result to be appropriate to get a full insight into the interaction process of hyperbolic waves. The leading idea of these methods was to provide exact solutions which inherit the striking wave features of the classical Riemann invariants existing for 2×2 homogeneous systems so that initial value problems which are appropriate to describe wave interactions can be solved in a closed form in a hodograph-like plane. For $(1 + 1)$ strictly hyperbolic systems involving $N > 2$ dependent variables the Riemann variables in general do not exist and in turn the hodograph transformation is no longer valid so that a detailed description of wave interactions in terms of exact or closed form solutions to initial value problems is a hard task. However a remarkable role is played by Hamiltonian homogeneous systems of hydrodynamic type which have been investigated thoroughly in [1]. Actually, these systems under strict structural conditions can be diagonalized in terms of suitable field variables which represent Riemann invariants. Such a class encompasses a number of governing models of relevant interest in engineering applications as chromatography. Moreover via a generalized hodograph method solutions of these systems can be obtained by integrating, in principle, a linear set of equations. Nevertheless within the latter framework wave problems in terms of exact or closed form solutions were not considered, although many evolution phenomena concerning chemical engineering and especially chromatography, in fact, fall into modelling nonlinear wave processes. The aim of this paper is to show that the approach worked out elsewhere [2, 3, 4] for 2×2 homogeneous systems also provides a suitable analytical tool for an accurate description of hyperbolic wave interactions governed by $(1 + 1)$ diagonalizable Hamiltonian homogeneous hyperbolic systems. Even though the study proposed herein can be performed for any of the systems belonging to the class in point, in this paper attention is focussed on a governing model of multicomponent chromatography which has been investigated thoroughly by several authors [5, 6] because of its relevance in fields of chemical engineering application.

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Reflection coefficient of a fractional reflector

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In this presentation, we study the reflection coefficient generated by the surface ($\Sigma \subset \mathbb{R}^3$) of equation $x_1 = 0$, such that the acoustic velocity has a C^α discontinuity (see [2]) on Σ. Such models occur in the study of seismic waves and interfaces between two media [1]. The model studied is given by $c^{-2}(x) = c_0^{-2}(1 + d(x_1)(\max(x_1, 0))^\alpha)$, $0 < \alpha$. The pseudo differential asymptotic parameter is

$$\theta = d(0)c_0^\alpha \omega^{-\alpha}(1-\eta^2)^{-\frac{\alpha+2}{2}}$$

where $\omega > 0$ is the frequency of the wave, $\omega\eta \in \mathbb{R}^2$ is the transverse wave number associated with the incoming wave, $\eta^2 := (\eta_2^2 + \eta_3^2)^{\frac{1}{2}}$, and $\theta \to 0$.

Under the assumption on c:

$$\int_0^{+\infty} |c(x_1)c''(x_1)|dx_1 < +\infty$$

one proves, by expressing exactly the family of outgoing solutions at $+\infty$ (defined by a limiting absorption principle in $\Im\omega < 0$) that the reflection coefficient R satisfies the estimate

$$\forall\alpha > 0, \quad \exists\beta > \alpha, \quad \forall\epsilon_0 > 0, \quad \exists\theta_0 > 0, \quad \forall 0\theta \in]0, \theta_0[, \quad \forall\eta, \quad 1-\eta^2 \geq \epsilon_0, \quad |R - K(\alpha)\theta| \leq M(\alpha)\theta^\beta,$$

where $K(\alpha)$ is an explicit constant. For example, in the case $1 \leq \alpha < 2$

$$1 < \alpha < 2, \quad K(\alpha) = i\int_0^{+\infty} \frac{\alpha(\alpha-1)}{8} X^{\alpha-2} e^{-2iX} dX, \quad K(1) = -\frac{i}{8}.$$

The family of outgoing solutions is calculated through a Volterra equation (which can be linked with Bremmer series [3]) on its Fourier transform in (t, x_2, x_3).

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One Hörmander formula in the Maslov canonical operator and localization of the Berry type solutions in the beam theory

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In some situations semiclassical methods give the (exact) integral representation for special functions and exact solution of some partial differential equations. Using the new integral representation for the Maslov canonical operator [1, 2] we establish this fact for the so-called Airy–Bessel beams, which are the product of the Berry–Balazs solution to 1-D Schrödinger equation [3] and the Bessel function (Bessel beams [4]) and also is the solution to the 3-D free particle Schrödinger equation known in optics as paraxial approximation [5]. The physical defect of this solution is its infinite energy. Using one Hörmander asymptotic formula for fast oscillating integrals and the Maslov canonical operator we show how to localize this solution in space preserving the global structure of the solution based on Airy and Bessel functions. We also show that the same ideas could be used if one wants to take into account the corrections connecting with the passage from original wave-type equation for beams to the Shcrödinger equations.

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Asymptotics of the solution of the Cauchy problem with localized initial data for a wave equation degenerating on the boundary

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Let $\Omega \subset \mathbf{R}^2$ be a domain with smooth boundary $\partial\Omega$. The Cauchy problem for the wave equation

$$\eta_{tt} - \langle \nabla, c^2(x)\nabla\rangle \eta = 0, \quad x \in \Omega, \qquad \eta|_{t=0} = \eta_0, \quad \eta_t|_{t=0} = \eta_1,$$

where $c^2(x) \in C^\infty(\Omega \cup \partial\Omega)$, $c^2(x) > 0$ in Ω, $c^2(x)|_{\partial\Omega} = 0$, and $\nabla(c^2(x))$ vanishes nowhere on $\partial\Omega$, is well posed in the class of functions with finite energy integral $J^2(t) = \|\eta_t\|^2_{L^2(\Omega)} + \|c(x)\nabla\eta\|^2_{L_2(\Omega)}$. Set

$$\eta_0 = V(\mu^{-1}(x - x_0)), \quad x_0 \in \Omega, \quad \eta_1 = 0, \quad \text{where } V(y) \text{ sufficiently rapidly decays as } |y| \to \infty.$$

We seek the asymptotics of the solution as $\mu \to +0$. One main idea going back to [1] and earlier used by the authors in [2] (the 1D case) and [3] (the 2D case with $c^2(x) = x_1$) is that the boundary $\partial\Omega$ can be treated as a caustic of special type. We combine this idea with the geometric constructions in [4] to introduce a generalization of the Maslov canonical operator [5] covering the degenerate problem and obtain efficient formulas for the asymptotic solutions. The results are partly published in [6]. In particular, if we use this problem to model tsunami run-up on the shore [7] in the linear approximation, so that $\eta(x,t)$ is the free surface elevation, $\partial\Omega$ is the shoreline, $c(x) = \sqrt{gD(x)}$, $D(x)$ is the depth at x, g is the acceleration due to gravity, and the parameter μ characterizes the relative size of the tsunami source, then, for the special elliptical source $V(y) = A(1+(y_1/b_1)^2+(y_2/b_2)^2)^{-3/2}$, our asymptotic formulas give the maximum wave amplitude $A_{max}(x)$ at $x \in \partial\Omega$ in the form

$$A_{max}(x) = \kappa A \frac{[2D(x_0)]^{1/2}}{[\tan\Theta(x)|X_\psi(\psi_*,\tau_*)|]^{1/2}} \frac{e}{\cos^2\psi_* + e^2\sin^2\psi_*}, \qquad x \in \partial\Omega,$$

where $X(\psi,\tau)$ is the x-component of the trajectory of the Hamiltonian system constructed in [4] with initial data $x|_{\tau=0} = x_0$, $p|_{\tau=0} = (\cos\psi, \sin\psi)$, (ψ_*, τ_*) is the solution of the system $X(\psi,\tau) = x$, $\tan\Theta(x)$ is the slope of the bottom at x, $e = b_1/b_2$ is the eccentricity of the source, and $\kappa = 1$ or $3\sqrt{3}/8$ depending on the Maslov index [5] of the trajectory $X(\psi_*,\tau)$, $\tau \in [0,\tau_*]$. (If the system $X(\psi,\tau) = X$ has several solutions, then the formula is modified appropriately, and if $X_\psi(\psi_*,\tau_*) = 0$ for at least one of these solutions, then a completely different formula should be used instead.)

Acknowledgments. The research was supported by RFBR grants 11-01-00973a and 14-01-00521a and by the RITMARE–CINFAI contract.

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Asymptotic solution of linearized shallow water equations on the sphere with localized initial data

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We consider linearized shallow water equations on the sphere. For velocity vector (u,v) and elevation of free surface η we have system of equations:

$$\frac{\partial u}{\partial t} - 2\Omega\sin\varphi\ v + \frac{g}{R_0\cos\varphi}\frac{\partial\eta}{\partial\lambda} = 0,$$

$$\frac{\partial v}{\partial t} + 2\Omega\sin\varphi\ u + \frac{g}{R_0}\frac{\partial\eta}{\partial\varphi} = 0,$$

$$\frac{\partial\eta}{\partial t} + \frac{1}{R_0\cos\varphi}\left(\frac{\partial(uH)}{\partial\lambda} + \frac{\partial(vH\cos\varphi)}{\partial\varphi}\right) = 0.$$

Here (λ, φ) are spherical coordinates (longitude and latitude), $2\Omega \sin\varphi$ is Coriolis parameter, H is depth. Besides, we have initial conditions

$$u|_{t=0} = v|_{t=0} = 0, \quad \eta|_{t=0} = f\left(\frac{r}{\mu}\right),$$

where r is distance to fixed point x_0, μ is small parameter.

Using the metod of Maslov canonical operator we can write the main term in asymptotic expansion of η by μ (see [1]). The influence of angle speed Ω will be demonstrated in examples.

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Self-action dynamics of single-cycle optical pulses

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The recent progress in the area of ultrafast optics has opened a door to the generation of radiation with durations down to a single optical cycles [1]. The study of propagation of intense ultrashort pulses led to the investigation of novel effects in nonlinear optics. Here we report on a theoretical analysis of spatio-temporal pulse dynamics in an isotropic dielectric medium with instant Kerr-type nonlinear response and normal group dispersion. We reveal a new feature of suppression of third-harmonic generation for single-cycle optical pulses.

We first investigate the harmonic generation depending on the pulse duration by considering a case when the effects of dispersion and diffraction can be neglected. Our results are based on approximate analytical solution of field equation [2] and summarized in Fig. 1. The nonlinearly-induced corrections to the pulse profile and spectrum are shown in Figs. 1(a) and (b), respectively. We show the frequency where the correction completely vanishes with white line in Fig. 1(b). For the normalized on central period T_0 pulse duration $\tau_p/T_0 \cong 0.3$, when the input pulse is single-cycle, the third-harmonic generation completely vanishes.

Then we analyze the self-action features of axisymmetric paraxial single-cycle optical pulses under the combined effects of cubic nonlinearity, temporal dispersion and spatial diffraction. It is shown that the minimum of spectral density at triple temporal frequencies is formed only at low spatial frequencies [Fig. 2]. At higher spatial frequencies the maximum of the spectral density can shift to quadruple temporal frequencies.

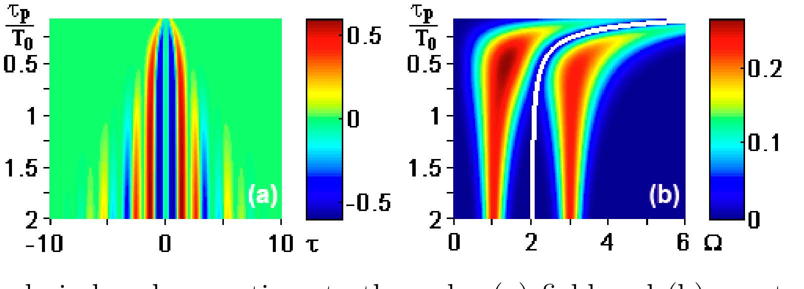

Fig. 1: Nonlinearly-induced corrections to the pulse (a) field and (b) spectrum vs. the normalized duration. τ, Ω are normalized time and frequency, respectively.

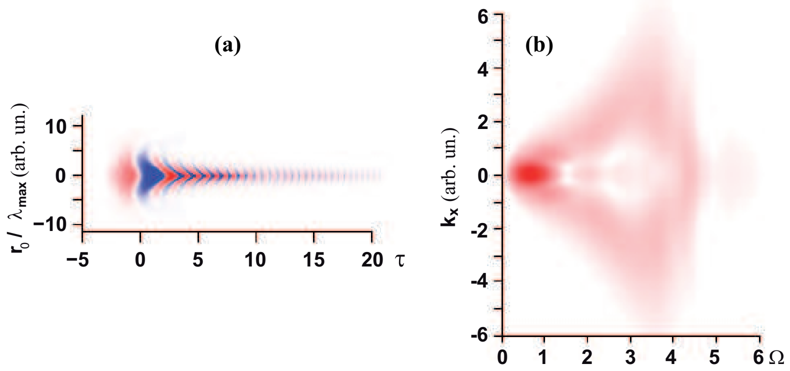

Fig. 2: Spatio-temporal distribution of (a) the electric field at the output of nonlinear medium and (b) the corresponding spatio-temporal spectrum. k_x is the spatial frequency.

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On uniformity of the field inside small scatterers

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Light scattering by particles small in comparison with the wavelength of the incident radiation is usually considered in the Rayleigh approximation. This approximation implies solution of the corresponding electrostatic problem. An exact solution of such problems is possible only when the particle surface coincides with a coordinate hypersurface. Then the separation of variables can be applied, and the solution can be written explicitly. It can be done for spheres, prolate and oblate spheroids, and ellipsoids. In all the cases the field inside the particles is uniform. A question arises whether other particles may have the uniform internal field in the electrostatic limit.

We consider the electrostatic problem for a homogeneous axisymmetric particle. The approach applied is similar to the extended boundary condition method often used in the light scattering theory. The surface integrals forming the elements of an analog of the well known T-matrix are studied in detail. The inverse problem is solved under the condition that the internal field is uniform. It is proved that this condition is satisfied for axisymmetric particles only if they are spheroids. Approximate solutions to the electrostatic problem that are based on the assumption of the uniform internal field are discussed as well.

Marryland equation, renormalization formulas and mimimal meromorphic solutions to difference equations

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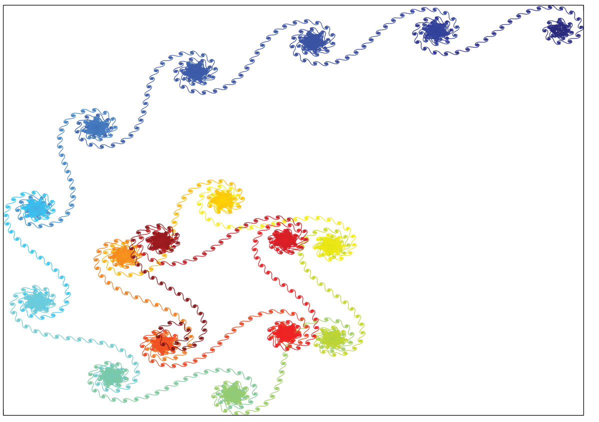
Consider the difference Schrödinger equation

$$\psi_{k+1} + \psi_{k-1} + \lambda \cot(\pi\omega k + \theta)\psi_k = E\psi_k, \quad k \in \mathbb{Z},$$

where λ, ω, θ and E are parameters. If ω is irrational, this equation is quasi-periodic. It was introduced by specialists in solid state physics from Marryland and is now called the Marryland equation. Computer calculations show that, for large k, its eigenfunctions have a multiscale, "mutltifractal" structure. We obtained renormalization formulas that express the solutions to the input Marryland equation for large k in terms of solutions to the Marryland equation with new parameters for bounded k. The proof is based on the theory of meromorphic solutions of difference equations on the complex plane, and on ideas of the monodromization method of V.S. Buslaev and A.A. Fedotov.



Our formulas are close to the renormalization formulas from the theory of the Gaussian expoential sums $S(N) = \sum_{n=0}^{N} e^{2\pi i(\omega n^2+\theta n)}$, where ω and θ are parametrs. For large N, these sums also have a multiscale behavior: in the Figure, we plot values $S(N)$, $N = 1, 2, 3, \ldots$, in the complex plane. The renormalization formulas lead to a natural explanation of the mutiscale structure that reflects certain quasi-classical asymptotic effects (Fedotov–Klopp, 2012).

Destruction of adiabatic normal waves for an adiabatic non-stationary Schrödinger operator

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We consider a one-dimensional non-stationary Schrödinger equation

$$i\frac{\partial\psi}{\partial t} = -\frac{\partial^2\psi}{\partial x^2} + v(x,\varepsilon t)\psi \tag{1}$$

on the half-line $x \geq 0$ with the Dirichlet condition at $x = 0$. In this equation, ε is a small positive parameter, and thus, the potential v slowly depends on the time t. A well-known problem is to get the asymptotic description of the solutions of Cauchy problems for (1) on the times intervals of the length of the order of $1/\varepsilon$.

The results strongly depend on spectral properties of the stationary Schrödinger operator $H(\varepsilon t) = -\frac{\partial^2}{\partial x^2} + v(x,\varepsilon t)$ where t is a parameter. One of the traditional directions of investigation is the analysis of the case when, for all τ, the spectrum of $H(\tau)$ is located on two fixed non-intersecting intervals. There are many papers devoted to the case where, for all τ, the spectrum of H is discrete; one then constructs the adiabatic normal waves, i.e., solutions having the asymptotics

$$U_n(x,\varepsilon t) \sim e^{\frac{i}{\varepsilon}\int_0^{\varepsilon t} E_n(\tau)\,d\tau} \sum_{k=0}^{\infty} \varepsilon^k \psi_{n,k}(x,\varepsilon t),$$

where $\psi_{n,k}$ are functions decreasing as $x \to \infty$, and $E_n(\tau)$ and $\psi_{n,0}(\cdot,\tau)$ are the nth eigenvalue and the nth eigenfunction of $H(\tau)$. There are several papers, e.g., by V. Buslaev and C. Sulem, devoted to the case where the spectrum of $H(\tau)$ is purely absolutely continuous.

We assume that $H(\tau)$ has a finite number of, say, negative eigenvalues and the absolutely continuous spectrum filling the positive half-line. In the talk, we concentrate on a model problem: we assume that v is the piecewise constant function defined by the formulas

$$v(x,\tau) = \begin{cases} -1 & \text{if } 0 \leq x \leq 1-\varepsilon t, \\ 0 & \text{otherwise.} \end{cases}$$

If τ increases, the (negative) eigenvalues of $H(\tau)$ move toward zero, i.e., the beginning of the absolutely continuous spectrum, and, arriving to zero, disappear one after another.

For small ε, we construct the adiabatic normal wave U_n and describe its destruction, i.e., the crucial changes in its asymptotic behavior that happen when the corresponding eigenvalue of $H(\varepsilon t)$ approaches the continuous spectrum and disappears.

After the destruction of the standard asymptotics of U_n, on the interval where the potential is non-zero, U_n becomes small. We call the corresponding domain in the (x, t)-plane the shadow region. One of the important effects is that, in the shadow region, there is a sequence of "lightened" zones. They exists near to the moments of "birth" of the $(n-1)$st, $(n-2)$th etc resonances of the operator $H(\varepsilon t)$, which are the moments of "death" of its $(n-1)$st, $(n-2)$th etc eigenvalues.

The method we use to solve the problem is close to the method of Sommerfeld and Malyuzhinets; one has to analyze on the complex plane difference equations with the translation parameters equal to the small ε.

The energy aspects of abnormal wave propagation in the cylinder shell submerged into the liquid

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The problem of oscillations of elastic constructions submerged into the water is one of the actual problems of modern techniques. Pipe lines, different supports of the hydro technical constructions can be modeled by cylinder shell. The report is devoted to energy aspects of propagation of the waves in such system. The free oscillations problem is considered in the rigorous mathematical statement. Abnormal waves (the group and faze velocities has the opposite signs) attract the special interest. The energy flux and its components are investigated.

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Novel representation of solutions of the Heun equation

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We propose a new series expansion of the solutions of the Heun differential equation and study their coefficients and convergence.

Dielectric concentrators for Cherenkov radiation

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Cherenkov radiation (CR) is widely applied in various areas of particle physics, including detection of charged particles [1], wakefield acceleration [2] etc. In recent years, possibilities of using this effect for bunch diagnostics are actively discussed [3]. However, in mentioned cases the complexity of radiator geometry usually does not allow constructing the rigorous solution for the field of radiation. Therefore, different approximate methods are elaborated for investigation of excited radiation [3–6]. Here we investigate the case of CR generated by a charge moving near a dielectric target with complex shape. We use the approximate method that allows calculating CR from the target with two main boundaries [6], where the first boundary (inner) interacts with a field of flying charge and the second one (outer) refracts the radiation generated inside the target. This technique has two steps. The first step implies the rigorous solution of the "etalon" problem, which is the problem without the outer boundary. At the second step, the interaction of the field with the outer boundary is taken into account in the frame of ray optics.

In this report, we present the cases where the radiation outside the target is convergent, i.e. this target can be called concentrator for CR. Since ray optics fails near focal points, we also involve the aperture integration technique [7] for the field investigation.

First, we deal with the case of a conical target. Under certain conditions, this radiation is concentrated near the symmetry axis of the target. Second, we present the specific shape of the target that concentrates CR in a small vicinity of given point (focus). Note that in both cases we suppose that targets have a cylindrical channel inside them where the charge moves. For such geometry of the inner boundary, the rigorous solution of the "etalon" problem is known [8]. We give typical field plots and energy distribution in the area surrounding the targets. Mentioned effects can be used for improvement of detectors and bunch diagnostics systems based on Cherenkov effect.

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Temporal soliton as a rotation of the Wigner function in the phase space

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The Wigner distribution function (WDF) allows representing any optical pulse as a function in the phase space "coordinate-spatial frequency". In a quadratic approximation, the propagation the pulse WDF through an optical medium with dispersion and nonlinearity can be described by using a matrix form [1]. At a certain combination of nonlinear and dispersive effects, the resulting matrix takes the form of the rotation matrix in the phase space with dimensionless coordinates [2, 3]. In this case, propagation of the laser pulse through this optical medium is accompanied by the rotation of its WDF in the phase space. From point of view of an outside observer, the rotation of the WDF looks like a pulse shape oscillates with the period of rotation in real space. If the form of the WDF in the phase is close to a circle, then the observer get impression that the pulse shape preserves. The pulse preserving its shape is called a temporal optical soliton [4].

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The analysis of scattering properties of the polished optical surfaces in view of multiscale of microtopography

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Questions of the description of optical surfaces microrelief characteristics play a key role in definition of properties of radiation scattering by optical elements. In turn, definition of quality of optical elements is an actual problem of optical instrument making.

By present time methods of contact measurements of surface structure and methods of tunnel microscopy are well developed. However these methods can lead to damage of the polished surfaces. That causes necessity of development of a contactless quality monitoring, for example, on the basis of the analysis of characteristics of radiation scattering.

Theoretical research of reflection of electromagnetic radiation by a rough surface was undertaken repeatedly. One of the method of monitoring of quality of the surface, based on the analysis of interrelation of characteristics of radiation scattering and statistical properties of a microrelief of a surface is offered and checked experimentally up in [1].

In the present work the method of the description of angular distribution of scatterer radiation depending on a spectrum of spatial frequencies of a microrelief of a surface is considered. This approach is perspectively in cases when the surface is characterized by several scales of microroughnesses, in particular, when the surface is described by fractal structure. As shown in [2], polished surfaces of metal mirrors can possess fractal properties.

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Source energy distribution and successive forwarding in layered and functionally graded elastic substructures

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Functionally graded materials (FGMs) are advanced composites consisting of two or more material phases with gradual transverse variation of composition, e.g., materials with strengthening gradient coatings. There exists an increasing need in reliable nondestructive wave methods for their control, which assumes a comprehensive study of wave propagation in such materials.

The present talk is a continuation of our previous research of dispersion properties and amplitudes of guided waves generated by a surface source in elastic half-spaces covered by various functionally graded coatings [1]. Now it is focused on the wave energy distribution in layered and FGM half-spaces and plates with soft inner channels. Guided waves propagating in such structures exhibit specific effects of wave energy localization observed at certain frequencies.

The analysis is performed using the integral approach, which is based on the Fourier transform technique and Green's matrix integral representations. The guided waves are obtained in terms of residues, while the body waves are described by the contribution of stationary points of the oscillate integrals derived. The time-averaged wave energy fluxes are calculated based on the Umov–Poynting energy vector. The energy balance in the source-substructure system as a whole is controlled via the representations for the source energy and for the amount of energy transferred by the guided and body waves through the side cylindrical surfaces and the horizontal planes in the solid volume considered. The energy streamlines and energy density plots visualize the trajectories of energy fluxes as well.

We will discuss the distribution of the source energy among the excited normal modes as well as the depth and frequency dependencies for each mode. Among them, the effect of successive forwarding of the main part of source energy to the every next appearing mode is of prime interest.

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Electric dipole antenna over a Fabry–Perot meshed parallel-plate resonator

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A diffraction problem of a short electric dipole antenna over a Fabry–Perot meshed parallel-plate resonator is solved. The resonator consists of two separated periodic lattices of thin metal wires, where inter-wire spacing is assumed to be much smaller than wavelength. Exact Image Theory approach [1] and Kontorovich Averaged Boundary Conditions [2] are employed in order to obtain a full-wave field solution in terms of averaged fields and surface currents. Two cases are considered: horizontal electric dipole (HED) over two lattices of parallel wires (Fig. 1a) and vertical electric dipole (VED) over two square-cell lattices of connected wires (Fig. 1b). Near- and far-field patterns depending on frequency are presented. Far-field approximations are given and spatial filtering performed by the considered structures is discussed.

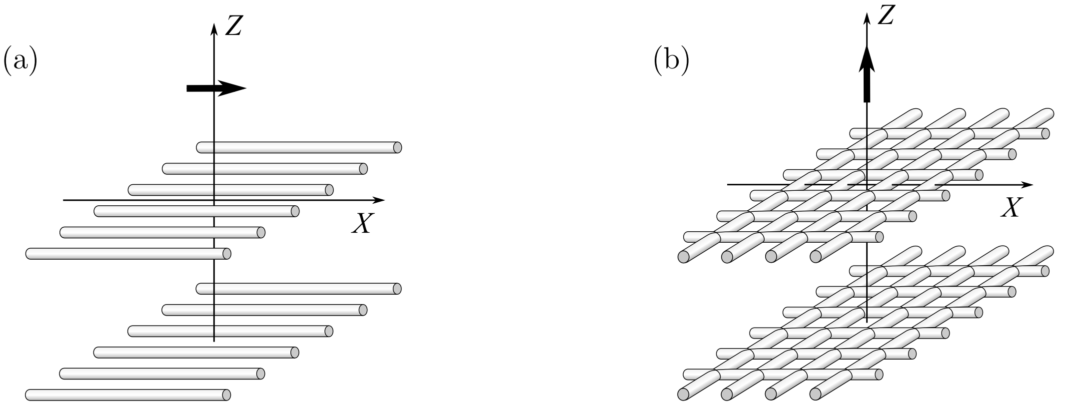

Fig. 1: Considered geometries of the problem: a — HED over parallel-wire lattices; b — VED over square-cell lattices

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Ray theory for acoustic-gravity waves in the atmosphere

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The ray and WKB approximations have long been important tools of understanding and modeling propagation of atmospheric waves. However, contradictory claims regarding applicability and uniqueness of the approximations persist in the literature. Here, we consider linear acoustic-gravity waves in a three-dimensionally inhomogeneous, moving atmosphere with time-independent parameters. The ray theory is understood to be an asymptotic technique to calculate wave fields in the atmosphere, where temperature, composition, and background wind velocity vary gradually on the scale of the wavelength. We derive the ray theory systematically from the first principles and compare the results to the eikonal, transport, and polarization equations proposed earlier for acoustic-gravity waves in horizontally stratified or three-dimensionally inhomogeneous media. Properties of low-order ray approximations are discussed in some detail. Contrary to better studied cases of acoustic waves and internal gravity waves in the Boussinesq approximation, the first-order ray solution is found to contain the geometric, or Berry, phase. The Berry phase is generally non-negligible for acoustic-gravity waves in a moving atmosphere. Put differently, knowledge of the dispersion relation is not sufficient for calculation of the wave phase in the ray approximation.

Wave energy dissipation through irreversible thermodynamic processes is a major factor influencing propagation of acoustic and gravity waves in the Earth's atmosphere. Accurate modeling of the wave dissipation is important in a wide range of problems from understanding the momentum and energy transport by waves into the upper atmosphere to predicting long-range propagation of infrasound to the acoustic remote sensing of mesospheric and thermospheric winds. Variations with height of the mass density, kinematic viscosity, and other physical parameters of the atmosphere have a profound effect on the wave dissipation and its frequency dependence. To characterize the wave dissipation, it is typical to consider an idealized environment, which admits plane-wave solutions. Here, we use an asymptotic approach to derivation of dispersion equations of acoustic-gravity waves in horizontally stratified dissipative fluids. The approach does not presuppose existence of any plane-wave solutions and relies instead on the assumption that spatial variations of environmental parameters are gradual. Linearized hydrodynamic equations for compressible fluids in a gravity field are solved asymptotically, leading to a self-consistent version of the WKB approximation for acoustic-gravity waves. Dissipative processes are found to affect both the eikonal and the geometric (Berry) phase of the wave. Newly found expressions for acoustic-gravity wave attenuation due to viscosity and thermal conductivity of the air are compared to results previously reported in the literature. Effects of the wind on the wave dissipation prove to be significant. Knowledge of the wind velocity profile is essential for accurate modeling of dissipation of the atmospheric waves.

Rayleigh scattering of spherical sound waves by spherically symmetric bodies

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Acoustic Green's functions for a homogeneous medium with an embedded spherical obstacle arise in analyses of scattering by objects on or near an interface, radiation by finite sources, sound attenuation in and scattering from clouds of suspended particles, etc. An exact solution of the problem of diffraction of a monochromatic spherical sound wave on a sphere is given by an infinite series involving products of Bessel functions and Legendre polynomials. This paper presents a simple, closed-form solution for diffraction of a spherical wave on a sphere with a radius that is small compared to the wavelength. Soft, hard, impedance, and homogeneous fluid and solid obstacles are considered. The solution is valid for arbitrary positions of the source and receiver relative to the scatterer. Low-frequency scattering is shown to be rather sensitive to boundary conditions on the surface of the obstacle. Low-frequency asymptotics of the scattered acoustic field are extended to transient incident waves. The asymptotic expansions admit an intuitive interpretation in terms of image sources and reduce to classical results in appropriate limiting cases.

An asymptotic technique is developed to extend the results to spherically symmetric inhomogeneous obstacles. The obstacle can be fluid, solid, or a fluid-filled solid shell. Physical properties of the obstacle are arbitrary piece-wise continuous functions of the distance to its center. The radius of the obstacle is assumed to be small compared to the wavelengths of sound in the surrounding fluid as well as of compressional and shear waves inside the obstacle. General properties of the sound scattering by spherically symmetric bodies are established. Resonant Rayleigh scattering is studied in detail. For plane and spherical incident waves, it is discussed which physical and geometrical parameters of the obstacle can be retrieved from the scattered acoustic field.

Simulation of plane 3D wave propagation in layered piezoelectric phononic crystals

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Composite materials with periodic internal structure become widely used in various fields [1]. In such structures, which are called phononic crystals, can be observed an effect of complete reflection of signal in certain frequency ranges (band-gaps). This effect allows to use periodic structures as filters, acoustic insulations for the vibrating structures, mechanical resonators, sensors etc. In addition to band-gap formation, phononic crystals can reveal phenomena of wave localization, "bending" of waves, negative refraction, etc.

Plane wave propagation in layered piezoelectric phononic crystals is considered, where composite structure consists of repeating layers. The wavefields are calculated in terms of the transfer matrix method [2], which takes into account the periodicity of the structure and provides a sustainable solution through the exploitation of the transfer matrix in Jordan form. Such representation based on eigenvalues of transfer matrix leads to numerical stability and to simple estimation of wave energy transmission and reflection coefficients for analysis of filtration properties of the piezoelectric phononic crystal [3]. The results of parametric studies of elastic wave propagation in layered structures are presented, the effects of material and geometrical properties, angle of incidence, frequency are considered.

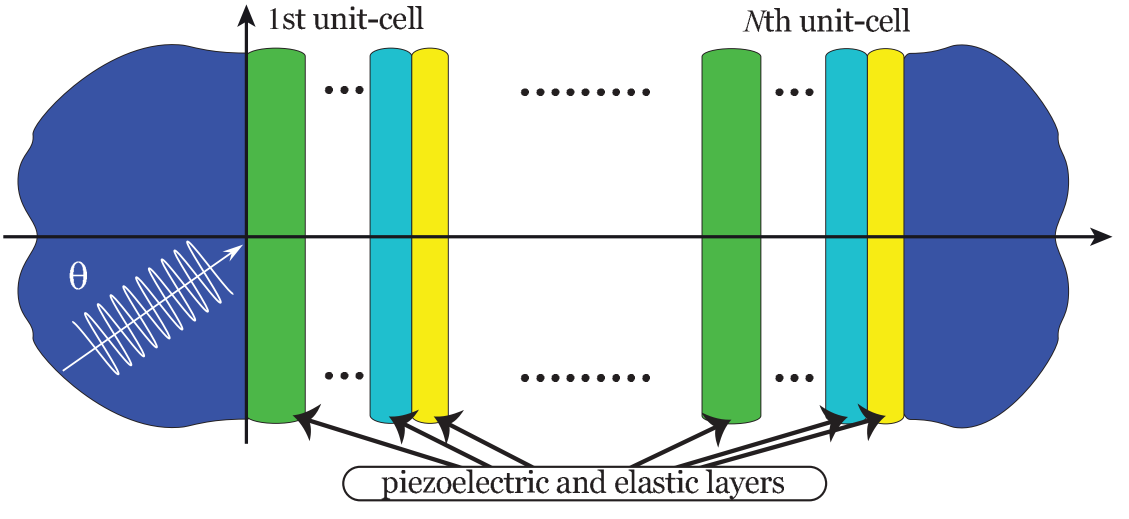

Fig. 1: Geometry of the problem considered.

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Boundary conditions effects on electronic states in quantum-well – nanobridge – quantum dot structures

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We consider a varied-dimension InGaAs/GaAs structure of quantum well – nanobridge – quantum dot (QW-NB-QD) embedded in a "virtual" cylinder Ω to obtain a realistic 3D electronic model. The radius R and the height H of the cylinder are free parameters in our model. We assume the QD as a truncated cone of height h_{QD}, with a (lower) base radius r_{QD} and the radius of the upper base equals the radius of the nanobridge. The NB is considered as a cylinder with diameter D and height h. Eigenstates are computed for electrons and holes of the whole QW-NB-QD system using the finite volume method and effective mass approximation for the single-particle. The strain is neglected, but it is considered that the NB can also confine states within it. Within our formalism we can consider a linearly graded In concentration in the cylindrical NB. The electronic states and the hole states are computed separately, as solutions of the Schrödinger-type equation for the envelope function $\Psi(r)$ [1]:

$$\left[-\frac{\hbar^2}{2}\nabla\cdot\left(\boldsymbol{M}(\mathbf{r})^{-1}\nabla\right)+V(\mathbf{r})\right]\Psi(\mathbf{r})=E\Psi(\mathbf{r}),\quad \mathbf{r}\in\Omega,$$

where $M(r)$ is the effective mass tensor and $V(r)$ contains the (conduction and valence) band offsets of the heterostructure materials. In order to describe the optical processes (i.e. absorption and emission of light) in the complex structure we also compute optical matrix elements as momentum matrix elements.

Dirichlet, or Neumann, or mixed boundary conditions can be applied on the surface of Ω. It was shown that hybrid states appear in this complex system under the Dirichlet boundary conditions ("hard wall" confinement) at defined small values of h for certain values of photon energies [2]. The interaction between the eigenvalues may be an explanation for the additional photoluminescence peak measured for inverted structures with smaller nanobridge heights [3]. Such hybrid electronic states which exist in the combined system for a specified range of values of h are not localized in the subsystems, i.e. QW, QD or NB. Also they have different optical oscillator strengths. However, for the Neumann boundary conditions which imposed on the end of the cylinder there are no such hybridizations for large values of R. We discuss whether the results obtained in the previous calculations with the Dirichlet boundary conditions are more realistic for such complex systems and single-particle approximation used or they are physical artifacts.

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Nonlinear acoustic wave propagation in the waveguide formed by the bottom bubble layer

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The formation of the bottom acoustic waveguide in the presence of the liquid layer containing suspended gas bubbles is investigated. Evolutionary equations describing the propagation of intense sound in the bubble layer at oblique incidence on its border are set up. It is shown that the presence of bubbles increases as the interval of angles for that the incident wave passes in the bubble layer and the interval of angles, for which total internal reflection occurs when propagating inside the layer, i.e. the bubble layer effectively captures acoustic waves and creates conditions for their waveguide propagation.

Numerical studies of the scattering of light from, and its transmission through, two-dimensional randomly rough surfaces

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Calculations of the scattering of light from, and its transmission through, two-dimensional randomly rough surfaces are difficult, and are still often carried out by means of small-amplitude perturbation theory, the Kirchhoff approximation, or the small slope approximation. However, accurate approaches to the solution of such problems are needed in a variety of contexts, and searches for such approaches are an active area of research in computational electrodynamics. Purely numerical solutions of the integral equations of scattering theory represent such an approach. In this talk we describe some of our recent work in which this approach is applied to produce nonperturbative, purely numerical, solutions to such scattering and transmission problems.

We first present solutions of the Rayleigh equations for the scattering of p- and s- polarized light from a two-dimensional randomly rough perfectly conducting surface. The solutions are used to calculate the reflectivity of the surface, the mean differential reflection coefficient, and the full angular distribution of the intensity of the scattered light. It is found that our results satisfy unitarity with an error no larger than 10^{-5} for surfaces whose root-mean — square slope is smaller than 0.28. These results are compared with the results of our earlier rigorous numerical solutions of the corresponding Stratton–Chu equations [Phys. Rev. A **81**, 013806(1-13)(2010)], and very good agreement is found. Through such comparisons insight into the limits of validity of the Rayleigh hypothesis can be gained.

The transmission of polarized light through a two-dimensional randomly rough interface between two dielectric media has been much less studied, by any approach, than the scattering of light from such an interface. We have solved numerically the reduced Rayleigh equations for the transmission of p- and s- polarized light through, and its reflection from, such an interface. The solutions are used to calculate the transmissivity of the interface, the mean differential transmission coefficient, and the full angular distribution of the intensity of the transmitted light. On adding the contribution from the scattered field to that from the transmitted field, it is found that the results of these calculations satisfy unitarity with an error smaller than 10^{-4}.

The results we have obtained can be used as benchmarks against which the results of approximate, perturbative or numerical, calculations of the scattering of polarized light from, and its transmission through, two-dimensional randomly rough surfaces can be compared.

We conclude this talk with a discussion of several directions future research in this field can take.

Stability of nonstationary Navier–Stokes flow and algebraic energy decay

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Let $V = V(x,t)$ be a given time-dependent Navier–Stokes flow of an incompressible fluid in the whole space $\mathbb{R}^3$. As important examples of this basic flow V, we have the following in mind: Self-similar solution, time-periodic solution and global mild (eventually strong) solution of the Cauchy problem. It is thus reasonable to assume that $V \in L^\infty(\mathbb{R}; L^{3,\infty})$, where $L^{3,\infty} = L^{3,\infty}(\mathbb{R}^3)$ denotes the weak-L^3 space. This presentation addresses the stability of V and provides specific rates of decay of disturbance (as weak solution) in L^2 as $t \to \infty$. The global stability of small V in the class above with respect to any initial disturbance in L^2_σ has been recently investigated by Karch, Pilarczyk and

Schonbek [1]. It would be interesting to find how fast the disturbance decays as $t \to \infty$ when the initial disturbance possesses better summability at space infinity. Concerning other results on the related issue, we refer to Kozono [2] and the references cited there, in which the global stability of large weak solution $V \in L^q(0,\infty; L^r)$ was established, where $2/q + 3/r = 1$ and $3 < r \leq \infty$ (the Serrin class), however, this class covers neither self-similar solution nor time-periodic solution.

When $V = 0$, the energy decay problem was raised in his celebrated paper by Leray and, fifty years later, it was solved by Kato and by Masuda, independently. The related studies, such as optimal rate of decay, have been well developed by several mathematicians. There are also some literature on the case where V is a stationary solution in $L^{3,\infty}$, while we know less when V depends on time variable. In this presentation it is shown that if the time-dependent Navier–Stokes flow V is small in $L^\infty(\mathbb{R}; L^{3,\infty})$ and if the initial disturbance is taken from L^1 as well as L^2_σ, then any weak solution satisfying the strong energy inequality to the perturbed Navier–Stokes system, which the disturbance should obey, decays like $t^{-3/4}$ in L^2 as $t \to \infty$. A weak solution with the strong energy inequality has been actually constructed in [1]. The essential step for the proof of the decay property is to deduce L^q-L^r estimates of the evolution operator generated by the linearized operator around the basic flow V.

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Fifteen classes of solutions of the quantum two-state problem in terms of the confluent Heun function

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We derive 15 classes of time-dependent two-state models solvable in terms of the confluent Heun functions. These classes extend over all the known families of 3- and 2-parametric models solvable in terms of the Gauss hypergeometric and the Kummer confluent hypergeometric functions to more general four-parametric classes involving three-parametric detuning modulation functions. The classes suggest a variety of families of field configurations possessing useful properties not covered by the previously known analytic models. In the case of constant detuning the field configurations defined by the derived classes describe excitations of two-state quantum systems by symmetric or asymmetric pulses of controllable width and edge-steepness. The particular classes out of the derived fifteen that provide constant detuning pulses of finite area are identified and the factors controlling the corresponding pulse shapes are discussed in detail. The positions of the pulse edges for the case of step-wise edges are determined. We show that the asymmetry and the peak heights are mostly defined by two of the three parameters of the detuning modulation function, while the pulse width is mainly controlled by the third one, the constant term in the detuning modulation function. It is shown that the pulse width diverges as this parameter goes to infinity. Furthermore, it is shown that rectangular box pulses, as well as infinitely narrow pulses are possible, and the conditions for these to be achieved are obtained. Several examples of such field configurations are mentioned and their basic properties are discussed.

Analyzing the physical field configurations for the general case of variable Rabi frequency and frequency detuning, it is mentioned that the most notable features of the models provided by the derived classes are due to the extra constant term in the detuning modulation function. Due to this term the classes suggest numerous symmetric or asymmetric chirped pulses and a variety of models with two crossings of the frequency resonance. The latter models are generated by both real and complex transformations of the independent variable. In general, the resulting detuning functions are asymmetric, the asymmetry being controlled by the parameters of the detuning modulation function. For some classes, however, the asymmetry may be additionally caused by the amplitude modulation function. We present an example of the latter possibility and additionally mention a constant amplitude model with periodically repeated resonance-crossings. Finally, we discuss the excitation of a two-level atom by a pulse of Lorentzian shape with a detuning providing one or two crossings of the resonance. Using a series expansion of the solution of the confluent Heun equation in terms of the Kummer hypergeometric functions we derive particular closed form solutions of the two-state problem for this field configuration both for the constant and variable detuning cases. The particular sets of the involved parameters for which these solutions are obtained define curves in the 3D space of the involved parameters belonging to the complete return spectrum of the considered two-state quantum system.

Second order finite volume scheme on structured meshes for Maxwell's equations with discontinuous dielectric permittivity

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Developing second order schemes for Maxwell's equations with discontinuous dielectric permittivity is a challenging problem. One such scheme was developed for unstructured meshes in [1, 2]. For many problems such as photonic crystal simulation using unstructured meshes is not necessary. Here we extend finite volume scheme from [1, 2] to structured meshes. The scheme employs a technique for gradient calculation and two alternative techniques for gradient limitation near dielectric permittivity discontinuity. Scheme was tested for problems with linear and curvilinear discontinuities for TE and TM waves. Test results support second order of approximation in space and time.

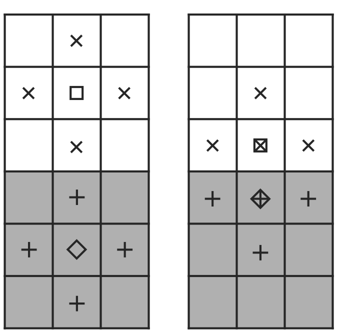

Fig. 1: Gradient calculation

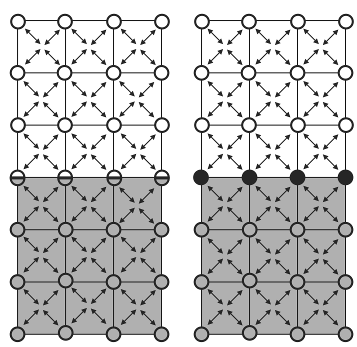

Fig. 2: Gradient limitation

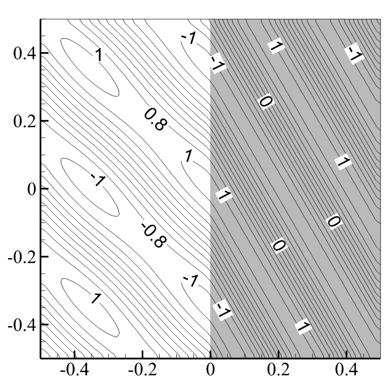

Fig. 3: Curvilinear discontinuity

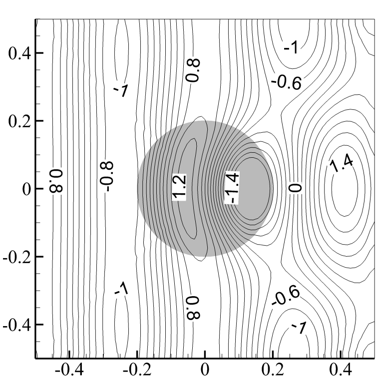

Fig. 4: Linear discontinuity

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An inverse eigenvalue problem of the theory of optical waveguides

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Inverse eigenvalue problems arise in a remarkable variety of applications, including system and control theory, geophysics, molecular spectroscopy, particle physics, structure analysis, and so on. An inverse eigenvalue problem concerns the reconstruction of a physical system from prescribed spectral data. The spectral data involved may consist of the complete or only partial information of eigenvalues or eigenvectors.

We present a new method for calculation of permittivity of dielectric materials using optical fiber's propagation constant measurements. This problem is an inverse eigenvalue problem of the optical waveguide theory. We obtain a mathematical model of eigenmodes of a weakly-guiding step-index arbitrarily shaped optical waveguide. By methods of the integral equations theory we prove that it is enough to measure the propagation constant of the fundamental eigenmode only at one frequency for the reconstruction of unknown dielectric constant of this waveguide. We present a new numerical algorithm for the calculation of the dielectric constant based on approximate solution of a nonlinear nonselfadjoint inverse eigenvalue problem for a system of weakly singular integral equations. The convergence and quality of this numerical method we prove by numerical experiments.

Monodromy of Heun equations with apparent singularities

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Heun and confluent Heun equations when one regular singularity is apparent singularity are under consideration. Corresponding connection matrices are calculated in explicit form.

The Dunkl–Darboux differential-difference operators and integrability

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Let $\mathfrak{F}$ be a space of the differentiable functions on $\mathbb{R}$; $\hat{s}$ be a reflection operator, so that $\hat{s}[f](x) = f(-x)$, $f(x) \in \mathfrak{F}$, and $\omega(x)$, $x \in \mathbb{R}$, be a even analytical function in an open domain $D_\omega \in \mathbb{R}$.

We consider the Dunkl–Darboux differential-difference operators in $\mathbb{R}$

$$\widehat{\nabla}_\omega = \frac{d}{dx} - (\ln|\omega(x)|)'\hat{s}. \tag{1}$$

Here the prime is a derivative of the function.

If $\omega(x) = |x|^k,\ k \in \mathbb{Z}_{\geqslant 0}$, then the operators (1) are the classical Dunkl operators [1]

$$\widehat{\nabla}_{|x|^k} \equiv \nabla_k = \frac{d}{dx} - \frac{k}{x}\,\hat{s}.$$

In a case which is connected with the Burchnall–Chaundy polynomials the operators (1) were studied in [2].

The question of integrability for the operators $\widehat{\nabla}_\omega$ is associated with the Cherednik algebra

$$\mathcal{A} = \langle 1, x, \frac{d}{dx}, \hat{s}\rangle.$$

In this algebra we consider the operator equality

$$\widehat{\nabla}_\omega V = V\frac{d}{dx}, \tag{2}$$

where

$$V = \sum_{i=0}^{N} f_i(x)\frac{d^i}{dx^i} + \sum_{i=N+1}^{2N} f_i(x)\frac{d^{N-i+1}}{dx^{N-i+1}}\,\hat{s}, \tag{3}$$

N is a natural number, $f_i(x) \in \mathfrak{F}$.

Next, let E_{N+1} be a unitary matrix of order $N+1$, $I_{N+1} = (a_{ij})$ be a matrix of $N+1$ order with the Kronekker elements $a_{ij} = \delta_{i,j+1}$.

Proposition. *The equality* (2) *on the operator V defined by the formula* (3) *is equivalent to matrix equation on functions* $f_i(x)$, $i = 0, 1, \dots, 2N$:

$$\frac{df}{dx} = \Omega_\varkappa f,\ \ \Omega_\varkappa = \begin{pmatrix} 0 & \varkappa E_{N+1} \\ \varkappa E_{N+1} & -2I_{N+1}\end{pmatrix},\ \ \varkappa = (\ln|\omega|)',$$

$$f = \begin{pmatrix} f_0 \\ \vdots \\ f_{2N} \\ 0 \end{pmatrix},\ \ \frac{df}{dx} = \begin{pmatrix} \frac{df_0}{dx} \\ \vdots \\ \frac{df_{2N}}{dx} \\ 0 \end{pmatrix}.$$

Particularly for $N = 1$ we have the following classical integrated families

$$\widehat{\nabla}_{|x|^k} = \frac{d}{dx} - \frac{k}{x}\,\hat{s},;\quad \widehat{\nabla}_{\tan(p|x|/2)} = \frac{d}{dx} \mp \frac{p}{\sin px}\,\hat{s},\ p \neq 0,$$

for the Dunkl operators of rational and trigonometrical types respectively.

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Nonlinear ring waves in a stratified fluid over a shear flow

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We study the propagation of a ring wave in a stratified fluid over a prescribed shear flow. A weakly-nonlinear 2+1-dimensional long wave model is derived from the full set of Euler equations for incompressible fluid with background stratification and shear flow written in cylindrical coordinates, subject to the free surface and rigid bottom boundary conditions. The boundary conditions are typical for the oceanographic applications.

In the absence of the shear flow and stratification, the derived equation reduces to the cylindrical Korteweg – de Vries type equations previously obtained by V.D. Lipovskii [1] and R.S. Johnson [2], respectively. The features of the ring waves are then studied both analytically and numerically, using a finite-difference code for the derived model.

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Self-action of single-cycle nonparaxial optical waves in nonlinear dielectric media

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In this paper, we introduce a system of spectral equations describing the diffraction dynamics of nonparaxial waves in transparent dielectric media with inertialless cubic nonlinearity in the form

$$\begin{cases} \dfrac{\partial^2 g_x}{\partial z^2} + (k^2 - k_x^2 - k_y^2) g_x = -\dfrac{\varepsilon_{nl}}{n^2(\omega)} \Big((k^2 - k_x^2)\Phi_x - k_x k_y \Phi_y + i k_x \Phi_z \Big), \\ \dfrac{\partial^2 g_y}{\partial z^2} + (k^2 - k_x^2 - k_y^2) g_y = -\dfrac{\varepsilon_{nl}}{n^2(\omega)} \Big((k^2 - k_y^2)\Phi_y - k_x k_y \Phi_x + i k_y \Phi_z \Big), \\ \dfrac{\partial g_z}{\partial z} + i k_x g_x + i k_y g_y = -\dfrac{\varepsilon_{nl}}{n^2(\omega)} \Big(i k_x \Phi_x + i k_y \Phi_y + \Phi_z \Big), \end{cases}$$

where $g_{x,y,z}$ are the Cartesian components of the radiation spectrum; $\omega, k_{x,y}$ are the frequencies of temporal and spatial spectra, respectively; $n(\omega)$ is the frequency-dependent refractive index of the medium; $k(\omega) = \omega n(\omega)/c$ is the wave number; c is the speed of light in vacuum; ε_{nl} is the nonlinear susceptibility; z is the direction of radiation propagation,

$$\begin{aligned} \Phi_{x,y} &= F\Big[F^{-1}[g_{x,y}]^3 + F^{-1}[g_{y,x}]^2 F^{-1}[g_{x,y}] + F^{-1}[g_z]^2 F^{-1}[g_{x,y}] \Big], \\ \Phi_z &= F\Big[2F^{-1}[\frac{\partial g_x}{\partial z}] F^{-1}[g_x] F^{-1}[g_z] + 2F^{-1}[\frac{\partial g_y}{\partial z}] F^{-1}[g_y] F^{-1}[g_z] \\ &\quad + (F^{-1}[g_x]^2 + F^{-1}[g_y]^2 + 3F^{-1}[g_z]^2)(-i k_x F^{-1}[g_x] - i k_y F^{-1}[g_y]) \Big], \end{aligned}$$

whereas F, F^{-1} are the operators of direct and inverse Fourier transform.

The figure shows the typical results of calculation of the longitudinal field component ($F^{-1}[g_z]$) TM polarized single-cycle terahertz pulse in fused silica with a nonlinear additive to the refractive index $\Delta n_{nl} = 10^{-3}$.

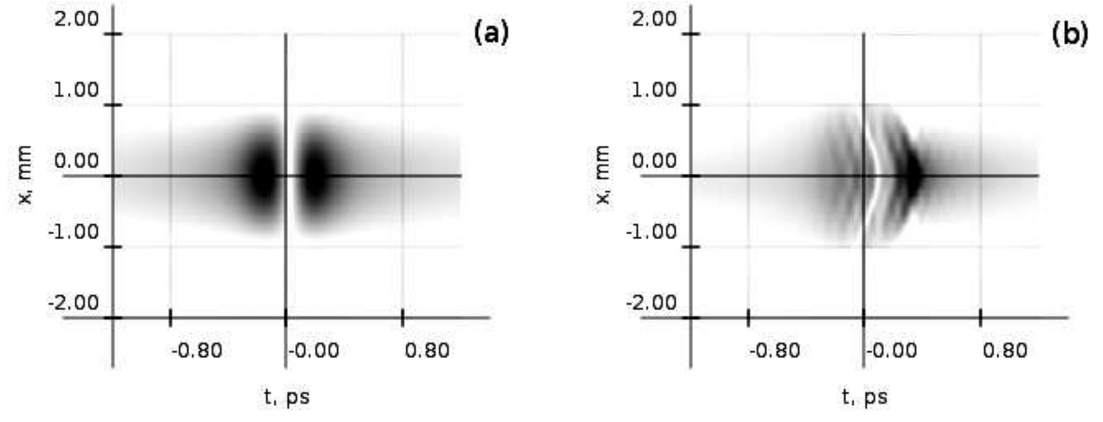

Fig. 1: Dependence of the longitudinal component of the electric field on the transverse coordinate x and time t at the input of the medium z = 0 (a) and its output z = 10 mm (b).

As can be seen from the figure, the phase surfaces of the longitudinal field component are curved, the pulse becomes asymmetric with respect to time and modulated by the radiation on tripled frequency. Pulse width at the center of the beam becomes larger than that at its periphery.

Using the spheroidal coordinates for solving the diffraction problems by pattern equation method

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A new method for solving the scattering problems scattering: Pattern Equation Method (PEM) has been proposed in [1–3]. In the present paper we used a prolate spheroidal coordinates in PEM for the numerical simulation of wave scattering by strongly elongated scatterer. We discuss the convergence and stability of numerical algorithms based on PEM. The calculations accuracy was controlled by means of calculating the balance of energy flows for the incident and scattered waves (verification of the “optical theorem”). It is shown that this method has high speed and universality. The presented results clearly demonstrated, that PEM has good numerical convergence due to the fact, that in this approach the unknown function is very smooth in the contrast to unknown function in the Boundary Integral Equation Method. The various examples demonstrated the effectiveness of the proposed method. As an example, we considered the diffraction of the plane wave by the super-ellipsoid with the ratio of semiaxes more than 100. The results obtained were compared with data obtained by other methods, in particular by the method of Extended Boundary Conditions.

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Simulation of Josephson antenna array in two dimensional electrodynamic waveguide.

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We present the mathematical model and algorithm of simulation of active Josephson antenna, which consists of a few lumped Josephson junctions, in two-dimensional electrodynamic system. First we discuss some problems, connected with the formulation of mathematical models of the considered system. We suggest a system of equations which contains the discrete model of Maxwell equations, known as *Yee scheme* [1, 2], the equations for the dynamics of lumped Josephson junctions biased by a.c. current, and conditions which connect the current and voltage on the junctions with electromagnetic field in the waveguide. We use so-called *Perfectly Matched Layer* boundary conditions [1, 3] to avoid the reflection of electromagnetic waves at the artificial boundary of calculating domain. To simulate the dynamic of electromagnetic field we use the known FDTD explicit method [1] and semi-implicit scheme for the Josephson equations. We also discuss the implementation of suggested algorithm. By using near-to-far field transformation [1], the antenna diagram at the Josephson frequency is calculated.

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Generation of high-frequency radiation in noncollinear collision of waves of a few oscillations in nonlinear media

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In this paper we analyzed the features of the interaction of light pulses with superwide temporal spectrum, propagating at an angle to each other, in dielectric media with inertialess cubic nonlinearity. This classical problem of nonlinear optics has been well studied in the last century for quasi-monochromatic waves. For waves of a few oscillations nowadays it is considered only for collinear interaction [1].

Dynamics of the spectral density g of linearly (TE) polarized radiation in a nonlinear medium is described by the equation [2]:

$$\frac{\partial^2 g}{\partial z^2} + (k^2 - k_x^2) g = -\frac{\varepsilon_{nl}}{n^2(\omega)} k^2 \frac{1}{(2\pi)^4} \iiiint\limits_{-\infty}^{+\infty} g(\omega - \omega', k_x - k_x', z) g(\omega' - \omega'', k_x' - k_x'', z) \times g(\omega'', k_x'', z) d\omega' dk_x' d\omega'' dk_x'', \quad (1)$$

where ω, k_x are the temporal and spatial frequencies, respectively, $k(\omega) = \omega n(\omega)/c$ is the wave number, c is the speed of light in vacuum, $n(\omega)$ is the frequency-dependent refractive index of the medium, ε_{nl} is the nonlinear susceptibility, z is the direction of radiation propagation.

The figure shows that the main result of the propagation and collision of pulses in a nonlinear medium is the generation of radiation on tripled frequencies. Radiation of these temporal frequencies is generated at the higher spatial frequencies.

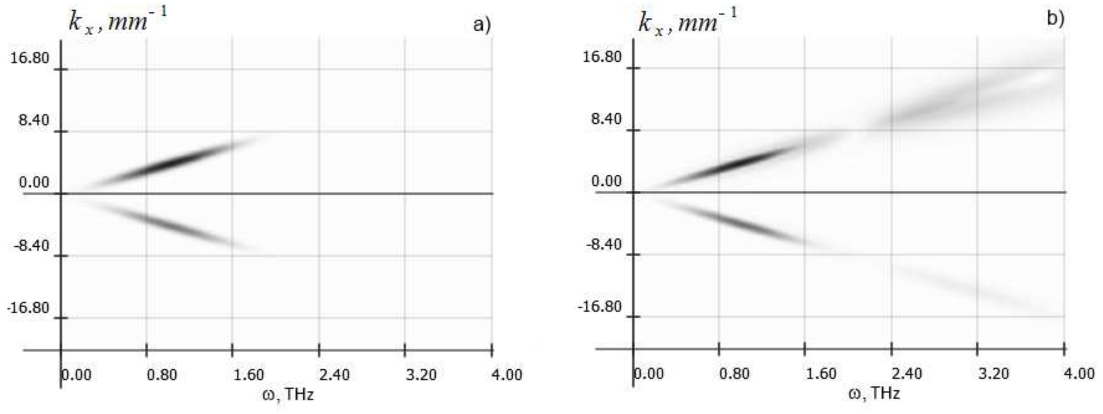

Fig. 1: Spatio-temporal spectra of the radiation at the input z = 0 of fused silica with a nonlinear additive to the refractive index $\Delta n = 4 \cdot 10^{-4}$ (a) and at its output z = 70 mm (b).

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Application of new family of atomic functions $\mathrm{ch}_{a,n}$ to solution of boundary value problems

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Atomic functions (AF) [1] are an effective mathematical apparatus for interpolation of functions. Given function is presented as sum of shifts of compactly supported AF. The simplest example is an interpolation formula based on AF $\mathrm{up}(x)$ (it is exact for any polynomial of degree n [1])

$$s(x) = \sum_{k=k_1}^{k_2} c_k \mathrm{up}(x - k2^{-n}). \tag{1}$$

For practical computation of c_k in (1) one have to solve the system of linear equations (SLE) with banded matrix. This system has fewer equations than unknowns. To complete it one can make some additional assumptions, for example, conditions on derivatives $s^{(m)}(x)$ on the boundary of interpolation interval, as is usual for splines. AFs are connected with their derivatives by the simple functional-differential equations (FDE), that simplifies construction of equations for such conditions.

For $n > 0$ the problem $s(x_i) = 0$ in all interpolation points (decomposition of zero) has not unique solution and doesn't require $s(x_i) \equiv 0$ (we call decomposition of zero good if $s(x_i) \equiv 0$ and bad otherwise). Existence of bad decompositions of zero makes interpolation (1) sensitive to the boundary conditions, and good ones allow in some cases smooth connection of two interpolations on the two connected segments. Nonuniqueness of zero decomposition implies nonuniqueness of solution of corresponding SLE. Then SLE can not be solved with standard direct method like Gauss or sweep method but can be solved with iterative methods. Additional advantage of iterative methods is that they suppress zero decompositions decreasing c_k. Some special properties of (1) are discussed in [2].

Note that derivatives of (1) $s^{(m)}(x)$ for $m < n$ are interpolations of $f^{(m)}(x)$. It is the basic property of such interpolation formulas. Then if we know c_k, we can compute not only $s(x)$, but $f^{(m)}(x)$ represented as $s^{(m)}(x)$ computed basing on FDE. On the other hand any linear differential equation of order less than n can be presented as corresponding SLE. Boundary conditions produce additional equations. Then AF may be used to solve boundary value problems.

Main disadvantage of (1) is that for degree n width of the matrix band is 2^n. More effective interpolation scheme with width of the matrix band n is presented by family of AF $\mathrm{fup}_n(x)$ [2, 3].

In the report similar to [3] method based on the family of AFs $\mathrm{ch}_{a,n}$ [4] is presented. Main advantage of new family is flexibility of computation methods provided by the large two parametric family of AFs. For big a $\mathrm{ch}_{a,n}$ tends to zero on the sides of support and considerably more than zero only in the middle. That allows us to omit some coefficients and decrease width of the matrix band.

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Asymptotics of solutions to wave equation in domain with a small hole

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Let Ω and ω be domains in $\mathbb{R}^3$ with compact closures and smooth boundaries. We assume that each of these domains contains the origin. We introduce a domain with small hole using the formula $\Omega(\varepsilon) = \Omega\backslash\overline{\omega(\varepsilon)}$, where $\omega(\varepsilon) = \{x : \varepsilon^{-1}x \in \omega\}$ and $\varepsilon > 0$ is a small parameter. In the cylinder $Q(\varepsilon) = \{(x,t) : x \in \Omega(\varepsilon), t \in \mathbb{R}\}$ we consider the wave equation $U_{tt} - \triangle U = F$ under the condition $U = 0$ on $\partial Q(\varepsilon)$. Our purpose is to obtain the asymptotics of solutions as $\varepsilon \to 0$. This situation is the simplest example of a hyperbolic boundary value problem in a singularly perturbed domain.

In order to obtain the asymptotics of solutions, we use the method of compound asymptotic expansions (for elliptic problems in singularly perturbed domains, the method of compound expansions was presented in [1]). The asymptotics of solutions is constructed from solutions of the "limit problems", which do not depend on ε.

The specific character of the situation considered in the present work is that one of the limit problems is hyperbolic. Therefore, when describing the asymptotics of a solution to this problem, it is necessary to use methods and results from the theory of hyperbolic boundary value problems in domains with piecewise smooth boundary, which were presented in [2]. The main result is the asymptotics of a solution to the original problem:

$$U(x,t,\varepsilon) = \sum_{n=0}^{N-1} \varepsilon^n \big(V_n(x,t) + \phi(|x|)W_n(\varepsilon^{-1}x,t)\big) + \tilde{U}_N(x,t,\varepsilon)$$

(where $\phi \in C_c^\infty(\mathbb{R}_+)$ and $\phi = 1$ near origin). The remainder $\tilde{U}_N(\cdot,\cdot,\varepsilon)$ for all $\delta > 0$ satisfies the estimate

$$\int_{-\infty}^{+\infty}\int_{\Omega(\varepsilon)} \big(|\nabla_{(x,t)}\tilde{U}_N(x,t,\varepsilon)|^2 + |\tilde{U}_N(x,t,\varepsilon)|^2\big)e^{-2\gamma t}dxdt = O(\varepsilon^{2(N-\delta)}).$$

The developed approach can be used for a considerably wider class of hyperbolic boundary value problems in singularly perturbed domains.

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Modeling of structures in 3D double-diffusive convection

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Three-dimensional double-diffusive convection in a horizontally infinite layer of an uncompressible liquid interacting with horizontal vorticity field is considered in the neighborhood of Hopf bifurcation points. Shape of the cells is given as a superposition of a finite number of convective rolls with different wave vectors $\vec{k}_j = (k_{aj}, k_{bj})$. The following family of amplitude equations for variations of convective cells amplitude is derived by multiple-scaled method:

$$\partial_{T_2} A_j = rA_j + \frac{\alpha_1}{k^2}(k_{aj}\partial_X + k_{bj}\partial_Y)^2 A_j - \alpha_0 \Delta_\perp A_j + \mathrm{i}k\alpha_3 \widehat{G}_j(\Psi)A_j + J(\Psi, A_j) + N_j(A)\,,$$

$$(\partial_{T_2} - \sigma\Delta_\perp)\Delta_\perp \Psi = J(\Psi, \Delta_\perp \Psi) - \frac{\pi^2}{k^2}\sum_{j=1}^{n} \widehat{G}_j(|A_j|^2)\,,$$

where $\Delta_\perp$ is Laplacian with respect to the slow variables, α_i are complex coefficients. Index $j = 1 \ldots n$ denotes the mode number, $\widehat{G}_j(f) = ((k_{aj}^2 - k_{bj}^2)f_{XY} + k_{aj}k_{bj}(f_{YY} - f_{XX}))/k^2$. The functions $N_j(A)$ are the combinations of cubic nonlinear terms ($\mathcal{D}(0) = 1$, $\mathcal{D}(x) = 0$ for $x \neq 0$):

$$N_j(A) = \alpha_2 A_j \sum_{q=1}^{n} |A_q|^2 + \sum_{m=1}^{n}\sum_{q=1}^{n}\sum_{p=q+1}^{n} \Big[\mathcal{D}(\vec{k}_q + \vec{k}_p - \vec{k}_m - \vec{k}_j)\alpha_{jmqp}^{(1)} A_m^* A_q A_p$$
$$+ \mathcal{D}(\vec{k}_q - \vec{k}_p - \vec{k}_m + \vec{k}_j)\alpha_{jmqp}^{(2)} A_m A_q^* A_p + \mathcal{D}(\vec{k}_q - \vec{k}_p + \vec{k}_m - \vec{k}_j)\alpha_{jmqp}^{(3)} A_m A_q A_p^*\Big]\,.$$

For numerical simulation of the obtained systems of amplitude equations a few numerical schemes based on modern ETD (exponential time differencing) pseudospectral methods were developed. The software packages were written for simulation of roll-type convection and convection with square and hexagonal type cells.

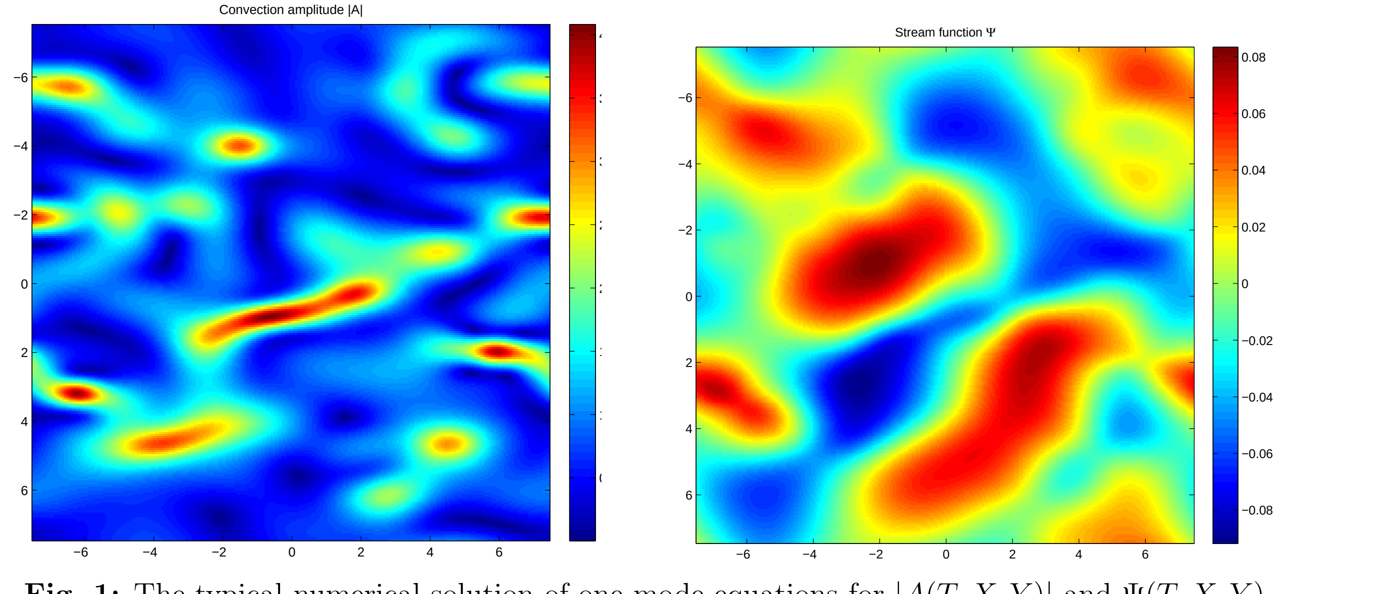

Fig. 1: The typical numerical solution of one-mode equations for $|A(T, X, Y)|$ and $\Psi(T, X, Y)$.

Numerical simulation has showed that the convection takes the form of “spots”, elongated “clouds” or “filaments”. It was noted that in the system quite rapidly a state of diffusive chaos is developed, where the initial symmetric state is destroyed and the convection becomes irregular both in space and time. At the same time in some areas there are bursts of vorticity.

The obtained results will help to describe more adequately the convective and vortex structures that arise in physical systems with convective instabilities, and may also be the basis for the construction of more advanced models of such kind.

Application of the optimized parabolic theory of acoustic beam propagation in anisotropic media for the quartz crystal

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In the general case of acoustic beam propagation in anisotropic media the problem of obtaining analytical solutions which could describe such propagation are of fundamental priority. The traditional approach to the problems of bulk waves propagation in anisotropic media is based on spatial Fourier decomposition of the wave fields into a series of plane waves. Nevertheless, a simple mental comparison between the form of a plane wave and that of a beam leads one to the apparent thought that such “plane wave” approach is not the physically justified one. Indeed the decomposition of a beam based on plane waves leads to infinite series i.e. to an integral between infinite limits of angular aperture. This sum could be reduced to analytical form only by use of adequate approximations. Having in view the aforesaid considerations, the logical way to develop the theory of acoustic beams would lie in attempting to obtain analytic solutions to the problem already in the form of a bounded beam. As it was stated earlier, an alternate approach which would be free of the mentioned drawbacks is the parabolic equation formalism. In the previous works of the authors [1, 2], a new method of an optimized derivation of such a parabolic equation was proposed for acoustic beams in anisotropic media. Herein, a more specific research is presented which consists of analysis of a special case of Y-propagation beams in the quartz crystal. The conditions of propagation were taken from the well-known paper [3], that allows to compare the results of calculations with a real case not only theoretically but also visually by matching the field patterns of the beams and the corresponding skew angles. The results of the current study would be used in future for the analysis of the eigenmodes of acoustic piezoelectric resonators. In that case the eigenmodes would be treated as a result of a constructive interference of reflected acoustic beams. From such point of view the study on the influence of the acoustic anisotropy on the beam propagation conditions appears to be even more crucial for current acoustic science.

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Stokes waves on rotational flows with counter-currents

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The nonlinear problem under consideration describes two-dimensional, steady waves with vorticity on the free surface of water occupying a horizontal open channel of uniform rectangular cross-section. The aim is to investigate the bifurcation mechanism resulting in the formation of Stokes waves on the horizontal free surface of a shear flow in which counter-currents may be present. The whole family of these flows was studied in a joint paper with N. Kuznetsov [see QJMAM, 64 (2011)], in which, in particular, the expressions for their depths were derived. Here, the explicit conditions are presented that guarantee the existence of Stokes waves on a shear flow. It occurs that there are two different sets of conditions: one of these sets describes the case when the Bernoulli constant is fixed and a bifurcating parameter is related to the wavelength; on the contrary, the wavelength is fixed in the second case, whereas the Bernoulli constant varies. For unidirectional subcritical flows both types of conditions are always satisfied. General theorems are illustrated by several examples. This is a joint work with N. Kuznetsov, Russian Academy of Sciences, St. Petersburg.

Concerning description of electromagnetic processes in a substance in relativistic invariant format

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J. Stretton [1] demonstrates that in case when a substance is free from all electric currents, i.e. $\mathbf{j}=0$, electromagnetic processes therein may be described with two Hertz vectors. One of these vectors is fully defined by magnetization $\mathbf{J}$, while the other one — by polarization $\mathbf{P}$. These two Hertz vectors $\mathbf{G}(J)$ and $\mathbf{G}(P)$ define two pairs of potentials $(\mathbf{A},\Phi)$ and $(\mathbf{A}_*,\Phi_*)$, used to express field vectors $\mathbf{E}$, $\mathbf{H}$, $\mathbf{D}$ and $\mathbf{B}$.

In work [2] the author demonstrated that limitation $\mathbf{j}=0$ is not necessary if solenoidal vector $\mathbf{D}_\bullet$ is used for description of electromagnetic processes instead of the electric induction vector $\mathbf{D}$:

$$\mathbf{D}_\bullet=\mathbf{D}+\int_{-\infty}^{t}\mathbf{j}\,dt'=\mathbf{E}+\mathbf{P}+\int_{-\infty}^{t}\mathbf{j}\,dt'\equiv\mathbf{E}+\mathbf{Q},\quad \operatorname{div}\mathbf{D}_\bullet=0. \tag{1}$$

This enables representing Maxwell equation solutions through Hertz vectors:

$$\mathbf{G}(J)=\frac{1}{4\pi}\int_V\frac{1}{r_{xx'}}\{\mathbf{J}\}dV',\quad \mathbf{G}(Q)=\frac{1}{4\pi}\int_V\frac{1}{r_{xx'}}\{\mathbf{Q}\}dV'. \tag{2}$$

in all cases where values $\mathbf{J}$ and $\mathbf{Q}$ can be considered specified in the V domain at time $t\in(-\infty,+\infty)$. Besides efficient representation of Maxwell equations' solutions such an approach provides more meaningful relativistic description of electromagnetic processes than usually done ([1, 3]).

In Minkowsky space the values $\mathbf{J}$ and $\mathbf{Q}$ are specified by two special antisymmetric tensors of $F_{\mu\nu}(\mathbf{J},i\mathbf{Q})$ and $F_{\mu\nu}(\mathbf{Q},-i\mathbf{J})$ ([3]). It is natural to expect that together with these tensors, $F_{\mu\nu}(\mathbf{G}(J),i\mathbf{G}(Q))$ and $F_{\mu\nu}(\mathbf{G}(Q),-i\mathbf{G}(J))$ tensors are specified in accordance with formula (2) and their 4-divergence just defines two 4-vectors of potentials:

$$\operatorname{Div}F_{\mu\nu}(\mathbf{G}(J),i\mathbf{G}(Q))=(\mathbf{A}+i\Phi),\quad \operatorname{Div}F_{\mu\nu}(\mathbf{G}(Q),-i\mathbf{G}(J))=(\mathbf{A}_*+i\Phi_*). \tag{3}$$

When electromagnetic field is described by vectors $\mathbf{E}$ and $\mathbf{B}$, the value of 4-rotor of potential $(\mathbf{A}+i\Phi)$ is used:

$$F_{\mu\nu}(\mathbf{B},-i\mathbf{E})=\operatorname{Rot}(\mathbf{A}+i\Phi).$$

And when it is described by vectors $\mathbf{D}_\bullet$ and $\mathbf{H}$, the value of 4-rotor of potential $(\mathbf{A}_* + i\Phi_*)$ is used:

$$F_{\mu\nu}(\mathbf{D}_\bullet, i\mathbf{H}) = \mathrm{Rot}(\mathbf{A}_* + i\Phi_*).$$

As consistent with these variants, Maxwell equations are written as follows:

$$\mathrm{Div}\, F_{\mu\nu}(\mathbf{B}, -i\mathbf{E}) = \mathrm{Div}\, F_{\mu\nu}(\mathbf{J}, i\mathbf{Q}), \qquad \mathrm{Div}\, F_{\mu\nu}(\mathbf{E}, i\mathbf{B}) = 0,$$
$$\mathrm{Div}\, F_{\mu\nu}(\mathbf{D}_\bullet, i\mathbf{H}) = \mathrm{Div}\, F_{\mu\nu}(\mathbf{Q}, -i\mathbf{J}), \qquad \mathrm{Div}\, F_{\mu\nu}(\mathbf{H}, -i\mathbf{D}_\bullet) = 0.$$

When electromagnetic field is described by vectors $\mathbf{E}$ and $\mathbf{H}$, or by vectors $\mathbf{D}_\bullet$ and $\mathbf{B}$, both potentials are used, while Maxwell equations are of symmetrical form. This approach enables deriving a number of other relations between the elements of Minkowsky space.

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Analytic Kravchenko–Kaiser wavelets and their physical properties

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On basis of the ideas and results of [1–9] a new class of Kravchenko–Kaiser weight and WA-systems of functions are proposed and justified. Its effectiveness in various physical applications has shown. At first on the basis of the theory of atomic functions (AF) [1] and Kaiser windows Kravchenko–Kaiser weighting function (WF) are constructed. Then new analytical constructions of Kravchenko–Kaiser WA-systems of functions are proposed and developed. Shown their advantages in comparison with Morlet wavelet. Their application to problems of weight averaging of difference frequency is considered.

Kravchenko–Kaiser weight functions. On the example of AF $fup_N(x)$ [1] the Kravchenko–Kaiser weight functions are considered. Their constructions we obtain by using the direct product operation of Kravchenko ($w(x)$) and Kaiser ($K(x)$) weight functions: $w_{KKa}(x) = w(x)K(x)$. Thus,

$$w_{KKa} = \frac{1}{fup_N(0)} fup_N\left(\frac{2x}{N+2}\right) \frac{I_0\left(\pi a\sqrt{1-x^2}\right)}{I_0(\pi a)}.$$

Kravchenko–Kaiser functions has compact support and depends on two parameters. Quality is determined by the physical characteristics of the filter [1, 9]. For more information is given by broadbandness parameter μ [3]. The larger value of the μ correspond the smaller number of the wavelet decomposition levels to cover a given frequency domain.

Wavelet modulated Kravchenko–Kaiser weight functions. The construction of analytic Kravchenko–Kaiser wavelets [3–9] are considered. The analytical expression [9] for the wavelet functions has the form

$$\psi(x) = \frac{1}{b} w_{KKa}(x) \left\{ \exp(i\eta x) - \frac{1}{q} A\left(\frac{\eta}{q}\right) \right\},$$

where η is modulation parameter, b and q are scaling parameters, the correction term $A(\eta) = \hat{w}_{KKa}(\eta)$. Here $\hat{w}_{KKa}(\omega)$ is Fourier transform of weight function $w_{KKa}(x)$.

Kravchenko–Kaiser functions to the problems of weighted averaging of the difference frequency signals are applied. Processing of the difference frequency signal with a frequency modulation rangefinder using the weighted averaging method which enables to significantly reduce the influence of noise and improve the accuracy of determination of the frequency shift.

In this paper we proposed and justified a new class of weight and WA-systems of Kravchenko–Kaiser functions. Numerical experiment and physical analysis of the results for physical models confirmed their effectiveness.

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Comparison of integral equation and transmission line methods for analysis of a loop antenna located on the surface of an axially magnetized plasma column

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Much previous work on studying the electrodynamic characteristics of loop antennas in magnetized plasmas either applies to antennas immersed in an unbounded homogeneous magnetoplasma (see, e.g., [1] and references therein) or to the case where such a source is located on the surface of an axially magnetized plasma column surrounded by a homogeneous isotropic medium [2, 3]. This is in particular explained by the fact that in the above-mentioned cases, the problem of finding the current distribution of the antenna is mathematically tractable and can be solved using an integral equation method. For a loop antenna operated on the surface of an axially magnetized plasma column in a background isotropic medium, the basic solution has recently been published in [2, 3]. In the present work, we compare the results of our calculations of the antenna characteristics obtained using the integral equation method [2, 3] with the results of some simpler approximate approaches such as the transmission line theory.

We determine conditions under which the closed-form expressions for the current distribution and the input impedance of the considered loop antenna, which were rigorously obtained within the framework of the integral equation method in some special cases [3], can be derived using an approximate approach based on the transmission line theory. Detailed calculations using the two approaches have been performed for these comparisons, for both a resonant and nonresonant magnetoplasma inside the column. In particular, our findings show that for the column filled with a resonant magnetoplasma, the transmission line theory, after some generalization, becomes capable of ensuring reasonably accurate predictions for the characteristics of a loop antenna placed on the surface of such a column.

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Propagation of TE waves in a plane dielectric waveguide with nonlinear permittivity

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Consider a monochromatic TE wave in the form $\mathbf{E}e^{-i\omega t}$, $\mathbf{H}e^{-i\omega t}$, where

$$\mathbf{E} = (0, \mathrm{E}_y(x)e^{i\gamma z}, 0)^T, \quad \mathbf{H} = (\mathrm{H}_x(x)e^{i\gamma z}, 0, \mathrm{H}_y(x)e^{i\gamma z})^T, \tag{1}$$

are the complex amplitudes; ω is a circular frequency; $(\ \cdot\ )^T$ is the transposition operation; γ is unknown (real) spectral parameter (propagation constant of a guided wave); $\mathrm{E}_y, \mathrm{H}_x, \mathrm{H}_z$ are unknown functions. The TE wave propagates along the surface of the lossless dielectric waveguide

$$\Sigma := \{(x, y, z) \in \mathbb{R}^3 : 0 < x < h\}.$$

The waveguide Σ is located in the Cartesian coordinate system $Oxyz$ and is filled with homogeneous isotropic medium. Inside the waveguide Σ $(0 \leqslant x \leqslant h)$ the permittivity ε is described by the formula

$$\varepsilon = \varepsilon_2 + \alpha\big(1 - e^{-\beta|\mathbf{E}|^2}\big),$$

where ε_2 is a constant part of the permittivity; $\alpha, \beta \geqslant 0$ are real constants. Half-spaces $x < 0$ and $x > h$ are filled with homogeneous isotropic media with constant permittivities $\varepsilon = \varepsilon_1 \geqslant \varepsilon_0$ and $\varepsilon = \varepsilon_3 \geqslant \varepsilon_0$, respectively, ε_0 is the permittivity of free space. Entire space contains no sources. It is supposed that everywhere $\mu = \mu_0$, where μ_0 is the permeability of free space.

Complex amplitudes (1) of the TE wave must satisfy Maxwell's equations

$$\begin{cases} \operatorname{rot} \mathbf{H} = -i\omega\varepsilon\mathbf{E}, \\ \operatorname{rot} \mathbf{E} = i\omega\mu\mathbf{H}; \end{cases} \tag{2}$$

the continuity condition for the tangential components of the field on the boundaries $x = 0$ and $x = h$; and the radiation condition at infinity: the electromagnetic field decays as $O(|x|^{-1})$ when $|x| \to \infty$. The solution is sought for in the entire space.

The continuity conditions for the tangential components are

$$[\mathrm{E}_y]|_{x=0} = 0, \quad [\mathrm{E}_y]|_{x=h} = 0, \quad [\mathrm{H}_z]|_{x=0} = 0, \quad [\mathrm{H}_z]|_{x=h} = 0. \tag{3}$$

Problem P_E: *it is necessary to determine eigenvalues* $\hat{\gamma}$ *for which there exist nontrivial functions* $\mathrm{E}_y(x;\hat{\gamma}), \mathrm{H}_x(x;\hat{\gamma}), \mathrm{H}_z(x;\hat{\gamma})$ *that are defined for* $x \in (-\infty, +\infty)$*, satisfies equation* (2) *and transmission conditions* (3) (results for similar problems see in [1–4]).

Numerical results are presented, comparison with the linear case is given.

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Freely floating bodies trapping time-harmonic water waves

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We study the coupled small-amplitude motion of the following mechanical system: infinitely deep water bounded above by a free surface and an immersed structure formed by an arbitrary finite number of surface-piercing bodies floating freely. The mathematical model of time-harmonic motion is a linear spectral problem with the frequency of oscillations considered as the spectral parameter. It is proved that there exist axisymmetric structures consisting of $N \geq 2$ bodies and having the following properties: (i) a time-harmonic wave mode is trapped by every of these structures; (ii) some of the structure's bodies (may be none) are motionless, whereas the rest of the bodies (may be none) are heaving at the same frequency as water. The construction of these structures is based on a generalization of the semi-inverse procedure applied earlier for obtaining trapping bodies that are motionless although float freely.

Transfer the OAM of light to SPP and chiral nanostructures formation

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There are few known experimental examples of laser-induced chiral nanostructures formation under the action of powerful polarized laser radiation. One of such examples is the formation of helicoidal nanostructures under the action of series of circular polarized femtosecond laser radiation inside quarts glass [1]. In this case the symmetrical array of helicoudal nanochannels of ~ 50 nm diameter are formed around the geometric axis of normally incident circular polarized laser beam. The number of channels depends on laser power density. The sign of chirality depends on either the left or right-hand circular laser polarization was used. But the explanation of this effect till now is lack.

We propose the explanation of this effect in frame of universal polariton model of laser-induced condensed matter damage. Under the action of powerful laser radiation the filamentation of laser beam inside transparent media can occurs. In case the material metallization inside the filament of micron or submicron diameter metallic cylindrical channel can supports the cylindrical surface plasmon polariton (CSPP) propagation [2]. Mutual interference of CSPP of opposite propagation directions produces the standing waves and nanostructures formation with period $d \leq \lambda/2n$, where n is refractive index of material, λ is the wavelength of laser radiation. Note that in the case of circular polarization of laser radiation the orbital angular momentum of light can be transferred to CSPP. In this case the self-produced metallic nanochannel became unstable and undergoes the helicoidal shape. The transfer of the orbital angular momentum (OAM) of laser light to CSPP will force the handedness of helicoidal spiral. So in case of left-hand light polarization the helicoidal spiral will has left direction, and right for the case of right-handedness , which was experimentally observed [1] and confirms our conclusion.

Another example of OAM transfer to surface plasmon polaritons following by chiral nanostructure formation is laser-induced helicoidal cone-shape metal nanostructure formation.

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Absorbing boundary conditions in numerical analysis of electrodynamic systems by the method of minimal autonomous blocks

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The method of minimal autonomous blocks (MAB) is used to solve a wide class of problems in applied electrodynamics [1]. In its basis decomposition of investigated area on system of the blocks which electromagnetic properties are described by scattering matrixes lies. For solving electrodynamic problems by the MAB method iteration, recomposition and hybrid algorithms are used.

Efficiency and accuracy of the external electrodynamic problems solution by the MAB method are depended from valid choice and implementation of absorbing boundary conditions on external borders of investigated area. Various variants of the absorbing boundary conditions considering specificity of the MAB method are offered. For recomposition variant of the MAB method a functional analog of the perfectly matched layer (PML), widely used in the finite element method and in the finite-difference time-domain method, is appropriate to use.

Adaptive absorbing boundary conditions are developed for an iterative variant of the MAB method. For their implementation at each iteration step for each of the boundary block the complex amplitude of the wave incident on its outer face is calculated, taking into account information about the excitation of internal faces. Potential possibilities of various variants of absorbing boundary conditions for cases of sliding falling of a wave on external border and close located local sources are investigated.

Features of realization of absorbing boundary conditions in problems of diffraction of plane electromagnetic waves on periodic gratings are considered. Realization of these conditions is based on the matching of external channels with a discrete spectrum of spatial harmonics and iterative transformation of channel waves and Floquet harmonics. The features of the implementation of absorbing boundary conditions for the solution of acoustic problems by the MAB method are considered.

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Solutions of the general Heun equation in series of incomplete Beta functions

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We show that for some parameters the solution of the general Heun equation allows expansions in terms of incomplete Beta functions. By means of termination of the series, closed-form solutions are derived for two infinite sets of special values of the involved parameters. The resultant solutions are reduced to elementary functions that are quasi-polynomials.

Light beam tunneling in 1D photonic crystal

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We study light beam propagation in the first forbidden gap of a planar photonic crystal with the help of mathematical modeling. Angle width of the gap $\Delta\theta_{Br}$ is determined by the dielectric contract of two component periodical structure. Unless values of contrast are small the angle spectrum width of a tilted beam $\Delta\theta_b$ may be comparable with $\Delta\theta_{Br}$ or greater than it. This regime of Gaussian beams tunneling is purpose of this research.

Solving 2D coupled equations for the amplitudes of the forward and backward waves and using spectral method we find analytical expressions for the field of the beam inside the photonic band gap and outside it as well.

If $\Delta\theta_b >> \Delta\theta_{Br}$ we observed the regime of tunneling. A weakened output beam preserves the initial Gaussian form, at the same time the center of beam shifts [1]. The value of its longitudinal shift is comparable with few wave lengths and saturates with increasing crystal length. This effect is a space analog of the well-known Hartman effect which manifests itself as time delay saturation when a pulse tunnels through a photonic barrier [2]. It is shown that the shift of the beam center is proportional to mean stored energy in the crystal. This energy is concentrated in a narrow surface layer if $\Delta\theta_b > \Delta\theta_{Br}$.

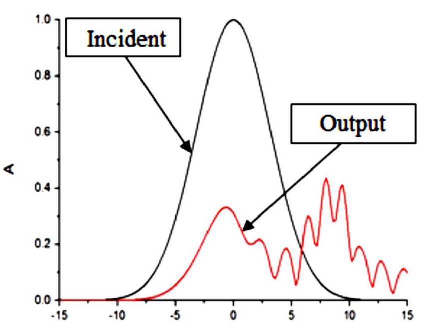

Fig. 1: Initially focusing beam breaks into two sub beam.

When $\Delta\theta_b \geq \Delta\theta_{Br}$, a part of central spectrum components falls outside of the gap which results in distortions of beam profile. The gravity center of the output beam displaces to the value of geometrical deviation with increasing barrier width. Field inside the layer appears to be an oscillating function along the normal to the incident plane. The oscillation depth decrease with growth of $\Delta\theta_b$. General regularities of tunneling hold for focusing and defocusing beams with the spectrum enlarging due to the phase modulation of front unless $\Delta\theta_b >> \Delta\theta_{Br}$. But if $\Delta\theta_b \geq \Delta\theta_{Br}$ we observed stronger distortion of beam profile, and in particular case the profile broke into two sub beams.

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Coupled electromagnetic TE-TE waves propagation. Numerical approach to determine coupled propagation constants

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Consider a sum of two TE waves propagating in two orthogonal directions (TE-TE wave) [1]

$$\mathbf{E} = \mathbf{E}_1 e^{-i\omega_1 t} + \mathbf{E}_2 e^{-i\omega_2 t}, \quad \mathbf{H} = \mathbf{H}_1 e^{-i\omega_1 t} + \mathbf{H}_2 e^{-i\omega_2 t}, \tag{1}$$

where

$$\begin{aligned} \mathbf{E}_1 &= (0, \mathrm{E}_{1y}(x)e^{i\gamma_1 z}, 0)^{\mathrm{T}}, \quad \mathbf{H}_1 = (\mathrm{H}_{1x}(x)e^{i\gamma_1 z}, 0, \mathrm{H}_{1z}(x)e^{i\gamma_1 z})^{\mathrm{T}}, \\ \mathbf{E}_2 &= (0, 0, \mathrm{E}_{2z}(x)e^{i\gamma_2 z})^{\mathrm{T}}, \quad \mathbf{H}_2 = (\mathrm{H}_{2x}(x)e^{i\gamma_2 z}, \mathrm{H}_{2y}(x)e^{i\gamma_2 z}, 0)^{\mathrm{T}} \end{aligned} \tag{2}$$

are the complex amplitudes; ω_1, ω_2 are circular frequencies; $(\,\cdot\,)^{\mathrm{T}}$ is the transposition operation; γ_1, γ_2 are unknown (real) spectral parameters (propagation constants of a guided wave); $\mathrm{E}_{1y}, \mathrm{H}_{1x}, \mathrm{H}_{1z}$, $\mathrm{E}_{2z}, \mathrm{H}_{2x}, \mathrm{H}_{2y}$ are unknown functions. The TE-TE wave propagates along the surface of the lossless dielectric waveguide $\Sigma := \{(x, y, z) \in \mathbb{R}^3 : 0 < x < h\}$. The waveguide Σ is located in the Cartesian coordinate system $Oxyz$ and is filled with homogeneous isotropic medium. Inside the waveguide Σ the permittivity ε is described by the Kerr law $\varepsilon = \varepsilon_2 + \alpha|\mathbf{E}|^2$, where $\varepsilon_2 > 0$ is a constant part of the permittivity; $\alpha > 0$ is a real constant. Half-spaces $x < 0$ and $x > h$ are filled with homogeneous isotropic media with constant permittivities $\varepsilon = \varepsilon_1 \geqslant \varepsilon_0$ and $\varepsilon = \varepsilon_3 \geqslant \varepsilon_0$, respectively, ε_0 is the permittivity of free space; $\max(\varepsilon_1, \varepsilon_3) < \varepsilon_2$. Entire space contains no sources. It is supposed that everywhere $\mu = \mu_0$, where μ_0 is the permeability of free space.

Maxwell's equations have the form $\mathrm{rot}\tilde{\mathbf{H}} = \partial_t \tilde{\mathbf{D}}$, $\mathrm{rot}\tilde{\mathbf{E}} = -\partial_t \tilde{\mathbf{B}}$, where $\tilde{\mathbf{D}} = \varepsilon\tilde{\mathbf{E}}$, $\tilde{\mathbf{B}} = \mu\tilde{\mathbf{H}}$ and $\partial_t = \partial/\partial t$; $(\tilde{\mathbf{E}}, \tilde{\mathbf{H}})$ represents the total field.

In the case under consideration Maxwell's equations depend on t in the same way as if ε would be a constant. This allows to write Maxwell's equations for fields (1)

$$\begin{aligned} \mathrm{rot}\,(\mathbf{E}_1 e^{-i\omega_1 t} + \mathbf{E}_2 e^{-i\omega_2 t}) &= i\mu\omega_1 \mathbf{H}_1 e^{-i\omega_1 t} + i\mu\omega_2 \mathbf{H}_2 e^{-i\omega_2 t}, \\ \mathrm{rot}\,(\mathbf{H}_1 e^{-i\omega_1 t} + \mathbf{H}_2 e^{-i\omega_2 t}) &= -i\varepsilon\omega_1 \mathbf{E}_1 e^{-i\omega_1 t} - i\varepsilon\omega_2 \mathbf{E}_2 e^{-i\omega_2 t}. \end{aligned} \tag{3}$$

Complex amplitudes (2) must satisfy equations (3), the continuity condition for the tangential components of the fields on the boundaries $x = 0$ and $x = h$; and the radiation condition at infinity: the electromagnetic field decays as $O(|x|^{-1})$ when $|x| \to \infty$.

The continuity conditions for the tangential components are

$$\begin{aligned} &[\mathrm{E}_{1y}]\big|_{x=0} = 0, \quad [\mathrm{H}_{1z}]\big|_{x=0} = 0, \quad [\mathrm{E}_{2z}]\big|_{x=0} = 0, \quad [\mathrm{H}_{2y}]\big|_{x=0} = 0, \\ &[\mathrm{E}_{1y}]\big|_{x=h} = 0, \quad [\mathrm{H}_{1z}]\big|_{x=h} = 0, \quad [\mathrm{E}_{2z}]\big|_{x=h} = 0, \quad [\mathrm{H}_{2y}]\big|_{x=h} = 0. \end{aligned} \tag{4}$$

Problem P_E: *it is necessary to determine coupled eigenvalues* $(\hat{\gamma}_1, \hat{\gamma}_2)$ *for which there exist non-trivial functions* $\mathrm{E}_{1y}(x;\hat{\gamma}_1,\hat{\gamma}_2), \mathrm{H}_{1x}(x;\hat{\gamma}_1,\hat{\gamma}_2), \mathrm{H}_{1z}(x;\hat{\gamma}_1,\hat{\gamma}_2),\ \mathrm{E}_{2z}(x;\hat{\gamma}_1,\hat{\gamma}_2), \mathrm{H}_{2x}(x;\hat{\gamma}_1,\hat{\gamma}_2), \mathrm{H}_{2y}(x;\hat{\gamma}_1,\hat{\gamma}_2)$ *that are defined for* $x \in (-\infty,+\infty)$*, satisfies equations* (3)*, transmission conditions* (4)*, and decay as* $O(|x|^{-1})$ *when* $|x| \to \infty$ (results for similar problems see in [1–3]).

Numerical results are presented, comparison with the linear case is given.

The work is partly supported by the Russian Federation President Grant (no. MK-90.2014.1), The Ministry of Education and Science of the Russian Federation (Goszadanie), and the Russian Foundation for Basic Research (no. 12-07-97010-r-A).

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Spectral bands for chain of ball resonators with Dirichlet condition

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In this paper a solvable model describing spectral properties of an electron inside infinite direct chain of quantum ball resonators is suggested. Elementary cell of this system can be presented as a ball of unit radius. It is also assumed that these resonators are weakly coupled. It means that neighbour balls interact with each other via point-like aperture (see, [1, 2]). Balls are also considered connected by δ-couplings with a parameter $\alpha \in \mathbf{R}$ at the touching points.

The stationary Shrödinger equation $\hat{H}(x)\psi(x) = \lambda\psi(x)$ is studied and its Hamiltonian is a orthogonal sum of Dirichlet Laplacians : $\hat{H} = \oplus_i(-\Delta_i)$, $\psi_i|_{\partial B_i} = 0$. The solution of spectral problem for this system is based on the theory of self-adjoint extensions of symmetric operators (see, [3]). In general, this approach consists of construction a restriction of the initial operator and its further extension. To obtain self-adjoint extension it is necessary to establish some linear relation that should be also modified to take into account the δ-coupling condition with coupling constant α:

$$\begin{cases} a_j^+ = -a_{j-1}^-, \\ b_j^+ - b_{j-1}^- = -\alpha a_{j-1}^-. \end{cases} \tag{1}$$

By substituting explicit form of coefficients $b_j^\pm$ into system (1), one can derive the transfer matrix for such system in the following form:

$$\mathbf{M} = \begin{pmatrix} 0 & -1 \\ 1 & \frac{-\alpha + 2\lim\limits_{\mathbf{x}\to\mathbf{x}_i}(G(\mathbf{x},\mathbf{x}_i,\lambda) - G(\mathbf{x},\mathbf{x}_i,\lambda_0))}{G(\mathbf{x}_i,\mathbf{x}_{i+1},\lambda)} \end{pmatrix}, \tag{2}$$

where $G(\mathbf{x}_i, \mathbf{x}_{i+1}, \lambda)$ is a Green's function.

Spectral properties of matrix (2) allow one to obtain systems that describe in general spectrum structure (for the system being under discussion) in the following form:

$$\begin{cases} \sum\limits_{k=1}^{\infty}\sum\limits_{l=1}^{\infty} \dfrac{(-1)^l \xi_{l,k}}{x_{l,k}^2 - \lambda} \lesseqgtr 0 \qquad (\Leftrightarrow \qquad G(\mathbf{x}_i, \mathbf{x}_{i+1}, \lambda) \lesseqgtr 0) \\ \sum\limits_{k=1}^{\infty}\sum\limits_{l=1}^{\infty} \dfrac{\left(1-(-1)^l\right)\xi_{l,k}}{x_{l,k}^2 - \lambda} \gtreqless \sum\limits_{k=1}^{\infty}\sum\limits_{l=1}^{\infty} \dfrac{\xi_{l,k}}{x_{l,k}^2 - \lambda_0} + \dfrac{\alpha}{2} \\ \sum\limits_{k=1}^{\infty}\sum\limits_{l=1}^{\infty} \dfrac{\left(1+(-1)^l\right)\xi_{l,k}}{x_{l,k}^2 - \lambda} \lesseqgtr \sum\limits_{k=1}^{\infty}\sum\limits_{l=1}^{\infty} \dfrac{\xi_{l,k}}{x_{l,k}^2 - \lambda_0} + \dfrac{\alpha}{2} \end{cases}, \tag{3}$$

where $\xi_{lk} = \pi^{-2}\ (2l+1)\ x_{lk}^3 \left|j_l'(x_l k)\right|^2 \left|J_{l+1/2}'(x_{lk})\right|^{-2}$, x_{lk} is a k-th root of the equation $j_l(x) = 0$ and $\lambda_0 \in \mathbf{R} : \lambda_0 < 0$.

In this work analytically derived conditions (3) on bands of continuous spectrum were numerically computed for different values of coupling constant α and parameter of aperture λ_0.

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Homogenization of the initial boundary value problems for parabolic systems with periodic coefficients

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Let $\mathcal{O} \subset \mathbb{R}^d$ be a bounded domain of class $C^{1,1}$. In $L_2(\mathcal{O};\mathbb{C}^n)$, we consider matrix elliptic second order differential operators (DO's) $A_{\flat,\varepsilon} = b(\mathbf{D})^* g(\mathbf{x}/\varepsilon) b(\mathbf{D})$, $\flat = D, N$, with the Dirichlet or Neumann boundary conditions, respectively. Here $0 < \varepsilon \leqslant 1$ is a small parameter, $g(\mathbf{x})$ is an $(m \times m)$-matrix-valued function which is assumed to be bounded, uniformly positive definite and periodic with respect to some lattice Γ. Next, $b(\mathbf{D})$ is a first order DO of the form $b(\mathbf{D}) = \sum_{j=1}^d b_j D_j$, where b_j are constant $(m \times n)$-matrices. It is assumed that $m \geqslant n$ and the symbol $b(\boldsymbol{\xi}) = \sum_{j=1}^d b_j \xi_j$ has maximal rank: $\operatorname{rank} b(\boldsymbol{\xi}) = n$, $0 \neq \boldsymbol{\xi} \in \mathbb{R}^d$ for $\flat = D$ and $0 \neq \boldsymbol{\xi} \in \mathbb{C}^d$ for $\flat = N$. The symbol $\partial_{\boldsymbol{\nu}}^\varepsilon$ stands for the conormal derivative. Formally, $\partial_{\boldsymbol{\nu}}^\varepsilon \mathbf{w}(\mathbf{x}) = b(\boldsymbol{\nu}(\mathbf{x}))^* g(\mathbf{x}/\varepsilon) b(\nabla)\mathbf{w}(\mathbf{x})$, where $\boldsymbol{\nu}(\mathbf{x})$ is the unit normal vector to $\partial\mathcal{O}$ at the point $\mathbf{x} \in \partial\mathcal{O}$.

We study homogenization for the operator exponential $e^{-A_{\flat,\varepsilon} t}$, $t > 0$, $\flat = D, N$. In other words, we are interested in the behaviour of the solution $\mathbf{u}_{\flat,\varepsilon}(\mathbf{x}, t)$ of the initial boundary value problem

$$\partial_t \mathbf{u}_{\flat,\varepsilon} = -b(\mathbf{D})^* g(\mathbf{x}/\varepsilon) b(\mathbf{D}) \mathbf{u}_{\flat,\varepsilon} \text{ in } \mathcal{O}, \quad \mathbf{u}_{\flat,\varepsilon}|_{t=0} = \boldsymbol{\phi}, \quad \mathbf{u}_{D,\varepsilon}|_{\partial\mathcal{O}} = 0 \text{ or } \partial_{\boldsymbol{\nu}}^\varepsilon \mathbf{u}_{N,\varepsilon}|_{\partial\mathcal{O}} = 0,$$

for small ε. Here $\boldsymbol{\phi} \in L_2(\mathcal{O};\mathbb{C}^n)$. Then $\mathbf{u}_{\flat,\varepsilon} = e^{-A_{\flat,\varepsilon} t}\boldsymbol{\phi}$, $\flat = D, N$.

The *effective problem* has the form

$$\partial_t \mathbf{u}_{\flat,0} = -b(\mathbf{D})^* g^0 b(\mathbf{D}) \mathbf{u}_{\flat,0} \text{ in } \mathcal{O}, \quad \mathbf{u}_{\flat,0}|_{t=0} = \boldsymbol{\phi}, \quad \mathbf{u}_{D,0}|_{\partial\mathcal{O}} = 0 \text{ or } \partial_{\boldsymbol{\nu}}^0 \mathbf{u}_{N,0}|_{\partial\mathcal{O}} = 0.$$

Here g^0 is the constant positive *effective matrix* (defined as usual in homogenization theory). The effective operator $A_\flat^0$, $\flat = D, N$, is given by $b(\mathbf{D})^* g^0 b(\mathbf{D})$ with the Dirichlet or Neumann boundary conditions, respectively. We have $\mathbf{u}_{\flat,0} = e^{-A_\flat^0 t}\boldsymbol{\phi}$.

Theorem [2]. *There exists a number $\varepsilon_0 \in (0, 1]$ depending on the domain $\mathcal{O}$ and the lattice Γ such that for $0 < \varepsilon \leqslant \varepsilon_0$ we have*

$$\|\mathbf{u}_{\flat,\varepsilon}(\cdot, t) - \mathbf{u}_{\flat,0}(\cdot, t)\|_{L_2(\mathcal{O};\mathbb{C}^n)} \leqslant C_{1,\flat} e^{-c_\flat t} \varepsilon (t + \varepsilon^2)^{-1/2} \|\boldsymbol{\phi}\|_{L_2(\mathcal{O};\mathbb{C}^n)},\ t \geqslant 0,\ \flat = D, N. \qquad (1)$$

We also have the following approximation of $\mathbf{u}_{\flat,\varepsilon}$ in the Sobolev space $H^1(\mathcal{O};\mathbb{C}^n)$ for $0 < \varepsilon \leqslant \varepsilon_0$

$$\|\mathbf{u}_{\flat,\varepsilon}(\cdot, t) - \mathbf{u}_{\flat,0}(\cdot, t) - \varepsilon \mathbf{v}_{\flat,\varepsilon}(\cdot, t)\|_{H^1(\mathcal{O};\mathbb{C}^n)} \leqslant C_{2,\flat} e^{-c_\flat t} \left(\varepsilon^{1/2} t^{-3/4} + \varepsilon t^{-1}\right) \|\boldsymbol{\phi}\|_{L_2(\mathcal{O};\mathbb{C}^n)},\ t > 0,\ \flat = D, N.$$

Here $\mathbf{v}_{\flat,\varepsilon}$ is the corresponding corrector. The positive constants $C_{1,\flat}$, $C_{2,\flat}$, and $c_\flat$ can be written explicitly in terms of the problem data.

Estimate (1) is order sharp for small ε and a fixed $t > 0$. The method is based on using the representation $e^{-A_{\flat,\varepsilon}t} = -(2\pi i)^{-1} \int_\gamma e^{-\zeta t}(A_{\flat,\varepsilon} - \zeta I)^{-1}\, d\zeta$ for a suitable contour $\gamma \subset \mathbb{C}$. For $e^{-A_\flat^0 t}$ we have a similar representation, so the problem is reduced to the study of the resolvent $(A_{\flat,\varepsilon} - \zeta I)^{-1}$ for small ε and $\zeta \in \gamma$. The required approximation of this resolvent was obtained in [1].

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Scattering of the electromagnetic wave by a shielded conducting sphere

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Consider an ideal conducting sphere located at the distance L (distance from the sphere's center) from an infinite conducting shield (see Fig. 1). Let an electromagnetic parallel polarized wave with the component $E_y^{(0)}(x, y, z) = A^{(0)}e^{ikz}$, where A^0 is the incident wave's amplitude; k is the wave number fall onto the sphere in the normal direction to the shield (the z axis is assumed to be parallel to the normal direction toward the shield). It is required to find the diffracted field.

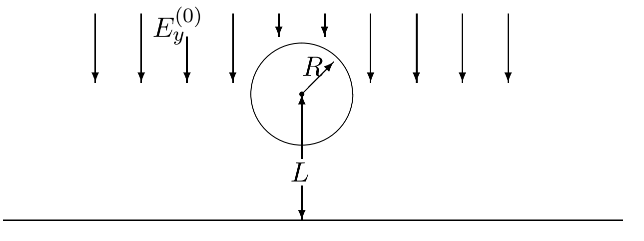

Fig. 1: Geometry of the problem.

We seek the diffracted field in the form of the solution to the system of the Maxwell equations. The boundary conditions are taken as for ideal conductors. More precisely, the shear components of the vector of electric intensity of the full field at the boundaries of the sphere and at the boundaries of the shield equal each other.

We represent a solution to the problem in the form of the sum of solutions of an infinite sequence of problems of diffraction by the sphere and the shield. We assume that the incident wave is first diffracted by the shield. Then the wave reflected off the shield is scattered by the sphere, then it is reflected off the shield again and again. One can expect that after every iteration from diffraction, the scattered field carries less and less energy compared to that of the previous iteration. The condition for shut-down of the iteration process is smallness of energy scattered at the current iteration.

The problem of scattering of the wave by the shield is replaced with an equivalent problem of eradiation by specular reflected sphere. Plots of calculation results for various ratios of incident wave length, sphere radius and distance between the sphere and the shield are given.

Dimension reduction for quantum waveguides: Which transmission conditions are asymptotically correct?

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An asymptotic analysis of the rectangular grating of quantum waveguides is performed. It is shown that, in contrast to the traditional Kirchhoff transmission conditions at nodes of the corresponding quantum graphs, the asymptotically correct transmission conditions become the Dirichlet conditions which split the graph into independent line segment units and endow the spectrum with specific band-gap structure, very different from the structure derived on the basis of the Kirchhoff conditions. Moreover, one spectral band is detected which has no relation to the graph model. The results are obtained by means of analysis of trapped modes and bounded solutions in the infinite cross-shaped waveguide. Other geometrical shapes will be discussed as well.

Multi-soliton solutions for non-integrable equations: asymptotic approach

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We describe an approach to construct multi-soliton asymptotic solutions for essentially non-integrable equations. The general idea is realized in the case of three waves and for the GKdV-4 equation:

$$\frac{\partial u}{\partial t} + \frac{\partial u^4}{\partial x} + \varepsilon^2 \frac{\partial^3 u}{\partial x^3} = 0, \quad x \in \mathbb{R}^1, \quad t > 0,$$

where the dispersion parameter ε is assumed to be small.

A brief review of asymptotic methods as well as results of numerical simulation are included.

Vector complex source beams carrying a screw phase dislocation

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A screw phase dislocation disguised as a dark singular spot, where the phase of the beam is undetermined and its amplitude vanishes is called an optical vortex. The order of such a singularity is associated with the orbital angular momentum per photon $m\hbar$ [1] and referred to as the topological charge of the vortex m. Coherent superpositions of optical vortices with different topological charges create optical fields with vortical structures richer than that of the individual beams [2]. Moreover, optical phase vortices are widely known for their applications in optical manipulation. The vector properties of the field introduce an additional complexity.

Optical beams focused down to subwavelength focal spots constitute one of the building blocks of modern optics due to the complexity of their polarization states [3] and their wide range of applications such as a selective excitation of the nanoparticles [4] or the generation of novel states of the light field [5]. To describe highly focused beams of various polarizations, one can start from the exact solution of the scalar wave equation by the complex source beam (CSB), which can be extended to accurately describe highly focused linearly, radially and azimuthally polarized light beams [6].

In this work we report on the accurate and unified vectorial description of highly focused vortex beams of various polarization states, which we derive from a scalar CSB vortex beam. We construct three different families of optical vector vortices originating from different symmetries. Optical vortices derived within Cartesian symmetry exhibit properties analogous to circularly and linearly polarized highly focused vortex beams (Fig. 1 (a–d)). The vortical CSB solutions derived within cylindrical symmetry behave as azimuthally (radially) polarized vortices in the far-field but their vortical structrure in the near-field is that of a circularly polarized vortex beam (Fig. 1 (e,f)). On the other hand, CSB vector vortex beams derived within spherical symmetry behave like azimuthally (radially) polarized vortices in their focal plane but their vortical structure in the far-field is rather complicated. The latter two families exhibit very special properties (Fig. 1 (g,h)).

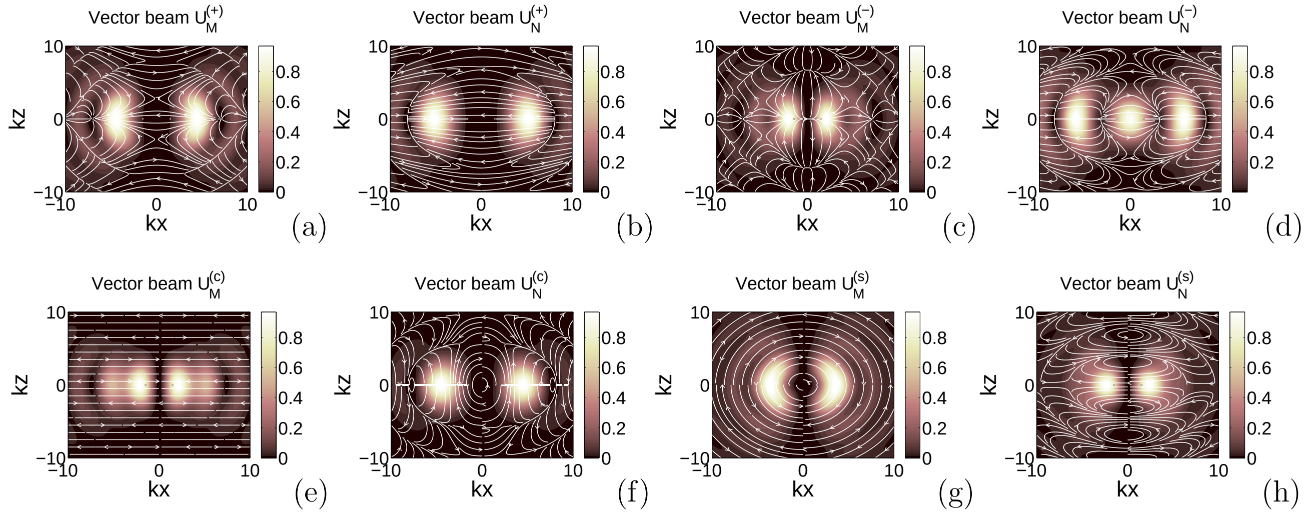

Fig. 1: Modulus of the electric fields for vector vortex beams derived within Cartesian symmetry (a-d) ($\mathbf{U}_M^{(\beta)}$,$\mathbf{U}_N^{(\beta)}$), cylindrical symmetry (e,f) ($\mathbf{U}_M^{(c)}$,$\mathbf{U}_N^{(c)}$), spherical symmetry (g,h) ($\mathbf{U}_M^{(s)}$,$\mathbf{U}_N^{(s)}$). The white arrows depict the direction of the in-plane electric field $\mathbf{E}$. The topological charge is $m = 2$, the handedness $\beta = 1$ (a,b) and $\beta = -1$ (c,d) and the complex displacement $kz_0 = 1.5$.

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About asymptotic approach to electromagnetic beams propagation in layered periodic medium

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Monochromatic solutions of Maxwell equations in half-space filled with periodic medium are studied asymptotically. The field on the boundary is assumed to be known and vary slowly with respect to the period of the structure. The method of two scale asymptotic expansions is used for the construction of full asymptotic expansion of the field. The leader-order term describes the transformation of modes of TM and TE polarization in the medium.

An asymptotic solution for the problem of adiabatic sound propagation in an underwater canyon

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The problem of 3D sound propagation in a shallow-water waveguide featuring a canyon-type bottom inhomogeneity is considered. If the canyon depth is sufficiently small and the propagation is adiabatic (i.e. the variations of depth do not cause the mode cut-off), an asymptotic solution to this problem may be obtained using the mode parabolic equations method (MPEs) [1, 2]. For the acoustic track directed along the canyon mode parabolic equations admit separation of variables. For example if the bottom relief is described by the function $z = H(x, y)$ (x and y are horizontal coordinates, z denotes depth), where

$$H = H_0 + \frac{H_1}{\cosh^2(\sigma y)}\,,$$

(H_0 is the depth of the sea and H_1 is the depth of the canyon with respect to the sea bottom) the solution to MPEs may be written in terms of the associated Legendre polynomials [3] (for the track aligned along the x axis). This solution allows us to answer the question whether the sound will undergo usual cylindrical spreading or become partially trapped inside the canyon for a given shape of the latter. It may be also useful for the benchmarking of the 3D sound propagation models.

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A method for single-hydrophone geoacoustic inversion based on the modal group velocities estimation: application to a waveguide with inhomogeneous bottom relief

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In recent years a new method of bottom properties inversion in shallow-water geoacoustic waveguides based on the modal group velocities estimation with a single receiver was developed in underwater acoustics community [1]. This technique relies on the mode dispersion curves filtering from the spectrogram of a pulse signal subjected to a so-called time-warping transform which allows to improve mode separability [2]. When the dispersion curves $\tau_k = \tau_k(f)$ (here τ_k is the delay time of k-th mode and f is the frequency) of individual modes are separated from each other, we may use the dispersion data (i.e. modal delays) as the input for geoacoustic inversion. For a given bottom model M we compute "theoretical" delay times $\tau^M = \tau_k^M(f)$ and the residue function $E(M) = \|\bar{\tau} - \bar{\tau}^M\|$. The inversion process is organized as the minimization of $E(M)$, i.e. we search M such that $E(M) \to \min$.

Assuming that the mode interaction is sufficiently weak, we can generalize this technique to the case of a waveguide featuring bottom relief inhomogeneities. To this end, we use the adiabatic theory of mode propagation. The modal delays in this case are computed by

$$\tau_k^M(f) = \int_{r=0}^{r=r_{max}} \frac{dr}{V_{gr}^k(f,r)}\,. \tag{1}$$

where r is range along the acoustic track and $V_{gr}^k(f,r)$ is the modal group velocity computed for the frequency f at the point r (modal group velocities vary with the range since the bottom depth $h = h(r)$ is range-dependent).

We apply our method to the estimation of bottom parameters near the cape Schultz in the Posyet bay, Sea of Japan.

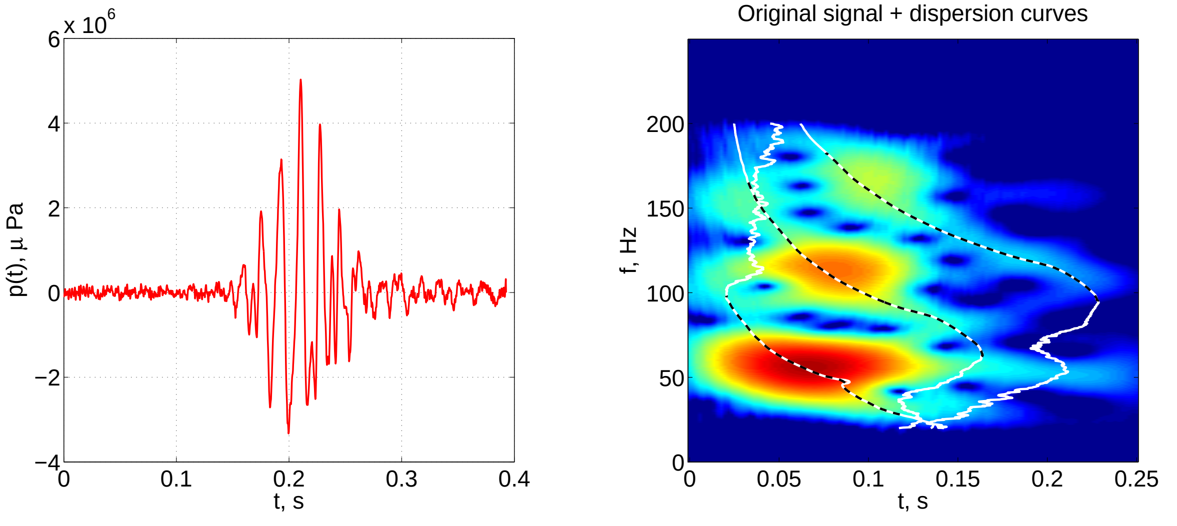

Fig. 1: A pulse sound signal (left) and its spectrogram with the filtered out modal dispersion curves $\tau_k = \tau_k(f)$ of the first three modes (right).

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Electromagnetic wave diffraction problem on shielded bi-periodical set of screens

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Let an electromagnetic wave fall down on the bi-periodical set of infinity thin ideally conducting screes (on the grating) placed over the conducting plane. It is necessary to find electromagnetic field arising by diffraction of this wave.

It is shown that if the initial wave is quasi-periodic then the diffracted wave can be quasi-periodic also. In this case the Floquet coefficients of components of unknown field must be solutions of a set of ordinary differential equations.

Boundary conditions and conjunction conditions on the grating plane are equal to vectorial pair summatorial equation. This equation is reduced to regular infinite set of linear algebraic equations by method of integral identities (or by method of over-determined boundary value problem).

Some results of numerical experiment are presented. The dependence of reflected field on the geometrical parameters of grating is investigated.

The optimization problem is considered when it is necessary to find parameters of layered structure furnishing extremum of the energy stream through the cross-section of planar layer between shield and grating.

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Synthetic aperture approach to microwave holographic image improvement

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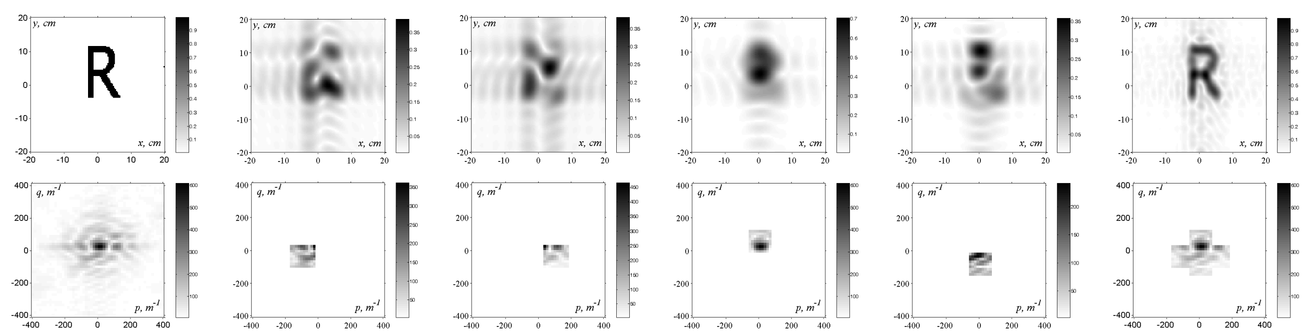
In our experiments with a prototype of microwave holographic near-field radar we met with a poor image quality due to diffraction effects. Within given technical bounds, the wavelength at the operating frequency is comparable to the typical target sizes and not small compared with the antenna array dimensions and probing range. In order to analytically describe microwave image formation and find a way to improve the holographic radar resolution we apply Fresnel–Kirchhoff diffraction theory uniformly treating target illumination, incident wave scattering, holographic data acquisition, and object reconstruction by means of numerical wave front conversion.

Consider a planar test object illuminated by a plane incident wave $E_i(x,y,l) = \exp\big[ik(x\sin\alpha_0 + y\sin\beta_0\big]$ at a finite distance l from the rectangular holographic antenna array. Let us characterize the object by reflection coefficient $f(x,y)$. In our numerical example, $f(x,y)$ equals to one over the stencil and zero outside it. Using narrow-angle Kirchhoff approximation to calculate "microwave hologram" $h(x_0,y_0)$ – scattered wave field registered in the antenna aperture $z_0 = 0$, $|x_0| < a$, $|y_0| < b$) and applying backward Kirchhoff transform to reconstruct the holographic image $g(x,y)$ from $h(x_0,y_0)$ we obtain an integral operator directly relating modified object image $\tilde{g}(x,y) = g(x,y)\,\exp\big(ik\,\frac{x^2+y^2}{2l}\big)$ with the original $\tilde{f}(\xi,\eta) = f(\xi,\eta)\,\exp\big(ik\,\frac{\xi^2+\eta^2}{2l}\big)$:

$$\tilde{g}(x,y) = \frac{1}{\pi^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\tilde{f}(\xi,\eta)\,e^{i(p_0\xi+q_0\eta)}\,\frac{\sin\mu(\xi-x)}{\xi-x}\cdot\frac{\sin\nu(\eta-y)}{\eta-y}d\xi\,d\eta$$

Here, $\mu = \frac{ka}{l}$, $\nu = \frac{kb}{l}$ are Fresnel parameters of the effective antenna aperture while $p_0 = k\sin\alpha_0$, $q_0 = k\sin\beta_0$ characterize the illumination conditions. In terms of spatial spectrum this reads: $\tilde{G}(p,q) = \tilde{F}(p-p_0,\,q-q_0)\cdot\Pi_\mu(p)\cdot\Pi_\nu(q)$ where $\Pi_\mu(p)\cdot\Pi_\nu(q)$ is a step function equal to unity inside the rectangle $|p| < \mu$, $|q| < \nu$ and to zero outside it. This formula makes obvious the action of this operator: it cuts from the target spatial spectrum a rectangular segment centered according to the illumination angles. For a successful object reconstruction the acquired rectangle must cover a significant part of the target spatial spectrum. If the antenna aperture is too small to meet this condition a drastic image distortion takes place. The following pictures illustrate microwave image formation.



(a) (b) (c) (d) (e) (f)

The upper row shows a 10×15 cm wide test object (a) and its distorted images obtained from numerical holograms registered by a rectangular 40×36 cm holographic antenna array placed at $l = 50$ cm from the target. Four successive images (b), (c), (d), (e) differ by illumination angles: $(\tan\alpha_0,\ \tan\beta_0) = (30/50,\ 10/50)$ – illumination from the left; $(-30/50,\ 10/50)$ – from the right; $(0,\ -10/50)$ – from above; $(0,\ 25/50)$ – from below. The corresponding spatial spectra of the target and of its distorted images, shown in the next row, clarify the nature of this distortion. The last picture (f) in the upper row depicts the synthetic image obtained by coherent superposition of the four numerical holograms (b)–(e). Its spectral "puzzle" shown below covers a substantial part of the target spectrum, which explains radical improvement of the image quality.

Matching of local asymptotics in the illuminated part of Fock domain

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The paper by V.A. Fock on the wave field in a vicinity of a conducting body [1, 2] is the earliest article where the Leontovich–Fock method of parabolic equation in the theory of diffraction and wave propagation was proposed and developed. However, there are some assumptions of physical character which restrict applicability of their results by a narrow vicinity of the incident plane where the limiting ray is located. Therefore the mathematical technique of that paper cannot be directly used for investigation of the shortwave diffraction problems for elongated bodies, compare with [3, 4]. Exploration of those problems requires detail consideration of matching of local asymptotics in the illuminated part of Fock domain. In the presentation we solve that task by means of direct construction of the reflected wave with the help of the ray method. The main problem on that way, which was estimated by V.A. Fock in [1, 2] as rather difficult, is the calculation of the eikonal and geometrical spreading in curvilinear coordinates used in the boundary layer method in the vicinity of scatterer. The mathematical technique proposed in the presentation can be used for exploration of shortwave diffraction by strongly elongated bodies.

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Influence dispersion of structural gas molecules models

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Many experimental facts tell us about the importance of gradients of physical values (density, linear momentum, energy, angular momentum). Currently, the angular momentum is not used, it is replaced by the symmetry of the stress tensor but it is responsible for the dispersion properties of the system. Taking into account the angular momentum law nonsymmetrical stress tensor is received. The method for calculation of nonsymmetrical part is suggested. In the previous studies, the problem of influence of dispersion on the models and equations of continuum mechanics was considered carefully for various applications of gas and solids without structure. In those papers one can find also historical facts concerning different approaches to this problem, as well as some examples; in particular, modified Navier–Stokes equations, connection to kinetic theory, boundary layer, shock waves, numerical solutions, asymptotical methods, etc., so there is no need to repeat all results. The modified equations for gas was found from the modified Boltzmann equation and from the phenomenological theory. For a rigid body the equations was used of the phenomenological theory, but changed their interpretation. Now for consideration of angular momentum are used the theory of brothers E. Cosserat, F. Cosserat and their modifications. Their theory contains additional constant with dimension of length that determined from experiments. The angular momentum does not contain the new dimension constants. Therefore, the equations do not have to describe it contain new constants. The order of the new equations (for the density and linear momentum, energy) more than in the classical case. If we are dealing with a continuous medium, the boundary condition at the outer edge of the boundary layer can be defined by the value of the vortex or the value of the vertical velocity. On the surface you want to set the friction. For the turbulent layer is usually given friction. For a rarefied gas flow must be set apart from the usual boundary conditions. In general, the formulation of the boundary conditions requires further study. A large number of experiments tell us about the bad turbulence theories. For the turbulent layer is usually given friction. Usually use the following theory: a direct numerical method, the large eddies, the Reynolds-averaged equations and others.

In classic theory we suggest that the energy is passing from big vortex to little vortex without back direction. The role of high-frequency spectrum on slowly basic flows is investigated for the modified Korteweg – de Vries – Burgers (KBGB) equation. At this example we received importance of back crossing. For structure molecules the influence of angular momentum is studied for quantum kinetic theory. New continued equations with the inclusion of angular momentum and cross streams contain only two dimensionless parameters: the Reynolds number and Prandtl number as angular momentum does not include new constants. It is interesting that angular momentum is written in Lagrangian coordinates. So we have not the stationary waves as it depend from t. The experiments give us this depending on t for elasticity theory, the flow becomes turbulent at large distance (the tired effects). It can be shown that the functional with the momentum for elasticity problems and for gas is a local minimum. The estimates of the contributions of surface and bulk quantities is a local minimum is reached, the global is the same. Thus, we propose to add the description of continuum mechanics for gas and solid without structure law of conservation of angular momentum and refine this conservation law for the structure of the gas.

The Wigner distribution function of three-Airy beams

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The Wigner distribution function (WDF) is one of the most famous so-called phase-space distributions in quantum mechanics and optics [1, 2]. The WDF is used for simultaneous description of an object and its Fourier image.

The WDF of a 2D coherent paraxial light field $f(\mathbf{r})$ is defined by the equation

$$W[f](\mathbf{r},\mathbf{k}) = \frac{1}{4\pi^2} \iint_{\mathbb{R}^2} e^{-i\langle \mathbf{k},\boldsymbol{\rho}\rangle} f\Big(\mathbf{r}+\frac{\boldsymbol{\rho}}{2}\Big)\overline{f}\Big(\mathbf{r}-\frac{\boldsymbol{\rho}}{2}\Big)\,\mathrm{d}^2\boldsymbol{\rho}, \tag{1}$$

where $\mathbf{r} = (x, y) = (r\cos\phi, r\sin\phi)$, $\mathbf{k} = (k_x, k_y)$, $\boldsymbol{\rho} = (\xi, \eta)$ are 2D vectors, $\langle \bullet, \bullet \rangle$ is the scalar product in the space $\mathbb{R}^2$, and an overline means complex conjugation.

Three years ago a novel type of 2D light fields, named three-Airy beams,

$$\mathrm{tAi}(\mathbf{r}; c) = \mathrm{Ai}\Big(\frac{x\sqrt{3}-y}{2} + c\Big)\mathrm{Ai}\Big(\frac{-x\sqrt{3}-y}{2} + c\Big)\mathrm{Ai}(y+c) \tag{2}$$

has been proposed [3]. It has been shown that the Fourier image of these fields has a cubic phase and a radially symmetric intensity with super-Gaussian decrease:

$$\frac{1}{2\pi}\iint_{\mathbb{R}^2} e^{-i\langle \mathbf{r}, \boldsymbol{\rho}\rangle}\,\mathrm{tAi}(\boldsymbol{\rho}; c)\,\mathrm{d}^2\boldsymbol{\rho} = \frac{1}{3^{5/6}\pi}\exp\Big(-\frac{2i}{27}r^3\sin 3\phi\Big)\mathrm{Ai}\Big(3^{2/3}c + \frac{2}{3^{4/3}}r^2\Big). \tag{3}$$

Recently various topics on three-Airy beams have been investigated theoretically and experimentally [4, 5, 6].

In this work we calculate the WDF of three-Airy beams:

$$\begin{aligned} W[\mathrm{tAi}](\mathbf{r}, \mathbf{k}) = \frac{1}{\pi^3\sqrt{3}}\int_{\mathbb{R}} &\mathrm{Ai}\Big(2^{2/3}\Big[c + \frac{x\sqrt{3}-y}{2} + \Big(t - \frac{k_x}{\sqrt{3}}\Big)^2\Big]\Big) \\ &\times \mathrm{Ai}\Big(2^{2/3}\Big[c - \frac{x\sqrt{3}+y}{2} + \Big(t + \frac{k_x}{\sqrt{3}}\Big)^2\Big]\Big)\mathrm{Ai}\big(2^{2/3}\big[c + y + (t-k_y)^2\big]\big)\,\mathrm{d}t \end{aligned} \tag{4}$$

and obtain some corollaries containing Bessel and Airy functions, for example,

$$\frac{3^{5/6}}{2}\int_0^\infty J_0\big(t^3\sqrt{2}\big)\mathrm{Ai}^2\big(3^{2/3}[c+t^2]\big)\,t\,\mathrm{d}t = \int_0^\infty \mathrm{Ai}^3\big(2^{2/3}[c+t^2]\big)\,\mathrm{d}t. \tag{5}$$

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Solitons in a dynamical billiard

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We consider interaction of a Bose–Einstein condensate soliton with a single oscillating barrier, regular or chaotic solitons' behavior in a dynamical billiard formed by harmonically oscillating barriers, and various scenarios of interaction of such solitons.

Dynamical billiards with classical point particles bouncing between oscillating barriers or walls were widely investigated. In the talk, we analyze features of scalar "longitudinal" and vector "transverse" solitons in a dynamical billiard. For definiteness, we analyze the atomic Bose–Einstein condensate (BEC) in a dynamical trap with oscillating walls.

The governing equation is the Gross–Pitaevskii equation for the wave function of weakly nonideal atomic gas at zero temperature. For BEC confined by the trap in the transverse directions and ideal longitudinal barriers with square potential moving along the longitudinal direction z, the BEC wavefunction ψ obeys the equations:

$$i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m_p}\frac{\partial^2\psi}{\partial z^2} + U_0|\psi|^2\psi, \quad L_{\text{left}}(t) < z < L_{\text{right}}(t), \quad \psi(z = L_{\text{left}}(t), t) = \psi(z = L_{\text{right}}(t), t) = 0.$$

Here t is time, $\hbar$ is the reduced Planck constant, m_p is the atom mass, and U_0 is the nonlinearity constant (positive or negative). Without the trap ($L_{\text{left}} \to -\infty$, $L_{\text{right}} \to +\infty$), this is well-known nonlinear Schrödinger equation solvable by the inverse scattering method [5]. For harmonically oscillating barriers, we demonstrate various scenarios of the interaction including soliton periodic and chaotic motion.

In regimes when the BEC wave packet diffuses over the whole trap length, the Schrödinger equation with $U_0 = 0$ and harmonically oscillating barriers has highly non-equidistant quasienergy spectrum. If the modulation frequency is close to the frequency of transition between a pair of the levels, it is possible to realize effective interaction in the framework of two-level scheme. For these levels' amplitudes, a_n and a_m, the transverse dynamics is governing by the following equations with coherent linear and incoherent nonlinear coupling:

$$\left(i\hbar\frac{\partial}{\partial t} + \frac{\hbar^2}{2m_p}\nabla_\perp^2\right)a_n + (-1)^{m-n}\mu nm E_1^{(0)} a_m - U_0\left(\frac{3}{4}|a_n|^2 + |a_m|^2\right)a_n = 0,$$
$$\left(i\hbar\frac{\partial}{\partial t} + \frac{\hbar^2}{2m_p}\nabla_\perp^2\right)a_m + (-1)^{m-n}\mu nm E_1^{(0)} a_n + \left[\hbar\delta\Omega - U_0\left(\frac{3}{4}|a_m|^2 + |a_n|^2\right)\right]a_m = 0.$$

Here $\nabla_\perp^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2$ is the transverse Laplacian, x and y are the transverse coordinates, μ is the barrier position modulation depth, $\delta\Omega$ is frequency detuning, and $E_1^{(0)}$ is the energy of the ground state of the linear quantum well for $\mu = 0$. For transversely 1D-geometry we present families of "transverse" two-component (vector) solitons corresponding to these equations and various scenarios of solitons' interaction including breather formation.

Optical vortices formation in mirror-symmetric structures

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Interference phenomena are widely used in technological processes and various-duty devices. For example, they show promise as he bases for the creation of photon crystals [1, 2]. Utilization of interference field provides the opportunity to proceed spatial order of ensemble of microparticles [3, 4]. Besides, regular interference pictures can be an instrument for influence on biological systems [5, 6]. The interference of decaying and surface electromagnetic waves is an effective method to move across the diffraction barrier in photolithography and form micro- and nanostructures with the dimensions several times smaller than the wavelength [7].

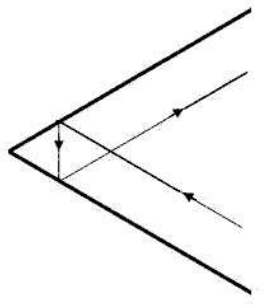
In the given work, the diffraction splitting of a wave is considered into partial waves with their subsequent interference in mirror-symmetric structures. Such structures are formed by intersection of mirror-plane surfaces at the angles π/s, where s is the integer number. They can be represented as di-, tri- or polyhedral ones, as closed as open. Here, we consider only dihedral structures.

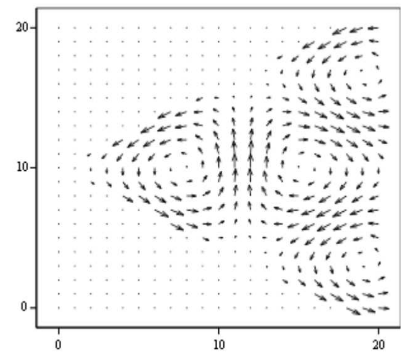
Let the edge of a dihedral angle $|x| \leq z\tan\alpha$, $z \leq 0$ is aligned along the y-axis. The angle of opening is equal $2\alpha = \pi/n$. Then the interference field of the E wave can be written in the form

$$u = \sum_{j=0}^{2n-1}(-1)^j \exp\left\{-ih\left[(-1)^j x\sin(\beta - j\pi/n) + z\cos(\beta - j\pi/n)\right]\right\}$$

up to a constant factor. The diffraction field for the H wave is written in much the same way.

The number of waves in interference is equal to $2n$. Waves arise as a result of the incident wave ($j = 0$) diffraction in mirror-symmetric structures. For even values of the number n, waves have opposite directions of propagation in pairs. Consequently, the interference picture is formed by superposition of standing waves, which are oriented in space in different ways. For odd values of n such a picture is observed only at $\beta = 0$. For another values of this parameter the interference picture includes running waves, and optical vortices can arise. Figure shows the directions of wave propagation and the Poynting vector at $n = 3$ and $\beta = \pi/6$.





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Estimates of spectral bands for Laplacians on periodic equilateral metric graphs

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We consider Laplacians operators on periodic equilateral metric graphs. It is known that the spectrum of these operators consists of an absolutely continuous part (which is a union of an infinite number of non-degenerated spectral bands) plus an infinite number of flat bands, i.e., eigenvalues of infinite multiplicity. We obtain the following results: 1) estimates of the Lebesgue measure of the spectrum on a finite interval in terms of geometric parameters of the graph, 2) localization of the spectral bands in terms of eigenvalues of discrete Laplacians on some finite graphs, 3) detailed analysis of all spectral bands, the existence and positions of the flat bands for specific graphs. The proofs are based on spectral properties of discrete Laplacians.

Asymptotic of linear water waves in a basin with fast oscillating bottom

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We consider the linear system of differential equations for the water waves in the basin with fast oscillating bottom. The bottom has a form of slow changing background with additional and fast oscillating. Using the homogenization method in an operator form we reduce original problem to a pseudodifferential equation for the potential on the free surface [1], [2], [3]. This equation includes two types of dispersion: the first one is a "standard" water wave dispersion and the second one is "anomalies" dispersion connected with the fast oscillation of the bottom. We compare the influence of of these two dispersion into the the different types of waves.

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Diffraction on a grating composed of absorbing screens. Asymptotic results

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In [1, 2] authors considered diffraction of high frequency grazing wave on gratings showed in Fig. 1 and Fig. 2. A recently developed approach [2, 3] based on a embedding formula and a "spectral" equation for the directivity of an edge Green's functions has been applied to the problems. An evolution equation was introduced for the problem showed on fig. 1. Asymptotic formula for the reflection coefficient for the principal order has been obtained:

$$R_0 = -1 - \zeta\left(\frac{1}{2}\right)\sqrt{\frac{ka}{\pi}}(1-i)\theta_{in} - \zeta\left(\frac{3}{2}\right)\sqrt{\frac{ka}{\pi}}(1+i)2^{-5/2}(2\sqrt{2}-1)\theta_{in}\eta^2 + o(\eta^2)o(\sqrt{ka}\theta_{in}), \quad (1)$$

where $\eta = y^*\sqrt{k/a}$, and ζ is Riemann zeta function.

In the present study we obtain an evolution equation for the problem showed on fig. 2. Asymptotic formula for the reflection coefficient R_0 as $|(a-b)/(a+b)| \ll 1$ is proved. Also for both problems we prove that reflection coefficient R_0 tends to -1 as $\theta_{in} \to 0$:

$$\lim_{\theta_{in} \to 0} R_0 = -1. \tag{2}$$

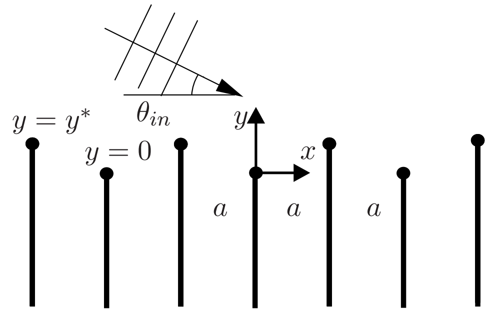

Fig. 1: Geometry of the array of scatterers in [1].

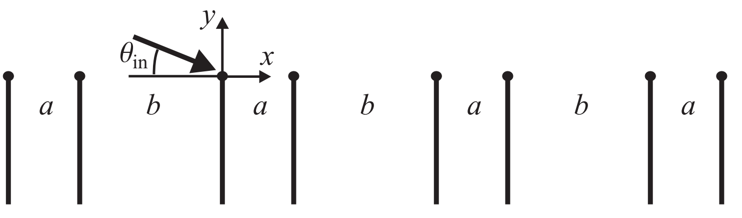

Fig. 2: Geometry of the array of scatterers in [2].

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On asymptotics for a resolvent in multidimensional problems with frequent alternation of boundary conditions.

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The work is devoted to studying the asymptotic behavior of resolvent for an elliptic operator in a multidimensional domain with frequent alternation of boundary conditions. We consider a domain with a piecewise smooth boundary. The domain can be bounded or unbounded. On the boundary of domain we select a subset depending on a small parameter. This subset consists of a large number of disjoint parts. As the small parameter tends to zero, the number of the disjoint parts increases while the sizes and the distances between them tend to zero. On these subsets we impose Dirichlet boundary condition, whereas Robin condition is imposed on the rest part of the boundary, so, we have alternating boundary conditions. The structure of alternation is assumed to be non-periodic and rather general. We consider the self-adjoint second order scalar differential operator the second order variable coefficients and deal with the case when the homogenized operator involves Dirichlet boundary condition instead of alternating ones. The main aim of the work is to study the asymptotic behavior of the resolvent of the perturbed operator. We establish the uniform resolvent convergence of the perturbed operator to the homogenized one in the sense of the norm of the operator acting from L_2 into W_2^1. The estimates for the rate of convergence are provided. We also construct complete asymptotic expansion for the resolvent assuming that the domain is unbounded, the alternation is periodic and the resolvent acts on sufficiently smooth functions.

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The application of spectral element method to the study of acoustic waves dispersion in non-cylindrical boreholes

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Conventional processing algorithms for acoustic logging data assume cylindrical geometry of borehole. In real life borehole drilled in the earth is often deformed due to natural or technological reasons. Previous studies showed that waveguide cross-section affects the propagation of monopole, dipole, and quadrupole modes. Analytical and semi-analytical methods, based on perturbation theory and boundary element method, can provide a quantitative assessment of dispersion curves for non-circular boreholes in isotropic media. However, direct numerical modeling is still more effective in complex cases, involving asymmetric geometry and anisotropy.

In this study computations are performed with spectral element method (SEM). Earlier it was successfully applied to the calculation of synthetic seismograms in 3-D global models of Earth. SEM is based on the finite element method and uses higher-degree Lagrange interpolants to discretize the wavefield on a mesh of hexahedral elements. Integration over an element is accomplished by Gauss–Lobatto–Legendre integration rule. Interpolation order can be chosen parametrically in order to optimize balance between speed and accuracy of calculation. Dispersion curves for guided modes are obtained by processing the calculated waveforms with a modified matrix pencil algorithm.

A series of numerical calculations were performed for elliptical and asymmetrical boreholes in isotropic elastic media. The material properties of the surrounding media correspond to typical fast and slow rocks (shear wave velocity in elastic media is higher or lower than the sound velocity in fluid). The obtained data shows good agreement with the results of previous studies for flexural and Stoneley wave dispersion in almost the entire range of considered frequencies. The flexural wave in non-cylindrical borehole split into two waves with radial polarizations parallel to the minor and major axes of the cross-section. In comparison with analytical solution for cylindrical boreholes ellipticity effect is approximately three times higher in fast formation than in slow formation in terms of relative slowness change. Quantitative values of these variations for each frequency are consistent with estimations, obtained by perturbation theory.

Thus, spectral element method proved to be fast and accurate enough for considered problems. The possibility of its application to anisotropic and inhomogeneous media gives an advantage over other mentioned approaches in future investigations.

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Longitudinal strain solitons in thin-walled shells

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We consider the evolution of a longitudinal strain soliton in a nonlinearly elastic thin-walled cylindrical shell, generalizing the previous results obtained for similar waves in cylindrical rods [1] and thin plates [2]. The axisymmetrical deformation of a cylinder is considered, and relationships between longitudinal and transversal displacements are found via asymptotic expansions with respect to a small thickness of the shell, satisfying boundary conditions on lateral surfaces. The deformation energy is described by Murnaghan's model, both geometrical and physical nonlinearities of the material are taken into account, and application of the Hamilton principle leads to the equation, which in case of homogeneous and isotropic wave guide has the form of the Double Dispersive Equation (DDE) [2] for the longitudinal strain component u:

$$u_{tt} - \frac{\alpha}{\rho}u_{xx} = \left[\frac{\beta}{\rho}u^2 + \frac{\nu^2 h^2}{12(1-\nu)^2}\left(u_{tt} - \frac{\mu}{\rho}u_{xx}\right)\right]_{xx}, \tag{1}$$

with coefficients $\alpha = \alpha(E, \nu, R_0, h), \beta = \beta(E, \nu, m, l, n, R_0, h)$, where E is Young's modulus, ν is Poisson's ratio, μ is shear modulus, (m, l, n) are the Murnaghan 3rd-order moduli, ρ is density, R_0 is the radius of the middle surface and h is the shell thickness. Note that even the linear wave speed depends on the complex value of α, not E. The equation (1) has a solitary wave solution

$$u = \frac{3(\rho V^2 - \alpha)}{2\beta}\cosh^{-2}\left(\frac{(1-\nu)}{\nu h}\sqrt{\frac{3(\rho V^2 - \alpha)}{\rho V^2 - \mu}}(x \pm Vt)\right), \tag{2}$$

where $z = x \pm Vt$ is the phase variable, and V denotes the wave velocity. It is worth to note that the soliton amplitude depends on both geometrical and material properties of the shell, that provides an opportunity to apply this model to nondestructive testing of solids.

We performed physical experiments, similar to [3], and found the bulk strain soliton in a duct-like polymer shell. A 300-mm-long duct shell with 3 mm thick walls and 10×10 mm cross section has been used to observe bulk strain soliton in experiments. In Fig. 1 the fringe shift shown below the holographic interferogram presents the soliton in the PMMA hollow duct at the distance 70–120 mm from the shell input.

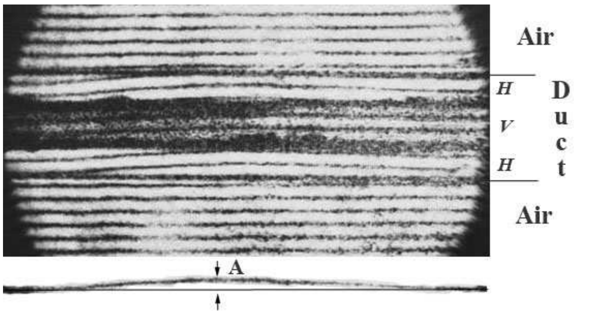

Fig. 1: The bulk soliton in a duct shell. H and V denote the horizontal and vertical duct walls.

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Tensor permittivity reconstruction of two-sectional diaphragm in a rectangular waveguide

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In this paper we consider tensor permittivity determination of two-sectional diaphragm in a rectangular waveguide. This study employs the technique developed in [1, 2] and deals with tensor permittivity reconstruction of layered materials in the form of diaphragms (sections) in a waveguide of rectangular cross section from the transmission coefficient. We perform a detailed analysis for two-sectional diaphragm. Numerical results are presented.

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The research of electromagnetic waves diffraction problem on the perfectly conducting arbitrary shaped screens by a subhierarchical method

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The diffraction problem of electromagnetic waves on the perfectly conducting screens is considered. We applied subhierarchical method. Discretization of the problem is made according to [1, 2]. The numerical solutions of diffraction problem on the arbitrary shaped screens in free space are presented.

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Scalar problem of diffraction of a plane wave on a system of two- and three-dimensional scatterers

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The scalar problem of diffraction of a plane wave on a system of two screens with the boundary conditions of the first and second kind and volume inhomogeneous body in quasi-classical statement is considered.

The initial boundary value problem for the Helmholtz equation leads to a system of integral equations on two- and three-dimensional manifolds with boundary. The volume integral operator is weakly singular as well as the operator on the surface of acoustically “soft” screen, integral operator on the surface of acoustically “hard” screen is hypersingular. These operators are treated as pseudodifferential operators in Sobolev spaces on manifolds with boundary.

Owing to known results on properties of the PsD operators some important results are received: the smoothness of the solution to the system of the integral equations in the interior points of the screens is proved; the equivalence of the integral equations to an initial boundary value problem under assumption of infinite differentiability of the incident field is established. Finally, the unique solvability of the BVP and Fredholm property of matrix integral operator results its invertibility.

Galerkin method for numerical solving of the integral equation is proposed. In the area of the volume body and on the surface of the "soft" screen piecewise constant basis functions are introduced, while piecewise linear functions on the "hard" screen are considered. The description of basic functions is provided in the work, the approximation property as well as the theorem of convergence of Galerkin method is proved.

The numerical results are provided.

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Propagation of TE waves in a double-layer nonlinear inhomogeneous cylindrical waveguide

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Consider a monochromatic TE wave in the form $\mathbf{E}e^{-i\omega t}$, $\mathbf{H}e^{-i\omega t}$, where

$$\mathbf{E} = (0, \mathrm{E}_\varphi(\rho)e^{i\gamma z}, 0)^T, \quad \mathbf{H} = (\mathrm{H}_\rho(\rho)e^{i\gamma z}, 0, \mathrm{H}_z(\rho)e^{i\gamma z})^T, \tag{1}$$

are the complex amplitudes; ω is a circular frequency; $(\ \cdot\ )^T$ is the transposition operation; γ is unknown (real) spectral parameter (propagation constant of a guided wave); $\mathrm{E}_\varphi, \mathrm{H}_\rho, \mathrm{H}_z$ are unknown functions. The TE wave propagates along the surface of the lossless double-layer dielectric waveguide

$$\Sigma := \{(\rho,\varphi,z) : 0 \leqslant \rho < R_1, 0 \leqslant \varphi < 2\pi\} \cup \{(\rho,\varphi,z) : R_1 \leqslant \rho \leqslant R_2, 0 \leqslant \varphi < 2\pi\}$$

with circular cross-section. The waveguide Σ is located in cylindrical coordinate system $O\rho\varphi z$ and is filled with inhomogeneous isotropic medium. The inner part $\rho < R_1$ of the waveguide Σ and the space $\rho > R_2$ are filled with homogeneous isotropic media with constant permittivities $\varepsilon = \varepsilon_1 \geqslant \varepsilon_0$ and $\varepsilon = \varepsilon_3 \geqslant \varepsilon_0$, respectively, ε_0 is the permittivity of free space. Entire space contains no sources.

It is supposed that everywhere $\mu = \mu_0$, where μ_0 is the permeability of free space. For $R_1 \leqslant \rho \leqslant R_2$ (inside the waveguide Σ) the permittivity ε is described by the formula

$$\varepsilon = \varepsilon_2(\rho) + \alpha\big(1 - e^{-\beta|\mathbf{E}|^2}\big),$$

where $\min\{\varepsilon_2(\rho) : \rho \in [R_1, R_2]\} > \max(\varepsilon_1, \varepsilon_3)$, $\varepsilon_2(\rho) \in C[R_1, R_2]$, and $\alpha, \beta \geqslant 0$ are real constants.

Complex amplitudes (1) of the TE wave must satisfy Maxwell's equations

$$\begin{cases} \operatorname{rot} \mathbf{H} = -i\omega\varepsilon\mathbf{E}, \\ \operatorname{rot} \mathbf{E} = i\omega\mu\mathbf{H}; \end{cases} \tag{2}$$

the continuity condition for the tangential components of the field on the boundaries $\rho = R_1$ and $\rho = R_2$; and the radiation condition at infinity: the electromagnetic field decays as $O(\rho^{-1})$ when $\rho \to \infty$. The solution is sought for in the entire space.

The continuity conditions for the tangential components are

$$[\mathrm{E}_\varphi]\big|_{\rho=R_1} = 0, \quad [\mathrm{E}_\varphi]\big|_{\rho=R_2} = 0, \quad [\mathrm{H}_z]\big|_{\rho=R_1} = 0, \quad [\mathrm{H}_z]\big|_{\rho=R_2} = 0. \tag{3}$$

Problem P_E: *it is necessary to determine eigenvalues* $\hat{\gamma}$ *for which there exist nontrivial functions* $\mathrm{E}_\varphi(\rho;\hat{\gamma}), \mathrm{H}_\rho(\rho;\hat{\gamma}), \mathrm{H}_z(\rho;\hat{\gamma})$ *that are defined for* $\rho \in (0, +\infty)$*, satisfies equation* (2) *and transmission conditions* (3) (further details and similar problems see in [1–4]).

Numerical results are presented, comparison with linear in- and homogeneous cases is given.

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Parallel computing for numerical calculations of step-index optical fibers eigenmodes by collocation method

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We study the natural modes of a weakly guiding optical fiber, which is a representative of typical optical circuits. In recent years, research on the natural modes of arbitrarily shaped optical fibers has been focused on the development of efficient and reliable computational methods. Many different numerical techniques are applied for computing eigenmodes of optical fibers, namely, Finite-element, Finite-difference, beam propagation, and spline collocation methods, as well as multidomain spectral approach.

The most rigorous efforts were connected with integral equation formulations. Particularly, the problem on surface and leaky eigenmodes of a weakly guiding step-index optical waveguide was considered in our previous works. The original problem was reduced to a nonlinear nonselfadjoint spectral problem for the set of weakly singular boundary integral equations. The integral operator was approximated by collocation method and by Galerkin method. The convergence and quality of these numerical methods was proved by numerical experiments. The collocation method demonstrated better speed of convergence.

In this work we develop the collocation method for numerical calculations of step-index optical fibers eigenmodes. The main difficulty with practical solution of nonlinear nonselfadjoint spectral problems is a calculation of good initial approximations for eigenvalues. We propose to use the singular value decomposition of the collocation method matrix for the initial approximation of eigenvalues. Our numerical experiments showed practical effectiveness of such approach, but singular value decomposition needs in high performance computations. Therefore, we have used the parallel computing technologies (OpenMP and MPI) and have calculated on APK-1 supercomputer. Our software package can be used for numerical simulations of new type's optical fibers.

Diffraction of electromagnetic wave on skin capillary

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Human skin is a very intricate tissue that exhibits complex material behaviour [1]. Due to potential applications in medical diagnostic, therapeutic and surgical procedures, the understanding of optical/electromagnetic properties of biological tissues is an active and important research topic. In this work, we have proposed a theoretical model for the diffraction of electromagnetic wave on the skin blood vessel. For this purpose, the Green's function formalism is used [2]. The wavelength of the incident electromagnetic field for medical purposes is around 1 mm [3], while the standard diameter of the blood vessel is between 0.01–0.2 mm. Therefore, in this paper the problem of diffraction on the capillary located in the layered media is investigated in the long-wave approximation. The basic concept of the model under consideration is shown in Fig. 1, which includes three layers and the blood vessel represented by the cylinder with a wall of uniform thickness.

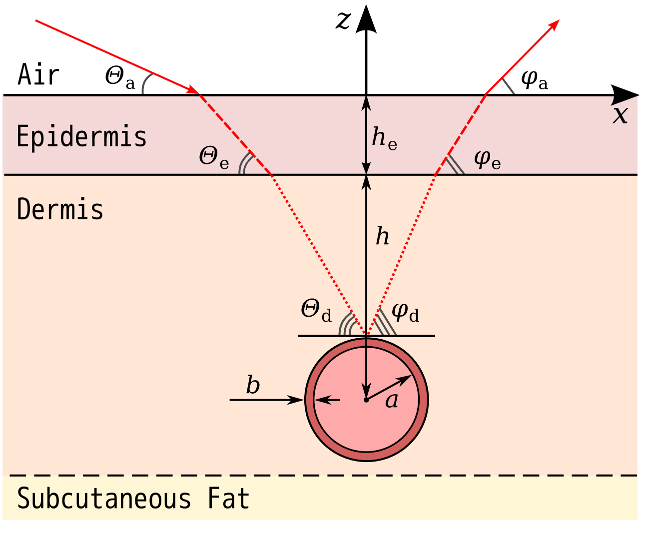

Fig. 1: The simplified three-layer skin structure.

The calculations performed in the paper are based on real experimental data, which makes it more compelling. As an important result of the study, it is established that the amplitude of the scattered field is proportional to the square of the capillary area. Besides, the comparison of simulation results for TE and TM waves allows to determine the dielectric constant of the vessel. For the case of long waves, the reflected field is linearly dependent on the depth position of the capillary. These findings of our work have significant implications and contribute valuable insights for the modeling of layered biological systems.

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Green's function asymptotic in periodic medium

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The study of propagation and scattering of waves is of fundamental importance in diverse areas of applications. The Green's function method is a well-developed technique used for this purpose and is widely adopted in investigations of liquid crystals and photonic band structures [1]. Here we have applied this approach to a one-dimensional periodic structure, e.g. with a sinusoidal or step-like form, Fig. 1. The solution is constructed by the stationary phase method [2]. In contrast to a homogeneous medium, the Green's function for the periodic structure has a number of features. There are areas in which the wave field is described not only by a single wave, but by a sum of wave fields with the maximal number defined by the properties of the structure. In addition, there exist allocated directions in which there is an occurrence or disappearance of an additional beam summand. The asymptotic of the wave field for these directions is described through the Airy function [3] and in these directions the Green's function and the power flux density decreases slowly. It is important to point out that the derived formulas do not depend on the type and magnitude of periodicity. As an example of the model application, the far field of a point source in a two-layered medium with a step-like spatial variation of wave numbers (see Fig. 1) is considered in details. The obtained results are illustrated by numerical calculations. One may conclude that the concept proposed in this work offers additional possibilities to control and manipulate the light flow.

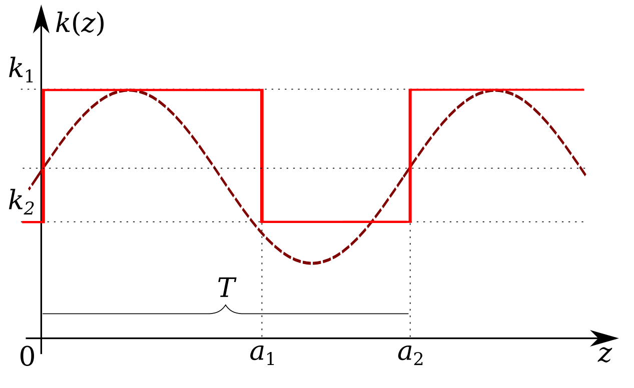

Fig. 1: Possible variants of the structure under investigation with the periodicity T. In the case of a two-layered medium (straight line) the values $\{a_1,\ a_2,\ k_1,\ k_2\}$ are the layer widths and the wave numbers of the medium, respectively.

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Forced oscillations of the elastic strip with a longitudinal crack

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One of the pressing problems in the field of propagation and diffraction of elastic waves is the the problem of wave scattering by a crack. Such problems are often encountered in ultrasound defect testing in metal frameworks as well as in studying the processes of propagation of elastic waves in creviced media.

In the present work, a two-dimensional problem of scattering of an elastic wave by a horizontal finite-size crack located inside the strip is considered. It is assumed that the oscillations are of the forced type and their source is located on one of the sides of the infinite plate.

The Cartesian coordinate system is chosen such that walls of the strip coincide with straight lines $y = 0$ and $y = H_2$. The straight line $y = H_1$ conditionally splits the strip of density ρ, longitudinal velocity v_P and transverse velocity v_S into two parts (see Fig. 1). The crack's location is on this straight line at $x \in (0, L)$. It is assumed that the crack edges oscillate freely. In addition, a material of the strip is assumed to be homogeneous and isotropic.

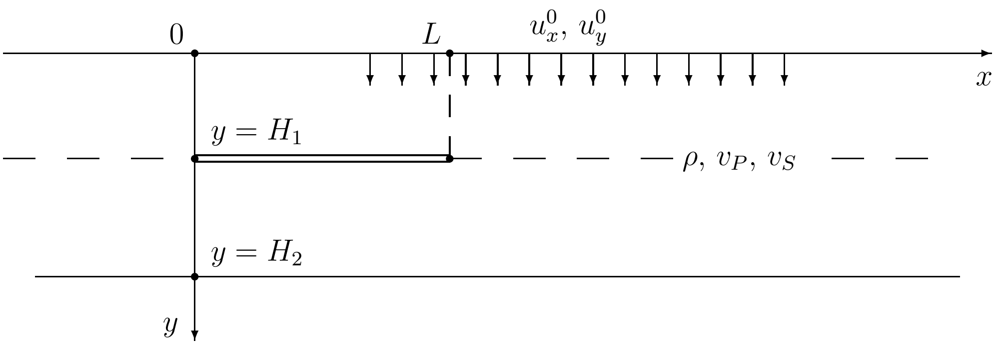

Fig. 1: A crack inside the elastic strip.

Suppose that a source of the oscillations is located on the upper boundary of the layer at $y = 0$: $u_x(x, 0) = u_x^0(x)$ and $u_y(x, 0) = u_y^0(x)$. The lower boundary is assumed to be a free boundary: $\sigma_y(x, H_2) = 0$, $\tau(x, H_2) = 0$.

Mathematical formulation of the problem of diffraction of the elastic wave by a longitudinal crack located inside the strip is following: it is required to find a solution to the system of equations from the dynamic theory of elasticity satisfying the boundary conditions and the condition at the crack: $\sigma_y(x, H_1) = 0$, $\tau(x, H_1) = 0$, at $x \in (0, L)$.

As a result, a representation of the field in every part of the strip is obtained [1]. Applying a condition of continuity in the layer, except for the region of the crack's location, reduces the diffraction problem to the system of paired integral equations. In turn, applying the Galerkin method allows reducing the system of integral equations to the system of linear algebraic equations.

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Stability of autoresonance under random perturbations

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The main object of research is the system of two differential equations:

$$\frac{dr}{dt} = \sin\psi, \quad r\left[\frac{d\psi}{dt} - r^2 + \lambda t\right] = b\cos\psi; \quad (\lambda, b = \text{const} > 0). \tag{1}$$

These equations describe an initial stage of capture in resonance for different nonlinear oscillating systems under weak excitation. The unknown functions $r(t)$, $\psi(t)$ are slowly varying amplitude and phase shift of fast harmonic oscillations. The solutions with growing amplitude $r(t) \approx \sqrt{\lambda t}$ and bounded phase shift $\psi \approx \pi$ as $t \to \infty$ correspond to autoresonance phenomenon [1].

The perturbed equations are considered in the form:

$$\frac{dr}{dt} = \sin\psi + \xi(r,\psi,t,\omega), \quad r\left[\frac{d\psi}{dt} - r^2 + \lambda t\right] = b\cos\psi + \eta(r,\psi,t,\omega).$$

Here, $\xi(r,\psi,t,\omega)$ and $\eta(r,\psi,t,\omega)$ are one-dimensional stochastic processes defined on a probability space $(\Omega, \mathcal{U}, \mathbf{P})$. Stability of autoresonance phenomenon under random perturbations is discussed in the report. We identify the class of random perturbations under which the resonance solution of (1) is steady [2]. Lyapunov function is applied to prove the stability.

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Homogenization of the elliptic operators in dependence of the spectral parameter

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In $L_2(\mathbb{R}^d;\mathbb{C}^n)$, we consider matrix elliptic differential operators (DO's) $A_\varepsilon = b(\mathbf{D})^* g(\mathbf{x}/\varepsilon) b(\mathbf{D})$, $\varepsilon > 0$. Here $g(\mathbf{x})$ is a bounded and uniformly positive definite $(m\times m)$-matrix-valued function periodic with respect to some lattice Γ. Next, $b(\mathbf{D})$ is a first order DO of the form $b(\mathbf{D}) = \sum_{j=1}^d b_j D_j$, where b_j are constant $(m \times n)$-matrices. Assume that $m \geqslant n$ and the symbol $b(\boldsymbol{\xi}) = \sum_{j=1}^d b_j \xi_j$ is such that

$$\operatorname{rank} b(\boldsymbol{\xi}) = n, \quad 0 \neq \boldsymbol{\xi} \in \mathbb{R}^d. \tag{1}$$

Let $A^0 = b(\mathbf{D})^* g^0 b(\mathbf{D})$ be the *effective operator*, where g^0 is the constant positive *effective matrix* (defined as usual in homogenization theory). We are interested in the behavior of the resolvent $(A_\varepsilon - \zeta I)^{-1}$ for small ε and $\zeta \in \mathbb{C} \setminus \mathbb{R}_+$.

Theorem 1 [1]. *Let $\zeta = |\zeta| e^{i\varphi} \in \mathbb{C} \setminus \mathbb{R}_+$. Let $c(\varphi) = |\sin\varphi|^{-1}$ if $\varphi \in (0,\pi/2) \cup (3\pi/2, 2\pi)$ and $c(\varphi) = 1$ if $\varphi \in [\pi/2, 3\pi/2]$. Then for any $\varepsilon > 0$ we have*

$$\|(A_\varepsilon - \zeta I)^{-1} - (A^0 - \zeta I)^{-1}\|_{L_2(\mathbb{R}^d) \to L_2(\mathbb{R}^d)} \leq C_1 c(\varphi)^2 \varepsilon |\zeta|^{-1/2},$$

$$\|(A_\varepsilon - \zeta I)^{-1} - (A^0 - \zeta I)^{-1} - \varepsilon K(\varepsilon;\zeta)\|_{L_2(\mathbb{R}^d) \to H^1(\mathbb{R}^d)} \leq (C_2 + C_3|\zeta|^{-1/2}) c(\varphi)^2 \varepsilon.$$

Here $K(\varepsilon;\zeta)$ is the corresponding corrector.

Let $\mathcal{O} \subset \mathbb{R}^d$ be a bounded domain of class $C^{1,1}$. In $L_2(\mathcal{O};\mathbb{C}^n)$, we consider DO's $A_{\flat,\varepsilon}$, $\flat = D, N$, given by $b(\mathbf{D})^* g(\mathbf{x}/\varepsilon) b(\mathbf{D})$ with the Dirichlet or Neumann boundary conditions, respectively. In the case $\flat = N$ we assume that $\operatorname{rank} b(\boldsymbol{\xi}) = n$ for $0 \neq \boldsymbol{\xi} \in \mathbb{C}^d$ (which is more restrictive than (1)). The effective operator $A^0_\flat$, $\flat = D, N$, is given by $b(\mathbf{D})^* g^0 b(\mathbf{D})$ with the Dirichlet or Neumann boundary conditions, respectively. We study the behavior of the resolvent $(A_\varepsilon - \zeta I)^{-1}$ for small ε. We start with the most interesting case where $\zeta \in \mathbb{C} \setminus \mathbb{R}_+$ and $|\zeta| \geq 1$.

Theorem 2 [1]. *Let $\zeta = |\zeta| e^{i\varphi} \in \mathbb{C} \setminus \mathbb{R}_+$ and $|\zeta| \geq 1$. There exists a number $\varepsilon_0 \in (0,1]$ (depending on $\mathcal{O}$ and Γ) such that for $0 < \varepsilon \leq \varepsilon_0$ we have*

$$\|(A_{\flat,\varepsilon} - \zeta I)^{-1} - (A^0_\flat - \zeta I)^{-1}\|_{L_2(\mathcal{O}) \to L_2(\mathcal{O})} \leq C_{1,\flat} c(\varphi)^5 \left(\varepsilon |\zeta|^{-1/2} + \varepsilon^2\right), \quad \flat = D, N,$$

$$\|(A_{\flat,\varepsilon} - \zeta I)^{-1} - (A^0_\flat - \zeta I)^{-1} - \varepsilon K_\flat(\varepsilon;\zeta)\|_{L_2(\mathcal{O}) \to H^1(\mathcal{O})} \leq C_{2,\flat} \left(c(\varphi)^2 \varepsilon^{1/2} |\zeta|^{-1/4} + c(\varphi)^4 \varepsilon\right), \quad \flat = D, N.$$

Here $K_\flat(\varepsilon;\zeta)$ is the corresponding corrector.

The following theorem gives another approximation of the resolvent which may be preferable for bounded $|\zeta|$ or small $|\sin\varphi|$.

Theorem 3 [1]. *Let $\zeta - c_\flat = |\zeta - c_\flat| e^{i\psi} \in \mathbb{C} \setminus \mathbb{R}_+$. Here $c_D > 0$ is such that $A_{D,\varepsilon} \geq c_D I$ and $A^0_D \geq c_D I$, and $c_N = 0$. Let $\rho_\flat(\zeta) = c(\psi)^2 |\zeta - c_\flat|^{-2}$ if $|\zeta - c_\flat| < 1$ and $\rho_\flat(\zeta) = c(\psi)^2$ if $|\zeta - c_\flat| \geq 1$. There exists a number $\varepsilon_0 \in (0,1]$ (depending on $\mathcal{O}$ and Γ) such that for $0 < \varepsilon \leq \varepsilon_0$ we have*

$$\|(A_{\flat,\varepsilon} - \zeta I)^{-1} - (A^0_\flat - \zeta I)^{-1}\|_{L_2(\mathcal{O}) \to L_2(\mathcal{O})} \leq C_{3,\flat} \rho_\flat(\zeta) \varepsilon, \quad \flat = D, N,$$

$$\|(A_{\flat,\varepsilon} - \zeta I)^{-1} - (A^0_\flat - \zeta I)^{-1} - \varepsilon K_\flat(\varepsilon;\zeta)\|_{L_2(\mathcal{O}) \to H^1(\mathcal{O})} \leq C_{4,\flat} \rho_\flat(\zeta) \varepsilon^{1/2}, \quad \flat = D, N.$$

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Generalized spherical waves

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We present a review of generalizations of classical spherical waves, which attract attention of researchers during the past four decades. We discuss 'complex source' wavefields focusing on their sources in the real space, both in time-harmonic and non-stationary versions [1]. We also pay attention to Sheppard–Saghafi solutions [2] and X waves [3].

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Acoustic mode equations with mode interaction

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We consider the propagation of sound in the two-dimensional waveguide $\Omega = \{(r,z)|0 \le x < \infty, -H \le z \le 0\}$ (z-axis is directed upward), described by the acoustic Helmholtz equation

$$(\gamma P_r)_r + \frac{1}{r}\gamma P_r + (\gamma P_z)_z + \gamma\kappa^2 P = \frac{-\delta(z-z_0)\delta(r)}{2\pi r}\,, \tag{1}$$

where $\gamma = 1/\rho$, $\rho = \rho(r,z)$ is the density, $\kappa(r,z)$ is the wave-number. We assume the pressure-release boundary condition at $P = 0$ $z = 0$ and rigid boundary condition $\partial u/\partial z = 0$ at $z = -H$.

The parameters of medium may be discontinuous at the nonintersecting smooth interfaces $z = h_1(r), \dots, h_m(r)$, where the usual interface conditions are imposed. It is sufficient to consider the case $m = 1$, so we set $m = 1$ and denote h_1 by h.

We introduce a small parameter ϵ, the slow variable $R = \epsilon r$ and postulate the following expansions for the parameters κ^2, γ and h: $\kappa^2 = n_0^2(R,z) + \epsilon\nu(R,z)$, $\gamma = \gamma(R,z)$, $h = h(R)$. Considering first a solution to the Helmholtz equation (1) in the form of the WKB-ansatz

$$P = (u_0(R,z) + \epsilon u_1(R,z) + \dots)\exp(\mathrm{i}\theta(R,z)/\epsilon)\,,$$

we collect the following information: θ is independent of z, the interface conditions up to $O(\epsilon^2)$ are

$$(u_0 + \epsilon u_1)_+ = (\text{the same terms})_-\,, \quad \gamma\,(u_{0z} + \epsilon u_{1z} - \epsilon\mathrm{i}\theta_R h_R u_0)_+ = (\text{the same terms})_-\,,$$

and u_0 satisfies the usual equation

$$(\gamma u_{0z})_z + \gamma n_0^2 - \gamma(\theta_R)^2 u_0 = 0\,. \tag{2}$$

Differentiating spectral problem (2) with respect to R we obtain the boundary value problem for

$$C_{jl} = \int_{-H}^{0} \gamma_0 \phi_{jR}\phi_l\,dz\,.$$

Consider now the following ansatz: $P = \sum_{j=M}^{N} (u_0^{(j)}(R,z) + \epsilon u_1^{(j)}(R,z) + \dots)\exp\left(\frac{\mathrm{i}}{\epsilon}\theta_j\right)$.

Proposition 1. *The solvability conditions for the problem at $O(\epsilon^1)$ are a system of equations for $l = M, \dots, N$*

$$2\mathrm{i}k_l A_{l,R} + \mathrm{i}k_{l,R}A_l + \mathrm{i}k_l\frac{1}{R}A_l + \sum_{j=M}^{N}\alpha_{lj}A_j\exp(\theta_{lj}) = 0\,, \tag{3}$$

$$\alpha_{lj} = \int_{-\infty}^{0}\gamma\nu\phi_j\phi_l\,dz - \mathrm{i}k_j\,(C_{lj} - C_{jl})\,, \quad \theta_{lj} = \frac{1}{\epsilon}(\theta_j - \theta_l)\,.$$

The acoustic energy flux is defined as

$$J(r,z) = \gamma\,\mathrm{Im}((\operatorname{grad}P(r,z))P^*(r,z))\,.$$

As is well known, if P is a solution of the Helmholtz equation (1) then the corresponding energy flux is conserved, that is $\operatorname{div}J(r,z) = 0$. With our boundary conditions we have also the 'modal' conservation property

$$\operatorname{div}\int_{-H}^{0} J(r,z)\,dz = 0\,.$$

Proposition 2. *Assume that* $\mathrm{Im}\,\bar\nu = 0$. *Let* $\{A_j|j = M, \dots N\}$ *be a solution to equations* (3). *Then for* $P = \sum_{j=M}^{N} A_j\exp\left(\frac{\mathrm{i}}{\epsilon}\theta_j\right)\phi_j$ *we have* $\operatorname{div}\int_{-H}^{0} J(r,z)\,dz = O(\epsilon^2)$.

The numerical comparisons with COUPLE program gives the excellent results.

Acoustical reflection by a concave paraboloid with a mixed boundary condition

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A scalar plane wave propagates along the axis of a concave paraboloid of revolution. The space inside the paraboloid is filled with a linear, homogeneous and isotropic medium, and the analysis is conducted in the phasor domain with a time-dependence factor $\exp(+j\omega t)$ and a propagation constant k. At the surface of the paraboloid, a mixed boundary condition is imposed, that includes soft (Dirichlet boundary condition) and hard (Neumann boundary condition) paraboloids, as well as perfectly absorbing paraboloids, as particular cases.

The analysis consists in writing the solution to the scalar wave equation as the product of the exponential function that appears in the expression of the incident plane wave times an unknown function of the coordinate that is constant on the surface of the paraboloid. An exact solution based on this type of Ansatz is possible only for two coordinate systems: the parabolic-cylinder coordinates, and the paraboloidal coordinates. The concave parabolic cylinder case was originally proposed by Lamb (1906), who, however, was unable to solve the problem; that problem was solved by Uslenghi (*IEEE Antennas Wireless Propag. Lett.*, 2012); the unknown function in this case is a Fourier integral. For the concave paraboloid of revolution considered in the present work, the unknown function is an exponential integral. The particular cases of soft and hard concave paraboloids were presented recently (P.L.E. Uslenghi, *USNC-URSI National Radio Science Meeting*, Boulder, Colorado, USA, Jan. 2013; P.L.E. Uslenghi, *Atti Acc. Scienze Torino*, Nov. 2013).

It is shown that the incident field cannot consist only of a plane wave, but must also contain an additive term that is the product of a plane wave and an exponential integral; this is due to the fact that the concave paraboloid is not a truly open surface, so that the incident plane wave is everywhere inextricably linked to its reflection at the paraboloidal surface. Similarly, the field reflected at the paraboloidal surface, after crossing the focus, consists of a plane wave propagating axially in a direction opposite to that of the incident plane wave plus a field that is the product of the reflected plane wave times an exponential integral. The incident and reflected fields are matched at the focus of the paraboloid, thus yielding the exact solution to the boundary-value problems. The energy integrability condition is analyzed.

Wave booms originated from fast line sources

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Long-range transmission of acoustic and electromagnetic signals using localized waves is a very topical area of research and development concerned with wave motion. Localized X-shaped waves are among the most prospective carriers, but their application is currently limited by the lack of realistic wave sources.

A decade ago Recami *et al.* discovered especially promising "localized X-shaped field generated by a superluminal electric charge" [1]. Here the specific properties of an X-shaped wave are additionally enhanced by a cumulative effect known in acoustics as a *sonic boom* — formation of strong pressure wave by supersonic bodies. Similar *electromagnetic boom* must accompany propagation of superluminal sources.

A lot is yet to be done for developing the ideas put forward in [1] into practical information and communication technologies as a) they are related to a steady-state process without beginning and end, so both retarded and advanced components are present in the solution; b) superluminal particle motion is involved; c) the simplified point-source model results in singularities, which impede making practical estimations of the critical parameters of the wave, the amplitude and rise time. In [2]–[4] Recami's model was extended to far more realistic scenarios of wave generation, replacing the charged tachyon by a transient superluminal current pulse, which can be created without violation of special relativity using shadow-style motion (see, e.g., [5]). A family of physically tenable, causal localized pulses (droplet-shaped waves) was constructed as unique solutions to the corresponding initial value problem using the spacetime triangle diagram (STTD) technique, overcoming impediments (a) and (b) mentioned above.

The report addresses the last impediment (c) on the way of constructing "engineerable" models, by getting rid of the *wave boom* singularity via description of the source as a pulse of finite rise time. Thorough investigation of STTDs depicting the arrival of the wave boom at the observation point (see, e.g., Figs. 2a,b of [2]) and expansion of the corresponding integral solutions into a series (with respect to a specially chosen small parameter characterizing time reckoned from the arrival of the wavefront at the observation point) demonstrated that relatively simple and realistic estimations of wave booms can be obtained even within the framework of line-source models.

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On the reflection phenomenon of quasi-stationary waves from the smooth boundary of an anisotropic elastic medium

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In this work, we tackle the problem of the reflection of elastic nonstationary waves off a stress-free smooth boundary surface of an anisotropic elastic body. A general anisotropic case of an inhomogeneous medium is assumed. Both incident and reflected waves are categorized as quasi-stationary high-frequency oscillations, which allow both the instantaneous frequency and instantaneous wave vector to be defined in time-space.

The time-space ray method is employed for the mathematical treatment of both incident and reflected waves. The analytical expressions for these waves can be construed as a generalization of the previously reported results [2, 3] to a similar boundary problem for the cases of monochromatic waves and the plane boundaries of a homogeneous anisotropic medium.

All waves propagating near the surface can be represented, to an allowable error, as locally plane waves with an angle of incidence formed by the instantaneous wave vector $\nabla\varphi$ and a normal n_0 to the boundary surface at a point M_0, where φ is the real phase function of the wave. An incident wave U_0 is treated as a quasi-transverse wave and represented by a time-space ray expansion.

We consider two ranges of angular incidence of the wave U_0: (1) where of the three reflected waves, only the quasi-longitudinal wave is an inhomogeneous one, i.e., its phase is complex; (2) the quasi-longitudinal wave and either of the quasi-transverse waves are both inhomogeneous. By contrast, waves with a real phase are referred to as homogeneous.

In the zeroth asymptotic approximation, the phases of the homogeneous reflected waves satisfy the Hamilton–Jacobi equation (or the eikonal equation) which is solved using the method of characteristics. Note that as the ray parameter scanned along the characteristics (which are space-time rays) we use the distance n of ray point M on the ray to the boundary. The amplitude of these waves is given by a standard ray method formula as a solution to the transport equation, which ensures the construction of higher asymptotic approximations.

In order to find the complex phases of inhomogeneous waves, a characteristic equation, rather than an eikonal equation, for a 3-row matrix with complex elements is solved by expanding the sought phase functions in integer powers of n, where n is the distance of point M to the boundary. The coefficients of these expansions computed at points on the boundary take the form of implicit functions defined by the algebraic relations derived from the above characteristic equation.

The amplitudes of inhomogeneous waves are found from the solvability conditions (necessary for higher asymptotic approximations to be constructed) by solving the Cauchy problem for the first-order linear differential equation in variable $n(M)$ with initial values on the boundary.

The stress-free boundary conditions for the problem under study include the contributions of all incident and reflected waves to the wave field and are reduced to a linear inhomogeneous system for arbitrary coefficients contained in the ray expressions for the homogeneous reflected waves and initial values of the Cauchy problem for inhomogeneous waves.

On application of the system of equations thus obtained over the two ranges of angular incidence of wave U_0 (see above), we are able to prove, using both the previously established phase functions and polarization vectors of the reflected waves, that the determinant of this system is nonzero within either angular range. Therefore, both the desired coefficients of the ray method formulae and the initial values of the Cauchy problem can be uniquely established in terms of the displacement of the incident wave U_0.

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Resonance excitation of acoustic Fabry–Perot antenna resonator formed by two parallel disks (GTD analysis)

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In the paper we study the problem of description of the acoustic field radiated by a Fabry–Perot resonator formed by two hard wall and infinitely thin, parallel, circular disks in 3D space. The resonator is excited by an acoustic monopole located at the exact center of cylindrical space between the two horizontal disks. The problem is considered in the high-frequency approximation, i.e., the diameter of the disks is much greater than the wave length [1]. In this diffraction problem we observe infinitely many multiply diffracted fields. Firstly, a radial waveguide modes excited by monopole and travelling towards to the circular open end due to diffraction partially radiate into the outer space and partially reflect back. This process of multiple diffractions continues until infinity. We apply Geometrical Theory of Diffraction (GTD) [2] with the help of uniform stationary method [3] to construct uniform asymptotic expansions for directivity of acoustic field irradiated into the far field zone of the outer space as well as the field representation inside the resonator. This GTD asymptotic analysis is based on the exact solution to the corresponding canonical problem — radiation of the open end of a waveguide formed by two parallel half-planes and obtained in [4] by means of the method of factorization (Wiener–Hopf method).

We also solve a homogeneous problem of asymptotic description of complex resonances in the high-frequency approximation. The corresponding complex eigen-frequencies are found from a dispersion equation obtained with the help of GTD by taking into account all multiple diffracted fields propagating from the centre of the resonator towards to the circular edges of the open end and back. It is shown that the maxima of the total radiation power takes place when the frequency of acoustictic field radiated by monopole coincides with the real part of one of the complex resonance eigen-frequency. We make comparison to test our GTD asymptotic formulas by means of finite element package. It is shown that we have a good agreement between GTD asymptotic and numerical results.

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Workshop on metamaterials

Giant second-harmonic generation enhancement in the presence of Tamm plasmon-polariton

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Tamm plasmon polariton (TPP) is an optical analogue of an electronic density localization at the boundary of periodic atomic potential and appears as electromagnetic field localization at the boundary of photonic crystal (PC) and metal [1, 2]. Contrary to the well-known surface electromagnetic waves and surface plasmon-polaritons, TPPs do not require phase-matching conditions for the tangential component of wave vector and can be excited at any angle of incidence. Experimentally, TPPs manifest themselves as narrow resonances in reflectance or transmittance spectra of metal/PC systems. Recently TPPs were proposed to be used in new types of compact lasers and sensors [3, 4, 5].

Enhancement of the second harmonic generation (SHG) due to electromagnetic field localization was explored in details in photonic crystals and in thin metal films in case of surface plasmon excitation. Thus the idea of second-harmonic generation enhancement straightforwardly derives from the localization of the electromagnetic field in case of TPP excitation.

Studied sample of one-dimensional PC consists of 6 pairs of ZrO_2/SiO_2 layers (average thicknesses 110 nm and 145 nm, respectively) covered with a semitransparent 30-nm-thick gold film. Fundamental wavelength was tuned in the range of 700–820 nm. Pump fluence at the sample was about $1\ \mathrm{GW\cdot cm^{-2}}$.

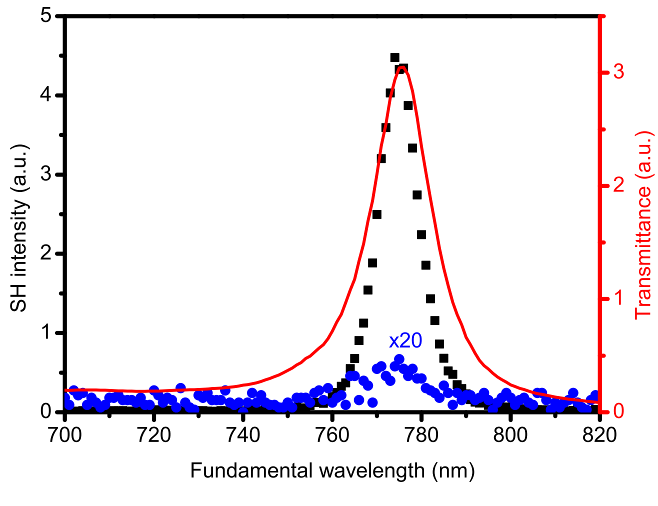

Fig. 1: (Color online) Solid red curve — experimental transmittance spectrum of the Au/PC sample. Black circles — experimental SH spectrum of the Au/PC sample in *pp* geometry. Blue triangles — experimental SH spectrum of the Au/PC sample in *ps* geometry multiplied by 20.

Figure 1 represents SHG spectra of the Au/PC sample for the *pp* and *ps* combinations of fundamental and second-harmonic radiation polarization. Both spectra demonstrate resonance near 774 nm associated with the TPP. For *pp* combination SHG is permitted and enhancement of the local field in the vicinity of the TPP resonance leads to the increase of the SH intensity by a factor of 240 compared to the intensity level far from resonance. Appearance of the peak in forbidden *ps* combination can be interpreted as a result of hyper Rayleigh scattering due to roughness of the gold surface. Experimental results are in an excellent agreement with the results of the numerical calculation.

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Study of guided modes of the wire medium slab

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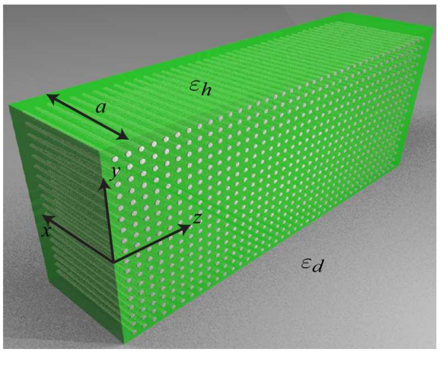
While the guided waves in the wire metamaterial slab have been studied previously [1, 2], there are no consistent studies of these modes. In our work, we present a theoretical and experimental analysis of the properties of the guided waves in the wire metamaterial slab (Fig. 1A). We get dispersion relations for wire medium hosted by the dielectric. We find, in particular, that the nonlocal effects lead to a strong coupling between "fast" and "slow" eigenmodes of the waveguide, manifested by anti-crossings of their dispersion curves (Fig. 1B). We also study how the losses in the host media affect the dispersion and the coupling of the guided modes. We illustrate the concept of waveguide modes in the experiment using the sample designed as a wired media for the microwave frequency range. We have extracted the waveguide numbers of the transmitted radiation from the phase maps and compared them with analytical results.

(A)



(B)

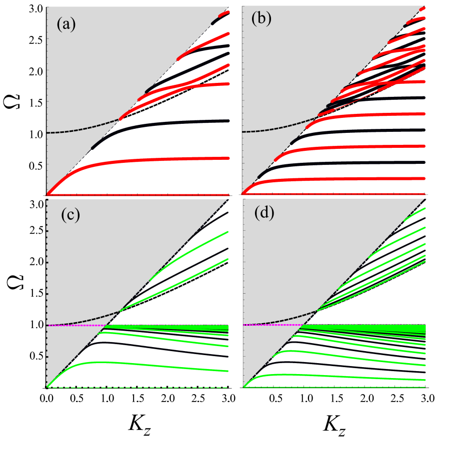


Fig. 1: A) Waveguiding structure: a planar slab of wire medium of thickness a, cladded on both sides by a uniform dielectric with a constant permittivity ε_d. B) Band structure of guided TM modes in nonlocal ((a,b), black curves for symmetric, red curves for antisymmetric modes) and local ((c,d)) WM slab models for different slab thicknesses.

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Quantum optics and quantum information with spatially-periodic microstructures

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Nowadays, the elaboration and investigation of hybrid quantum devices and artificial nanostructures represent a huge area of experimental and theoretical research [1]. In particular, quantum memory devices are proposed for mapping the quantum state of light onto the matter by using a slow light phenomenon through the coupling between matter excitation and quantized field. In this sense, polaritons, linear superpositions of quantized field and collective excitations in matter, provide a very elegant way for optical information storage, where the group velocity of the wave packet could be low enough due to a large value of polariton mass.

We consider the formation of lattice polariton solitons in the array of weakly coupled cavity-QED arrays, with the ensembles of two-level atoms embedded in each cavity [2]. The extended tight-binding model will be established in this case, we introduce a coupled atom-light excitation basis that is the polariton basis for the lattice system. In particular, LB polaritons occurring at each site of the cavity array are a subject of our study. We use a time-dependent variational approach to obtain polariton wave-packet behavior. Basic equations for the wave-packet parameters and their general properties in the QED cavity array are established. With the introduction of the next-nearest photonic tunneling effects, five different dynamical regimes are revealed; they include the diffusion, self-trapping, soliton, and two breather states.

We propose novel physical algorithm of storage of optical information by using lattice polariton localized states. Transformation between matterlike and photonlike lattice polariton solitons paves the way to the storage and retrieval of optical information through the adiabatic manipulation of detuning frequency. In order, the fidelity of storage for various polariton dynamical regimes is examined. We have shown that maximal value of fidelity is achieved for switching between two steady-state soliton regimes for solitonic polariton wave packet. Fidelity vanishes and goes to zero for the transitions involving self-trapping or diffusive regimes. The local maxima of fidelity obtained for breather states of the polariton wave packet.

We hope that predicted dynamical regimes for LB polaritons and storage protocol could be implemented with some other two-level systems. In particular, we discuss cavity QED with excitonic qubits in QWs too.

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Magnetic resonance imaging meets microwave engineering

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Magnetic resonance imaging (MRI) became a powerful tool in medical diagnostics over the last three decades. This relatively young imaging technique is highly innovative and its impact continues to grow. MRI combines three types of magnetic fields. First, the main static magnetic field (in the order of several Tesla's) to create Zeeman magnetization. Then, time dependent (pulsed) magnetic field gradients are played out for spatial signal encoding (in the order of milli Tesla). Finally, a pulsed radiofrequency (RF) magnetic field (order of micro Tesla) induces the nuclear magnetic resonance (NMR) signal. The RF operating frequency depends on the gyromagnetic ratio of excited nuclei (e.g. hydrogen) and is directly proportional to the main static magnetic field strength. In human MRI mainly hydrogen nuclei (proton) distributions are imaged. At 7T (the highest magnetic field strength in the pre-clinical human practice) the RF frequency for proton excitation is about 300 MHz. The RF signal wavelength at 7T becomes comparable to the circumference of the human MR scanner bore (1 m vs. 1.9 m) and the bore starts to act as a cylindrical waveguide for the RF signal. Moreover, in human tissue the RF wavelength shrinks to about 15 cm only. The operating wavelength shrinkage at 7T challenged the traditional design of large volume RF probes which was based on near field coupling and was successfully used at lower magnetic field strengths. On the other hand, because of the relatively small wavelength, microwave engineering principles could be borrowed to design novel RF probes. This opened an exciting new RF research area for investigation in MRI world and was called travelling wave NMR or waveguide MRI. In our work we explored waveguide principles and applied them for human body MRI at 7T. Three key attributes for any high field RF probe were addressed from the waveguide perspective: efficiency, signal uniformity by means of shimming and RF tissue heating.

Toroidal all-dielectric metamaterial

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Toroidal moment is one of the components of the multipole expansion, first considered by Zel'dovich to explain parity violation in the weak interaction . Toroidal response arises due to the poloidal currents flowing on the surfaces of the torus along the meridians and producing circulated magnetic field along the toroid. Toroidal moment is different from standard electric and magnetic moments due to its non-radiating configuration and the very strong field localization inside the torus. Here, we discuss the idea of employing all-dielectric metamaterial clusters for demonstrating toroidal response. Moreover, we study some promising phenomena in all-dielectric metamaterials with toroidal response: Electromagnetically induced transparency (EIT), Fano-like resonance, Strong localization of electromagnetic fields and etc.

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Acoustic metamaterials: modeling, general properties, examples

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Acoustic metamaterials (AMM) are artificially created periodic structures whose periodicity cells are implemented in the form of mechanical oscillatory systems with many DOFs, part of which are internal (hidden). At frequencies where the wavelengths of normal waves exceed the cell dimensions, the structures behave as continuous media and, depending on the cell construction, can demonstrate unusual wave properties.

In the presentation, AMM is modeled with another periodic structure having a periodicity cell of the simplest form of discrete elements (mass, springs) which are taken as the effective parameters of the modeled AMM. These parameters are determined from condition of equality of wave dispersion in AMM and the model. They can easily be obtained from the characteristics of an individual AMM cell, computed or measured. It is shown that all the wave properties — phase and group velocities, cell energy, power flow vector, etc. are exactly calculated via thus determined effective parameters. The corresponding equations are derived. Analysis of them revealed some general properties and restrictions for AMMs. One of them states, e.g., that AMMs with negative effective parameters independent of frequency cannot exist. Serious restrictions are formulated for the so-called complementary media and for sound crystals.

Also, by spreading the discrete effective parameters over the periodicity cell, the transition to continuous media is carried out and the corresponding "wave equations" are written for negative acoustic media. Examples of AMMs are presented.

Langmuir modes in hyperbolic media

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Dielectric function of many optically uniaxial composite materials can be described within effective medium approximation, i.e. by a tensor [1, 2] with following main components of the tensor:

$$\varepsilon_{||,\perp}(\omega) = \varepsilon_\infty\left(1 - \frac{\Omega^2_{||,\perp}}{\omega(\omega + i\gamma_{||,\perp})}\right). \qquad (1)$$

Here $\Omega_{||,\perp}$, $\gamma_{||,\perp}$ are plasma frequency and inverse relaxation time corresponding to the directions along and across to the optical axis of the material.

In the present work we analyze spectrum of electromagnetic waves in the waveguide which filled up by the material whose dielectric function is described by equation (1). It is shown that in the range between $\Omega_{||}$ and $\Omega_\perp$ composite material is hyperbolic medium. This leads to the possibility of propagation the Langmuir modes which are degenerated in the isotropic case. At the frequencies close to $\Omega_{||}$, Langmuir modes are nearly transverse and at frequencies close to $\Omega_\perp$ they are nearly longitudinal. Dispersion for the Langmuir modes is positive if $\Omega_{||} < \Omega_\perp$ and negative if $\Omega_{||} > \Omega_\perp$. In our analysis we take into account the losses and spatial dispersion within hydrodynamic approximation.

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Purcell effect in metal-dielectric metamaterials with elliptic isofrequency contours

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Recently effect of spontaneous emission enhancement in metal-dielectric metamaterials was demonstrated [1, 2]. However Purcell effect in this case appear because hyperbolic isofrequency contours of structure. It means that such phenomenon is caused by evanescent waves, which have big wave vectors. In this paper we have shown that a big Purcell factor can be obtained for the case when metamaterial's isofrequency contour is an ellipse (see Fig. 1).

It implies that this Purcell factor is absolutely radiative. Firstly multilayered metal-dielectric metamaterial with a bilayer unit cell was considered. But in this case, effects of spatial dispersion do not allow to obtain a big Purcell factor in structure with elliptic isofrequency. Then we shown that this issue can be overcome by using 4-layers unit cell.

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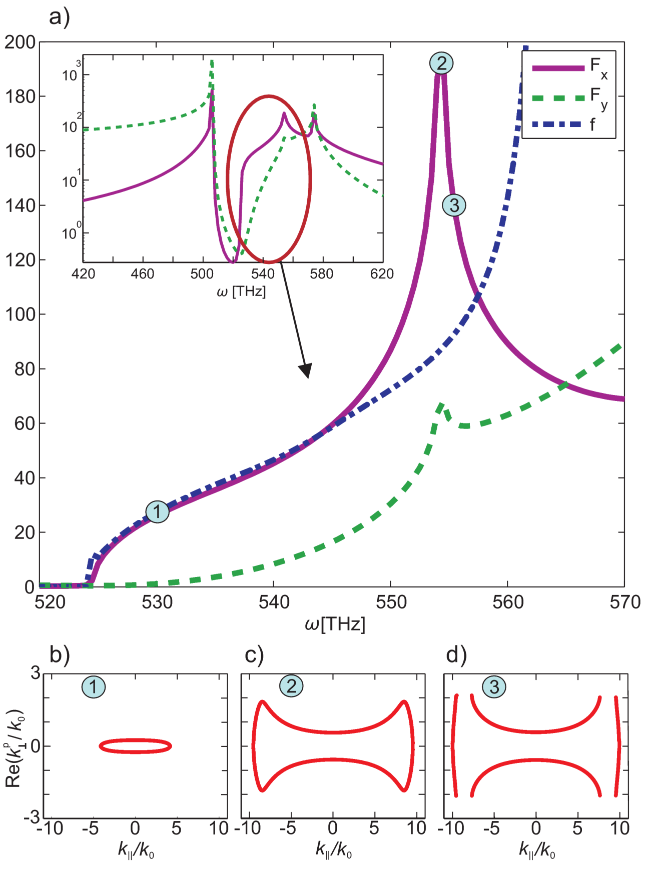

Fig. 1: a) Dependence of Purcell factor from the frequency. Inset — logarithmic scale. F_x — dipole oriented along the layers, F_y — dipole oriented across the layers, f — function $(Re(\varepsilon_{||})+3Re(\varepsilon_\perp))/8Re(\sqrt{\varepsilon_{||}})$. b)–d) isofrequency contours of metamaterials on 530, 554, 556 THz respectively.

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Active meta-materials based on liquid crystals and phase-change materials

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Today it is possible to engineer the building blocks, meta-atoms, of artificial materials with feature sizes smaller than the wavelength of light. The ability to design the meta-atoms in a largely arbitrary fashion adds a new degree of freedom in material engineering, allowing to create artificial materials with unusual physical properties rare or absent in nature. Examples include media with negative refractive index, hyperbolic media, and artificial media based on the concept of transformation optics. Recently research focus is shifting towards active meta-materials. Incorporating switching and modulation capabilities to meta-material based devices will drive advancement of their functionalities. In this presentation, our recent results on design and modeling of active meta-materials based on plasmonic and phononic materials and incorporating liquid crystals or phase-change materials as active elements will be discussed.

Narrowband plasmonic resonances and their applications

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Narrowband plamonic resonances are of great interest for a wide range of the potential applications from telecom to bio sensing. The narrowband resonances can be designed and achieved by a loss compensation (spaser), a diffraction coupling (coincidence of the eigen modes with Wood anomaly), and a combination with the high Q dielectric microresonators. An alternative way is to utilize the low loss and low radiative structures — as an example, a toroidal structures made from silicon will be considered. The physics behind the functioning of these proposed narrowband structures, as like as the recent experimental achievements will be the main focus of the presentation.

Checking accuracy of numerical and approximate analytical calculus of symmetrical multiports by group-theoretical methods

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Accuracy of numerical methods and approximate analytical ones in analysis of electromagnetic structures can be validated by different methods. In optical and microwave multiports, the usual tool is energy conservation criterion and, in case of reciprocal multiports, reciprocity test. The situation is complicated in cases of nonreciprocal structures. In this work, we will show that the exact results of scattering matrix calculation by group-theoretical methods can also be used for this purpose. The suggested method is not limited by reciprocity condition and lossless approximation. As an example, we consider square and circular waveguides filled with ferrite or semiconducting material magnetized azimuthally (Fig. 1). A section of such waveguides can be considered as a four-port. These waveguides are used in microwave technology as nonreciprocal phase shifters, cut-off switches and isolators. The complexity of analysis of these waveguides stems from the tensor nature of the ferrite medium and large number of the parameters. In practice even in case of a homogeneous ferrite medium, magnetization by a line current on the axis of the waveguide has a radial dependence, i.e. the magnetization is nonuniform. The problem is complicated for multilayer structures. For these cases exact analytical solutions are not known.

The magnetic group of the magnetized cylinder in Fig. 1a is $D_{\infty h}(C_{\infty v})$. The magnetic group of section of the magnetized square waveguide in Fig. 1b is $D_{4h}(C_{4v})$ which is a subgroup of $D_{\infty h}(C_{\infty v})$. The four ports in both cases of Fig. 1 can be analyzed using the group $D_{4h}(C_{4v})$. Using the theory presented in [1], we calculate the scattering matrix and then we use the obtained results for checking accuracy of numerical calculation by commercial software COMSOL [2] for different discretization levels.

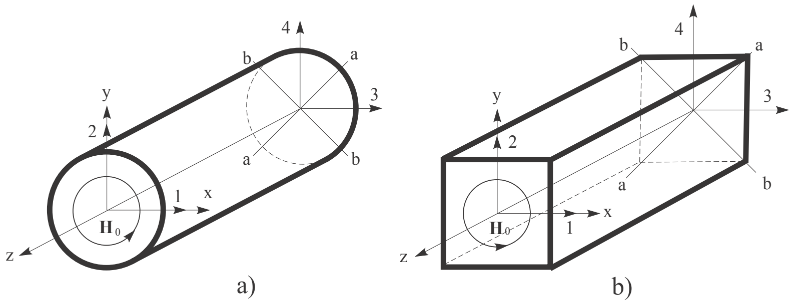

Fig. 1: a) Section of circular waveguide, b) section of square waveguide, $\mathbf{H}_0$ is DC magnetic field.

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Nonreciprocal and control optical components based on 2D photonic crystal resonators with magneto-optical material

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In this paper, we review some of our recent results in the field of nonreciprocal and control optical components based on 2D photonic crystal resonators with magneto-optical material. Among these components are circulators, 2-way and 3-way nonreciprocal dividers, different types of switches and new multifunctional components.

We consider 2D photonic crystals with hexagonal lattice. As a first step, we suggest a general classification scheme of 2-, 3-, 4-, 5- and 6-ports based on group-theoretical calculations of their scattering matrices. Such analysis helps to choose structures which can fulfill the discussed specific functions.

In the second part of our presentation we analyze physical mechanisms of functioning of the components. All of them are based on magneto-optical resonator coupled with 2–6 waveguides. In the circulator and the two types of the dividers, the mechanisms are related with the effect of splitting of rotating dipole modes in the resonator by a DC magnetic field. In the switches, we consider two types of mechanisms. Two new components which fulfill simultaneously three functions, namely, equal division, isolation and switching will be presented as well. These components will allow one to reduce the dimensions of the highly integrated optical circuits.

In the third part of our paper we present some results of numerical analysis of W-circulator, 2-way and 3-way nonreciprocal dividers, switches and multifunctional components. Among the switches we consider components without bending, with 600, 1200 and 1800 bending of the output waveguide with respect to the input one. Such geometrical modifications can provide a more flexibility in design of integrated circuits. Depending on the type of the components, at the level of isolation $(-15 \div -20)$ dB and the insertion loss $(-1 \div -2)$ dB, the bandwidths of them are about $(60 \div 230)$ GHz at the wavelength $\lambda = 1.55$ μm.

Radiation and scattering of thin wires of arbitrary geometry in chiral media

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Chiral medium is a type of meta-medium. Particular interest to investigation of electromagnetic properties of chiral structures is mainly connected with the possibility to use them in microwave equipment. Cross polarization phenomenon gives an opportunity to create frequency-selective and polarization-selective filters, polarization convertors, frequency-selective protective screens etc. It is also known, that chirality leads to increase of absorption and decrease of the level of direct and reverse scattering of electromagnetic waves in comparison to non-chiral medium. This feature is associated with the perspective of creating low-reflecting and masking coating for SHF band. It is also demonstrated in the literature that it is possible to use chiral structures as the elements of integral circuits, lens, horn and printed circuit antennas.

The problem of electromagnetic waves radiation in chiral medium is poorly covered in the known literature. Generally, only the tasks of radiation of elementary sources and straight-line vibrator antennas located in chiral medium are investigated. Solution of the general task of electromagnetic waves radiation by antennas, located in different natural and artificial meta-media, as well as creation of adequate theoretical models of characteristics calculation for such antennas, remains one of the actual problems.

The goal of this work is to consider peculiarities of radiation of an arbitrary system of sources, located in biisotropic chiral medium.

If some sources of current with defined volume density are located in infinite medium, then from Maxwell equations it possible to derive unrelated inhomogeneous differential equations of second order for electric-field vector for waves of left and right circular polarization. To resolve the derived equations electrodynamic potentials are introduced, which are represented via Green's function for arbitrary space. As the result, the obtained correlations allow to calculate electromagnetic field, created by an arbitrary system of sources in chiral medium.

On the basis of obtained results an important applied task of excitation of electrically thin curvilinear conductors with gradually changing geometry is considered. In this situation a curvilinear cylindrical coordinate system is used, such that the basic vectors of the 1st and 2nd coordinates lie within the section plane of the conductor, and the 3rd coordinate is tangential to conductor axis. Within the limitations of the so-called thin-wire approximation, when conductor radius is significantly smaller than the wavelength of electromagnetic field, an assumption is made that current does not depend on azimuthal coordinate and is uniformly distributed along azimuth. This gives a possibility to switch to approximation of axis current.

Then, using the boundary condition on ideal conductor surface, an integral equation is obtained relative to an unknown function of current distribution for such a thin conductor, located in chiral medium, which for the case of non-chiral medium transforms into known Pocklington equation [4].

To estimate the authenticity of results, obtained with the help of the derived integral equation, a calculation of current distribution for a straight-line vibrator is made. To ways of vibrator excitation were considered: by a concentrated source, located in the center of vibrator, and by a plane electromagnetic wave, which propagates orthogonally to vibrator axis. In both cases the character of current amplitude dependency on chirality factor corresponds to the data, obtained in [3].

It is ascertained that increase of chirality factor leads to exponential decrease of current amplitude. Calculation of radiation filed shows that it leads to narrowing of the main lobe of directivity pattern both in radiation mode and in scattering mode, and, correspondingly, to increase of vibrator directed properties.

The use of the suggested equation allows in a relatively easy way to calculate electrodynamic characteristics of thin-wire radiators in a chiral medium.

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Stochastic integrable system: optical resonance in Λ-configuration atomic medium

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Maxwell–Bloch system is known to be an accurate model of the electromagnetic wave interactions with ensembles of atoms. In the particular case of two-level atoms with non-degenerate upper and degenerate ground levels, (Λ-configuration) this system is integrable. There are several formulations of physical problems involving structural disorder, such as pulse generation from initial fluctuations within the medium polarization or polarization switching due to non uniformity of the relative ground level populations. We describe the statistical properties of the optical pulses in such materials by exploiting the integrability of the system despite the presence of structural disorder.

Extreme plasmonics in atomically thin materials

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The recent observation [1–4] and extensive theoretical understanding [5–7] of plasmons in graphene has triggered the search for similar phenomena in other atomically thin materials, such as noble-metal monolayers [8] and molecular versions of graphene [9]. The number of valence electrons that are engaged in the plasmon excitations of such thin layers is much smaller than in conventional 3D metallic particles, so that the addition or removal of a comparatively small number of electrons produces sizeable changes in their oscillation frequencies. This can be realized using gating technology, thus resulting in fast optical modulation at high microelectronic speeds. However, plasmons in graphene have only been observed at mid-infrared and lower frequencies [1–4], and therefore, small molecular structures [9] and atomically thin metals [9] constitute attractive alternatives to achieve fast electro-optical modulation in the visible and near-infrared (vis-NIR) parts of the spectrum. We will discuss several approaches towards optical modulation using atomically thin structures, as well as the challenges and opportunities introduced by these types of materials, including their application to a new generation of quantum-optics and electro-optical devices.

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Nonlinearities in plasmonics and metamaterials

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Nonlinear optics has triggered the evolution of modern optics, yielding discoveries of important phenomena, deep understandings of fundamental optical effects and, moreover, serving as a source for a large variety of applications. Nonlinear optical interactions are relatively weak but can be significantly enhanced using various approaches. Generally, nonlinear optical phenomena are proportional to higher orders of the driving field, motivating the quest for local electromagnetic field enhancement for which various nanostructures have been proven to be beneficial. In particular, noble metals with negative permittivity at optical and infrared wavelengths can support the so-called surface plasmon modes with the deep-subwavelength localization of the electromagnetic energy, overcoming the conventional diffraction limit and leading to the field enhancement effects. Plasmonic nanostructures are perfect candidates for the realization of various concepts for the enhancement of nonlinear effects.

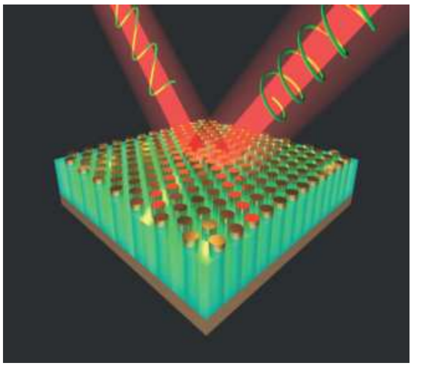

Fig. 1: Plasmonic nanostructure for polarization control of reflected and transmitted beams.

In this talk, we will overview nonlinear plasmonic effects due to intrinsic, hydrodynamic metal nonlinearity, enhanced by sub-wavelength field confinement and interaction between plasmonic resonances. In particular, harmonic generation, solitonic effects and Kerr-nonlinearity-induced switching will be discussed. In addition to conventional intensity and phase modulations, active control of light polarization, presenting an important alternative, will be demonstrated.

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Microscopic model of the self-induced torque in metamaterials

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We consider a torque acting on an electric dipole placed in an anisotropic metamaterial. The torque arises due to the response of the anisotropic structure polarized by the field of the dipole. This effect was predicted in Ref. [1] and it was called the self-induced torque. In our study, the metamaterial is modeled as a cubic lattice of anisotropic uniaxial polarizable particles. We calculate the self-induced torque employing the discrete dipole approximation. This model allows to take into account the effects of frequency and spatial dispersion in the metamaterial. We demonstrate that the maximal absolute values of the torque are achieved in so-called mixed dispersion regime [2], that is the transition region between elliptic and hyperbolic dispersion regimes. The obtained results are compared with the model based on the effective medium approximation [3].

We show that the self-induced torque is equal to

$$\langle T\rangle = \frac{\langle d^2\rangle \sin 2\theta}{a^3} f ,$$

where d is the dipole moment of the probe dipole, a is the lattice period, θ is the angle between the dipole and the anisotropy axis, and f is the dimensionless anisotropy factor.

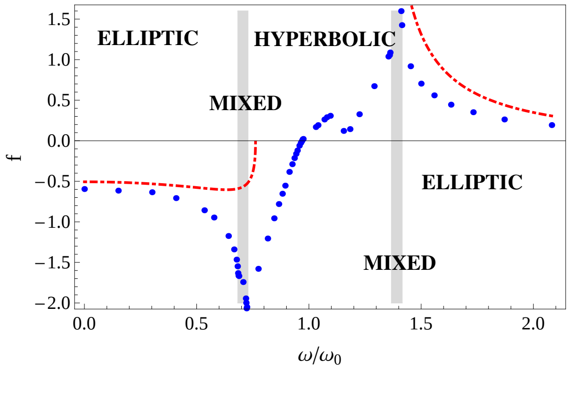

Fig. 1: The calculated values of the self-induced torque in a discrete metamaterial. Dot-dashed curve shows the prediction of the simplified model [3] based on the effective medium approximation, dots show the results of calculations.

The calculated values of the self-induced torque for the case of the uniaxial scatterers with the polarizability tensor $\hat{\alpha} = \alpha_z \, \vec{e}_z \otimes \vec{e}_z$ are shown in Fig. 1. Here, $\alpha_z = \alpha_0 \, \frac{\omega_0^2}{\omega_0^2 - \omega^2}$, where $\alpha_0/a^3 = 0.1$, $\omega_0 \, a/c = 0.3$. The obtained results may provide important insights into the spectroscopy of intermolecular interactions and physics of nonlinear metamaterials.

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On one class of theoretically constructed isotropic single negative continuous acoustic metamaterials

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We show that linear elastic complex media of certain type have forbidden bands of frequencies, i.e. they are single negative acoustic metamaterials.

Let us consider an elastic complex medium with the density of strain energy U. A point-body of this medium consists of several point masses or infinitesimal rigid bodies. All constraints are ideal and holonomic. The motion of a point body is described by several generalised co-ordinates. We use Lagrange formalism to obtain results for complex point bodies of various nature. There is at least one ("special") co-ordinate q_0 such that

$$\frac{\partial U}{\partial q_0} \neq 0, \qquad \frac{\partial U}{\partial \nabla q_0} = \mathbf{0}. \tag{1}$$

If we imagine the medium as a continuum of inertial objects and elastic springs, in each point there is a "broken spring".

We prove that for some cases there exist a regime of "quasi-independent oscillators" when all other co-ordinates are zero, and at each point q_0 performs harmonic oscillations at its partial frequency ω_0. For other cases, on the contrary, there are no waves, standing or propagating, at ω_0. The corresponding wave number does not exist.

We can eliminate the special co-ordinate. Resulting equations will look as equations of a history-dependent medium if it is possible to satisfy the principle of material objectivity for effective stress tensors.

Then we consider the case when the point body is described by two generalized *vectorial* co-ordinates, one of them "special", another one is coupled to it. The principle of material objectivity is supposed to be satisfied. We also require some symmetry conditions. We prove that there exist a forbidden band of frequencies for the propagation of the wave corresponding to the non-special co-ordinate, just below or above the partial frequency of the "special" co-ordinate. Thus the continuum under consideration demonstrates behaviour of so-called acoustic metamaterial ("effective negative density" in terminology of some researchers). We show several isotropic examples of such media. One of them is the reduced Cosserat continuum that can be used as a model for granular materials.

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Compton scattering in hyperbolic media

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In this work we study the process of the Compton scattering inside the hyperbolic medium as shown in Fig. 1(a). We show that unconventional dispersion relations for the photons in hyperbolic media lead to the drastic change of the Compton shift and corresponding cross-section dependence on the scattering angle. Particularly, we show that the Compton shifts of the order of meVs can be achieved for the incident photons in visible range, which is at least one order of magnitude larger than the corresponding shifts in conventional materials. We derive the spectral intensity function of the reflected radiation taking into account losses and finite period of the hyperbolic metamaterial.

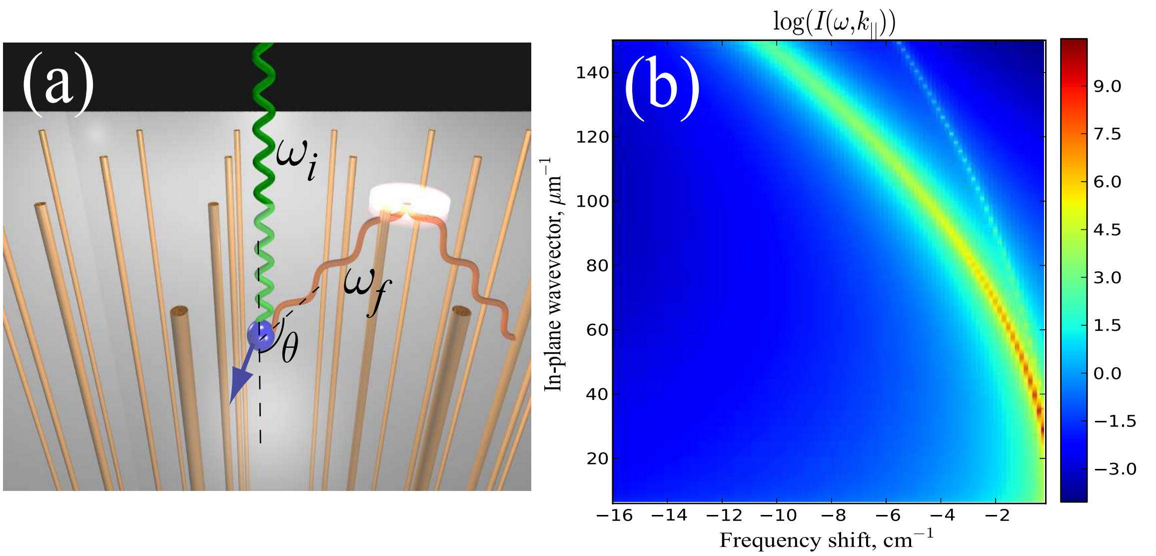

Fig. 1: (a) Geometry of the problem a photon falls normally at the interface of the semi-infinite hyperbolic medium, then scatters non-elastically on the free electron inside the medium, and then its near-field is detected at the interface of the structure. (b) Spectral intensity map of the reflected radiation vs frequency shift and in-plane wavevector for the light reflected from lossy wire medium with period 20 nm.

Multi-stable switchable metamaterial employing Josephson junctions

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The field of metamaterial research revolves around the idea of creating artificial media that interact with light in a way unknown from naturally occurring materials. This is commonly achieved by creating sub-wavelength lattices of electronic or plasmonic structures, so-called meta-atoms, that determine the interaction between light and metamaterial. One of the ultimate goals for these tailored media is the ability to control their properties in-situ which has led to a whole new branch of tunable and switchable metamaterials. Many of the present realizations rely on introducing micro-electromechanical actuators or semiconductor elements into their meta-atom structures. We show that superconducting quantum interference devices based on Josephson junctions can be used as fast, intrinsically switchable meta-atoms. We found that their intrinsic nonlinearity leads to simultaneously stable dynamic states, each of which is associated with a different value and sign of the magnetic susceptibility in the microwave domain [1]. Moreover, we demonstrate that it is possible to switch between these states by applying a nanosecond long pulse in addition to the microwave probe signal. Apart from potential applications such as, for example, an all-optical metamaterial switch, these results suggest that multi-stability, which is a common feature in many nonlinear systems, can be utilized to create new types of meta-atoms.

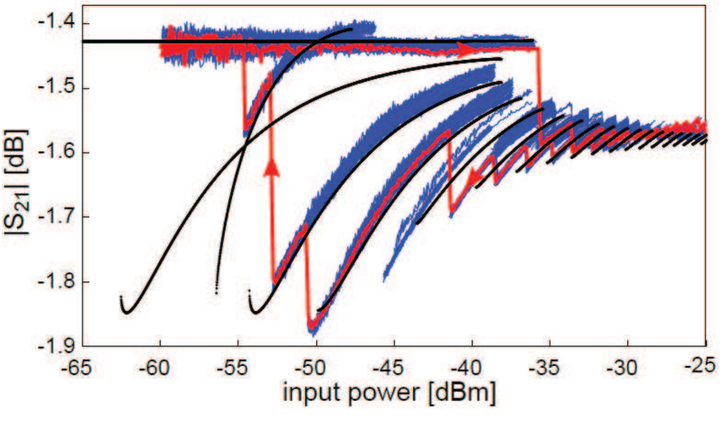

Fig. 1: Calculated (black) and measured (blue and red) transmission through the sample arrangement containing only a single SQUID. The red data show a hysteresis loop from low to high power and back. The red arrows indicate the direction of the sweep. The blue data are a collection of different power sweeps of varying length and initial conditions [1].

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Sub-diffraction-limited imaging using metamaterial-hyperlens

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In common imaging systems the resolution is restricted to approximately half the wavelength as a result of the diffraction limit and the exponentially decaying nature of evanescent waves with high spatial frequencies that carry subwavelength information. There have been several approaches to overcome this limit, like Pendry's perfect lens, but one of the most promising approaches is the metamaterial hyperlens composed of an array of subwavelength scaled metal wires [1, 2, 3]. By using electric fields that are polarized orthogonal towards the interface of the wire medium, perfect imaging can be achieved since they are constituted exclusively by extraordinary waves, filtering out all ordinary waves.

We have demonstrated sub-diffraction-limited imaging and focusing through straight and tapered wire arrays down to $\lambda/28$ over optically long distances [4]. The wire arrays were fabricated using a fibre drawing technique [5]. As a next step, the transmission of purely TM-polarized near-field distribution of different structures through our wire array metamaterial fibre is investigated. As an example a complementary split ring resonator (CSRR) is used. We simulated with a 3D finite element package (CST MWS) a linear polarized, plane wave propagating through a single CSRR exciting the structure (Fig. 1a). The CSRR has a side length of 500 μm, a gap size of 150 μm and slit width of 10 μm. On the other side of the CSRR an array of 1.5 mm long, perfectly conducting wires embedded in Zeonex with a diameter of 1 mm was placed collecting the near-field distribution of the CSRR and transmitting it in the image plane of the hyperlens.

In Figure 1b, c and d the z-component of the electric field distribution is shown at the 3rd-order eigenmode of the CSRR at 225 GHz [6]. All subwavelength details of the characteristic pattern representing a quadrupolar charge distribution along the CSRR are well reconstructed at the image plane of the wire medium (Fig. 1d). Note, that the field strength is normalized in each picture to its maximum value which is an order of magnitude smaller at the image plane in comparison to the source plane.

In order to characterize the electric field distribution experimentally terahertz near-field microscopy is used [7]. An electro-optical detection scheme with a ZnTe crystal cut in (100)-direction allows measuring the longitudinal E-field component. Our approach allows us to scan the sample pixel by pixel and reconstruct the field pattern in the time domain and the frequency domain. Our measurements will be compared with the simulations.

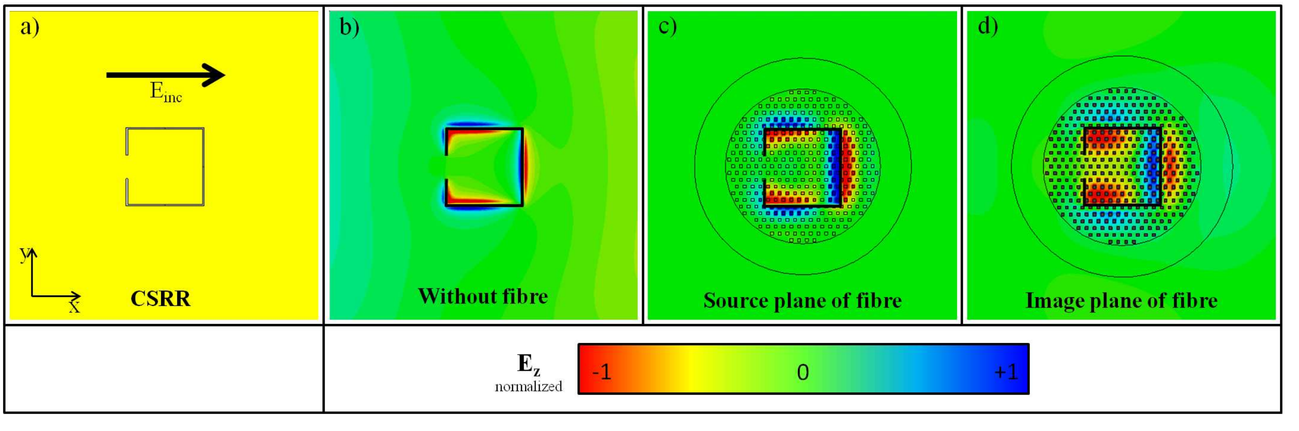

Fig. 1: a) Structure design of the CSRR; normalized longitudinal component of the electric field Ez: b) 2 μm above the CSRR without fibre, c) at the source plane of fibre, d) at the image plane of fibre.

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Tailoring radiation patterns in planar metamaterials

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Hyperbolic metamaterials, being a particular class of indefinite media [1], are described by the electric or/and magnetic tensors with the components of the opposite sign. Due to the hyperbolic isofrequency contours in the wave-vector space, such structures exhibit a number of unusual properties. First, waves at their boundaries may exhibit negative refraction, similarly to the case of double-negative metamaterials. Second, they have a diverging density of photonic states that allows enhancing the strength of light-matter coupling [2, 3, 4]. This makes a concept of hyperbolic media very promising for tailoring broad-band light-matter interaction, nanophotonics applications, including single-photon generation, sensing, and photovoltaic [5, 6, 7]. Here, we consider an uniaxial anisotropic hyperbolic medium characterized by the scalar permittivity ϵ and longitudinal and transverse permeabilities μ_{xx} and μ_{yy}. In the radio-frequency regime we mimic such a medium by artificial two-dimensional transmission lines based on lumped elements [8]. We demonstrate that a circularly polarized emitter near an anisotropic hyperbolic metamaterial unidirectionally emits in extraordinary modes of the metamaterial with the directionality of energy propagation controlled by the circular dipole handedness [9]. The effect is numerically demonstrated and confirmed by the experimental investigation of the hyperbolic metamaterial prototype operating at 36 MHz frequency.

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Terahertz/infrared waveguide modulators using graphene metamaterials

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Active developement of terahertz (THz)/ infrared (IR) science and technology has created a growing demand for new electronic and quasi-optical devices. In particular, the promising opportunities for broadband high-speed terahertz communication require new techniques for real-time manipulation of radiation. Extrodinarily high carrier mobility [1] and pico-second-scale photocarrier generation and relaxation [2] have attracted attention to graphene as an active material for high-speed modulation solutions at various frequencies.

We study and classify the electromagnetic regimes of multilayer graphene/dielectric artificial metamaterials in the THz/IR range. Placed inside a hollow waveguide (Fig. 1), they provide high-speed modulation of radiation and offer novel concepts for terahertz modulators and tunable filters.

We demonstrate an efficient, compact and ultrasensitive modulation at high-THz frequencies around the CO_2 laser emission line (30 THz) and analyse three examples of resulting tunable devices. The first one is a modulator with excellent ON-state transmission and very high modulation depth: > 38 dB at 70 meV graphene's electrochemical potential (Fermi energy) change (Fig. 1). The second one is a modulator with extreme sensitivity towards graphene's Fermi energy — a minute 1 meV variation of the latter leads to > 13.2 dB modulation depth. The third one is a tunable waveguide-based passband filter. The narrow-band cut-off conditions around the ON-state allow the latter to shift its central frequency by 1.25% per every meV graphene's Fermi energy change. We believe that graphene-dielectric multilayer composites will constitute a useful functional element for the THz-IR waveguide-integrated devices.

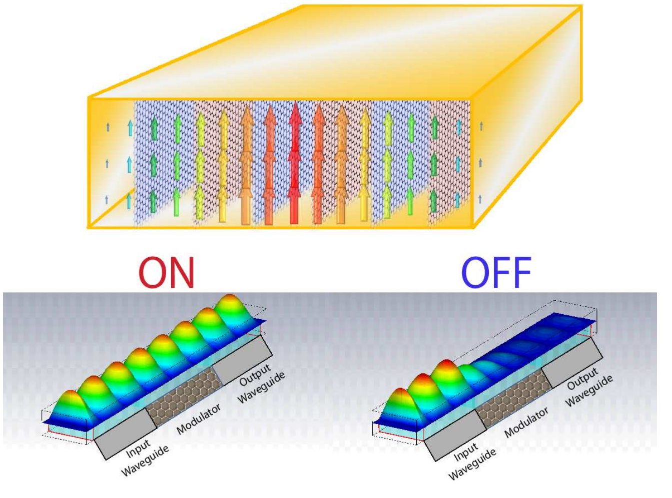

Fig. 1: Above: Graphene metamaterial inside a hollow waveguide. The arrows show the TE_{10} mode profile. Below: Electric fields in graphene/CsBr metamaterial modulator in the ON- and OFF-states. The modulating section is coupled to input/output NaCl-filled waveguides.

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All-dielectric nanophotonics: "magnetic light", Fano resonances, nanoparticle oligomers, and metasurfaces

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This talk will review the recent results from the Nonlinear Physics Center in Canberra and Metamaterial Laboratory in St. Petersburg on the topic of all-dielectric nanophotonics based on the magnetic resonances of high-index dielectric nanoparticles. The subjects will include the studies of nanoparticle antennas, nanoantenna arrays, oligomers, and metasurfaces. First, we will discuss useful functionalities and radiation efficiencies of nanoparticle antennas in the form of isolated nanoparticles or Yagi–Uda-type structures, and summarize the useful strategies to achieve the super-radiative performance via the excitation of multipole magnetic resonance modes. Then, we will discuss the scattering properties of nanoparticle oligomers and address the subject of the polarization-independent Fano resonances. This analysis is able to provide a useful insight into the use of polarization to offer non-trivial control over the near field distribution and hot spots of symmetric nanoparticle oligomers, despite the associated invariance of the far field transmission and all radiative and dissipative losses. Then, we will address the problem of the resonance engineering in silicon nanodisk-based metasurfaces and also study nonlinear properties of nanoparticle clusters including the third-harmonic generation spectroscopy based on contributions of both magnetic and electric resonances.

Superdirective dielectric nanoantennas for NV center photoluminescence collection enhancing

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In this report we apply a concept of all-dielectic nanoantennas [1, 2] for light collection, emitted by the single NV center. We demonstrate that using of this concept can lead to fluorescence collection efficiency increasing and zero phonon line (ZPL) emission enhancement. In addition we demonstrate that all-dielectric nanoantenna can provide the spatial selective excitation of a few tens of nanometers spaced NV centers without mechanical excitation scanning.

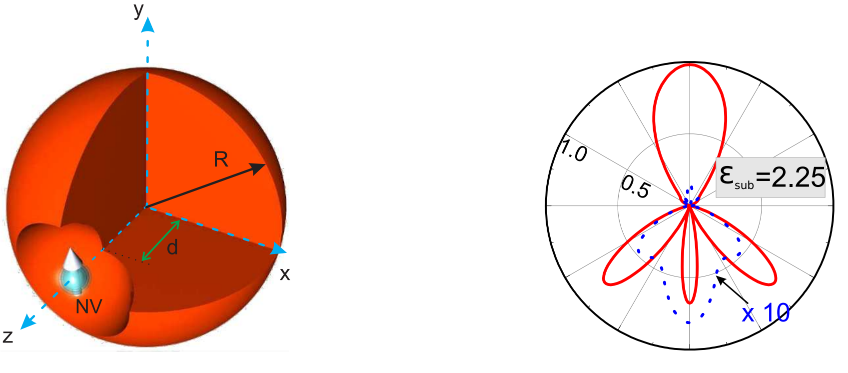

Fig. 1: Geometry sketch of the superdirective dielectric nanoantenna (left). Normalized power patterns for the emitter placed inside the notch above the substrate ($\epsilon_{sub} = 2.25$) with (solid red line) and without nanoantenna (dashed blue line) (right). Emitted light wavelentgh $\lambda = 637$ nm.

The nanoantenna is an optically small spherical dielectric nanoparticle with a notch excited by a quantum emitters located in the notch. The NV center is placed in free space inside the notch. The nanoantenna parameters were selected to maximize the average directivity in ZPL spectral region and get the highest Purcell factor at NV center ZPL. The directivity and Purcell effect demonstrate the resonance behavior. The resonance frequencies are defined by antenna geometry, and absolute values are defined by dipole position. The geometrical parameters can be tuned making the resonances coincide in frequency. This allows effectively collect the emitted light from quantum emitter and enhance ZPL emission simultaneously.

Contrary, to the generally accepted opinion that general nanoantennas direct emitted light into the substrate, this special antenna allows to direct light outside the substrate (fig. 1). In spectral region around ZPL frequency the power flow is enhanced up to two orders of magnitude compared to plain case.

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Magnetic Purcell factor in wire metamaterials

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Wire metamaterial or so-called wire medium is a broad class of electromagnetic metamaterials, which are usually composed of an array of parallel conducting wires embedded in a dielectric matrix [1]. These materials are very promising due to simple fabrication technologies and various breakthrough applications, such as: superlensing [2], improvement of magnetic resonance imaging systems [3], biosensing applications [4], shaping of waves at deep subwavelength scales [5] and design of high quality cavities [6].

Here we present an experimental study of the magnetic Purcell effect in finite arrays of the wire metamaterial. By directly measuring the spatial-frequency map of the Purcell factor we explicitly demonstrate how the Purcell factor is enhanced at the Fabry–Pérot resonances of the wire metamaterial block in microwave frequency range. The experimental results are in a good agreement with theoretical and numerical estimations.

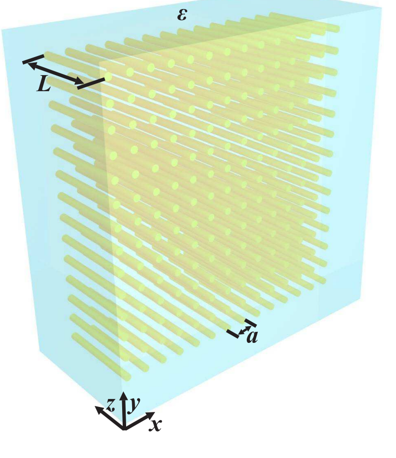

Fig. 1: Artist's view of the finite size wire metamaterial resonator embedded in a dielectric matrix.

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Multi-refringence phenomena in bi-periodic plasmonic multilayers

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Propagation of electromagnetic waves in stratified media has been studied for a very long time [1]. It was realized that plasmonic multilayers, optical metamaterials formed by a multilayered metal-dielectric nanostructure shown in Fig. 1(a), possess a range of striking electromagnetic phenomena that include broadband all-angle negative refraction, anomalous birefringence, k-dependent precession of the optical axis, and ultra-high values for the Purcell factor [2].

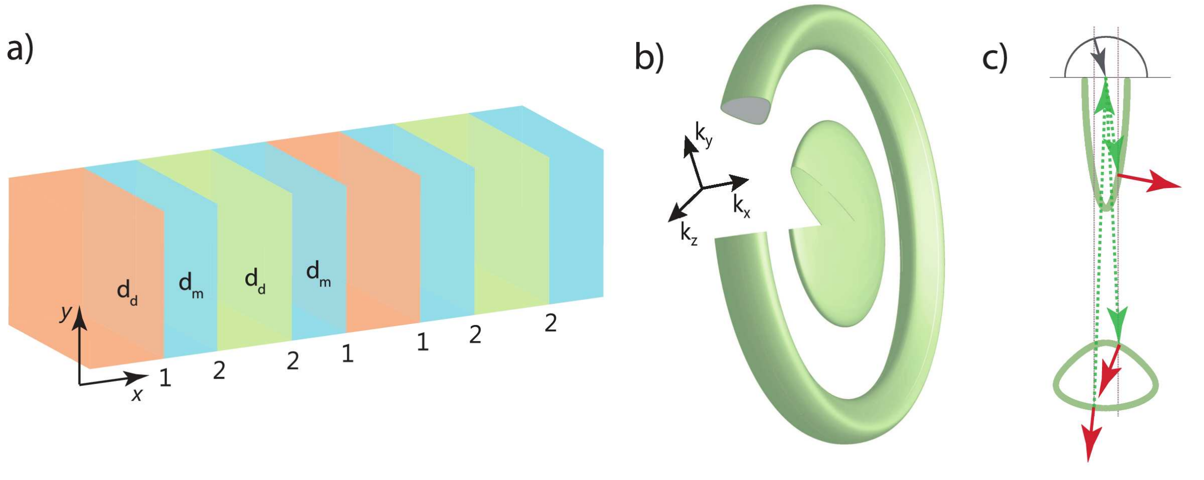

Fig. 1: (a) Geometry of a bi-periodic plasmonic multilayer, with interfaces between different layers of dielectric and metal marked with numerals. (b) One of the isofrequency surfaces realizing multi-refringence with (c) corresponding refration diagram.

In the present work we demonstrate multi-refringence in bi-periodic plasmonic multilayers. By bi-periodicity we mean that a multilayer is formed with different kinds of metal-dielectric interfaces — two in our case. We analyze isofrequency surfaces of such plasmonic multilayers and demonstrate the presence of multiple dispersion sheets leading to occurrence of the multi-refringence phenomena. Tri-refringence is demonstrated for bi-periodic multilayers both theoretically and numerically.

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Nanoplasmonic split-ball resonators

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A new concept, split-ball resonator, is introduced which allows experimental demonstration of a strong omnidirectional magnetic dipole response tuneable throughout the visible spectral range using standard plasmonic metals such as gold and silver.

One of the main challenges in the field of plasmonics and metamaterials during last decade is to engineer nanostructures with strong magnetic and electric dipole resonances at optical frequencies. Getting these two resonances together in the same frequency range can lead to unique material properties associated with near-zero or even negative effective refractive index. The concept of split-ring resonator (SRR), which provides strong magnetic dipole response of metallic structures, was theoretically introduces by Pendry et al. in 1999 [1]. Since then many efforts has been given to experimentally demonstrate magnetic resonance of metallic structures, first at GHz, then at THz, and finally at optical frequencies. It has been shown that scaling down the sizes of the split-ring resonator linearly increases the magnetic resonance frequency. However, this linear dependence saturates close to the visible spectral range mainly due to non-ideality of plasmonic metals at these frequencies. Further control of the resonance position is possible using optimization of the design of SRRs.

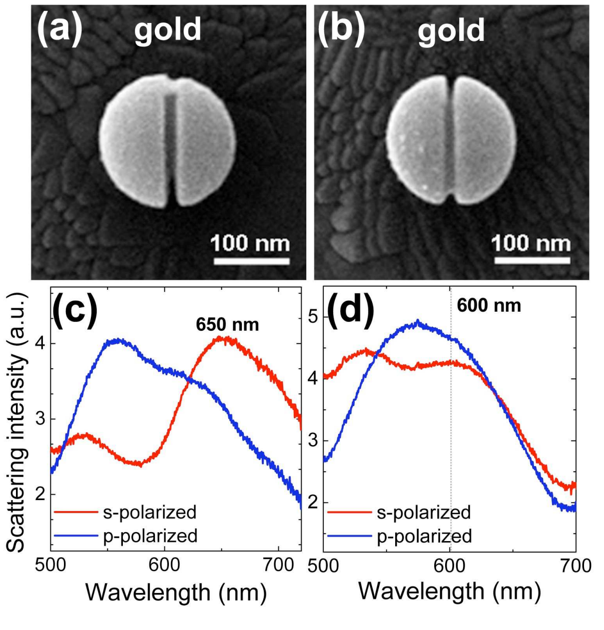

Fig. 1: Experimental realization of gold SBRs. (a)&(b) – HIM images of two gold nanoparticles with similar diameters of 170 nm and cuts width of 15 nm. Cut depth in (a) is larger than in (b), which is obtained by 25% longer helium ion beam milling time. (c)&(d) Scattering spectra of the particles shown in (a)&(b) for incident light with s (perpendicular to the cut) and p (parallel to the cut) polarizations.

In this paper, we introduce and experimentally demonstrate a new concept of a strong magnetic dipole resonance tuneable almost throughout the whole visible spectral range using standard plasmonic metals such as gold and silver. The key aspect is a nanometer-size cut fabricated inside an almost perfectly spherical plasmonic nanoparticle. Such 3D spherical design allows shifting magnetic dipole resonance down to electric dipole resonance wavelength in the visible spectral range. Experimentally this novel design is realized using laser-induced transfer method (LIT) to produce almost perfect spherical nanoparticles and helium ion beam milling (HIM) to introduce small cuts with nanometer-scale resolution [2]. The size of the nanoparticles is controlled using initial lithography step while their spherical shape and low surface roughness is assured by strong surface tension forces of molten metal during the laser processing. Then nanocuts with straight side walls have been produced on top of the nanoparticles using focused helium (He) ion beam milling. In comparison to standard FIB systems, which use gallium (Ga) ions, HIM can provide significantly higher structuring resolution. Also, in contrast to standard gallium-ions based FIB systems, HIM does not significantly dope side walls of the materials during milling and thus may keep good plamonic properties of the nanoparticles. HIM images of two gold nanoparticles with cuts fabricated by this combined method are shown in Fig. 1. These nanoparticles have a similar diameter of around 170 nm and cut width of around 15 nm. Cut depth was experimentally controlled by a dwell time of the helium ion beam in the milled area at a fixed ion flow.

Strong field-enhancement and magnetic dipole resonance of the split-ball resonators make them promising for future applications in metamaterials, surface-enhanced Raman scattering (SERS), heat-assisted magnetic recording (HAMR), and nanoantennas.

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Sphere cloaking using thin all-dielectric multilayer coatings designed by stochastic optimizer

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We used the adaptive differential evolution method [1] and Mie calculations [2] to design index spatial distribution in the thin all-dielectric multilayer coating. Optimization aim was to reduce total scattering from a coated spherical object made of a perfect electric conductor material. We investigated cases of several coating widths, target sizes, and number of layers in the coating. The best designs achieve total scattering reduction by a factor of two in case of target ball diameter one and a half wavelength. We found two types of design and their critical thickness needed to achieve significant scattering reduction. Full-wave simulation has shown field concentration in the coating with phase switching to reverse in radial direction and phase plane moving preferentially in tangential direction inside the switched regions. Physical description of such waveguide-like cloaking was proposed.

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Large penetration depth of near-field heat flux in hyperbolic media

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Theoretically it is well-known for a long time that heat radiation at the nanoscale can surpass the blackbody limit by orders of magnitude [1]. In particular, surface phonon polaritons can enhance the radiative heat flux by several orders of magnitude due to the very large number of contributing modes [2]. However, it has been shown very recently that also so called hyperbolic modes can lead to an enormous increase in the radiative heat flux [3–4]. The main advantage of the hyperbolic modes with respect to the surface modes is that they are propagating inside the hyperbolic medium, whereas the surface modes are bound to the surface of the material. The purpose of this work is to analyze the penetration depth (PD) of thermal photons, resulting from the heat exchange between two bodies close to each other. This quantity is very important for possible near-field thermophotovoltaic (TPV) applications. It defines the effective thickness of the layer in which electron hole pairs can be generated. But also for cooling applications larger PDs are preferable because photons transport heat much faster than phonons. Furthermore it is better to have a larger volume where the heat is absorbed to avoid local overheating.

In the following we consider the heat exchange by thermal radiation for the three different structures: (i) two gallium nitride (GaN) half spaces, (ii) two multilayer HMM half spaces composed of GaN/Ge bilayers and (iii) two nanowire HMM half spaces consisting of GaN nanowires immersed in a Ge host, separated by a vacuum gap with width l. The optical response of GaN is in the infrared dominated by the optical phonons so that the relative permittivity of GaN can be described by a Drude–Lorentz model. Germanium (Ge) is assumed to have a refractive index of 4. In order to describe the optical response of the structures (ii) and (iii) we use effective medium theory (EMT) which gives reliable results if the unit-cell size of the underlying structure is much smaller than the wavelength and the gap.

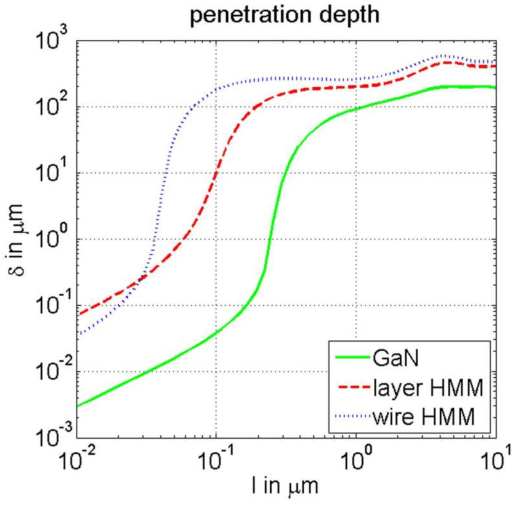

Fig. 1: Thermal penetration depth at T = 300 K for the systems: bulk GaN, GaN/Ge layer HMM and GaN/Ge wire HMM.

The total PD δ is depicted in Fig. 1. In the far-field it is more or less constant for all materials. Most importantly in the intermediate regime the PD in the HMMs can be two to three orders of magnitude larger than in GaN and in the strong near-field regime it can be more than one order of magnitude larger than in GaN. Hence, our numerical results suggest that hyperbolic materials are preferable to phonon-polaritonic media when larger near-field PDs are needed as in the case of near-field TPV.

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Ruling the rings: Consequences of strong interaction

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In this talk, we review some unusual properties of metamaterials assembled as regular lattices of densely packed conducting rings, including capacitively loaded resonators and closed loops. We argue that such systems feature exceptionally strong interaction between the elements, which brings some effects not available in other systems, e.g. regular lattices of dipoles, or wire media.

Strong mutual interaction was found to manifest itself both in the frame of an effective medium description [1] and analysis of magnetoinductive waves [2], and the effect of lattice parameters and symmetry on the properties of such metamaterials is well understood.

More recently, we have analysed the performance of the super-lenses based on three-dimensional isotropic assembly of subwavelength ring resonators [3], and found that its behaviour differs significantly from the predictions based on a continuous medium approximation. It turned out that no direct coincidence to the imaging properties of a $\mu = -1$ slab can be obtained in practice, although analogous phenomena, suitable for imaging, are available in a certain frequency range. We have also assessed the resolution, achievable with a discrete lens of finite size, and conclude that it is generally limited to 5–7 lattice constants. It is important to emphasise that this limitation is entirely structural and cannot be improved by decreasing losses.

More generally, finite systems containing a few thousands elements or less, cannot be described with an effective medium approach if the mutual interaction is sufficiently strong. Surface effects are quite significant in this case, and specific structure of a boundary of metamaterial sample is crucial in determining its observable properties [4]. The surface effects can be reduced by choosing a boundary structure such that the boundary elements have a more similar environment compared to the ones in the bulk: the so-called “ragged” surface instead of a flat one.

Strong mutual coupling is equally crucial for non-resonant systems. A detailed analysis of the diamagnetic properties of metamaterials made of closed conducting loops [5] have shown that with an appropriate choice of the structural parameters in anisotropic lattices, rather low effective permeability — down to at least 0.05 — can be reached. The low values of magnetic permeability are expected in a very wide frequency range, easily spanning several decades in frequency, and are accompanied by rather weak dissipation. In achieving such characteristics, lattice structure is most crucial, as it provides an efficient control over the mutual interaction, leading to a significant enhancement of diamagnetic properties.

At the same time, we point out that from a practical point of view, artificial diamagnetics with sophisticated internal structure are not optimal for certain applications. In particular, when only the external fields are of interest (as it is the case for magnetic levitation), bulk conductors offer an easier solution. In fact, the lowest magnetic polarisability (the strongest diamagnetism) is achieved when a body is filled by a good conductor. Any structuring of the conducting object, such as splitting it into closely packed cubes [6] leads to a higher polarisability (weaker diamagnetism), as it leads to a current confinement, and in the region of the confined current the fields are enhanced and the magnetic energy grows.

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Recycling radio waves with smart walls

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An increasing amount of data is nowadays conveyed wirelessly by radio waves in dense environments such as cities or buildings [1]. While propagating, the electromagnetic waves carrying the information are reflected several times off obstacles such as walls and furniture. Due to this multiple scattering and multipathing, the associated electromagnetic energy is spread evenly throughout those complex and reverberating media, which degrades the quality of the communications, raises health issues and wastes energy. In this talk we will show that part of this seemingly lost energy can be recycled by reflecting more intelligently these multiply scattered waves using smart walls rather than bare ones [2]. To do so we propose to use ultrathin metasurfaces [3] that we design to be electronically reconfigurable in real time as spatial microwave modulators. We will show that they can be utilized to cover part of the walls of a typical office room, hence transforming these dumb surfaces into smart ones. We will demonstrate that, akin to the spatial light modulators which can focus light through complex media [4, 5], those spatial microwave modulators can passively turn a random electromagnetic field resulting from reverberated and multiply scattered waves into a shaped one using a simple energy feedback. Specifically, we will prove that such smart walls can, in real time, increase by orders of magnitude the energy of a wireless signal received by any antenna or locally conceal a volume from penetration of microwaves. We will finally propose a quantitative estimation of the benefits brought by the approach, by introducing and modeling the notion of wavefront shaping in reverberating media. The spatial microwave modulators proposed in this talk as smart walls have obvious applications in green wireless communications and electromagnetic protection, but they are also amazing tools for fundamental physics related to the propagation of waves in complex, highly scattering or reverberating media.

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Locally resonant metamaterials: focusing, imaging and manipulating waves at the deep subwavelength scale

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In this talk I will present some of our recent works on metamaterials based on resonant unit cells.

I will show how the use of time dependent and broadband wavefields, in conjunction with those metamaterials, permits to beat the diffraction limit from the far field for imaging or focusing purposes. I will introduce the idea of resonant metalens, first demonstrated in the microwave domain, and explain its principles. In particular, I will show how the concept of time reversal can be utilized to focus in this metamaterial based lens and from the far field, onto focal spots much smaller than the diffraction limit [1]. I will then prove the generality of the approach by demonstrating its transposition to the acoustic domain [2] thanks to a very simple setup: an array of soda cans (Figure a). Then I will present our latest theoretical and numerical results obtained using a resonant metalens made out of plasmonic nanorods in the visible part of the spectrum [3].

Finally I will then prove that since some of those media are solely governed by interference effects, it is possible to go beyond the effective medium theory usually used in this field, and adopt a microscopic approach to these metamaterials. In particular I will show that those media can be modified locally at will in order to confine, guide, bend, or split waves (Figure b), just like it is realized in photonic or phononic crystals, yet on dimensions that are much smaller, i.e. that are deeply subwavelength. This approach, which fills the gap between photonic crystals and metamaterials, will be experimentally demonstrated with acoustic and electromagnetic waves [4, 5].

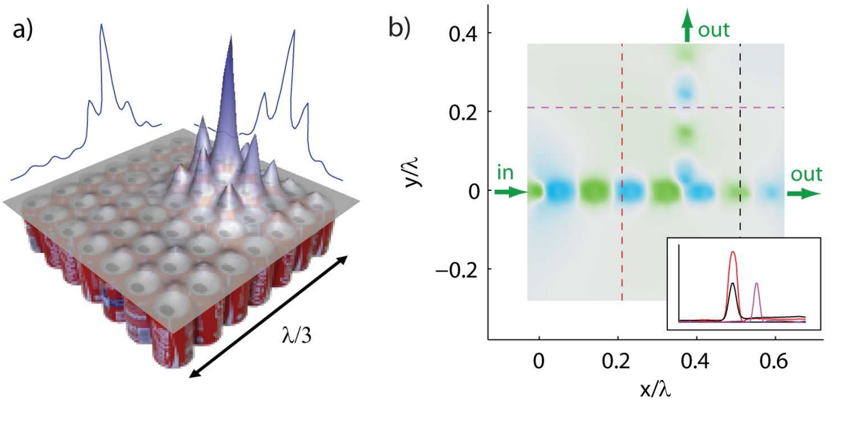

Fig. 1: a) deep subwavelength focal spot obtained using far field time reversal on top of an array of soda cans, and b) waveguiding microwaves at the deep subwavelength scale in an array of locally modified resonant electric wires.

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Manipulating beams with metamaterials

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In this talk we discuss novel approaches to manipulating electromagnetic waves on micro- and macro-scales using linear and nonlinear metamaterials. In particular, we discuss matamaterial route to structuring light itself and consider propagation and interactions of structured light with metamaterials.

Singular optics (or Structured Light) is a fascinating emerging area of modern optics that considers spin and orbital angular momentum properties of light and brings a new dimension to the science of light and physics in general. Optics facilitates the realization of many spin- and orbital angular momentum related effects that were predicted in a myriad of other physical systems where direct experimental observations are challenging or impossible. Moreover, recent developments in the field of metamaterials and transformation optics enable unprecedented control over light propagation and a possibility of "engineering" space for light propagation, opening a new paradigm in structured light related phenomena in optical physics.

Light beams with orbital angular momentum have significant potential to transform many areas of modern photonics, from imaging to classical and quantum communication systems. We proposed and experimentally demonstrated an ultra-compact array of nano-waveguides with a circular graded distribution of channel diameters that coverts a conventional laser beam into a vortex with configurable orbital angular momentum. The proposed nanoscale beam converter is likely to enable a new generation of on-chip or all-fiber structured light applications.

In the second part of this talk we discuss so-called virtual hyperbolic metamaterials that we proposed for manipulating microwaves in air. Microwave beam transmission and manipulation in the atmosphere is an important but difficult task. One of the major challenges in transmitting and routing microwaves in air is unavoidable divergence because of diffraction. We proposed and designed virtual hyperbolic metamaterials formed by an array of plasma channels in air as a result of self-focusing of an intense laser pulse, and show that such structure can be used to manipulate microwave beams in air. Hyperbolic, or indefinite, metamaterials are photonic structures that possess permittivity and/or permeability tensor elements of opposite sign with respect to one another along principal axes, resulting in a strong anisotropy. Our proof-of-concept results confirm that the proposed virtual hyperbolic metamaterial structure can be used for efficient beam collimation and for guiding radar signals around obstacles, opening a new paradigm for electromagnetic wave manipulation in air.

Spontaneous symmetry breaking in nonlinear metamaterials

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Spontaneous symmetry breaking is a prominent effect closely related to a number of fundamental phenomena in a wide range of areas. Here we show theoretically and experimentally that this effect can also be available in magneto-elastic metamaterial systems, in which enantiomeric nonlinear meta-molecules with opposite handedness are electromagnetically coupled. Our study provides a new possibility for creating and designing artificial phase-transition effects in metamaterials.

Spontaneous symmetry breaking is an underlying mechanism of such fundamental phenomena as spontaneous magnetization [1], the homo-chirality of bio-molecules [2], the bulk of mass of nucleons [3], and the recently discovered Higgs bosons, etc. Such prominent effect now can be found in magneto-elastic metamaterials [4].

In this work, we present the study of the nonlinear behavior of two torsional meta-molecules with opposite handedness (the so-called *metamaterial enantiomers*), which are electromagnetically coupled. Intuitively, under chiral symmetry, torsional meta-molecules with opposite handedness should have identical magnitudes of electromagnetic response if they are equally excited; the net chirality of the whole system should also vanish due to chiral symmetry. However, our study explicitly shows that the interaction between meta-molecules becomes sufficiently strong, the system undergoes spontaneous symmetry breaking and becomes chiral.

We study the underlying mechanism of this effect and analyze the evolution of the system stability, revealing that intermolecular interaction is indispensable for the existence of the asymmetric states. We further study such processes in enantiomeric necklace rings and infinite arrays of meta-molecules, showing that spontaneous chiral symmetry breaking provides a novel and feasible mechanism for creating artificial phase-transitions in assemblies of meta-molecules without using naturally occurring phase-change materials.

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Life time and photon statistics of a single dye molecule near hyperbolic metamaterials

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In a hyperbolic medium, the principal components of the permittivity have opposite signs causing the medium to exhibit a 'metallic' type of optical response in one direction, and a 'dielectric' in the other. A nanolithography pattern was demonstrated with a hyperbolic slab made of many thin planar layers of a metal and dielectric [1]. Improving the radiative decay rate for dye molecules with hyperbolic metamaterials was shown with a similar multilayer structure by measuring their life time and quantum yield [2]. We study also the hyperbolic dispersion of metamaterials with magnetic response. Recently we explored the group delay dispersion of metamagnetic gratings by the multiphoton intrapulse interference phase scan technique [3], which enables their use for ultrafast pulse shaping. The metamagnetic films have properties of biaxial anisotropic materials. In contrast to uniaxial hyperbolic metamaterials, metamagnetics represent a more general example of indefinite media [4] where both permittivity and permeability components have opposite signs depending on the wavelength. We show that the dye luminescence life time depends on the relative position of the emission resonance and magnetic resonance of the grating. Anti-bunching effect in photon correlation function is also studied for single dye molecules near the grating surface. Our findings on the life time and photon statistics of dye emission hyperbolic metamaterials will be discussed.

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The concept of invisibility and imitation of objects based on the method of minimal autonomous blocks

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Masking of objects in the microwave and optical frequency ranges is complex electrodynamic and technological problem. To solve it, the following approaches are used: a method of transformation optics, interference compensation of reflected waves, the system reradiating antennas, etc. Effectiveness of the practical use of these approaches depends on the wave size, shape and material composition of the masked objects. The problem of quality evaluation masking objects for arbitrary sources of electromagnetic radiation is very complicated and actual. A new approach to the evaluation of radar and optical visibility, based on the method of minimal autonomous blocks [1], is proposed. This approach includes the following steps:

— selection the space region containing the investigated object;
— decomposition of the space region on the system of minimum autonomous blocks;
— calculation of scattering matrices for all main and auxiliary blocks;
— recomposition of internal channels and multichannel scattering matrix calculation relatively channels onto the outer boundary of the region;
— comparative analysis of multichannel scattering matrix of region containing the masked object, with multichannel scattering matrix corresponding to the region without of an object (the problem of invisibility) or the region with a emulated object (imitation task).

Algorithms for calculating multichannel matrix for large wave sizes region containing composites and metamaterials are presented. For the description of electromagnetic properties of composites and metamaterials it is offered to use the average scattering matrixes of non-uniform macroblocks [2, 3]. Efficient algorithms are presented for calculating multichannel scattering matrices in multivariate analysis.

Examples of using multichannel scattering matrix to evaluate the effectiveness of three-dimensional objects masking are presented.

The possibility of using the MAB method for evaluating the effectiveness of acoustic masking and imitation of objects was considered.

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Optical properties of high-index dielectric nanoparticles tailored by substrates

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Nanoparticles of high refractive index materials can possess strong magnetic polarizabilities and give rise to artificial magnetism in the optical spectral range. While the response of individual dielectric or metal spherical particles can be described analytically via multipole decomposition in the Mie series, the influence of substrates, in many cases present in experimental observations, requires different approaches. Here, the comprehensive numerical studies of the influence of a substrate on the spectral response of high-index dielectric nanoparticles were performed. In particular, glass and perfect electric conductor substrates were investigated. Optical properties of nanoparticles were characterized via scattering cross-section spectra. The presence of substrates was shown to introduce significant impact on particle's magnetic resonances and resonant scattering cross-sections.

Introduction. Optical magnetism can be artificially created via carefully engineered subwavelength structures, such as arrays of ordered split-ring resonators. One of the fundamental bottlenecks, limiting the performance of plasmonic components and metamaterials is inherent material losses that is of great importance for various nanophotonic components, such as nanolenses, antennas, particle-based waveguides, ordered particle arrays, and biosensors. At the same time, high refractive index dielectric nanoparticles have shown to be promising in the context of artificial magnetism and the majority of aforementioned components can be implemented with all-dielectric elements, as was already demonstrated in case of ordered particle arrays [1] and antenna applications [2]. Strong resonant light scattering associated with the excitation of magnetic and electric dipolar modes in silicon nanoparticles has recently been demonstrated experimentally using dark-field optical microscopy [3, 4]. While the majority of theoretical studies consider isolated spherical particles or their clusters in free space, very often experimental geometries involve the presence of substrates where nanoparticles are placed [3, 4]. In this work, we performed numerical studies of optical properties of high-index dielectric nanoparticles on various types of substrates.

Methodology. One of the commonly used techniques for numerical analysis of scattering processes is the so-called "total-field scattered-field" (TFSF) approach [5]. The key advantage of this method is the separation of relatively weak scattered field (SF) from predominating high amplitude total field (TF) which contains both incident and scattered fields, in a distinct simulation domain. The TFSF method also allows to subtract the electric field, reflected backwards by the substrate in the SF domain, enabling the calculation of a scattering cross-section.

Results. Scattering cross-section spectra were calculated for the case of a dielectric particle in free-space, on a dielectric substrate, and on a PEC substrate. Scattering cross-section spectra (Fig. 1(a)) shows 3 distinctive resonances in the case of free-space, corresponding to magnetic dipole (MD), electric dipole (ED), and magnetic quadrupole (MQ) at 644 nm, 472 nm, and 444 nm. The influence of dielectric substrate results in significant suppression of the high-order multipoles (Fig. 1(b)), with both electric and, especially, magnetic dipolar resonances being less affected. A PEC substrate (Fig. 1(c)) leads to the spectral shift of the nanoparticle resonances and significant modifications of the spectrum and magnitude of the scattering cross-sections.

Conclusions. Different types of substrates such as flint glass and perfect electric conductor, were investigated. The presence of substrates was shown to introduce significant impact on particle's magnetic resonances and resonant scattering cross-sections. We can observe that dispersionless dielectric substrate preserves natural properties of individual particles, PEC substrate gives a relatively strong MQ resonance and a significant enhancement of the ED resonance.

a)

Free space			
λ, [nm]	444	472	644
σ / S	2.8	1.8	3.4

b)

Dielectric substrate			
λ, [nm]	446	475	645
σ / S	1.5	2.3	3.0

c)

PEC substrate			
λ, [nm]	437	492	648
σ / S	5.2	10.0	6.0

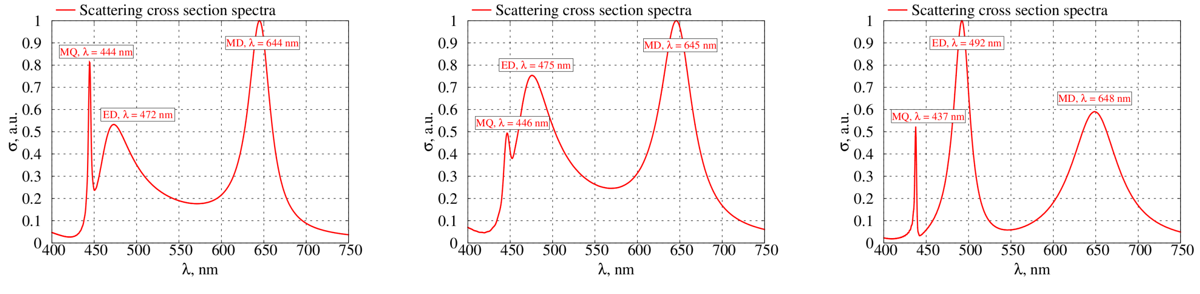

Fig. 1: Scattering cross-section spectra and scattering cross-section normalised to geometric cross-section at resonant wavelengths of a dielectric ($\epsilon_{particle} = 20$) nanoparticle of 70 nm radius (a) in free space, (b) on a dielectric substrate ($\epsilon_{dielectric} = 3.1$), and (c) on a PEC substrate ($\epsilon_{PEC} = 1 + 1e^6 i$).

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Theory of super-Planckian metamaterial thermal emitters

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As is well known, thermal emission of hot macroscopic bodies is governed by Kirchhoff's emissivity law, which connects absorptive and emissive properties of the bodies. The same law also defines the concept of the ideal black body — the perfectly absorbing body — which attains maximum emissivity at a given wavelength under given temperature.

The spectral power density of thermal radiation produced by an ideal black body is given by the well-known Planck's law. Until recently, it had been believed that this law sets a strict upper bound for the thermal power radiated by a macroscopic body of a fixed radius $R \gg \lambda$, where λ is the radiation wavelength. It had been also known that microscopic thermal radiators with $R \lesssim \lambda$ could emit significantly more power than the Planck law would predict for a black body of the same radius. For such emitters, the super-Planckian radiation effect was explained by resonant interaction of a microscopic object with the electromagnetic field, which dramatically increases the scattering cross-section of the object, which implies also the same increase in the emitted power [1].

In this work we develop a theory [2] applicable to both microscopic and macroscopic objects and demonstrate that even in the case when $R \gg \lambda$ there is a theoretical possibility to overcome Planck's black body limit. Moreover, we propose a conjugate-matched metamaterial thermal emitter that, at a given wavelength, may outperform the ideal black body of the same dimensions by up to 10 times, depending on the size of the emitter (see fig. 1).

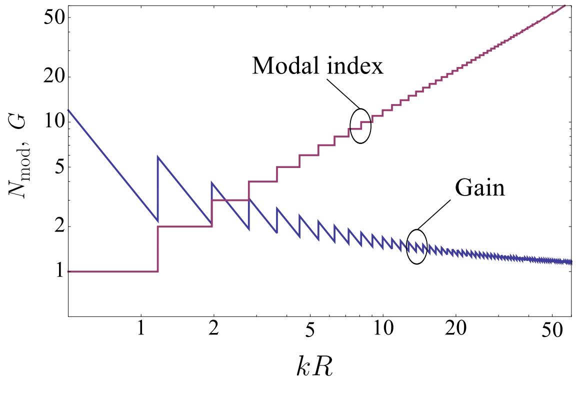

Fig. 1: The gain G of a spherical metamaterial emitter with the normalized radius kR and the modal index N_{mod} of the highest spherical harmonic radiated from the emitter with the figure of merit $Q \sim 10$ (log scale in both axes).

This result demonstrates that conjugate-matched metamaterial emitters characterized with a moderate figure of merit $Q \sim 10$ can be at least twice more efficient than a black body up to $kR \lesssim 2\pi$. Emitters with higher Q may have higher gain, however, because emitter's bandwidth is inversely proportional to Q, they are more narrowband.

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Effective-medium model of wire metamaterials in the problems of radiative heat transfer

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Heat harvesting and its conversion into electricity is one of the most important challenges for the modern world. Dielectric hyperbolic materials [1], which are one of the most interesting classes of electromagnetic metamaterials, have attracted considerable attention corresponding to this modern challenge, especially to thermo-photovoltaic (TPV) systems. Investigations of TPV systems enhanced by metamatetrials are becoming now a new branch of the modern literature. A strongly super Planckian radiative heat transfer (SSP RHT) can be achieved by using the dielectric hyperbolic metamaterials in micro-gap TPV systems. The gap, which separates the emitter and the PV cell, is filled with arrays of aligned metal nanowires [2, 3]. This wire medium (WMM), which posesesses a hyperbolic dispersion, is one of the classes of the dielectric hyperbolic metamaterials. The radiative heat transfer grows dramatically because the spatial harmonics of radiative heat which are evanescent in free space are propagating across the gap.

One fundamental problem related to improving the performance of TPV systems by using metamaterials, is the problem of the adequate homogenization of metamaterials in the context of RHT. For WMMs, the theory of the effective-medium model (EMM) has been developed quite well. In [4], two known EMMs of WMMs (the quasi-static one and the non-local one) are reviewed. Both of them describe the optical properties of these media through a uniaxial dyad of effective permittivity. The bounds of validity of the EMM for WMMs have been already outlined for all practically important variants of WMMs operating in many ranges from microwaves to visible light. However, the previous studies were targeted to the imaging applications of WMMs. The adequacy or inadequacy of the EMM for these applications can not be transferred directly to the adaptability of EMM for RHT purposes and TPV systems. In the present work, we study the applicability of EMM for RHT over the micron-scale path through WMMs of metal nanowires operating in the near IR range (1.5–6 μm or 50–200 THz). Here, we investigate several geometries that may correspond to micro-gap TPV systems. These geometries seem to be promising in view of our previous researches related to these systems.

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Electromagnetic wave coupling through single sub-wavelength ($\sim \lambda/100$) apertures: application for terahertz (THz) imaging and spectroscopy

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Coupling of short terahertz (THz) pulses through single apertures in metallic screens as small as 1/100th of the wavelength will be considered for applications in near-field THz imaging and local THz spectroscopy. Effects of metallic surfaces and dielectric objects present near the aperture will be demonstrated in THz imaging experiments.

Introduction. Near-field microscopy with a sub-wavelength aperture allows investigations of electromagnetic (EM) fields with spatial resolution beyond the diffraction limit. It is widely used in applied and fundamental research across the spectrum from the optical waves to radiowaves. The problem of EM wave coupling through a single sub-wavelength aperture is central in near-field microscopy: it determines resolution limits and provides insight into interpretation of near-field images.

For applications with terahertz (THz) waves, due to the long wavelength, the aperture size can be in the range of 1/100th of the wavelength. Here we will discuss the coupling of the EM waves through such small apertures, for which the effects of evanescent waves become increasingly important, focusing on the impact of the basic metamaterial building blocks, metals and high-dielectric constant materials, on the process of coupling through the sub-wavelength aperture.

Discussion. According to Bethe's theory, the transmission coefficient of a sub-wavelength aperture follows a strong power dependence on aperture size a: transmission decreases as a^6. This dependence, in practice, limits applications of small apertures in near-field microscopy. Although corrugations around the aperture can be used to improve the transmission coefficient, their use in near-field probes for imaging and spectroscopy can also degrade spatial resolution and impose frequency-selective response.

An alternative approach for enabling the use of $\lambda/100$ apertures is an integrated near-field probe, which contains a detector in the near-field zone of the aperture [1]. In the near-field zone the transmission coefficient changes from the a^6 dependence to a much weaker (a^2-a^3) dependence. It allowed us to realize THz near-field probes with apertures as small as 3 μm and to investigate amplitude and phase changes, which occur on transmission of EM waves through apertures [1].

In application to metamaterials, the integrated near-field probes enable detection of THz surface plasmon waves. We will discuss the coupling mechanism, surface wave mapping and characterization of strongly confined THz waves using the sub-wavelength aperture probes [2, 3].

Dielectric materials that exhibit a large value of the relative dielectric constant, e.g. TiO_2 with an average $\varepsilon \sim 100$ in polycrystalline form, can be used to make low-loss metamaterials. Dielectric spheres and ellipsoids made of TiO_2 can act as high quality THz resonators of sub-wavelength dimensions with both the magnetic and electric-type resonances, providing building blocks for THz dielectric metamaterials, alternative to metallic split-ring resonators. We will discuss applications of sub-wavelength aperture probe for characterizing single TiO_2 spheres, in which the Mie resonances are predicted at 1–2 THz [4].

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Optics and magneto-optics in 2D magnetoplasmonic crystals

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The use of periodic nanostructured materials due to plasmonic, photonic and other resonant effects allows one to enhance magneto-optical response in various spectral regions [1, 2]. Periodic nanostructuring of magnetoplasmonic crystals allows one to control excitation and propagation of surface plasmon-polaritons (SPP). 2D-periodicity provides the opportunity of spectral tuning of the phase-matching conditions via superposition of two vectors of the reciprocal lattice at various azimuthal angles. There are structures where simultaneous excitation of two intersecting magnetoplasmons can be achieved forming a standing plasmonic wave that leads to formation of plasmonic band gap [3]. In this work, the transversal magneto-optical Kerr effect (TKE) is studied in two-dimensional magnetoplasmonic crystals. Resonant enhancement of the Kerr effect is observed and can be attributed to the resonant excitation and the interaction between magnetoplasmonic modes, waveguide modes and effects that are emerging when diffraction maxima lays into the plane of structure.

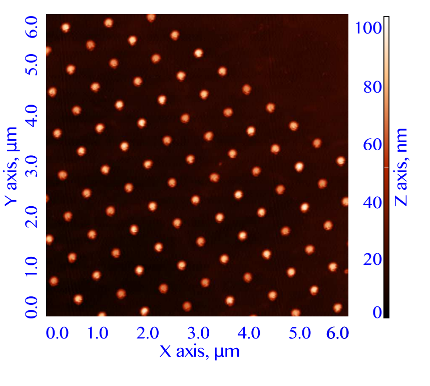

Fig. 1: An AFM-image of the sample.

The sample is 2D square lattice of Au disks placed on 1-mm-thick quartz substrate. 100-nm-thick Bi:YIG layer covers the structure. The sample is characterized by an atomic force microscopy (Fig. 1). The period of the lattice is 600 nm. The Au disks height is about 60 nm and their diameters are about 100 nm. According to vibrating sample magnetometry measurements the saturation field is 1 kOe.

Optical and magneto-optical spectra of the sample were measured at different azimuthal angles and polarizations. There is a distinct correlation between the spectral positions of plasmonic resonances and the resonances in TKE spectra.

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Graphene-based asymmetric hyperbolic metamaterials for photonics applications

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Asymmetric hyperbolic metamaterial (AHM) is an uniaxial material with different signs of the diagonal components of the permittivity tensor and the optical axis, tilted with respect to a medium interfaces [1]. Asymmetry appears as a difference in properties of waves, propagating upward and downward with respect to the metamaterial interface. The most important feature of AHM is a possibility to excite very slow waves in AHM by a plane wave, incoming from free space with a minimal reflection. In this paper we present three effects in graphene-based AHM, which can find applications in photonics. The first one is a possibility of the total absorption in an optically ultra-thin layer, see Fig. 1 (left) [2]. The second example illustrates a high-directive thermal emission, produced by the AHM (center). The last is a possibility to use the graphene-based AHM for amplification of the terahertz radiation (right). Effect of plasmonic THz lasing in a planar array of graphene resonant nanocavities was described in [3]. We suggest a more efficient way how to provide the THz amplification in plasmonic graphene structures. Our approach gives much higher gain of an incoming plane terahertz wave within a considerably broader band than in the strip-like graphene structure, considered in [3].

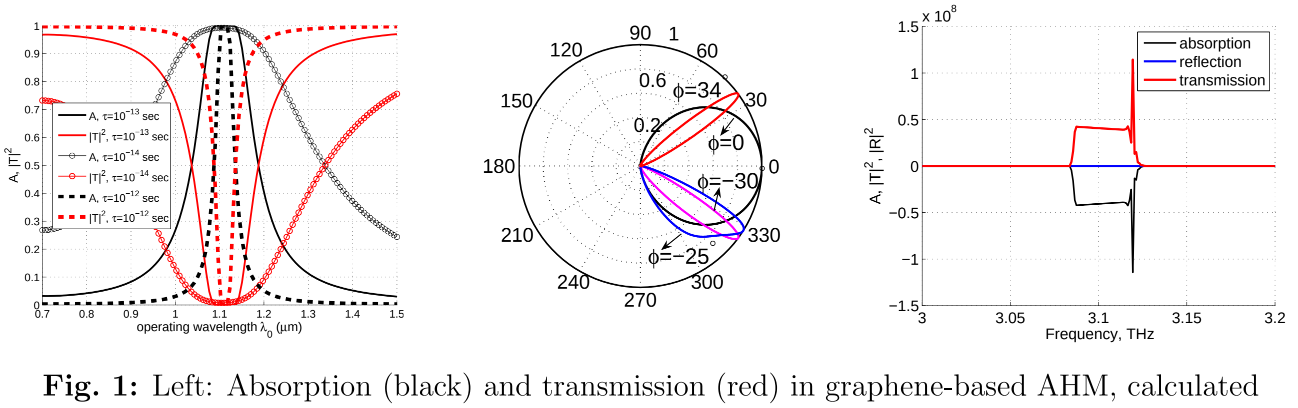

Fig. 1: Left: Absorption (black) and transmission (red) in graphene-based AHM, calculated for different relaxation times τ. Center: Radiation pattern in polar coordinate, calculated for different tilt angles ϕ. Right: Transmission, reflection and absorption.

Thus we have demonstrated that our concept of asymmetric hyperbolic metamaterials brings benefits in different areas related to graphene nanophotonics.

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Light interaction with linear and nonlinear hyperbolic metamaterials

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Hyperbolic metamaterials (HMMs) are anisotropic composite structures ensuring hyperboloid isofrequency surface for electromagnetic waves [1]. This means that the wavenumbers can be large, and the HMM slab resolves subwavelength features transmitting both propagating and evanescent waves. HMMs can be applied in nanolithography and engineering of the density of states for absorption and photoluminescence enhancement, lifetime shortening, etc.

We consider an HMM as alternating set of dielectric and metallic slabs, supposing linear and nonlinear materials of dielectric layers. We investigate the scattering properties and optical forces for the spheres made of the linear HMM. We notice that the effective parameters well describe the HMM using dielectric permittivity tensor of the form $\hat{\varepsilon} = \mathrm{diag}(\varepsilon_r, \varepsilon_t, \varepsilon_t)$, where the real parts of the radial ε_r and transverse ε_t permittivities have opposite signs. For anisotropic spherical beads with effective permittivity $\hat{\varepsilon}$ we study the pulling force effect [2, 3, 4] for the particles in non-paraxial Bessel beams. Pulling force is a non-conservative optical force arising due to the interaction of multipoles (in the simplest case, interaction of the electric and magnetic dipoles). We reveal that the HMM particles can be pulled by the light without intensity gradient only if $\mathrm{Re}(\varepsilon_t) > 0$ and $\mathrm{Re}(\varepsilon_r) < 0$. Hyperbolic metamaterials containing metal are inherently lossy. Nevertheless, the presence of the losses is not fatal for pulling, so that the HMM particles can still be attracted by the light source.

As to nonlinear case, we are interested in temporal dynamics of radiation propagating in the HMM, i.e. the metal-dielectric multilayer system. Using finite-difference time-domain numerical methods, we study the effects of light-structure interaction in both steady and pulse regimes. In particular, we analyze the possibility to obtain optical bistable response and the effects of pulse reshaping (such as compression) in the HMM. Another topic of our research is connected with the introduction of disorder in the multilayers considered. In particular, we study the peculiarities of the Anderson localization in the HMM and, especially, the interplay of disorder and nonlinearity in such systems.

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Short pulse dynamics in nonlinear disordered photonic crystals

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Disordered photonic structures have become the object of active study in recent years. Such interest is substantially due to observation of new optical effects such as the coherent backscattering, the Anderson localization of light, and the deviations of statistical properties of light from the classical diffusion [1, 2]. The possible realizations of optical disordered system include the microstructured waveguides, metamaterials [3], and photonic crystals [4]. Introduction of nonlinearity to disordered systems strongly enhances the complexity and richness of possible optical dynamics. Though there are many works devoted to this topic, one cannot say that the problem of disorder/nonlinearity interplay is fully understood.

In this paper, we consider ultrashort pulse propagation in a one-dimensional disordered multilayer (photonic crystal) with instantaneous and noninstantaneous nonlinearity. We are interested in the regime of both strong disorder, when the range of Bragg-like reflection is effectively extended beyond the band gap, and strong nonlinearity, when the effects of pulse shape transformation occur. The interplay of disorder and nonlinearity is studied on both the short timescale (pulse shape transformation) and at large times (pulse decay pattern transformation). In the latter case, the nonexponential behavior of the pulse "tail" can be interpreted as the evidence of diffusion violation and the sign of the Anderson localization. Numerical simulations based on the finite-difference time-domain method for both instantaneous and relaxing nonlinearity imply that nonlinearity and disorder are the competing factors: when one of them is strong, the influence of the other becomes very limited. Special attention is given to the modification of the self-trapping effect [5] which still takes place at the low enough disorder strengths. However, the patterns of light distribution along the photonic crystal are substantially changed as demonstrated by various realizations of disordered system. At higher disorders, the usual reflection prevails diminishing the part of energy trapped inside the structure.

Possible further directions of research include studies of spectral transformations and multi-pulse dynamics inside disordered photonic crystals.

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Characterization of zero-index plasmonic multilayers using retrieval of the constitutive parameters from S-parameters

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Plasmonic multilayers are periodic layered metamaterials formed by alternating layers of metal and dielectric. It has been shown that the permittivity tensor of the plasmonic multilayer is nonlocal and have off-diagonal components [1]. The only component that is not connected to other components is the component parallel to layer interfaces. Consequently, it may seem that this is the only component that can be retrieved easily from information contained in reflection R and transmission T coefficients by means of the method called Nicolson–Ross–Weir extraction [2].

Having a finite plasmonic slab with thickness of L we can calculate its ε and μ as outlined beyond. First, using the following equations the wave impedance Z and the refractive index n are calculated:

$$Z = \pm\sqrt{\frac{(1+R)^2 - T^2}{(1-R)^2 - T^2}}, \quad n = \frac{\pm \arccos\left(\frac{1}{2T}(1 - R^2 + T^2)\right) + 2\pi m}{k_0 L}. \tag{1}$$

Next, since $Z = \sqrt{\mu/\varepsilon}$, $n = \sqrt{\varepsilon\mu}$ permittivity and permeability can be yielded as $\varepsilon = n/Z$, $\mu = Zn$. It can be seen that there is a singular point in (1) where $R = |R|e^{i\phi_R} = 1$ and $\mu \to \infty$ and $\varepsilon \to 0$ at the same time.

At a glance, for the total reflection there always epsilon-near-zero, mu-near-pole medium will be extracted. However, there is a way how correct permittivity and permeability can be retrieved from reflection and transmission coefficients. The concept is based on a cycle shift operator being applied to the structure. The operator acts like a frame of the length L moving across an infinite multilayer and cutting out a region it covers. It turns out that for a certain value of l_s that is the position of the frame we are able to retrieve correct ε and μ for a plasmonic multilayer overpassing the singularity in wave impedance originating from $R = 1$ that leads to retrieval of spurious constitutive parameters of plasmonic multilayers.

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Quantum theory of a spaser-based nanolaser

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We describe the behaviour of the spaser-based nanolaser below and above generation threshold, and demonstrate that the spectral line narrows considerably, when passing through the threshold. We find the average number of plasmons in the cavity and show that this number near the generation threshold can be of the order of unity. In the case the coherence is preserved in a state of active atoms, which relax slowly than the damping of cavity mode occurs. This fact fundamentally distinguishes the behaviour of the bad-cavity nanolasers in comparison with the good-cavity lasers, where the coherence is preserved in a state of photons. The evaluation for the number of plasmons is in accordance with the experimental observations [1]. Moreover, we obtain second-order correlation function $g^{(2)}(\tau)$, and find that above the generation threshold the amplitude fluctuations of polarization of active atoms lead to the damped oscillations in $g^{(2)}(\tau)$. A similar dependence was observed in numerical simulations in the paper [2], and it is usual for bad-cavity lasers [3]. However, in the case of good-cavity lasers there is no oscillations in second-order correlation function. We assume that the shape of the curve $g^{(2)}(\tau)$ indicates the mechanism of the spectral line narrowing, and therefore we investigate at what relationship between cavity decay rate κ and homogeneous broadening of active atoms Γ the oscillations occur. We think that obtained results are important for understanding the fundamental principles of operation of spaser-based nanolasers.

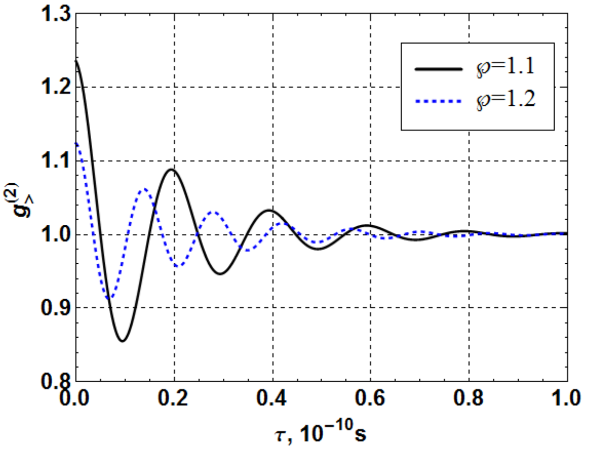

Fig. 1: The second-order correlation function above the generation threshold.

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Numerical simulation of experiment on detection of Langmuir modes in a hyperbolic medium

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It is shown theoretically in [1] that an anisotropic plasma slab can provide propagation of Langmuir waves. According to [2] dielectric function of wire medium at radiofrequencies within effective medium approximation can be described by the dielectric function of anisotropic plasma. Therefore, we can reveal propagation of Langmuir waves in the slab of 3D wire medium. To confirm our proposal we made a numerical simulation of the forthcoming experiment. In the experiment, Langmuir modes are supposed to be excited by total reflection method within Otto configuration [3].

Reflection spectrum obtained numerically has dips corresponding to the excitation of Langmuir modes. Dispersion of Langmuir modes can be determined by the position of the dips. Revealed dispersion is in good agreement with the theoretical results and direct numerical simulation of the spectrum of electromagnetic waves in the hyperbolic medium. For the simulation we use MatLab and CST microwave studio.

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Dyson singularity in disordered nanoparticle chains

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The role of disorder in photonics has been recently reconsidered with the breakthrough results in random lasing [1], SERS enhancement [2], and with development of cost-effective self-assembly fabrication techniques [3]. The special case of a disordered one-dimensional plasmonic nanoparticle chain is considered in this paper. In addition to previous studies [4] we observe the fine structure in the density of resonant states (DOS) in such systems. One of the main results is the DOS divergence at the frequency of plasmonic resonance of an individual nanoparticle. Such peculiarity in the DOS spectra is often referred to as "Dyson singularity" that is typical for low dimensional disordered systems [5].

The divergent DOS allows one to utilize the disorder chains structures for nanophotonics purposes. In the present manuscript we claim that the Dyson singularity affects local DOS either allowing to enhance coupling of light emitter to resonant states of disordered chains. This can give a new perspective on the origin of spontaneous emission enhancement in disordered structures and to explain already observed chain resonances in nanoparticle aggregates [3].

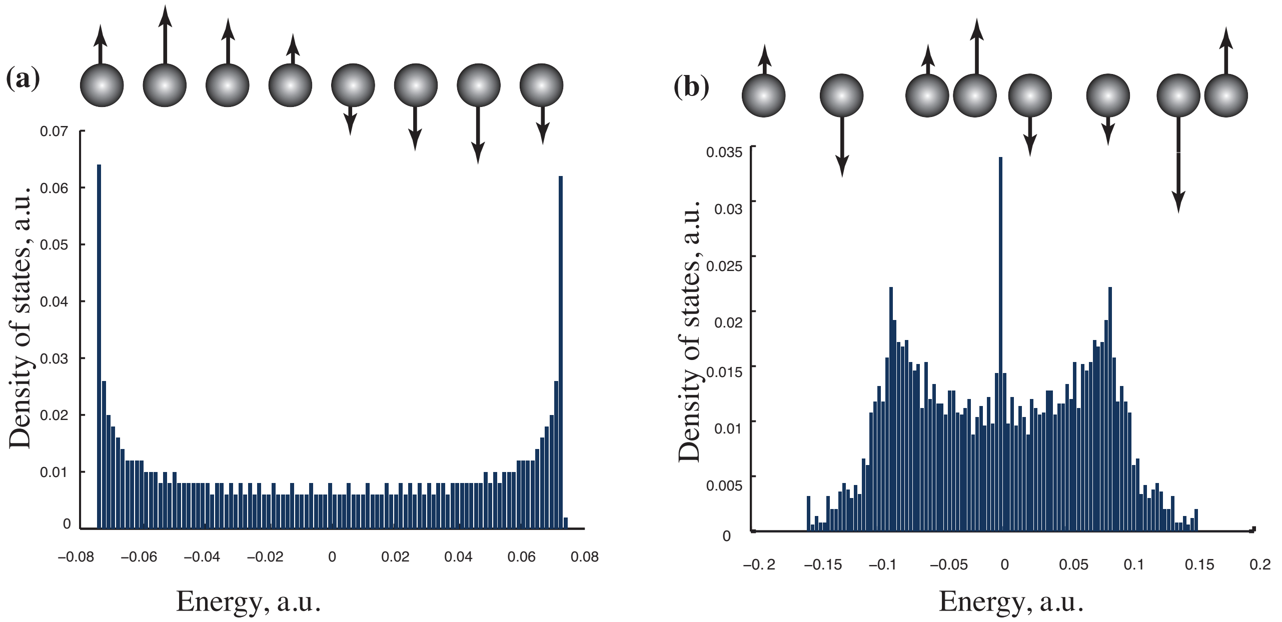

Fig. 1: Comparison of DOS in periodic nanoparticle chain (a) and nanoparticle chain with longitudinal disorder (b). The energy is centered at the energy of surface plasmon resonance of an individual nanoparticle.

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Topological Majorana edge states in zigzag chains of plasmonic nanodisks

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The study of topological insulators is one of the most rapidly developing branches of modern physics. Such structures possess bandgaps in the bulk and special edge states in the gap. Contrary to traditional Tamm states, these edge states are topologically protected, i.e. robust against a certain class of perturbations conserving the general symmetry of the system. Recently, a significant progress has been made in the study of the topological edge states of photons in various structures, such as photonic crystals, coupled cavities and waveguides, quasicrystals, and metamaterials. Here, we present a novel approach for the realization of the topological edge states in the zigzag chains of plasmonic nanoparticles [1].

While various nanoparticle clusters are now readily realized and demonstrate rich physics, the special topological symmetry of the zigzag chains has not been addressed so far. In our study, we demonstrate a one-to-one correspondence between coupled dipole equations for localized plasmon modes in zigzag chains of nanodisks and the Bogoliubov – de Gennes equations for the Majorana edge states of the Kitaev model for a quantum wire on top of the superconductor. Majorana quasiparticles are now actively sought in solids and optics, and they are promising candidates to realize robust qubits. In our work [1] we have proposed a simple realization of topological edge states in zigzag chains of plasmonic nanoparticles mimicking the Kitaev model of Majorana fermions. We have demonstrated that the plasmons can be excited on any edge of the zigzag depending on the incident wave polarization (see Fig. 1). In this work we demonstrate this effect experimentally for the fabricated plasmonic nanostructure by means of scanning near-field optical spectroscopy.

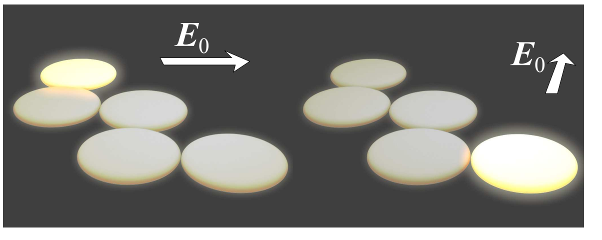

Fig. 1: Schematic illustration of the excitation of left and right edges of the zigzag chain depending on the linear polarization of the incident plane wave $\boldsymbol{E}_0$.

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Directly determining the modes of open electromagnetic resonators

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For meta-atoms, nano antennas and similar particles, strong resonant features are often observed which clearly correspond to the excitation of some mode. In closed electromagnetic systems, where losses can be treated as a perturbation, these modes are easily found by casting Maxwell's equations into an eigenvalue problem, where the modal field corresponds to the eigenvector. Meta-atoms are in a completely different regime from such closed cavities, and their strong radiation losses mean that modes should be considered in terms of the scattering problem. For special cases such as spheres, we can rely on the closed-form Mie solution, and certain two-dimensional geometries have a well-developed formal theory [1].

It is shown in this work that the modes of arbitrary small three-dimensional scatterers can be determined robustly and independently of any excitation field. This is based on an integral equation model of the meta-atom, which only needs to consider the polarisation within the object, automatically accounting for the far-field radiation conditions. Such approaches are well-established in microwave modelling, and are increasingly finding application in nano-photonic systems [2]. Utilising concepts from the singularity expansion method [3], a search is made for the complex eigenfrequencies where this integral operator becomes singular.

The currents satisfying the homogeneous equation are the modes, and an arbitrary incident field can be projected onto them. Each mode obeys a simple scalar impedance equation, which fully describes its dynamics over a broad frequency range. This gives a simple yet highly accurate model of the excitation and radiation of meta-atoms, including interaction between near or far neighbours, local stored energy, coupling to a local emitter, scattering and interference. This method is implemented in an open source code [4].

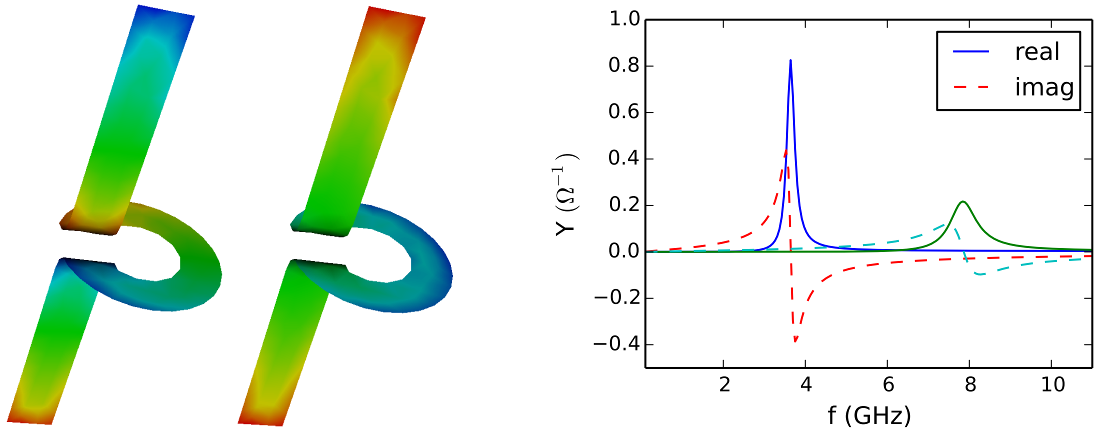

Fig. 1: The two lowest-order modes of a canonical spiral made from PEC strips, and the admittance functions which govern their dynamics.

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Fano resonance and anticrossing regime in high-index dielectric crystals

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Here, we study the spectroscopic properties of photonic structures composed of high-index dielectric rods, by varying the value of the refractive index from low to high values, and analyzing the interplay between the scattering features associated with photonic crystals and metamaterials. These structures are attractive due to low losses and the possibility of scaling the geometrical sizes of dielectric objects in wide range from microwaves to the visible. In 2002, it was shown [1] that high index dielectric cylinder possesses magnetic resonance in TE polarization (H-field along the cylinder axis).

Our aim is to study a crystal with square lattice of dielectric cylinders in air for TE polarization mode. When permittivity is small enough, such structure is a good example of photonic crystal. Another extreme case of very high permittivity gives us an all-dielectric metamaterial. Our goal is to study the intermediate values of permittivity. Periodicity of structure is responsible for the Bragg scattering which is the key concept of photonic crystal. On the other hand, there are strong Mie resonances in high-index dielectric cylinders. In the area of our interest Bragg bands and Mie bands intersect with each other so we can expect the anticrossing behavior if the band widths are comparable or Fano resonance regime [2] if one of band is wider than the other.

The main parameters of the crystal are cylinder permittivity ε and lattice constant a. To calculate transmission spectra we consider a structure that is infinite in y direction and has 10 layers in x direction. We simulate the transmission spectra [3] for various ε and a parameters. The transmission spectra show different interaction regimes: both Fano-like and anti-crossing. Additionally, we calculate the photonic band structures by means of plane wave expansion method.

Also we construct a model of such crystal for the microwave range. The sample consists of 5 by 10 cylinders filled with water ($\varepsilon \approx 60$). The structure is applicable to measure the transmission spectra in dependence on the lattice constant. All three methods (simulations, experiment, and photonic band structure calculations) demonstrate a good agreement.

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Subwavelength guiding and routing with high-index dielectric nanoparticles

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A new type of optical waveguides, based on the guiding properties of an array of coupled high-Q optical resonators was suggested in Ref. [1]. One of the realizations of such novel waveguides are periodic dielectric waveguides – one dimensional arrays of dielectric nanoparticles with high refractive index [2]. They are characterized by small cross-section sizes, low material losses, and (theoretically predicted) low bending losses. It was also experimentally demonstarted, that silicon spherical subwavelength nanoparticles can support both magnetic and electric dipole resonances in optical frequency range [3, 4], which gives an additional control possibility over the light scattering, and waveguides composed of such nanoparticles support several modes of different types [5].

Here we study the subwavelength guiding properties of arrays of high-index dielectric nanoparticles, originating from the long-range coupling of the effective electric and magnetic Mie resonance modes supported by individual nanoparticles. We analyze the dispersion properties of straight coupled-resonator optical waveguides by using coupled-dipole approach, and then verify the validity of the coupled-dipole model by comparing the results with direct numerical simulations and also with microwave experiments. We reveal that a chain of silicon nanoparticles with realistic material losses can guide light for the distances exceeding several tens of micrometres, and it can transmit the energy through sharp bends and defects, which confirms a promising perspective of using them as waveguides with the subwavelength guiding in optical integrated circuits.

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Non-plasmonic light trapping for thin film solar cells

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Photovoltaics (PV) is one of the most promising trends in the modern energy industry. A large amount of toxic waste generated by the high-purity semiconductor production forces to pay attention to thin-film solar cells (TFSC) [1, 2]. The thickness of the PV layer can be as small as 150–300 nm. The disadvantage of TFSCs is parasitic optical losses due to a small thickness of the PV layer: the solar radiation transmitting through the PV layer is then absorbed in the substrate. As a result, the conventional anti-reflecting coating (ARC) can be less efficient than so-called light-trapping structures (see e.g. in [3–5]).

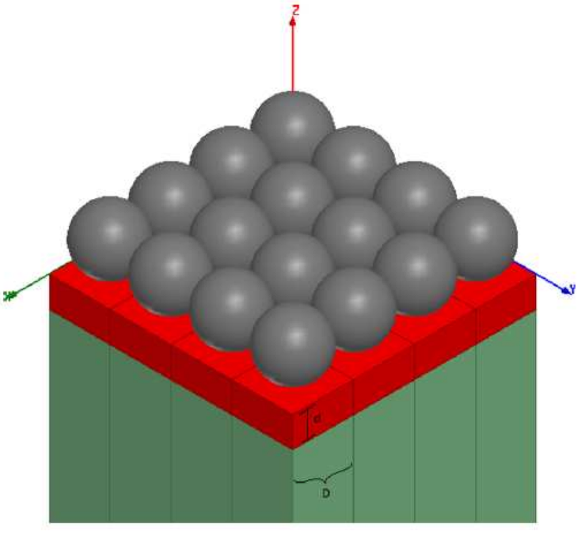

Fig. 1: A schematic view of thin film solar cell with nanosphere coating. Polystyrene particles ($\varepsilon = 2.53$) are packed in a square array on the surface of the doped crystalline silicon film of thickness $d = 300$ nm. The density of carriers in the sillcon layer is assumed to be $3{\cdot}10^{18}$ cm^{-3}. Substrate – AZO ($\varepsilon_s = 3.53 + i0.004$ at the wavelength of 600 nm).

A schematic view of our photovoltaic structure is shown in Fig. 1. A layer of closely packed micron or submicron dielectric (e.g. polystyrene, silica, etc) spheroidal particles is placed on the surface of a PV layer (e.g. doped crystalline silicon) of thickness $d << 1000$ nm. The PV film is deposited on the aluminum-doped zinc oxide (AZO) substrate. As follows from Fig. 2, the proposed easily-fabricated nanostructure provides a significant increase of the PV absorption. Thus, the maximal PV conversion is achieved via the deal between two mechanisms — maximal field concentration and minimal reflection.

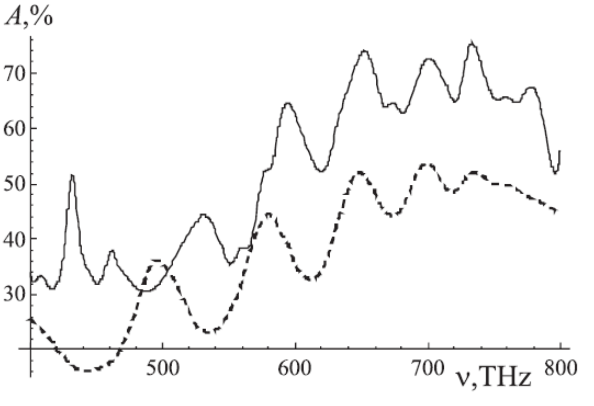

Fig. 2: Absorption coefficient for the pure surface solar cell (dashed line) and at the presence of the nanospheres (solid line). Radius of spheres is equal to 450 nm. The external wave is normally incident. Numerical calculations are performed in Comsol Multiphysics and CST Microwave Studio.

The most pronounced spectral peaks of absorption in Fig. 2 correspond to the regime of a standing wave in the PV layer. To see it we have analyzed the observed Fabry–Perot resonances of reflection, transmission and absorptions.

We proposed the use of an array of non-resonant submicron dielectric particles (e.g. polystyrene spheres) with negligible optical losses for a significant increase of the useful absorption in thin-film solar cells. Instead of the utilization of the resonator regime we suggest the broadband regime which is the deal between the reduction of the transmittance through the thin PV layer and the reduction of the reflectance.

We hope that physical mechanisms we have revealed will help to realize a significant increase of the PV absorption in TFSC in a way more attractive for industrial adaptation than LTS based on photonic crystals, plasmonic or dielectric nanoantennas and other resonant nanostructures.

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Discrete ripples in Green function of hyperbolic medium

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The hyperbolic medium is a strongly anisotropic unixial medium, which described by the electric or/and magnetic tensors with the components of the opposite sign [1, 2]. Due to the hyperbolic isofrequency contours in the wave-vector space, such structures exhibit a number of unusual properties: negative refraction, diverging density of photonic states, ultra-high rate of spontaneous emission and increasing of subwavelength fields. This makes a concept of hyperbolic media very promising for tailoring broad-band light-matter interaction, nanophotonics applications, including single-photon generation, sensing, and photovoltaics. Recently, hyperbolic media became a subject of active research in the field of artificial structured systems, metamaterials. Model systems for hyperbolic metamaterials include layered metal-dielectric structures, arrays of wires, arrays of metallodielectric nanopyramids, graphene layers and transmission-line (TL) arrays. In our earlier work [3] we have realized hyperbolic metamaterials with artificial two-dimensional TL metamaterials. We have demonstrated experimentally that the emission pattern of a current source, being a fingerprint of hyperbolic media, has a pronounced cross form. Here, we study the Green function, i.e. the distribution of the field, excited by a point-like source in a hyperbolic medium. Knowledge of the Green function is instrumental for calculation of arbitrary electromagnetic properties of the structure [4]. We choose the simplest nearest-neighbor two-dimensional tight-binding model of the hyperbolic metamaterial based on artificial TL metamaterials. It is demonstrated, that the field pattern has pronounced ripples, which are characteristic for the hyperbolic regime and can not be described within the effective medium approximation.

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Third harmonic generation in metamaterials: a probe for optical magnetism

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Optical magnetism — or magnetic dipolar response of materials to light — is a phenomenon not met in natural materials and predicted [1] and demonstrated [2] recently in artificial nanocomposites made of either plasmonic [3] or dielectric [4] nanoparticles. Magnetic resonances responsible for the non-unitary effective magnetic permeability rise due to the circulating free-carrier/displacement currents in nanoparticles. In both cases, optical magnetism is an important precursor to negative-index metamaterials [2], Fano resonances [5] and general tailoring of constitutive relations.

Specific current distribution in magnetic resonances implies peculiarities in both linear and nonlinear optical response. However, there are few works dedicated to indicating the role of magnetic resonances in forming of optical nonlinearities of metamaterials. In this work, we summarize our efforts in disclosing the contribution of magnetic resonances to the third-order optical nonlinearities of various metamaterials. Specifically, we demonstrate that it is possible to distinguish between electric and magnetic resonances of the fishnet metamaterial by measuring the angular spectra of the third harmonic generation (THG) from the metamaterial, i.e., without measuring the field distribution of the modes directly [5]. Moreover, applying the THG spectroscopy technique to silicon nanodisk oligomers reveals constructive far-field interference of the THG from electric and magnetic modes — a phenomenon that could be utilized to tailor the nonlinear response of dielectric materials via nanostructuring. We conclude that, due to extreme sensetivity of nonlinear optical response to the local-field structure, nonlinear spectroscopy is a powerful tool for far-field characterization of optical metamaterials.

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Laser trapping and photonic-force microscopy for optical manipulation of functional micro- and nanoparticles

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This work is devoted to recent achievements in using of optical tweezers' technique as photonic force microscope for diagnostics of force interactions between functional micro- and nanoparticles. Optical tweezers have been proved to be an efficient tool for micro-objects manipulating and has found lots of applications, mainly in biology. The main idea of optical tweezering lies in trapping of microobjects utilizing forces of optical pressure and formation of a potential well for dielectric microobjects located near the focus of the tightly focused laser beam that allows trapping of microobjects and manipulation of them. If the trapped particle is transparent this method turns out to be non-disturbing way of diagnostics of single objects of micrometer scale. As an example, we discussed the studies of magnetic interaction of paramagnetic Brownian submicron-sized particles by optical tweezers technique. Correlation analysis allows one to extract magnetic interaction of two particles of 0.4 μm in size, which are optically trapped at the distance of 3 μm one from each other and placed in a static magnetic field of 30 Oe, from the background of their Brownian motion. The magnetic interaction force is estimated to be of approximately 100 fN in both configurations of the mutual orientation of the magnetic field vector and the line connecting two centers of optical traps are used in the experiment. Then, we present a novel approach to probe viscoelastic properties of biological cells based on double trap optical tweezers. Frequency dependence of the tangent of phase difference in the movement of the opposite erythrocyte edges while one of the edges is forced to oscillate by optical tweezers appeared to be highly dependent on the rigidity of the cellular membrane. Effective viscoelastic parameters characterizing red blood cells with different stiffnesses (normal and glutaraldehyde-fixed) are determined. Direct measurements of aggregation forces in piconewton range between two red blood cells in pair rouleau are performed under physiological conditions using double trap optical tweezers. Aggregation and disaggregation properties of healthy and pathologic (system lupus erythematosis – SLE) blood samples are analyzed. Strong difference in aggregation speed and behavior is revealed using the offered method which is proposed to be a promising tool for SLE monitoring at single cell level. Finally, we present recent results in trap position control in the subwavelength vicinity of metallic surfaces. Shift of the trap position from the laser beam waist of optical tweezers is studied experimentally in the presence of a reflecting surface in the vicinity of the focal plane. A standing wave is shown to be formed owing to the interference of waves forming the waist and reflected from the surface. The standing wave is shown to affect significantly the resulting trap position. The distance between the surface and the stable optical trap as a function of the trapped particle size is studied numerically. A new method to stabilize the position of the microparticle relative to the surface is proposed. The localization accuracy is determined by the Brownian fluctuations in optical tweezers and is about 10 nm for effective trap stiffness of $4 \cdot 10^{-5}$ N/m.

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Fabrication of submicron structures by three-dimensional direct laser writing

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The fabrication of three-dimensional micro- and nano- objects of free arbitrary shape and formation of spatially ordered arrays composed of such objects remains complicated and challenging problem. One of the methods which allows one to create such structures is the three-dimensional direct laser writing. The highly nonlinear process of the two-photon absorption of light is an underlying principle which allows one a truly three-dimensional structuring in a microscale. By scanning with the tightly focused laser beam in the volume of the photoresist along a predefined motion trajectory one can reproduce the shape of desired object with the resolution beyond diffraction limit.

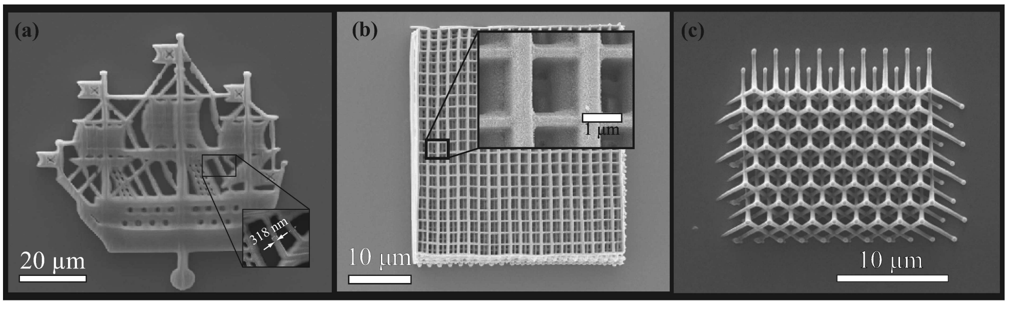

Fig. 1: SEM images of complex structure fabricated by using 3D model of the structure as a template **(a)** or by defining translation vectors **(b)**. Defining the crystal geometry by lattice translation vectors gives higher structal fidelity compared to definition via 3D model.

There are two basic ways to define the desired pattern within resist. The first one relies on the layer-by-layer fabrication based on the in-plane cross-sections of three-dimensional model of the object with the constant or variable step along z-axis. This method is best suited for the shapes of relatively high complexity as shown in **fig. 1(a)**. The second method is best suited for ideal periodic structures, such as photonic crystals. In this case, the structure is defined by the translation vectors along which ones the focused laser beam moves. This approach is suitable for fabrication of large PhCs suitable for optical measurements due to the speed of fabrication.

The woodpile (**fig. 1(b)**) and inverted Yablonovite photonic crystals (**fig. 1(c)**) were fabricated by defining them via lattice translation vectors. The study on the dependency of photonic properties of woodpile and inverted Yablonovite photonic crystals on the dielectric permittivity contrast and the filling factor of the structures was performed numerically using plane-wave method following [2].

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Third-harmonic generation spectroscopy of Mie resonances in silicon nanoparticles

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It was realized from Mie scattering theory that high-index dielectric particles can exhibit artificial magnetic resonances due to the excitation of circular displacement currents [1]. Recently, this fundamental phenomenon was observed experimentally throughout the entire visible and infrared spectral ranges for silicon nanospheres [2, 3]. Also it was predicted that in multi-particle silicon oligomers, the electric and magnetic dipolar moments can interfere, leading to a novel type of Fano resonances [4], which are highly sensitive to the changes of the environment. These results have prompted towards the investigation of nonlinear effects in all-dielectric resonant structures.

In this work we perform the third harmonic generation (THG) spectroscopy of silicon disk trimers in the vicinity of their electric and magnetic dipolar resonances. We show that by engineering of the modes of such trimers we can control the nonlinear interference of the magnetic and electric dipoles and strong enhancement of THG.

Different silicon trimer arrays with different diameter of nanodisks and with different distance between nanodisks in the trimers were considered. By changing the oligomer geometry we bring both electric and magnetic resonances to a partial overlap, which causes strong THG enhancement for wavelengths of the strongest overlap. THG is found to be increased in the spectral ranges that correspond to excitation of the electric and magnetic dipolar resonances of the high-index silicon trimers. Simultaneous excitation of the partially-overlapping electric and magnetic resonances produces strongly enhanced nonlinear signal in the area of spectral overlap, which is connected with constructive far-field interference of THG from the resonances. These results have prompted towards the investigation of nonlinear effects in all-dielectric resonant structures opening completely new functionalities for nanophotonics devices.

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Multiple refractions in the Dallenbach layer

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The problem of reflection of microwaves from Dallenbach's layer — a plane layer of homogeneous material, based on a metal foundation — is considered. Traditionally the task is solved using impedances of reflecting layer and a medium of propagation of microwaves [1–3]. In that approximation, only single reflections are taking into account.

In the case, when losses are big enough, the model provides correct results of calculation. When there are no losses, model of single reflections leads to incorrect results. A big amount of calibrating sheets, using in the case when the reflection coefficient from radar absorbing materials is measured, are matched to the case.

The model of multiply reflections in the Dallenbach's layer without losses, analogical to the model, was accepted in [4] is used in the report. The difference consists in approximation, that low border is supposed to be absolutely reflecting. The following expression for reflection coefficient is received for normal incidence of a wave on a plane homogeneous Dallenbach's layer.

$$\rho_w \approx \rho + (1-\rho)^2 \sum_{m=1}^{\infty} \rho^{2m-1} \cos(2knh), \tag{1}$$

where ρ is Fresnel's reflection coefficient, n is refraction coefficient in the material of a layer, h is thickness of a layer, $k = 2\pi/\lambda$, where k is wave length of a field in the air. From (1) follows that minimums of wave dependence are observed in the points, where arguments of cosine are multiple to π. When a layer is located in the air, then $\rho = (\varepsilon - 1)/(\varepsilon + 1)$, and minimums are approximated to zero, fig. 1.

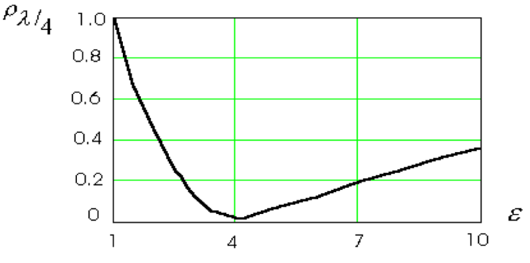

Fig. 1: Reflection coefficient from Dallenbach's layer without losses from $\lambda/4$ layer versus ε.

Summation of the row demonstrates that exist minimum of reflection coefficient when permittivity of the layer is $\varepsilon \approx 4$. Increasing or decreasing of permittivity leads to the growth of reflection coefficient towards one. Received dependence of reflection coefficient from the Dallenbach layer can be used for choosing of the material for calibrating sheets with defined reflection coefficient.

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Narrow-band super-Plankian radiative heat transfer in micron-gap thermophotovoltaics systems

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Experimental samples of micron-gap (MG) thermophotovoltaic systems (TPVS) were fabricated by the group of R. Di-Matteo in 2003–2004. This novel technique promised a breakthrough in the microscale electric generation for which near-field TPVS are not very appropriate. However, theorists rather weakly responded to this challenge of engineers and the main drawback of first demonstrators – low ratio of radiative heat transfer (RHT) integrated over the photovoltaic frequency region to RHT beyond this region – has persisted in the next samples. Since this drawback results in the modest efficiency of TPV conversion this technique has been weakly developing in 2004–2014. Recently, the idea of MG TPVS got a new pulse for development from theorists. Work [1] has shown that strongly super-Plankian RHT (which holds in near-field TPVS) is achievable in MG TPVS using realistic hyperbolic metamaterials (HMM). Work [2] has theoretically demonstrated the possibility to achieve frequency selectivity of this RHT, so that RHT is super-Plankian in the operational band of the photovoltaic layer and suppressed beyond this band. This means that the intrinsic drawback of MG TPVS can be overcome and they may combine advantages of near-field TPVS and far-field TPVS.

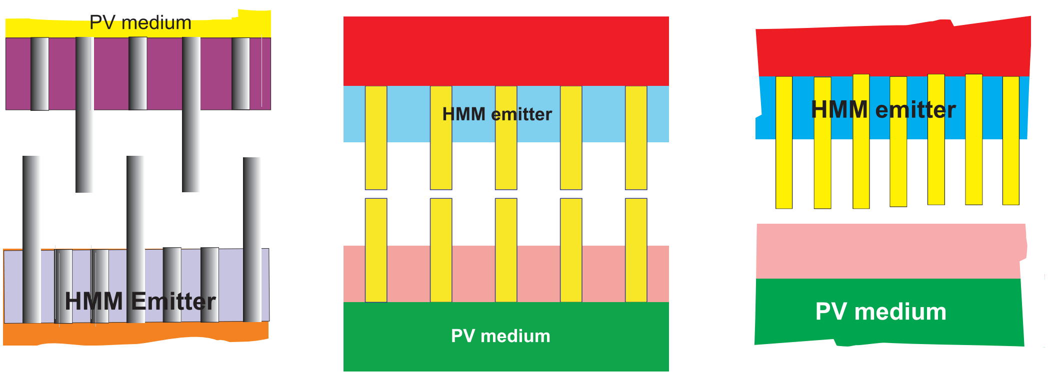

Fig. 1: Schematic view of three types of MG TPVS: (a)— interdigital with two-side nanowires, (b)— two-side nanowires with vacuum nanogap, (c)— one-side nanowires with vacuum nanogap.

In [2] super-Plankian RHT was achieved due to the presence of free-standing metal nanowires in the gap, as shown in Fig. 1(a), where the interdigital arrangement was essential for the high frequency selectivity, as well. This structure is however difficult to fabricate (in practice most of hot wire may intersect with cold ones and the HMM will be damaged). Therefeore in the present work we suggest two new geometries of MG TPVS which both promise high frequency selectivity. Design solutions depicted in Fig. 1(b,c) allow more modest gain in RHT compared to the Plankian limit, than interdigital arrays of nanowires do, however still one order of magnitude is achievable for this gain. Structures sketched in Fig. 1(b,c) are feasible using the existing technologies and should be attractive for experimentalists interested in the development of MG TPVS.

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Near-field investigations of arrays of non-resonant plasmonic nanoantennas

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Domino modes are highly-confined collective modes that were first predicted for a periodic arrangement of metallic parallelepipeds in far-infrared region [1]. The main feature of domino modes is the advantageous distribution of the local electric field, which is concentrated between metallic elements (hot spots), while its penetration depth in metal is much smaller than the skin-depth. Therefore, arrays of non-resonant plasmonic nanoantennas exhibiting domino modes can be employed as broadband light trapping coatings for thin-film solar cells [2]. However, until now in the excitation of such modes was demonstrated only in numerical simulations.

Here, we for the first time demonstrate experimentally the excitation of optical domino modes in arrays of non-resonant plasmonic nanoantennas [2]. We characterize the nanoantenna arrays produced by means of electron beam lithography both experimentally using an aperture-type near-field scanning optical microscope and numerically. To associate the calculated field components with measured near-field signal we use the approach based on optical reciprocity theorem [3, 4]. Good agreement between experimental and numerical near-field patterns allows us to interpret the observed near-field modes as domino modes.

The proof of domino modes concept for plasmonic arrays of nanoantennas in the visible spectral region opens new pathways for development of low-absorptive structures for effective focusing of light at the nanoscale.

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Near-field manipulation by metasurface for increased sensitivity of magnetic resonance imaging

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Metasurfaces were studied by many groups and have shown spectacular applications in long-wavelength regime for communication and in optics in order to control the flow of light and to provide subwavelength imaging. Here we introduce the concept how to effectively apply unique properties of ultra thin metasurface resonator in order to improve the MRI efficiency. Metasurface is formed by the array of wires and placed inside the scanner under the studied object. Coupling of the nuclear magnetic resonance to the Fabry–Perot modes of the metasurface strongly increases the scanner sensitivity, signal-to-noise ratio and the image quality. This concept leads to a new possibilities of the most affordable in hospitals MRI devices. We experimentally demonstrate the effect for a commercially available MRI machine and a biological tissue sample. Our results are corroborated by measured and simulated characteristics of the metasurface resonator.

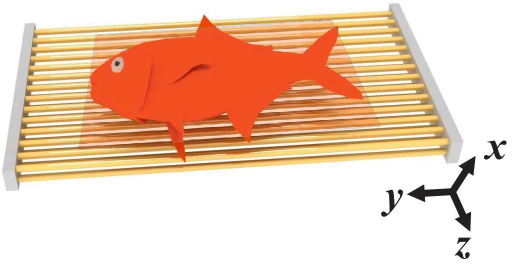

Fig. 1: Artist's view of a metasurface resonator and a fish as the test object.

Coherent radiation of relativistic electrons in metamaterials

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We investigated the interaction of the relativistic electron field with metamaterial, which is constructed from thin copper wires and split ring resonator structure on a dielectric substrate with period of 3 mm. The measurements of the spectral-angular coherent radiation intensity were performed in millimeter wavelength region (10–40 mm) in far field zone on relativistic electron beam with energy of 6 MeV. The measured characteristics demonstrate, that the Cherenkov radiation is observed in the backward semi-sphere in contrast to the conventional Cherenkov radiation from dielectric target in the similar geometry of experiment [1].

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Comparison of the coherent radiation intensity of relativistic electrons in a periodic wire structure in the geometry of the transition and Cherenkov radiation

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We present in this report the experimental investigation of the interaction of electromagnetic field of relativistic bunched electron beam with one-dimensional periodic wire structure. The measurements were carried out in two geometries. In the first geometry the electron beam passed close to the wire structure. As is known, in this case the Cherenkov radiation is emitted [1]. In the second case the electron beam passed through the wire structure with generation of a backward transition radiation. The measurements were done in millimeter wavelength region on the relativistic electron beam with energy of 6.2 MeV in far field zone. Based on the angular dependencies, a comparison of the Cherenkov radiation and transition radiation from the wire structure and from the continuous conductive target (conventional transition radiation) was made.

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Metamaterials based on self-assembled arrays of ferromagnetic nano-wires: magnonic, photonic and magneto-optic properties

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Magnetic properties of arrays of metal (Co, Ni, permalloy) nanowires in a dielectric substrate have been intensively studied during the last decades both theoretically and experimentally. Major physical properties, including magnetic, optical and magneto-optical (MO) behavior can be varied over a very wide range by independently modifying the wire sizes and aspect ratio, their separation and composition. Not surprisingly, their unique features make them potential candidates for numerous applications in such major fields as spintronics, microwave devices, photonics and even plasmonics.

Two techniques of their elaboration, the conventional one relying on electro-deposition of metals in an anodized aluminum oxide self-assembled template [1] (typical diameter varies from 20 nm to 200 nm, see Fig. 1a, b) and a novel one allowing fabricating really ultrathin wires with a diameter as small as 2–6 nm [2] (see Fig. 1c) have been employed.

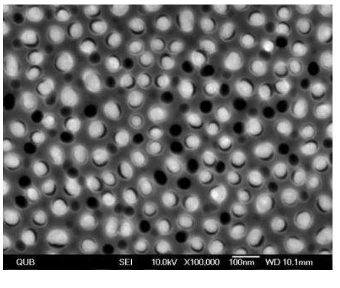

Fig. 1a: SEM image of a typical electrodeposited diluted Co sample.

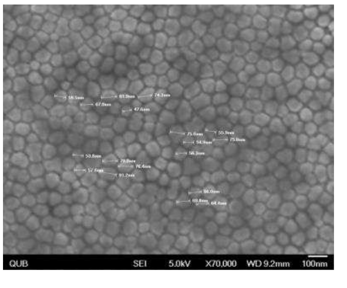

Fig. 1b: SEM image of a typical electrodeposited concentrated Co sample.

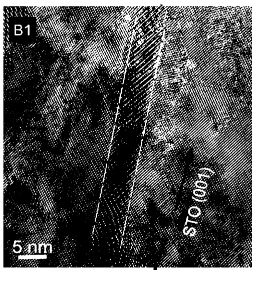

Fig. 1c: HRTEM image of ultrathin Co wires fabricated by laser ablation codeposition

More specifically, in this talk, based on the results obtained by means of Brillouin Light Scattering (BLS), we will address the following important features. First, magnonic behavior in general and the transition from magnetic oscillations localized on individual wires in dilute samples (Fig. 1a) to collective modes in concentrated ones (Fig. 1b) will be considered. BLS spectroscopy is universally considered as a technique of MO characterization of magnetic properties of different materials. Our previous BLS studies have, however, revealed that this conventional logic can be inverted [3, 4]. In other words, through BLS one can gain access to crucial information, otherwise unattainable, on MO and optical properties of the studied structures. More specifically, this will discuss the cross-section of MO interactions between magnons and light, fine structure of BLS spectral lines as well as the Stokes/Anti-Stokes asymmetry.

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Experimental study of nonlocal effects in plasmonic structures with electron energy loss spectroscopy

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Recent experiments have demonstrated that the plasmon resonances of metallic nanostructures with feature sizes below 10 nm is drastically different to the predictions of classical electrodynamics [1, 2]. This peculiar behavior of the plasmon resonances is usually attributed to the emergence of the quantum wave nature of the electron gas at these scales leading to effects such as nonlocal effects, quantum confinement, and quantum tunneling [1, 2]. However, the exact mechanism at work is far from understood and this calls naturally for dedicated experiments using high-end near field microscopy with subnanometer spatial resolution like Electron Energy Loss Spectroscopy in Transmission Electron Microscopes.

After reviewing the potential quantum effects influencing the plasmon resonance on the nanometer scale, I will present ongoing experiments performed at the Technical University of Denmark on metallic nanoparticles with sizes below 10 nanometers [3]. I will then conclude by discussing potential implications of these quantum effects on the design of plasmonic nanostructures like metallic Metamaterials.

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Strong angular magneto-induced anisotropy of Voigt effect and other magneto-optical phenomena in ordered metal-dielectric metamaterials

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Recently we predicted that the strong field *dc* macroscopic or bulk effective conductivity $\hat{\sigma}_e$ of periodic metamaterials [where the cubic or square array of insulating (conducting) inclusions are placed inside a conducting (insulating) host medium] exhibits a strong dependence on the precise orientations of the external magnetic field $\mathbf{B}_0$ (with respect to the main crystal lattice axes) and the volume averaged current density $\langle \mathbf{J} \rangle$ [1, 2]. This is very similar to its behavior in certain metallic single crystals. Since $\hat{\sigma}_e$ can be measured directly, our predictions for $\hat{\sigma}_e(\mathbf{B}_0)$ were already verified experimentally [3, 4]. A similar magneto-induced anisotropy should exist in the case of *ac* conductivity, i.e., for the macroscopic or bulk effective permittivity tensor $\hat{\varepsilon}_e$ of metal-dielectric metamaterials and consequently for their optical properties [5, 6]. However, since $\hat{\varepsilon}_e$ cannot be measured directly, our prediction for $\hat{\varepsilon}_e(\mathbf{B}_0)$ has not yet been tested experimentally. What can be measured directly is the Faraday-like rotation. However, for the case of an in-plane magnetic field $\mathbf{B}_0$ in a metamaterial film the relevant effect is Voigt rotation, for which general exact analytical expressions (as far as we know) are not published. In this work we derive such exact expressions for the general case and verify our predictions numerically.

In summary, we have studied analytically and numerically the rotation and ellipticity of polarization of the light propagating through a metamaterial film with periodic nanostructure for arbitrary direction of the applied static magnetic field, including both Voigt (when the static magnetic field is in the film plane) and Faraday (when that field is perpendicular to the film) configurations. In the Voigt configuration we found strong dependencies of the above mentioned effects on the direction of the applied field $\mathbf{B}_0$. We hope that the results presented here will stimulate experimental studies aimed at verification of our predictions and continued exploration of the MO properties of such systems. This could also form the basis for a new type of magneto-optical switch and other MO devices.

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Adaptive nonconservative forces on scattering objects

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Since first demonstrated by Ashkin [1], optical manipulation has found a number of applications in biology and nanoscience. Stimulated by remarkable progress in an ability to control properties of electromagnetic fields, there is renewed interest on manipulating of matter with different degrees of freedom of light.

During any scattering event, the wave imparts a certain amount of momentum to the scattering body creating a force. Depending on the scattering phase function, the radiation pressure can act in a direction different than the incidence. However, in a majority of situations, some amount of momentum along the direction of light incidence is lost and this makes it impossible for a scattering object to move in a direction opposite to that of the incident beam. To overcome the difficulty of creation of reversely directed forces, one needs to increase the linear momentum of light during scattering event [2]. We will discuss several possibilities to achieve this goal. Some of the suggested approaches are illustrated in Figure 1. One way to achieve the momentum increase is by illuminating with waves that propagate at some angle θ with respect to overall beam propagation direction (z-axis) and have the nonabsorbing scattering object (the "black box") redirect these partial waves along the z-axis [3] (Figure 1a). We will show that this method works even for complex scattering objects. Another approach uses the fact that light momentum changes when the light goes from one medium into another with different refractive index [4] (Figure 1b). We will show how this easily scalable method allows simple manipulation of surface bound particles. Detailed discussion of these and other methods of micromanipulation and their application will be given in a talk.

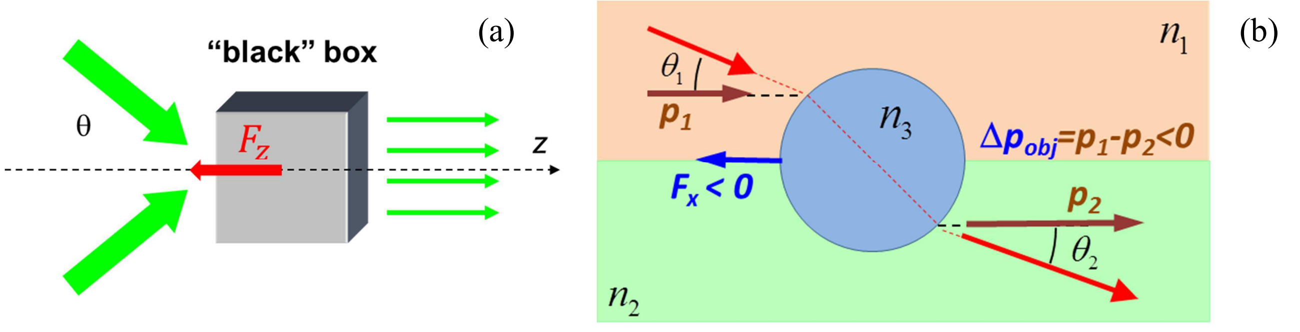

Fig. 1: a) A generic optical device ("black box") converts the incident waves into waves propagating along the z-axis; b) Light momentum **p** is amplifyed when a ray propagates into a medium with higher refractive index. As a result of momentum conservation, a negative scattering force is generated in both cases.

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Effects of femtosecond laser pulses propagation in 1D photonic crystals in the Laue diffraction geometry

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Photonic crystals are being actively studied over the last decades and the number of new observations of their unique optical properties is continuously increasing. For a particular case of 1D photonic crystals the majority of the experiments are performed for the Bragg diffraction geometry, which is more easy in the experimental realization. In our talk we survey the recent results on a few effects that were observed recently under the propagation of the laser radiation in the Laue diffraction scheme in porous-quartz based 1D photonic crystals. Namely, we discuss the appearance of (i) the so called pendelosung effect, (ii) temporal splitting of the femtosecond laser pulses and (iii) selective compression of chirped femtosecond laser pulses in linear PC.

For the realisation of these experiments, 1D photonic crystals (PC) composed of hundreds of structural periods were made. We used the electrochemical method of the fabrication of porous-silicon based PC. The samples consist of 300–400 porous silicon layers, the period of the structure being about 800 nm, so that the overall PC thickness being about 300 μm. After thermal annealing, the porous silicon structure transformed to porous quartz PC with the same periodicity and lower refraction indices of the adjacent layers. Large thickness of the PC along with high transparency made it possible to study the propagation effects in the Laue geometry, with the tunable femtosecond Ti-sapphire laser or a nanosecond OPO laser system used as a probe light source. In consistence with the dynamical Bragg diffraction theory and for Laue geometry, the laser radiation after passing through the PC structure is divided into two beams that propagate either collinear to the incident beam or is diffracted in accordance with the Bragg conditions.

We show that light propagating through a PC sample of approximately 300 μm in length reveals the pendelosung effect, that appears as periodic redistribution of the light energy between the two (direct and diffracted) beams with varied probe wavelength. The spectral period of the oscillations is about 6 nm, while the phases of the dependencies for the direct and diffracted beams are shifted to 180 deg. We observe that the period of oscillations depends on the polarization of the incident beam.

Temporal splitting of short (40 fs) laser pulses was studied in similar PC samples using linearly polarized probe beam. The effect consists in the appearance of pairs (or even more number) of pulses outgoing from the PC in both the direct and diffraction directions. It is demonstrated that the splitting time, which is determined by the characteristic length of the laser pulse path within the photonic crystals, and the number of the outgoing pulses are influenced by the polarization of the laser pulses. We show both experimentally and theoretically that the splitting time in porous quartz-based photonic crystals for s-polarized probe pulses is smaller as compared with the case of p-polarized ones. This polarization sensitivity of the diffraction-induced pulse splitting effect is caused by a large lattice-induced PC dispersion [1].

The effect of the selective laser pulses' compression under the experimental conditions similar to described above was studied using the chirped fs laser pulses. It was obtained that the scenario of the temporal pulse splitting differs significantly depending on the chirp value and sign.

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Photoemission of hot electrons from plasmonic nanoantennas

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Strongly enhanced photoemission of electrons from plasmonic nanoantennas in Schottky-barrier photodetectors has been of great interest during the last several years [1]. The enhanced photoelectric effect from plasmonic nanoantennas with generation of hot electrons can be used in photoconductive plasmonic metamaterials and light-harvesting devices such as, photodetectors, solar and photochemical cells, etc., in order to improve their characteristics.

We studied the physical mechanisms of photoelectron emission from plasmonic nanoparticles into a surrounding semiconductor matrix (Fig. 1a-b). We theoretically compare surface- and volume-based mechanisms from spherical nanoparticles, obtaining analytical expressions for the emission rate in both cases [2]. We show that the surface mechanism prevails because the volume one is affected by detrimental hot electron collisions (Fig. 1c), which agrees with the results for a flat metal surface [3].

Furthermore, we show that photoconductive metamaterials based on asymmetric plasmonic nanoparticles exhibit giant bulk photovoltaic effect [4]. Analogies between photogalvanic properties of crystals and nanoparticles can be further exploited to provide new detection mechanisms both for measuring the properties of incident light and for probing the detailed mechanisms of photoemission.

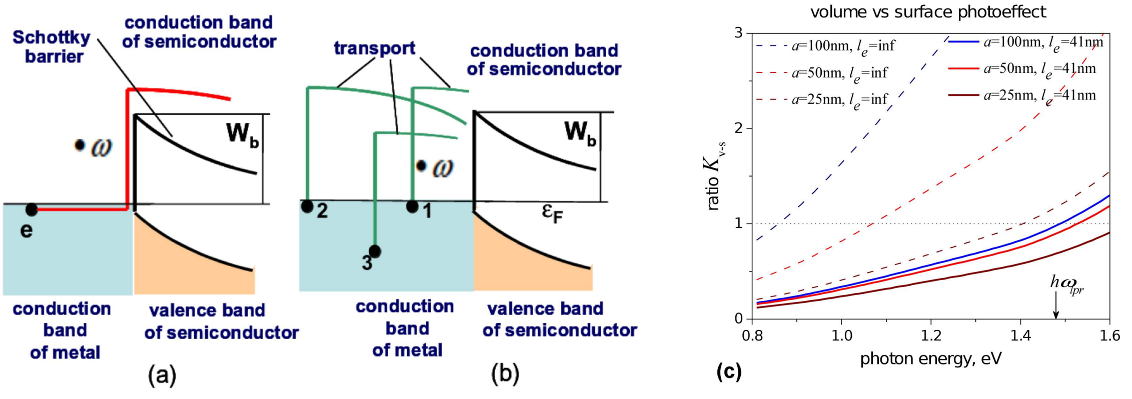

Fig. 1: Illustration of the two mechanisms for the photoelectric effect. (a) Surface effect: an electron collides with the Schottky barrier, absorbs a photon and leaves the metal. The photoemission rate is proportional to the square of the electric field component normal to the metal surface. (b) Volume effect: an electron receives energy from a photon, moves to the Schottky barrier and overcomes it, leaving the metal. The rate is proportional to the local light absorption rate in the bulk metal and strongly depends on the rate of cooling of hot electrons due to electron-electron collisions during transport to the nanoparticle boundary. (c) The two mechanisms for photoelectron emission from nanospheres are compared by considering the ratio of photoelectron emission cross-sections, $K_{v\text{-}s} = \sigma_v/\sigma_s$ (a is the nanoparticle radius, l_e is the electron mean free path). For the plasmonic resonance, $\hbar\omega \approx \hbar\omega_{lpr} = 1.48$ eV.

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Femtosecond intrapulse evolution of the magneto-optical Kerr effect in iron-based magnetoplasmonic crystal

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Surface plasmon-polaritons (SPPs) are believed to be a candidate for everyday-use information units as opposed to diffraction-limited photons and bandwidth-limited electrons. However, for plasmon-based technologies to be realized, efficient techniques for active manipulation of SPP signals are required. One of the most prominent opportunities of using SPP nanostructures is to shape femtosecond laser pulses. A femtosecond laser pulse interacting with a short-living excitation like SPP gets coherently modified as indicated in many preceding works [1, 2, 3]. However, the active control of the laser pulse shaping effect with plasmonic nanostructures has not been demonstrated yet. Here, we experimentally demonstrate ultrafast manifestations of magnetoplasmonics by observing the non-trivial evolution of the transverse magneto-optic Kerr effect (TMOKE) within 45-fs pulses reflected from an iron-based magnetoplasmonic crystal.

The idea of creating the appropriate conditions for observing the intra-pulse time-dependent TMOKE is illustrated in Fig. 1. The femtosecond pulse excites an SPP in a one-dimensional iron grating the so-called magnetoplasmonic nanostructure. SPP that is excited in magnetic metal has a magnetization-dependent dispersion relation. Lifetime of SPP is limited by radiative and dissipative losses down to the values of no greater than 1 ps. Therefore, the resultant reflected pulse is perturbed with respect to the initial shape and contains information about SPP towards the end of the pulse, that is depicted with the elongated tail of the reflected pulse.

We experimentally demonstrate that exciting SPPs with magnetization-dependent dispersion law allows one to control the shape of the reflected pulse. TMOKE evolution is shown to have either positive or negative time derivative depending on the spectral position of the incident pulse carrier wavelength with respect to the SPP resonance wavelength.

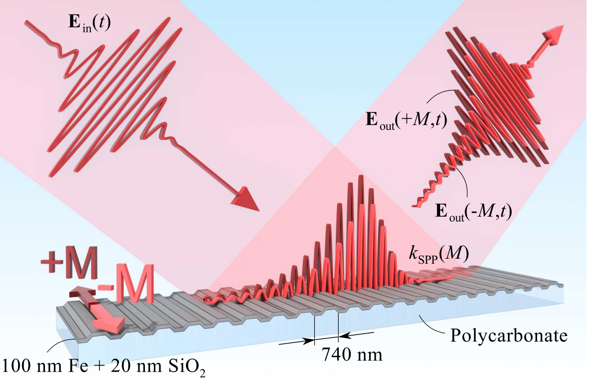

Fig. 1: Illustration of the ultrafast time-dependent TMOKE. The incident pulse is transformed into an SPP wave that has the dispersion law depending on the magnetization direction of the sample. The influence of SPPs is stronger at later time moments. Consequently, the reflected pulse profile depends on the sample magnetization, thereby yielding an intra-pulse time-dependent TMOKE.

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2D self-organized metal nanostructures for plasmonic applications

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Physical vapor deposition of coinage metals on the transparent dielectric substrates is one of the simplest and most popular rotes to obtain materials with bright plasmonic properties. Unfortunately, self-organization leads to broad shape and size distributions of the metal nanoparticles comprising the granular metal films obtained in this way. Because of that absorption bands associated with the plasmon excitations localized in metal nanoparticles are much broader than it may be expected from the point of view of optical properties of corresponding metals. Another possible cause of broadening is interaction between the metal nanoparticles. Although a number of computations have been performed for dimers, trimers and oligomers that simulate the absorption spectra of different ensembles of identical particles, a controversy still exists and leads to the ambiguities when it comes to the interpretation of experimental data as well as in using of optics to characterize the nanostructures. We believe that experimental resolution of this controversy may be obtained with the aid of nonlinear optical methods, namely, persistent spectral hole burning technique. The results of these experiments will be presented and interpreted in favor of the inhomogeneous broadening model. At the same time, this technique leads to the estimations of the homogenous widths and the lifetimes of the plasmon excitations localized in the nanoparticles of particular shape.

To defeat the inhomogeneous broadening we suggest to employ a new technique based on the laser induced desorption of adsorbed atoms from the substrate surface. In particular, we have demonstrated that intense laser irradiation of the surface can avoid nucleation and growth of metal deposits in the course of physical vapor deposition. At the same time, in the dark places the atoms are adsorbed, diffuse over the surface reach each over and form nuclei of the metal phase. Then additional material comes from the vapor phase and finally a metal nanoparticle appeases. It may be expected that besides the arbitrary configuration of the dark and illuminated places laser irradiation defines also the borders between the dark places and prevent the atoms from diffusion from one dark place to another. Hence, the size distribution of the particles obtained in this way will be narrower that in the case self-organization without laser illumination. Proof of concept was obtained in the experiments with sodium.

As the regular and homogeneous in size and shape ensembles of plasmonic metal nanoparticles are not available yet, a number of experiments were performed with the self-organized silver nanoparticles. Broad absorption bands of such structures facilitate resonance interaction of different strong absorber like semiconductor quantum dots, organic dyes or carbon nanotubes with at least one subensemble of nanoparticles. The results concerning rather strong enhancement of absorption and fluorescence will be presented. An interpretation of the obtained results will be given taking into account not only the field enhancement due to the plasmon excitation but also the changes in the dielectric surrounding of metal nanoparticles due to the nearby organic molecules and semiconductor quantum dots.

Planar metal nanostructures possess also very unusual electrical properties. Controlled evaporation and annealing lead to the production of the planar arrays of metal nanoparticles that demonstrate hysteresis of conductivity and resistive memory effects. We have obtained resistance switching of granular silver films at relatively low voltage of 5 V. Both reversible and irreversible switching has been observed depending on the structure of the metal film. Photoconductivity of granular silver films was also observed and studied.

Optical nanoantennas for enhanced light trapping in thin-film solar cells

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We propose a light-trapping structure offering a significant enhancement of photovoltaic absorption in thin-film solar cells. The mechanism of the light trapping is related with the excitation of so-called *domino modes* [1, 2] in silver nanoantenna arrays, which represent non-plasmonic collective oscillations in array of metal nanostrips or nanobars. In the frequency range of domino modes the solar energy is transformed into a set of hot spots partially located in between the metal elements, partially in their substrate with negligible penetration of enhanced electric field inside the metal. The penetration depth of the local field into metal is much smaller than the skin-depth and the incident light energy is practically not dissipated in the metal. The array supporting these modes operates beyond plasmon resonances as if metal elements were perfectly conducting metal. These modes are observed only for substantial metal nanobars or nanostrips whose thickness exceeds the skin-depth. A part of every hot spot located inside the active layer corresponds to the enhanced power absorption which decreases the transmission of light in the operation band. In the spectrum of enhanced absorption the reflectance is also suppressed.

We have shown that the light-trapping structure based on silver nanoantennas supporting so-called domino-modes can be used for the significant enhancement of photovoltaic absorption in organic solar cells with photovoltaic layers as thin as 40 nm. A practical design solution is suggested for an organic solar cell which operates in the NIR range and keeps rather transparent in the visible range [3, 4]. We studied the suggested structure and theoretically proved that domino modes are excited in the same range as they were predicted for same nanoantennas located on a substantial semiconductor substrate [5]. We performed extended numerical simulations and optimized the structure achieving a significant (3.6 times) increase of photovoltaic absorption. The reduction of the transparency in the visible range from 74% to 50% due to the presence of nanoantennas is fully justified by more than triple gain expected for the photocurrent. The light-trapping structure is feasible [5], and we envisage opportunities for the experimental demonstration of our claims.

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Generalized Clausius–Mossotti homogenization for the permittivity of electric quadrupolar media

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An approximate equivalent macroscopic permittivity of a 3D cubic array of point electric quadrupoles is derived using a generalized Clausius–Mossotti (C-M) derivation. At low frequencies the C-M approximate permittivity is shown to equal the exact permittivity of the fundamental Floquet mode of the array. As far as we are aware, this generalized C-M derivation is the first application of the C-M method to media containing multipoles of higher order than dipoles.

We shall consider an array of point $q(\hat{\mathbf{x}}\hat{\mathbf{y}} + \hat{\mathbf{y}}\hat{\mathbf{x}})$ electric quadrupoles with propagation constant β in the x principal direction. The amplitude of any one point quadrupole is given in terms of the quadrupolarizability times the relevant spatial derivatives of the local electric field (with that quadrupole removed). For the array to behave as a continuum satisfying Maxwell's equations for electric quadrupolarization, field and source averages over each unit cell must closely approximate their respective fundamental Floquet modal coefficients and this requires that $\beta d \ll 1$ and $k_0 d \ll 1$ (k_0 denoting the free-space propagation constant). In such a continuum array, we can choose a spherical macroscopic volume ΔV, as shown in Fig. 1, of radius r_0 centered at the location of the quadrupole at the origin (O) where we want to compute the local fields. The radius r_0 of the

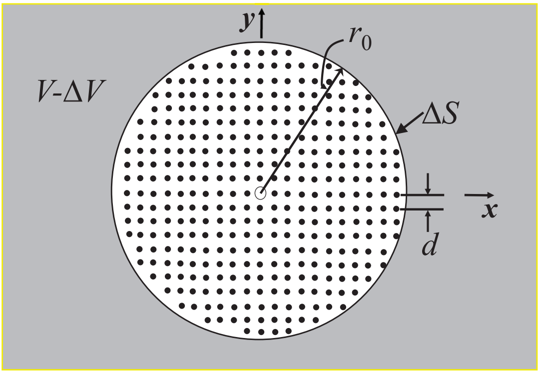

Fig. 1: Spherical macroscopic volume ΔV with radius $r_0 \gg d$ but $\beta r_0 \ll 1$ and $k_0 r_0 \ll 1$.

spherical macroscopic volume ΔV is chosen large enough to contain many quadrupoles but small enough that $\beta r_0 \ll 1$ and $k_0 r_0 \ll 1$, and thus the fields contributed by the quadrupoles inside the sphere at the center O of the sphere are well-approximated by the quasi-static fields. The surface ΔS of ΔV is chosen so as not to intersect any of the point electric quadrupoles. The quadrupoles located outside the sphere are far enough away (many unit cells) from O that their contribution can be well-approximated by that of a continuum outside ΔV as depicted in Fig. 1 by the outside shaded region. For this continuum, the required local fields can be calculated from the electric dyadic Green's function.

In addition to these continuum local fields, the contribution to the local fields from the point electric quadrupoles inside ΔV must be calculated. Although the local fields of these point quadrupoles inside ΔV are predominantly quasi-static fields, the ones in the vicinity of O are too close to be approximated by a continuum. The fields at O from each point electric quadrupole inside ΔV can be found by summing the quasi-static fields of all the quadrupoles inside ΔV. The summation computes rapidly and accurately with just a few terms.

With the local fields from the quadrupoles outside and inside ΔV, we find the electric quadrupole moment q of each quadrupole in terms of its electric quadrupolarizability α_q and applied local fields. This expression is then used to derive a causal effective relative electric permittivity of the electric quadupolar array continuum, namely

$$\epsilon_r \approx \frac{1}{1 - \frac{(k_0 d)^2}{4\left(\mathrm{Re}\left[\alpha_q^{-1}\right] d^5 - 2a_0\right)}}\,, \quad a_0 = 1.484\,.$$

Symmetry breaking and electromagnetic spatial solitons in a liquid metacrystal

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Liquid metacrystals (LMC) have recently been suggested [1] as a new type of resonant metamaterials which potentially possess a number of unique properties such, for example, as high tunability and very strong nonlinearity, absent in natural materials as well as in most known metamaterials. These properties are caused by meta-atoms reorientation under the action of both static bias and high-frequency electric fields resulting in reorientation of optical axis of LMC. Furthermore, resonant response of meta-atoms enhances LMC tunability and nonlinearity which are in no way regarded with tunability and nonlinearity of media composing LMC. In fact, any new metamaterial expands the capability of electromagnetic radiation control that makes them to be very attractive for further applications. Indeed, in the last decade a lot of different kinds of metamaterials have appeared operating in frequency ranges from microwaves up to visible light. In this respect, LMC might also find its niche among other metamaterials and a comprehensive study of their electromagnetic potentiality in different frequency domains is seemed to be highly important. In this report we show that transverse electromagnetic wave propagating along polarization axis of LMC can trigger an instability leading to the meta-atoms reorientation and, consequently, to the changing of the effective refraction index of the metamaterial. In the frequency range where the effective refraction index increases due to the meta-atom re-orientation, the conditions of the photons confinement into the self-supported waveguide channels (spatial solitons) arise. One should notice that nonlinear change of refractive index on the frequencies close to resonance can be of the same order, greater, or even much greater than its unperturbed value that, according to the terminology accepted in optics of conventional liquid crystals, corresponds to colossal or maybe super colossal nonlinearity. We study in detail the transverse structure of soliton and its stability. By means of numerical simulations the stationary solution for the problem of soliton excitation by incident electromagnetic beam is found. We also study the collisions of in-phase and out-of-phase spatial solitons.

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Surface waves in liquid meta-crystals

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We have recently introduced a new type of metamaterial, which we called liquid metacrystal (LMC) [1], consisting of elongated resonant metal particles (meta-atoms) suspended in viscous liquid. A static electric field applied to LMC aligns all of the meta-atoms in the same direction that makes this medium anisotropic. The axis of anisotropy can be reoriented, and this type of tunability resembles that of liquid crystals in the nematic phase. Moreover, meta-atoms also reorient in response to the high-frequency electromagnetic waves, suggesting strong nonlinear properties of the LMC. We should also note, that electromagnetic response strongly depends on the frequency shift relatively to the resonant value. The anisotropy of the LMC induced by the external field causes specific properties of the propagation of electromagnetic waves of different kinds.

This report is devoted to the study of linear electromagnetic surface waves localised at the interface between dielectric medium and LMC. We derive the expressions for the dispersion relations for such waves. In this work we study the propagation of the surface waves for different direction of the constant electric field. We show that surface polaritons have significantly different transverse structure from what are on the surface between air and normal metal. It is shown that such surface waves at the interface between LMC and dielectric medium penetrates into LMC not purely exponentially but with oscillations. We also study surface waves supported by the thin film of the LMC.

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Complex conformal transformations in plasmonics

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Transformation optics (TO) has become one of the effective tools to design materials with specially tailored properties for controlling electromagnetic waves [1]. Developing of TO concept has led to the growing interest in engineering of various TO devices, in particular, coatings with scattering-free and low observability behavior. Up to now, the ability to achieve invisibility at optical frequencies is challenging, but there has been significant progress towards this using the idea of a carpet-cloak of Li and Pendry [2]. This approach proved to be especially important in plasmonics where the application of the ideas of carpet-cloak leads to experimental demonstration of considerable reduction of plasmon scattering by surface roughness.

One of the challenging issues in optics and plasmonics is that of material losses, and natural way to overcome this problem is exploiting the gain to compensate for the losses. Full compensation of losses for an arbitrary spatial point is often complicated. Considering the case with spatially separated losses and gain one can meet a problem of the scattering of the radiation from this spatially nonuniform structure. So, it might be desirable to design nonreflecting and nonscattering permittivity distributions including the loss-gain materials. Recently, it was suggested a way based upon PT invariance which extends the possibilities of the transformation optics to a case of complex transformation of spatial coordinates and results in a media with balanced but spatially separated losses and gain [3].

In this work, we propose a method for the design of electromagnetic media that is based on *complex conformal mapping* and gives the structure with *complex* permittivity. This generalization of conformal transformations extends the variety of conventional applications to the regime of using gain and loss materials. The example of absorbing but "non-disturbing" layer is analyzed for the case of the surface plasmon propagation along the plane metal-dielectric interface and also along the deformed one that corresponds to a ground-plane cloak similar to Ref. [2]. Although this work on complex conformal transformations was implemented in the context of plasmonic devices, we believe the potential impact can be significant in other fields.

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Beyond the light line: large-wavevector wave engineering in hyperbolic metamaterials

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Hyperbolic metamaterials (HMMs) – highly anisotropic media with permittivity tensor eigenvalues of different signs – have attracted avid interest in recent years due to several promising applications such as far-field subwavelength imaging (hyperlensing) and anomalously large photonic density of states (DOS) [1]. These properties were found to result from the existence of large-wavevector or high-k waves. They are evanescent in isotropic media but propagating in presence of extreme anisotropy characteristic for HMMs [2], and serve to populate the large DOS (Fig. 1a).

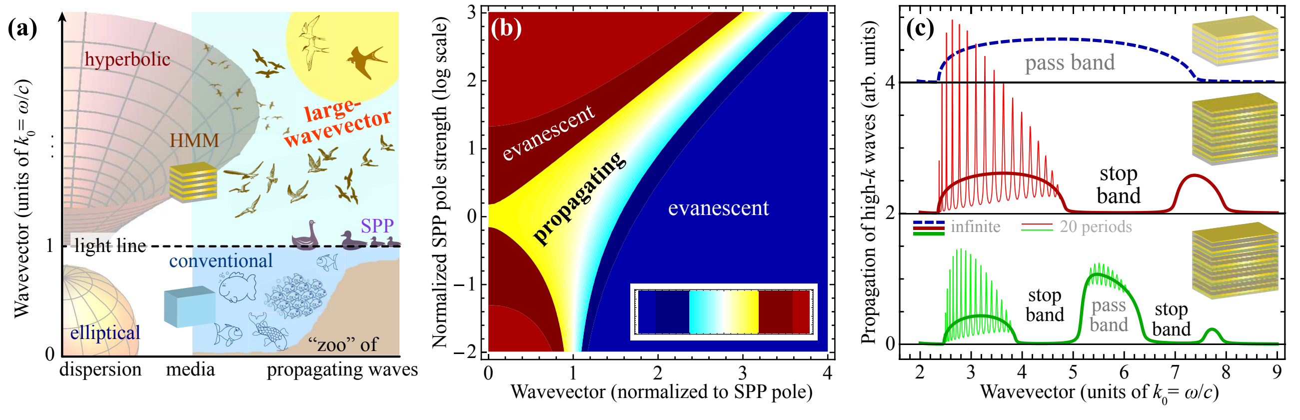

Fig. 1: (a) Artistic impression of conventional, SPP, and high-k propagating waves in k-space. (b) Existence diagram of Bloch high-k waves stemming from pole-like surface excitations such as SPPs. (c) Multiperiodic plasmonic multilayers with selective high-k wave propagation.

In this paper, we investigate high-k wave formation and propagation in one embodiment of HMMs – subwavelength metal-dielectric multilayers. We analyze the process of volume propagating wave formation by Bloch-theorem coupling of surface excitations in individual layers, such as surface plasmon polaritons (SPPs). Using the pole expansion approach [2], we derive the general analytical requirements for SPPs to bring about high-k waves that span a large portion of the k-space (Fig. 1b), explaining why short-range SPPs do give rise to a high-k band while long-range SPPs do not.

We further explore how high-k wave propagation can be tailored when the structure of a plasmonic multilayer becomes more complicated than simple periodic. Combining the subwavelength structure (which underlies the HMM behavior) with multiperiodic superstructure (which acts as a photonic multilayer for high-k waves) is seen to result in stop bands in the large-wavevector domain that can be rearranged by changing the multiperiodicity (Fig. 1c). More complicated multiscale structures can provide an even better flexibility in engineering the high-k wave propagation [3], which can be used for the design of HMM-based devices with desired properties.

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Frequency-resolved optical gating for surface plasmons ultrafast spectroscopy

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The resonant interaction between surface charge and the electromagnetic field of the light constitutes the surface plasmons polaritons (SPP) and gives rise to its unique properties [1]. Oscillations of such nature allow one to use plasmonic nanostructures as devices that control the optical radiation on micro- and nanoscale. Several techniques can be used to excite SPP. One of them is the use of a periodic perforation of the metallic surface. The intensity autocorrelation scheme is the most common technique to examine ultrashort pulses because it is easy to set up and use. But the information of the electric field phase is lost in such a measurement. Frequency-Resolved Optical Gating (FROG) which was invented by R. Trebino et al. [2] is one of the technique for retrieving both the amplitude and phase of the field. FROG is mainly based on the intensity autocorrelation of pulses. In this work ultrafast dynamics of surface plasmons in one-dimensional plasmonic crystals is observed by using FROG technique.

To study the excitation of surface plasmon-polaritons the one-dimensional perforated metal film was fabricated by laser interference lithography. There is a 50-nm-thick gold film deposited onto the grating of the 0.8-μm-thick photoresist on a quartz substrate. The optical spectra of the sample demonstrates features in the reflection spectrum for p-polarized light whereas there are no resonances for s-polarized light. Frequency-resolved optical gating (FROG) technique was used to measure the plasmon polaritons dynamics. Spectrograms were measured for the pulses normally reflected from the sample. The wavelength was chosen near edges of the band gap. One of the distinctive spectrogram pairs is shown in figure.

There are significant changes both for the pulse width and for spectral characteristics at wavelengths near edges of the plasmonic band gap for p-polarized incident light. The features are associated with resonant excitation of surface plasmon polaritons. The amplitude and phase of the pulses were extracted from the spectrogram series.

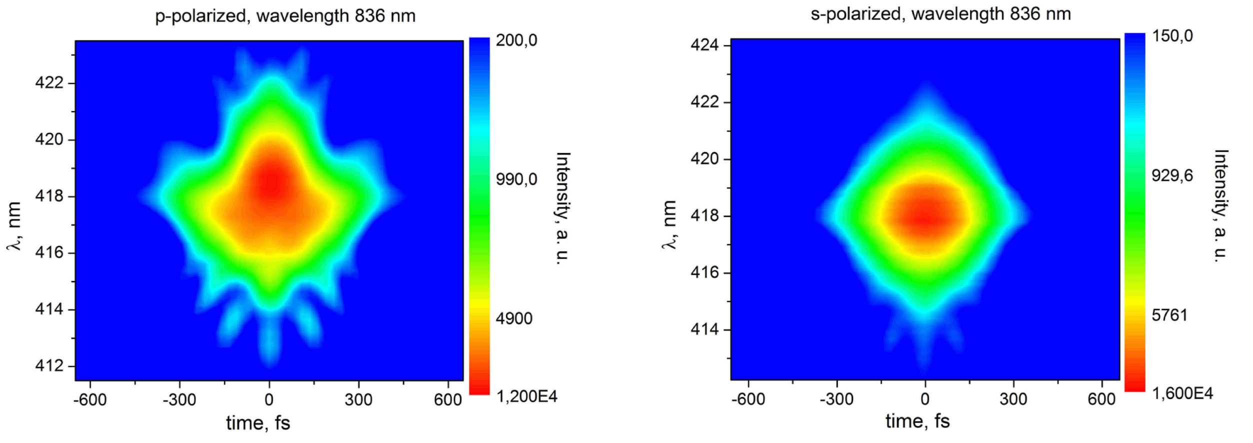

Fig. 1: Spectrograms for the reflected pulses at 836 nm for p- and s-polarized incident light.

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