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DAYS ON DIFFRACTION 2015

ABSTRACTS

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FOREWORD

"Days on Diffraction" is an annual conference taking place in May–June in St. Petersburg since 1968. The present event is organized by St. Petersburg State University, St. Petersburg Department of the Steklov Mathematical Institute, the Euler International Mathematical Institute and the ITMO University.

The abstracts of 237 talks to be presented at oral and poster sessions during 5 days of the Conference form the contents of this booklet. The author index is located on the last pages.

Full-length texts of selected talks will be published in the Conference Proceedings. They must be prepared in LaTeX format and sent not later than 14 June 2015 to `diffraction15@gmail.com`. Format file and instructions can be found at `http://www.pdmi.ras.ru/~dd/proceedings.php`. The final judgement on accepting the paper for the Proceedings will be made by the Organizing Committee after peer reviewing.

As always, it is our pleasure to see in St. Petersburg active researchers in the field of Diffraction Theory from all over the world.

Organizing Committee

List of talks

Workshop on metamaterials

Mathematical methods of realization for nonlinear model of deformation of crystal media with complex lattice

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In last years nonlinear theory of crystal media with complex lattice consisting of two sublattices was developed [1]. Displacement of the center of inertia of atoms of the elementary cell is described by acoustic mode and mutual displacements of atoms inside the cell — by optical mode. Equations of motion were obtained from the Lagrange variation principle. They are represented by the system of six coupled nonlinear partial differential equations. Three equations for acoustic mode are written in the form of standard elasticity theory equations. Equations for optical mode have the form of system of three coupled sine-Gordon equation with coefficients before sines (amplitudes) depending on tensor of macrodeformations. The last could be expressed through the macro- and microstress tensor. It results in the system of three coupled equations of double sine-Gordon. Making simplifying assumptions (homogeneous deformation, thin layer approximation) system of three equations could be reduced to the solution of one sine-Gordon (double sine-Gordon) with constant or variable amplitude.

Two-dimensional solutions of sine-Gordon equation with a constant amplitude are found. Two types of solutions, regular and singular, are received. The second solutions, unlike the first ones, describe deformation of the crystal medium with formation of various defects. The analysis of the received solutions is given.

Functionally invariant solutions are found for sine-Gordon equation with variable amplitude [2, 3]. They are received by method which was developed for the solution of the wave equation [4, 5] and have a form of arbitrary function depending on special function (ansatz). The solutions depending on one α or two α, β ansatzes are found. Ansatzes (α, β) are defined as roots of the algebraic or mixed (algebraic and differential in private derivatives of the first order) equations. The equations defining ansatzes, also contain arbitrary functions depending on (α, β). Features of the received solutions are discussed.

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On proof of the reduction method of an infinite set of linear algebraic equations

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In their papers N. B. Pleshchinskii, I. E. Pleshchinskaya, D. N. Tumakov proposed to solve the diffraction problem by the over-determined Cauchy problem solution. The electromagnetic wave diffraction problem on a thin conducting screen is equivalent to a regular infinite set of linear algebraic equations

$$-A_k + \sum_{n=0}^{+\infty} A_n \gamma_n \lambda_n \sum_{m=0}^{+\infty} \frac{1}{\gamma_m} I_{n,m} J_{m,k} = \sum_{n=0}^{+\infty} A_n^0 \gamma_n \lambda_n I_{n,k}, \quad k = 0,\, 1,\, \ldots \tag{1}$$

relative to unknown coefficients A_k of an expansion into Fourier series. The infinite set of equations (1) can be solved by the reduction method. An approximate equation will have the following form

$$-A_k + \sum_{n=0}^{N} A_n \gamma_n \lambda_n \sum_{m=0}^{M} \frac{1}{\gamma_m} I_{n,m} J_{m,k} = \sum_{n=0}^{N} A_n^0 \gamma_n \lambda_n I_{n,k}, \quad k = 0, \ldots N,$$

where N, M are reduction parameters.

In the report we consider the infinite set of linear algebraic equations. This set of equations was obtained in the diffraction problem on a thin conducting screen in a plane waveguide. We prove the reduction method. The proof is based on the abstract scheme of approximate methods for solving linear operator equations [1], [2]. The reduction method gives an approximate solution converging to the exact when $N \to +\infty$ if $M > (1+\theta)N$, $\theta > 0$.

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Vectorial effects in fiber acousto-optics

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We study the effect of light's spin-orbit interaction (SOI) [1] on the acousto-optic resonance in circular optical fibers with a flexural acoustic wave [2]. Analytical expressions for the modes and their propagation constants were established by solving the vector wave equation. It is shown that a strong SOI ensures the splitting of the well-known single scalar resonance into the three new ones. Yet, we report on a novel type of the optical mode conversion in fiber acousto-optics [3]. The all-fiber narrowband complete transformation of the fundamental mode into the frequency downshifted optical vortex beam [4] of topological charge +1 or −1 is theoretically demonstrated. Moreover, such a process is found to be polarization-dependent: both topological charge and polarization state of the produced optical vortex are governed by the circular polarization handedness of the input mode. It creates the new possibility of all-fiber narrowband vortex beams generation and fast dynamic control of its spin and orbital angular momentum [5] through changing the input polarization.

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Scattering by a fast moving body

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The acoustic problem of high-frequency diffraction by a fast moving body is considered. In the moving coordinate system the problem of diffraction of a harmonic of frequency ω plane wave incident at and angle ϑ_0 to the direction of movement, is described by Helmholtz equation with the wavenumber

$$k' = \frac{\omega(c - v\cos\vartheta_0)}{c\sqrt{c^2 - v^2}},$$

where v is the velocity of the ball and c is the sound velocity. The body is represented as an elongated body, in particular a ball of radius a becomes the prolate spheroid with the minor axis a and the major axis $b = a/\sqrt{1 - v^2c^{-2}}$.

When v approaches to c the spheroid becomes strongly elongated in the terminology of [1] and the results of [2, 3] can be used to describe the near field distribution and the results of [4, 5] allow the representation of the far field to be given.

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Analysis of the generation properties of nonlinear layered media

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Nonlinear dielectrics with controllable permittivity have a great application prospect in electronics and device technology. We develop a model of resonance scattering and generation of waves on an isotropic nonmagnetic nonlinear layered dielectric structure excited by packets of plane waves in the resonance frequency range in a self-consistent formulation [1–3].

The paper presents the results of the numerical analysis characterizing the scattering/generation and spectral properties of the considered structures. An effective way to describe the processes of generation of oscillations via the variation of the relative Q-factor of the eigen-oscillations corresponding to the eigen-frequencies of the scattering and generating structures, when the intensity of the excitation field changes, is given. Moreover, the proposed approach applies equally well for sufficiently weak/strong energy generation in ranges from a few percent to dozens percents of generated energy. For the first time, two-sided acting fields at the scattering frequency were taken into account and a type-conversion of the oscillations could be observed. The latter effect was observed at a symmetry violation of the nonlinear problem caused by different amplitudes of the excitation fields. This effect may serve as a basis for numerical and analytical methods for the synthesis and analysis of nonlinear structures in the vicinity of critical points of the amplitude-phase dispersion, similar to the approach developed in the papers [4, 5]. That is, mathematical models for the control of anomalous scattering and generation properties of nonlinear structures via the variation of amplitudes in a two-sided excitation of a nonlinear structure at scattering and generation frequencies near the resonance frequencies of the linearized spectral problems can be created.

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Investigation of nonlinear cubically polarizable layered media with a controllable permittivity

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The present paper focuses on the development of a mathematical model, an effective algorithm and a self-consistent numerical analysis of the multifunctional properties of resonant scattering and generation of oscillations by nonlinear, cubically polarizable layered structures.

The multifunctionality of the nonlinear layered media will be caused by the interference mechanism between interacting oscillations — the incident oscillations exciting the nonlinear layer from the upper and lower half-spaces as well as the scattered and generated oscillations at the frequencies of excitation/scattering and generation.

The study of the resonance properties of scattering and generation of oscillations by a nonlinear layered structure with a controllable permittivity in dependence on the variation of the intensities of the components of the exciting wave package is of particular interest. In the present paper we extend our former results [1–3] and furthermore we analyze the realizability of multifunctional properties of nonlinear electromagnetic objects with a controllable permittivity.

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Semi-classical asymptotics for spectral bands of quantum periodic dimers

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We study a quantum dimer on a periodic one-dimensional substrate in a semi-classical approximation. This is a 2D Schrödinger operator

$$\hat{H} = -\frac{h^2\Delta}{2} + U(x,y), \qquad U = \frac{y^2}{2} - \alpha\cos x\cos(y - y_0) \tag{1}$$

with the potential periodic in x and raising at infinity in y. We deal with the part of the spectrum near the bottom of the potential (where harmonic oscillator approximation works). We calculate the eigenvalues associated with the Bloch in x eigenfunctions, i.e. such that $\psi(x+2\pi, y) = e^{2\pi iq}\psi(x,y)$, where the number $q \bmod 1$ is called quasimomentum.

We show that the spectrum consists of bands (sometimes overlapping) and calculate their widths. We also find the asymptotics for dispersion relation between energy and quasimomenta.

Our motivation comes, first, from the physical applications (see [1, 2, 3]). On the other hand, this system presents an example of a 2D quantum system with a potential which is periodic in one variable, and raises at infinity in the other. Luckily, the system is not very complicated and allows to calculate some asymptotics, and yet not trivial (the variables do not separate).

The structure of the spectrum bands depends on parameters α and y_0. In a typical case when $y_0 \neq \pm\frac{\pi}{2}$ (Case 1) the potential has a single point of global minimum. There is also a special case $y_0 = \pm\frac{\pi}{2}$ (Case 2), when potential has two points of global minimum. The latter case becomes even more interesting due to additional symmetry.

Roughly speaking the spectrum in Case 1 is close to that for the potential $V_1 = \frac{y^2}{2} - \alpha\cos x$. Here variables separate, and the spectrum is a sum of the harmonic oscillator and one-dimensional periodic Sturm–Liouville spectra. The former one is discrete $E_n = \frac{h(2n+1)}{2}$, and the latter has well-known band structure. The position of the m-th band is again described by a harmonic oscillator approximation $\mathcal{E}_m = \frac{\sqrt{\alpha}h(2m+1)}{2} + O(h^2)$. The widths of bands are exponentially small in h.

In Case 2, the spectrum is alike to that for the potential $V_1 = \frac{(y^2-1)^2}{2} - \alpha \cos x$. The variables again separate here, and the spectrum is a sum of the symmetric double well and one-dimensional periodic Sturm–Liouville spectra. The former spectrum is discrete but consists of pairs $E_n^{\pm} = \frac{h(2n+1)}{2} + O(h^2)$ with exponentially small distance. Thus, with each pair $n, m \in \mathbb{Z}_+$ two bands are associated. They may overlap, depending on the value of exponentially small gaps.

We also point out an important effect taking place in Case 2, which is not seen in a described model example. Actually the rich symmetry in Case 2 mentioned earlier leads to the exact degeneration of some eigenvalues. Namely, the ends of bands corresponding to $q = \frac{1}{2}$ (anti-periodic in x eigenfunctions) happen to (exactly!) coincide.

The work is prepared in collaboration with M. I. Katsnelson, S. Yu. Dobrokhotov and A. Fasolino.

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On some of the peculiarities of propagation of an elastic wave through a gradient anisotropic layer

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Peculiarities of propagation of longitudinal waves through inhomogeneous anisotropic layers with gradient-like distribution of density and elastic parameters are of interest for modeling elastic wave propagation in the real media. In particular, problems of reflection and propagation of longitudinal waves through heterogeneous alloys, composite materials and spatially confined porous structures were under investigation by a number of researchers in the past.

In this study we investigate the problem of diffraction of an elastic wave by the inhomogeneous anisotropic layer with a continuous distribution of elastic parameters. Peculiarities of propagation of the plane wave through the gradient isotropic and transversely isotropic layers were already considered by these authors in the past in [1] and [2]. Our goal is to detail characteristic features of frequency-response characteristics of the elastic wave diffraction by anisotropic layers.

Differential equations for describing the diffraction problem are considered separately for half-planes and for the layer. The elastic parameters in the layer are defined through the elasticity tensor. Problems in the half-planes are overdetermined, which allow establishing a connection between traces of the required functions at media interfaces. Thus, the original problem reduces to the boundary value problem for the system of partial differential equations with boundary conditions of the third type. The Fourier transformation is applied with respect to the variable for which homogeneity of the problem is preserved. The obtained boundary value problem for the system of ordinary differential equations is solved using the grid method.

Results of numerical calculations are presented for "synthesised" environments. Characteristic extrema in transmittance ratios of the elastic wave are determined.

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Wideband computationally-effective worst-case model of twisted pair radiation

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Twisted pairs are widely used in modern equipment, therefore estimation of their radiation is actual problem for analysis of electromagnetic compatibility (EMC). Radiation of twisted pair can be calculated by numerical methods, e.g., method of moments [1]. But the use of numerical methods is unacceptable for express-analysis of EMC between equipment of complex systems by the following reasons: 1) it is required to model thousands of spurious couplings on dense grids (containing up to 104–106 frequencies); 2) at high frequencies, the field is jagged by resonances, and result of computation becomes unstable to errors in model parameters definition.

We have developed a worst-case model for amplitude-to-frequency characteristic (AFC) of the field radiated by a finite-length rectilinear piece of twisted pair (operating in balanced or unbalanced mode). The model is applicable in near- and far-field zones. The following conditions and simplifications were introduced during the model development. Twisted pair is placed in parallel to perfectly-conducting infinite ground plane. Current distribution over the cross-section of each wire of the twisted pair is considered to be symmetrical with respect to the wire center. It is assumed that the power radiated by the twisted pair is many times less than the power transmitted from source to load. Transmission line formed by the twisted pair is considered as lossless.

Worst-case estimation for AFC of the field radiated by the twisted pair is computed by the following algorithm: 1) currents at the source side and at the load side are calculated for one wire of the twisted pair (by a worst-case technique [2]); 2) effective currents are defined (to do that, the currents found at stage 1 are multiplied by correcting coefficients depending on the line parameters and its mode of operation); 3) the twisted pair is replaced by a thin single wire of the same length and with effective currents (found at stage 2); 4) complex amplitudes of current waves [3] are computed using the effective currents; 5) obtained complex amplitudes are substituted into the worst-case radiation model of single wire placed above ground plane [4]. The balanced mode of the twisted pair operation is accounted by an additional correcting multiplier.

By comparison of computed worst-case estimations of radiation field AFCs with results of numerical modeling, it is shown that the model is worst-case and computationally-effective in a wide frequency band. Developed model can be used for diagnostics (express-analysis) of EMC of radio and electronic equipment installed in big systems (cars, aircrafts, ships, etc.) [2].

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Semiclassical limit for the nonlinear Schrödinger equation

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Semiclassical limits of nonlinear Schrödinger equations appear as mean field limits [2, 3], in the long distance propagation of nonlinear waves [6], in optics, and other problems. We focus on power nonlinearities; a particular example included in our analysis is the 3D Gross–Pitaevskii equation for Bose–Einstein condensates,

$$i\hbar\partial_t u^\hbar + \frac{\hbar^2}{2}\Delta u^\hbar - \hbar^2|u^\hbar|^2 u^\hbar = 0, \qquad \|u_0^\hbar\|_{L^2(\mathbb{R}^3)} = 1.$$

For this particular problem no semiclassical asymptotics were known in the state of the art; what is more instability results have been proved [7]. Using the Wigner measures method, and a new semiclassical estimate along with appropriate embeddings, we prove well-posed semiclassical asymptotics for a large family of problems. A phase-space localisation condition on the initial data naturally appears; it is shown that if it is violated then instabilities can appear. For the 3D Gross–Pitaevskii in particular, standard coherent state initial data satisfy this condition. A comparison is given with the standard coherent states method [5], and we explain why the latter can only work for short times and very small ("critical" or below [4]) nonlinearities,

$$i\hbar\partial_t u^\hbar + \frac{\hbar^2}{2}\Delta u^\hbar - b|u^\hbar|^2 u^\hbar = 0, \qquad \|u_0^\hbar\|_{L^2(\mathbb{R}^3)} = 1, \qquad 0 < b \leqslant \hbar^{\frac{5}{2}}.$$

This work is the subject of [1].

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Nonlinear irrotational water waves over variable bathymetry. The Hamiltonian approach with a new efficient representation of the Dirichlet to Neumann operator

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The observation of Zakharov [1], that the nonlinear irrotational water-wave problem (in deep water) admits of a Hamiltonian formulation, involving only surface fields as canonical variables, initiated a sound and efficient way of studying this problem both theoretically and numerically. Craig and Sulem [2] introduced the Dirichlet to Neumann (DtN) operator associated with the substrate boundary value problem (BVP) for the potential, and exploited its analyticity with respect to the surface elevation field in order to expand DtN in a functional Taylor series, facilitating the calculations. However, in the presence of a variable bathymetry, this approach becomes quite complicated [3], and the treatment of wave-irregular bottom interaction problems remain a demanding issue.

In the present work, starting from the unconstrained variational formulation of Luke [4] and a rapidly convergent series representation of the velocity potential [5], we derive the Hamiltonian evolution equations along with a new characterization of the DtN operator over general bathymetry. The utilized series representation is constructed by using explicitly defined vertical functions (forming local H^2-bases) multiplied by unknown horizontal functions (modal amplitudes). The key issues of the present approach are i) the DtN operator is easily expressed in terms of one modal amplitude, ii) because of the rapid convergence of the series expansion, only a few modes (6–8) suffices for very accurate computations.

Numerical results, obtained by using a fourth-order finite difference method, verify the convergence and the accuracy of the new representation of the DtN operator for a flat-bottom test case where an exact solution is available. Computations of the time evolution of highly nonlinear waves over general bathymetry are also presented.

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Physical problems admitting Heun-to-hypergeometric reduction

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The Heun's equation is a Fuchsian equation that can be written as

$$\frac{d^2H}{dz^2} + \left(\frac{\Gamma}{z} + \frac{\Delta}{z-1} + \frac{\varepsilon}{z-d}\right)\frac{dH}{dz} + \frac{abz-q}{z(z-1)(z-d)}H = 0.$$

It has four regular singularities at $\{0, 1, d, \infty\}$ (where $d \neq 0, 1$) with exponents $\{0, 1-\Gamma\}$, $\{0, 1-\Delta\}$, $\{0, 1-\varepsilon\}$ and $\{a, b\}$ respectively [1, 2, 3]. The relation $a + b + 1 = \Gamma + \Delta + \varepsilon$ should hold to ensure the regularity at infinity.

The Heun's equation and its confluent forms emerge in many physical applications [4, 5]. However, the literature related to the mathematical analysis of the Heun's equation is far from complete in comparison with the hypergeometric equation which is known in detail. As a result, studying the reduction methods from Heun to hypergeometric equation is substantial for understanding the physical system better. For instance, the occurrence of the hypergeometric equation can be related with conformal invariance [6, 7]. Apart from their physical importance, the reduction methods can yield a way to probe the mathematical features of the Heun's equation.

In this work, Maier's Heun-to-hypergeometric reduction cases [8] are studied for several physical problems giving rise to the Heun's equation in their mathematical formulation. It is shown that the reduction cases imply special limits of the physical problems such as specific energy spectra or limit density functions in quantum mechanics.

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An interference head wave and concept of localization

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The interference head wave was considered by V. S. Buldyrev [1, 2] (see also [3]). Some formulas for the wave of this class contain an illusory contradiction with concept of localization. We demonstrate, that the contradiction is fictitious and obtain a mathematical description of the wave using the localization principle.

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Inverse problems and sharp eigenvalue asymptotics for Euler–Bernoulli operators

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We consider the Euler–Bernoulli operators $\frac{1}{b}(au'')''$ on the unit interval with the boundary conditions $u(0) = u(1) = u''(0) = u''(1) = 0$, where a, b are positive coefficients. These operators describe the relationship between the pinned-pinned beam's deflection and the applied load. We prove the following results:

i) Ambarzumyan type theorem about the inverse problems for the Euler–Bernoulli operator.

ii) The sharp asymptotics of eigenvalues for the Euler–Bernoulli operator when its coefficients converge to the constant function.

iii) The sharp eigenvalue asymptotics at high energy.

Spectra of open waveguides in some periodic structures

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We discuss the essential spectra of the Neumann problem for Laplace operator on double periodic 3-dimensional media consisting of the bodies connected by long thin ligaments (see fig. 1a) and perturbed in such a way that "open waveguide" appears (see fig. 1b and 1c). The perturbation may influence the essential spectrum of the problem, and the description of this spectrum is the main goal of the talk. We construct a simple examples of media with open waveguides with the additional components of the essential spectrum. Our construction based on a well investigated case of periodic media consisting of the bodies connected by long ligaments with the small width of order $\varepsilon > 0$. Moreover, we give an asymptotic behaviour on ε of the new bands. Also we discuss some cases in which a discrete component of the spectrum may appear.

This is a joint work with Sergey Nazarov from Institute of Mechanical Engineering Problems (Saint Petersburg).

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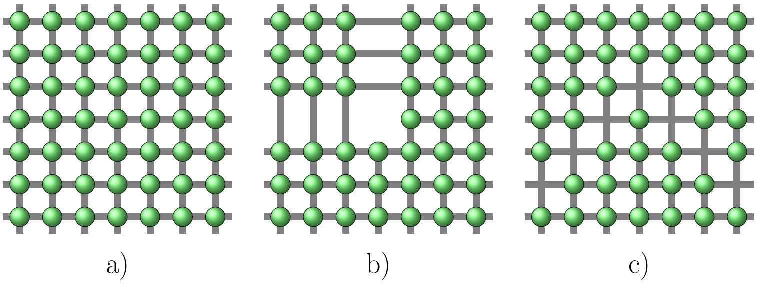

Fig. 1: Unperturbed periodic medium (a) and media with open waveguides (b, c).

Grid methods for boundary layer problems

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Variety of significant applied problems exist in electrodynamics in which boundary layers appear at the interface between two media. Among the examples there are problems of scalar diffraction on metallic surfaces; surface induction heating in steel parts hardening; magnetic field diffusion into compression frame of ultrahigh field magnetic cumulation generator. In all these cases the external field hardly penetrates inside the domain G, and there appears a skin-layer near the domain boundary Γ.

The skin-layer problems are described by diffusion equation with small parameter

$$\mu^2 \operatorname{div}(k(\mathbf{r}) \operatorname{grad} u) - \kappa(\mathbf{r})u = -f(\mathbf{r}), \quad \mathbf{r} \in G, \ \kappa(\mathbf{r}) > 0, \tag{1}$$

where the external media is replaced by boundary conditions on Γ and $\mu \ll 1$. The solution of (1) contains a μ-thick boundary layer and a regular internal part. Due to small value of μ the numerical calculation of such problems is traditionally considered to be difficult.

The paper states that 1) considerable difficulties in computation are delivered also by the transition zone between the boundary layer and the regular solution part; 2) such problems can be uniformly solved by using of an adaptive mesh that contains large enough amount of points in boundary, transition and regular zones; 3) it is possible to construct an algorithm with guaranteed (asymptotically precise) mathematical error estimation by using adaptive meshes and multi-grid Richardson accuracy estimating procedure. It is worth mentioning that numerous solver packages do not include such estimations.

The method proposed here includes following steps. 1) An infinitely smooth adaptive grid generating function is constructed providing approximately equal number of grid points in boundary, transition and regular zones. Thus it is possible to acquire a sequence of nested semi-uniform grids with doubling number N of steps in each direction. 2) A finite-difference scheme is constructed basing on the grids obtained. The best results are achieved with conservative schemes (in foreign literature they are sometimes referred to as neutral). 3) The corresponding linear systems are solved on each grid from the sequence. In case of arbitrary grids this can be done via the conjugate gradients method; for the rectangular nets there exist much faster methods. 4) By comparing the solutions on sequential nested grids, the a posteriori asymptotically precise (and thus rigorous and unimprovable) accuracy estimation is obtained.

Numerical examples show that problems in simple shaped (rectangular) domains are easy to be solved even for extremely small $\mu \sim 10^{-7}$. The solution possesses high relative precision $\sim 10^{-3} \div 10^{-5}$ in all zones on quite moderate grids with $N \sim 200 \div 500$ steps in each direction.

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Axisymmetric flows of non-Newtonian fluids

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Flow of an incompressible fluid with variable viscosity and with furling between two coaxial cylinders is analyzed. Two component structured liquid whose viscosity depends on the degree of structural transformations adopted as a model of non-Newtonian fluid.

This model is described by the equation of motion flow and kinetic diffusion equation. Stationary flows with potential furling around the axis of symmetry are considered.

Qualitative and numerical analysis of the system of equations for the stream function and the degree of structural changes was performed. The formation of vortex structures near the furling axis whose existence for fluids with constant viscosity was shown analytically [1], is investigated.

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Characteristic matrices of layered periodic structures

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Method of symmetric polynomials is developed to calculations of high powers of matrices. It is based on the new type of recurrence relations between symmetric polynomials of n-th order $\mathscr{B}_j(n)$ [1]:

$$\mathscr{B}_{j+l}(n) = \sum_{h=0}^{n-1} \mathcal{C}_{jh}\mathscr{B}_{h+l}(n), \quad \text{where} \;\; \mathcal{C}_{j\,l} = \sum_{g=0}^{l} p_{n-l+g}\mathscr{B}_{j-1-g}(n)\,, \quad l = 0, 1, \ldots, n-1,$$

p_g, $g = 1, \ldots, n$, are coefficients of the characteristic equation of the matrix and polynomials $\mathscr{B}_j(n)$ are defined by the equations: $\mathscr{B}_j(n) = \sum_{l=1}^{n} p_l \mathscr{B}_{j-l}(n)$; $\mathscr{B}_j(n) = 0,\; j = 0, \ldots, n-2$; $\mathscr{B}_{n-1}(n) = 1$.

New algorithm for computation matrix M^j of order n is more effective and accurate than the ordinary matrices multiplication for $j > n$. For example, the method presented in this paper, for the calculation matrix $M^{2^{10}}$ of the sixth order uses the number of arithmetic operations of addition and multiplication in 1.34 times less than the method of ordinary repeated squaring of matrices.

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Coefficients of *SH*-wave conversion into *SV*- and *P*-waves by the crystal layer

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Scattering of SH-wave in crystalline layer bounded by isotropic media is considered. Computation of reflected (marked in Fig. 1 by the index ϱ) and transmitted (marked in Fig. 1 by the index τ) waves are made by means of the transfer matrix method.

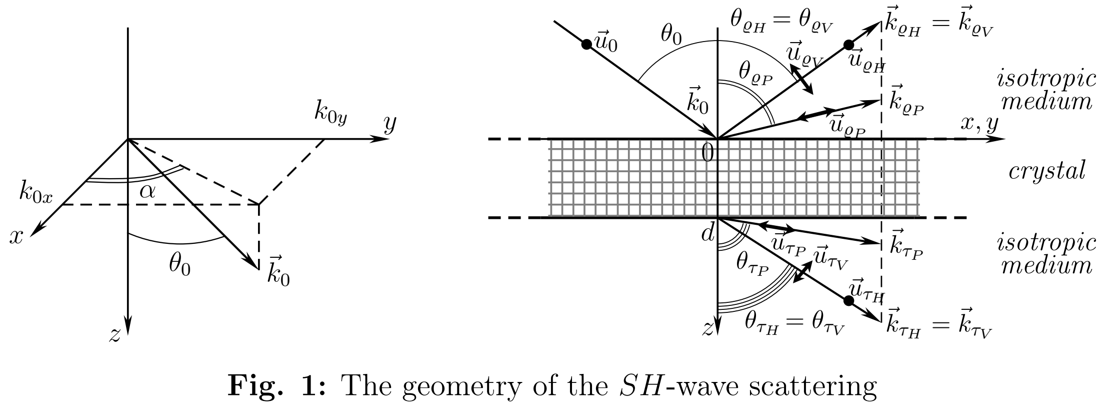

Fig. 1: The geometry of the SH-wave scattering

Wave amplitudes u_{ϱ_S}, u_{ϱ_V}, u_{ϱ_P}, u_{τ_S}, u_{τ_V}, u_{τ_P} are represented by elements of the transfer matrix. The transfer matrix of the sixth order is found by the method of symmetric polynomials [1]. Dependences of the reflected and transmitted through the crystal SH-, SV- and P-waves from the angles of incidence θ_0 and α (Fig. 1) are investigated.

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Impedance theory of scattering and absorption of sound

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An impedance approach is applied to the problem of interaction between a linearly elastic body or structure of finite volume V and interface A with surrounding fluid medium. Three fundamental impedance matrices are needed for solving the problem. One matrix, Z, characterizes the body. It is a matrix of the input surface impedances of the body in vacuo: it relates a vector of external forces applied to the small surface elements with the vector of the normal velocity responses of these elements. Two other impedance matrices, Z_i and Z_r, characterize the fluid. Matrix Z_i of the internal impedances of the fluid is defined similarly to the body matrix Z and represents the matrix of the input surface impedances of isolated volume V filled with fluid. Matrix Z_r of the radiation impedances is defined as a matrix of the input surface impedances of the fluid in the exterior of surface A with all the acoustic sources in it switched off. Introducing two scattering matrices that relate the normal velocity and pressure vectors of the scattered field component to the normal velocity and pressure vectors of the incident field component on A, $v_s = Qv_i$, $p_s = Sp_i$, one can obtain the following solution to the scattering problem, $Q = (Z_r + Z)^{-1}(Z_i - Z)$, $S = (Y_r + Y)^{-1}(Y_i - Y)$, where the mobility matrices Y_k are the inverse impedance matrices Z_k, $k = \emptyset, i, r$. These equations generalize the classical Fresnel's equation for the plane wave coefficient of reflection from a plane interface. This solution is very convenient for treating some unsolved problems, in particular, variational problems of bodies with extreme acoustic properties. One of them is the problem of the best absorber. It is formulated as follows: find such an impedance matrix $Z = R + iX$ that renders maximum to the absorbed power. Giving matrix Z a small variation ΔZ and going through the matrix algebra, one can obtain that the absorbed power has one stationary value (maximum) if the absorber impedance matrix is equal to Hermitian conjugate of the radiation impedance matrix, $Z = Z_r^*$. The body that has such surface impedance matrix is the best absorber which, among the variety of all bodies of the same configuration, absorbs maximum energy of the incident field. Interestingly, this maximum is many orders of magnitude greater than the absorption power of available absorptive materials and structures. Relation of the best absorber to the black body and sonic black hole is discussed. Similar variational problem is solved for the best scatterer. It is shown that the scattered power of the best scatterer is four times the absorbed power of the best absorber. One of the consequences of the impedance theory is the existence of a finite region of allowable values for the absorption and scattering powers of passive bodies. A plot of the region is given. Presented are also some results, theoretical and experimental, concerning transparent, i.e. nonscattering and, hence, acoustically invisible bodies.

Singular value decomposition in application to a scattering problem in a plane channel with sharp corners

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2-D classical scattering problem in a strip region with a sharp ledge is considered. The problem is governed by the Helmholtz equation together with the Neumann condition on the boundary and the radiation condition in infinity. To solve the problem considered the method of discrete sources is applied.

An unknown solution is sought as a linear combination of the Green (source) functions of the infinite strip domain. Substitution of such an anzats into the boundary condition leads to a set of linear algebraic equations (SLAE). The situation becomes dramatically ill for domains with sharp corners. Obviously separated minimal eigenvalues can appear. In other words, the matrix of SLAE becomes very close to singular.

For such "zero" problems there exists a very powerful technique known as singular value decomposition (SVD). First of all, SVD gives us a clear diagnosis of the situation: it explicitly constructs orthonormal bases for the nullspace and range of a matrix. Moreover, this numerical tool allows to solve SLAE effectively in the case that a matrix is singular: a solution is obtained by means of SVD immediately. This property permits us to get explicitly "zero" eigenfunctions corresponding to nearly zero eigenvalues.

The new way of source allocation for sharp-pointed domains are suggested. An algorithm, permitting us to find the optimal source placement, based on the singular value decomposition technique, is presented. Numerical experiments illustrate our investigations.

Perturbation of threshold of essential spectrum for three-dimensional layer with window

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Let $x = (x_1, x_2, x_3)$ be Cartesian coordinates, $\omega \in \mathbb{R}^2$ be a bounded domain with a smooth boundary. We consider a three-dimensional layer $\Pi_\omega := \{x : x_3 \in (-d, 0) \cup (0, \pi)\} \cup (\omega \times \{0\})$, where $d \geqslant \pi$ is a fixed constant. This layer consists of two parts corresponding to $-d < x_3 < 0$ and $0 < x_3 < \pi$ and these parts are coupled by the set $\omega \times \{0\}$. In what follows this set is referred to as window. The main object of our study is the magnetic Schrödinger operator

$$\mathcal{H}_\omega := (i\nabla + A)^2 + V$$

subject to Dirichlet boundary condition. Here A and V are magnetic and electric potential assumed to be smooth enough and compactly supported.

The essential spectrum of operator $\mathcal{H}_\omega$ is independent of window and potentials and it is the set $[1, +\infty)$. Our first main result says that there infinitely many critical shapes of the window with the following property: any infinitesimally small increasing of the window produces a discrete eigenvalue of $\mathcal{H}_\omega$ below the threshold of the essential spectrum. The criterion for the criticality of a given window is the existence of bounded non-trivial solution to the problem

$$\big((i\nabla + A)^2 + V - 1\big)\psi = 0 \quad \text{in} \quad \Pi_\omega, \qquad \psi = 0 \quad \text{on} \quad \partial\Pi_\omega. \tag{1}$$

Our second main result concerns the situation when problem (1) has non-trivial decaying solutions only. In this case we consider a small perturbation of the window shape. It is shown that under such perturbation, the threshold of the essential spectrum produces either a bound and anti-bound state or a pair of resonances. We provide simple sufficient condition identifying each of these situations and calculate the leading terms in the asymptotics of the corresponding spectral values.

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On representations of the generalized oscillator for two systems of orthogonal polynomials connected by linear relations

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We consider a generalized oscillator algebra A connected with a sequence of monic orthogonal polynomials $\{P_n(x)\}_{n=0}^{\infty}$ with respect to a positive Borel measure μ on the real line. Let $\{Q_n(x)\}_{n=0}^{\infty}$ be the sequence of monic polynomials such that

$$Q_n(x) = P_n(x) + a_1 P_{n-1}(x),$$

and the sequence $\{Q_n(x)\}_{n=0}^{\infty}$ is orthogonal with respect to a positive Borel measure ν on the real line. We discuss the question: under which conditions the generalized oscillator algebra connected with the polynomial system $\{Q_n(x)\}_{n=0}^{\infty}$ be the same as the algebra A.

Explicit transition between classical and semiclassical regimes for a periodic Schrödinger operator with a non C^1 potential

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Let L_0 be a characteristic length and V_0 a reference potential. We describe the spectral bands of the $2L_0$-periodic Schrödinger operator acting on $L^2(\mathbb{R})$, $H = -\frac{\hbar^2}{2m}\frac{\mathrm{d}^2}{\mathrm{d}x^2} + V$ associated with a sawtooth-like potential V defined for $|x| \leq L_0$ by $V(x) = \frac{|x|}{L_0}V_0$.

This model is equivalent to a infinite periodized sawtooth junction PN in crystals, giving an explicitly solvable situation in a quantum setting (see [M] for a wave analog in photonic crystals).

Let θV_0 be the dimensionless semiclassical parameter of the model, deduced from the Planck constant, the mass of the considered particle, V_0 and L_0 through

$$\theta V_0 = \left(\frac{2mV_0L_0^2}{\hbar^2}\right)^{\frac{1}{3}}.$$

Introduce $-\tilde{a}_1 \approx -1.019$ the greatest zero of Ai' and denote by $-c \approx -1.515$ the largest zero of the function $Bi'(\cdot)Ai(0) - Bi(0)Ai'(\cdot)$.

We consider equations for the energy level E, namely

$$Ai'(-\theta V_0 - \theta E)Bi'(-\theta E) - Bi'(-\theta V_0 - \theta E)Ai'(-\theta E) = 0 \tag{1}$$

and three other similar equations.

1. The edges of the spectral bands are the ordered roots in E of the four equations (see [RS]).
2. The bottom of the spectrum $E_{\min}$ is the unique solution of equation (1) with E negative and $-\theta V_0 - \theta E$ in $(-\tilde{a}_1, 0]$. More precisely we have, for every $\theta > 0$, $-1 < E_{\min}/V_0 < -1 + \tilde{a}_1/(\theta V_0)$.
3. For every $E \in [-V_0, 0]$ there is a classical localized solution of the motion whereas for $\theta V_0 \geq c$, there exist an energy E in $[-V_0, 0]$ which is not in the spectrum of H. As a consequence, *the constant c marks the transition between classical and semiclassical regimes.*

4. Let $\{\tilde{c}_p\}_{p\geq 0}$ be the ordered set of roots in x of $(Bi(-x)Ai'(0) - Bi'(0)Ai(-x))(Bi'(-x)Ai'(0) - Bi'(0)Ai'(-x))$. For $\theta V_0 \geq \tilde{c}_p$, we have precise estimates on the position of the upper and lower edges of the $p-1$ first spectral bands and on the width of the spectral gaps in terms of the semiclassical parameter and of the roots of the Airy function and its derivative.

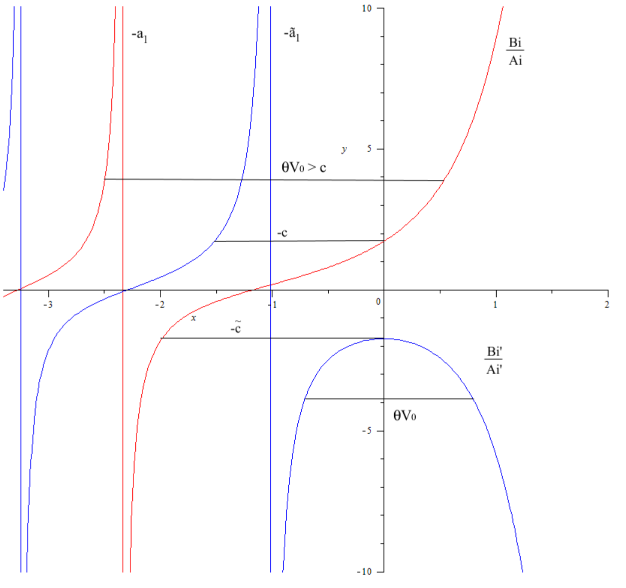

Fig. 1: The constant c.

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Modeling asymmetric Mössbauer spectra of superparamagetics

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Magnetic moment evolution of superparamagnetic particles is a Markov process and satisfies the Fokker–Plank equation on the unit sphere with a drift field generated by the magnetic anisotropy [1]. Thus it is a quantum system with the Hamiltonian depending on stochastic variable being the position of the representative point on the unit sphere. Mössbauer spectra one can express in terms of resolvent of the generator of quantum evolution operator of a nucleus, averaged along all diffusion process trajectories. Computation is connected with solving complicated differential equation system of partial derivatives and can not be done in analytical form. At Discrete Orientation Model (DOM) the movement looks like a Markov chain with finite number of states — easy magnetization directions coinciding with points (edges) of lattice [2]–[3]. Discrete Markov chain is an asymptotic limit at large times of continuous motion [4]–[5]. Potential drift field leads to symmetrical line shape in any case. For more correspondence to experiment one can introduce at DOM stochastic distributions inside attraction regions of easy magnetization directions [6]–[7] which reduce effective magnetic field on a nucleus, but the spectrum remains symmetrical one. The opportunity to get asymmetrical spectrum in frames of discrete approximation corresponding to solenoidal drift field is introducing as a generator of dynamics non-symmetric stochastic matrix (with a zero sum in lines — so called Laplace matrices [8]), which leads to precession of magnetic moment not inside but between easy magnetization directions. Computations show results like at Fig. 1.

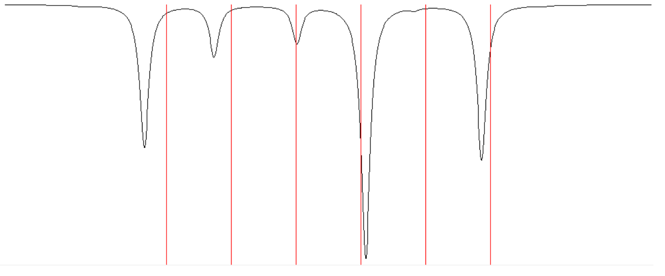

Fig. 1: Example of calculated hyperfine splitting line shape in the case of magnetic moment precession between easy magnetization axes.

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Spatiotemporal field transformation and spatiotemporal Fano lineshape in guided-mode resonant gratings

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Spatiotemporal optical pulse transformations are of great interest for a wide variety of applications, including ultrafast all-optical data processing, optical computing, beam and pulse shaping. In recent papers various types of Bragg gratings were proposed as temporal integrators and differentiators (see, e.g., Ref. 1 and references therein). The use of resonant grating allows one to implement both temporal and spatial transformations of the pulse envelope [2, 3].

In this work, we investigate *spatiotemporal* transformations implemented by guided-mode resonant gratings (Fig. 1a). We develop an analytical model describing these transformations. In particular, we show that the pulse envelope transformation is described by a linear time-invariant (LTI) system. The transfer function of the LTI system is defined by the transmission coefficient of the grating. We derive an analytical approximation of the transmission coefficient, taking into account the symmetry of the grating, the energy conservation law, and the causality condition [4]. The proposed approximation, being a function of both temporal and spatial frequency of the incident light, can be considered as a spatiotemporal generalization of the Fano lineshape [4]. The use of the proposed spatiotemporal Fano lineshape allows us to derive a second-order partial differential equation that describes the general form of the optical pulse spatiotemporal transformation implemented by a resonant grating.

We verify our analytical model by means of rigorous simulation based on the Fourier modal method. The simulation results presented in Fig. 1b, c are in good agreement with the proposed model.

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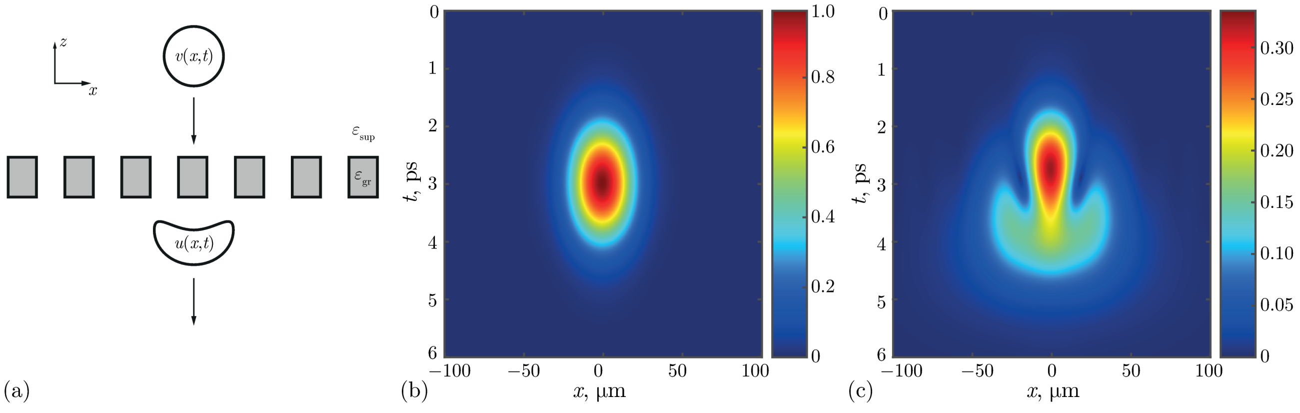

Fig. 1: (a) Pulse diffraction on resonant grating, (b) incident pulse envelope $v(x, t)$, (c) transmitted pulse envelope $u(x, t)$.

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Spectral stability of special discontinuities

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The stability of special discontinuities corresponding to solutions of model generalized Korteweg – de Vries – Burgers equation is studied. The discontinuity is a special one when its structure is represented by a phase heteroclinic curve connecting two singular point of the saddle type (they are states in front of and behind the discontinuity). The well known deflagration fronts in gases, discontinuities with structures in nonlinear elasticity theory and structures arising when gas crossing a discontinuity acquires or looses heat conductivity in the presence of magnetic field are examples of such discontinuities.

The model equation under consideration is one of the simplest equations with solutions including special discontinuities. Depending on the dispersion–dissipation relation for the K–dV–Burgers equation with a non-monotone potential there is a finite number of solutions corresponding to special discontinuities [1, 2]. The number of special solutions of different types grows with growing relative dispersion influence as compared to that of dissipation, this leads to many non-unique solutions of self-similar problems and the more different special discontinuities exist, the more solutions we obtain. In the case of non-uniqueness the solutions can include special discontinuities of different types as well as sequences of such discontinuities.

In the paper presented the spectral (linear) stability of special discontinuity structure is studied. It is shown that only one special discontinuity with monotone structure is stable. Special discontinuities with non-monotone structures are unstable.

Analysis of spectral stability for special discontinuities gives a criterion to choose solutions in the non-uniqueness region. According to this criterion solutions of self-similar problems including special discontinuities with non-monotone structures are unstable. Hence, in the non-uniqueness case only two solutions are possible — one of them includes stable special discontinuity, while the other solution is an one non-special discontinuity.

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The new laws of the Rayleigh, resonance and diffuse scatterings

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The problem of the Rayleigh surface acoustic wave by the deterministic and statistical surface roughness and subsurface inhomogeneity is solved in the Born (the Rayleigh–Born) approximation of the perturbation theory [1] in the amplitude of a roughness or inhomogeneity. The new laws of the Rayleigh scattering, when the character size of the rough or inhomogeneous region a is much smaller tnan the wavelength λ of the incident Rayleigh wave $a \ll \lambda$, of the resonance scattering, when $a \sim \lambda$, and of the diffuse scattering, when $a \gg \lambda$, are obtained and investigated.

It is obtained that the strong modulation of the Rayleigh, resonance and diffuse scatterings by the roughness or inhomogeneity form obtained in the present work enables amplification and suppression of scattering in all the range of the a/λ ratio. The phenomena of the Rayleigh, resonance and diffuse scatterings amplification and suppression are obtained and investigated in the present work.

A strong modulation of scattering by a roughness or inhomogeneity form giving rise to violation of the Rayleigh law of scattering [1] enables to construct basis linear independent functions of the variables $p = a/\lambda$ and φ_s — the angle of scattering. Each of these functions describes the angular distribution of scattering $G^{(R)}(p, \varphi_s)$ or coefficient of scattering $1/l^{(R)}$ [2]–[4] and corresponds to the scattering by the definite form of the roughness or inhomogeneity.

From the mathematical point of view a strong modulation of scattering by a roughness or inhomogeneity form obtained in the present work means that the principal physical laws of the Rayleigh, resonance and diffuse scatterings are defined not only by the one parameter p — character size of roughness or inhomogeneity to wavelength ratio but by the variety of parameters as well. These parameters are relations between partial roughnesses or inhomogeneities character sizes, i.e. between partial character sizes of the partial roughnesses or inhomogeneities constituting the summary roughness or inhomogeneity and so defining the form of a summary roughness or inhomogeneity. The physical value defining all the variety of these parameters, consequently the roughness or inhomogeneity form and thus the principal laws of the Rayleigh, resonance and diffuse scatterings is the topological characteristics of a roughness or inhomogeneity $C^{(n)}$ [2]–[4].

So the principal physical laws of the Rayleigh, resonance and diffuse scatterings completely depend on the space configuration of the physical objects interacting in scattering, that is on the form of the incident wave defined by the wavelength and on the form of the roughness or inhomogeneity, defined by the topological characteristics $C^{(n)}$.

This basis of scattering, i.e. a strong modulation of scattering by a roughness or inhomogeneity form, enables to construct a definite form of the scattering spectrum not only for the Rayleigh scattering $p \ll 1$, but for the resonance $p \sim 1$ and for the diffuse $p \gg 1$ scatterings, violating the main laws of the Rayleigh, resonance and diffuse scatterings as well.

Results of the present work can be used in seismology, solid state physics, acoustic microscopy and acoustoelectronics.

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Implementation of modified Sinc-collocation method to linear two-point boundary value problem

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Recently [1] was considered one-dimensional boundary value problem (BVP) in the following form:

$$p(x)y'' + q(x)y' + r(x)y = f(x) \tag{1}$$

with boundary conditions

$$y(x_0) = y_0,\, y(x_1) = y_1. \tag{2}$$

Let us discuss model problems of the form $-y'' + r(x)y = f(x)$ with boundary conditions $y(0) = 0$, $y(1) = 0$ and exact solution $\bar{y}(x)$. Numerical solution of (1), (2) was constructed with help of atomic functions family $\mathrm{ch}_{a,n}$. For the other hand, Sinc-collocation method [2], [3] is a powerful tool for treat boundary value problems for both ordinary and partial differential equation. In present report we investigate a modified Sinc-collocation transform. Modification based on the usage of modified Cardinal series which consist of shifts of the modified Sinc kernel functions [4]–[8]. We pay our attention to the linear two-point problem and notice some properties of modified Sinc-collocation with respect to ordinary one.

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Complex waves and waveguide resonance excitement

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The solution of hollow waveguide stationary excitation problem reported in many classical courses (for example [1]). The stabilization problem was considered in [2] and resonance case in [3]. The growth of field is proportional to $\sqrt{t}$ for waveguides in spite of t [3].

The problem becomes much more complex for waveguides with filling. Even for homogeneous but anisotropic filling the complex waves and backward waves may take place. We consider some simple example of the problem [4] which can be exactly solved. We demonstrate for this problem the origin of backward waves and consider the resonance excitation of waveguide. We found the frequencies for which the resonance field has growth t but not $\sqrt{t}$. We consider the problem for nontraditional choosing the first and second pairs of Maxwell equations which gives some advantages for spectral problem [4]–[6]. The first pair of equations takes form

$$\begin{pmatrix} 0 & -\frac{1}{c}\frac{\partial}{\partial t} & \frac{\partial}{\partial x} \\ \frac{1}{c}\frac{\partial}{\partial t} & 0 & \frac{\partial}{\partial y} \\ -\frac{\partial}{\partial x} & -\frac{\partial}{\partial y} & 0 \end{pmatrix} \begin{pmatrix} H_x \\ H_y \\ E_z \end{pmatrix} = \begin{pmatrix} \varepsilon_1^{-1} & 0 & 0 \\ 0 & \varepsilon_2^{-1} & 0 \\ 0 & 0 & 1 \end{pmatrix} \frac{\partial}{\partial z} \begin{pmatrix} \varepsilon_1 E_x \\ \varepsilon_2 E_y \\ H_z \end{pmatrix}, \tag{1}$$

$$\begin{pmatrix} 0 & \frac{1}{c}\frac{\partial}{\partial t} & \frac{\partial}{\partial x} \\ -\frac{1}{c}\frac{\partial}{\partial t} & 0 & \frac{\partial}{\partial y} \\ -\frac{\partial}{\partial x} & -\frac{\partial}{\partial y} & 0 \end{pmatrix} \begin{pmatrix} \varepsilon_1 E_x \\ \varepsilon_2 E_y \\ H_z \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \varepsilon_3 \end{pmatrix} \frac{\partial}{\partial z} \begin{pmatrix} H_x \\ H_y \\ E_z \end{pmatrix} + \frac{4\pi}{c} \begin{pmatrix} -j_y \\ j_x \\ -c\rho \end{pmatrix}. \tag{2}$$

The second group consists of the equations

$$(\mathrm{rot} E)_z = -\frac{1}{c}\frac{\partial}{\partial t} H_z, \tag{3}$$

$$(\mathrm{rot} H)_z = \frac{1}{c}\frac{\partial}{\partial t}\varepsilon_3 E_z + \frac{4\pi}{c} j_z. \tag{4}$$

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Asymptotic behavior of singular values of the acoustic observation problem

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Suppose $\mathbb{R}^d_+ = \{x_d > 0\}$, $d \geqslant 2$, $q \in C_0^\infty(\mathbb{R}^d_+)$. Consider the following initial boundary value problem for scalar function $u(x,t)$ in $\mathbb{R}^d_+ \times (0,\infty)$:

$$\begin{aligned} &\partial_t^2 u - \Delta u + qu = 0, \quad \partial_{x_d} u|_{x_d=0} = 0, \\ &u|_{t=0} = 0, \quad \partial_t u|_{t=0} = v. \end{aligned} \tag{1}$$

If $v \in C_0^\infty(\mathbb{R}^d_+)$, the solution u is smooth. Now suppose that Ω is an open bounded set such that $\overline{\Omega} \subset \mathbb{R}^d_+$. Put $\Sigma_0 = \partial\mathbb{R}^d_+ \times (0,\infty)$ (Σ_0 is "time-space" boundary) and let Σ be an open bounded subset of Σ_0 such that $\overline{\Sigma} \subset \Sigma_0$. Consider the operator

$$\mathcal{O}_\Omega^\Sigma : v \mapsto u|_\Sigma,$$

where u is the solution of (1) for the initial data $v \in C_0^\infty(\Omega)$. The operator $\mathcal{O}_\Omega^\Sigma$ can be continued as compact operator acting from $L_2(\Omega)$ to $L_2(\Sigma)$.

Consider the following questions: is it possible to recover v by $\mathcal{O}_\Omega^\Sigma v$ (q is fixed) and is the recovering stable? Certain geometric conditions on Ω and Σ guarantee that the operator $\mathcal{O}_\Omega^\Sigma$ is invertible. The stability issue is connected with the behavior of the singular values $s_n(\mathcal{O}_\Omega^\Sigma)$, i.e. the square roots of positive eigenvalues of $(\mathcal{O}_\Omega^\Sigma)^*\mathcal{O}_\Omega^\Sigma$ numbered in non-increasing order with the multiplicity taken into account. We prove that $s_n(\mathcal{O}_\Omega^\Sigma)$ decrease as a negative power of n (relation (2)). The result is obtained without any geometric conditions on Ω and Σ (we do not need the invertibility of $\mathcal{O}_\Omega^\Sigma$ in (2)). We apply the theory of Fourier integral operators (which are close to Maslov's canonical operator) and make use of the result [1] on the spectral asymptotics of pseudodifferential operators. Note that our problem is close to certain problems of integral geometry considered by V. P. Palamodov [2] as well as E. T. Quinto, G. Uhlmann, L. V. Nguyen, etc.

Introduce the following mapping from $\Omega \times \mathbb{R}^d$ to Σ_0:

$$\gamma(y,\eta) = \left(y' - y_d \frac{\eta'}{\eta_d},\, y_d \frac{|\eta|}{|\eta_d|} \right), \quad y \in \Omega,\, \eta \in \mathbb{R}^d.$$

Here $y' = (y_1, \ldots, y_{d-1})$, $\eta' = (\eta_1, \ldots, \eta_{d-1})$. The function γ is defined if $\eta_d \neq 0$, i.e. almost everywhere in $\Omega \times \mathbb{R}^d$. The first component of $\gamma(y,\eta)$ is an intersection point of the line $\{y + s\eta,\, s \in \mathbb{R}\}$ and $\partial\mathbb{R}^d_+$, while the second component is the distance between this intersection point and y. Define the *audible zone* as follows

$$AZ = \big\{(y,\eta) \in \Omega \times \mathbb{R}^d \mid \eta_d \neq 0,\, \gamma(y,\eta) \in \Sigma\big\}.$$

Denote by $n(\mathcal{O}_\Omega^\Sigma, \lambda)$ the number of singular values $s_n(\mathcal{O}_\Omega^\Sigma)$ greater than $\lambda > 0$.

Theorem. *Let Σ satisfy $\mu_d(\partial\Sigma) = 0$ (μ_d is the d-dimensional Hausdorff measure, $\partial\Sigma$ is taken in the topology of Σ_0). Then*

$$\lim_{\lambda\to 0} \lambda^d n(\mathcal{O}_\Omega^\Sigma, \lambda) = (2\pi)^{-d} \int_{AZ} dy d\eta\, \theta\left(\frac{1}{|\eta|\,|\eta_d|} - 1 \right) \tag{2}$$

(θ is the Heaviside function).

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Global solvability of the problem on two-phase capillary fluid motion in the Oberbeck–Boussinesq approximation

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Unsteady motion of a drop in another incompressible fluid bounded by a rigid surface is considered in the Oberbeck–Boussinesq approximation. The liquids are separated by a closed unknown interface Γ_t where surface tension is taken into account.

Global existence theorem for the problem is stated in Hölder classes of functions provided that the data have small norms and the initial configuration of the drop is close to a ball B_{R_0} with the center in the origin. It is shown that velocity vector field and temperature deviation from the mean value decay exponentially as $t \to \infty$, the interface between the liquids tending to a sphere $\{|x - h_\infty| = R_0\}$ with a center h_∞, the limiting position of drop's barycenter. It is established that if the initial data are small enough, the inner liquid will remain strictly inside the other one during all the time.

Our study is based on a local classical existence theorem for the problem which was proved in [1]. Using the idea of constructing a function of generalized energy for the fluid proposed by M. Padula in [2], we show that the L_2-norms of the velocity and temperature deviation decrease exponentially with respect to time. In the proof, we apply a similar result without including temperature obtained in the joint paper of the author and V. A. Solonnikov [3]. Finally, energy estimate allows us to repeat our considerations in a small time interval step by step and to prolong the solution as long as we like.

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Inverse problem of the reconstruction of electromagnetic and geometrical parameters of a multi-sectional diaphragm in a rectangular waveguide from the transmission or reflection coefficients

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Reconstruction of electromagnetic and geometrical parameters of dielectric body is an urgent problem as far as investigation of the properties of nanocomposite materials are concerned. As a rule, these parameters cannot directly be measured. For this reason it is necessary to develop methods of mathematical modeling which allow one to calculate these parameters from easily measured data, like reflection and transmission coefficients. This paper focuses on the reconstruction of permittivity and thickness of a multi-sectional diaphragm placed in a rectangular waveguide from the transmission coefficient or reflection coefficient, measured at different frequencies. First results in this problem have been found in papers [1, 2].

This work is a continuation of series papers [3, 4]. Using the developed recursive method we obtain solution to the inverse problem with one, two, and three diaphragm sections. For the case of one-sectional diaphragm we obtain analytical solution of the problem. Numerical results are presented and confirm the efficiency of the developed method.

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The study of scaling properties of the human brain electrical activity in various states of consciousness

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It is well known that the voltage fluctuations resulting from ionic current flows within the neurons of the brain are measured by electroencephalograms (EEG). In this work we study the scaling properties of multichannel EEG for a group of subjects to find the difference between the states of rest and meditation. Namely, we introduce and calculate the so-called self affine index. It differs from the traditional ways of studying the scaling properties in electroencephalography. The self affine index has been calculated for 20 fragments consisting of 1000 samples of each EEG channel for the group of subjects. This group consisted of five experienced in the practice of meditation subjects and ten inexperienced subjects. EEG have been recorded in the rest state and during meditation. We then statistically processed the obtained results by means of repeated ANOVA. The main results are the following. For experienced in the practice of meditation subjects the self affine index in a state of meditation was significantly higher as compared to the same index at rest state. For inexperienced subjects difference in self-affinity index at rest and a state of meditation was not statistically significant. For all subjects in meditation state the self affine index was significantly higher in the left anterior frontal region. Thus it can be assumed that the higher self-affine index in the left frontal region reflects higher compared to other channels synchronization of sources involved in the generation of the EEG. More detailed neurophysiologic explanations of the obtained results will be presented in the talk.

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Asymptotic solutions to the Cauchy problem with localized initial data for linear strictly hyperbolic systems

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We construct the asymptotic solutions to the Cauchy problem with localized initial data of linear strictly hyperbolic systems in multidimensional situation. Using the new representations for the rapidly oscillating functions and the Maslov canonical operator in the neighborhood of the caustic and focal points we construct the maximum explicit and effective formulas for these asymptotics based on the wave fronts in the phase space. We illustrate general formulas by the localized asymptotic solutions of 3-D wave equation with variable velocity. This work was done together with A. Allilueva, S. Sergeev and A. Shafarevich, and was partially supported by RFBR grant № 14-01-00521.

Localized vortical solutions of linear and nonlinear shallow water equations

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Quite explicit asymptotic formulas for localized vortical solution of linear shallow water equations are found in [1, 2] (using approach based on Maslov canonical operator). It was proved that the asymptotic solution has the same form as the initial function iff the Cauchy–Riemann conditions are satisfied along the trajectory of background flow. To construct asymptotic formulas for localized vortical solution of nonlinear shallow water equations we can use the method of fast variable.

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Surface plasmon-solitons in heterostructures with Kerr nonlinearity

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Recently, there is a significant interest in study of nonlinear optics in metallic nano-structures, where the surface plasmon polaritons (SPP) provide strong nonlinear effects under low power excitations. The most remarkable properties of SPPs are their abilities of strong field confinement and consequent field enhancement at the dielectric-metal interface, which lead to an extraordinary sensitivity of SPPs to surface conditions. Basic guiding nanostructures, such as metallic gratings, can be assembled in more complex plasmonic systems, which finds applications in a wide range of optical devices exploiting the main virtue.

In the proposed plasmonic nanostructure, which consists of an array of closely spaced parallel metallic nanorodes embedded in an optical medium with the Kerr-type nonlinearity solitary waves (plasmonic lattice solitons) are studied. It has been shown in [1] that in similar configuration the vector plasmonic lattices solitons are found theoretically. We propose an extended method from [2] to derive the nonlinear Shrodinger equation and study solitons that are directed athwart metallic grating. To estimate the method accuracy we compared obtained values of soliton's parameters with the calculations received in an effective medium approximation. Dependence of soliton's peak power on geometrical parameters is studied. We developed a method of selection of optimal geometrical parameters for such metamaterials to minimize estimated excitation power and enhance localization. We compared numerical solutions based on direct derivation of nonlinear equation with varying coefficients directly from Maxwell equations with solitons obtained with our approximate general method. In addition to previous results it can lead to improved guiding of plasmonic lattice solitons in metallic nano-structures.

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On the relativistic constraint dynamics of electromagnetic beams

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The classical field in an electromagnetic beam is usually thought of as a simple function of 4-position in Minkowski space but it also depends on the coordinates of a focal point of the beam. In a complete relativistic treatment the position of the focal point must also be represented as a 4-vector creating the problem that the field depends on two time coordinates. This problem is familiar from relativistic treatments of two interacting particles. The resolution is to restrict the relative coordinates between the two points to a three-dimensional constraint space such that the relative time can be expressed as a pure function of relative position. This idea is investigated here using exact Bateman–Hillion solutions to the wave equation to represent the electromagnetic field. Two constraint conditions are considered. One relates the relative time to the axial distance from the focal point. It is found this approximation is mathematically equivalent to the paraxial approximation of the wave equation for continuous wave beams in the sense the constrained Bateman–Hillion solution exactly satisfies the paraxial wave equation. The other relates the relative time to the radial distance from the focal points in spherical polar coordinates. It is proposed this radial constraint condition is appropriate for approximate calculations beyond the paraxial limit. Results are presented to illustrate how more rapidly diverging beams differ from paraxial beams and the limitations of the method are discussed.

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Semiclassical quantization rules for a periodic orbit of hyperbolic type

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We consider semi-excited resonances for a Schrödinger type operator $H(x, hD_x)$ on $L^2(\mathbb{R}_n)$ induced by a periodic orbit of hyperbolic type, as arises in the AC Stark effect. It is known [NoSjZw], [FaLoRo] that these resonances are given by the zeroes of a determinant associated with Poincaré map. We make here this result more precise, in providing a first order asymptotics of Bohr–Sommerfeld quantization rule in terms of the (real) longitudinal and (complex) transverse quantum numbers, including the action integral, the sub-principal 1-form and Gelfand–Lidskii index. To this aim we bring the operator to its Birkhoff normal form microlocally near the periodic orbit, and introduce a Hermitian structure on the fibre bundle $K_h(E)$ of WKB solutions of $(H-E)u=0$. Then we construct the monodromy operator as an h-Fourier Integral Operator; as in Van Vleck formula, stationary phase selects from all trajectories the periodic orbit that supplies with the main contribution to the oscillatory integrals representing forward and backward parametrices; the residue of their phase functions modulo exact forms gives the generalized action integral and we obtain the quantization condition by expressing that $K_h(E)$ should be trivial whenever E is a resonance.

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Analysis of the extended boundary condition method in electrostatic problems for non-spherical particles: the region of applicability and the Rayleigh hypothesis

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The electrostatic problem of excitation of a homogeneous axisymmetric particle is considered. The approach applied is based on surface integral equations analogous to those given by the extended boundary condition method (EBCM) for wave problems [1, 2]. The electrostatic fields are related to scalar potentials that are represented in the form of expansions in terms of eigen functions of the Laplace operator in spherical coordinates. The unknown expansion coefficients are derived from infinite systems of linear algebraic equations (ISLAEs). Analytical study of ISLAE solvability is performed and the expansion convergence radii are found. It is shown that for applicability of the EBCM in the far-field zone, i.e. for construction of the T-matrix, one does not need the Rayleigh hypothesis fulfillment (convergence of the expansions everywhere up to the particle boundary). However, there appears a weaker constraint that can be reduced to the demand of existence of a spherical shell where the expansions of the exciting and internal fields converge. The cases of prolate and oblate spheroids as well as pseudo-spheroids $\big(r(\theta) = ab/r_{\rm spheroid}(\theta) = a\sqrt{1-\epsilon^2\cos^2\theta},\ \epsilon^2 = 1-(b/a)^2\big)$ produced from spheroids by inversion are investigated in detail. It is shown that the EBCM is applicable to spheroids with any aspect ratio a/b and to pseudo-spheroids with $a/b < 1+\sqrt{2}$. The external Rayleigh hypothesis, i.e. convergence of the "scattered" field upto the particle boundary, is valid for $a/b < \sqrt{2}$. The internal hypothesis, i.e. convergence of the internal field, is correct always as the field inside the spheroid is uniform. The hypotheses are valid for pseudo-spheroids when $a/b < \sqrt{5}$ and $a/b < \sqrt{2}$, respectively. The electrostatic problem for axisymmetric Chebyshev particles $r(\theta) = a(1+\epsilon\cos n\theta)$ is considered. The main attention is paid to the case $n = 1$. For the considered particles, numerical calculations have been performed. In this process, integral ISLAE elements have been represented as sums the summands of which depend on gamma functions. Calculations of matrix elements by explicit formulas have made it possible to considerably increase the dimension of the solved reduced ISLAE with preservation of the necessary accuracy. Analysis of results of numerical calculations verified their agreement with theoretical conclusions. The relation of results obtained for wave and electrostatic problems is considered and their similarity is demonstrated.

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Excitation of electromagnetic waves in a dynamical cavity

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We consider excitation of electromagnetic waves in a cavity with oscillating mirrors. First, conditions of cavity modes stability are found for the case of not plane, but spherical mirrors. Second, effect of finite spectral width of mirror reflection coefficient is studied and stabilization of field growth under conditions of parametric resonance is demonstrated.

We consider excitation of electromagnetic waves in a high-quality cavity with periodically oscillating mirror. These oscillations serve as the source of electromagnetic field. In classical approach, this phenomenon is an example of field parametric excitation [1], and in the framework of quantum electrodynamics it is referred as the dynamical Casimir effect [2].

In the paraxial approximation, stationary cavities formed by two motionless spherical mirrors are classified as stable or unstable depending whether there are or no localized modes; in another approach, for stable cavities rays close initially to the axial one continue to be close to the axis after many trips, and they move off the axis in unstable cavities [3]. For dynamical cavities, the notion of modes loses its meaning, however the approach of geometric optics retains its validity. The case of plane parallel mirrors corresponds to the boundary between stable and unstable cavities when even weak perturbation can change the dynamics radically. We present the criterion of stability of cavities with periodically oscillating mirrors in the approximation of geometrical optics.

For stable cavities, it is possible to consider the field temporal dynamics in the approximation of plane waves. In the classical approach, it is necessary to solve one-dimensional Maxwell's equations with boundary conditions at moving mirrors. When neglecting frequency dispersion of mirrors reflection and under conditions of parametric resonance [1], the initial pulse in the cavity transforms with time into progressively narrowing pulse with exponentially increasing energy. However, the finite spectral width of reflection limits the pulse narrowing. For the mirror frequency dispersion characterization, we use the following relation between the electric field E of incident (subscript i) and reflected (subscript r) pulses at time moment t: $E_r(t) = \int_0^\infty D(\tau)E_i(t-\tau)\,d\tau$. In the absence of dispersion, D is proportional to the Dirac delta-function, $D(\tau) = D_0\delta(\tau)$. We demonstrate the pulse stabilization and its main parameters for the case of Lorentz-type dispersion.

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Stark–Wannier resonances and cubic exponential sums

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We discuss the Schrödinger operator $H = -\frac{\partial^2}{\partial x^2} + v(x) - \lambda x$ acting in $L^2(\mathbb{R})$. Here, v is a 1-periodic function, and λ is a positive constant. This operator is a model used to study a Bloch electron in a constant electric field. The parameter λ is proportional to the value of the electric field. The spectrum of H is absolutely continuous and fills the real axis.

The operator attracted attention as of physicists so of mathematicians after the discovery of the Stark–Wannier ladders. These are λ-periodic sequences of resonances, i.e., of the poles arising in course of analytic continuation of the resolvent across the spectrum, see review [1]. A series of papers was devoted to the asymptotics of the ladders in the case of small λ, i.e., of the small electric field, see, for example, [2]. The complexity of the problem is related to the fact that, in this case, there are ladders exponentially close to the real axis. Actually, only the case of finite gap potentials v was understood relatively well: for these potentials, there is only a finite number of ladders that are close to the real axis. It appeared that the ladders non-trivially "interact" as λ changes, and physicists conjectured that the behavior of the resonances strongly depends on the arithmetic nature of λ, see, for example, [3].

We assume that $v(x) = 2\cos(2\pi x)$ and describe the asymptotics of the reflection coefficient $R(E)$ in the lower half-plane of the complex plane of the spectral parameter E as $\operatorname{Im} E \to -\infty$. The resonances are poles of the reflection coefficient. Our main result is that, roughly, the leading term of the asymptotics of $1/R(E)$ is the regularized cubic exponential sum of the form

$$\sum_{n\geq 1} \exp\left(-2\pi i\omega n^3 + 2\pi n i E/\lambda - 2n\left(\ln(2\pi n) - 1\right)\right), \quad \omega = \left\{\tfrac{\pi^2}{3\lambda}\right\},$$

where $\{x\}$ denotes the fractional part of $x \in \mathbb{R}$. The cubic exponential sums $\sum_{n=1}^{N} e^{-2\pi i\omega n^3}$ are well known objects of the analytic number theory, see, for example, [4]. They were extensively studied for large N and were proved to depend strongly on the arithmetic nature of ω. This appears to be true in our case too. As a first step in the asymptotic study of the resonances, we describe their behavior far from the real axis for rational ω.

Note that our analysis strongly relies on the quasiclassical asymptotic methods and on the analytic theory of difference equations on the complex plane.

The talk is based on a joint work with Frederic Klopp, Université de Pierre et Marie Curie.

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Axisymmetric vibrations of the semiinfinite cylindrical shell partially submerged into the liquid

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The mathematical model and exact analytical solution of the forced axisymmetric vibrations of semiinfinite cylindrical shell partially submerged into the liquid is constructed. The protruding part of the shell is infinite. The source of the vibrations in the system shell – liquid is incendiary wave propagating from the infinite side of the shell. The joint oscillations of the shell and ideal compressible liquid are considered. The influence of parameters of the system on vibrational and acoustical fields is analyzed. The energy flux reflected from the place of submerging of the shell into the liquid is considered.

A resonance mechanism of earthquakes

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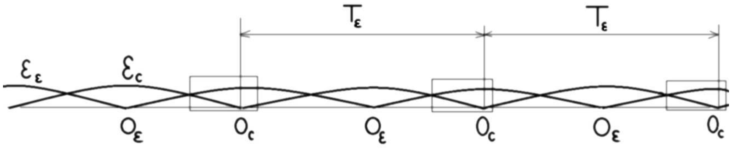
Beating of spectral modes on tectonic plates was discussed recently in [1]. We guess that the manifestation of the phenomenon may bring new understanding to the study of seismo-gravitational oscillations (SGO) and pulsations (SGP) of tectonic plates, see [5], which are non-randomly (95%) registered prior to powerful earthquakes. Using the bi-harmonic model for relatively thin tectonic plates, we interprete SGO as spectral modes of the plates [8]. The SGO dynamics is stable under stretching tension, see [7], but may be unstable, [2], under contraction caused by additional pressure on localized active zones, due to arising new low frequency spectral modes localized on the zones, which may be in resonance with the modes of the complement on the tectonic plate or other active zones. Alternation of the modes in the perturbed dynamics similar to one of the Wilberforce pendulum, [9], was clearly registered as an energy migration between the corresponding unperturbed modes, prior to powerful earthquake (EQ) 26 September 2005 in Peru. Mathematical modeling of the alternation process is suggested in [3], based on zero-range approach [6], modified with regard of dissipation of energy, see [4], resulted the below diagram of the energy beating on the active zone and on the complement C. Small rectangles mark time intervals with maximal energy on the active zone. The total energy is conserved. The suggested above interpretation of pulsations may help to develop a new algorithm of short-time prediction of powerful earthquakes based on monitoring of the pulses.



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On two-parameter boundary value problem for the Schrödinger operator with fast oscillating and delta-like potentials

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We consider the following boundary value problem:

$$
\begin{gathered}
-\frac{\mathrm{d}^2 u_{\mu,\varepsilon}}{\mathrm{d}x^2} + \left(q\left(x, \frac{x}{\mu}\right) + \varepsilon^{-1} Q\left(\frac{x}{\varepsilon}\right)\right) u_{\mu,\varepsilon} = f(x), \quad x \in (a,b), \\
l_a u_{\mu,\varepsilon} := h_a u_{\mu,\varepsilon}(a) - H_a u'_{\mu,\varepsilon}(a) = 0, \qquad l_b u_{\mu,\varepsilon} := h_b u_{\mu,\varepsilon}(b) + H_b u'_{\mu,\varepsilon}(b) = 0,
\end{gathered} \tag{1}
$$

where $q(x,\zeta)$ is a 1-periodic on ζ function from $C^{2,0}([a,b]\times(-\infty,\infty))$, $f(x)\in L_2(a,b)$, $a<0<b$, $Q(\xi)\in C_0(-\infty,\infty)$, $0<\mu,\varepsilon\ll 1$, $q(x,\zeta)>0$, $Q(\xi)\geq 0$, h_a, h_b, H_a, $H_b\geq 0$, $h_a+H_a>0$, $h_b+H_b>0$.

We define

$$\langle Q\rangle=\int_{-\infty}^{\infty}Q(\xi)\,\mathrm{d}\xi,\qquad [q]\,(x)=\int_0^1 q(x,\zeta)\mathrm{d}\zeta.$$

Combination of the method of matching asymptotic expansions [1] and the homogenization method [2] allowed to prove the following statement.

Theorem. *Let $u_{\mu,\varepsilon}(x)$ is a solution of the boundary value problem* (1). *Then*

$$\|u_{\mu,\varepsilon}-U\|_{L_2(a,b)}\leq(\mu+\varepsilon)C\|f\|_{L_2(a,b)},$$

where $U(x)$ is the solution of the boundary value problem

$$-\frac{\mathrm{d}^2U}{\mathrm{d}x^2}+[q](x)U=f(x),\quad x\in(a,b)\backslash\{0\},\qquad l_aU=l_bU=0,$$
$$U'(+0)-U'(-0)=\langle Q\rangle\,U(0).$$

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Interaction of relativistic electron beam focused by constant magnetic field with microwave field of periodic overmode waveguide

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In the present work we considered a diffraction problem of proper radiation of relativistic tubular electron beam modulated on reference frequency ω_0 on periodic obstacles of axisymmetric slow-wave structure (SWS). In the model of beam we assumed also that it is focused by magnetic field of finite value, so the transversal interaction is governed by gyrocyclotron frequency ω_c. For consideration of nonstationary nonlinear selfconsistent problem and processes in time iteration we applied Matrix Multiwave Method [1] which is based on eigenmodes of axially symmetric periodic structures. We consider only axisymmetric modes of E_{0n}-type. For numerical simulation we adopted parameters of experimental sample of Multiwave Cherenkov Generator(MVCHG) operating near π-type high-frequency cutoff. Here, taking into account the cyclotron gyration of beam particles, one can achieve the synchronization of cyclotron gyration of beam with the field spatial harmonics. The Interaction between cyclotron gyration of beam and "+1" spatial harmonics makes the main contribution to output power. The condition for cyclotron synchronization can be expressed as $\frac{\omega_0}{\mathbf{v_0}}\mathbf{d}+\frac{\omega_\mathbf{c}}{\mathbf{v_0}}\mathbf{d}\approx\mathbf{3}\pi$, where $\mathbf{d}$ is period of structure, $\mathbf{v_0}$ is longitudinal component of electron velocity in beam. Thus gyroresonance frequency in MVCHG is some twice as high as reference frequency. Slightly above this giroresonance close to double reference frequency we proved and studied cyclotron absorption caused by energy transfer from SWS field to cyclotron wave. For some range of cyclotron frequency the electromagnetic field profiles, trajectories of electrons in the beam, mode structure and radiation frequency spectrum were investigated. Some results of the numerical experiments will be presented at the conference.

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Radiation from the open-ended cylindrical waveguide with dielectric filling

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Terahertz radiation (with frequencies 0.1–10 THz) is a promising tool for a number of scientific and practical applications. One promising scheme to obtain powerful and efficient THz emission is usage of beam-driven dielectric loaded structures [1]. Recently we have considered the problem where a waveguide mode with a high number is transformed at the open end of a cylindrical waveguide with a dielectric layer (it is supposed that the waveguide end can be not orthogonal to the structure axis) [2]. Such high-order mode can be generated by a microbunched ultrarelativistic charge exiting the waveguide. For calculation of the field produced by the aperture of the waveguide, we utilized Stratton–Chu formulas, while the field at the external aperture calculated using Kirchhoff approximation and Fresnel coefficients formalism. Now, to investigate the applicability of approximations utilized for solving this problem, we consider the rigorous formulation of the problem where single waveguide mode is transformed at the orthogonal open end of this structure. Note, that a method is known for solution of corresponding problem in plane geometry [3]. This method is a combination of the Wiener–Hopf technique and tailoring technique which leads to the infinite linear system for magnitudes of reflected waveguide modes. This system can be solved numerically using the reduction technique. In this paper, we apply mentioned technique to the case of cylindrical geometry.

First, we deal with the structure without vacuum channel. We deduce the infinite linear system for magnitudes of reflected modes and discuss the limiting process to the case of a waveguide without dielectric where this system can be solved analytically and the obtained solution can be compared with known result [4]. For the structure with dielectric, we present typical dependencies for reflecting coefficients, typical patterns describing the field distribution over the aperture of the structure and discuss applicability of approximations used in [2].

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Superluminal non-diffracting pulses applied to FSO systems: theoretical description

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In recent years, some authors have focused on improving the free-space optical (FSO) systems' performance by using the so-called Nondiffracting Waves (NW) [1]. These waves have the ability of self-reconstruction, i.e., after being partially obstructed, they can rebuild their spatial form and continue propagating. Owing to this property, the NWs would be an interesting alternative to mitigate some undesirable effects in FSO due to adverse atmospheric conditions (rain, fog, pollution, etc).

By using a new and unusual approximation, but very effective, to the ordinary zero order Bessel function, we develop an analytic approach capable of providing superluminal ideal nondiffracting pulses (INPs) from suitable frequency spectra, which are generally used in the field of optical communication. Using OOK-AMI modulation, we verify that superluminal INPs would be suitable for data signal transmission without distortion, i.e. without intersymbol interference (ISI), over long distances. So far, this verification has only been made by means of numerical analysis [2].

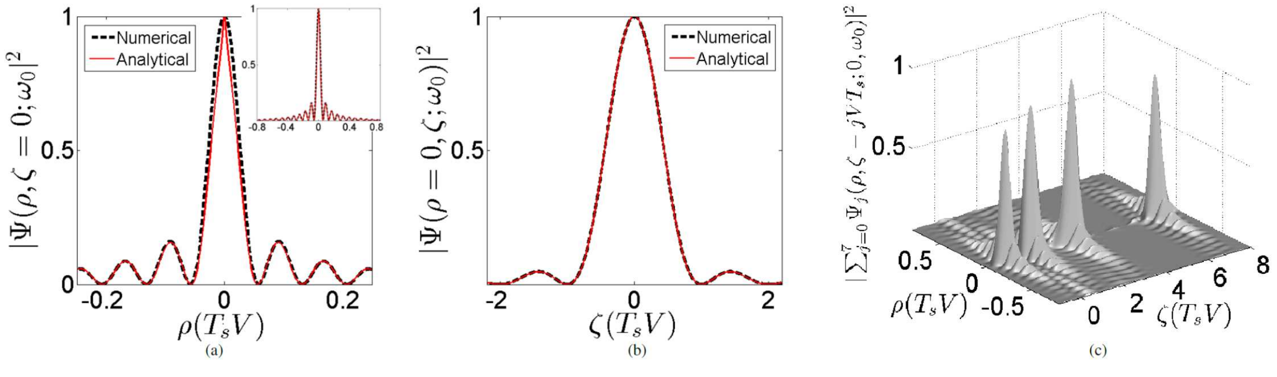

Fig. 1: Normalized intensity of the superluminal ($V = 1.2c$) INP for $\zeta = z - Vt = 0$ in **(a)** and for $\rho = 0$ in **(b)**. This solution was obtained from a square spectrum centered on a carrier frequency of 477.5 THz ($\omega_0 = 3 \times 10^{15}$ rad/s or $\lambda = 629$ nm) with a spectral symbol length $W_s = 0.1\omega_0$. $T_s = 2\pi/W_s$ is the sampling time. The red continuous lines represent results obtained from our analytical method, while the black dotted ones represent results of the numerical simulation. The modulation of this pulse through the OOK-AMI technique encoded for the transmission of one bit per symbol is shown in **(c)**, which is a sample 8-bit stream.

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Scale-invariant initial value problems with applications to the dynamical theory of stress-induced phase transformations

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We deal with the 1D non-stationary wave propagation in an elastic phase-transforming bar. The stress in the bar σ is assumed to be a piecewise linear function of the strain ε containing a "negative slope segment" (Fig. 1), thus the strain energy is a non-convex function of the strain. It is known that the problem of elastostatics for such a kind of material can have solutions with discontinuous deformation gradients. In the framework of the model of stress-induced phase transitions, the surfaces of the strain discontinuity are considered as the phase boundaries, and the domains of continuity are considered as zones occupied by different phases of the material. The solution of both statical and dynamical problems are generally non-unique; therefore, an additional thermodynamic boundary condition at the phase boundary is required.

The comparative analysis for two types of problems is under consideration. The first problem is concerned with a new phase nucleation in a phase-transforming bar caused by a collision of two non-stationary waves [1, 2]. The second one is a new phase nucleation caused by an impact loading applied at the end of a semi-infinite phase-transforming bar [3]. Both of the problem can be formulated as a scale-invariant initial value problem with additional restrictions in the form of several inequalities involving the problem parameters. The aim of the investigation is to determine the domains of existence of the solution in the parameter space.

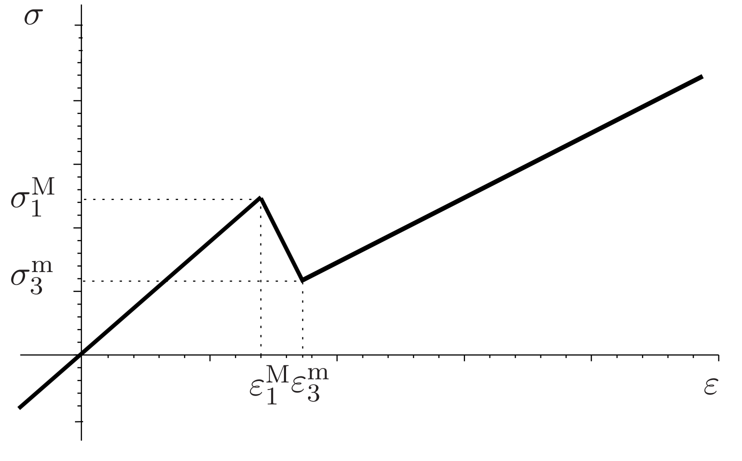

Fig. 1: The stress-strain curve for the material of a phase-transforming bar.

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Opportunity of reception of the information on an acoustic speech signal on the reflected laser radiation

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It is known, that there is an opportunity of reception of the speech information on the reflected laser radiation from a windowpane [1, 2]. But the decision of a problem on definition of registered information quality is actual. The principle of action of the optico-acoustic equipment is based on fact that it is present three kinds of modulation optical radiation at the bunches of laser radiation reflected from glass: 1) the frequency modulation caused by Doppler Effect, produced by oscillatory movements of a windowpane under influence of acoustic signals; 2) the phase modulation caused by presence in the reflected signal as mirror reflected radiation, and diffraction components; 3) the amplitude modulation caused by fluctuations of the highlighting bunch concerning a direction of mirror (maximal) reflection. These fluctuations are caused by influence of an acoustic signal on glass. All three kinds of modulation can be used for extraction acoustic speech information from the scattered laser radiation. However it is connected with the greater technical complexities caused by that amplitudes bending vibration of glass are too small: from several nanometres up to several hundreds nanometers for various intensity and frequency of an acoustic signal.

In work the mathematical model of reception of the information on an acoustic speech signal on laser location channel is considered. The theoretical analysis of all effects of modulation of a probing optical signal by fluctuations of a windowpane is lead.

Theoretical parities for indexes of peak, phase and frequency modulation, and also the attitude a signal/noise on an input of the photoreception module laser location devices are received. The lead analysis has allowed to receive expression for verbal legibility of the speech message received on laser location channel.

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The interaction of sound and ultrasound waves with multilayered elastic plates immersed in acoustic fluid

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Wave interaction with elastic plates immersed in acoustic fluid (water or air) is a classical problem of structural acoustics. It relates to numerous technical applications ranging from the vibration isolation and noise control to the acoustic stealth of submarines. An important motivation for the present research is a study of wave processes underlying the work of acoustic microscopes and ultrasonic air-coupled transducers. It is assumed that the wave fields are generated by a point source located in the acoustic half-space or by a point force applied to the plate. The traveling waves, excited in the plate, re-emit the wave energy into the environment in the course of their propagation, operating as moving sources. The study is focused on the source energy distribution among the reflected and transmitted spherical acoustic waves propagating in the fluid and cylindrical guided waves excited in the plate. Besides that, the directional diagrams of reflected and transmitted waves obtained using both integral transform technique and ray representations are also analyzed.

To estimate the range of applicability of different theoretical approaches, a hierarchy of mathematical and computer models have been developed. The models differ in the governing equations for the sounded elastic laminate structure (from a simplified Kirchhoff–Love plate to a multilayered anisotropic composite plate obeying the full set of 3D Navier–Lame equations in each sublayer). In most, the models are based on the explicit integral representations for the generated and scattered wave fields that have been derived using the Fourier transform technique. In the far-field, the acoustic and guided waves are described by asymptotic representations obtained from those integrals using the stationary phase method and the residual technique. The ray technique and the incident plane wave formalism have been also implemented for comparisons. The propagation of transient wave packets is visualized via additional integration of the time-harmonic solutions in the frequency domain.

Validating comparisons against known results of other authors have exhibited good coincidence. Numerical results obtained within different models show the influence of plate structure and material properties on the energy distribution, reflection and transmission coefficients and directional diagrams.

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Electromagnetic wave propagation along a thin wire over an arbitrary isotropic interface

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The problem of a transmission line formed by an infinitely long and thin metal wire over a realistic ground surface ("ground return effect") has been studied for many decades [1]. Solution of such a problem means determination of a complex propagation constant of a travelling wave, which is localized between the wire and the ground. However analytical solutions were obtained only in a number of particular cases (e.g. a boundary between two dielectric half-spaces [2]).

In this work we introduce a new general approach, which makes it possible to determine the propagation constant of a thin infinite wire over an arbitrary isotropic interface (Fig. 1). The interface is characterized by its reflection coefficients R^{TE} and R^{TM} with respect to incident TE- and TM-polarized plane waves that are assumed to be known as functions of the incidence angle. Then the propagation constant can be found as a root of the general characteristic equation, which was derived based on the Exact Image Theory [3] and the Integral Equation technique. We derive the characteristic equation and find its asymptotic solutions in order to compare with particular cases studied in the literature. The only restriction of the present approach is isotropy of the interface, i.e. the reflection coefficients do not depend on the incidence plane orientation, and the cross-polarization coefficient is zero. Therefore the approach is attractive for design of low-frequency wire antennas located near an arbitrary realistic ground surface, for which the reflection coefficients can be easily measured.

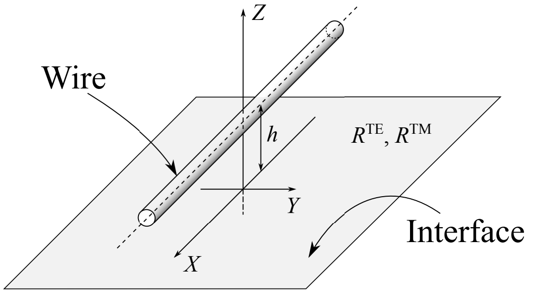

Fig. 1: Geometry of the problem: a section of an infinite thin wire over the interface at $z = 0$.

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Diffraction of ultrashort pulse on a nanoscale conductive cone

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Generic spacial diffraction limits for the laser field localization can be overcome with the help of the secondary near field radiation. The effect of amplification of a field near a cone tip depends on the field polarization. It is optimal for TM waves, focused on the base of a cone.

The main difference from the classical diffraction problem is connected with the fact that usual assumption a thin skin-effect is not fulfilled for nano-needles. In addition, we take into account the real experimental frequency dependences for the complex-valued dielectric constants of metals [1] using adequate analytical approximations [2]. The analytical problem solution for the propagation and self-focusing of surface plasmon-polaritons by a tapered metallic needles have been obtained in the frame of improved approach for the solution of Maxwell equations [3]. The small opening angle approximation allows obtain analytical solution of the problem in terms of Macdonald functions and Bessel functions with imaginary indexes [4].

The results of numerical simulations for femtosecond pulse propagation along a silver nano-needle are discussed. It is shown, that the space distribution of a near field strongly depends on a linear chirp of the laser pulse which can partially compensate the wave dispersion. Field amplification has been calculated for different chirp values, opening angle $\alpha = 0.01$, distances $r_i = 1000$ nm and $r_f = 250$ nm. For a pulse with a chirp $\beta = -0.03$ fs a duration became shorter and has been changed from 31.2 fs to 18.8 fs, the coefficient of amplification was equal 38.

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Oscillations in some gene networks models

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In order to prepare a theoretical basis for numerical simulations of gene networks functioning oscillating modes, we describe geometrical and topological properties of phase portraits of some low-dimensional piecewise linear dynamical systems considered as hypothetical models of some biological processes. In these portraits, we construct damping oscillating trajectories and invariant piecewise linear submanifolds containing these trajectories and cycles.

The mathematical results and approaches of this and our previous publications are used in natural gene networks numerical modeling in the frame of long-term cooperation with Institute of Cytology and Genetics of SB RAS, see for example [5, 7]. Some results of numerical experiments with nonlinear dynamical systems analogous to the considered one are described there and in [1, 3, 4, 6].

A large collection of graphic representations of most of these results is contained in https://github.com/AndreyAkinshin/DynamicalSystemsPortraits. See also http://ppanalyzer.ru/

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Weak formulation of energy conservation for diffraction by lossy bi-periodic gratings

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Consider the general case of vector diffraction by an arbitrary crossed grating with periods d_1 and d_2 directed, in general, non-orthogonally. Let a time-harmonic (with time dependence e^{-iwt}) electromagnetic plane wave incident above (+) on a bi-periodic lossy, in general, structure G bounded in

$\mathbb{R}^3$ and separated by two homogeneous half-spaces $G_+ := \{x_3 \geq H\}$ and $G_- := \{x_3 \leq -H\}$, $H > 0$ in Cartesian coordinates $(x_1, x_2, x_3) = \mathbf{x} \in \mathbb{R}^3$. We assume constant electric permittivity $\epsilon_\pm$ and constant magnetic permeability $\mu_\pm$ such that $\operatorname{Re}\epsilon_+ \wedge \operatorname{Re}\mu_+ > 0$, $\operatorname{Im}\epsilon_+ \wedge \operatorname{Im}\mu_+ = 0$, $\operatorname{Im}\epsilon_- \wedge \operatorname{Im}\mu_- \geq 0$, $|k_\pm| = \omega\sqrt{\epsilon_\pm}\sqrt{\mu_\pm}$, $\epsilon_\pm \neq 0 \wedge \vee \mu_\pm \neq 0$, where w is a fixed positive frequency. Otherwise, the permittivity $\epsilon(x)$ and permeability $\mu(x)$ functions of the grating region G are given by nonsingular 3×3 matrices with doubly periodic, complex-valued L^∞ (bounded) components. In physics, these components are usually piecewise continuous or piecewise constant functions corresponding to material refractive indices. Thus, we allow rather general bi-periodic anisotropic structures including edges, corners, intersected boundaries, inclusions and metamaterials. As it is usual in the treatment of periodic problems, we restrict the consideration to one unit-cell $\Omega := \{\mathbf{x} \in Q \times \mathbb{R} : |x_3| < H\}$ for one bi-period $Q := [0, d_1) \times [0, d_2)$ and uniform regions $\Omega_\pm$ above and below Ω such that $\Omega_+ := \{\mathbf{x} \in Q \times \mathbb{R} : x_3 \geq H\}$ and $\Omega_- := \{\mathbf{x} \in Q \times \mathbb{R} : x_3 \leq -H\}$. The incident electric plane wave $\mathbf{E}^i$ is specified by $\mathbf{p}e^{\mathrm{i}(\alpha_1 x_1 + \alpha_2 x_2 - \alpha_3 x_3)}$ with $\alpha_3 = |k_+| \sin\phi > 0, |\alpha|^2 = |k_+|^2$ and $\alpha \cdot \mathbf{p} = 0$. The total electric fields are given by $\mathbf{E}^i + \mathbf{E}_+$ in G_+ and $\mathbf{E}_-$ in G_- and also satisfy quasiperiodicity, outgoing wave conditions in the sense of Rayleigh series and boundary conditions in the form of transmission conditions on upper and lower boundaries $\partial\Omega_+$ and $\partial\Omega_-$. In the following we need vector fields $\mathbf{E}_\pm$, $\mathbf{H}_\pm$ of locally finite energy satisfying the time-harmonic Maxwell equations (the solvability of such a problem is discussed in [1, 2])

$$\nabla \times \nabla \times \mathbf{E} - k_\pm^2 \mathbf{E} = 0, \quad \mathbf{H} = -\mathrm{i}(w\mu_+)^{-1} \nabla \times \mathbf{E} \text{ in } G_\pm. \tag{1}$$

The variational (weak) formulation follows the classical lines and is based on the construction of a weighted residual of the so called vectorial Helmholtz propagation equation introduced in (1), left for $\mathbf{E}$. This equation is multiplied scalarly by the complex conjugate of a weighted vector $\mathbf{E}'$ chosen in the space of bi-quasiperiodic functions of $L_2(\mathbf{curl})$ in Ω. Then, integrating by parts and making the use of the Green–Ostrogradsky theorem lead to

$$\int_\Omega \nabla \times \nabla \times \mathbf{E} \cdot \overline{\mathbf{E}}' - k_\pm^2 \mathbf{E} \cdot \overline{\mathbf{E}}' dv - \int_{\partial\Omega} (\mathbf{n} \times \nabla \times \mathbf{E}) \cdot \overline{\mathbf{E}}' ds, \tag{2}$$

where $\mathbf{n}$ refers to the exterior unit vector normal to the surface $\partial\Omega$ enclosing Ω. The first term of (2) concerns the volume behavior of the unknown vector field, whereas the right-hand term can be used to set boundary conditions in the explicit form. The similar can be applied to the Helmholtz equation for $\mathbf{H}$. The values of the field on $\partial\Omega_+$ and $\partial\Omega_-$ give valuable information on the absorbing power [3]. After simple manipulations using quasiperiodicity, transmission boundary and outgoing wave conditions one can derive the energy balance using some normalization

$$A + \sum_{(n,m)} e_{n,m}^+ + \sum_{(n,m)} e_{n,m}^- = 1, \tag{3}$$

where $e_{n,m}^\pm$ are efficiencies of reflection $(+)$ and transmission $(-)$, if exist, propagating modes (n, m), $n \wedge m \in \mathbb{Z}$ which can be deduced from the Pointing theorem. A is called the absorption in the given diffraction problem and it can be calculated as a difference of two integrals using field values on $\partial\Omega_\pm$:

$$A = 0.5 \operatorname{Re}\left[\int_{\partial\Omega_+} (\mathbf{E}_+ \times \overline{\mathbf{H}}_+) \cdot \mathbf{n}\, ds - \int_{\partial\Omega_-} (\mathbf{E}_- \times \overline{\mathbf{H}}_-) \cdot \mathbf{n}\, ds\right] \Big/ \left(0.5 d_1 d_2 \sin^{-1}\phi \sqrt{\epsilon_+\mu_+/\epsilon_v\mu_v}\right), \tag{4}$$

where ϵ_v, μ_v are vacuum constants and the losses is normalized to the time-averaged incident power flux across Q that calculates explicitly.

A generalization of the energy balance (3)–(4) presented for a general bi-periodic absorbing grating is based on computations of quantity A by the respective absorption integrals using values of the field on upper and lower boundaries. It has not only intuitive significance but the same rigor, namely in the sense of generalized functions (distributions). A derivation of expressions for finding the absorption as well as the interpretation of the results obtained bear only on the Green–Ostrogradsky theorem and respective boundary and outgoing wave conditions. The computation of A itself is not connected with a specific rigorous method which is used for near-zone field calculus. Thus, the present energy balance generalization for absorbing gratings can be considered as much universal and useful as well known energy conservation laws for perfectly conducting and lossless gratings.

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Study of elastic wave propagation in multilayered structures with uniform cross sections by the reflection coefficients method

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Study of elastic wave propagation in multilayered structures with uniform cross sections by the reflection coefficients method.

A new method is suggested for the simplification of analytical calculation of the axisymmetric elastic wave propagation in systems of several infinite coaxial cylindrical shells with various filling of gaps. The reflection coefficients similar to those known in the theory of transmission lines are introduced. Four coefficients are required at each cylindrical boundary.

A conical wave expansion of waves excited by a force exerted on the ring-shaped area within a central pipe wall is used (the spatial Fourier transform along the axial coordinate).

As an example of application of the method two similar multi-pipe coaxial structures with various filling of an external gap (water or cement) were numerically simulated. Other gaps are filled with water in both structures.

The differences between the received signals in both variants are shown. Those differences are higher in case when an excitation source and detector, both located inside the central pipe, are placed at some distance.

Mode transformation in circular waveguide with transversal boundary between vacuum and partially dielectric area

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The promising method in accelerator physics is wakefield acceleration in dielectric structures [1]. One of difficulties of this technique is that available dielectric structures are rather short, and a future wakefield accelerator should have a lot of dielectric sections. Moreover, facilities based on dielectric waveguide structures are considered as promising candidates for generation of terahertz radiation [2]. In any case, we should deal with cylindrical structures with partial dielectric filling. Thus, the problems of interaction of the fields with transversal boundaries in such structures are of essential importance.

We consider a circular waveguide with a vacuum part ($z < 0$) and a part having a vacuum channel and a dielectric layer ($z > 0$). The problem is considered for the axially symmetrical incident mode in two cases: (1) the mode propagates from the vacuum part of waveguide to the part with dielectric, and (2) the case of opposite propagation of the incident mode. Analysis is carried out by expanding the reflected (r) and transmitted (t) fields in series of waveguide modes [3]. We perform analytical investigation and construct numerical algorithm to analyze mode transformation. Furthermore, some approximate results are obtained in two specific cases: thin vacuum channel and thin dielectric layer.

Figure 1 presents the example of computation of reflection $R_{zj} = E_{zj}^{(r)}/E_z^{(i)}\big|_{\rho\to 0,\, z=0}$ and transmission $T_{zj} = E_{zj}^{(t)}/E_z^{(i)}\big|_{\rho\to 0,\, z=0}$ coefficients in the case when the mode propagates from the vacuum part of waveguide. As one can see, for relatively small channel radius transmitted field has two propagating modes. With increase in channel radius only one propagating mode is excited.

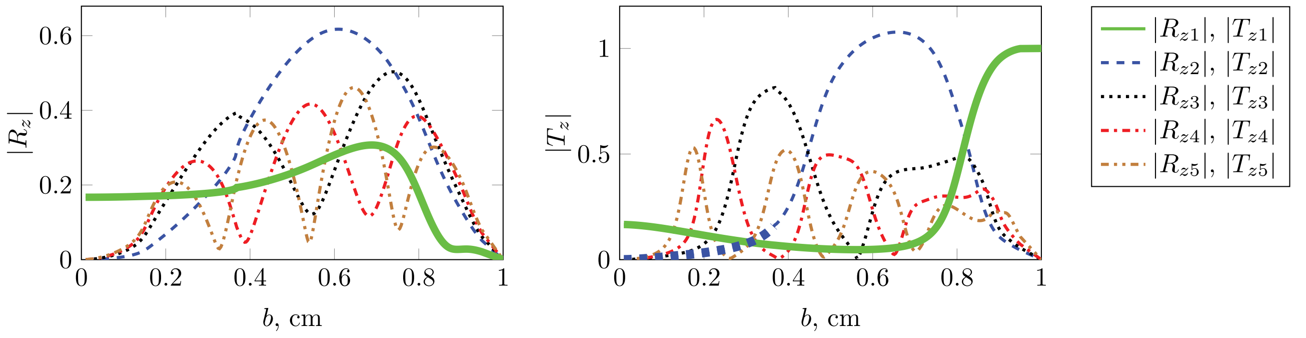

Fig. 1: Dependence of $|R_z|$ (left) and $|T_z|$ (right) on channel radius b in the case of incidence of the 1st propagating mode; dielectric permittivity is equal to 7, waveguide radius is 1 cm, the frequency is 13 GHz, phase velocity of initial mode is equal to 2 c. Thick line — propagating modes, thin line — evanescent modes.

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Nonlinear sound beam propagation in the porous viscoelastic medium

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The high intensity acoustic wave propagation in structurally inhomogeneous viscoelastic medium containing highly compressible inclusion — gas cavities — is investigated. The system of equations describing the self-consistent acoustic field in such medium is suggested. Nonlinear effects associated with the response of strongly compressible inclusions are taken into account. It is shown that a strong contrast of inclusions and medium compressibilities leads to a significant increase of the effective nonlinear parameter. The simplified equation for high intensity bounded acoustic beams in such media is derived. There is suggested the method of constructing the analytical solution for the field of nonlinear beams which is suitable in the focal region and adequately take into account diffraction effects. Analytical solutions for the field along the beam axis and the transverse structure of the field is obtained.

The adiabatic regime of the asymmetric diffraction of atoms in the field of a standing wave

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A model of asymmetric coherent diffraction of two-state atoms interacting with a standing-wave field of laser radiation taking place at large detuning and adiabatic course of the interaction is discussed. The asymmetry is caused by initial atomic wave-packet splitting in the momentum space. We show that the same form of the initial wave-packet splitting may lead to different, in general, diffraction patterns for opposite, adiabatic and resonant, regimes of the standing-wave scattering. We show that the scattering of the Gaussian wave packet in the adiabatic case presents refraction (a limiting form of the asymmetric scattering) in contrast to the bi-refringence (the limiting case of the high-order narrowed scattering) occurring in the resonant scattering.

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Numerical simulations of electron transport in nanowires near the thresholds

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In the recent times, the study and design of electronic components, based on ballistic electron transport in nanowires, show steep increase of interest. The components can be field effect transistors, resonant tunneling [1] diodes [2], lasers [3], cubits [4], etc. Previously we carried out numerical simulations of electron transport for energies far from the thresholds (in the case of multichannel scattering).

Here we present the results of numerical simulations of electron transport in nanowires near a threshold and compare the conventional and augmented scattering matrices elements (see [5] for the theory).

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Fiction asymptotics and justification theorems

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In the report we provide examples of the problems with a small parameter, for which an approximate (asymptotic) solution gives approximation to none of exact solutions [1]. By asymptotic solution we mean a function satisfying all the equations in the problem with a high degree of accuracy. As such functions, in the asymptotic constructions one usually takes the partial sums of the series being the formal asymptotic solution w.r.t. the powers of the small parameter. Sometimes the series is called the asymptotic solution. The coefficients of the series appears from the series of simpler (approximate) equations being more treatable than the original problems. Such simplification, whose realization looks rather like an art [2]. is the main advantage of the asymptotic approach. However, it turns out to be an awkward situation if one does not control the error term of the asymptotics. An example of such situation for a particular problem was analyzed in detail in work [3].

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Confluent Heun equation with 2 added apparent singularities

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Confluent Heun equation with 2 added apparent singular points has has a rich set of symmetries. These symmetries connect solutions of this equation with different parameters. Elementary symmetries can be introduced by s-homotopic transforms of solutions. Gauge and integral symmetries (Euler integral symmetry, Laplace integral symmetry, hypergeometric integral symmetry) can be constructed using generating systems — linear systems, whose reduction produces confluent Heun equation with 2 added apparent singularities. All these symmetries can be expressed in terms of the monodromy group.

A neural network technique for reconstruction of a homogeneous dielectric layer's parameters

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An inverse problem of diffraction of an electromagnetic wave by a uniform dielectric layer is considered. Most of the methods used for solving this problem are based upon the multiple solution of a direct problem [1]. In the present work, the neural network method is used for recovery of unknown parameters of the layer. Some relevant subproblems are considered. The case with the known layer thickness is considered first; recovery of dielectric permeability is carried out based upon both the known reflected and passed waves and each wave separately. The second complex subproblem arises in case of absence of any aprioristic information. In addition, three cases are considered for known reflected and/or passed waves.

In the present work, three standard activation functions of neurons are used: linear, sigmoid and gaussian. Numerical experiments for each activation function are carried out. Conclusions are drawn on what of the functions provide the best approximation by the neural network for a concrete subproblem.

Training of the network is carried out by means of two methods: the back-propagation algorithm and the genetic algorithm. Comparison of the two methods of training of the network is carried out using various examples. The conclusion that in most cases the genetic algorithm yields the best results is drawn.

Graphs depicting dependences of accuracy of recovery of the unknown parameters on parameters of a neural network are given.

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Laplace–Gauss and Helmholtz–Gauss modes in a medium with a quadratic refraction index

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We are concerned with paraxial propagation of Gaussian-type beams in an axisymmetric medium described by

$$u_{xx} + u_{yy} + u_{zz} + \omega^2[n_0^2 - n_0 n_2(x^2 + y^2)]u = 0, \qquad (1)$$

with $n_0 > 0$ and n_2 constants. The cases of $n_2 > 0$, $n_2 < 0$ and $n_2 = 0$ are allowed. The standard paraxial procedure starting with separating out the oscillating factor $u = e^{i\omega n_0 z}U(x, y, z)$ allows the approximate parabolic equation (PE). Denote its well-known axisymmetric fundamental mode solution by $g = A(z)e^{i\omega B(z)(x^2+y^2)}$, where expressions for A and B can be found, e.g., in [1]. Seeking a solution of PE in the form

$$U = H(X, Y, z)g, \quad X = A(z)x, \quad Y = A(z)y, \qquad (2)$$

we come up with either $H_{XX} + H_{YY} = 0$, for H independent of z, or

$$H(X, Y; z) = e^{\frac{iK}{n_0}\int A^2(z)\,dz}\Psi(X, Y)\,, \qquad (3)$$

where K is a free constant and $\Psi(X,Y)$ is an arbitrary solution of the Helmholtz equation

$$\Psi_{XX} + \Psi_{YY} + K^2\Psi = 0. \tag{4}$$

We thus got a complete analog of the theory of higher-order Laplace–Gauss and Helmholtz–Gauss modes developed earlier for a homogeneous medium (that is $n_2 = 0$) in [2].

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Pattern equation method in the spheroidal coordinate basis

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Nowadays the great attention is devoted to the problems of the electromagnetic wave scattering by the scatterers with complicated shape [1]. In this paper we present the further development of the ideas of the book [2], in which a new method for solving the scattering problems — the Pattern Equation Method (PEM) — was discussed. It has been shown that this approach has important advantages over of many universal techniques and is very effective in solving a wide class of problems. Later, PEM was generalized to the case of impedance boundary conditions; it was shown that the method maintains its high efficiency in the case where the surface of the scatterer has the edges.

In [3, 4] we suggested the scalar modification of PEM, which was based on the using of special coordinate system for solving the problem of the wave scattering by bodies with extremal geometry. In this paper we present a vectorial modification of the PEM in the spheroidal basis for the numerical simulation of electromagnetic wave scattering. It should be noted that the use of vectorial spheroidal wave functions for solving scattering problems has a long history. However, in most cases, only the diffraction of electromagnetic waves by a spheroid was considered [5–7]. The review of these researches is given in the well-known book [8]. In our paper we consider the solution of the problem of scattering of a plane electromagnetic wave by a strongly elongated bodies of revolution. We investigated various examples demonstrating the effectiveness of the proposed method. It is shown that the accuracy of the calculations, which is checked by calculating the balance of power flows for the incident and scattered waves, is sufficient for practice: in all examples, the balance of power flows was fulfilled with relative accuracy not worse than 10^{-5}.

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Inertial waves and instability of the Rankine vortex with axial viscous flow

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The model of the Rankine vortex is simple: undisturbed column consist of a core of uniform vorticity (solid-body rotation), surrounded by irrotationally revolving liquid, but nonetheless in many cases this is a good approximation for real columnar vortices and vortex filaments.

Kelvin showed in his pioneering paper (1880) that the Rankine vortex is neutrally stable for small disturbances (both for axisymmetrical and spiral modes). The disturbances that do not depend on the axial coordinate have been reviewed by H. Lamb (1932). The instability develops if the axial flow in the core is imposed upon the Rankine vortex. Some aspects of the problem of temporal and spatial instability have been studied by Moore, Saffman, Uberoi et al., Lessen et al., Drazin and Reid, Loiseleux et al.

The effect of fluid viscosity on the stability of hydrodynamic flows is not trivial. On the one hand, at low undisturbed flow velocities the viscosity has often a stabilizing effect but, on the other hand, it can also be destabilizing (i.e. a cause of instability). In this report we shall restrict consideration to the effect of the viscosity of only inner fluid on the stability. Inviscid liquid occupy the external region. The surface of separation between these two liquids would be a surface of tangential discontinuity. We also take into account different densities of immiscible liquids in the inner region and in the external region. It is found exact solution represented an inertial normal mode. The absolute/convective nature of the instability and effects of varying the Kibel number and the Ekman number are investigated.

Double boundary layer in asymptotics of nonlinear problem

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We study the boundary value problem with a singular perturbation

$$\varepsilon^2 u_{xx} = f(u,x,\varepsilon), \quad 0 < x < 1, \quad 0 < \varepsilon \ll 1,$$
$$u(0) = \alpha, \quad u(1) = \beta.$$

The problem is an asymptotic expansion of the solution $u(x;\varepsilon) \approx ?$ as $\varepsilon \to 0$, uniform on the interval $0 < x < 1$.

A specific feature of the problem is the multiple root $u_0(x)$ of the limit equation:

$$f(u_0(x),x,0) \equiv 0, \quad f_u(u_0(x),x,0) \equiv 0, \ x \in [0,1].$$

The novelty in the result: there are double boundary layers near left and right boundaries as well.

A peculiarity of the method: the matching method [1] is applied instead of the boundary layer method [2].

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Scattering in the semirestricted cylinder

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We consider scattering theory for the Laplace on semicylinders. Denote (y, x) a vector in $\mathbb{R} \otimes \mathbb{T}^d$. Let $\gamma(x) \in C(\mathbb{T}^d)$ be a bounded function (without loss of the generality $|\gamma(x)| \le 1$). We define the semicylinder by

$$\mathcal{C} = \{(y, x) : y > \gamma(x)\}.$$

Let $k \in \mathbb{T}^d$. Define the operator H on $L^2(\mathcal{C})$

$$H = -\nabla_y^2 - (\nabla_x + ik)^2,$$

with the Dirichlet boundary condition on the cylinder base. We study spectral and scattering properties of the operator H. The unperturbed operator($\gamma \equiv 0$) is denoted H_0. We consider wave operators $W_{\pm}(H, H_0; J)$ for some natural embedding J. The main results are existence and completeness of wave operators $W_{\pm}(H, H_0; J)$. These results are obtained by the Enss method.

Diffraction of a plane wave with an amplitude linearly varying along its front by wedges

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Exact analytic solutions of a number of diffraction problems for a special wave incidence are obtained. Incident wave is a plane acoustic wave with the delta-function profile and linear wave distribution along its wavefront. Diffraction is considered for the next cases:

– half-infinite screen with the Dirichlet boundary conditions on it,

– half-infinite screen with the Neumann boundary conditions on it,

– half-infinite wedge with Dirichlet boundary conditions on it.

The problems are solved by Smirnoff–Sobolev method. The standard approach is modified to deal with the new type of the incident field.

Transmission of a flexural-gravitational wave through an obstacle in an elastic plate floating atop a two-layered fluid. Thin upper layer approximation

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Flexural-gravitational waves in an ideal incompressible two-layered fluid are considered. Densities in layers are different. The surface of the fluid is covered by a thin elastic plate. The upper fluid layer is assumed thin in comparison with the characteristic wavelengths. A narrow infinitely long straight-line obstacle divides the plate into two semi-infinite identical parts. The obstacles under consideration are a rigid clamp, a sliding fastening and a crack. A surface flexural-gravitational wave propagates along the interface between the fluid and the plate and is orthogonally incident upon an obstacle in the plate [1]. The transmission and reflection of this wave are studied analytically. Transmission and reflection coefficients are compared with those found earlier for the non-thin thickness of the upper fluid layer. The main goal of the paper is an approximate replacement of the two-layered fluid by a single-layered one with a special boundary condition at its upper surface. The idea of the derivation of the approximate boundary condition is borrowed from [2].

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Restrictions on pp scattering amplitude by the first diffraction minimum TOTEM LHC data

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Recently the fine experimental data on the behavior of the elastic scattering cross-section in the region of diffraction peak and first diffraction minimum was obtained by TOTEM experiment for 7 and 8 TeV pp collisions at LHC [1, 2]. The first restrictions on pp scattering amplitude imposed by this data were discussed in [3, 4]. In this work we present the systematic analysis of the problem in the framework of the dipole model [5] and quasi-eikonal Regge approach [6] taking into account the unitarity condition for the amplitude. We show that to restore the real and imaginary parts of the pp scattering amplitude in the region of the first diffraction minimum we need also the information on the elastic scattering cross-section in vicinity of the second minimum. So we model the behavior of the elastic scattering cross-section in wide range of momentum transfer at various initial energies, using the impact parameter profile function as a starting point. We discuss which further restrictions on the elastic pp-scattering amplitude can be obtained from future information on elastic cross-section in the second diffraction minimum.

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Velocity of light pulse propagation in a forbidden gap of 1D photonic crystal

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The present paper concerns Gaussian light pulses propagation in the first bandgap of a 1D photonic crystal (PC) described as a two component planar dielectric structure, which may be thought as a photonic barrier. In [1, 2] it was shown theoretically and experimentally that the tunneling time (group delay τ_d) of a narrowband pulse saturates with the increase of the PC length. Therefore, if a propagation velocity of a pulse is determined as the ratio of the PC length and τ_d, it can exceed the velocity of light. In the papers [3, 4] authors noted, that when a dielectric contrast of PC is small, τ_d is not the propagation time and PC behaves as a reactive element for the pulse envelope. Since interpretations of experimental results are controversial so far [5], we report our results illustrating a quasi-stastic nature of tunneling and also discuss the limits of applicability of the approximation of PC as a reactive element.

We obtained spatial distributions of the pulse field inside and outside PC with the help of the transfer matrix method. We demonstrated the tunneling regime when the spectrum width of the incident pulse $\Delta\omega_{inc}$ is smaller then the forbidden gap width $\Delta\omega_{br}$. PC behaves as a modulator reproducing a weakened input signal with a delay. The equivalent scheme is proposed, which consists of RC-circuits and describes "reflected" and "transmitted" radio pulses. The group delay in the equivalent scheme is determined as in PC [2], which means the ratio of the stored (reactive) energy and the input power.

The increase of the pulse duration makes $\Delta\omega_{inc}$ comparable or exceeding $\Delta\omega_{br}$, and the shape of the output pulse differs from Gaussian one since a part of spectral components is out of the forbidden gap. Then the velocity of energy transfer, which is determined as the velocity of the gravity center of radiation, acquires physical meaning and its value turns out to be sublight.

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Model of the convection induced vertical microstructure in the ocean

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We consider thermohaline convection [1] in a horizontally infinite layer of water at large Rayleigh numbers in the neighbourhood of Hopf bifurcation points. At the layer boundaries constant values of temperature and salinity are maintained, higher at the lower boundary. So that the system is in slightly supercritical regime. The starting point for the model are the equations of hydrodynamics of the liquid mixture in the gravity field, where the diffusing components are heat and salt. On the horizontal boundaries the free boundary conditions are posed. They are well suited to describe convection in the inner layers of the liquid. Also we demand for the convection amplitude to be limited throughout the region in horizontal directions.

The system of the basic equations is investigated by the multiscale expansions method. Linear stability analysis of the considered system shows, that the convection tends to have a form of travelling tall thin rolls with the width considerably less than the height. So we have a small parameter ε as the relation of the convective cell width to its height. According to multiscale expansions method we introduce slow vertical $Z = \varepsilon z$ and time $T = \varepsilon^2 t$ variables, and also the prolonged derivatives. So the dependent variables also depend on the slow variables. The solution is sought in the form of asymptotic series in powers of ε. At $O(\varepsilon^n)$ for each n different systems of equations are obtained, and then investigated separately. From the resolution conditions for the system at $O(\varepsilon^3)$ we get the system of ABC type amplitude equations [2], in which the amplitudes B and C describe the average vertical temperature and salinity fields defining variations of the water density. Also we incorporate into these equations some terms arising at $O(\varepsilon^5)$ to simulate vertical step-like structure more correctly.

$$\begin{aligned} \partial_T A &= rA + \alpha_0 \partial_Z^4 A + \alpha_1 \partial_Z^2 A - \alpha_2 A \partial_Z B + \alpha_3 A \partial_Z C + N_1(A,B,C)\,, \\ \partial_T B &= \partial_Z^2 B - \alpha_4 \partial_Z(|A|^2) + N_2(A,B,C)\,, \\ \partial_T C &= \tau \partial_Z^2 C - \tau \alpha_5 \partial_Z(|A|^2) + N_3(A,B,C)\,. \end{aligned} \tag{1}$$

Here τ is Lewis number, r, α_i, $i = 0...5$ are the complex coefficients and $N_i(A,B,C)$, $i = 1...3$ are the nonlinear operators, acting on amplitude variables $A(T,X)$, $B(T,X)$, $C(T,X)$.

We developed a numerical model for the equations (1), based on modern ETD (exponential time differencing) pseudo-spectral methods. As a rule, the initial condition is an arbitrary noise with the amplitude 10^{-4} for $A(T,X)$ to obtain spontaneously arising structures. And the boundary conditions are periodic to describe convection in the inner layers of liquid. The numerical simulation shows that for a wide range of parameters system (1) has solutions, describing a vertical fine structure with a characteristic time of formation of the order of several hours. The fine structure has a very irregular shape, and all the layer is split into 10–30 thinner layers. The vertical structure of the buoyancy frequency has peak emissions with the amplitude and width in reasonable agreement with the observed data, even for the relatively small basic gradients of the temperature and the salinity over the layer.

The obtained results can be useful for the explanation of fine structure parameters in some important oceanographical systems like thermohaline staircases.

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On bounds and non-existence in the problem of steady waves with vorticity

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For the problem describing steady gravity waves with vorticity on a two-dimensional unidirectional flow of finite depth the following results are obtained.

1. Bounds are found for the free-surface profile and for Bernoulli's constant.

2. If only one parallel shear flow exists for a given value of Bernoulli's constant, then there are no wave solutions provided the vorticity distribution is subject to a certain condition.

Applications of these results will be given. This is joint work with N. Kuznetsov (St. Petersburg, Russia) and E. Lokharu (Linköping University, Sweden).

Kirchhoff formula, its analogs and modifications

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Kirchhoff's formula [1, 2] for the d'Alembert operator plays the same role as the Green's formula for the Laplace operator. The report shows that the following identity holds:

$$p_v F(x,t) = \Box \frac{1}{4\pi} \int_V \frac{1}{r_{xx'}} F(x', t - r_{xx'}/c) dx', \tag{1}$$

where V is the area with continuously differentiable boundary S, p_v is the characteristic function of V, $\Box$ is D'Alembert operator, F is a twice continuously differentiable function of its arguments. Identity (1) becomes Poisson identity when the function does not depend on t. In general, the identity (1) is equivalent to the Kirchhoff formula:

$$\begin{aligned} p_v F(x,t) = \frac{1}{4\pi} \int_V \frac{1}{r_{xx'}} \left(\frac{1}{c^2} \frac{\partial^2}{\partial \tau^2} F(x',\tau) - \Delta' F(x',\tau) \right)_{\tau = t - r_{xx'}/c} dx' \\ + \frac{1}{4\pi} \int_S \left(\frac{1}{r_{xs}} \left(\frac{1}{c} \frac{\partial}{\partial n} F(s,\tau) + \frac{1}{c} \frac{\partial}{\partial \tau} F(s,\tau) \frac{\partial}{\partial n} r_{xs} \right) - F(s,\tau) \frac{\partial}{\partial n} \frac{1}{r_{xs}} \right)_{\tau = t - r_{xx'}/c} ds, \end{aligned} \tag{2}$$

which turns into Green's formula if F does not depend on t. Vector analog of (1) can be reduced to the following identity:

$$\begin{aligned} p_v \mathrm{A}(x,t) = \frac{1}{4\pi} \int_V \frac{1}{r_{xx'}} \left(\frac{1}{c^2} \frac{\partial^2}{\partial \tau^2} \mathrm{A}(x',\tau) - \Delta' \mathrm{A}(x',\tau) \right)_{\tau = t - r_{xx'}/c} dx' \\ + \frac{1}{4\pi} \int_S \left(\frac{1}{r_{xs}} (\mathrm{n} \operatorname{div}' \mathrm{A}(s,\tau) + [\operatorname{rot}' \mathrm{A}(s,\tau), \mathrm{n}] + \frac{1}{c} \frac{\partial}{\partial \tau} \mathrm{A}(s,\tau) \frac{\partial}{\partial n} r_{xs} + \left[\frac{1}{c} \frac{\partial}{\partial \tau} \mathrm{A}(s,\tau), [\mathrm{n}, \operatorname{grad} r_{xs}] \right] \right) \\ + A_n(s,\tau) \operatorname{grad} \frac{1}{r_{xs}} + \left[\operatorname{grad} \frac{1}{r_{xs}}, [\mathrm{A}(s,\tau), \mathrm{n}] \right] \Bigg)_{\tau = t - r_{xx'}/c} ds. \end{aligned} \tag{3}$$

Formula (3) is also equivalent to a vector analog of formula (2), but differs from it in shape and moreover formula (3) has no direct scalar counterpart. If the vector function A does not depend on t, equation (3) becomes one of the analogs of Green's formula, which follows from Maxwell's identity ([3], p. 21).

Modification of formula (3) for the case when it is determined also in the exterior region of V and it satisfies the homogeneous wave equation there, can be carried out in the same manner as the modification of the scalar Kirchhoff formula given in [4].

Such a modification of the formulas (2) and (3) allows us to give a simple rationale for representations of pairs of electromagnetic potentials than it was done in [4].

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Application of $\mathrm{ch}_{a,n}$ atomic basis to solution of scalar hyperbolic equation

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It is known that numerical solution of hyperbolic equation (1) is important in computational fluid dynamics [1]:

$$u_t + a(u)u_x = 0, \quad u(0,x) = u_0(x),\ x \in (x_1, x_2),\ t \geq 0, \tag{1}$$

where (x_1, x_2) is a segment of real line.

This report presents comparison between the numerical method which is constructed by using of interpolation of $\mathrm{ch}_{a,n}$ basis functions [2] are belong to atomic functions family [3, 4] and of the cubic B-spline quasi-interpolant [1] as well.

Spatial derivatives are approximated by using of the $\mathrm{ch}_{a,n}$ interpolant (or cubic B-spline quasi-interpolant) derivative and first order forward difference to approximate the time derivative of the dependent variable. The method for advection equation and one-dimensional Hopf equation (Burger's equation without viscosity) is verified with some numerical examples.

The advantage of the resulting scheme is that the algorithm is very simple and easy to implement.

In this report we solve hyperbolic PDEs using this approach and then compare the results. Some examples have been given at the end of study.

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Construction of filter systems with reference areas of complex geometry in the frequency and time spaces on the basis of R-functions theory

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In this report on the basis of the atomic and R-functions [1–9] the synthesis of two-dimensional filters with the finite pulse characteristic (FIR) and complex shape of reference areas in time or frequency domain is considered. The two-dimensional filters synthesized on the reference areas which geometry is essential. It affects the physical characteristics of filters and also results of signal processing. The classical methods [10] allow synthesizing the 2D filters only on the ordinary reference areas like circle and square. On the basis of the theory of R-functions it is possible to describe the reference area of any shape at an analytical level. Such approach allows constructing the two-dimensional weight functions (windows) and wavelets [9] on reference areas with non-standard geometry. The proposed algorithms allow carrying out the filter construction of both the frequency and time domains. Such filters can be widely used in the problems of the digital signal and image processing for various physical natures.

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FIR-filters on basis of atomic functions in problems of boundary value problems of diffraction and signal processing

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In work on the basis of atomic functions [1–10] the filters with finite impulse response (FIR) by means of weighing method [1, 2] and [7–12] are synthesized. One of their advantages over infinite impulse response filter is the possibility of zero phase shifts. Furthermore, the method of FIR filters synthesis eliminates the difficulty of ensuring their stability. Phase responses of obtained filters are linear. This provides the feasibility and stability of the proposed systems. Built low-frequency, high-frequency and band-pass filters which with good accuracy satisfy the required amplitude and frequency parameters. On the physical characteristics [1, 2] and [8, 9] the results are compared with the known [11, 12]. So, the possibilities of their effective application in boundary value problems, including problems of diffraction, as well as digital processing of one-dimensional and multidimensional signals are shown.

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Generalization of Kravchenko wavelets based on the family of atomic functions $\mathrm{ch}_{a,n}$

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Application of family of atomic functions $\mathrm{ch}_{a,n}$ to obtain new wavelets is discussed. Construction of orthogonal Kravchenko wavelets (also known as WA-systems) [1, 2, 3] is based on the Riesz basis and Daubechies theorems. It consists of four main steps.

At fist we define the spectra of scaling function $\hat{\varphi}(\omega)$. To produce orthogonal multiresolution analysis it must meet the following condition $\sum\limits_{n\in\mathbb{Z}} |\hat{\varphi}(\omega+2\pi n)|^2 = 1$.

For Kravchenko wavelets the square roots of scaled sums of shifts of atomic functions $\mathrm{up}(x)$, $\mathrm{fup}_n(x)$, $\mathrm{h}_a(x)$, $\Xi_n(x)$ and others were used [1, 2, 3]. Each atomic function leads to corresponding wavelet. In our work we use $\hat{\varphi}(\omega) = \sqrt{\mathrm{ch}^{sum}_{a,n}(\omega)}$, where $\mathrm{ch}^{sum}_{a,n}$ is sum of several scaled shifts of atomic function $\mathrm{ch}_{a,n}$ [4]. Family of atomic functions $\mathrm{ch}_{a,n}(x)$ is native generalization of $\mathrm{up}(x)$, $\mathrm{h}_a(x)$ and $\Xi_n(x)$. Each atomic function $\mathrm{ch}_{a,n}(x)$ is supported on $\left[-\frac{n}{a-1}, \frac{n}{a-1}\right]$ and satisfies equation

$$y^{(n)} = a^{n+1}2^{-n}\sum_{k=0}^{n} C_n^k(-1)^k y(ax+n-2k).$$

On the second step conjugation filter H_0 is presented as

$$H_0(\omega) = \begin{cases} \hat{\varphi}(2\omega), & \omega \in \left[-\frac{2\pi}{3}; \frac{2\pi}{3}\right], \\ 0, & \omega \in \left[-\pi; -\frac{2\pi}{3}\right] \cup \left[\frac{2\pi}{3}; \pi\right] \end{cases}$$

continued periodically with period 2π. It meets additional condition $|H_0(\omega)|^2 + |H_0(\omega+\pi)|^2 = 1$. The spectra of wavelet function is presented in the form

$$\hat{\psi}(\omega) = e^{\frac{i\omega}{2}} \overline{H_0\left(\frac{\omega}{2} + \pi\right)} \hat{\varphi}\left(\frac{\omega}{2}\right).$$

The last step is computation of $\varphi(t)$ and $\psi(t)$ as an inverse Fourier transform of $\hat{\varphi}(\omega)$ and $\hat{\psi}(\omega)$.

Rich family of atomic functions $\mathrm{ch}_{a,n}(x)$ allows to obtain a large variety of new wavelets which will be useful for digital signal processing, boundary value problems and other physical applications.

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Radiation from a loop antenna located on the surface of an anisotropic plasma cylinder and excited by a wideband signal

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Radiation from loop antennas in gyrotropic media has received much careful study (see, e.g., [1] and references therein). Enhanced attention has been paid to the radiation from circular loop antennas in the presence of a gyrotropic cylinder. This interest is explained by some important practical applications as well as the fact that the geometry of the problem in such a case admits using the integral equation method to find the current distribution of the antenna. Recently, the electrodynamic characteristics of a loop antenna located on the surface of a gyrotropic cylinder and excited by a time-harmonic given voltage have been considered using this method in [2, 3]. However, there exists very little theory of the corresponding antenna excited by a nonmonochromatic voltage. It is the purpose of the present work to discuss the energy radiation characteristics of such an antenna excited by a wideband signal.

We consider an antenna having the form of an infinitesimally thin, perfectly conducting narrow strip, which is coiled into a circular loop. The antenna is located coaxially on the surface of a uniform gyrotropic cylinder placed in free space and aligned with the gyrotropic axis. As the gyrotropic medium inside the cylinder, we consider the magnetoplasma described by a dielectric tensor with nonzero off-diagonal elements. The elements of the tensor are functions of frequency. The antenna current is excited by a given wideband pulsed voltage applied to a narrow gap of the strip. The main attention is paid to the case where the frequency spectrum of the excitation voltage is concentrated in the resonant frequency intervals of a magnetoplasma [1].

To evaluate the energy radiated by the loop antenna with pulsed voltage excitation, we use the Fourier transform technique and the results obtained earlier for an antenna with a time-harmonic voltage source [3]. We have studied the distribution of the radiated energy over the spatial and frequency spectra of the excited waves. It has been found that in the case where the frequency spectrum of the excitation voltage is concentrated in the resonant part of the whistler range [1, 3], the main contribution to the radiated energy is ensured by quasi-electrostatic modes that are guided by the magnetized plasma cylinder surrounded by free space under these conditions. The radiation characteristics of the considered antenna have been studied analytically and numerically, and the results of the numerical calculations will be reported for some cases of interest.

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High-throughput femtosecond laser nanofabrication: basic principles and prospective applications

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This talk provides an overview of our recent research on high throughput femtosecond laser fabrication of nanoscale surface elements, using multi-beam and high repetition rate direct writing at tight focusing conditions (high focusing numerical apertures), as well as nanoscale surface relief self-organization during its wet nanostructuring at moderate focusing conditions. Basic physical mechanisms underlying the resulting surface nanostructures and their prospective applications are discussed.

Electromagnetic TE wave propagation in a two-layered waveguide with nonlinear permittivity

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Let us consider a monochromatic TE wave $\mathbf{E}e^{-i\omega t}$, $\mathbf{H}e^{-i\omega t}$ given in the form

$$\mathbf{E} = (0, \mathrm{E}_y(x)e^{i\gamma z}, 0)^T, \quad \mathbf{H} = (\mathrm{H}_x(x)e^{i\gamma z}, 0, \mathrm{H}_z(x)e^{i\gamma z})^T, \tag{1}$$

where $\mathbf{E}$, $\mathbf{H}$ are the complex amplitudes; ω is a circular frequency; $(\,\cdot\,)^T$ is the transposition operation; γ is an unknown (real) spectral parameter (propagation constant of a guided wave); $\mathrm{E}_y, \mathrm{H}_x, \mathrm{H}_z$ are unknown functions.

Let us also consider a double-layer dielectric waveguide $\Sigma = \Sigma_1 \cup \Sigma_2$, where

$$\Sigma_1 := \{(x,y,z) \in \mathbb{R}^3 : 0 \leqslant x \leqslant h_1\}, \quad \Sigma_2 := \{(x,y,z) \in \mathbb{R}^3 : h_1 \leqslant x \leqslant h_1 + h_2\}.$$

The waveguide Σ is located in the Cartesian coordinate system $Oxyz$. The permittivity in the whole space is described by the formula

$$\varepsilon = \begin{cases} \varepsilon_s, & x < 0, \\ \varepsilon_1 + \alpha_1 |\mathbf{E}|^2, & 0 \leqslant x < h_1, \\ \varepsilon_2 + \alpha_3 |\mathbf{E}|^2, & h_1 \leqslant x \leqslant h_1 + h_2, \\ \varepsilon_c, & x > h_1 + h_2, \end{cases}$$

where $\varepsilon_s, \varepsilon_1, \varepsilon_2, \varepsilon_c, \alpha_1, \alpha_2$ are real positive constants. In addition, we assume that $\varepsilon_s \geqslant \varepsilon_0$ and $\varepsilon_c \geqslant \varepsilon_0$, where ε_0 is the permittivity of free space, and $\max\{\varepsilon_s, \varepsilon_c\} < \varepsilon_1 < \varepsilon_2$. There are no sources in the entire space. It is supposed that everywhere $\mu = \mu_0$, where μ_0 is the magnetic permeability of free space. We also consider the case in which the waveguide Σ has a perfectly conducted wall at the boundary $x = h_1 + h_2$.

We look for guided waves which propagate along the boundaries of the double-layer dielectric waveguide Σ and decay when they move off from the boundaries of the waveguide.

Complex amplitudes (1) of the TE wave must satisfy Maxwell's equations

$$\begin{cases} \operatorname{rot} \mathbf{H} = -i\omega\varepsilon\mathbf{E}, \\ \operatorname{rot} \mathbf{E} = i\omega\mu\mathbf{H}; \end{cases} \tag{2}$$

the continuity condition for the tangential components of the field at the boundaries $x = 0$, $x = h_1$, and $x = h_1 + h_2$; and the radiation condition at infinity: the electromagnetic field decays as $O(|x|^{-1})$ when $x \to \infty$. The solution is sought for in the entire space. In the case in which the waveguide has a perfectly conducted wall, tangential components of the TE wave must vanish at the wall.

Problem $P_E(\alpha_1, \alpha_2)$ *is to determine eigenvalues* $\widehat{\gamma}$ *for which there exists a nontrivial field* $\mathbf{E}e^{-i\omega t}$, $\mathbf{H}e^{-i\omega t}$ *with components* (1) *that satisfies equation* (2) *and aforementioned conditions.*

An analytical approach and numerical method to study the problem are suggested. Numerical results are presented, comparison with the linear cases is given. Results for similar problems see in [1, 2, 3].

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Gyrotropic plasmonic slot waveguides

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One of the most discussed topics in modern optics is the manipulation of light at the nanoscale. Tremendous decrease in size of optical systems and waveguides can be achieved with excitation of surface plasmon polaritons, or plasmon modes. Such phenomena taking place in metal-insulator structures which can be of use in wide range of applications, for example supersensitive optical sensors or quantum optical devices. Strong confinement of light in a small dielectric gap between two metal layers serves as plasmonic slot waveguide that allow to concentrate optical field inside dielectric core of metal-insulator-metal nanocavity. It also significantly increases propagation distance of SPP mode due to coupling plasmons at the nearby interfaces in comparison with a two layer metal-insulator waveguide.

In this work we introduce a novel metal-MO insulator-metal structure. Its main part consists of buried Ag layer and Au layer, which constitutes a nanocavity filled with two layers: rare earth iron garnet ($Bi_2Dy_1Fe_4Ga_1O_{12}$) and Al_2O_3. The considered slot waveguide can support different types of eigenmodes: forward and backward (with the group velocity opposite to the phase velocity) waveguide modes and surface plasmon polaritons. While the electromagnetic field of the former is mostly concentrated in the bulk of the nanocavity, the optical field of the plasmonic modes is concentrated at the metal interfaces. The asymmetric dielectric filling of the nanocavity was chosen deliberately. Presence of such asymmetry is necessary to have odd in magnetization magneto-optical effect in transversal magnetization. The transversal magneto-optical effect is defined by relative difference of the propagation constant β of the waveguide or surface plasmon-polariton modes for their forward (f) and backward (b) propagation: $\delta = (\beta_f - \beta_b)/(\beta_f + \beta_b)$. Consequently, it can be called as the magneto-optical nonreciprocity effect (MONRE). In this work we derived the dispersion equation for the eigenmodes of the considered slot waveguide and analyzed peculiarities of the MONRE for different types of the eigenmodes of the structure. It is shown that the MONRE is maximal at the plasmonic modes. Optimization was done in order to get its largest values. However, in order to observe large MONRE it is necessary to fabricate structure of sufficiently good quality. At the first place, interface roughness must be minimized.

Main technological obstacle for the implementation of these structures is related to the annealing of the iron garnet layer at temperatures higher than 650°C that is inevitable for making the iron garnet layer crystallized. However, at the annealing temperatures the metal layers (either Au or Ag) lose their uniformity and tend to form into nanoparticles. We overcome this problem by adding Al_2O_3 dielectric layers to thermally isolate the Ag layers and to keep them smooth. Also, the entire multilayer structure is prepared on a pre-crystallized garnet layer that prevents overheating of the garnet layer from the substrate side. The composition of the pre-crystallized garnet layer is the same as for the top garnet layer. In order to make even better performance of the multilayered structure we polished the surfaces of the iron garnet layers by the argon plasma assisted ion bombardment process. At this, an oxide-base-mixed sputtering target ($BaTiO_3$) in RF magnetron sputtering system with no extra oxygen or nitrogen input was used.

Experimental data shows that metallic nanocavity provides 6 time enhancement of the Faraday rotation in comparison to the non-resonant case. It justifies the adequacy of the used technological method for fabrication of the high quality gyrotropic nanocavities and slot waveguides.

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When no axisymmetric modes are trapped by a freely floating moonpool

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We analyse the spectral problem describing the coupled time-harmonic motion of the following mechanical system: an axisymmetric moonpool (toroidal, surface-piercing body) floats freely in infinitely deep water bounded above by a free surface. The system's motion is of small amplitude near equilibrium which allows us to apply the linearized model developed by John [1] and substantially simplified in [3]. The essential point of the problem under consideration is that one of the coupling conditions is linear with respect to the spectral parameter (the frequency of oscillations), whereas two other problem's relations (another coupling condition and the free surface boundary condition) are quadratic with respect to it.

It is shown in [4], that every positive number is a point eigenvalue when a motionless (but floating freely) body belongs to a certain family depending on the eigenvalue. The family is constructed by virtue of the so-called semi-inverse procedure and a characteristic feature of bodies belonging to it is that their axisymmetric immersed parts divide the free surface into at least two connected components and are bulbous on both sides (directed to infinity and to the inside).

Our present aim is to describe frequency intervals containing no point eigenvalues provided a freely floating moonpool satisfies a geometric condition that excludes the bulbous geometry described above. This condition was used in [2] for guaranteeing the absence of point eigenvalues in the problem when the same moonpool is fixed instead of floating freely. The frequency intervals obtained here only partly coincide with those in [2] because an extra condition is involved — the equation of body's motion.

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Modified T-matrix method on the basis of continued boundary conditions

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In [1] a modified variant of the T-matrix method (see., for example, [2, 3]), based on a modified null-field method [4, 5], in which the surface (in two-dimensional case — curve) for which the null field condition in the corresponding integral equation is fulfilled, is constructed by analytic deformation boundaries of the scatterer up to the analytic continuation singularities of the diffraction field. Then the corresponding integral equation was solved using the method of discrete sources [5, 6]. Was demonstrated high efficiency and universality of the proposed approach is applicable to the solution of diffraction problems on the scatterers rather arbitrary geometry. However, this approach is not applicable to the solution of diffraction problems on the bodies with nonanalytic boundary. At the same time body this kind are the most common and nonidealization type scatterers. Therefore, it seems very promising development of technology T-matrices with respect to such scatterers. In the present paper we propose a modification of the T-matrix on the basis of continued boundary conditions method (CBCM) [5]. This method is among the most universal and can effectively solve the diffraction problems on the bodies with breaks boundaries and thin screens. This universality is achieved by an approximate formulation of the problem. In fact, in the framework of the CBCM the boundary condition is not on the boundary S of the scatterer, and on a surface (curve) S_δ, moved from S a small distance δ. We have investigated the accuracy of the calculation of the T-matrix elements in the framework of the proposed approach is demonstrated on several examples of the universality and efficiency of the proposed method.

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A posteriori error control and adaptivity for Schrödinger equations

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We consider Crank–Nicolson-type fully discrete approximations for Schrödinger equations. For the spatial discretisation we use finite element spaces that are allowed to change in time. In the case of linear Schrödinger equations we derive optimal order a posteriori error estimates using appropriate time-space reconstructions, [1, 3] and a novel elliptic reconstruction, [2]. The obtained estimates reflect correctly the physical properties of the continuous problem and are also valid for time-dependent and rough potentials. Our theoretical results are validated by numerical experiments on various model problems using both uniform and adaptive meshes. Using the derived a posteriori error estimator, we develop and analyse a time-space adaptive algorithm, [2]. The adaptive algorithm reduces the computational cost substantially and provides efficient error control for the solution and the observables of the problem, especially for small values of Planck's constant. Finally, we present some preliminary results on the a posteriori error control for the one- and the two-dimensional nonlinear Schrödinger equations with nonlinearities of the form $|u|^{2p}u$, up to the critical exponent.

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Free oscillations of viscous fluid in rectangular volume with elastic wall

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The problem of search of frequencies and forms of free oscillations of the viscous liquid filling rectangular volume $-A < x < A$, $0 < y < B$ is considered. The speeds and pressure fields in fluid submits linearized Navier–Stokes equations. One wall of volume is thin elastic plate, and its bending displacement satisfy Kirchhoff equation. The plate edges are clamped. The displacement continuity condition in a plate–fluid interface is set. Three other walls of volume are rigid. An adhesion condition of fluid and rigid walls surfaces is set. The oscillatory processes dependence on time t is meant by factor $\exp(-i\omega t)$. Natural frequency has the form $\omega = \omega_0 - i\omega_1$. The magnitude ω_0 is frequency of periodic oscillatory process. The magnitude ω_1 is an attenuation constant. The natural frequency equation of the resonator is constructed as

$$\det C(A, B, \omega) = 0.$$

$C(A, B, \omega)$ is an infinite linear algebraic equations matrix concerning decomposition coefficients of required fields in Fourier series. The natural frequencies evolution at reduction of size B is investigated. It was detected, for each natural frequency ω there is a critical size $B^\star$ such, that at $B < B^\star$ process is aperiodic. At $B = B^\star$ process is critical, that is intervening between periodic and aperiodic. The system of the equations

$$\begin{cases} \det C(A, B, \omega) = 0, \\ \dfrac{\partial}{\partial \omega} \det C(A, B, \omega) = 0 \end{cases}$$

is applied to search of the critical size and corresponding frequency. The critical size search algorithm is offered. The approximate formula for natural frequencies at small fluid viscosity is deduced.

Electro-elastic wave scattering matrix on the tunable phoxonic crystal

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At present time the electrical and mechanical properties of the phoxonic crystals becomes very attractive for researchers [1]. Phoxonic crystal is an artificial periodic structure formed in piezoelectric material. In this structure propagates the coupled elastic and electromagnetic waves. We propose the electrical field tunable phoxonic crystal. Periodic structure of the phoxonic crystal (see Figure) is created by the control electrodes. Electric field biasing to the electrodes lead to changing of the piezoelectric waveguide properties.

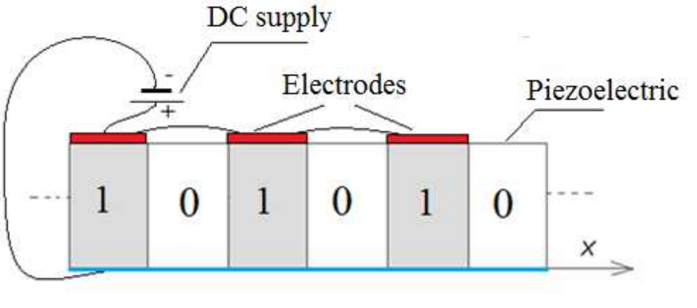

Fig. 1: Elastic, piezoelectric and permittivity properties of Area 0: $\boldsymbol{C}$, $\boldsymbol{e}$, $\boldsymbol{\varepsilon}$; Area 1: $\boldsymbol{C}(E)$, $\boldsymbol{e}(E)$, $\boldsymbol{\varepsilon}(E)$.

The wave spectrum in phoxonic crystal is investigated. The band gaps and transmission gaps were found. The control electrical field magnitude influence on the band gaps and transmission gaps has been investigated. The problem of electro-elastic waves scattering on the semi-infinite phoxonic crystal has been solved analytically. Scattering matrix of the electro-elastic waves on the interface of piezoelectric material and phoxonic crystal was obtained. Investigation of dependence of the reflection and transmission coefficients on the frequency of wave which incident on semi-infinite phoxonic crystal was conducted. Also influencing of the control electrical field magnitude on the scattering matrix coefficients was investigated.

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Geometrical conditions of effective acousto-optic diffraction of interference imaging light beams

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Nowadays, optical coherence tomography (OCT) is an effective method for the analysis of optically inhomogeneous objects in biology, medicine and industry [1]. Full-field spectral-domain OCT is the technique which provides high spatial resolution by means of spectral interferometry without any mechanical scanning. Usually this approach has been implemented by the integration of the high-resolution ($\delta\lambda < 0.1$ nm) spectral element in the illumination channel of the Michelson interferometer. In this case large number of spectral channels ($N \sim 1000$) ensuring comfortable thickness of the detected layer ($\Delta z \sim 1$–3 mm) requires rather high light source intensity and hampers the high-speed operation as the total time of the data collection is proportional to the number of spectral images N. In the applications, where such a high spectral resolution is excessive while the speed of image registration and processing is vital, it is reasonable to reduce the number of spectral channels N. That is why the spectral tuning implemented by means of specialized imaging acousto-optic tunable filters (AOTF) located in the registration channel may be an acceptable solution in this cases [2,3]. Despite the successful application of imaging AOTFs for OCT [2] and profilometry [3] applications, the theoretical description of AO filtration of interference images is still lacking. It is not a priori evident that the diffraction of two interfering optical beams via the ultrasound wave maintains their optical coherence. First of all, the diffraction effect is not localized to a certain surface but is distributed across the crystal cell filled with ultrasonic waves. Second, the diffraction grating induced by ultrasound is not static, but active, so that, for example, diffracted radiation is Doppler-shifted by the ultrasonic frequency. Moreover, imperfections of the grating caused by ultrasound beam divergence, attenuation and instability may have an effect upon the interfering diffracted waves. The additional question is if the coherence keeps its state uniformly across the light beam or the Bragg diffraction affects the amplitude-phase structure of the output interference image.

In this paper we present the calculation of the interfering light waves' propagation under diffraction via acoustic wave in uniaxial crystal. It is based on the approach described in [4] which is already in use for the optical performance optimization of AOTF-based imagers. It allows computation of 2-dimensional transmission function of the AOTF and spectral-spatial geometrical distortion that is enough to get the amplitudes and directions of the diffracted image-transferring beams after AO diffraction. Interference pattern of these waves may be determined by calculation of superposition integral over the AOTF aperture. These patterns are calculated for the main wide-angle AOTF configurations. It is shown that sensitivity of amplitude-phase structure of the output interference image is dependant on the parameters of AO interaction.

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Origin of anomalous nanostructures formation under linear polarized femtosecond laser irradiation of condensed matter

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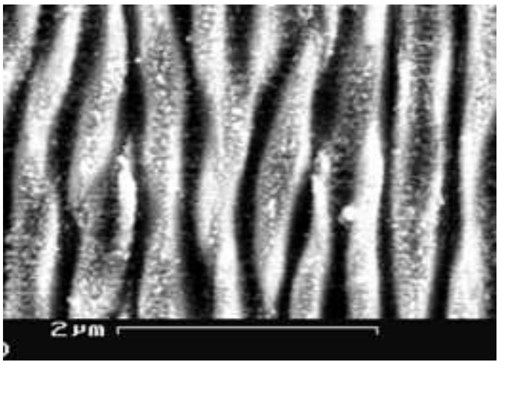
Recent time the nature of spatial ordered micro- and nano-structures formation on condensed matter surfaces and inside the transparent materials under femtosecond laser pulses irradiation attracts sufficient interest. Theoretically, the formation of the set of the spatial periods of the structures is proportional to wavelength of laser radiation (λ) multiplied to 1/2 (Feigenbaum's universality) and normal grating orientation $\vec{g} \parallel \vec{E}$ has been suggested in the framework of universal polariton model [1] and nonlinear mathematical model for spatial periods of structures [2]. Here $\vec{E}$ is the electric field strength vector of laser radiation. It is essential that predicted periods of the structures can overcome the optical diffraction limit value (Abbe criterion). Here n is the refractive index of transparent medium. This prediction satisfies the nonlinear Abbe theory [3] and has been confirmed by number of experiments [4].

In addition to the normal structures the nanostructures with anomalous grating orientation $\vec{g} \perp \vec{E}$ were observed firstly by us on tungsten (see Fig. 1) and titanium surfaces [5] and later for semiconductors and dielectrics [6, 7]. The origin of their formation was unclear for a long time and has been made on the basis of excitation and mutual interference of wedge surface polaritons polaritons (SPP) guided by hills of the main resonant relief, $d = \lambda/\eta$ [5]. Here η is the real part of SPP's refractive index.



In this work our own and published experimental results causing the anomalous nanostructures formation are analyzed. It was shown that anomalous nanostructures formation can be explained as a result of mutual interference of wedge or channel surface plasmon polaritons (SPP) guided by wedge or channel SPP taking into account Feigenbaum's universality for spatial periods formation. So our approach well explains the laser radiation wavelength dependent linear grating of anomalous orientation both on hills and drops of main resonant relief also on semiconductors [6] and dielectrics as small as (40÷60) nm (tungsten and $\lambda = 800$ nm).

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Thermocapillary liquid flows under axi symmetric surface laser heating

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The dynamics of liquid flow is important to many technological and biological processes [1, 2]. Here an example of unstable near surface liquid flow driven by temperature gradient of surface tension is reported.

In the process of continuous wave CO_2 laser beam action by the normal to surface of water (cylindrical cavity with typical diameter L and liquid depth h (L, $h \gg d$)) the sub-synchronous structure of flow were observed. The intensity distribution of laser radiation in the beam has axisymmetric character of the spot diameter of $d \cong 8 \div 10$ mm. The absorption coefficient of laser radiation ($\lambda = 10.6\,\mu$) in water is $\alpha = 3.10^3$ cm^{-1}, which gives the surface source of heat and induces thermocapillary liquid flows.

Experiments with the power density of laser radiation varying near the value of $q_0 = 50$ W/cm^2 have shown the formation of an azimuthally subsymmetric surface liquid flows. Experimentally observed areas on the surface of water are schematically shown on the Fig. 1 (plan view), where 1 is irradiated spot, 2 is an area of thermocapillary flow, 3 is an area of almost unperturbed water. The area 2 has the diameter about 25÷50 mm and was visualized due to refractive index temperature change. Then the boundary of the areas 2 and 3 brings the shape of axially symmetric figure with "protuberances". The directions of fluid movement outside and near the boundary are shown on Fig. 1. In the side view the liquid movements in modes of vortices were seen. The depth of flow area was about (1.5÷2) mm. The causes of circle front liquid boundary instability in mode of symmetric fingering and liquid movement out of the heated central zone need further inquiry [3].

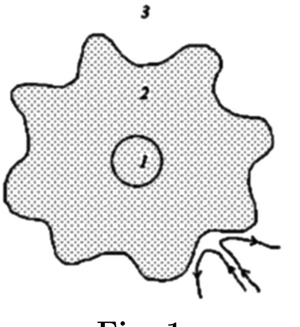

Fig. 1.

For laser power density value $q = 1.4\,q_0$ the plane water-air boundary instability arises in mode of propagating out of laser beam center circular capillary waves. The origin of their appearance is blowing gas bubbles from underwater heated layer. The local long areas of evaporated water were observed under white light visualization by small condensed water droplets scattering along tracks of evaporation. Sometimes the tracks of water condensation look like twisted whirlwind.

In conclusion, the complex thermocapillary liquid flows were observed under the action of axisymmetric surface heat source induced by continuous wave CO_2 laser radiation.

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Direct methods for solving systems of linear algebraic equations with sparse matrices for wave fields modeling by the minimal autonomous blocks method

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The minimal autonomous blocks (MAB) method is used for solving problems of electrodynamics, acoustics and other problems of mathematical physics [1]. There exist several approaches to implementing this method: recompositional and iterative algorithms and direct solving systems of linear algebraic equations [2].

Recompositional approach has high computational complexity and does not allow studying field distribution in the inner domains. Iterative algorithm does not have these drawbacks, but the use of this algorithm is problematic for modeling wave fields in domains with self resonances because of large number of iterations necessary for iterative process convergence.

Direct solving systems of linear equations allows to find an accurate solution with the opportunity to investigate fields in the inner domains. Besides, this technique takes into account resonances correctly. It is important that the matrix of the system has very specific sparse structure which is determined by the MAB and virtual channels numbering.

In this work the system matrix structure for two- and three-dimensional problems depending on the blocks and channels numbering is investigated. The most general methods, algorithms and libraries for solving linear systems with sparse matrices [3] applied to the MAB method are studied. The ways to improve efficiency of existing algorithms using the information about the sparse matrix structure are proposed.

The results of the practical use of direct methods for solving acoustic and electrodynamic problems are presented.

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Modelling of long wireless communication channels by the method of minimal autonomous blocks

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The wide class of problems of applied electrodynamics (land and space communication, remote sounding etc.) is connected with calculation of electromagnetic fields along long channels. Usually for calculation of a field along channels methods of geometrical optics and the geometrical theory of diffraction are used. In the presence of non-uniform objects of the resonant sizes, use of these methods can lead to essential errors.

The use of direct 3D methods of computational electrodynamics (FD-TD, FE, FI) in the simulation of long wireless channels leads to excessive computational cost. A new efficient approach to the calculation of long inhomogeneous channels based on using the method of minimal autonomous blocks [1] and the recursive algorithm of recomposition is proposed. It includes the following main steps:

– partition channel into several longitudinally homogeneous fragments;
– for each longitudinally homogeneous fragment channel allocated transverse layer;
– decomposition of selected layers to the system of minimum autonomous blocks;
– computation of multichannel scattering matrix of layers by recomposition of internal channels;
– computation of multichannel scattering matrices of longitudinally homogeneous fragments of channel using a recursive algorithm of recomposition;
– calculation of the total scattering matrix of the channel by recomposition of the fragments;
– calculation of the transmitted and reflected fields distribution on the transverse boundaries of the channel for a given source of the electromagnetic fields.

Features of realization of absorbing boundary conditions on the longitudinal boundaries of the layers and their influence on the accuracy of the simulation were investigated. Variants of absorbing boundary conditions for arbitrary angles of incidence have been proposed. If the channel comprises scattering objects, the spatial region containing these allocated to individual fragments. Matrix for these fragments are calculated according to standard algorithm of recomposition.

Examples of using the developed approach to the modeling of long channels are provided. Evaluation of the computational efficiency of the proposed method is obtained.

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Specra of 3D cruciform quantum waveguides

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The spectral Dirichlet boundary value problem for the Laplace operator on the union of two infinite circular cylinders of unit width (see Fig. 1a) is considered. It is well known that the essential spectrum of this problem is the ray $[\lambda_\dagger, +\infty)$ and the cut-off value $\lambda_\dagger$ is the first eigenvalue of the Dirichlet problem on the disk with unit diameter. The discrete spectrum contains at least one eigenvalue below the cut-off value. If the waveguides are orthogonal to each other, we prove that total multiplicity of the discrete spectrum is equal to one. Moreover, if the angle between cylinders is equal to ε or cross sections of orthogonal cylinders are ellipses with the semi-minor axis equal to ε (see Fig. 1b), the number of eigenvalues below the cut-off value tends to infinity as $\varepsilon \to 0$.

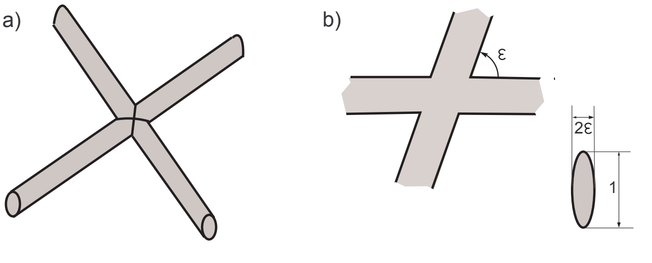

Fig. 1: The cruciform waveguide (a); nonorthogonal and elliptic cross section cases (b).

This is a joint work with Fedor Bakharev from St. Petersburg State University and Sergey Nazarov from Institute of Mechanical Engineering Problems (St. Petersburg).

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Twoparametric operator error estimates for homogenization of elliptic systems

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Let Γ be a lattice in $\mathbb{R}^d$ and let Ω be the cell of Γ. In $L_2(\mathbb{R}^d;\mathbb{C}^n)$, we consider matrix selfadjoint second order differential operators B_ε, $\varepsilon>0$. The coefficients of B_ε are periodic with respect to Γ and depend on $\mathbf{x}/\varepsilon$. So, they oscillate rapidly, as $\varepsilon\to 0$. The principal part of B_ε is given in a factorized form $b(\mathbf{D})^*g(\mathbf{x}/\varepsilon)b(\mathbf{D})$. It is assumed that the matrix-valued function $g(\mathbf{x})$ is bounded and uniformly positive definite. Next, $b(\mathbf{D})$ is a first order differential operator with constant coefficients. The symbol $b(\boldsymbol{\xi})$ is subject to some condition which ensures strong ellipticity of the operator B_ε. The coefficients of the lower order terms belong to some $L_p(\Omega)$-spaces. It is assumed that $B_\varepsilon \geqslant 0$.

Let Q be a Γ-periodic uniformly positive definite and bounded matrix-valued function, and let Q^ε be an operator of multiplication by $Q(\mathbf{x}/\varepsilon)$. Our goal is to study the behaviour of the generalized resolvent $(B_\varepsilon-\zeta Q^\varepsilon)^{-1}$ in dependence of the small parameter ε and the spectral parameter ζ.

Theorem 1. *Let $\zeta=|\zeta|e^{i\phi}\in\mathbb{C}\setminus\mathbb{R}_+$, $|\zeta|\geqslant 1$. Let $c(\phi)=|\sin\phi|^{-1}$ if $\phi\in(0,\pi/2)\cup(3\pi/2,2\pi)$, and $c(\phi)=1$ if $\phi\in[\pi/2,3\pi/2]$. Then for $\varepsilon>0$ we have*

$$\|(B_\varepsilon-\zeta Q^\varepsilon)^{-1}-(B^0-\zeta\overline{Q})^{-1}\|_{L_2(\mathbb{R}^d)\to L_2(\mathbb{R}^d)}\leqslant C_1c(\phi)^2\varepsilon|\zeta|^{-1/2}. \tag{1}$$

Here B^0 is the so-called effective operator with constant coefficients and $\overline{Q}=|\Omega|^{-1}\int_\Omega Q(\mathbf{x})\,d\mathbf{x}$. We also obtain approximation for $(B_\varepsilon-\zeta Q^\varepsilon)^{-1}$ in the $L_2\to H^1$-operator norm:

$$\|(B_\varepsilon-\zeta Q^\varepsilon)^{-1}-(B^0-\zeta\overline{Q})^{-1}-\varepsilon K(\varepsilon,\zeta)\|_{L_2(\mathbb{R}^d)\to H^1(\mathbb{R}^d)}\leqslant C_2c(\phi)^2\varepsilon. \tag{2}$$

Here $K(\varepsilon,\zeta)$ is the correction term. It contains rapidly oscillating factors and so depends on ε.

Now, let $\mathcal{O}\subset\mathbb{R}^d$ be a bounded domain of class $C^{1,1}$. We also study operators $B_{D,\varepsilon}$ acting in $L_2(\mathcal{O};\mathbb{C}^n)$ and given by the same differential expression as before with the Dirichlet boundary condition.

Theorem 2. *Let $\zeta=|\zeta|e^{i\phi}\in\mathbb{C}\setminus\mathbb{R}_+$ and let $|\zeta|\geqslant 1$. For sufficiently small ε we have*

$$\|(B_{D,\varepsilon}-\zeta Q^\varepsilon)^{-1}-(B_D^0-\zeta\overline{Q})^{-1}\|_{L_2(\mathcal{O})\to L_2(\mathcal{O})}\leqslant C_3c(\phi)^5(\varepsilon|\zeta|^{-1/2}+\varepsilon^2), \tag{3}$$

$$\|(B_{D,\varepsilon}-\zeta Q^\varepsilon)^{-1}-(B_D^0-\zeta\overline{Q})^{-1}-\varepsilon K_D(\varepsilon,\zeta)\|_{L_2(\mathcal{O})\to H^1(\mathcal{O})}\leqslant C_4(c(\phi)^2\varepsilon^{1/2}|\zeta|^{-1/4}+c(\phi)^4\varepsilon). \tag{4}$$

Here B_D^0 is the effective operator with the Dirichlet boundary condition and $K_D(\varepsilon,\zeta)$ is the corrector.

The constants C_j, $j=1,\dots,4$, can be controlled explicitly in terms of the problem data. Inequalities like (1)–(4) are called operator error estimates in homogenization theory. For a fixed ζ, estimates (1)–(3) are of sharp order $O(\varepsilon)$, while estimate (4) is of order $O(\varepsilon^{1/2})$. The order of estimate (4) becomes worse compared with (2) because of the boundary influence.

The proof of Theorem 1 is based on the results about homogenization of the generalized resolvent of B_ε at a fixed point (see [1]) and introduction of some auxiliary parameter. The proof of Theorem 2 relies on Theorem 1, introduction of the boundary layer correction term and a careful analysis of this term (cf. [2]).

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On some application of boundary control method in inverse problems

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We consider applications of the Boundary Control (BC) method to inverse source problems and to the problems of the recovering of the spectral data from the dynamical one. We derive the equations of the BC method for this problems and show that solvability of this equations crucially depends on the controllability properties of the corresponding dynamical system and properties of corresponding families of exponentials.

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Asymptotics for 2D stationary Dirac equation with mass and the Berry phase

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The 2D stationary Dirac equation (system) with potential and mass is considered. Using adiabatic approximation it can be decomposed to two Schrodinger-like equations with perturbation which defines the Berry phase. For these equations asymptotics in the form of Maslov canonical operator are constructed. To this end an invariant Lagrangian manifold with eikonal coordinates is considered, which greatly simplifies the derivation of final formulas near caustics. At the end, a trick is presented that allows us, in principle, to generate any Berry phase we want in order to fully utilize the difference between Dirac equation and 2D wave equation (without Berry phase).

A numerical finding of a 2D surface by its mean curvature

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An important part of investigation of a flow with a free boundary is a finding of a surface with a given mean curvature in a neighborhood of the given smooth surface.

We consider the smooth surface S_0 determined by the equation

$$z = u_0(x, y), \quad (x, y) \in \Omega \subset \mathbf{R}^2$$

in Cartesian coordinates (x, y, z). Ω is the domain with smooth boundary, $u_0 \in C(\bar{\Omega})$. The double mean curvature of the surface S_0 is

$$2H_0 = \left[1 + \left(\frac{\partial u_0}{\partial x}\right)^2 + \left(\frac{\partial u_0}{\partial y}\right)^2\right]^{-3/2} \cdot \left[\Delta u_0 + \left(\frac{\partial u_0}{\partial x}\right)^2 \frac{\partial^2 u_0}{\partial y^2} + \left(\frac{\partial u_0}{\partial y}\right)^2 \frac{\partial^2 u_0}{\partial x^2} - 2\,\frac{\partial u_0}{\partial x}\,\frac{\partial u_0}{\partial y}\,\frac{\partial^2 u_0}{\partial x \partial y}\right].$$

Let H is a given smooth function such that L_p-norm of $H - H_0$, is small for $p > 2$. We have to find the function $u(x, y)$ satisfying to the condition

$$u = 0 \quad \text{on } \partial\Omega$$

such that the mean curvature of the surface S determining by the function u equals to H. We find the function u in the form

$$u = u_0 + v.$$

Function v is the solution to the Dirichlet boundary value problem

$$K_0 v + K_1 v = 2(H - H_0) \text{ in } \Omega, \quad v = 0 \quad \text{on } \partial\Omega, \tag{1}$$

where K_0 is the second order linear elliptic operator, K_1 is the nonlinear second order differential operator. Coefficients of the operators K_0 and K_1 depend of the function u_0. If $u_0 = 0$ then $K_0 v = \Delta v$. v is found by the successive approximations. v_1 is the solution to the Dirichlet boundary value problem

$$K_0 v_1 = 2(H - H_0) \text{ in } \Omega, \quad u_1 = 0 \quad \text{on } \partial\Omega,$$

v_n for $n \geq 2$ is the solution to the Dirichlet boundary value problem

$$K_0 v_n = 2(H - H_0) - K_1 v_{n-1} \text{ in } \Omega, \quad v_n = 0 \quad \text{on } \partial\Omega.$$

It is proved that the sequence $\{v_n\}$ tends to the solution of the problem (1) if the L_p-norm of $H - H_0$ is sufficiently small. The above algorithm is realized numerically by MATLAB for $u_0 = 0$ and $\Omega = \{0 < x < a, 0 < y < b\}$. The solution to the Dirichlet boundary value problem is constructed as a Fourier series.

On computation of the Heun functions

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In this work we deal with the Heun functions. They are solutions of the Heun equation, which is the most general Fuchsian equation of second order with four regular singular points. Despite the increasing interest to the equation (see references at [1]) and numerous applications of the functions in a wide variety of physical problems (see also [2]), it is only Maple amidst known software packages which is able to evaluate the Heun functions numerically. However, the Maple routine is known to be imperfect; even at regular points it may return infinities or end up with no result; the choice of branch cuts is unclear. Since the code is not publicly available, improving the situation is virtually impossible. The purpose of the work is to suggest and develop alternative algorithms and program code for numerical evaluation of the Heun functions. Since the Heun functions are generally multi-valued functions, for the computer encoding it is vital to define single-valued counterparts and we pay special attention to the choice of branch cuts. Further a procedure based on power series expansions and analytic continuation is suggested which allows us to avoid numerical integration of the differential equation and to achieve reasonable efficiency and accuracy. We also suggest an improvement of the algorithm for points close to the singularities, using representations [3]. Results of numerical tests are presented.

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Stabilizing solutions at thresholds of the continuous spectrum and scattering anomalies in a waveguide

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A criterion for the existence of stabilizing solutions of the homogeneous Dirichlet and Neumann spectral problems at thresholds of the continuous spectrum in a perturbed cylindrical or periodic waveguide will be presented on the base of the energy radiation conditions. The relationship of these solutions with the following spectral anomalies will be explained:

• the appearance of near-threshold eigenvalues, isolated in the discrete spectrum and embedded into the continuous spectrum;

• classical Wood's anomalies of the diffraction patterns;

• almost full transmission and reflection at above-threshold frequencies;

• failure of the limit absorbtion principle at the threshold frequencies.

Some open questions, especially in elasticity, will be formulated.

Data analysis in laser diffractometry of red blood cells in shear flow conditions

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One of the main microrheologic parameters of blood is erythrocyte deformability, which is defined as a measure of the ability of cells to change their shape under the influence of external forces. The deformability of the erythrocytes is conventionally measured using different methods [1]. One of the most convenient methods is based on laser diffraction by red blood cells in shear flow (ektacytometry). In ektacytometry, the information about red cells shapes is extracted from the diffraction patterns that are formed due to low-angle scattering of a laser beam while passing through a sheared low hematocrit RBC suspension. Our studies [2] show that laser ektacytometry can be used not only to measure the mean deformability of the cells in suspension but also to measure the parameters of the distribution of erythrocytes in deformability. However, implementation of these possibilities requires a specific calibration of the ektacytometer. The calibration involves measuring the level of light intensity at various points of the observation screen relative to the light intensity in the central diffraction maximum. Direct calibration can be performed by regular photometry and digitization of the diffraction pattern in a wide area of the observation screen, including the area of the central diffraction maximum. However, such measurements are complicated by the fact that the central diffraction maximum is located in the vicinity of the point of incidence of the direct laser beam on the observation screen. Therefore, we need to search for other ways to solve this problem. One such method is proposed in this paper. Our method is based on measuring the geometric parameters of the diffraction patterns obtained in the experiments with specially prepared samples of blood ("blood sample calibration"). In this paper, we derive the necessary diffractometric equations, as well as describe the procedure for the preparation of blood samples. In addition, we test the data processing algorithm [2], which allows for estimating the parameters of the distribution of erythrocytes in deformability.

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Atom channelling with Bessel–Gauss beams

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Ideal Bessel beams are non-diffracting waves [1] which suffer the drawback of having infinite power flux. This problem can be addressed through a Gaussian apodization of their initial fields, resulting in the so-called Bessel–Gauss beams [2]. These beams can resist the diffraction effects for long (finite) distances, even if suffering a progressive intensity decay. Here, we use higher order Bessel–Gauss beams for guiding ^{85}Rb atoms through the dipole optical potential [3]. According to our results, we can choose appropriate depth of field for these beams to canalize the atoms with a kind of optical funnel.

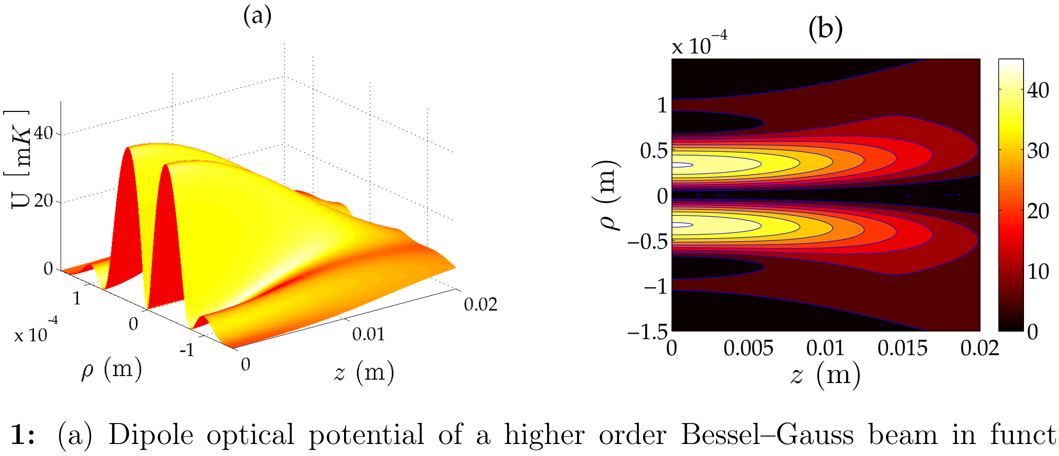

Fig. 1: (a) Dipole optical potential of a higher order Bessel–Gauss beam in function the transverse distance and the propagation distance. (b) Orthogonal projection.

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An approach to a multi-dimensional tunneling

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A stationary Schrödinger operator with one well was considered. Some useful theorems were proven. They allow us to describe tunneling effects in multi-dimensional case.

On decay of periodic entropy solutions to scalar conservation laws

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We study a multidimensional conservation law

$$u_t + \mathrm{div}_x \varphi(u) = 0, \tag{1}$$

where $u = u(t,x)$, $(t,x) \in \Pi = (0,+\infty) \times \mathbb{R}^n$, and the flux vector $\varphi(u) = (\varphi_1(u), \dots, \varphi_n(u)) \in C(\mathbb{R}, \mathbb{R}^n)$ is supposed to be merely continuous. Recall that the function $u(t,x) \in L^\infty(\Pi)$ is an entropy solution (e.s. for short) of (1) in Kruzkov's sense [1] if for all $k \in \mathbb{R}$

$$|u-k|_t + \mathrm{div}_x[\mathrm{sgn}(u-k)(\varphi(u) - \varphi(k))] \le 0$$

in the space of distributions on Π. As was shown in [2], any e.s. $u(t,\cdot) \in C([0,\infty), L^1_{loc}(\mathbb{R}^n))$, after possible correction on a set of null measure. In particular, the strong trace $u(0,x) = u_0(x) \in L^\infty(\mathbb{R}^n)$ is well-defined. This means that $u(t,x)$ is an e.s. of the Cauchy problem for (1) with the initial data

$$u(0,x) = u_0(x). \tag{2}$$

We assume that u_0 is a periodic function (with the standard lattice of periods $\mathbb{Z}^n$). It is known (see for instance [3]) that an e.s. of problem (1), (2) always exists, is unique (in the class of all e.s., not necessarily space-periodic), and x-periodic. Let $\mathbb{T}^n = \mathbb{R}^n/\mathbb{Z}^n$ be the standard torus,

$$I = \int_{\mathbb{T}^n} u_0(x)dx$$

be the mean value of initial data (here dx is the normalized Lebesgue measure on $\mathbb{T}^n$). Our main result is the following.

Theorem 1. *Suppose that for all $\xi \in \mathbb{Z}^n$, $\xi \neq 0$ the function $\lambda \to \xi \cdot \varphi(\lambda) = \sum_{i=1}^n \xi_i \varphi_i(\lambda)$ is not affine in any vicinity of I. Then*

$$\lim_{t\to+\infty} u(t,\cdot) = I \quad \text{in } L^1(\mathbb{T}^n)$$

(decay property).

This theorem generalizes results of recent papers [4, 5]. The proof is based on new localization principles (extending results of [6]) for H-measures corresponding to the scaled sequence $u(kt,kx)$, $k \in \mathbb{N}$. As a simple application of Theorem 1, we obtain that the necessary and sufficient condition for the decay of periodic e.s. of the equation $u_t + (|u|)_x = 0$ is that either $u \equiv \mathrm{const}$ or $I = \int_0^1 u_0(x)dx = 0$.

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On band-gap structure of spectrum in network double-porosity models

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We study spectral properties of periodic divergent-type elliptic operator A_ε with high contrast coefficients on ε-periodic thin network F_ε, which is asymptotically singular and can be obtained from 1-periodic graph F by means of fattening and contraction. The network F_ε is divided into stiff and soft parts, also 1-periodic, where the coefficients of A_ε are of order 1 and ε^2, respectively; the stiff part is connected and the soft part is dispersive. We prove that the spectrum of the operator A_ε has the band-gap structure and show the existence of non-degenerate spectral bands and open gaps, the number of which grows to infinity as $\varepsilon \to 0$. We establish connection between the endpoints of gaps and eigenvalues of two operators defined on the cell of periodicity. The first is the Laplace–Dirichlet operator on the "soft" part of the graph F within the unit cell and the second is its electrostatic extension onto the whole cell. Moreover, the band-gap structure of the spectrum can be described asymptotically exactly on each finite interval under additional geometric condition which is the "smallness" of the soft phase with respect to the stiff phase.

Transparent boundary conditions for the high-order parabolic approximations

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The wide-angle parabolic approximations to the solution of the 2D acoustical Helmholtz equation

$$p_{xx} + p_{zz} + \kappa_0^2 p = \delta(x)\delta(z - z_s) \tag{1}$$

are usually obtained by the method of the approximation of the operator square root by the Padé or Taylor series. Another approach was proposed in the paper [1], where the solution of (1) was approximated by the series of the form

$$p_n(x,z) = \exp\left(\int_0^x \kappa_0 dx\right) \sum_{j=0}^{j=n} A_j(x,z)\,,$$

where the amplitudes A_j are determined from the system of the parabolic equations (PEs)

$$2\mathrm{i}\kappa_0 A_{j,x} + A_{j,zz} + \nu A_j + A_{j-1,xx} = 0\,, \tag{2}$$

where the input term in the j-th equation is obtained from the solution of the previous equation (we set $A_{-1} \equiv 0$).

In many practical problems of the underwater acoustics the sound propagation is studied in the open waveguides. In such cases the numerical solution of the PEs (2) requires the artificial domain truncation. This truncation may be accomplished by introducing an artificial boundary, say $z = L$, and imposing the transparent boundary conditions at $z = L$ [2]. For example, such conditions for the standard narrow-angle PE were derived in [2].

In this work we generalize the TBCs [2] to the case of the system of PEs (2). The resulting TBCs read as

$$\frac{\partial A_j^t(x,z)}{\partial \mathbf{n}} = -\sqrt{\frac{2\kappa_0}{\pi}} \mathrm{e}^{-\mathrm{i}\frac{\pi}{4}} \sum_{k=0}^{j} \alpha_{k,1} (-2\mathrm{i}\kappa_0)^{-k} \frac{d}{dx} \int_0^x \frac{\partial^k A_{j-k}^t(y,z)}{\partial y^k} \frac{dy}{\sqrt{x-y}} \text{ at } z = 0, L\,, \tag{3}$$

where $\alpha_{k,1}$ are certain constants ($\alpha_{0,1} = -1$, $\alpha_{1,1} = \frac{1}{2}$, $\alpha_{2,1} = \frac{1}{8}, \ldots$), and $\mathbf{n}$ denotes the outward unit normal vector at $z = L$, $z = 0$ respectively.

The work was accomplished during P. S. Petrov's visit to the Bergische Universität Wuppertal sponsored by the DAAD. P. S. Petrov was also partially supported by the RFFI (Proj. No. 14-05-3148614_mol_a), and the RF President grant MK-4323.2015.5.

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On the computation of the Morse index on rays between the source and observation points by means of Gaussian beam technique

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The ray method is, perhaps, the simplest and most visual method from physical point of view and therefore it is widely used for the short wave propagation problems. In particular, it is in intensive use in laborious migration problems, where one deals with the wave propagation in complicated inhomogeneous media.

Though it is well known that the ray method fails on caustics due to the ray amplitudes getting singular there and the rays forming numerous caustics in inhomogeneous media, we normally observe many subdomains free of caustics, i.e. the ray field remains regular there. That situation gives rise to the following problem. Suppose the observation points are located in an area free of caustics, but the rays from the source of the wave field to an observation point meet a number of caustics.

So, how to correctly compute the phase shift in ray formulae between the source and observation points? The answer to this question consists in the computations of the Morse indices, see [1] and also [2], on those rays, that is the number of focal points on the rays between the source and observation points.

In the report we propose a method of computation of the Morse index which is based on a Gaussian beam technique.

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A method for computing waveguide scattering matrices of the Maxwell system

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We consider the stationary Maxwell system in a 3D waveguide G having several cylindrical outlets to infinity and perfectly conductive boundary. We propose and justify a method for approximate computing the scattering matrix of the problem. A minimizer of a quadratic functional $J_l^R(\cdot)$ serves as an approximation to a row S_l of the scattering matrix. To construct such a functional, we solve an auxiliary boundary value problem in the bounded domain G^R obtained from G by cutting off, at a distance R, the waveguide outlets to infinity; the auxiliary problem is proved to have a unique solution. The minimizer tends to S_l at exponential rate as R tends to infinity.

The talk is based on the joint paper [1].

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Effects of dispersion on models of mechanics

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The modified energy equation of motion, continuity and momentum for structureless particles, taking into account the effects of the change of the angular momentum in the elementary volume were previously obtained. They followed from a modified of the Boltzmann equation. This equation was received from modified the Liouville equation. The classical phenomenological theory was used for a solid body but interpretation was varied in this case [1–3]. The construction of continuum mechanics equations used conditions of equilibrium of forces. This leads to a symmetric stress tensor and a violation of "continuity" of the environment that a rigorous analysis requires additional conditions. The laws provide a balance of power conservation of mass, momentum and energy. In addition the fundamental conservation law is the law of conservation of angular momentum. It should be noted that the kinetic theory (in the Boltzmann equation) the law of conservation of angular momentum is not executed. The importance of these effects observed for fluid mechanics, and the plasma gas and the solid body. Exact solution was received for the barometric Boltzmann formula. Gas is at stationary condition in field of force which have potential φ. Besides, some new variants influence of the boundary conditions in the Falkner–Skan problem with angular momentum are investigated. Essentially that elementary volume can be rotated around the axis of inertia or to be involved in the rotational movement. In both cases, the density of the flow across the border changes by $\frac{d(\rho u)}{dr}(r' - r) + \cdots$ by rotation of the elementary volume. The contribution of other components is small, taking into account the smallness of the volume and the absence of rotation on axis. In classic elementary volume is rotating near arbitrary point, so interpretation about velocity that it sum of divergent and rotation is not real, as we need have inertia axis. Another important results are effects of angular momentum for potential that is not central. It is important if we want to find interaction among compound molecules or to calculate potential in crystal having defects. The Lagrange equation is building with influence of angular momentum. The second important issue, delay in mechanics and physics. Effect of delay due to the nature of the definition of the derivative as a limit, while environment is discrete. Differently for writing derivative in condition of finite the mean free path of the molecules (rarefied gas) we are taking into consideration only molecules with great velocity as slow collisions does not have the time for collision. Therefore there is a need to use the second member of sum of the Taylor series for the remaining components, or to use the mean value of the time derivative, i.e. we must take the average values over time for all the terms in the equations in space due the average value. For a finite mean free path of the molecules when the Knudsen number is of order unity in the collision integral is also necessary to take into account the "delay". That is done in this work.

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Use of oblate spheroidal coordinates for modeling of quantum ring spectra

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A new approach for the problem of quantum ring and quantum dot spectra is proposed. The exact separation of variables [1] for the problem allows us to study the variations of the spectra for a broad range of geometries while keeping an advantage of high computational efficiency. Our approach is illustrated by the calculations of a single particle spectrum of a quantum ring for varying geometry of the quantum ring. The spectrum can resemble or deviate from the one-dimentional ring spectrum according to the choice of the ring geometry.

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Caustics in graphene: asymptotic and numerical analysis

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We study above-barrier scattering of Dirac electrons by a smooth electrostatic potential combined with a coordinate-dependent mass in graphene. We assume that the potential and mass are sufficiently smooth, so that we can define a dimensionless semiclassical parameter $h \ll 1$. This setup naturally leads to focusing and the formation of caustics. In this talk, I will discuss how the semiclassical asymptotics, obtained with the Maslov canonical operator, can be implemented to numerically construct the wavefunction Ψ near the caustic. The matrix character of the Dirac equation gives rise to a nontrivial Berry phase in this problem, which makes it different from a scalar wave equation. I will show how this Berry phase influences the wavefunction, by comparing the results for graphene with those for a scalar wave equation.

In my talk, I will place particular emphasis on the region near the cusp, where the main focus occurs and the maximum of the intensity $|\Psi|^2$ is found. In the simplest approximation for the wavefunction in this region, only the leading order term proportional to $h^{1/4}$ is considered and the wavefunction is expressed in terms of the Pearcey function. It turns out that, at the length scales that are relevant in physics, this approximation does not give very good results. Furthermore, it is unable to fully capture the effect of the Berry phase, since it predicts that $|\Psi|^2$ is independent of the Berry phase. I will show that a much better description can be obtained by employing the uniform approximation, which also includes the terms of order $h^{1/2}$ and $h^{3/4}$, and expresses the wavefunction not only in terms of the Pearcey function, but also in terms of its derivatives. Using this approximation, we find that the Berry phase can have a significant effect on the maximal intensity at the focus. The accuracy of both approximations will be illustrated by comparing them to numerical results.

This work was done in collaboration with D. S. Minenkov, M. I. Katsnelson and S. Yu. Dobrokhotov.

Algorithm for reconstruction of inhomogeneous permittivity in optical fibers using propagation constant measurements

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Optical fibers are regular dielectric rods having various cross-sectional shapes where generally the permittivity may vary in the waveguide's cross section. The eigenvalue problem for natural modes of inhomogeneous optical waveguides in the weakly guiding approximation is formulated as the problem for Helmholtz equation with partial radiation conditions at infinity in the cross-sectional plane. The original problem is reduced to a nonlinear spectral problem with compact integral operator. The problems of existence, localization, and dependence on parameters of the spectrum are investigated. We prove that for each positive frequency there exists exactly one fundamental mode. First, we approximate the integral operator by a spline-collocation method. Then we present new numerical method for reconstruction of inhomogeneous permittivity using measurements of propagation constants of the fundamental mode for different frequencies. To solve our inverse eigenvalue problem, we are not requiring the information about specific values of eigenfunctions. In our algorithm, it is enough only to know that the fundamental mode is excited and then to measure its propagation constant for several frequencies. This approach satisfies to the practice of physical experiments because usually in real-life applications the fundamental mode is excited in waveguides. Moreover, the fundamental mode can be excited only for enough wide range of frequencies. The convergence and quality of the numerical method are confirmed by numerical experiments. Main applications of our algorithm are, for example, in detection of defects in metamaterials and in nanoelectronics, as well as in microwave imaging technology in remote sensing.

Longitudinal and transverse solitons in a dynamical trap

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We consider dynamics of solitons of Bose–Einstein condensate in a trap with harmonically oscillating walls. For the transverse solitons — structures localized along the trap axis — approximate

Newton-type equations are derived describing regular and chaotic soliton motion, as well as periodic parametric instability of soliton regular dynamics. For transverse solitons, under condensate resonance excitation, found are internal modes, their weak nonlinear damping, and their excitation after collisions of solitons.

We consider dynamics of Bose–Einstein condensates of cold gas of weakly interacting atoms in a trap with oscillating walls. The dimensionless governing Gross–Pitaevskii equation (GPE) for the condensate wavefunction ψ and boundary conditions at not-penetrating walls are:

$$i\frac{\partial\psi}{\partial t} = -\nabla^2\psi + |\psi|^2\psi, \quad L_{\text{left}}(t) < z < L_{\text{right}}(t), \quad \psi(z = L_{\text{left}}(t), t) = \psi(z = L_{\text{right}}(t), t) = 0. \quad (1)$$

Here t is time and z is the longitudinal coordinate. In 1D-geometry — the case of a cigar-type trap — the Laplacian $\nabla^2 \to \partial^2/\partial z^2$. Then, if the distance between soliton and walls are much larger than soliton width w, the soliton centre coordinate z_c obeys the Newton-type equation:

$$\frac{d^2 z_c}{dt^2} = -\frac{w}{4}\omega_0^2 \exp\left(\frac{1}{w}\right) \times \left[\exp\left(-\frac{2}{w}(L_{\text{right}}(t) - z_c)\right) - \exp\left(-\frac{2}{w}(z_c - L_{\text{left}}(t))\right)\right], \quad (2)$$

where $\omega_0 = 8w^{-2}\exp[-(2w)^{-1}]$. Under conditions indicated above, the Newtonian equation agrees with GPE with high precision. For harmonic oscillations of walls, we demonstrate regular and chaotic soliton dynamics depending on the system parameters and initial conditions. For antiphased oscillations of two opposite walls, there is solution corresponding to motionless soliton located in the trap centre. We show that under conditions of parametric resonance this regime is unstable and there are periodically alternating growth and decay of soliton distance from the centre. Further simplification of the Newton-type equation solution is in the case of fast walls oscillations (asymptotic two-scale solution). In the case of stochastic motion of walls, Maxwell-like distribution of soliton velocity is found.

Transverse phenomena are described, in the resonance approximation (two-level model), by two coupled equations of type Eq. 1, with replacement $\nabla^2 \to \partial^2/\partial x^2$ (1D) or $\nabla^2 \to \partial^2/\partial x^2 + \partial^2/\partial y^2$ (2D). For 1D-geometry we found soliton internal modes, demonstrate their weak damping and their excitation due to collisions of solitons. For 2D-geometry, typical is condensate collapse when the number of atoms in condensate exceeds the critical value that depends on relative populations of the levels and difference of their phases.

The research is supported by the Russian Scientific Foundation grant 14-12-00894.

Approximation of a compressible Navier–Stokes system by models of the non-linear acoustics

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We analyse the derivation of different models of the non-linear acoustics as Kuznetzov, Westervelt, nonlinear progressive wave equation (NPE) and Khokhlov–Zabolotskaya–Kuznetzov (KZK) equations which are perturbative and par-axial approximations of small perturbations around a given state of a compressible nonlinear isentropic Navier–Stokes (for viscous media) and Euler (for the non-viscous case $\nu = 0$) systems:

$$\partial_t\rho_\epsilon + \text{div}(\rho_\epsilon u_\epsilon) = 0\,, \quad \rho_\epsilon[\partial_t u_\epsilon + (u_\epsilon\cdot\nabla)u_\epsilon] = -\nabla p(\rho_\epsilon) + \epsilon\nu\Delta u_\epsilon \quad (1)$$

with the approximate state equation for $\rho_\epsilon = \rho_0 + \epsilon\tilde{\rho}_\epsilon$, not including terms of the order of $o(\epsilon^2)$,

$$p = p(\rho_\epsilon) = p_0 + c^2\epsilon\tilde{\rho}_\epsilon + \frac{(\gamma-1)c^2}{2\rho_0}\epsilon^2\tilde{\rho}_\epsilon^2, \quad (2)$$

where ϵ is a dimensionless parameter which characterizes the smallness of the perturbation. For instance, in water with an initial power of the order of 0.3 W/cm^2 ϵ is equal to 10^{-5}.

The direct derivation shows that the Kuznetzov equation is the first order approximation of the Navier–Stokes system (1)–(2), the KZK and NPE equations are the first order approximations of the Kuznetzov equation and the second order approximations of the Navier–Stokes system (1)–(2). All times, from Kuznetzov or Westervelt or NPE equations it is possible to obtain the KZK equation. In addition, there is a bijection between the NPE equation and the KZK equation [1].

Therefore, using the well-posedness results for the Navier–Stokes/Euler systems and for the KZK equation [4], we find the domains, where we validate the approximation of the exact solution of the Navier–Stokes/Euler systems by the solution of the KZK equation.

To be able to do it, we need to ensure that the derivation of our model, the KZK equation, allows us to reconstruct the solution of the initial Navier–Stokes system from the solution of the KZK equation. In this aim, we modify the initial physical derivation, given in Ref. [2], introducing a corrector function in the derivation *ansatz* [3, 5].

We prove the validation of the KZK-approximation for the non-viscous case, as well as for the viscous nonlinear and linear cases. The results are obtained in Sobolev spaces for functions periodic in one of the space variables and with a mean value of zero. The existence of a unique regular solution of the isentropic Navier–Stokes system in the half space with boundary conditions that are both periodic and mean value zero in time is also obtained [3].

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Simulation of diffraction image formation processes in optical lithography and evaluation of their quality on the bases of rigorous diffraction theory

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The method for integrated evaluation of quality of optical diffraction image, arising under diffraction distortions on the output of a transmitting optical device, is described. This method uses several real scalar parameters, determining by means of comparison of two functions on a plane image surface. The first function is the given energy-phase distribution and describe a required image. The second one is a real distribution of energy density and phase of the electric field, obtaining from experiment. The latter can be simulated theoretically as a solution of a corresponding exact or approximate diffraction problem on image formation. Application of the given method is demonstrated by the example of evaluation of images formed by contactless printing from a mask on a photoresist in optical lithography. In this case, the part of a required image is played by a step amplitude function, reproducing a geometro-optical image of openings in a mask, and a real image is a field, arising as a result of plane wave transmission through openings and diffraction on their edges. It is important that opening images are projected on photoresist not in the far zone, but in the near one, therefore one should use rigorous diffraction methods for computation of image field. The most simple model of such a process corresponds to transmission of a wave through isolated infinite slot in an absorbing screen. That is why we calculate an image function on the bases of the rigorous solution of plane wave diffraction by a slot in a conducting screen of finite thickness. With the help of this method, we have evaluated diffraction image quality for a slot in dependence on slot dimensions (width of an opening), the thickness of a screen (mask) and the distance between a mask and photoresist in comparison with the wavelength. Analogously, the theoretical model of multiwave optical lithography is considered, when several different waves take part at exposure, one of which is incident along the normal to a mask and the remaining are incident obliquely and symmetrically on both sides. We have studied the total diffraction field, arising in photoresist as a result of superposition of different diffraction fields caused by differently directed waves incident on the screen with a slot.

Nonlinear elastic surface waves: drift of the initial wave speed

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A nonlinearity can be put into the model of elastic deformation in different ways. This study is based on the Murnaghan elastic potential, which can be meant as the most general representation, that introduced the quadratic nonlinearity into the constitutive equations and further into the basic equations in displacements,

$$W = (1/2)\lambda(\varepsilon_{mm})^2 + \mu(\varepsilon_{ik})^2 + (1/3)A\varepsilon_{ik}\varepsilon_{im}\varepsilon_{km} + B(\varepsilon_{mm})^2\varepsilon_{mm} + (1/3)C(\varepsilon_{mm})^3. \tag{1}$$

It includes five elastic constants λ, μ, A, B, C and therefore is called sometimes the five-constant potential. Next assumption is associated with the wave motion, namely, with propagation of some types of classical surface waves (Rayleigh, Stoneley, Love, Mozhaev). In the study in hand, the corresponding nonlinear wave equations are taken in the form [1, 2] for Rayleigh and Stoneley waves

$$\begin{aligned}\rho u_{1,tt} - (\lambda+2\mu)u_{1,11} - (\lambda+\mu)u_{3,13} - \mu u_{1,33} &= [3(\lambda+2\mu) + 2(A+3B+C)]u_{1,1}u_{1,11} \\ &+ [\mu + (A/2) + B](u_{1,1}u_{1,33} + u_{1,3}u_{3,11} + u_{1,3}u_{3,33} + u_{3,3}u_{1,33}) + \dots \end{aligned} \tag{2}$$

(the second equation can be obtained from the first one by a change of indexes $1 \rightleftharpoons 3$), for Love and Mozhaev waves

$$\begin{gathered}\rho u_{3,tt} - \mu(u_{3,11} + u_{3,22} = T_1[(u_{3,1})^2 u_{3,11} + (u_{3,2})^2 u_{3,22}] + T_2(u_{3,2})^2 u_{3,11} + \dots \\ T_1 = (3/4)[4(\lambda+\mu) + A + 2B], \quad T_2 = (1/2)(2\lambda + \mu + A + B).\end{gathered} \tag{3}$$

The nonlinear problem on determination of the wave number (phase velocity) from the boundary conditions is solved for each kind of surface wave independently on the finding of nonlinear solutions of nonlinear wave equations. The nonlinear Rayleigh equation is as follows

$$\begin{gathered}-l_1 l_4 + l_2 l_3 - \left[\frac{l_3^2 n_2^2 + l_1^2 n_5^2 - 2 l_1 l_3 n_2 n_5}{l_2 n_5 - l_4 n_2} - \dots\right] A_{phi} + \dots = 0, \\ l_1 = 2\mu k_R^2 - (\lambda+2\mu)k_1^2, \quad \dots, n_5 = ((1/2)A + B) i k_R k_2^2 \sqrt{k_R^2 - k_2^2} e^{i(k_R x_1 - \omega t)}.\end{gathered} \tag{4}$$

The most unexpected wave phenomenon, which is described by new nonlinear Rayleigh equations, is dependence of phase speed of the nonlinear Rayleigh wave on the initial amplitude. The main phenomenon is displayed in that in conditions of fixed frequency and other wave parameters (for example, physical properties of material), the phase speed is changing with changing the initial amplitude. This effect is identical for all the mentioned above kinds of surface waves. The main part of the presentation is devoted to comments of detected theoretically effect.

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Effective masses for Laplacians on periodic graphs

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We consider Laplacians on periodic both discrete and metric equilateral graphs. Their spectrum consists of an absolutely continuous part (which is a union of non-degenerate spectral bands) and flat bands, i.e., eigenvalues of infinite multiplicity. We estimate effective masses associated with the ends of each spectral band in terms of geometric parameters of graphs. Moreover, in the case of the beginning of the spectrum we determine two-sided estimates of the effective mass in terms of geometric parameters of graphs. The proof is based on the Floquet theory, the factorization of fiber operators, the perturbation theory and the relation between effective masses for Laplacians on discrete and metric graphs.

Temporal and spatial correlations in semiconductor microcavities

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Description of purely quantum effects require modelling tools which keep track of all quantum correlations. The exponential growth of the Hilbert space versus the number of modes of the bosonic system forbids any direct master equation treatment and one has to cope with a cumbersome hierarchy of coupled equations for the operator averages accompanied by the truncation at a certain operator order [1]. A possible alternative is to use the so-called truncated Wigner approximation [2] where a noise is accounted for on top of the classical field evolution. This approach, however, is based on certain severe approximations that limit the range of applicability and cannot easily account for conditional backaction effects.

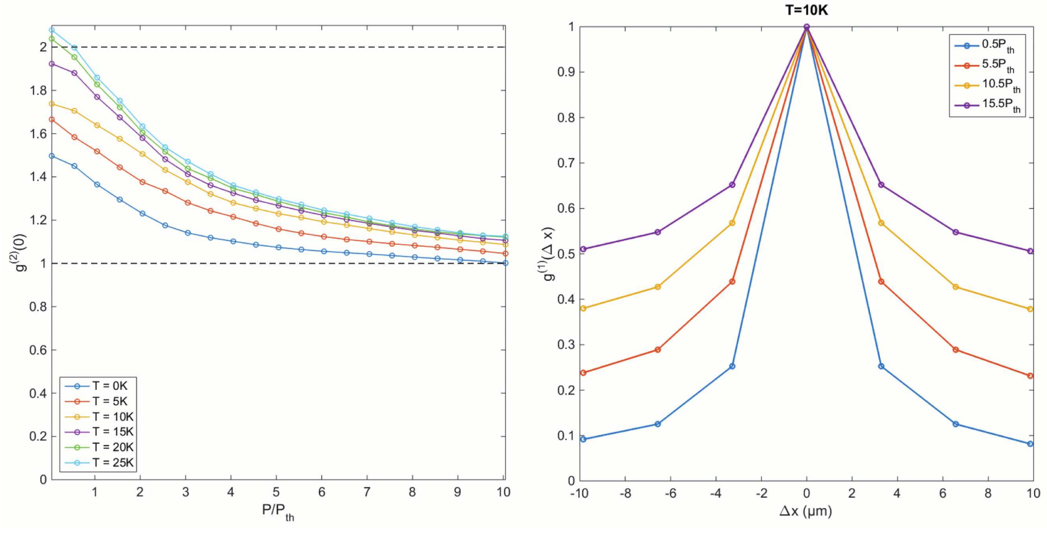

Fig. 1: a) Ground-state second-order temporal coherence function of the system of polaritons at zero delay as a function of the pump power for several temperatures. b) First-order spatial coherence as a function of the displacement from $x = 0$ point at different pump powers.

In this context, we propose a quantum jump approach to exciton-polariton condensation based on a stochastic evolution of the system wave function subject to interaction with the exciton reservoir and coherent/incoherent driving. We directly compute both the temporal second-order correlation function and spatial first-order coherence function (see Fig. 1) of the system keeping track of the excited state dynamics. We discuss the limitations and prospectives opened by this model that, as opposed to for example, DMRG algorithms, involves an intuitive and simple numerical implementation which is highly parallelizable and easily scalable to clusters.

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Dissipative exciton-polariton solitons in semiconductor microcavities

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We investigate a system of exciton polaritons in a specifically designed semiconductor microcavity which has an embedded saturable absorber in the heterostructure growth direction (e.g. within one of its Bragg mirrors) [1]. It results in emergence of additional (beside the Kerr-like) nonlinearity in the system with the possibility of bistability of the condensate particles number on the nonresonant (electrical or optical) pump intensity. Further, we propose a bright spatial dissipative soliton of a new kind in the system under nonresonant excitation. The soliton spatial extension lies in the submicron range and it is an order (or even several orders) of magnitude smaller than typical widths observed with regular optical dissipative solitons. The result of our modelling are presented in Fig. 1 showing the solitonic solutions in the absence (left-hand pamel) and in presence (right-hand panel) of particles interaction in the system.

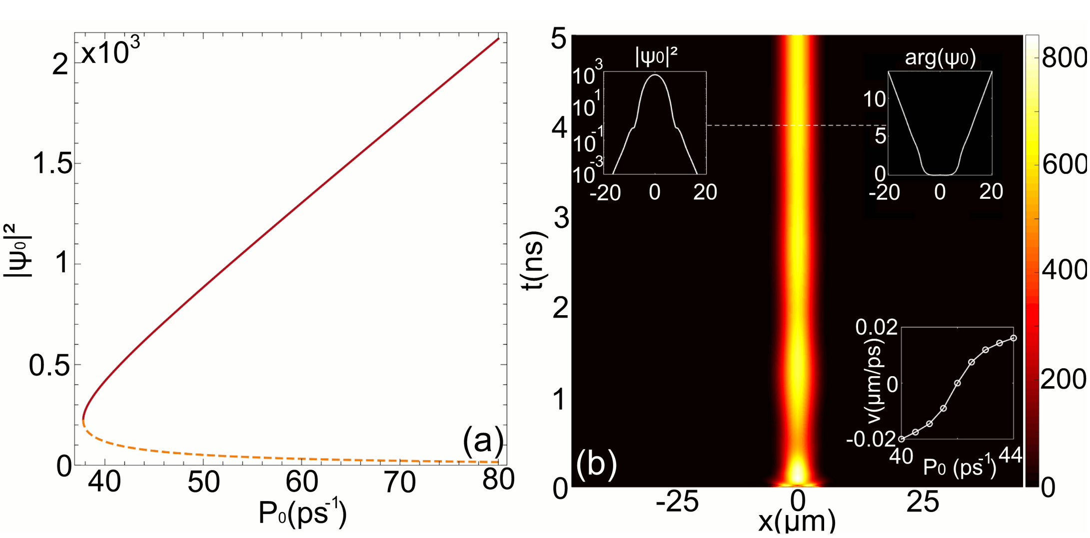

Fig. 1: (a) Bistability in the steady state: dependence of the exciton-polariton concentration, $|\psi_0|^2$, on the intensity of nonresonant pump, P_0, for the parameters: $\gamma_0 = 0.025$ ps^{-1}, $R =$

$5 \cdot 10^{-6}$ ps^{-1}, $\gamma_R = 1/200$ ps^{-1}, $\beta = 5$, $\sigma = 0.1$, dx is discretization length, w_y is the 1D microwire width. The 0-branch corresponding to stable trivial solution is not shown. (b) DS formation in the absence of the particle self-scattering, $\alpha = 0$; the intensity of pump is $P = 42$ ps^{-1}. The colormap shows the number of particles. Upper left-most inset shows the DS density profile (with the width ≈ 10 µm) in the steady state. Upper right-most inset illustrates the phase distribution. Lower inset illustrates the switching wave velocity dependence on the intensity of pump. At some value of pump, the switching waves stop ($v = 0$).

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On homogenization for non-self-adjoint periodic differential operators on an infinite cylinder

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We investigate the homogenization problem for operators on $L_2(\Xi)$, $\Xi = \mathbb{R}^{d_1} \times \mathbb{T}^{d_2}$ with $d_1 > 0$ and $d_2 \geq 0$, of the form

$$\mathcal{A}^\varepsilon = D^* A(x_1/\varepsilon, x_2) D + a_1^*(x_1/\varepsilon, x_2) D + D^* a_2(x_1/\varepsilon, x_2) + q(x_1/\varepsilon, x_2).$$

The coefficients are assumed to be periodic in the first variable and smooth in some sense in the second. We do not require $A(x_1/\varepsilon, x_2)$ to be Hermitian and $q(x_1/\varepsilon, x_2)$ to be real, so the operator $\mathcal{A}^\varepsilon$ is not generally self-adjoint.

Our goal is to study the asymptotic behavior of $(\mathcal{A}^\varepsilon - \mu)^{-1}$ as ε goes to 0. We show that the resolvent of $\mathcal{A}^\varepsilon$ converges in $\mathcal{B}(L_2(\Sigma))$ to the resolvent of an operator $\mathcal{A}^0$ whose coefficients depend only on x_2, and prove a sharp-order estimate on the difference. We also obtain an approximation to $(\mathcal{A}^\varepsilon - \mu)^{-1}$ in $\mathcal{B}(L_2(\Sigma), H^1(\Sigma))$, with a sharp-order error estimate.

Dispersion effects in the propagation of long linear water waves over fast oscillating bottom

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We consider the Cauchy–Poisson problem with localized initial data for the linear water wave equation over nonuniform bottom with rapidly oscillating parts. The characteristic size of the initial perturbation is much more than the characteristic depth of the basin, so we study the propagation of long waves. Physical reasons say that these long wave should be described by the wave equation Interaction of these long waves create the set of short waves which are described by the full water wave equation, which in turn change the structure of wave profile of the main propagated wave. Using the adiabatic approximation and homogenization we show that one should use the linearized Boussinesq type equations for its description. This means that the fast oscillations create effects similar to small dispersion ones. We discuss the following questions: 1) what type of bottom oscillation give the main influence into the profile of the propagated wave and 2) do these type of the dispersion comparable with the standard water wave dispersion and what dispersion is larger? This work was done together with V. Grushin, S. Dobrokhotov, B. Tirozzi, and was partially supported by RFBR grant 14-01-00521 (Russia) and Project RITMARE-CINFAI (Italy).

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Analytic solutions of the quantum time-dependent two-state problem in terms of confluent Heun functions

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We review the solutions of the quantum time-dependent two-state problem in terms of the single- or multiple- confluent Heun functions. We present fifteen classes of models solvable in terms of the single-confluent Heun functions, five classes solvable in terms of the double-confluent Heun functions, five other classes solvable in terms of the bi-confluent Heun functions, and a class solvable in terms of the tri-confluent Heun functions. These classes generalize all the known families of two-, three- or four-parametric models solvable in terms of the confluent hypergeometric functions to more general four- or five-parametric classes involving three-parametric detuning modulation functions. The particular models derived describe different non-linear (parabolic, cubic, sinh, cosh, etc.) level-sweeping or level-glancing processes, double- or triple-level-crossing processes, as well as periodically repeated resonance-glancing or resonance-crossing processes. Finally, we show that more classes can be derived using the equations obeyed by certain functions involving the derivatives of the Heun functions. We present an example of such a class for each of the discussed confluent Heun equations.

Diffraction by an impedance strip. Embedding formula

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A 2D problem of acoustic wave scattering by a segment bearing impedance boundary conditions is considered. The impedances of the sides are assumed to be equal. Due to this, problem can be symmetrized. Symmetrical and antisymmetrical problem are studied in parallel. Following [1] edge green functions are introduced for the symmetrical problem and embedding formula expressing the directivity of the scattered field in terms of the edge green functions directivities is derived. In the symmetrical case it is shown that embedding formula cannot be derived using method of edge green functions. To overcome this difficulty embedding formula derived using other method based on the determinant technique.

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On the resolvent of multidimensional operators with frequently changing boundary conditions

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The work is devoted to studying the asymptotic behavior of resolvent for an elliptic operator in a multidimensional domain with frequent alternation of boundary conditions. We consider a domain with a piecewise smooth boundary. The domain can be bounded or unbounded. On the boundary of domain we select a subset depending on a small parameter. This subset consists of a large number of disjoint parts. As the small parameter tends to zero, the number of the disjoint parts increases while the sizes and the distances between them tend to zero. On these subsets we impose Dirichlet boundary condition, whereas Robin condition is imposed on the rest part of the boundary, so, we have alternating boundary conditions. The structure of alternation is assumed to be non-periodic and rather general. We consider the self-adjoint second order scalar differential operator the second order variable coefficients and deal with the case when the homogenized operator involves Robin boundary condition instead of alternating ones. The main aim of the work is to study the asymptotic behavior of the resolvent of the perturbed operator. We establish the uniform resolvent convergence of the perturbed operator to the homogenized one in the sense of the norm of the operator acting from L_2 into W_2^1. The estimates for the rate of convergence are provided. We also construct complete asymptotic expansion for the resolvent assuming that the domain is unbounded, the alternation is periodic and the resolvent acts on sufficiently smooth functions.

On the high-frequency asymptotic for the solution of hyperbolic systems with multiple characteristics

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Let us consider system

$$\frac{\partial \mathbf{u}}{\partial t} = A\left(\frac{\partial}{\partial \mathbf{x}}\right)\mathbf{u}$$

with high-frequency initial condition

$$\mathbf{u}|_{t=0} = e^{i\omega S_0(\mathbf{x})}\varphi(\mathbf{x}), \quad \omega >> 1.$$

By $\lambda_1(\mathbf{p}),\ \lambda_2(\mathbf{p}), ..., \lambda_m(\mathbf{p})$ denote eigenvalues of matrix $A(\mathbf{p})$. Suppose $\lambda_k(\mathbf{p})$ $(k = 1, ..., m)$ are real so that the system is hyperbolic.

Fourier transformation reduces this problem to Cauchy's problem for ordinary differential equations. With help of the Cayley theorem we obtain the special representation for its solution. After that, method of stationary phase gives the asymptotic expansion for solution, admissible in the case multiple characteristics.

For example, if $\lambda_3(\mathbf{p}), ..., \lambda_m(\mathbf{p})$ are simple and $\lambda_1(\mathbf{p}),\ \lambda_2(\mathbf{p})$ have variable multiplicity, then the asymptotic expansion of solution is

$$u(t,\mathbf{x}) = e^{i\omega S_1(t,\mathbf{x})}\sum_{l=0}^{\infty}(i\omega)^{-l}\theta_{1l}(t,\mathbf{x}) + i\omega\int_0^t e^{i\omega S_{12}(t,\tau,\mathbf{x})}\sum_{l=0}^{\infty}(i\omega)^{-l}\theta_{2l}(t,\tau,\mathbf{x})d\tau + \sum_{k=3}^{n} e^{i\omega S_k(t,\mathbf{x})}\sum_{l=0}^{\infty}(i\omega)^{-l}\theta_{kl}(t,\mathbf{x}).$$

Here $S_k(t,\mathbf{x})$ are solutions of Hamilton–Jacobi equations

$$\frac{\partial S_k}{\partial t} = \lambda_k\left(\frac{\partial S_k}{\partial \mathbf{x}}\right)$$

with initial conditions $S_k(0,\mathbf{x}) = S_0(\mathbf{x})$, $S_{12}(t,\tau,\mathbf{y})$ is solution of Hamilton–Jacobi equation

$$\frac{\partial S_{12}}{\partial t} = \lambda_1\left(\frac{\partial S_{12}}{\partial \mathbf{x}}\right),$$

with initial conditions $S_{12}(\tau,\tau,\mathbf{x}) = S_2(\tau,\mathbf{x})$.

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Calculation of shielding effectiveness by duplex using the method of partial waves

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In the article the problem of microwave shielding by duplex, formed by the glass sheet with thin metal mesh at glass's surface, is solved using the method of partial waves. Duplexes are applied in microwave ovens and in other instruments for protection from electromagnetic emission. Traditionally Shielding Effectiveness (SE) is estimated as ratio of tensions of incident and transmitted fields. The SE value in duplex is consist of reflected, transmitted and multiple reflected repulses. This SE items is usually calculated on the base of analog model or equivalent electrical schemes. In both cases, phases of fields are not taking account, and it leads to discrepancy with experiment result.

In the report the SE value is not separated on items. It turns out the solution of electromagnetic task. The problem is solved by the method of partial waves, had reviewed Arthur R. Von Hippel in 1954. But in that time the problem did not get analytical solution, only the drawing of partial waves. The reason served negative Fresnel's coefficient, coming into existence at transition from more tight media to the less tight media. The difficulty was overcome in authors' previous papers. In the works analytical solutions were obtained in the tasks of microwaves reflection from the dielectric free spaced band and the dielectric on a metal foundation. The same method was used for SE estimation by duplex from glass with thin metal mesh.

Numerical simulation of the bulk strain solitons in cylindrical inhomogeneous shells

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Recently we have developed a refined theory of longitudinal strain waves in a thin-walled cylindrical shell made of a material, governed by the Murnaghan model, and derived the Doubly Dispersive

Equation (DDE) for a linear strain component [1, 2]. This equation has, amongst others, a solution representing a solitary bulk wave, which has been observed in our physical experiments in a duct-like shell. The strain soliton parameters have been estimated and it has been shown that the soliton propagates at the distances many times longer than its wavelength without any significant losses in shape and amplitude.

To study possible applications of the elastic strain solitons in the nondestructive testing of inhomogeneous shells, we consider the evolution of the bulk strain solitary waves in the infinite cylindrical shell with variation of both geometrical and physical properties along the shell axis. We derived the DDE with variable coefficients for the linear strain component u in the case of the variable geometric parameters $h = h(x)$ — width of the shell and $R = R(x)$ — radius of the middle surface:

$$u_{tt} = \left[\frac{1}{hR}\left(\frac{1}{1-\nu^2}(hR\,u)_x + \beta(hR\,u^2)_x + \frac{\nu^2}{12(1-\nu)^2}(h^3R\,u_{tt})_x - \frac{\nu^2}{24(1-\nu)^2(1+\nu)}(h^3R\,u_x)_{xx}\right)\right]_x, \tag{1}$$

and in the case of the variable physical parameters $\rho = \rho(x)$ – density of the shell material and $E = E(x)$ — Young's elasticity modulus:

$$u_{tt} = \left[\frac{1}{\rho}\left(\frac{1}{1-\nu^2}(E\,u)_x + (\beta\,u^2)_x + \frac{\nu^2}{12(1-\nu)^2}(\rho\,u_{tt})_x - \frac{\nu^2}{24(1-\nu)^2(1+\nu)}(E\,u_x)_{xx}\right)\right]_x. \tag{2}$$

In the above formulas ν is the Poisson ratio and $\beta = \beta(\nu, E, m, l, n)$ is the nonlinearity coefficient, which depends on the Murnaghan 3rd order moduli.

To solve the Cauchy problem for DDE with variable coefficients (1) or (2) we proposed, following [3], a conservative finite-difference scheme, which satisfies the integral conservation laws for "pseudo-mass" and "pseudo-energy" of the solitary wave. In numerical simulations, we considered both monotonic and abrupt variations of the geometric and physical properties of the shell and studied the nonlinear wave propagation through the interfaces between different waveguides as well as through local defect areas of the shell. We have shown that the change of the waveguide cross-section or physical properties of the material leads to remarkable variations of the soliton shape and velocity, as well as appearance of the accompanying soliton trains and reflected solitons travelling in the opposite direction. We believe that the results of the numerical simulation are important for understanding both possibilities and limitations of non-destructive testing of shells using elastic bulk strain solitons in technical applications.

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Structured polaritonic environment for nonlinear applications

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In my talk I am going to discuss a variety of problems related to propagation of localised polaritonic wavepacket in semiconductor resonators and waveguides where light and excitons are interacting in the regime of strong coupling. Bulk of this results have been obtained through the collaboration with experimental group in the Univesity of Sheffield. In particular, we are reporting novel type of planar waveguides with the record strong nonlinear response, where we have observed and described theoretically propagation of new temporal, spatial and spatio-temporal solitons. We have also studied resonator wire waveguides, where we have found a novel multi-stability behaviour and solitonic propagation regimes in configurations with pump and in free propagation. Regrading planar strongly coupled resonators, we are reporting observations and theory of the multi-soliton bound states and also reviewing our past results on vector and two-dimensional effects observed in these structures. Our results pave a way towards developing polariton photonic devices for nonlinear processing at ultralow powers.

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Antiquantization of deformed Heun class equations

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Deformed Heun class equations or more precise Heun class equations with added apparent singularity are considered. It is proved that any deformed Heun class equation realizes under antiquantization the transfer from a Heun class equation to the corresponding Painleve equation. The complete list of these transforms is given.

The research of electromagnetic waves diffraction problem on systems of bodies and screens by subhierarchical method

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The three-dimensional vector electromagnetic wave diffraction problem on systems of bodies and screens with irregular geometrical shape is considered. Research of similar problem on the screens is presented in [1].

The problem is devoted to solving of Maxwell equations, which satisfy the boundary conditions and the radiation conditions at infinity. This problem is reduced to the system of integro-differential equations.

From integro-differential equations we obtain the system of linear algebraic equation by Galerkin method. We use the "rooftop" basis function, is introduced in [2].

Modulus of solution of the integro-differential equations, that describe surface current distribution on the systems of bodies and screens, are presented. We use subhierarchical method to solve the system on the bodies and screens of irregular shape.

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Scalar problem of diffraction of a plane wave from a system of two- and three-dimensional scatterers

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The scalar problem of diffraction of a plane wave on a system of two screens with the boundary conditions of the first and second kind and volume inhomogeneous body in quasi-classical statement is considered.

The initial boundary value problem for the Helmholtz equation leads to a system of integral equations on two- and three-dimensional manifolds with boundary. The volume integral operator is weakly singular as well as the operator on the surface of acoustically "soft" screen, integral operator on the surface of acoustically "hard" screen is hypersingular. These operators are treated as pseudo-differential operators in Sobolev spaces on manifolds with boundary.

Owing to known results on properties of the pseudo-differential operators some important results are received: the smoothness of the solution to the system of the integral equations in the interior points of the screens is proved; the equivalence of the integral equations to an initial boundary value problem under assumption of infinite differentiability of the incident field is established. Finally, the unique solvability of the boundary value problem and Fredholm property of matrix integral operator results its invertibility.

Galerkin method for numerical solving of the integral equation is proposed. In the area of the volume body and on the surface of the "soft" screen piecewise constant basis functions are introduced, while piecewise linear functions on the "hard" screen are considered. The description of basic functions is provided in the work, the approximation property as well as the theorem of convergence of Galerkin method is proved.

The numerical results are provided.

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On the problem of propagation of nonlinear coupled TE–TM waves in a double-layer nonlinear inhomogeneous cylindrical waveguide

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The problem of simultaneous propagation of two types of electromagnetic waves (TE and TM) in a double-layer cylindrical dielectric waveguide filled with a nonlinear inhomogeneous medium is considered. These polarized waves have different frequencies and different propagation constants. Similar problems about coupled wave propagation see in [1–3]. The physical problem is reduced to a nonlinear two-parameter transmission eigenvalue problem for Maxwell's equations on a semiaxis.

A numerical method is proposed for determination of the propagation constants for TE and TM cases (see [4, 5]) can be applied for numerical study of propagation of coupled TE–TM waves. Numerical results are reported. Comparisons with linear and nonlinear cases are given. One of the interesting results is that there are new eigenvalues that correspond to a new nonlinear propagation regime. These eigenvalues can not be obtained from the corresponding linearised problem.

The work is partly supported by the RFBR (no. 14-01-31234).

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Estimates of the solutions of the Navier–Stokes equations for viscous compressible fluids

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We are concerned with the problem

$$\begin{cases} \rho(x,t)(\boldsymbol{v}_t + (\boldsymbol{v}\cdot\nabla)\boldsymbol{v})) - \nu\Delta\boldsymbol{v} + \nabla p(\rho) = 0, \\ \rho_t + \nabla\cdot(\rho\boldsymbol{v}) = 0, \quad x \in \Omega, \\ \boldsymbol{v}(x,t) = 0, \quad x \in S = \partial\Omega, \\ \boldsymbol{v}(x,0) = \boldsymbol{v}_0(x), \quad \rho(x,0) = \rho_0(x), \quad x \in \Omega, \end{cases} \tag{1}$$

where $\Omega \in \mathbb{R}^3$ is a bounded domain. The pressure function $p(\rho)$ is positive and strictly increasing. The initial data are close to the equilibrium state, i.e.,

$$\|\boldsymbol{v}_0\|_{W_2^{1+l}(\Omega_0)} + \|\rho_0 - \bar{\rho}_0\|_{W_2^{l+1}(\Omega_0)} \ll 1,$$

where $1/2 < l < 1$ and $\bar{\rho}_0$ is the mean value of the density. We prove that the problem is uniquely solvable in the infinite time interval $t > 0$ and the solution decays exponentially in the Sobolev norms, as $t \to \infty$. The proof is based on the "free work" method due to M. Padula [1] (estimate of an auxiliary function of Lyapunov type) combined with the classical method of localization. It permits to avoid the analysis of explicit solution for the model problems in the whole space and in the half-space. It is also applicable to some free boundary problems.

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Analytical regularization of a generalized eigenwave problem for weakly guiding step-index optical fibers by Muller boundary integral equations

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Muller boundary integral equations (BIEs) are reliable and efficient tools for analysis of the electromagnetic field in the presence of a 2D homogeneous dielectric object with an arbitrary smooth boundary [1]. Particularly, Muller BIEs were used in [2] for numerical calculations of surface eigenwaves of weakly guiding step-index optical fibers. In present work we solve a generalized eigenwave problem on surface and leaky natural waves of such fibers and perform analytical regularization [3] of this problem. We prove that the original problem for Helmholtz equation on the plane with Reichardt radiation condition is equivalent to a nonlinear eigenvalue problem for Muller BIEs with compact operator. We prove a theorem on spectrum localization of propagation constants on an appropriate Riemann surface, and also prove that the set of all eigenvalues of the obtained operator-valued function can be only a set of isolated points on the Riemann surface. Each eigenvalue depends continuously on nonspectral parameters and can appear and disappear only at the boundary of this surface. The integral operator we approximate by a spline-collocation method. The convergence and quality of the numerical method we confirm by numerical experiments.

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Real-time computer visualization of dynamic fluid jets

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In this article is considered the problem of developing an efficient algorithm and its optimization for visualization of fluid jet in dynamics. Visualization is done by approximating the desired surface by triangles. We solve the problem of triangulation. There are many algorithms for numerical solution of this problem, but most of them not so fast that necessary for real-time applications. The resulting program is used in simulation environment and the specificity is that the program must be called at every frame, in addition there are much other logic and computing. This puts severe constraints on the operation time and the number of generated triangles.

Fed to the input of program served an array of global point coordinates, which are centers of the molecules of the liquid. Output we obtain 3D Object Data, which consists of triangles, texture coordinates and normal for each vertex in triangulation. Obviously implies that the problem is divided into two independent subproblems: calculating values of the function at each points of area and triangulation of three-dimensional scalar field in the required grid cells. These issues have been solved in different ways that, giving the desired speed.

There are many ways to optimize the source code. The triangulation in each cell is independent from the adjacent. This allows performing calculations in parallel. We can create as many threads as many cells in partition we have. The knowledge of certain properties of the input points also offers some advantages. For example it is known that the input data may includes many multiple individual spheres and separate figures. So we can quickly triangulate separate spheres by the template and build the grid so that it does not take into account the empty spaces formed by separate figures. Can also exclude duplicate connection of entry points for ease the calculation of values of the function.

Program was successfully tested and widely used in imaging of physics engine and a special simulation environment is connected with medicine.

Asymptotic solution of the heat conduction equation with weak nonlinearity and fast oscillating heat source

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Study of layered structures becomes one of the most intensely investigated areas of the material research in the last decades. However, the problem of numerical calculation of the thermal regime in such systems with a large number of layers (more than 15) becomes almost unsolvable. As a consequence, in this case, it is natural to use asymptotic methods. To be more precise, consider the problem of finding the asymptotic solutions of the nonlinear heat equation

$$\frac{\partial T}{\partial t} = a\Delta T + \varepsilon\omega\cos(\omega t)q(T,z). \tag{1}$$

Here T is temperature, t is time, ε is small parameter, and q is a heat source. Some initial and boundary conditions are specified. The purpose of our study is to find the asymptotic solution of the equation (1) for $\varepsilon \to 0$ and $t \to \infty$. Note that this class of problems typically arises in the theory of solid-state cooler [1, 2]. To asymptotically solve the equation (1), we have successfully used the two-scale expansion method [3]. The performed calculations demonstrate that the time-to-steady-state condition is proportional to ε^2. The results of the numerical calculations are shown in Fig. 1. Finally, it is important to emphasize that the proposed method can also be used for the description of the asymptotic behavior of a point source of high frequency oscillations [4] with nonlinear dependence of the amplitude.

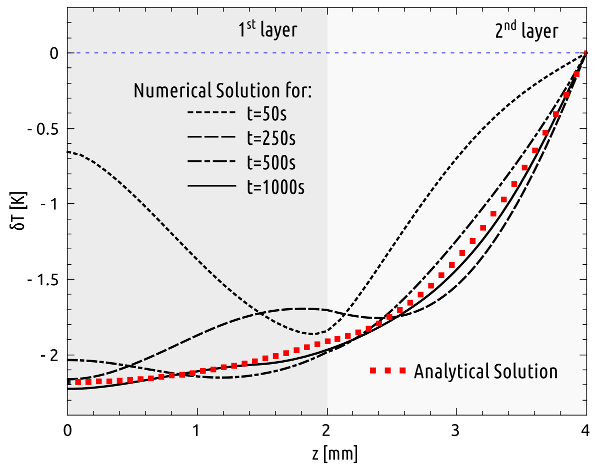

Fig. 1: The comparison of the analytical and numerical solutions of the heat conduction equation in the two-layered structure.

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Diffraction of plane wave on a thin/narrow body: influence of the curvature and torsion

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Analysis of the plane wave diffraction on thin bodies is a classical problem of the diffraction theory. In our paper we investigate a relatively neglected but important aspect: the influence of curvature and torsion of the body. The need to consider these parameters occurs, for example, in the study of reflection from skin structures: blood capillaries, sweat glands, etc [1]. Therefore, for definiteness, we consider the diffraction of an acoustic wave by a capillary. Let S be a smooth curve which describes the capillary tube. All the calculations are carried out in the special orthogonal coordinate system (s, r, φ) [2], determined by S. Here s is the arc length of S, and $\{r, \varphi\}$ are the polar coordinates in a plane perpendicular to S, see Fig. 1. We suppose that the capillary has a round cross-section and denote by $a(s)$ its radius. Moreover, we assume that the capillary is thin, and the radii of the curvature and torsion of the capillary are much larger than the wavelength of the incident field. Acoustical field u is represented as the sum $u^{\text{inc}} + u^{\text{sc}}$, where u^{inc} is an incident wave and u^{sc} is a scattered wave. The scattered field must satisfy the radiation conditions at infinity.

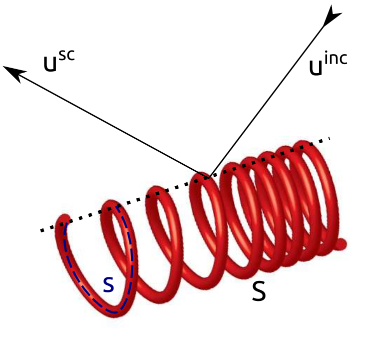

Fig. 1: The schematic representation of the capillary.

The goal of our paper is to determine the scattered field far from the object S, if the incident one is known. The solving approach is based on integral relations and does not require an exact expression for the acoustic field near S. The findings of the work have different implications and provide useful insights for the modeling of biological systems.

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On the solution of a mixed system of Laplace equations for convex domains

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The paper considers a problem of determining the internal and external solutions of a mixed system of Laplace equations with mixed boundary conditions of III kind on a smooth closed boundary. Using potential theory allows us to reduce the problem to solving a system of Fredholm equations of II type. Solvability of the problem follows from the Fredholm theory, but an analytical solution for an arbitrary domain can not be built. In this paper we propose a numerical method for the solution implemented in the environment of Python.

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Homogenization for the wave equation with rapidly oscillating coefficients and small dispersion effects

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We consider the n-D wave equation with rapidly varying coefficients and study the asymptotic solutions with localized initial data. We construct the homogenization procedure and show that under some assumption about the coefficients and the initial perturbation the asymptotic solution is described by the linearized generalized Boussinesq equation with slow varying coefficients. We discuss some explicit asymptotic solutions of the homogenized equation.

This work was partially supported by grants RITMARE-CINFAI (Italy) and RFBR (Russia).

Propagation of laser pulse with a few cycles duration in multi-level media

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Using computer simulation we studied the propagation of a laser pulse with few cycles in multi-level media in 1D case. Maxwell's equations were used for electromagnetic field and the medium was described in the framework of density matrix [1, 2]. We chose the pulse shape with Gaussian envelope and a cosine filling. Influence of the pulse length and pulse initial absolute phase on field-matter interaction was investigated. The spectrum of the pulse that passed through the medium is compared with the original one.

In the case when energy level structure of a medium and the spectrum of the incident laser pulse were chosen so that only transitions between neighbouring energy levels were covered, sequences of transitions were observed. These sequential transitions resulted in population increase at levels where the direct transition from ground level was not possible in such a setup, as well as in a presence of higher than initial frequency harmonics in spectra of final pulses. The passage of the pulse with a wider spectrum and the same energy caused a shorter delay between population upsurges of different levels.

Incident absolute phase dependence was examined for few-cycle pulses propagating in a five-level medium in which many concurrent transition frequences were covered by Fourier spectrum of the initial signal. Additional phase shifts of $\pm\pi/4$ and $\pm\pi/2$ were put inside cosine argument of the initial field distribution. While the initial spectra did coincide, the final ones were different near the transition frequencies. Populations of levels were different for various absolute phases as well.

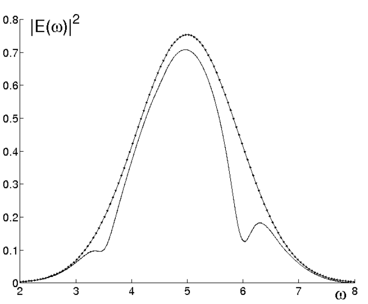

Fig. 1: Comparison of the original (asterix line) spectrum and final spectra (solid line) in phase dependence studies.

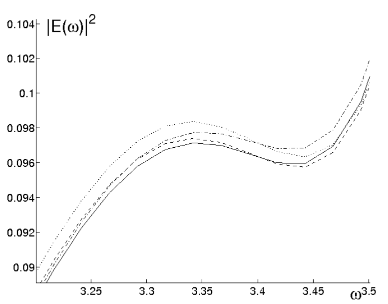

Fig. 2: Zoomed view illustrates spectra differences of pulses with various absolute phases.

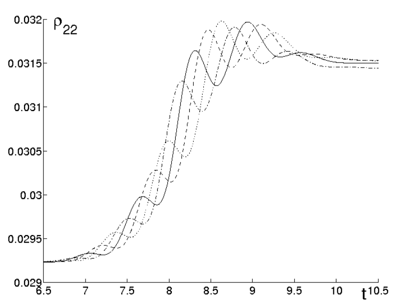

Fig. 3: Time evolution of the second level population in a certain point of a medium for pulses with different phases.

Note that we revealed a cascade mechanism of excitement in a multi-level medium when the mechanism of multi-photon absorption was not involved. This study was made under support of the Russian Science Foundation (Grant N 14-21-00081).

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Spacetime triangle diagram technique for line sources with finite cross section

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The spacetime triangle diagram (STTD) technique, also known as the Smirnov method of incomplete separation of variables [1] is a method for solving problems of electromagnetic and scalar wave motion directly in spacetime. Recent application of this technique to assessment of electromagnetic pulses emanated by transient line sources [2, 3] resulted in several promising technical solutions.

In particular, the phenomenon of wave localization was discovered and roughly described for the models involving slightly subluminal [4], luminal [5] and superluminal [3, 6] line source-current pulses. Unfortunately, these models (as well as a plethora of similar models discussed by other authors — see, e.g., [7] and references therein) results in singularities, which impede making estimations of the critical parameters of the emanated wave. Recent results [8] demonstrated how to reduce the area of singularities (more specifically, getting rid of “lateral”, V-shaped singularities) for superluminal sources, passing to a more complicated model of a line source pulse with a linearly growing front. However, the (highest) singularity at the verge of the V-shape, also appearing in luminal and subluminal solutions, persists. Thus, further developments towards each time more feasible, singularity-free solutions are possible only by taking into account the finiteness of the source-current cross-section. In this case, separating radial variable by the Fourier–Bessel transform, we cannot benefit from the closure equation involving two Bessel functions (like Eq. (37) in [2]), which reduces by two the number of integrals in the Riemann–Volterra formula and eliminates the oscillating factors in the remaining integrand, drastically simplifying the solution.

The report focuses on the alternative possibilities for the wavefunction simplification, which for the case of a finite source-current cross-section contains three, rather than two, Bessel functions (J_0), originated from the direct and inverse Fourier–Bessel transforms and the *ad hoc* Riemann function. It is shown that for a wide range of problems one can rearrange the integrals in the Riemann–Volterra solution, obtaining an analytical expression for the innermost one, which (1) in contrast to its parent Bessel functions, has no oscillating behaviour and (2) is of limited support.

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About the limits of applicability of the model of linear current in gaseous environment

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In the work connected with calculations of electromagnetic pulses resulting from the interaction of ionizing radiation with an environment was used the model of linear current [1, 2]. The physical ground of the model consists in that all interaction Processes take place inside a cylinder of a small radius compared to its height when a narrow beam of ionizing radiation moves through matter. The real total current is a sum of the currents of free electrons resulting from the interaction of gamma radiation with the gaseous environment. The calculation of real field generated with electrons moving inside the cylinder is replaced with calculation of a model field generated by the current pulses moving along a segment directed as cylinder's axis. In this work the components of real magnetic field are presented. They calculated as the sums of the fields of free electrons resulting from an interaction of gamma radiation with a gaseous environment. The comparison manifests the qualitative similarity of the both fields along with some quantitative differences of components of fields. The calculations were fulfilled for gaseous environment at pressure of 10 atm and gamma quanta with an energy 10 MeV. The said differences decrease with an increase of pressure.

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Time-dependent simulations of two-dimensional quantum waveguides of arbitrary shape

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Today's electronic devices like the nanoscale conducting and split-gate devices are rapidly shrinking in their size. In this context, modeling and numerical simulations play an important role in the development and design of new devices. If electronic density is low enough, it is possible to model the system by using a simple one-body Schrödinger equation [1]:

$$i\hbar\frac{\partial}{\partial t}\Psi\left(\mathbf{r},t\right) = -\frac{\hbar^2}{2m}\nabla^2\Psi\left(\mathbf{r},t\right) + V\left(\mathbf{r},t\right)\Psi\left(\mathbf{r},t\right). \tag{1}$$

The topic of this paper is two-dimensional structures (planar waveguides). A potential V can have some arbitrary shape but typically, the structure of the device causes the potential to be very large outside and very small inside quantum waveguide. Originally, the potential is represented by grayscale bitmap image, which can be drawn in any graphic editor (Fig. 1a). Resolution of the image defines uniform grid discretization of the domain. Color (in numerical representation) of the pixel with coordinates $[x,\, y]$ is proportional to the potential V at that point (black represents zero potential). Image then converted to the matrix $\mathbf{V}$ that approximates potential function (Fig. 1b).

We simulate propagation of Gaussian wave packet (Fig. 1c) by solving Schrödinger equation using matrix form of Alternating Direction Implicit (ADI) method. Approximation of the second derivative using the three-point rule gives Hamiltonian of the form:

$$\mathbf{H}_{yj} = -\frac{\hbar^2}{2m(\Delta x)^2}\begin{bmatrix} -2+V_{1j} & 1 & 0 & \dots \\ 1 & -2+V_{2j} & 1 & \dots \\ 0 & 1 & -2+V_{3j} & \dots \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix}. \quad (2)$$

The time evolution operator is obtained by taking matrix exponential of Hamiltonian matrix $\mathbf{H}$:

$$\mathbf{U}_{yj} = \exp\left(-\frac{i}{\hbar}\mathbf{H}_{yj}\Delta t\right). \quad (3)$$

We have N y-directional and M x-directional operators from $N\times M$ potential matrix $\mathbf{V}$.

Solution at time $t+\Delta t$ is obtained by simply multiplying $\mathbf{U}_{yi}$ matrices with corresponding j-columns of wave packet matrix and then multiplying appropriate i-raws by $\mathbf{U}_{yi}$ matrices. Fig. 1d shows probability distribution of finding particle in the waveguide at later time.

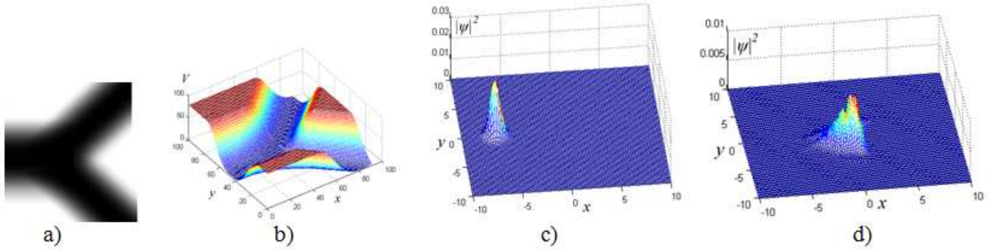

Fig. 1: The graphical representation of the calculated results.

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Quantum walks of photon pairs in twisted waveguide arrays

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Quantum walks involving several particles can be used to implement quantum algorithms, which can perform faster then classical analogues. Of particular interest are quantum walks, where interference of several walkers can be used to realize various simulations, including database search [1]. Quantum walks of photons can be implemented in arrays of coupled waveguides. In particular, coherent quantum phenomena can be potentially simulated in closed-loop arrangements of waveguides [2]. Recently, a new type of coupled waveguides with a twisted geometry were demonstrated in a photonic-crystal fiber [3]. In this work, we study the effect of twist on quantum walks of photons.

We consider an array of closely spaced optical waveguides, which are twisted around a central axis along the propagation direction. Such structure composed of three waveguides is schematically shown in Fig. 1(a). We derive Schrödinger-type equation of the biphoton wavefunction, taking into account the waveguide bending through the appearance of additional phase in the coupling coefficients [4]. We present an example of the evolution of quantum photon-pair state in Figs. 1(b–d). We consider an input condition in the form of Einstein–Podolsky–Rosen (EPR) entangled state, where two photons are present together with equal probability in any of the waveguides, but photons cannot be in different wavegudies, see correlation plot in Fig. 1(b). As the state evolves, the correlation properties can become reversed, and the photons are most likely to appear in different waveguides after a certain propagation distance, see Fig. 1(c). Interestingly, at a longer propagation distance the input state can be almost exactly restored, see Fig. 1(d). These features can be controlled by the amount of twist.

We also analyse integrated photon generation and quantum walks [5], implemented through spontaneous four-wave mixing in optical fibers. Such system can further tailored to produce entangled optical-angular-momentum states, which have applications for quantum communications and imaging.

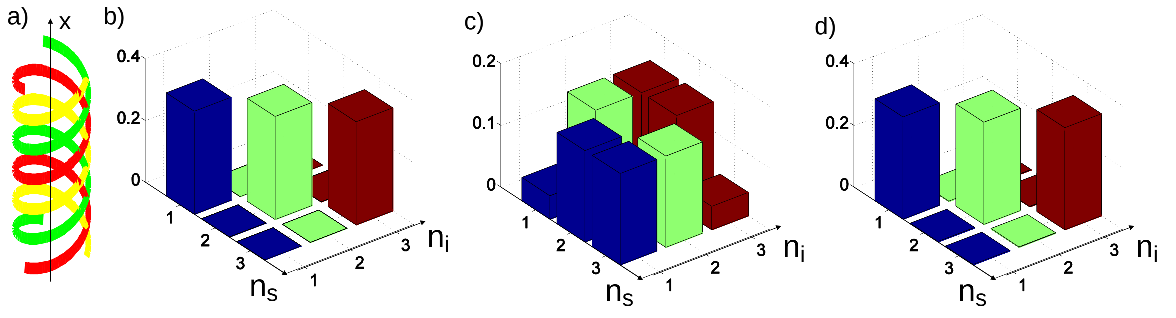

Fig. 1: (a) Scheme of coupled twisted waveguides. (b–d) Two-photon correlations ($|\psi|^2$) between different waveguides at (b) $z = 0$, (c) $z = L/2$, (d) $z = L$.

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Interaction of electromagnetic surface waves guided by dielectric cylinder surrounded by an anisotropic plasma

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In recent years, considerable attention has been paid to studying the propagation of electromagnetic waves in cylindrical channels located in magnetized plasmas. This interest is explained by the use of guiding properties of channels for solving some applied problems related to plasma diagnostics. In this report we consider the waves guided by uniform infinitely extended dielectric column surrounded by a homogeneous magnetoplasma. The axis of dielectric cylinder is aligned with an external static magnetic field. The plasma outside the column is described by the dielectric tensor with nonzero off-diagonal elements. Expression for the tensor elements can be found in [1, 2]. The frequency of the electromagnetic waves belongs to the whistler range, i.e. the frequencies between the lower hybrid resonance frequency and approximately half the electron-cyclotron frequency [2]. These waves play an important role in various basic physical phenomena in magnetoplasma. We consider the case of a resonant magnetoplasma outside the column when the diagonal elements of the tensor have opposite signs.

The dispersion characteristics of guided whistler waves both axisymmetric and nonsymmetric waves are analyzed. It is shown that, under certain conditions, such dielectric column is capable of guiding proper (eigen) modes. The main attention is given to the parametric instability of electromagnetic surface waves propagating in the opposite directions. The instability of the waves may be observed in the presence of the intense time-harmonic external radial electric field. Recall that a three wave interaction can occur if the space-time condition between the external field and the guided waves takes place. The equations for the amplitude of the surface waves can be obtained from the hydrodynamic and the Maxwell's equations. The expression for nonlinear current is found in the approximation of a week nonlinearity. We obtained the expressions for instability increment of guided waves and the threshold value of the external electric field. For the some practically interesting cases, numerical analyses have been performed and the results of computations will be reported.

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Resonant scattering of waves on solitons and vector instabilities in fiber cavities

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We consider the resonant four wave mixing between dissipative solitons and dispersive waves in the ring cavities supporting two modes with different polarizations. The mathematical model of the system can be written in the following form

$$\left[i\partial_t + i\gamma_{1,2} - \delta + \hat{D}_{1,2}\right] A_{1,2} + \left[|A_{1,2}|^2 + \frac{2}{3}|A_{2,1}|^2\right] A_{1,2} + \frac{1}{3}A_{2,1}^2 A_{1,2}^* = P_{1,2}, \tag{1}$$

where A_1 is the complex amplitude of the first mode, A_2 — the amplitude of the second mode, $\hat{D}_1 = \frac{1}{2}\partial_x^2$ and $\hat{D}_2 = \beta^{(0)} + i\beta^{(1)}\partial_x - \frac{\beta^{(2)}}{2}\partial_x^2$ are the operators describing the dispersion of the waves of the first and the second modes, $\beta^{(0)}$ and $\beta^{(1)}$ are the differences of the frequencies and the group velocities of the second and the first modes at $k = 0$, $\beta^{(2)}$ is the dispersion of the second mode, $\gamma_{1,2}$ are linear losses, $P_{1,2}$ are the resonant pumps. The boundary conditions for the annular systems are periodic $A_{1,2}(x) = A_{1,2}(x + L)$ where L is the length of the system.

It is known that the equation (1) allows for single-component soliton solutions stable against the perturbations belonging to the mode of the soliton. We consider the process of the resonant scattering of dispersive waves on the solitons and show that the scattering can destabilize the solitons. We study the development of the instability and show that the dispersive-wave mediated interaction can change the dynamics of the solitons drastically. For example, the interaction can cause the mutual attraction and collision of the solitons, see Figure 1 illustrating this phenomenon.

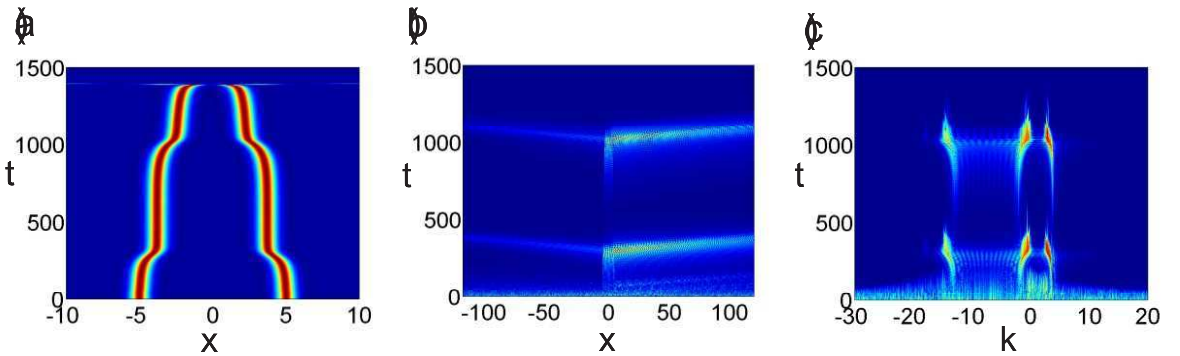

Fig. 1: Panel (a) shows the temporal evolution of the first mode. The initial condition for this mode is two dissipative solitons separated by the distance $d = 10$. The evolution on the field and the spectrum of the second mode are shown in panel (b) and (c). The second mode is initially perturbed by a weak noise. The parameters are $\gamma_1 = 0.1$, $\gamma_2 = 0.01$, $P_1 = 0.25$, $P_2 = 0$, $\beta^{(0)} = -2$, $\beta^{(1)} = 1.5$, $\beta^{(2)} = 0.2$, $\delta = 4.1$.

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Hydrodynamic model for laser swelling of polymer surfaces

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Laser swelling is an increase in volume of the laser-irradiated domain. This phenomenon is of particular interest for laser micro- and nano-structuring of the material surfaces. However, it can also be used in a broader field of applications.

This paper addresses the development of a relaxation model for laser swelling of polymers, which describes this phenomenon as a volume relaxation in glassy materials within the glass transition region. In order to analyze nano-swelling, a hydrodynamic model is presented. Here, the material heated by a laser is considered as a compressible liquid with the shear and bulk viscosities strongly dependent on temperature and pressure.

For numerical solution of the above hydrodynamic model, we used a new approach to the approximation of convective fluxes in the problem with dominating transport phenomenon. This approach is based on the ideas of "KABARE" scheme. The advantage of this approach realized at the Euler grid, is its ability to model very large deformations.

The solutions of the corresponding hydrodynamic equations allow understanding of the features of the nano-swelling process provided by the highly localized laser heating of the material surface. This localized heating can be obtained using near-field optical systems.

An example of calculations by the ray mode parabolic equation

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We consider the propagation of time-harmonic sound described by the acoustic Helmholtz equation

$$\left(\frac{1}{\rho}P_x\right)_x + \left(\frac{1}{\rho}P_y\right)_y + \left(\frac{1}{\rho}P_z\right)_z + \frac{1}{\rho}\kappa^2 P = 0\,. \tag{1}$$

By the ray mode parabolic equation in the ray coordinates [1] we have derived one-mode Helmholtz equation [1]

$$D_{jxx} + D_{jyy} + k_j^2 D_j + \alpha_j D_j = 0\,, \tag{2}$$

where D_j and k_j are the amplitude and wavenumber of the jth mode, and then we have obtained in the ray centered coordinates (t, N) [2, 3] (arc length and distance to the ray) the parabolic equation [1]

$$2ik_{j0}u_{jt} + ik_{j0t}u_j + u_{jNN} + (2(k_{j0}k_{j2} - k_{j1}^2)N^2 + \alpha_{j0})u_j = 0\,,$$

where u_j is the main term of WKB-expansion of D_j and for a given function $f = f(t, N)$ we use denotations $f_0 = f|_{N=0}$, $f_1 = f_N|_{N=0}$ and $f_2 = f_{NN}|_{N=0}$.

We have provided various examples of calculations. Thus, we consider the cross-slope propagation in the wedge environment. The solid curve in Fig. 1 represents a case of such a solution. The dashed curve in Fig. 1 is an analytical solution obtained on the basis of the work [4] results (method of source images).

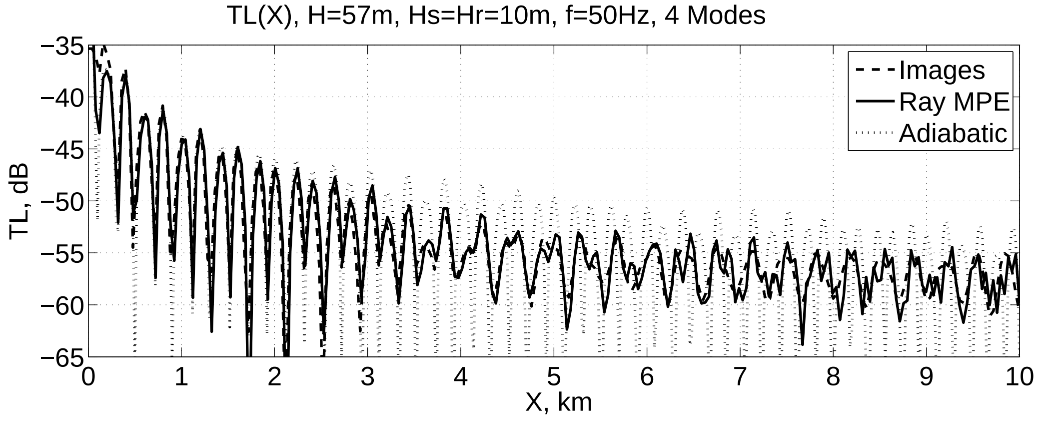

Fig. 1: The transmission losses for the cross-wedge sound propagation obtained by the three methods: source images, ray mode parabolic equation, adiabatic mode parabolic equation.

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Iterative approach in asymptotic analysis of electromagnetic scattering from 2D periodic arrays of thin conductors

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In this work we present a short description of iterative approach in asymptotic analysis of electromagnetic plane wave scattering from 2D arrays of parallel thin cylindrical conductors of finite length, perfectly conducting and absorbing. The analysis is based on the method of boundary integral equations [1]. These arrays could be infinite and finite in size. In the analysis we also assume that the ratio between the conductor radius and the wave length is a small parameter (thin conductor asymptotic approximation). Thus, as a starting point, the method of boundary integral equations is reduced to a system of 1D integral equations. In theoretical electrodynamics study of scattering from 2D arrays of parallel thin cylindrical conductors has been very popular for the last five decades and presented in a huge number of papers and monographs. Historically, firstly in the beginning of radio science we have to mention the theory of thin wire antennas, made of active and passive vibrators, and secondly, the analysis of phased antenna arrays (see, for example, [1], [2]). Nowadays the modern theory of meta-materials and meta-surfaces includes the analysis of wave propagation through 2D and 3D arrays of cylindrical thin conductors in the long wave length limit as a basic and important part (see, for example, [3]).

In the model we also assume that the conductors length and the periods of the 2D array are equal or greater than half of the wavelength. In this case the interaction between the conductors leads to a smaller contribution into the magnitude of electric current in comparison with a term induced by the incident plane wave. In the case of a very large size arrays the proposed approach of iterative techniques of electric current computation gives advantages because it is faster and requires less computer resources than known solvers of boundary integral equations method, for example ANSYS HFSS and Microwave CST studio. The key point in the approach of iterative techniques is usage of asymptotic approximation for a single finite length cylindrical conductor, developed by L. Weinstein [4] by means of the factorization method. To be more exact, we employ the approximate solution for electric current induced by a incident plane wave along a thin conductor as a sum of current wave generated by cylinder tips (see [4]). Weinstein solution is the first iteration in the proposed approach of iterative techniques for electric current computation. The next iteration takes into account the interaction between conductors and based on a new analytical asymptotic approximation developed by authors, thus generalizing Weinstein solution. We demonstrate that the numerical data for directivity pattern for one version of finite array obtained by means of our iterative asymptotic approach agrees with the results computed with the help of boundary integral equation method on the basis of the system ANSYS HFSS.

This work could be considered as one of the ways of studying electromagnetic waves reflection from coated metallic surfaces. In future analysis multi layers of 2D arrays of absorbing thin conductors will be incorporated into combinations with different dielectric layers.

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Propagation of finite energy Airy pulses in dispersive media

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In this paper, we propose an analytic method capable of describing time truncated Airy-type pulses in material media with quadratic dispersion. The method is based on the superposition [1] of exponentially apodized Airy pulses [2, 3] and provides a simple, elegant and fast way to obtain the pulse evolution and its characteristic resistance to the dispersion effects for long distances.

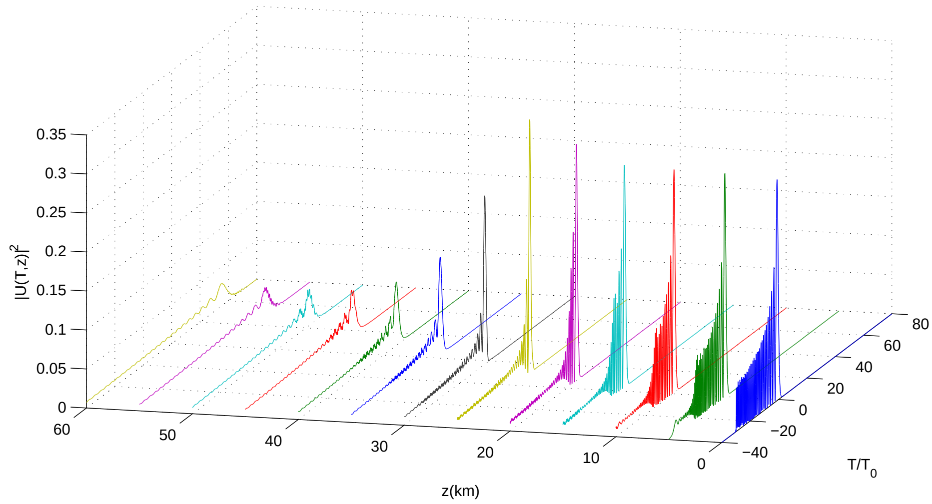

Fig. 1: Temporal evolution of time-truncated Ideal Airy pulse at different propagation distances

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Leaking P-SV modes of high-velocity elastic layer embedded in elastic medium

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Elastic waves comprising leaking modes attract more and more attention of geophysical prospecting community due to the potential of P-wave velocity retrieval in contrast to conventional normal Rayleigh modes based inversion techniques [1, 2].

Described by complex roots of dispersion equations leaking modes demand efficient and automatic numerical algorithms for their evaluation as parametrized curves on Riemann surface. We develop an approach suggested by G. I. Petrashen called the outpost method [3]. The natural idea is to catch roots numerically during their motion from complex infinity to the real axis (in frame of the Lamb method) at some curve called outpost and continue them further with another algorithm inside the domains of significant wavefield effect.

We supplement the approach with a convenient choice of the outpost line. Closed curve at the main (++) sheet is not enough because roots appear from behind the branch cuts. The concept is to take closed on Riemann surface curve surrounding all the vicinities of branch points at every sheet. It allows to control all the leaking modes arising up to a certain frequency of interest. Also we introduce a semi-analytical approach for tracing through peculiarities of leaking modes curves like roots fusion points or passing through the branch points. Computation of all the significant modes excited in the model both leaking and normal at once is available now. In this sense, roots on complementary sheets act as auxiliary tool. Nonetheless, they have independent applications [4].

Despite undoubted importance of model problems with one elastic layer, at the moment there is a comprehensive notion only for one velocity (acoustic) layer environments [3]. Several papers concern the problem with low-velocity layer [5, 6, 7]. We apply our edition of outpost method for a problem with high-velocity elastic layer. Leaking modes curves for that type models were firstly published apparently in [8]. For a representative set of material parameters leaking modes curves are studied at 4-sheet Riemann surface of dispersion equation. Obtained dispersion curves are compared with velocity-frequency spectra of numerically modeled seismograms.

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Workshop on metamaterials

Femtosecond spectroscopy of the electron thermalization in gold in the vicinity of Tamm plasmon resonance

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Tamm plasmon-polaritons (TPPs) have attracted huge interest since their first experimental observation in 2008. TPPs are surface states which arise at the boundary of 1D photonic crystal and metal film [1]. They are greatly analogous to the well-known surface electromagnetic waves and surface plasmon-polaritons, yet have distinctive features. TPPs exist for both s and p polarizations of the incoming light and do not require satisfaction of in-plane phase-matching conditions, thus they can be excited at any angle of incidence. Experimentally, TPP can be detected as a narrow absorption resonance in reflectance spectrum of a PC/metal structure. Electromagnetic field localization [2, 3] in the TPP mode leads to the enhancement of the nonlinear-optical phenomena, as it was demonstrated recently [4, 5].

Studied sample of one-dimensional PC consists of 6 pairs of ZrO_2/SiO_2 layers (average thicknesses 110 nm and 145 nm, respectively) covered with a semitransparent 30-nm-thick gold film. Measurements of the reflectivity change were performed in a conventional cross-polarized pump-probe scheme with a Ti:Sa laser as a source of radiation (fig. 1a). Pump pulses had an energy of 60 pJ, while probe pulses were approximately 25 times weaker. Pump and probe beams were independently focused into the 40-µm wide spots on the sample. Wavelength was tuned in the range of 700–820 nm.

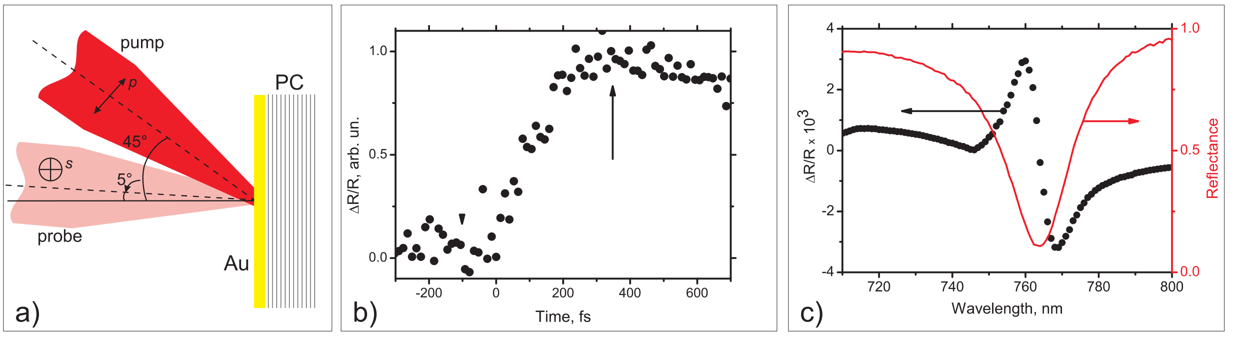

Fig. 1: (Color online) (a) Sketch of the experiment. (b) Transient reflectivity measured in the Au/PC sample at 760 nm. (c) Dots — experimental spectrum of the relative change in reflectivity of the Au/PC sample. Solid red curve — experimental reflectance spectrum of the Au/PC sample.

Figure 1b shows measurements of the time-dependent change in reflectivity of the Au/PC sample. Before the pump pulse have illuminated the sample (time less than zero) no change in reflectivity was observed. When probe pulse reflected from them pump-illuminated sample, the change in reflectivity appeared, reaching its maximum at approximately 250 fs after the zero time, and decaying for approximately 4 ps (not shown). Arrow indicate point where data was acquired for a spectroscopic measurements.

Figure 1b shows reflectance spectrum of the Au/PC sample with a solid line. A TPP-associated resonance is observed with a minimum at 765 nm. Spectrum of the relative change in reflectivity $\Delta R/R$ is shown with black dots. It exhibits a derivative-like shape in the vicinity of the TPP resonance. Maximal absolute values of the $\Delta R/R$ are observed at 760 and 770 nm at the falling and rising edges of the TPP resonance respectively. Corresponding values of the reflectivity change are in the order of $3{\cdot}10^{-3}$, which is approximately 10 times larger than the previously reported $\Delta R/R$ values for a bare gold film [6].

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Possible realizations of spaser operation

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Recently, the impact of the interaction of excited quantum systems such as atoms, molecules, quantum dots etc. with plasmonic structures on the optical properties of the latter has attracted a great deal of attention because of the rapid development of nanoplasmonics [1, 2]. Among applications of nanoplasmonics, the spaser is of particular interest [3]. The spaser is a nano-sized coherent source of near-field of optical frequencies. After its theoretical prediction [3], there have been many attempts to realize spasers experimentally. A subwavelength spaser, in which plasmons are confined in all directions, was realized in the pulse regime [4]. Such a spaser may consist of a spherical plasmonic nanoparticle surrounded with pumped dye molecules. The continuous wave regime was demonstrated only for distributed spasers [5–7].

We give a comparative analysis of different realization of surface plasmon amplification: subwavelength spaser, distributed spaser and spaser driven by an external field. It is shown that transition to driven spaser makes easy the achievement of coherent near field generation.

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Calculating of effective permittivity and permeability of metallic and metallized dielectric particles

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In this work, we modelled metallic particles and dielectric spherical particles covered by metallic layer. We obtained dependencies of real and imaginary parts of the effective permittivity and permeability of powders from metallic and metallized dielectric particles from frequency, radius and permittivity of the particle, thickness and conductivity of the metallic shell. We used finite element method for calculation of the electromagnetic fields in the investigated model and combination of impedance lows for perfect electric conductor and perfect magnetic conductor to calculate effective electrodynamics parameters. This calculation technique allows to take into account not potential field inside the particles. Calculation results compared with previous results obtained in a quasi-stationary approximation [1] and with experimental data.

According to obtained real part of the dynamic effective permeability the powder from such kind of particles exhibits diamagnetic properties. The imaginary part of the effective permeability is large enough compared with imaginary part of the effective permittivity and reaches its maximum values at the thickness of the metallic layer less than the skin depth. Therefore, microwave heating of such particles is due to artificial magnetic losses. Magnetic losses of metallized dielectric particles are about twice greater than of pure metallic particles. Moreover, real metal particles coated with a layer of absorbing oxide are heated not only due to electric losses in the oxide as was proposed in [1] but also owing to magnetic losses in the conductive core.

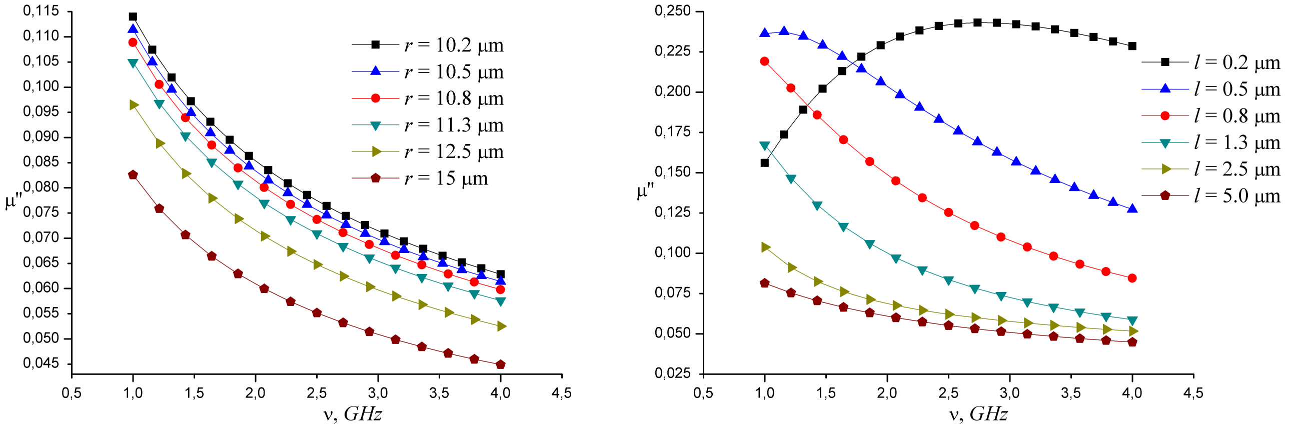

Fig. 1: Imaginary parts of the dynamic effective permeability of pure metallic conductive particles (left) and dielectric-coated conductor particles at different thickness of the conductive shell. The radius of the dielectric core is $r = 10$ μm. The conductivity is $\sigma = 5.8e^7$ S/m.

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Dark-field spectroscopy of whispering gallery mode cavities

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Optical microcavities are extensively used nowadays in various application areas, such as signal filtering, wavelength division multiplexing, generation of optical combs and multiple-wave mixing and high-sensitivity detection [1]. Most of these applications have become possible due to ultra-high quality factors of the cavities that have been achieved. The performance characterisation of such devices rely on obtaining their transmission spectra. Both commonly used methods of excitation of wispering gallery mode (WGM) resonators — by prism or by tapered fiber — rely on near-field coupling with the mode in resonator. In this work we report on observation of WGM excitation in a microdisk resonator via free-space coupling. The cavities under study were fabricated by two-photon polymerization, this method offers one high versatility in both shape and sizes of the fabricated cavities [2].

The excitation of whispering gallery mode was implemented in dark-field geometry, i.e. the incident excitation wave falls at an oblique incidence on the cavity. The scattered radiation was collected using dark-field confocal scheme, which collects only scattered light from the spot with lateral dimensions of several microns. The scattered radiation spectra and spatial field intensity distribution were studied. Since the distance between the adjacent maxima in the field distribution is equal to the wavelength of the light inside the cavity, one can observe the scattered field spatial distribution directly. This distribution corresponds to the interference pattern in WGM resonators and can provide the information about the effective parameters of the microresonator. By using the confocal scheme for the spectroscopy, we have measured the spectrum of the scattered light, which can give one the information regarding the Q-factor and finesse of the resonator.

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Plasmonic and all-dielectric nanoparticle anti-reflective coatings

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Thin-film solar cells is rapidly growing sector of photovoltaics industry, but the wide range of possible applications is limited by their low efficiency of light absorption in the thin active layer [1]. Plasmonic anti-reflective coatings were suggested as promising candidates for overcoming this problem [2]. Despite the active studies on the topic and great promises in the applications, practical use of plasmonic nanoparticles is still hindered by many challenges: large ohmic losses, oxidizing under ambient conditions [3]; plasmonic resonance depends on the particle shape rather than its size. Recently it has been proposed to implement all-dielectric nanoparticles with high refractive index for anti-reflective coatings [4]. Silicon is considered as one of the most promising materials for all-dielectric photonics as possesses high refractive index in the visible and near-infrared wavelength ranges with relatively small optical losses. We present a comparative analysis of plasmonic and all-dielectric anti-reflective coatings based on periodic arrays of silver and silicon spherical nanoparticles (see Fig. 1(a)). We show that silicon coatings demonstrate strong anti-reflecting properties due to interference of light scattered by magnetic and electric dipole that can be considered as Kerker-like condition [5]. Contrary to the anti-reflection effect observed with plasmonic coatings where the magnetic dipole moment arises from the displacement charge induced in the substrate, the silicon particles possesses their own intrinsic magnetic resonance. Moreover, silicon based coatings have narrow banded transmission enhancement in the spectral range between the magnetic and electric resonances. The strong dependence of transmission resonance on nanoparticle radius can be utilized for narrow band detection. The dependence of the integral enhancement on the nanoparticle radius is plotted for different types of coatings in the Fig. 1(b). For the case of Si-NPC there is a maximum of integral enhancement (7.6%) for nanoparticles of 35 nm radius, which support mainly MD resonance.

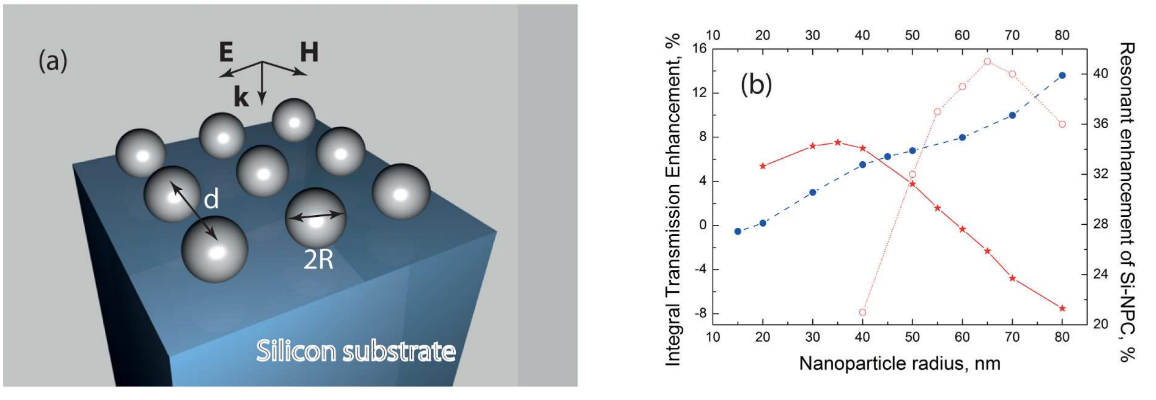

Fig. 1: (a) Square periodic array of spherical nanoparticles with radius R on the top of a-Si:H surface. (b) Left axes: integral enhancement for different type of coatings: Ag-NPC (blue circles) and Si-NPC (red stars). Right axes: the resonant enhancement for silicon coatings (red circles).

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Lifting and revival of degeneracy of surface plasmons localized in a nanosphere crossing an interface

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Surface plasmon resonances localized in the noble metal nanoparticles found many applications in science, technology and medicine [1, 2]. Being due to the collective oscillations of all free electrons in the nanoparticle they are characterized by extremely large oscillator strength. Hence, the scattering and absorption cross sections are also very large. As compared to other strong absorber like organic dye molecules and semiconductor quantum dots, noble metal nanoparticles are much more robust, stable and biologically inert. Taking into account that the noble metal nanoparticles with different sizes are commercially available, it is tempting to explore different possibilities of their use. In this contribution, we concentrate on the peculiarities of splitting of the plasmon resonance when the noble metal nanosphere approaches, crosses, and finally, leaves the interface of two transparent media with different refractive indices and show that this splitting may be used to monitor the particle motion with the nanoscale resolution.

The dipole surface plasmon resonance localized in a nanosphere is three-fold degenerate. This degeneracy disappears when the spherical symmetry is broken by any reason. Numerous results were obtained regarding the splitting of plasmon frequencies in the nanoparticles whose form deviates from the sphere, in nanosphere agglomerates, and in nanospheres approaching an interface. We concentrate our attention on one peculiarity of the last splitting that was not elucidated previously, namely, on the restoration of the degeneracy in the middle of the nanosphere passage from one medium to another. First, this revival of degeneracy was pointed out in [3]. Due to the perturbation approach used in [3] the degeneracy revival was obtained when the sphere center coincides with the interface plane. We performed numerical simulations that showed that the point at which all three dipole modes are degenerate belongs to the media with smaller refractive index.

Both resonances may be observed in the far field optical absorption experiments with p-polarized light. Depending on the difference between the refractive indices of two adjacent media extinction spectra would have two separate maxima or just one broadened maximum. These splitting or broadening being the same order of magnitude as the shift common to both dipole modes are useful additional parameters for estimating the particle position relative to the interface.

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Left-handed wired material

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Numerical 2D simulation method based on the solution of Maxwell's equations shows that the ordered wire structure is the left-handed medium under the certain conditions. The dependences of the effective permeability and permittivity of the structure on the frequency of the electromagnetic wave, on the lattice period, on the permittivity and permeability of the medium in which the wire structure are calculated. It is shown that effective permeability and permittivity simultaneously negative in a wide frequency domain. Moreover, investigated structure shows the effects of left-handed medium in this area.

Consider the composite consists of periodically spaced rectilinear conductive wires, placed in a medium with permittivity ε_m, permeability μ_m. The distance between the wires (a), and their length is much greater than the radius (r). Previously, such a structure is considered in [1] and [2]. By PMC/PEC Boundary Conditions method the dependences of the effective permeability and permittivity from values ε_m, μ_m, the period of the wires (a) and the wavelength of the electromagnetic field in the medium with ε_m, μ_m were obtained (see Fig. 1). Value λ_p is the plasma length is taken from [2].

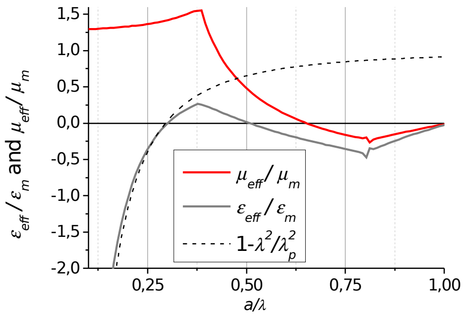

Fig. 1: Dependence of the effective permeability and permittivity of the structure on the ratio of the period (a) to the wavelength (λ), $\lambda = 2\pi c/\left(\omega\sqrt{\varepsilon_m \mu_m}\right)$, $r = 0.05a$. The conductivity of the wires over 10^4 S. Imaginary components of effective permeability and permittivity at these conditions are small.

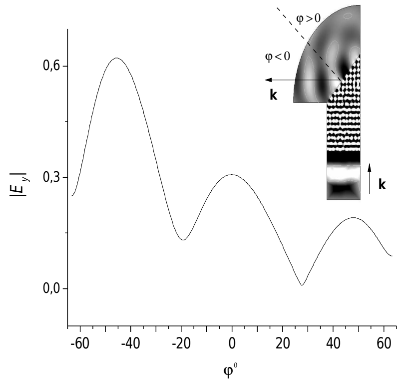

Fig. 2: The angular distribution of the absolute value of the vector of the field strength (E_y) of the electromagnetic wave transmitted through the prism of the wired material. The dark area — the maximum values of E_y, light area — minimal.

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Bistability in rf-SQUID based meta-surfaces

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We analyze the interaction of electromagnetic wave with two dimensional array of superconducting split ring resonators containing Josephson junction in the gap. These resonators are known in the literature as superconducting quantum interference device (rf-SQUIDs). The effective thickness h of such "meta-film" is much smaller than the wavelength ($h \ll \lambda$). We demonstrate that despite of small thickness in the linear approximation, such array effectively controls polarization of reflected and transmitted fields. In particular, it changes polarization of the reflected wave and angle of rotation is determined only by the orientation of the rf-SQUIDs. This effect is similar to the Kerr's effect in a gyrotropic medium. The polarization of transmitted wave also changes and depends both on the carrier frequency and on the orientation of rf-SQUIDs. This is similar to the Faraday's effect. Increasing the intensity of the incident field causes a larger current in the ring and subsequently a nonlinear response of the rf-SQUIDs. We showed that nonlinear response of rf-SQUIDs leads to bistability in the field refraction on such meta-surface and found key parameters characterizing this bistability.

Spatial dispersion in metamaterials based on three-dimensional arrays of spheres and disks

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Artificial structures based on metamaterials provide new unprecedented possibilities in the control of light-matter interactions. One of such novel degrees of freedom is a possibility to tailor metamaterial spatially dispersive response. In the present work, we study the properties of two artificial three-dimensional discrete structures employing the discrete dipole model that allows one to describe the nonlocal effects in a self-consistent way [1, 2].

The first example is a structure consisting of plasmonic spheres located in the sites of a cubic lattice. From the standpoint of effective medium model such structure is an isotropic local dielectric. However, the analysis of the structure properties demonstrates that in the frequency interval in the vicinity of the individual particle resonance the structure acquires anisotropic properties due to spatial dispersion effects. This phenomenon known as spatial-dispersion-induced-birefringence was observed experimentally for natural materials but was sufficiently weak. In the case of metamaterials, the anisotropy due to spatial dispersion effects can be enhanced significantly and can be directly detected by measuring the reflection coefficients for two different polarizations of the incident wave [3]. Moreover, the discrete dipole model predicts also that the eigenmodes of the structure are neither transverse nor longitudinal but have mixed polarization state.

The second example is a structure consisting of dielectric disks placed in the sites of a cubic lattice. We assume that the disks possess both electric and magnetic polarizabilities, magnetic polarizability being due to the circular displacement currents that can be excited in the disk. Effective medium model predicts topological transitions in the structure when either ε or μ changes its sign (Fig. 1). On the other hand, it is known that spatial dispersion effects are especially pronounced in the vicinity of the topological transition [1]. We investigate features introduced by spatial dispersion effects in more details. Additional modes arising due to nonlocality (namely, longitudinal magnetic mode) are explored.

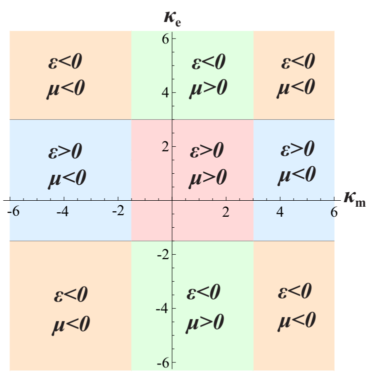

Fig. 1: Topological transitions in the structure composed of dielectric disks. $\kappa_{e,m} = 4\pi\alpha_{e,m}/a^3$, $\alpha_{e,m}$ is electric or magnetic polarizability of the disk.

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Nanophotonic components for telecom applications

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Optical components, exploiting properties of surface plasmon polaritons (SPP) as guided waves, unlock one of the ways to on-chip photonics, which demands high data bandwidth, low power consumption, and small footprint of the integrated circuits [1]. SPP represents a surface wave, propagating along a boundary between two media, which possess opposite signs of the real part of permittivity (for instance, metal at the frequencies lower than the plasmonic frequency and dielectric). Electrons on a metal boundary can perform coherent with electromagnetic field collective motion [2], and thus support a strong mode confinement at the subwavelength scale, which is an attractive feature allowing to scale down the size of integrated optical circuitry.

Various of nanoplasmonic devices starting from passive components like waveguides [3], couplers and polarization beam combiners [4], to active components, e.g. ring and disk resonators [5], and modulators [6] have been recently demonstrated. Authors of [3] reported a successful transmission of 480 Gbit/s aggregated data traffic (12 channels $\times$ 40 Gbit/s) over dielectric loaded plasmonic waveguides.

Dielectric-loaded SPP waveguides (DLSPPWs) consist of a dielectric stripe deposited on top of a metallic film and represent one of the basic configurations of the plasmonic waveguides [7]. DLSPPWs demonstrate sub-wavelength confinement of SPPs with the typical propagation distance of 50 mm at 1550 nm wavelength [8]. The propagation length can be significantly improved in the so-called long-range DLSPPWs (LR-DLSPPWs) that provide both mm-long SPP guiding and relatively tight mode confinement [9]. Experimental investigations of LR-DLSPPWs operating at continuous-wave telecommunication band signal (1550 nm) demonstrated propagation length 500mm with the mode size 1 mm [10]. In this paper we report successful transmission of 10 Gbit/s on-off-keying (OOK) modulated signal through the LR-DLSPPWs with almost negligible degradation of the data flow consistency.

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Electromagnetic bandgap metasurfaces for decoupling of MRI body coil-array elements at 7 Tesla

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Electromagnetic bandgap (EBG) structures are an important class of metamaterials that prevent wave propagation at certain frequencies [1]. One of promising application areas of such structures is magnetic resonance imaging (MRI). Nowadays MRI technology develops towards using higher static magnetic field strengths. Particularly development of novel receive and transmit body arrays for prospective 7T scanners operating at 298 MHz is of a great importance. Such devices allow achieving higher image resolution and better diagnostic accuracy. As an alternative to conventional magnetic coils in [2] electric dipole antennas were proposed in order to reduce SAR keeping a high penetration depth. One of challenges in using dipole antennas as array elements in MRI is their high mutual coupling, which limits the reachable image resolution. In the present work we develop EBG metasurfaces with highly miniaturized unit cells for suppression of the mutual coupling between two closely spaced dipole antenna elements of a 7T MRI body coil array. Metasurface samples were optimized by means of full-wave numerical simulations and manufactured as multi-layered structures containing PCB-boards. The metasurfaces exhibit measured isolation of better than −15 dB for a pair of half-wave dipoles with the separation of 1/12 of the wavelength. The developed metasurface samples can be employed as decoupling structures for each pair of neighbouring elements in 7T-MRI dipole-type body coil arrays, which can lead to better reachable resolution of deep scan areas with low SAR.

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Broadband antireflective coatings based on 2D-periodic arrays of subwavelength nanopores

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Antireflective coatings (ARC) have a wide range of uses in optics and semiconductor devices; however traditional homogeneous thin-film ARCs have a number of limitations related to material availability and fabrication processes [1]. Therefore, alternative ARCs are being actively developed generally based on some type of surface nanostructuring [2].

In Refs. [3] and [4] a novel concept of ARC, demonstrating a qualitatively different mechanism of antireflection, was theoretically proven to be viable. The ARC consists of a sparse 2D periodic grid of nanopores in a dielectric substrate, which introduce spatial dispersion of the refractive index, allowing for broadband suppression of reflection. Analytically, the array of nanopores is modeled as an array of point dipoles embedded in the substrate; as the nanopores have negative electric polarizability, they introduce an additional phase shift in the reflected waves, allowing for antireflection more broadband than is possible with traditional homogeneous thin-film ARCs. The ARC was fabricated by electron beam lithography in a layer of polymethylmethacrylate. Experimental reflection spectra show a reduction of reflection from the nanostructured samples down to a minimum of 0.5% and broadband reduction to less than 1%, performing at least as well as a matching single-layer homogeneous ARC. FDTD simulations of the pore arrays, whose geometrical properties were measured by AFM, show good agreement with experimental results. By varying the geometrical parameters of the nanopore array we can tune its antireflective parameters, even surpassing traditional single-layer homogeneous ARCs in antireflection bandwidth.

An important advantage of our ARC is that it can be optimized for and manufactured in any dielectric material — meaning that it can be used to create ARCs for materials for which traditional single-layer homogeneous ARCs are not available.

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Analysis of plasmonic nanostructures of different geometries in organic solar cells

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It is known that the insertion of periodic cylindrical silver nanoantennas on organic thin-film solar cell (OSC) increases its efficiency of light absorption [1, 2]. However it is needed the analysis of other geometries in order to achieve devices that are more efficient. In this study, it was performed the calculation of efficiency of nanoantennas placed in solar cell by using finite element method. Four types of nanoantennas with different geometrical parameters were used, preserving the superficial area of the cylindrical nanoantenna. The geometries are conical antennas, “barrel shaped", semi-spherical and spherical. We have found that the spherical antenna showed a better solution with greater efficiency compared to the conventional cylindrical case, where the increase in light absorption efficiency is for the TM-polarized light in accordance with AM1.5G spectrum.

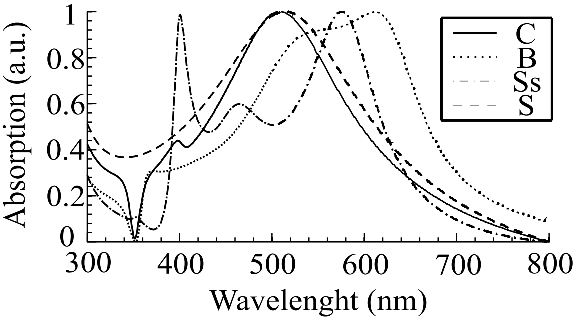

Fig. 1: Absorption of nanostructures for normal incidence (C — Conical, B — Barrel, Ss — Semi-sphere, and S — Sphere).

From the initial results of numerical studies, we confirmed of 8.13% OSCs increase in the efficiency of absorption (Fig. 1) at the wavelength range 300–800 nm with the use of conical antenna. The spherical geometry showed good absorption, but, in accordance with AM1.5G factor, had less efficiently than the conical.

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Combination of experimental and analytical approaches in the design problem for metafilms

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Currently the design of 2D metamaterials (metafilms) is among the most actual problems of applied optics. Many works using numerical algorithms are dedicated to calculation of their electrodynamic characteristics (i.e. to the solution of the direct problem). As an alternative one can use analytical approach [1–3] in which transmission T and reflection R coefficients of a metafilm are computed via known polarizability matrix $\|\alpha\|$ of single subwave particles. This approach is much less labor-consuming and can easily be integrated with natural experiment. In this work, the algorithm based on such analytical approach is constructed. It allows to design metafilms with given spectral properties.

The solution of the design problem for the metafilm is reduced to the multiple solutions of the direct problem. The latter consists of two stages. 1) Polarizability coefficients of single particles are determined. 2) One passes to the surface polarizability density. Thus it is possible to write down the effective boundary conditions for the metafilm [4]. It leads to the equations for transmittance T and reflectance R. The solution of the system can be presented in a simple analytical form. In the specific case of normal incidence on the metafilm made of spherical particles for temporal dependence $e^{i\omega t}$ they can be written as follows [2, 5]:

$$T = \frac{1 + em}{1 - em + e - m}; \quad R = \frac{-e - m}{1 - em + e - m}, \tag{1}$$

where $e = ik_0\alpha_{es}/2$, $m = ik_0\alpha_{\mathrm{ms}}/2$, $k_0 = \omega/c$ is the free-space wave number, ω is the incident wave frequency, α_{es} and $\alpha_{\mathrm{m}s}$ are the surface polarizability densities which have the following form:

$$\alpha_{es} = \frac{n\langle\alpha_e\rangle}{1 - \frac{n\langle\alpha_e\rangle}{4r}}; \quad \alpha_{ms} = \frac{n\langle\alpha_m\rangle}{1 - \frac{n\langle\alpha_m\rangle}{4r}}. \tag{2}$$

Here $n = 1/l^2$ is the concentration of meta-atoms, $r \approx 0,6956\,l$ is the so called impact domain radius [1], l is the metafilm period, α_{e} and α_{m} are electric and magnetic polarizabilities; angle brackets mean the averaging over the metafilm surface [6].

Spectra of T and R can be measured in both numerical and natural experiments. Having these measurements been carried out for several fixed values of incidence angle θ, one obtains the system of equations in the matrix $\|\alpha\|$ elements. Having enough amount of measurements (for the system not to be sub-definite) the system (1) can be reversed [7, 8]

$$e = -\frac{T+R-1}{T+R+1}; \quad m = \frac{T-R-1}{T-R+1}. \tag{3}$$

After substituting these values into general formulae for transmittance and reflectance [5], T and R can be calculated for arbitrary θ. In similar manner, one can conduct measurements with a set of metafilms with different periods l and get spectral characteristics by means of reversion for arbitrary l.

Thus the algorithm that makes it possible to solve the design problem for metafilms consisting of spherical resonators is proposed. Implication of analytical formulae for determination of polarizability matrix components for constituent particles via transmittance and reflectance spectra allows to sufficiently increase the productivity of numerical calculations. The algorithm can be easily generalized for the case of more complicated geometry of particles (for instance, bianisotropic U-shaped resonators). Also it is possible to account for statistical straggling of the particles' geometric parameters.

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Theory of nontypical low-threshold optical nonlinearity of a dielectric nanoparticles

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In recent years, experimental investigations of the nonlinear optical properties of nanocomposites containing small concentrations of nanoparticles of wide bandgap semiconductors and dielectrics dispersed in a dielectric matrix showed that these media can exhibit a low-threshold nonlinear response (threshold < 0.5 kW/cm^2) to both nanosecond pulses and continuous radiation in the visible and near IR spectral range. The great value of bandgap gave reason to think that the nonlinear response of nanocomposite media occurs under ultraviolet light. Nevertheless, recent studies have shown [1–3 and other] (i) that the low-threshold nonlinear response can be observed in dielectric nanocomposites in the visible and infrared light, (ii) it can be observed under pulsed and cw laser modes and disappears at high intensities, (iii) it takes place on frequencies of bands of light absorption of transmission spectra of nanoparticles array. The intensity threshold and nature of nonlinear response depend on characteristics of nanoparticles material as well as of the matrix material. This fact allows us to vary the response parameters by changing nanocomposite components and nanoparticles concentration, size, and shape. One can expect fabricating dielectric metamaterials based on similar nanoparticles and possessing unique nonlinear optical properties controlled by a weak radiation. In this report, we propose the theory of nonlinearity of dielectric nanoparticles in weak laser radiation fields, which explains of the experimentally observed features in the nonlinear optical properties of these nanomaterials. The main idea of consists in that the main factors responsible for the appearance of low-threshold nonlinearity in are (i) dipole electric moments of nanoparticles, which is induced by difference in the population of states of charge carriers in nanoparticle by optical radiation, and (ii) orientation of nanoparticles along the direction of polarization in the external optical field. We also assumes that the polarization properties of nanoparticles can be described in terms of the polarization tensor.

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Strong coupling between excitons and plasmons in a metallic slot waveguide

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We consider a metallic slot waveguide having a quantum well embedded in the dielectric core. This structure allows strong coupling of excitons with guided plasmonic modes at the excitonic resonance. We investigate the physics behind the formation of these hybrid modes and discuss their propagation properties depending on the control parameters of the system.

A remarkable progress in the area of quantum phenomena and strongly correlated Bosons last decade was related to an intensive study of exciton-polaritons occurring inside a high quality semiconductor microcavity operating in the strong light-mater coupling regime [1]. Alternatively, by virtue of the extreme subwavelength confinement of light and the recent advancement in nano-fabrication technology, plasmonic devices stand out as a promising candidate for even tighter light-matter coupling supporting the nano-scale integration of optical devices [2]. However the non-avoidable losses in the metal are known as a serious obstacle which induces dephasing mechanisms and thus hinders the formation of the coherent states between excitons and plasmon modes. Nevertheless the interaction between localized surface plasmons and excitons has been reported recently [3]. In this contribution we discuss possible schemes realizing the strong interaction between guided surface plasmons and quantum well (QW) excitons.

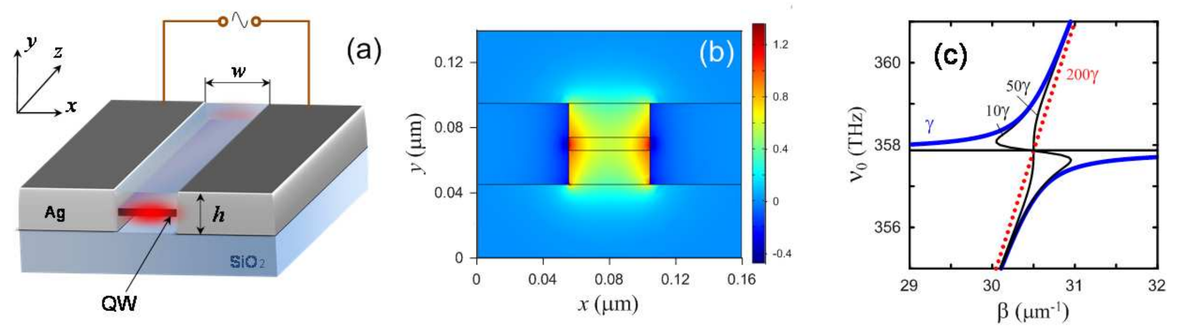

Fig. 1: (a) Schematic cross-section of the metallic slot waveguide configuration. QW denotes the quantum well embedded in the dielectric core (Al0:9Ga0:1As). (b) Poynting vector distribution is shown in the direction of plasmonic mode propagation at a frequency slightly below excitonic resonance. (c) Dispersion of the hybrid waveguide for different dephasing times of excitons. Real part of propagation vector.

Hereby, we propose a scheme for coupling QW excitons to propagating surface plasmon polaritons (SPP) in a metallic slot waveguide [see Fig. 1(a)]. Surface plasmon polaritons (SPPs) in metallic waveguides are the hybrid modes formed by the coupling of electromagnetic radiation and free surface electrons at the interface between metal and dielectric. The electric component of plasmonic field perpendicular to the metallic surface overlaps strongly with the excitons of the QW embedded into the metallic slot [see Fig. 1(b)]. The formation of hybrid exciton-plasmon-polariton state, i.e. a coherent state between SPP and the excitons, change drastically the fundamental optical properties of the system modifying the dispersion relation of the guided mode [see Fig. 1(c)]. To illustrate this we plotted typical dependencies of the frequency on the propagation wave-vector for different damping constants of the excitons. For very strong damping the hybrid exciton-plasmon mode transforms into the conventional plasmonic one.

The coherent exciton-plasmon-polariton states are quasi-particles which share simultaneously properties of the surface plasmons and the QW excitons. The latter ones open a unique channel for the inter-particle interactions and, more important, allows for an ultra-fast sensitive tuning of the optical properties. For instance, by applying an external electric field one can effectively control the absorption line of the QW excitons in accordance with the physical mechanism similar to the Franz–Keldysh effect [4]. Switching or scattering times of such plasmonic devices operating in the strong coupling regime are fundamentally limited by the period of the Rabi oscillations which realistically is of order of picoseconds.

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Strong coupling in the near field of a resonant meta-surface

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Meta-surfaces are the bidimensional analogues of metamaterials [1]. They are made on resonant elements periodically disposed on a surface. They have the ability of controlling the polarization of light and to allow generalized refraction laws as well. They have also been used to enhance the generation of the second harmonic. It seems however that their near-field properties have not been investigated. In this work, the coupling of an emitter with a meta-surface made of a periodic set of resonant linear dipoles was studied [2]. Bloch surface modes localized on the meta-surface exist due the resonance of the dipoles. The strong coupling regime with a emitter can be reached when the Bohr frequency of the emitter is in resonance with the Bloch modes of the meta-surface [3].

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Magnetoplasmonic analog of Borrmann effect

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There is a great fundamental and applied interest in magneto-optic effects because the state of polarization and intensity of light can alter when interacting with magnetic materials. For magneto-optic effects to be applied in modern optical devices, lager efficiency of light-matter interaction is required than that observed in common ferromagnetic materials. Recently, the relationship between the distribution of the electromagnetic field inside a magnetophotonic crystal (MPC) and the magneto-optic Faraday effect has been observed [1]. It was found that different values of Faraday effect on different edges of the photonic band gap of the MPC are connected with the Borrmann-type mechanism: when the electromagnetic field is localized inside the magnetic layers of the MPC, the Faraday rotation is enhanced.

Significant enhancement of the transverse magneto-optic Kerr effect (TMOKE) in comparison with that observed in ferromagnetic thin films was found in one-dimensional magnetoplasmonic crystals under excitation of surface plasmon polaritons (SPP) [2]. The enhancement appears due to the fact that the dispersion law of an SPP depends on the magnetization of the medium.

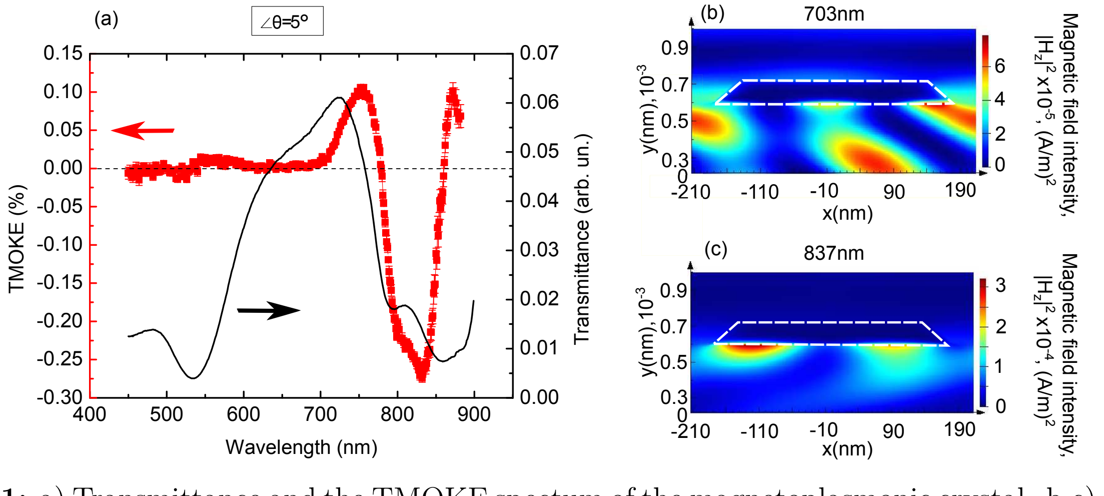

Fig. 1: a) Transmittance and the TMOKE spectum of the magnetoplasmonic crystal. b,c) Numerical calculations of the intensity distribution of the magnetic field inside a strip of the magnetoplasmonic crystal for wavelengths 703 nm (+1st SPP mode) and 837 nm (-1st SPP mode). The white dashed curve is a strip profile. The incident angle of light is 5°.

Here, the correlation of the spectral dependences of the TMOKE (Fig. 1a) and the intensity of the electromagnetic field inside the magnetic layer of the magnetoplasmonic crystal consisted of Au/Ni/Au layers is demonstrated. It was experimentally found that the excitation of different SPP modes in the magnetoplasmonic crystal leads to different maximum values of the TMOKE. The integrated value of the intensity of the magnetic field of the -1st SPP mode is larger than that for the +1st SPP mode as shown by numerical calculations of the electromagnetic field inside the magnetic layer (Fig. 1b,c). The larger value of the TMOKE and intensity of the magnetic field in the magnetic layer when the -1st SPP mode is excited comapred it for the +1st mode can be considered as a first manifestetion of the optical Borrmann effect in magnetoplasmonic crystals.

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Magneto-optic Fraunhofer diffraction on 2D spatially homogeneous magnetic domain patterns

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Magnetic materials with regular domain structures can be considered as tunable phase gratings [1]. Occurrence of the magneto-optical Faraday effect in transparent thin-film ferrites with a through domain structure allows to consider such objects as analogs of the phase diffractive optical element — the magneto-optical deflector. In magnetic garnet films, different spatially arranged domain structures can consist of 2D domain patterns. Patterns consist of topologically modified cylindrical domains that belong to different spatial groups of symmetry. Controlling the 2D domain patterns parameters by magnetic field opens the possibilities for creation of reconfigurable 2D phase diffractive optical elements [2–4].

In this work magneto-optical Fraunhofer diffraction on 2D magnetic domain patterns with Cmm6, Pab2 and P6 symmetry was studied. 2D domain patterns were formed in uniaxial iron garnet films under the action of pulsed magnetic fields in the presence of DC and AC bias fields. 2D domain structures are statically stable after removal of the external fields. They consist of more than 10^2 elementary cells and have high spatial homogeneity.

Fraunhofer diffraction patterns were experimentally registered using a helium-neon laser. The analyzer was used to reduce the backlight. Diffraction pattern on the regular set of micron-sized magnetic areas gives a clear understanding of the configuration and spatial symmetry of the magnetic domains (Fig. 1). Possibilities of space-time modulation of optical beam and multiplication of the modulation frequency (both under change of diffraction order as well as magnetic bias field strength) were shown. Mathematical modeling confirms the experimentally registered transformation of the diffraction pattern under the magnetic field. 2D domain patterns may serve as a basis for tunable magnetophotonic crystals and high-speed magneto-optical devices.

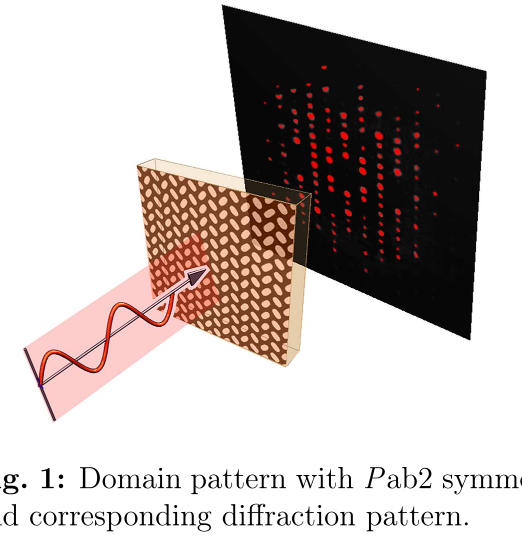

Fig. 1: Domain pattern with Pab2 symmetry and corresponding diffraction pattern.

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Variations in extinction spectra of plasmonic nanoparticle aggregates upon deformation during deposition on planar dielectric substrate

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The interaction of colloidal aggregates with a substrate may be accompanied by variations in their structure. In turn, the structure of nanoaggregates is a key factor that governs the pattern of their plasmon absorption spectra. In most cases the optical properties of aggregated disperse systems are studied in bulk hydrosols where disordered fractal aggregates are three-dimensional, while the structural properties are studied for such aggregates deposited onto a planar surface of an object holder in an electron microscope.

The initial absorption spectrum of an aggregated hydrosol and a quasi-planar colloidal structure, which is examined with an electron microscope, may be far from closely related to one another. However, it is the electron micrographs of these flattened aggregates from which their characteristics, are experimentally determined. It is obvious that a close correlation between the structural and electrodynamic parameters of nanoaggregates cannot be established under these conditions.

The goal of this work is to study the peculiarities of the structural transformation of a three-dimensional disordered aggregate of Ag nanoparticles into a quasi-planar structure by the Brownian dynamics method and to investigate the effect of this process on the extinction spectra of the deformed aggregate. In order to estimate the magnitude of this effect, the calculations were performed for both the extinction spectra of nanoparticle aggregates during the transformation of their structure and the factor of the local anisotropy of particle environment in aggregates, which predetermines the electrodynamic characteristics of these systems [1, 2].

The local character of the optical responses of fractal aggregates of nanoparticles, which governs a number of their unique electrodynamic properties, was studied in [1, 2] and experimentally confirmed in [3]. As applied to plasmon-resonant particles, it was reliably established that the higher the local anisotropy factor the larger the broadening of the plasmon absorption band.

In this paper the deformation of the disordered aggregate is simulated by solving the Langevin equation separately for each particle. The forces applied to particles include components directed toward the substrate. These forces simulate the gravitational sedimentation accelerated under the effect of free fall acceleration enhanced by several orders of magnitude.

The interaction potential comprises the Van der Waals attraction of particles to each other and to the substrate, and the elastic repulsion caused by the deformation of the adsorption layers of particles.

Calculated with coupled dipole method absorption spectrums shows that absorption in the long-wave spectral range noticeably increases during aggregate deposition. This finding agrees with the results calculated for the local anisotropy.

In order to verify the results of the simulations, the plasmon absorption spectra were experimentally studied for Ag 3D nanoparticle aggregates in hydrosols and the same aggregates deposited onto a planar quartz substrate.

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Exciton optics in transition metal dichalcogenides monolayers

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The synthesis and studies of graphene, an atom-thick layer of Carbon, has opened up an era of novel truly two-dimensional materials. Such systems combine unusual optical and transport properties and attract a lot of interest nowadays [1, 2]. Monolayers of transition metal dichalcogenides (TMDCs), like MoS_2 or WSe_2 have, similarly to graphene, hexagonal crystalline lattice, see Fig. 1(a), but, unlike graphene, possess direct band gaps on the order of 2 eV realized at the edges of the Brillouin zone denoted as the $\mathbf{K}_+$ and $\mathbf{K}_-$ valleys. An extremely strong spin-orbit coupling in TMDCs yields the spin-valley locking: Certain spin components of individual charge carriers correspond to the specific valley of the energy spectrum [3]. Interestingly, both the electron and hole states are already spin-split at the $\mathbf{K}_+$ and $\mathbf{K}_-$; spin-valley locking results also in the chiral selection rules for interband optical transitions, Fig. 1(b). Namely, the absorption of right- or left-handed circularly polarized light excites transitions in the $\mathbf{K}_+$ or $\mathbf{K}_-$ valley, respectively [4].

Recent experiments demonstrate valley-selective optical orientation [4–6], however, the valley/spin polarization decays rapidly in time in the case of resonant generation of excitons. On the other hand, spin-valley locking and spin splitting strongly suppresses spin relaxation of individual charge carriers.

We develop the theory of the neutral exciton fine structure in TMDCs monolayers governed by the long-range exchange interaction between the electron and the hole. This coupling is responsible for the rapid spin/valley depolarization observed in MoS_2 [5]. The theory is successfully applied also to WSe_2 monolayers, where the temperature dependence of spin/valley decoherence time has been addressed [6]. The fine structure of excitonic states in an external magnetic field is discussed [7].

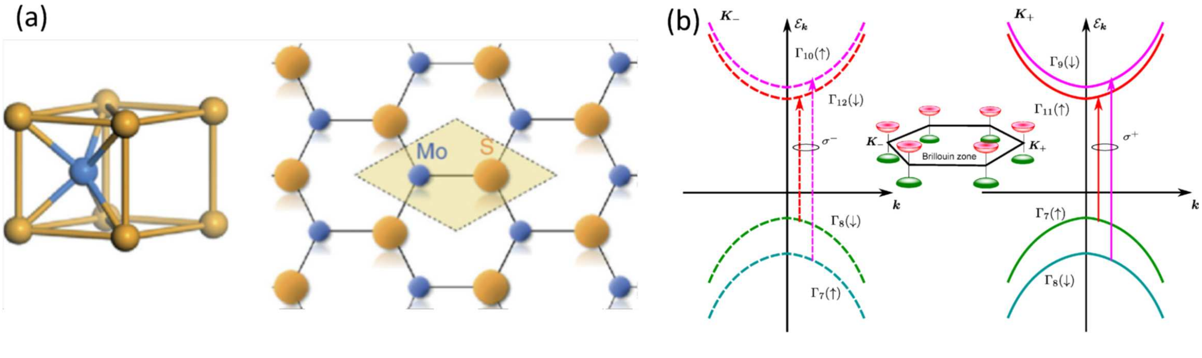

Fig. 1: (a) Schematics of the crystalline lattice of TMDCs on example of MoS_2. (b) Sketch of the band structure and selection rules for the direct transitions in the vicinity of the $\boldsymbol{K}_+$ and $\boldsymbol{K}_-$ points.

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Dispersion properties of coated wire medium

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Metamaterials are materials with unique physical properties usually not inherent to the natural ones [1]. One of the type of metamaterial is hyperbolic metamaterial [2]. Its name is explained by the shape of the equal frequency surface in the space of wave vectors, which represents a hyperboloid. Hyperbolic metamaterial interesting because of their unusual optical properties, such as a significant enhancement of spontaneous emission or negative refractive. This type of metamaterial can be realized using an array of parallel wires in a dielectric medium [3]. But this realization is not yet completely correct since such a medium has a flat equal-frequency surface but not hyperbolic.

In this work we put forward the model of true hyperbolic metamaterial (with hyperbolic surfaces of equal frequencies) based on the array of parallel wires coated by a dielectric. We analyzes the shape of equal frequency contours analytically and using a numerical simulation. Different cases of the dielectric contrast between the host material and coating of the wires are considered. It was shown that the equal-frequency surfaces of the structure under consideration is hyperboloid and changes their curvature depending on the dielectric contrast between the host material and the coating shells. We analyze behavior of radiofrequency beam incident on the coated wire medium. We have shown that refraction angle can be manipulated by the dielectric contrast and negative refraction is possible.

Therefore, we have shown that true hyperbolic metamaterial with hyperbolic equal-frequency surface can be realized as wire medium with dielectric coating.

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Coupled plasmon resonances and graphene plasmonics

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Coupled plasmon resonances in various plasmonic metamaterials attracted a lot of attention recently. We consider several coupling mechanisms: i) near-field coupling in self-similar structures, ii) diffractive coupling in arrays of metallic nanoparticles and iii) resistive coupling through conductive sublayer and discuss experimental properties of the corresponding metamaterials. We demonstrate diffraction coupled plasmonic resonances with the spectral line full-width-half-maximum as low as 5 nm and quality factors Q reaching 300, at important fibre-optic telecom wavelengths around 1.5 µm. Using these resonances, we demonstrate a hybrid graphene-plasmonic modulator with the modulation depth of 20% in reflection operated by gating of a single layer graphene, the largest measured so far.

Optical Tamm state at the cholesteric liquid crystal/metal interface

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Optical Tamm state (OTS) is an optical analogue of Tamm state which is electronic density localization at the boundary of periodic atomic potential. The OTS appears as electromagnetic field localization at the interface of photonic crystal and metal [1]. Experimentally the OTS appears as a narrow dip in reflectance spectrum in the photonic band gap spectral range. Cholesteric liquid crystals (CLCs) are self-organized photonic crystals formed by rod-like molecules arranged in a periodical helical structure [2]. The CLCs have a stop-band for light with the one direction of circular polarization (the same as the CLC twist). Therefore, the appearance of OTSs is expected in CLC/metal structures [3, 4]. In this work the properties of the OTS at the CLC/gold interface are studied using 4×4 Berreman matrix method.

The optical parameters [5] and structure of studied CLC/gold model are shown in Fig. 1a. The plane-parallel anisotropic dielectric layer was introduced between the CLC and the metal layers. The optical axis of the anisotropic layer was laid along the planes of its interfaces with the CLC and gold layers.

The calculated reflectance spectra of the structure are shown depending on thickness of anisotropic layer (Fig. 1b) and angle of incidence (Fig. 1c). The dependence of the OTS resonance depths and spectral positions on the thickness of the anisotropic layer and the wavelength of the incident light is periodic. The OTS is shown to be excited if phase of the wave transmitted through the anisotropic layer changes by $\pi/2$ and the anisotropic layer acts as a quater-wave plate. The maximum value of the resonance dip is achieved at the wavelength of 780 nm when the thickness of the anisotropic layer is 2030 nm. Fig. 1c demonstrates that the OTS excitation occurs over a large range of angles of incidence. It also shows that photonic band gap center and the OTS dispersion curve shifts to shorter wavelengths with incident angle increase.

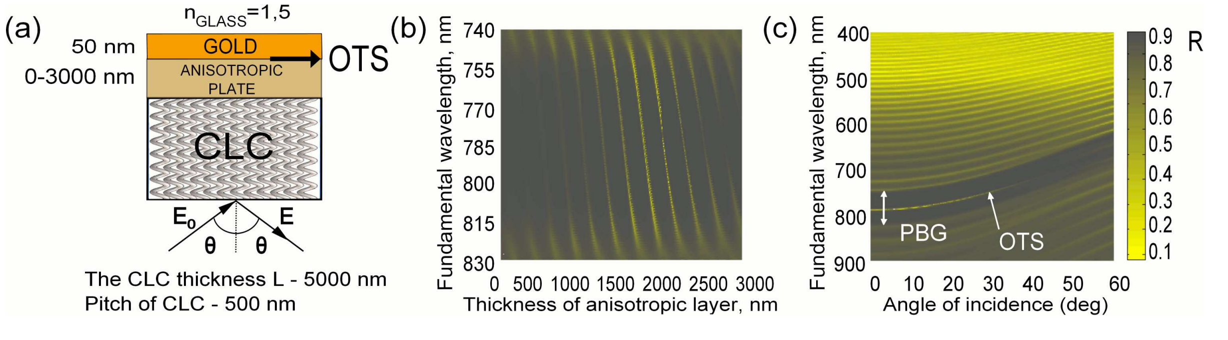

Fig. 1: a — The sketch of the studied CLC structure. E_0 and E are the incident and reflected waves, respectively, θ is angle of incidence. b — Reflectance spectrum of the modeled structure depending on the thickness of the anisotropic layer; c — Reflectance spectrum versus angle of incidence and wavelength of the radiation.

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Manipulating electromagnetic wave with meta-surfaces

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Metamaterials are artificially engineered materials whose optical properties arise primarily from their micro-structure (“meta-atom”) and its macroscopic order. Metasurface, as a new emerging field of metamaterials, have aroused considerable interest due to their capability of arbitrary manipulation of the phase and amplitude profile at the interface.

In this talk, we briefly summarize our recent efforts in employing meta-surfaces to control electromagnetic waves, including realizing high-efficiency photonic spin-hall effect [1] and surface-plasmon couplers [2], and controlling phases with graphene-based meta-surfaces [3].

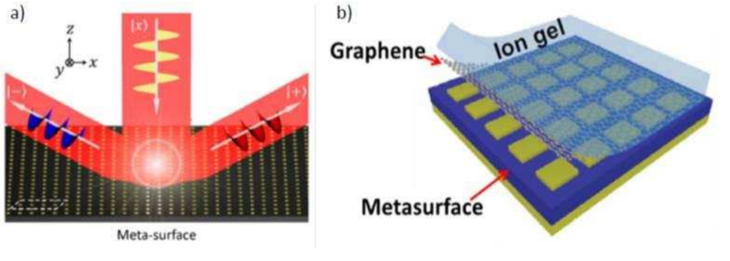

Fig. 1: a) Schematics of the 100%-efficiency photonic spin-hall effect realized at our reflective meta-surface, b) full-range phase modulator based on graphene metasurfaces.

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Characterization and application of resonant properties of out-diffused silver nanoislands

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Plasmonic nanoparticles used as nanoantennas [1] are nowadays undergoing deep investigations as structures, which are able to enhance the local electromagnetic field thus, leading to an increased resolution and intensity of various microscopic and spectroscopic methods, such as surface enhanced Raman spectroscopy (SERS) [2, 3] or surface enhanced fluorescence (SEF) [4].

Nanoisland films, composed of multiple nanoparticles, are mainly produced using electron beam lithography, thermal evaporation and others. In our recent SERS experiments we used silver nanoisland films prepared by out-diffusion of silver atoms from an ion exchanged glass substrate [5]. Compared to electron beam lithography, the out-diffusion process is more random, leading to a variety of different nanoisland shapes such as circular, elliptical and others. Recently the possibilities of 2D pattering of these films were investigated [5]. The opportunity to obtain single silver nanoislands separated from each other by a distance of several microns, gives the possibility to investigate spectroscopic plasmonic properties of each nanoisland independently.

In our experiments, polarization resolved dark field spectroscopy was used to get an insight into the resonance properties of different islands. The measured scattering spectra showed excellent agreement with the simulations carried out using discrete-dipole approximation method [6]. Atomic force microscopy and scanning electron microscopy were than used to determine the size and shape of the studied out-diffused islands. A numerically predicted shift of the resonance frequency of single islands from about 550 nm up to about 750 nm was confirmed experimentally. This tunability of the resonance wavelength allows one to use different excitation sources optimized for given types of samples in spectroscopic characterizations, e.g. different Raman analytes.

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The nature of Fano resonances in nanoparticle oligomers

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Here we present recent theoretical and experimental work on the distinctive scattering behavior that can be produced from interference in nanoparticle oligomers. The underlying requirement for interference phenomena, such as Fano resonances, are nonorthogonal eigenmodes of the associated scattering geometry, which are made possible by the inherent radiative loss in open scattering systems [1]. We follow the formation of nonorthogonal eigenmodes from hybridization in symmetric nanoparticle trimers, allowing us to investigate how high-index dielectric nanoparticles are able to produce a bianisotropic response space that enhances a geometry's capacity to produce Fano resonances. In particular, we realize multiple, polarization-independent, Fano resonances in an all-dielectric trimer. We then explore the magnetic responses produced from the interaction between collective and individual magnetic response in all-dielectric quadrumers. This allows us to produce sharp Fano resonances between magnetic responses only. To conclude, we show that it is possible to utilize the strong coupling into a dark subspace, commmonly associated with Fano resonances, to circumvent the constraints of reciprocity on circular dichroism in planar (two-dimensional) chiral structures. Reciprocity is known to constrain extinction and, in effect, we present a new form of circular dichroism produced by switching the loss mechanism of a plasmonic nanoparticle oligomer between far-field radiation and near-field material absorption [2]. Planar chiral oligomers are shown to be ideal candidates for exhibiting this form of, Fano-enhanced, circular dichroism. The formation of a Fano resonance in a gold planar chiral heptamer, and its association to circular dichroism in absorption and scattering cross sections, is demonstrated in the figure below.

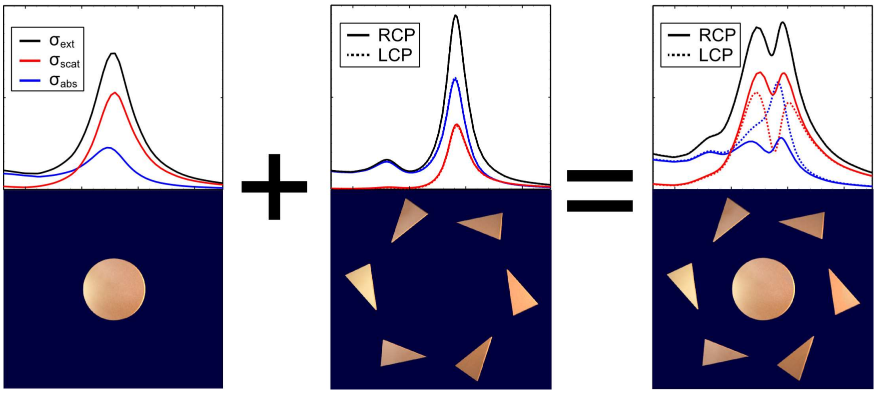

Fig. 1: Simulations of circular dichroism produced in the scattering and absorption losses of a planar chiral oligomer as a result of its Fano resonance.

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Electric field enhancement in the surface photonic crystal

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The current mission that researches are trying to accomplish is to develop novel devices for manipulating light on the nanoscale [1]. In the optical sensing the most common approach is to use the metallic nanostructures which exhibit huge enhancement of the electromagnetic field due to the excitation of the collective plasmon resonances at the surface of metallic nanostructures. In spite of all the evident advantages, the optical response of metal nanostructures is damped due to the absorption in such structures [2]. In our work we consider the Brag filter comprising ten dielectric bilayers based on aluminum substrate. Each bilayer contains two sculptured thin films (STFs) consist of silicone dioxide (SiO_2) and zirconium dioxide (ZrO_2) correspondingly. The surface of the multilayer has a form of periodic system of rectangle structures segregated by the cavities, which were obtained by FIB-assisted modification. We use the computer simulation to find remittances of STF as a function of the wavelength and EM field distribution. It was shown that the local electric field is much enhanced at the resonance frequences and it concentrates in the small spaces at the surface. The Raman enhancement factor $G = |\frac{E}{E_0}|^4 = 10^2$. The results of our computer simulations are in a good agreement with the experimental results shown in Fig. 1(a). The reflectance has a wide maximum which is usual for Bragg filter [3]. We speculate the proposed multilayer could be the effective in the optical sensing due to low losses.

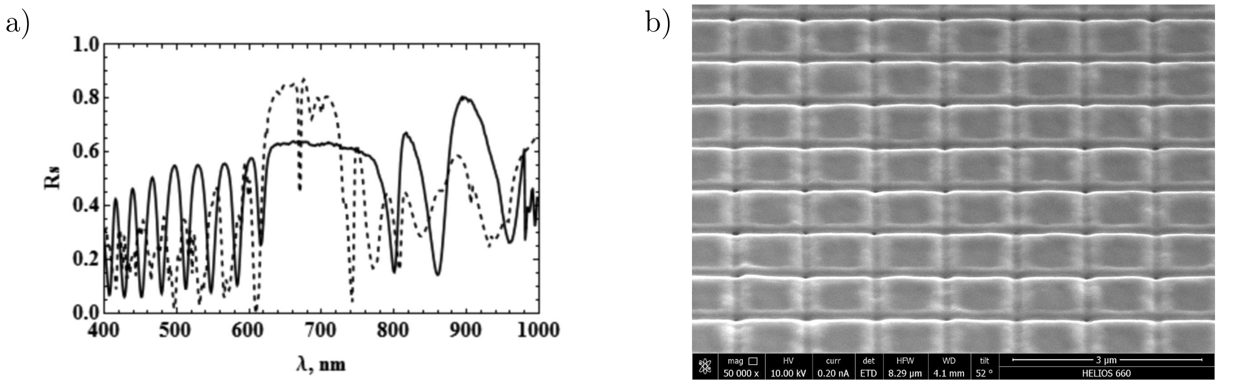

Fig. 1: (a) Reflection of a Bragg filter comprising 10 bilayers: angle of incidence $\alpha = 45°$, thickness SiO_2 layer $h_1 = 135.35$ nm; ZrO_2 layer $h_2 = 96.68$ nm; thickness Al substrate $h_3 = 150$ nm. The length and width of the cavities 300 nm and 100 nm correspondingly. Dashed line — computer simulation, solid line — experimental results. (b) REM picture of the surface of the multilayer.

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Multi-stable switchable metamaterial employing Josephson junctions

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The field of metamaterial research revolves around the idea of creating artificial media that interact with light in a way unknown from naturally occurring materials. This is commonly achieved by creating sub-wavelength lattices of electronic or plasmonic structures, so-called meta-atoms, that determine the interaction between light and metamaterial. One of the ultimate goals for these tailored media is the ability to control their properties in-situ which has led to a whole new branch of tunable and switchable metamaterials. Many of the present realizations rely on introducing micro-electromechanical actuators or semiconductor elements into their meta-atom structures. We show that superconducting quantum interference devices based on Josephson junctions can be used as fast, intrinsically switchable meta-atoms. We found that their intrinsic nonlinearity leads to simultaneously stable dynamic states, each of which is associated with a different value and sign of the magnetic susceptibility in the microwave domain [1]. Moreover, we demonstrate that it is possible to switch between these states by applying a nanosecond long pulse in addition to the microwave probe signal. Apart from potential applications such as, for example, an all-optical metamaterial switch, these results suggest that multi-stability, which is a common feature in many nonlinear systems, can be utilized to create new types of meta-atoms.

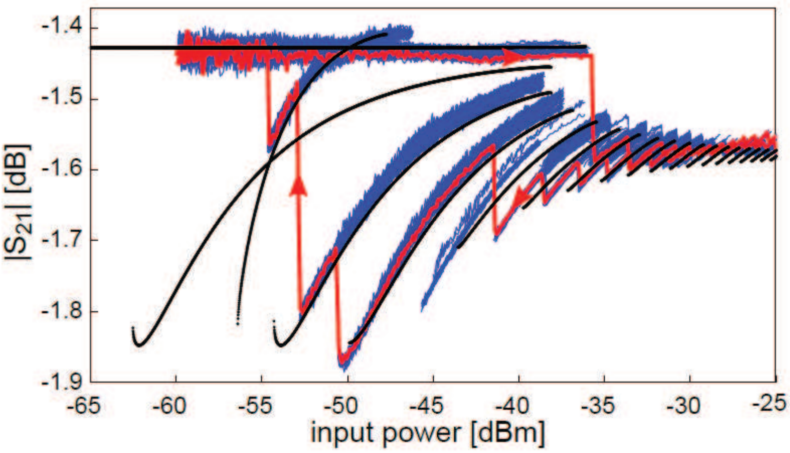

Fig. 1: Calculated (black) and measured (blue and red) transmission through the sample arrangement containing only a single SQUID. The red data show a hysteresis loop from low to high power and back. The red arrows indicate the direction of the sweep. The blue data are a collection of different power sweeps of varying length and initial conditions [1].

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Photonic topological insulators: from theory to practical realization

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The past three decades have witnessed the discovery of the Quantum Hall Effect (QHE), Quantum Spin Hall Effect (QSHE) and Topological Insulators (TIs) which transformed our views on the quantum states of matter. These exotic states are characterized by insulating behavior in the bulk and the presence of the edge states contributing to charge or spin currents which persist even when the edge is distorted or contains impurities. In the last few years, a number of research groups have realized that the same "robust" conducting edge states can be implemented in photonic systems. An early theoretical prediction [1, 2] and experimental demonstration [3–5] of the topologically protected states and transport of light opened a new direction in photonics. In this talk we will review developments in this field with focus on photonic topological insulators with preserved time-reversal symmetry that we have recently proposed [6] and realized experimentally with the use of bianisotropic metamaterials. We will present new designs of photonic topological insulators based on different geometries that has been implemented at microwave frequencies and will discuss perspectives for applications. It will be shown that photonic topological insulators offer an unprecedented platform for controlling light: deliberately created distribution of the bianisotropy, which plays the role of the effective magnetic field, allows routing of photons along arbitrarily shaped pathways (Fig. 1) without scattering and back-reflection [6].

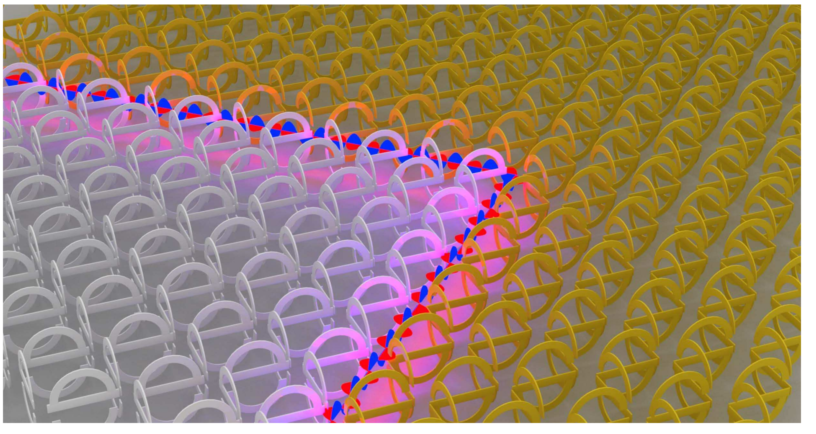

Fig. 1: Photonic topological insulator.

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Homogenization of quantum metamaterial

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We show that effective medium approximation is not applicable for superlattice with layers which thickness is less than carriers mean free path . We use more accurate approach for dielectric function considering energy spectrum of carriers modified by superlattice potential and carrier distribution function. We propose a new model of ultra homogeneous temperature tunable THz metamaterial based on semiconductor superlattice with doped quantum wells. Efficiency of thermal tunability is a result of high sensitivity of semiconductor conductivity to the temperature.

Superlattice (SL) represents uniaxial crystal. Dielectric function of uniaxial crystals is described by tensor with two different components corresponding to the directions along ($\varepsilon_{\parallel}$) and across ($\varepsilon_{\perp}$) the optical axis. In this case, where the superlattice consists of coupled quantum wells separated by thin tunnel transparent barriers, quantum effects are particularly relevant and, therefore, it is incorrect to describe dielectric function of each layer separately and then apply the homogenisation procedure. Quantum homogenisation should take into account wave functions of the carriers, their energy spectrum modified by SL potential, and carrier distribution function. We show that away from interband transition and reststrahlen band each permittivity tensor component has Drude–Lorentz dispersion with modified plasma frequency:

$$\Omega_\alpha^2 = \frac{4\pi e^2}{\varepsilon_\infty}\frac{2}{(2\pi\hbar)^3}\iiint f(E,\mu,T)\frac{\partial^2 E}{\partial p_\alpha^2}d^3p, \qquad \alpha = \perp, \parallel. \tag{1}$$

Here E is energy of carriers which depends on the momentum $\mathbf{p}$, $f(E,\mu,T)$ is the Fermi–Dirac distribution function, μ is the chemical potential, T is the temperature and ε_∞ is a permittivity of the lattice without free carriers.

In our model we use formulas for permittivity tensor which take into the energy distribution function of the carriers and their spectrum modified by quantum confinement effects. In opposition, in the work [1] permittivity tensor components are calculated within the effective medium approximation, where the dielectric function of layers supposed to be equal to the one of the bulk material. The effective medium approximation is applicable for a case of thick layers and, in contrast, quantum approach is convenient for a case of thin layers. This is the difference between the classical and the quantum model.

We propose a new concept of ultra homogeneous temperature tunable metamaterial based on semiconductor superlattice (SL) for THz applications. Tunability mechanism in proposed metamaterials is the following. Free carrier concentration and, therefore, plasma frequency of semiconductors is sensitive to the temperature in contrast to dielectrics and metals. Implementation of SL in semiconductors makes carrier spectrum and plasma frequency anisotropic. So, in SL we can distinguish plasma frequency along ($\Omega_\parallel$) and across ($\Omega_\perp$) the optical axis. Permittivity tensor signature and, therefore, optical properties of SL depends on the relation between frequency of the electromagnetic wave ω and plasma frequencies $\Omega_{\perp,\parallel}$.

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Non-regular wire-medium structures as dipole radiator enhancers

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We suggest and theoretically explore a possibility to strongly enhance radiation of small dipoles using two different types of hyperbolic wire-medium (WM) structures. The hyperbolic metamaterial converts emitter's near fields into propagating waves which are efficiently irradiated from the structure backward interface. Infinite WM as a particular example of hyperbolic metamaterial also can be used for significant enhancement of small dipole radiation. However, extracting this radiation from the WM sample into free space is not a simple task. The excessive radiation of a submerged dipole is almost (except Fabry–Perot maxima) totally internally reflected at the sample boundaries if wires are parallel. Wire-medium hyperlens comprise divergent wires and can be used for significant enhancement of radiation from small sources [1] in the wide frequency range, that average value of radiated power grows up largely and approaches to the value from [2]. However multiple reflections between two boundaries of the hyperlens still should obviously result in Fabry–Perot resonances. Our next idea is to create a non-regular wire-medium random structure with flat interfaces and achieve effects of enhancement in the same order, as it can be done with the cumbersome hemispherical hyperlens. Thus, with the flat interface structure the radiation of the same emitter in free space is enhanced more than the parallel WM slab allows. We believe that this study can open new doors to solar thermophotovoltaic emitters.

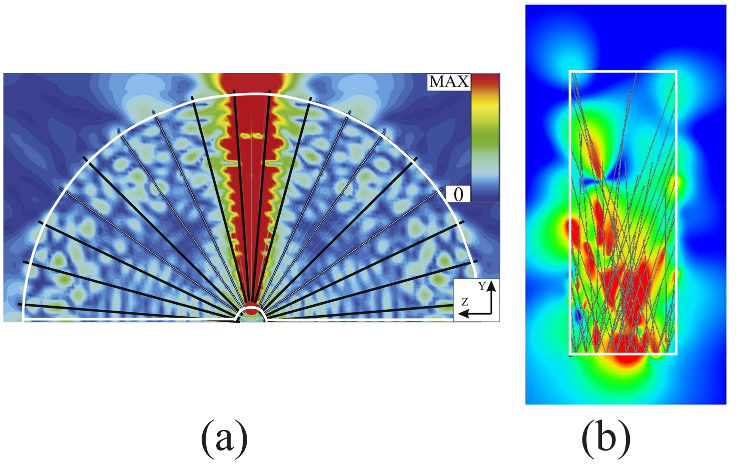

Fig. 1: Field distribution of hemispherical hyperlens (a) and randomized non-regular wire-medium (b) demonstrating the effect of radiating to free space energy from the poor radiating electrically small dipole. White line shows the contour of the structures.

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2D-tunnel bifurcations for interacting quantum molecules in the matrices of metamaterials

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Currently, a new class of materials, known as metamaterials, with unique properties in a certain frequency range, is of great interest. Metamaterials are the artificial composite materials consisting of a dielectric or conductive elements that form a regular structure, which are characterized by a negative effective permittivity and permeability (ε and μ) and, accordingly, a negative refractive index. On the basis of such materials it is possible to develop a number of unique devices, such as flat electromagnetic lenses that do not have the diffraction limit (superlens), masking the shell, and so such properties cause the increased interest in their practical implementation. In addition, the problem of control for nanostructures, which are in the matrices of metamaterials, is of great interest. In this paper we investigate the problem of controllability for 2D-tunnel bifurcations in systems with quantum molecules in a dielectric matrix of the metamaterial in the external electric field at finite temperature. Usage of the quantum tunneling with dissipation theory [1–3] to study the interaction of QM with contact medium is productive, because, despite of usage of the instanton approach it is possible to get the main results in an analytical form with account of the environment effects on the tunneling process, which it is not possible in other often used approaches. It is experimentally possible to observe earlier theoretically predicted 2D-bifurcations for QD from colloidal gold in the matrix of a conventional insulator on the tunneling current-voltage characteristics in the system of combined AFM / STM. An important problem is to reveal the experimentally realizable range of values of the relative dielectric constant for the matrix environment, allowing 2D-bifurcations regime, including negative values, which corresponds to the matrix of metamaterials.

The aim of this work is to theoretically study the range of control parameters (electric field strength, temperature, and the relative dielectric constant of matrix of the metamaterial), in which the regime of stable 2D-bifurcations in the QM system can be realized, as well as in system with "the cantilever needle of combined AFM / STM - QD or QM". Special attention is paid to the problem of control in a two-dimensional dissipative tunneling for system of interacting QM, simulated by 2D-oscillator potential in the medium with negative dielectric permittivity in an external electric field.

The "phase diagram" for the stable 2D-bifurcations regime for tunnel current in the matrix of a metamaterial in dependence on controllable parameters (the inverse temperature, electric field intensity and values of the (negative) relative permittivity for the heat–bath environmental matrix), has been represented. It is shown that in contrast to usual dielectric matrices in case of the metamaterial matrix the region of stable 2D-bifurcation is significantly narrowed, which is probably due to inversion of the sign of the tunneling particles interaction.

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Fabrication of functional plasmonic nanostructures using focused nano- and femtosecond laser pulses

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Functional plasmonic nanostructures made of noble metals owing to their unique optical and spectral properties are currently of great scientific interest. A number of promising applications in such areas as plasmonics, nanophotonics, biosensing, etc. were theoretically predicted and experimentally demonstrated. For these applications it is often necessary to fabricate both single element at a given point and sufficiently large ordered arrays of nanostructures, which can be realized by a number of relatively expensive or time-consuming techniques. Laser material processing potentially provides the easiest and cheapest way to fabricate single and large ordered arrays of different functional metallic nanostructures through initiating various hydrodynamic instabilities excited on the metal film surface under nano- and femtosecond pulse irradiation. In our paper we discuss the implementation of laser-based techniques for fabrication of different functional plasmonic nanostructures: nanorings, nanorods, nanocrowns, nanoparticle's chains etc. Underlying physical mechanisms responsible for the formation of all these nanostructures are discussed in terms of hydrodynamic instabilities induced by tightly focused nano- and femtosecond pulses. Possible implementation of the fabricated nanorings, nanorods, nanocrowns, nanoparticle's chains as promising plasmonic elements for photoluminescence and SERS signal enhancement in molecular sensors, hot electron generation in solar cells and light localization at nanoscale are also discussed in this paper.

Optophononic devices based on semiconductor multilayers

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The idea of trapping and manipulating light, sound and heat has fascinated mankind since ancient times. The development of micro- and nanofabrication techniques enabled the study of nanostructures where it is possible to confine photons (light) and acoustic phonons (ultrahigh frequency sound) in a single resonant cavity of about 100 nanometers, and control both the dynamics and the interactions between them. Through the engineering of semiconductor and metallic nanostructures displaying controlled acoustic impedance modulations, a new area is growing in nanoscience: nanophononics. This area is devoted to the study of the vibrational and thermal properties at the nanometer scale, and the interaction of these engineered phonons with photons and electrons at high frequencies.

Resonators that confine light and acoustic phonons provide a platform to study novel quantum phenomena and for the development of new ultrafast devices. We show that GaAs/AlAs microcavities designed to confine photons are automatically optimal to confine acoustic phonons of the same wavelength, strongly enhancing their interaction. Optomechanical coupling at sub-THz acoustic frequencies is reported in these monolithic devices bridging the gap between optomechanics and optoelectronics in systems that have been used to demonstrate single-photon emitters and the most efficient (vertical cavity surface emitting) lasers. In this presentation I will first introduce the basics of opto-phononic design, including the engineering of nanostructures and the study of novel optical/acoustic confinement strategies based on topological considerations.

Performing pump-probe experiments, sub-THz coherent acoustic phonons (nm wavelengths) can be generated and detected in semiconductor nanostructures. The coherent generation of acoustic phonons in nanometric multilayers relies on the space-dependent ultrafast strain induced by a femtosecond laser pulse. A delayed probe pulse senses the time-dependent change of the optical reflectivity. We study following the time evolution of the phonon strain in picosecond-laser experiments the impulsive generation of intense coherent and monochromatic acoustic phonons. Efficient optical detection is assured by the strong phonon backaction on the high-Q optical cavity mode. We demonstrate that micropillars based on these structures, that exploit the unsurpassed growth quality of molecular beam epitaxy, provide optomechanical devices that can attain very high mechanical and optical Q-factors ($Q \approx 105$), very low mechanical effective masses (meff $\approx$ fg), large optomechanical coupling factors (gom $\approx$ THz/nm), and ultra-high vibrational frequencies (sub-THz), while maintaining very efficient extraction efficiencies of both photons and phonons. These structures constitute the building blocks for more complex artificial structures based on coupled cavities like optophononic molecules and Bloch resonators, which will also be briefly presented.

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Femtosecond laser fabrication of plasmonic nanoantennas

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We demonstrate new simple methods of all-laser fabrication of nanoantenna (nano-jet) with an additional elements for coupling/focusing of surface plasmon-polaritons (SPPs). The first method is realized by using an aluminum plasmonic lens irradiated by linear polarized femtosecond laser pulses at fuences higher than the ablation threshold of aluminum. These plasmonic lenses are surface rings with tunable mean diameter in the range of 1.0–2.5 µm, which are easily fabricated by a single fs-laser pulse over the whole laser spot via self-organization processes. Following the first fs pulse, the next one irradiates these plasmonic lenses and initially excites quite intense SPPs which are focused in their center. The remaining part of the pulse interferes with the focused SPPs, resulting in field enhancement in the very center of the lenses and causing strong local (nanoscale) heating. The resulting plasmonic lenses contain single nanojets in their centers owing to melt expulsion in the locally heated area [1]. Such surface structure resembles a parabolic antenna, which has a receiver and focusing reflector.

The second method is based on double-shot femtosecond laser nanoablation of thin supported metallic (Au) film. The first fs-laser pulse produces nanojet, standing on bump of microscale diameter. Irradiation by spatially shifted (on several microns) second laser pulse results in the bump removing, transition of nanojet into nanosphere and formation of concentric periodical semi-rings. Resulted surface structure represents nanoantenna (gold nanospere), surrounded by plasmonic lens, delivering more incident energy to the nanoantenna [2]. The proposed new principle of all-laser nanoantenna fabrication is very simple and high-productive technology to be applied in nanophotonics soon. For instance, we showed almost 30-fold enhancement of fs-laser induced ultrafast electron emission from such nanoantennas, which is promising for time-resolved electron microscopy.

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Enlarging the shadow: getting past Planck's black body limit

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Our everyday experience tells us that macroscopic opaque bodies when illuminated by a very remote light source cast shadows, which are limited in diameter by the geometric cross section of the bodies. The same experience tells us that wave diffraction (which manifests itself only at distances $r_D \gtrsim D^2/\lambda$, where $D \gg \lambda$ is the characteristic diameter of the body, and λ is the wavelength of the incident light) does not affect the shadow region in the vicinity of a macroscopic body.

It is actually widely believed that such situation holds for *arbitrary* optically large opaque bodies. Indeed, to the majority of people — including specialists in diffraction — it would appear naturally impossible for a macroscopic body to cast a shadow (on a perpendicularly oriented screen) with size greater than body's largest diameter, in the scenario outlined in the previous paragraph. In this presentation, however, I am going to prove that, at a given wavelength, an optically large body of a given finite size can cast *an arbitrary large shadow*, if it has certain electromagnetic scattering characteristics. Moreover, the shadow region formed by such a body can have a diameter significantly greater than body's diameter already at distances much smaller than r_D (in practice, at distances comparable to the shadow diameter, see Fig. 1).

Besides shattering commonly accepted beliefs, the above-mentioned result has important consequences in general physics. In fact, it is straightforward to show that a macroscopic absorbing body which, at a given wavelength, casts a shadow which is larger than body's geometric cross section absorbs more electromagnetic energy at this wavelength than the ideal Kirchhoff–Planck's black body with the same geometric size and shape. Consequently, when considering thermal radiation spectrum produced by such a body, one finds that the spectral density of power emitted by this body at this wavelength *exceeds* the one predicted by Planck's emission formula!

Moreover, it is possible to prove that the thermal radiation from a finite region of space (filled with some hot matter) is *not* limited in its *spectral density at a given wavelength*, which, theoretically, can be made *arbitrary* high [1]. Nevertheless, the second law of thermodynamics is not violated by such thermal emitters. Thus, commonly accepted limits on the spectral density of thermal radiation produced by finite-size bodies — which seem to follow directly from Planck's law — are, in fact, only approximate results. Other related physical effects will be discussed in the presentation.

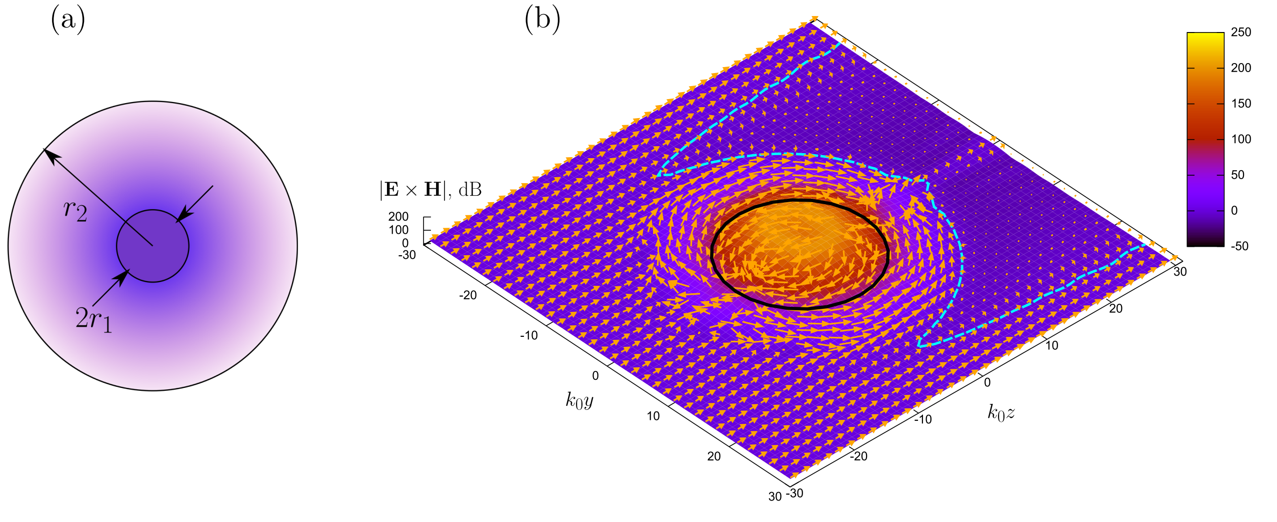

Fig. 1: (a) A spherical body with $\varepsilon(r)/\varepsilon_0 = \mu(r)/\mu_0 = -r_2^2/r^2$, $r_1 \le r \le r_2$, and $\varepsilon(r)/\mu(r) = \varepsilon_0/\mu_0$, $\mathrm{Im}[\varepsilon(r)] \to +\infty$, $r < r_1$. (b) Poynting vector plotted in the vicinity of a spherical body with structure shown in panel (a) which casts a shadow twice bigger than body's diameter.

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Interaction of light with a hyperbolic cavity in the strong-coupling regime with Fano resonance

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Propagation of the photon from a quantum emitter to the environment can be unidirectional, the energy from the atom decays in a monotonous way (i.e. feeding energy to the environment), or it could be bi-directional, i.e. the emitted photon is reflected back to the original atom and then re-emitted. These two different mechanisms are exclusively dependent on the degree of how the atom is coupled to the external medium. In the former case, the coupling is weak (hence the name weak-coupling), while in the latter case, the coupling is rather strong, thus strong-coupling [1].

In this report, we present light-matter interaction for a hyperbolic cavity that is realized by alternating layers of two different dielectrics in the strong-coupling regime. Non-perturbative local and far-field spectrum is calculated without any rotating wave approximation [2, 3]. Two distinct anti-crossings are reported in the near-field spectrum. Large Rabi oscillation is evident in the local spectrum along with Fano resonance at the spectral position of large Purcell factor and signature of ENZ mode is reported in the far-field.

The hyperbolic cavity that has been realized with alternating layers of different dielectrics, is shown to have very large Purcell factor and Lamb shift for frequency near epsilon near zero (ENZ), from the effective medium description and another frequency that arises due to the contribution from higher order harmonics. The local spectrum of a quantum dot shows two distinct anti-crossings and the anti-crossing near the ENZ originates due to Fano resonance of the cavity and the QD. The null points of the Fano resonance are explained. The far-field spectrum has some rather interesting characteristics — the ENZ only contributes to the far-field. Harmonic decomposition of the Green's function provides the reasoning for the disappearance of modes which are rather present in the near-field spectrum. Thus the propagation to far-field acts as a spectral filter [3]. As a remark, we then show the fidelity of the cavity with respect to loss mechanism that provides us some heuristic idea about the onset of strong coupling and the loss mechanisms in the system.

In conclusion, interaction of light from a quantum dot near a hyperbolic cavity is studied in the strong-coupling regime and the near-and far-field spectral characteristics are calculated. Each result is explained from physical and analytic point of view.

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Homogenization of dielectric wire media

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Electromagnetic metamaterials as artificial sub-wavelength-structured media are effectively homogeneous composite media which exhibit unusual features that can not be found in nature. One important and broad class of metamaterials is called wire medium [1]. This medium is formed by an array of parallel wires which are embedded artificially in a host dielectric material as shown in Fig. 1. The electromagnetic properties of a wire medium can be described through its effective parameters. The effective permittivity of a wire medium in which the wires are perfect electric conductors was derived in Ref. [2]. However, in THz and infrared frequency ranges, the metal is not perfect and the real part of its relative dielectric constant is negative. Reference [3] generalized the case and introduced a nonlocal homogenization model for an array of long negative-dielectric constant rods. In that nonlocal model, under the assumption that the relative dielectric constant of the rod is minus infinity (perfect electric conductor wire), the formula for the effective permittivity will be simplified to the equation expressed in Ref. [2]. Nonlocal homogenization model will be valid if the skin depth of the wire is at least two times smaller than its radius, and the wire diameter is small compared to the effective wavelength.

In the present work, we further improve previous studies assuming a very general case, i.e., the dielectric rods can possess negative or positive relative dielectric constant, and the wires radii are not very small compared to the effective wavelength. This causes that the transverse component of the effective permittivity ($\varepsilon_\perp$) becomes more complex compared to Maxwell–Garnett model, and also the axial component of the effective permeability ($\mu_\parallel$) is not unity. In order to calculate the effective parameters of the structure, first we solve the problem of plane wave scattering from a dielectric cylinder and derive the scattering coefficients. Next the electromagnetic fields inside the cylinder are integrated in order to achieve the corresponding electric and magnetic polarizabilities per unit of length. In the end, the effective permittivity and permeability are computed using the mixing formula. We can see that the obtained effective parameters depend on the axial component of the wave vector which shows the medium is strongly spatially dispersive.

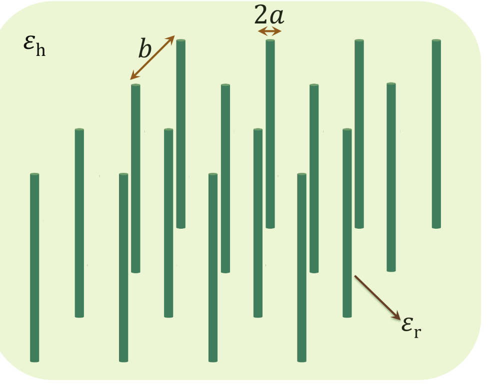

Fig. 1: Geometry of a square lattice wire medium. b and a are the lattice constant and the radius of the dielectric rod, respectively. In addition, ε_h is the host medium permittivity and ε_r represents the relative dielectric constant of the wire.

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Transport properties of a two-dimensional electron gas dressed by light

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Transport properties of two-dimensional electron gas (2DEG) in nanostructures exposed to a high-frequency electromagnetic field have been studied in the deep past and taken deserved place in textbooks [1, 2, 3]. However, the most attention in previous studies on the subject was paid to the regime of weak light-matter interaction.

We show theoretically that the strong interaction of a two-dimensional electron gas (2DEG) with a dressing electromagnetic field drastically changes its transport properties [4]. Particularly, the dressing field, through the modulation of electron scattering from impurities, leads to the giant increase of conductivity (which can reach thousands of percents), results in nontrivial oscillating dependence of conductivity on the field intensity, and suppresses the weak localization of 2DEG

(Fig. 1). As a consequence, the developed theory opens an unexplored way to control transport properties of 2DEG by a strong high-frequency electromagnetic field. From experimental viewpoint, this theory is applicable directly to quantum wells exposed to a laser-generated electromagnetic wave.

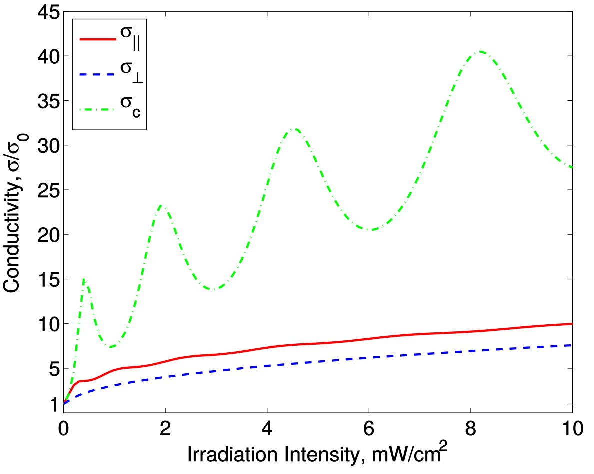

Fig. 1: The conductivity of 2DEG in a GaAs quantum well irradiated by a dressing electromagnetic field with the frequency $\omega = 10^{11}$ rad/s at the temperature $T = 0$ for $\epsilon_F = 10$ meV.

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Stochastic resonance in driven spaser

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Plasmonics deals with the nanoscale phenomena at optical frequencies [1]. Numerous applications of plasmonic devices cannot be realized due to the high level of Joule losses. To compensate them gain media can be included in matrix of metamaterials. This leads to formation of spaser [2, 3]. Schematically [2, 3] spaser is a system of excited two-level quantum dots (QD) surrounding plasmonic nanoparticles (NP). The principle of operation of spaser is similar to the laser. Excitement of plasmon mode of NP by QD near-fields leads to the possibility of further stimulated emission of QDs surrounding this NP in the same plasmon mode and, ultimately, to the development of the plasmon generation. Due to heavy losses in the metal the consideration of the noise is extremely important. The high level of noise can severely limit the possible applications of spaser.

We consider the dynamics of spaser in an external field, taking into account noise in the metal nanoparticle. By virtue of the fact that the dynamics of spaser is nonlinear, effects of noise may lead both to deterioration of spectral characteristics (signal/noise) of spaser and to improvement of them. In the latter case we speak about stochastic resonance [4]. We show that under certain values of the amplitude and frequency of the external field stochastic resonance appears in spaser, which is manifested in the improvement of spectrum of phase. It is interesting to note that this mode is observed on the compensation curve when spaser compensates the losses of the external field. This effect is particularly important in relation to the currently ongoing discussion about the possibility of using spaser in various applications, in particular, as an active component of the metamaterial.

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Pulling optical force for anisotropic cylindrical particles

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Indefinite dielectric permittivity and magnetic permeability tensors correspond to those with simultaneously positive and negative diagonal components [1]. We consider the effect of the recently proposed pulling optical forces [2, 3, 4] for cylindrical particles, which permittivity tensor $\hat{\varepsilon} = \mathrm{diag}(\varepsilon_r, \varepsilon_\varphi, \varepsilon_z)$ is indefinite. Pulling force is a negative radiation pressure acting on the objects in the light field without intensity gradient. As it was discussed previously for the particles of ordinary isotropic material, the pulling effect can be explained as the enhancement of the forward light scattering resulting in the backward force owing to the momentum conservation. Then the material loss making an inelastic momentum transfer should be prejudicial for the pulling effect. The great forward light scattering and, therefore, the pulling force is achieved using complicated field structures known as nonparaxial light beams. The natural example of such a beam is the Bessel light beam with a great cone angle, what is challenging for experimentalists. Moreover, the Rayleigh particles, which radiuses are much smaller than the radiation wavelength, can be attracted only if their material possesses both dielectric and magnetic properties.

We show that the simplification of the electromagnetic beam inevitably leads to the complication of the material of the particle. Both non-absorbing and lossy anisotropic cylindrical particles can be pulled by the light without intensity gradient. For some complex and/or indefinite permittivity tensor $\hat{\varepsilon}$, the pulling effect is achieved due to the complex values of the polarizability tensor of the anisotropic cylinder. Though isotropic magnetodielectric cylinders can be attracted by the nonparaxial light beams, which partial plane waves are characterized by the angle of incidence bigger than 45°, a nonparaxial beam is no more required for an anisotropic cylindrical particle with $\varepsilon_\varphi \neq \varepsilon_z$. In this case even a plane wave can be a tractor beam. Anisotropy $\varepsilon_\varphi \neq \varepsilon_z$ also lifts restrictions on the dimensions and magnetic properties of the cylinder: Rayleigh nonmagnetic anisotropic particle can be considered.

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Light-trapping metamaterial structure for the enhancement of thin-film solar cells

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Recently, we have suggested a dielectric metamaterial composed of an array of submicron dielectric spheres located on top of an amorphous thin-film solar cell. We have theoretically shown that this metamaterial can decrease the reflection and simultaneously suppress the transmission through the photovoltaic layer because it transforms the incident plane wave into a set of focused light beams. This theoretical concept [1] has been strongly developed and experimentally confirmed in the paper [2]. Here we consider the metamaterial for oblique illumination, redesign the solar cell and present a detailed experimental study of the whole structure. In contrast to our previous theoretical study we show that our omnidirectional light-trapping structure (LTS) may operate better than the optimized flat coating. Our LTS is not more expensive than the flat coating obtained by PECVD. The main mechanism of the additional improvement is the so-called cascade focusing. A part of the wave beam obliquely incident on a sphere after focusing enters the gap formed by the adjacent spheres and refracts into the PV layer (Fig. 1). Finally, it reaches the PV substrate beneath this sphere and refracts to it. So, due to the presence of the substrate these partial reflections do not direct the reflected waves back as it happens in the classical theory of rainbow (where similar rays go around the water drop and finally go nearly backward). On the contrary, this mechanism allows the advantageous re-distribution of the focused light in the PV layer so that nearly the whole volume of the semiconductor becomes illuminated (Fig. 1). Cascade focusing is a special mechanism of focusing which is possible only for mutually touching spheres on a substrate.

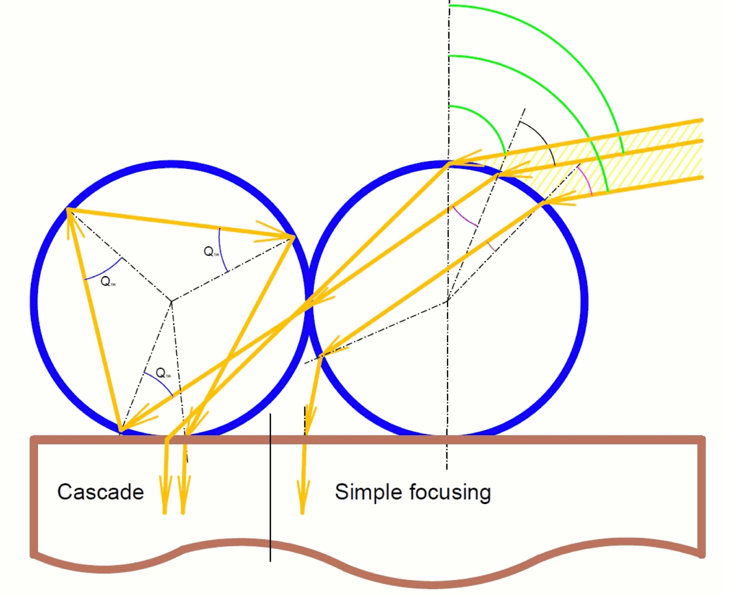

Fig. 1: Schematic view of mechanism of cascade focusing.

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Electrical field tunable acoustic metamaterial

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Periodic domain structures in piezoelectric and ferroelectric materials are very attracted for researchers. There are various acousto-optics and laser devices based on periodic domains [1]. For the periodic domain structures formation commonly used strong piezoelectrics, such as lithium niobade and lithium tantalate doped iron Fe^{2+} and Fe^{3+}. Appling to the crystal direct electric field ($E \sim 10^7$ V/m), resulting in the formation of sufficiently stable periodic domains in accordance with the sign of the electric field [1].

It this paper we are proposed a novel type of surface acoustic wave metamaterial based on the electrical field induced piezoelectric effect phenomena in ferroelectric [2–6]. For the metamaterial tunability used the voltage magnitudes 1–5 volts. Phononic band gaps formation in presented metamaterial is discussed. Basic equations describing the surface acoustic wave propogation on alternating periodic areas under electric field impact are derived. Finite element modeling revealed the possibility using the proposed metamaterial as electrical tunable microwave band ($\sim$ 1–2 GHz) surface acoustic wave filter.

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Negative refraction in microwave photonic crystal

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Electrodynamics of media with negative index of refraction was first studied by a Russian theorist Victor Veselago in 1967. The proposed left-handed or negative index materials were theorized to exhibit optical properties opposite to those of glass, air, and other transparent media. Such materials were predicted to exhibit counterintuitive properties like bending or refracting light in unusual and unexpected ways.

More and more attention is drawn to the possibility of creating materials with negative refractive index on the basis of ordered structures called microwave photonic crystals [1]. Electromagnetic crystal (EMC) — artificial medium formed of parallel conductive rods are periodically arranged in a dielectric matrix [2]. Of great importance is the experimental study of such structures in the microwave range and comparing them with theoretical data. Of great importance is the experimental study of such structures in the microwave range and comparing them with theoretical data.

The sample was a dielectric prism with copper rod of length $l = 12$ mm, 0.7 mm in diameter, arranged in a dielectric $\varepsilon = 2.9$, lattice constant $a = 12$ mm. In this paper we investigated the dependence of the transmission coefficient of the EMC prism at different angles (Fig. 1a). It is experimentally shown that the frequency $f = 11,34$ GHz ($ka/2\pi = 0.9$) refraction in the EMC has anomalous peculiar media with negative refractive index (Fig. 1b).

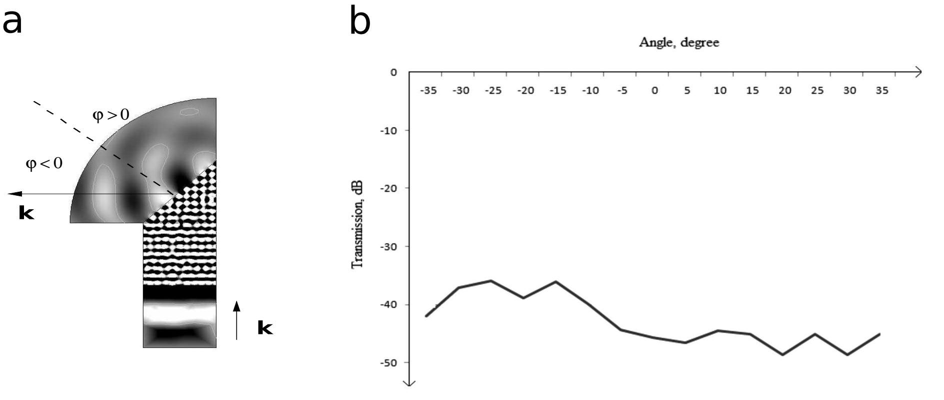

Fig. 1: a) sample under investigation b) the transmission factor of the angle of refraction.

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Electromagnetic properties of liquids at GHz frequencies for medical tasks and metamaterial applications

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The knowledge of the electromagnetic properties of liquids for problems of modern radio physics and medicine are important. The dispersion characteristics of fluids in the GHz frequency range required to characterize properties of metamaterials with fluid-filled unit cells. Concerning the medical applications in such important tasks like diagnosis of the state biological object, the same dispersion properties plays an important role[1].

For measurements of electrodynamic parameters of liquids two main techniques are used: Nicholson–Ross–Weir (NRW) [2] and the active near-field diagnostics [3]. These methods have similar physical principles of operation, but they differ in the practical execution and information that can be obtained from the test object. The NRW technique is based on the principle of extracting material parameters [2, 4] from S-parameters(reflection S_{11} and transmission S_{21}) of the sample with the thickness d, located in the waveguide section. Active near-field diagnostic technique allows to define the material parameters by measuring the conductivity of the test sample and the impedance changes in the near field zone of an antenna. In this case it is enough to locate the probe at the surface of a liquid. In this work measurements of liquid material properties were made by both techniques, and compared to each other showing good correspondence.

Generally the permeability value practically does not change being almost equal to one in comparison with the dielectric permittivity. Since the method of active near-field diagnostics is very comfortable in measuring permittivity and conductivity of liquids, it was preferred method of NRW for the most part of measurements. As a result of the experimental studies we obtained the values of permittivity (the real and imaginary parts) of several different types of liquids, including salt solutions.

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Direct mapping of magnetic and electric optical responses from silicon nanoparticles

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Dielectric nanostructures composed of materials with high refractive index are known to possess strong magnetic and electric optical responses [1, 2, 3]. Contrary to plasmonic, they have much lower losses, being able to shift the wavelength of the magnetic resonance down to the visible spectral range, bringing an additional degree of freedom for light manipulation at the nanoscale. These properties make the subwavelength dielectric nanoparticles promising candidates for building blocks of all-dielectric optical metamaterials and nanoantennas.

The basic nanostructure which possess both electric and magnetic responses in optical range is a silicon spherical nanoparticle [2, 3]. Commonly, nanoparticles are studied in the far-field by measuring the scattering spectra. Combination of these experimental spectra with the theoretical ones allows to distinguish the contributions of dipole and higher-order multipole modes and observe such extraordinary phenomena as the excitation of toroidal or anapole modes [4, 5]. Alternatively, scattering near-field scanning optical microscopy technique provides the opportunity for direct observation of high-order multipole modes in the near-field [6].

Here, we perform an alternative experimental approach allowing to distinguish the optical magnetic dipole and electric dipole resonances directly from experiment. This is made possible by combining the polarization-resolved dark-field spectroscopy and near-field scanning optical microscopy (NSOM) measurements. The first method allows to determine the spectral positions of optical scattering resonances and demonstrates strong sensitivity of electric dipole response to the incident polarization. In turn, the magnetic dipole response of the nanoparticle can be directly identified through the asymmetry in the NSOM patterns obtained using an aperture-type probe operating as an effective magnetic field analyzer. The presented results confirm that the proposed experimental approach allows for discrimination of the dipole optical responses of both magnetic and electric nature. Importantly, this can be done directly from the experimental data without employing complementary numerical simulations.

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Asymmetric hyperbolic metamaterials

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Hyperbolic metamaterials (HMMs) are artificial structures representing an ultra-anisotropic limit of the traditional uniaxial crystals. HMMs can be described by the effective permittivity tensor $\hat{\varepsilon}_{\text{eff}} = \text{diag}[\varepsilon_o, \varepsilon_o, \varepsilon_e]$, $(\Re[\varepsilon_o]\Re[\varepsilon_e] < 0)$, here the subscripts o and e indicate effective tensor's components perpendicular and parallel to anisotropy axis respectively. Permeability tensor is assumed to be unit. Unusual properties of HMMs originate from the hyperbolic form of its isofrequency surface for extraordinary waves. Nowadays, there are two types of HMMs: Type-1 with $\Re[\varepsilon_o] > 0$ and $\Re[\varepsilon_e] < 0$ (isofrequency surface is a two-sheeted hyperboloid) and Type-2 with $\Re[\varepsilon_o] < 0$ and $\Re[\varepsilon_e] > 0$ (isofrequency surface is a one-sheeted hyperboloid) [1].

The common realization of an HMM is a periodic layered metal-dielectric (LMD) structure composed of isotropic metallic and dielectric slabs. We study periodic LMD structures composed of dielectric and metallic slab of absorbing and conducting "uniaxial" crystals, respectively. Executed analysis allows us to treat such the LMD structures as a realization of metamaterials which can be called as generalized HMMs (GHMMs).

Effective medium approximation for layered GHMMs follows from Maxwell–Garnett approach [1]. If we consider the set of alternating dielectric and metallic slabs (permittivity tensors $\hat{\varepsilon}_d$ and $\hat{\varepsilon}_m$, respectively) with metal fill fraction ρ (ratio of the total thickness of metallic slabs to the thickness of the whole structure) and apply the Maxwell–Garnett approach, we obtain the set of nine equations for the nine components of the effective permittivity tensor $\hat{\varepsilon}_{\text{eff}}$.

In the following analysis we assume that metallic slabs are made of isotropic metal and $\hat{\varepsilon}_d$ takes diagonal form (e.g. $\hat{\varepsilon}_d = \text{diag}[\varepsilon_o, \varepsilon_o, \varepsilon_e]$). Then effective permittivity tensor takes diagonal form as well $\hat{\varepsilon}_{\text{eff}} = \text{diag}[\varepsilon_{xx}, \varepsilon_{yy}, \varepsilon_{zz}]$ and represent a biaxial structure. The components of $\hat{\varepsilon}_{\text{eff}}$ depend on seven real parameters ($\rho, \varepsilon_o, \varepsilon_e$ and ε_m). We have shown that it possible to obtain any combination of signs on the diagonal of the effective permittivity tensor due to an appropriate choice of the parameters. We imply signs combination of the real parts of effective permittivity tensor's components. In order to find such parameters we have introduced absorbing crystal parameter's diagram for fixed values of ρ and ε_m. Each point of the diagram corresponds to an eigenvalue of $\hat{\varepsilon}_d$. The diagram is divided into four or three regions (in dependence on values of ρ and ε_m) in correspondence with certain combinations of signs of the effective tensor's components.

The possibility to choose combination of signs on the diagonal of $\hat{\varepsilon}_{\text{eff}}$ allow us to obtain unique isofrequency surfaces. For instance, there are layered GHHMs such that in a certain plane of incidence isofrequency curve can represent either Type-1 or Type-2 dispersions but in the plane of incidence which is orthogonal to the previous one a layered GHMM does not support large wave-vectors already. This let us treat some kind of the layered GHMMs as asymmetric HMMs with arbitrary oriented anisotropy axis. We have also revealed that for similar dispersion properties (in the fixed plane of incidence for layered GHMMs) layered GHMMs and HMMs have basically different transmission properties, i.e. Type-2 dispersion of the layered GHMM possesses high transmission and Type-1 dispersion of the layered GHMM possesses low transmission in contrast to HMMs.

The comparison of transmission and reflection coefficients for a layered GHMM and the corresponding effective medium slab of the same thickness have shown that the developed effective medium approximation works worse for a LMD structure composed of anisotropic components and requires more layers than for a LMD structure composed of isotropic slabs.

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Propagation of surface plasmon polaritons in chains of non-spherical nanoparticles: the influence of the dielectric substrate

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In this work we study propagation of surface plasmon polaritons (SPPs) in linear chains of $N = 1001$ silver oblate spheroids located on the flat quartz substrate.The longer semi-axes of spheroids are assumed to be perpendicular to the chain. We use the Drude model for dielectric permittivity ε_m of metal and experimental values for permittivity of the substrate: $\varepsilon_s = 2.5 + 0.01i$.

We have found that the presence of the dielectric substrate leads to conversion of SPP's polarization: linear–circular, linear–linear and circular–linear conversions are possible. This effect is enhanced at the shorter distances from the substrate.

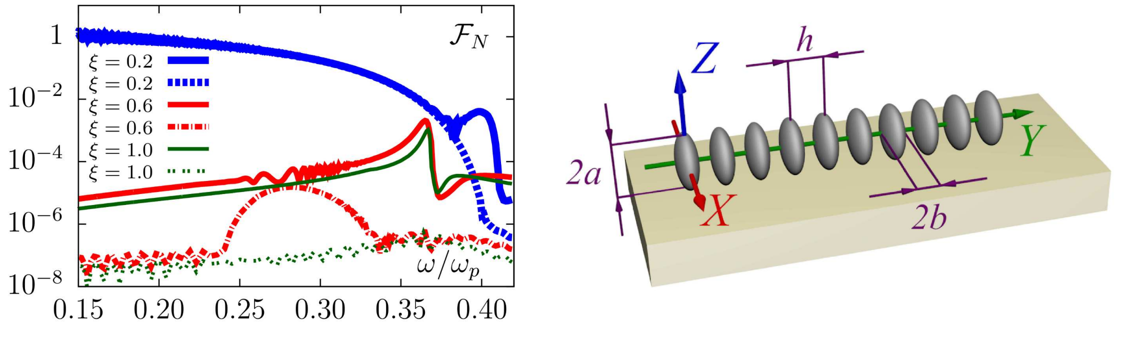

Fig. 1: Transmission spectra for chain consisting of $N = 1001$ oblate nanospheroids located on a dielectric substrate (dot-dashed line) and in its absence (solid lines). The calculations were performed for polarization along X axis and for various values of the aspect ratio $\xi = b/a$. The shorter semi-axis is $b = 8$ nm and the center-to-center interparticle distance is $h = 24$ nm.

The dielectric substrate exerts significant influence on transmission properties of the metal nanoparticles chains. We have found that attenuation of SPP strongly depends on its polarization and the aspect ratio of nanospheroids. In most cases, the presence of substrate increases the attenuation of SPP. However, the technological substrate does not violate the nondecaying propagation of SPP that takes place in chains of oblate spheroids with sufficiently small aspect ratios. In particular, for oblate spheroids with aspect ratio $b/a < 0.25$ (where a is the longer semi-axis) in the low-frequency spectral range the effect of dielectric substrate is negligible. The absence of strong decay can be explained by high values of the depolarization factor of oblate spheroids with small aspect ratios and by increase of Q-factor of surface plasmon resonance of non-spherical Ag nanoparticles for low frequencies.

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Plasmonic and diffraction structures via thermal poling of glass

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To date there are many efforts performed to find simple ways of the formation of 2D or 3D submicron structures for optics and plasmonics. An effective technique to fabricate such media is annealing of a glass containing ions of noble metals in hydrogen atmosphere [1]. This principally allows wide-scale manufacturing of metal-island films (MIF) on the glass surface and nanoparticles in the bulk glass. It has been recently shown [2] that combining thermal poling with this technique allows nanostructuring of both MIF and bulk nanocomposites. For example poling an ion-exchanged glass with a nanoprofiled electrode allowed us to make diffraction grating with period of 400 nm (the resolution can be even higher), and so called "plasmonic molecules" — isolated groups of several metal nanoislands (see Fig. 1). Experimental details are presented in [3].

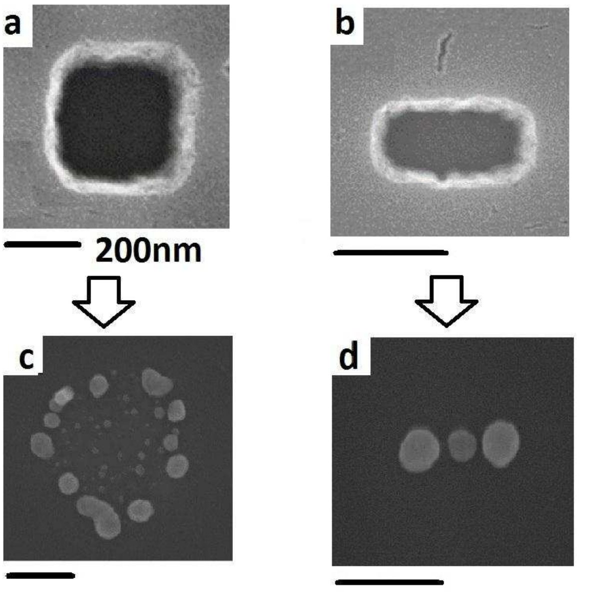

Fig. 1: SEM images of used electrodes: square (a), rectangle (b), and corresponding MIF-structures grown on the glass surface (c,d). Black scale bar below each figure corresponds to 200 nm.

In this work we study formation and growth of structured MIFs on the basis of the theory of phase transitions. We introduce the numerical model developed and examine the effect of the MIF growth conditions and glass parameters on final distribution of metal nanoislands. The results presented allow predicting the structure of MIF to be formed.

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Magneto-optical switching of Bloch surface electromagnetic waves in magnetophotonic crystals

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Bloch surface electromagnetic waves (SEW) at the interfaces separating photonic crystal and dielectrics are shown to be important in many applications from integrated optical circuits to biosensors [1]. The ability to realize the controllable switching of SEW is prospective for fabrication of the new types of tunable optoelectronic devices. One of the effective ways to control properties of the optoelectronic system is the application of magnetic field. Magnetic field applied to the magneto-optical active structure leads to the effects of the light state switching such as Faraday rotation of light polarization. The enhancement of the Faraday rotation can be achieved due to electromagnetic-field localization inside the structure in presence of surface plasmon [2] or at the edge of the photonic band gap of magnetophotonic crystal (MPC) [3]. Application of magnetic field allows one to manipulate surface electromagnetic state, for example, control the wavelength of optical Tamm state resonance [4]. The drastic enhancement of Faraday effect is expected in case of the SEW because of high localization of the SEW electromagnetic field that is why it looks prospective to study the SEW at the surface of the MPC. Excitation of the SEW is expected to be controlled by application of magnetic field.

Magnetic-field induced response of SEW at the surface of magnetophotonic crystal is studied using 4×4 transfer matrix method [5, 6]. One-dimensional two-component MPCs with 6 elementary cells consisting of one isotropic (SiO_2) and one magneto-optical active (Bi:YIG) $\lambda/4$-layers are considered. Reflectance spectra and the distributions of electromagnetic field inside the structure were calculated at the angle of 50° in Kretschmann configuration of attenuated total internal reflection scheme. If the magnetic field is absent, the reflectance spectra of s-polarized light have a typical narrow dip corresponds to the SEW resonance. The reflectance spectra of p-polarized light do not demonstrate the SEW excitation. If the magnetic field is applied, the Faraday rotation induces mixing of two polarizations and leads to p-polarized SEW excitation. Light localization close to the resonance of the SEW excitation leads to the giant enhancement of Faraday rotation. Excitation of the SEW at the surface of the MPC for p-polarized light depends on magnetization in the Bi:YIG layers and therefore is controllable. These features of the MPC can be of use for the design of for tunable optoelectronic devices.

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Controlling light emission and fields with metamaterials

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Metamaterials have been proposed as a potential solution to control emission of light for more than a decade [1, 2]. The Near Zero Index metamaterials have been proposed to design directive emitters. More recently so-called hyperbolic metamaterials have been intensively studied. These structures possess effective permeability or permittivity tensors components such that one principal component is opposite to the two others.

It has been shown that hyperbolic metamaterials could lead to negative refraction and superlensing [3, 4]. The dispersion relation display hyperbolic features (i.e. isofrequency dispersion relation is a hyperbola) and, thus, could lead to diverging density of states and enhancement of spontaneous emission [4].

We will present a detailed study including modeling and experiments of the emission in hyperbolic metamaterials and near zero index metamaterials. Modeling and measurements allows us to determine both effective permittivity and permeability values for the studied structure. The retrieval procedure will be described. Then, measurement of the electric and magnetic field inside the hyperbolic metamaterial will be shown (when excited by a source located inside). We will also present results of local density of states modeling and measurement.

Research conducted within the context of the International Associated Laboratory "ALPhFA": Associated Laboratory for Photonics between France and Australia.

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Multiscale modeling of all dielectric metamaterials

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Metamaterials as a material composed from other materials possess optical features related to properties on different scales. We consider four scale levels: material properties, structural element shape, element arrangement, and sample boundaries. Table 1 shows the scale levels and corresponding responses on electromagnetic radiation. The properties of whole sample are result of interference of resonances on each scale levels. The main strategy in design of such complicated systems is to use multiscale modeling approach [1].

Table 1: Scale levels in metamaterials and photonic crystals

Scale level	Examples of response
Material properties	Crystalline lattice, interband transitions, plasmonic resonances.
Structural element shape	Mie-like resonances
Arrangement	Bragg resonances, cavity modes, surface modes
Sample boundaries	Fabry-Perot interference

We study all dielectric metamaterials constructed from infinite cylinders arranged in square lattice. Let us consider the problem on different scale levels. We begin with material properties. To yield strong response of the whole system we choose material with a high dielectric permittivity in visible and near IR region. Most convenient materials are Si or chalcogenide materials. On the next levels we have to account frequency dependence of these materials. The second scale level is connected to the cylindrical form of the structural elements. The Lorenz–Mie solution is ready for frequency dependent permittivity [2].

To describe properties on arrangement scale level we calculate electromagnetic eigenproblem. Note, that usual tools to calculate eigenfrequency to wavevector dispersion [3] are not suitable to account frequency dependence of permittivity, which is a part of the Maxwell operator now. To overcome this issue we developed an inverse dispersion technique that calculates wavevector to frequency dispersion. Because of the spatial dispersion of typical material permittivity are negligible we able to separate wavevector as an eigenvalue. As a result we obtain a method that ready to calculate metamaterials composed from high dielectric index materials with inevitable frequency dependence.

On the last scale level we consider a sample with infinite number of cylinders in one direction and $N = 10$ layers in another. The boundaries can be described as a line defect which generate Bloch waves in backward directions. As a result boundaries give rise to Fabry–Perot resonances. We perform full wave simulations to study the sample.

We can effectively design metamaterial-based devices by dividing problem on different scales to choose or adjust properties on these levels.

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Sensor application of microwave metamaterial absorber to food products

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In this study, sensor ability of a metamaterial absorber (MA) based on rectangular resonator(RR) is investigated and studied. The proposed MA shown in Figure 1 provides perfect absorption at the resonant frequency of 2.4 GHz. Numerical results for the reflection (S_{11}) and absorption are shown in Figure 2. Proposed MA enables myriad potential applications in S band such as sensor, harvesting, and etc. Among the several potential applications, sensor application is selected to show the sensor functionality of the MA design. Carrot content is defined first and sensing application is performed for S-band frequency region. The temperature of the carrot content is varied and the influence of this variation on S_{11} is examined. The result of this examination is shown in Figure 3. When the temperature of the content is changed the spectral location of the resonance of the structure shifts to the lower frequencies. This shift can be explained by the change of the dielectric constant of the overall sensor system. As a result, the sensor device can be used in many sensing applications such as temperature-, pressure-, biological-sensing and so on. Additionally, it can also be utilized in different food sensing applications, too. Consequently, the proposed device will be very effective and can also be used as sensor for various nutrition and similar products.

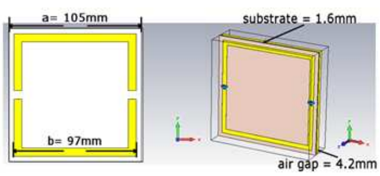

Fig. 1: Configuration and dimensions of the MA.

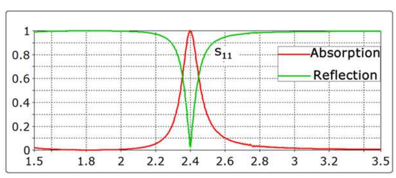

Fig. 2: Simulated reflection and absorption for the suggested MA.

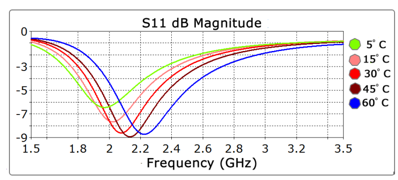

Fig. 3: Sensor application of the proposed MA with respect to temperature change.

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Fishnet-based-metamaterial loaded waveguide for sensing applications

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In this work, a fishnet-based-metamaterial loaded waveguide for sensing application is investigated for microwave frequencies. For this aim, a fishnet-like metamaterial structure for waveguide environment is designed and then it is used for single- and double-sided sensing applications. The model is configured to have a simple pattern and resonates at around 10 GHz (to be used in X-band waveguides). The proposed structure is shown in Figure 1. For sensing application, the unknown material to be detected is arranged as an over-layer. The frequency response of the metamaterial sensor is examined when the thickness of over-layer is changed. Both the single- and double-sided sensing situations are studied. Figure 2 shows the results of the mentioned situations. The transmission resonance with magnitude reduction shifts to the lower frequencies when the over-layer thickness increases. The downshift in the transmission resonance can be explained as follows: the entire sensor system (metamaterial + over-layer) can be treated as a resonant RLC circuit and its resonant frequency can be determined by effective inductance and effective capacitance values. When the thickness of over-layer is changed, the effective RLC values of the system changes too (increases in the present case). This yields a downshift in the transmission resonance since the resonant frequency is directly related with the thickness of over-layer(or effective inductance and effective capacitance values). Comparing the data for the single- and double-over-layer cases, the relative down-shift for each thicknesses with respect to the main resonance is enhanced when double-over-layer is used. This means that double-side sensing capability of the proposed sensor system provides sensitivity enhancement compared with single-sided sensors. As a result, the proposed sensor system can be used as a potential spatial selective sensing device. By detecting the targets at different spatial locations on the structure, the rather different sensing behaviors could be obtained and could be with more desired for different applications (i.e. spatial sensing and imaging) [1, 2].

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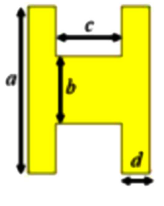

Fig. 1: Configuration fishnet-based-metamaterial with dimensions (a = 7.5 mm, b = 4 mm, c = 3 mm, and d = 1.25 mm).

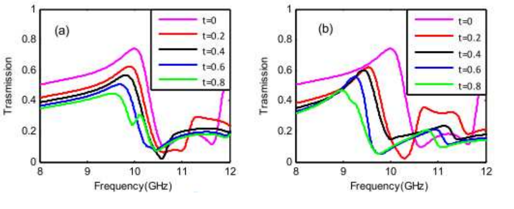

Fig. 2: Frequency response of the transmission for a) single-over-layer and b) double-over-layer for different over-layer thicknesses (t).

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Thermal phenomena in quantum plasmonics

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Plasmon nanolasers, also known as SPASERs, were suggested by Bergman and Stockman in 2003. Quantum plasmonics attract much attention in recent years due to the numerous potential applications in the plasmonics. We consider thermal effects in the metal nanoresonator immersed in the active, laser medium. The size of the resonator is much less than the wavelength. The plasmon field inside the nanoresonator operates as a quantum object. Due to the nanosize of the resonator, the internal plasmon electric field is about the atomic field even for few plasmon quants. The coupling between the plasmon field and plasmon resonator is anomalous strong. We develop the quantum dynamics of the plasmon field and show that the SPASER may be the subject of thermal instability. The loss in SPASER increases with increasing the temperature when the average number of the plasmons is maintained at the stationary level. Therefore, the heat generation increases with increasing the temperature. This positive feedback results in the thermal instability. When the energy, accumulated in the plasmon nanoresonator, exceeds the instability threshold the temperature increases exponentially. We find the increment of the temperature growth and lifetime as function of the loss in metal and the structure of the plasmon resonator. It is considered how the thermal instability influences the luminescence and find how the lasing threshold is changed. The coherence of the light emitted by the plasmon laser is also considered. The thermal stability of the nanolaser is crucial for any practical application.

Fano resonance in all-dielectric nanoparticle chains with side-coupled resonator

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Highly efficient integrated optical circuits require waveguides with subwavelength light localization and low overall losses. Recently, realization of such structures was suggested in the form of chains of high-index dielectric nanoparticles [1]. It is known that such nanoparticles with linear size less than the operating wavelength exhibit magnetic dipole resonance, and its resonance frequency depend on the particles size and shape [2]. Interest in control of transmission of light in waveguides led to proposition of several practical schemes allowing its realization. One of them — coupled resonator induced reflection – was suggested and studied in photonic crystal waveguides [3, 4].

Here we study the resonant transmission of light in a waveguide formed by a chain of high-index dielectric nanoparticles with side-coupled defect nanoparticle. We choose the permittivity of dielectric particles $\varepsilon = 16$, radius $R = 70$ nm and radius of additional side located particle $R_d = 75$ nm. By adjusting the size of additional particle we shift its magnetic dipole resonance frequency ω_r to the pass band of the uniform chain, thus providing the conditions for Fano-type resonance, where a single side-coupled particle resonance interferes with the pass band of the straight chain. The chain is excited with a small (point) electric dipole oriented in z direction, therefore magnetic moments induced in the particles oscillate in $x - y$ plane. In Figs. 1(a,b) we show magnetic field distribution at the resonance frequency ω_r in the chain consisted of 30 spheres, with and without additional sphere, respectively. We observe that additional particle almost completely blocks the propagating mode, which in turn induces a rotating dipole moment in it [see Fig. 1(c)]. We note that such induced reflection is not possible when $R_d = R$, since in this case resonance frequency of a single particle lies outside the pass band of transversely polarized modes [1].

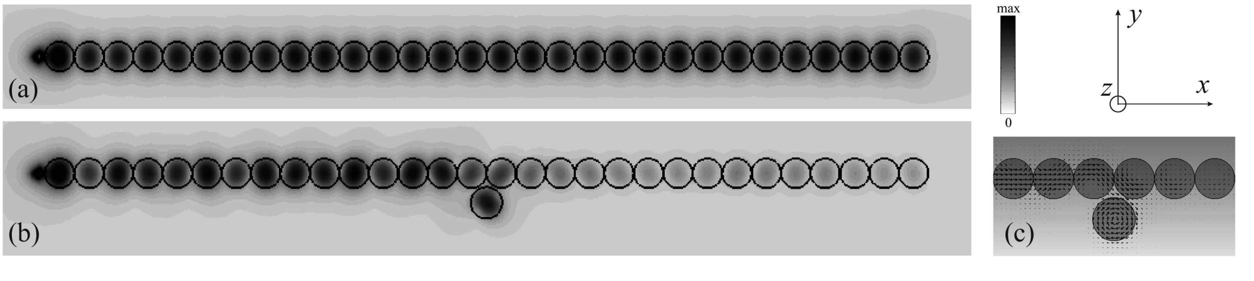

Fig. 1: Magnetic field distribution $|\mathbf{H}|$ in (a) straight chain of dielectric particles and (b) chain with additional particle. (c) Power flow distribution in the center of the chain.

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Quantum hyperbolic metamaterials with exciton-polaritons in semiconductor Bragg mirrors

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We propose a novel approach for emulating quantum effects in curved space-time using resonant semiconductor Bragg mirrors. We show that planar periodic semiconductor Bragg mirror structures with embedded quantum wells (QWs) allow for controlling the signs of effective masses of mixed light-matter quasiparticles termed Bragg exciton-polaritons in order to create hyperbolic metamaterials (HMMs) [1]. The magnitude and sign of the polariton effective mass in such structures affect the effective dielectric permeabilities which are crucial for designing HMMs. The discussed structure consists of the periodic array of alternating dielectric layers with QWs placed in the centres of the layers of one type. We demonstrate mapping of the polaritonic Gross–Pitaevskii equation onto a nonlinear Ginzburg–Landau–Higgs equation, that is typically discussed in connection with the Universe properties. In the liner case we obtain a polariton X-wave solution that is reminiscent of a non-diffractive (spatially localized) matter wave packet. We predict formation of kink-shaped states for weakly interacting polaritons. Small amplitude oscillations (oscillons) occur in a perturbed polariton Higgs field. Polaritonic nonlinear HMMs have a high potentiality for simulation of fundamental cosmological processes.

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Atomic Bose–Einstein condensates as a nonlinear hyperbolic metamaterials

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We consider the problem of formation of small-amplitude spatially localized oscillatory structures for atomic Bose–Einstein condensates confined in two- and three-dimensional optical lattices, respectively. Our approach is based on applying the regions with different signs of atomic effective masses where an atomic system exhibits effective hyperbolic dispersion within the first Brillouin zone. By using the **kp** method we have demonstrated mapping of the initial Gross–Pitaevskii equation on nonlinear Klein–Gordon and/or Ginzburg–Landau–Higgs equations, which is inherent in matter fields within ϕ^4-field theories. Formation of breatherlike oscillating localized states — atomic oscillons — as well as kink-shaped states have been predicted in this case. Apart from classical field theories atomic field oscillons occurring in finite lattice structures possess a critical number of particles for their formation. The obtained results pave the way to simulating some analogues of fundamental cosmological processes occurring during our Universe's evolution and to modeling nonlinear hyperbolic metamaterials with condensed matter (atomic) systems.

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Optical cloaking with spatially dispersive ENZ-metamaterials

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Suppression of scattering cross-sections in a specifically design material environment leads to reduced detectability of objects, and may, ultimately, result in "invisiblity" if the scattering is absent. The concept of "cloaking" was introduced in [1, 2] and gained considerable attention due to continuous demand to achieve invisibility for radar waves and visible light. The general approaches for cloaking rely either on transformation optics concepts [1] or conformal optical mapping of complex electromagnetic potentials [2]. While the former generally results in the requirements of highly anisotropic (and sometimes singular) electric and magnetic susceptibilities of a medium of a cloak, the latter approach requires a position-dependent refractive index variation but is restricted for two-dimensional geometries.

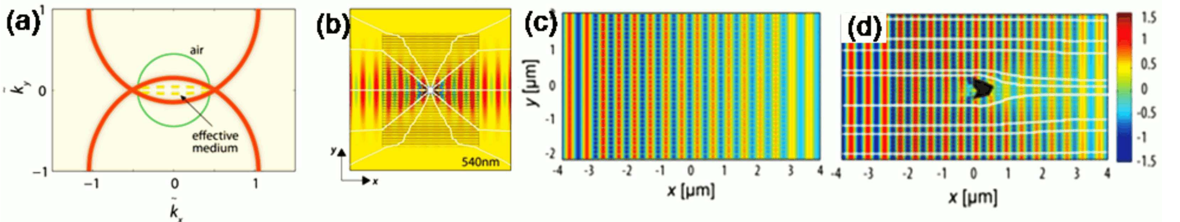

Fig. 1: (a) Isofrequency contours for metamaterial at the wavelength 540 nm. Red curves show propagation modes. Yellow — isofrequency contours, simulated in the effective medium theory. (b) The electric field distributions of the radiating y-polarized dipole in the metamaterial at the wavelength 540 nm. White lines represent the power flow. (c) Electric field (y-component) distributions for the plane wave illuminating the empty cloak. (d) PEC object (larger than the wavelength) of an arbitrary shape inside the cloak.

Here we demonstrate cloaking of arbitrary shaped and not necessarily subwavelength sized objects placed inside a layered metamaterial. We show that a metamaterial realization has major influence on the cloaking phenomenon since the electromagnetic response of those plasmonic multilayers is substantially affected by spatial dispersion effects. Investigation of the exact numerical model, shows the possibility of nearly perfect cloaking of arbitrary shaped bodies with the minimal variations of the phase front of the transmitted/reflected optical wave and highly suppressed scattering. Analysing the dipolar emission inside spatially dispersive layered metamaterial by considering its spatial dispersion

(Fig. 1(a)), we show that in contrary to the free space scenario the radiation patterns have flat uniform wavefronts (Fig. 1(b)). The dipole paves a way for investigation more complex geometries. Fig. 1(c,d) shows the electromagnetic scattering in the empty cloak and the one with an object inside. The similarity between transmitted wave fronts verifies the validity of the general concept.

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Femtosecond Faraday evolution in one-dimensional photonic structures

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Faraday effect is a phenomenon widely used for light propagation control. The effect is non-reciprocal and proportional to the thickness of a magnetic medium. Thus, it can be enhanced in multilayered structures with artificial dispersion and group delay, such as magnetophotonic crystals. In case of short femtosecond laser pulse its front part interacts with the structure less effectively than the rear one. Until now, however, ultrafast dynamics of magneto-optical effects has always been connected with magnetization changes. The goal of the work is to demonstrate Faraday effect evolution caused by the pulse interference in the layered structure.

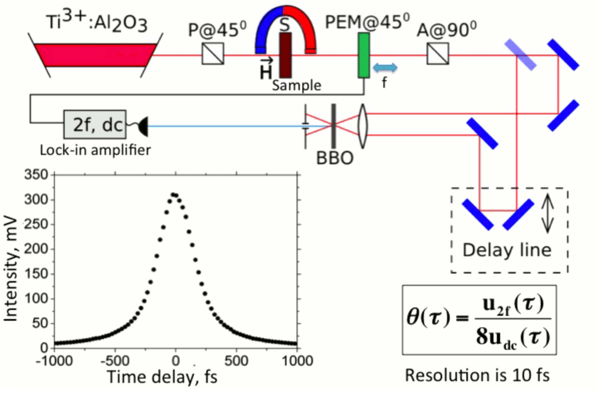

Fig. 1: Up: Experimental setup consists of correlation technique, augmented with polarization-sensitive elements. Down: Autocorrelation function of the laser pulse.

To detect the time dependence of Faraday rotation, autocorrelation technique has been used (see Fig. 1). We measured autocorrelation function of the 130-fs laser pulse with tunable central wavelength, which passed through magneto-photonic crystal with the center of microcavity mode at the 895 nm. We detected ultrafast Faraday rotation dynamics in a magnetophotonic microcavity. It is caused by the pulse self-interference inside its layers. The character of dynamics strongly depends on spectral position of the pulse central wavelength due to the strong artificial dispersion of the medium. The time derivative of Faraday rotation is the greatest when pulse central wavelength is at the center of microcavity mode and much less when it is in the vicinity of this point.

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Nonlinear dielectric metasurfaces and oligomers: harmonics generation and all-optical switching

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Compact yet efficient nonlinear-optical devices are a cornerstone of modern photonics. Wave-mixing and all-optical switching processes are sought for in micro- and nanostructures as those effects pave the way for a new optics-based telecommunication and data manipulation paradigm [1]. Silicon photonics is believed to dominate the area: microring resonators, Raman lasers and other devices are readily available to be integrated into photonic circuits [2]. However, shrinking down the size of silicon-based devices was hardly considered feasible until it was recently realized that highly localized Mie-type modes can be excited in silicon nanoparticles [3]. Utilizing the Mie-type resonances in silicon nanospheres, nanodisks and other geometries is a promising strategy to achieve high nonlinear processes efficiencies at low mode volumes.

In this talk we review our recent investigations of nonlinear optical properties of silicon-based nanoparticles and metasurfaces. The nanostructures under study are distinguished by fundamental localized magnetic Mie-type resonances with mode volumes as low as $\lambda^3/100$. It is shown by third-harmonic generation (THG) spectroscopy and THG microscopy that effective third-order nonlinearities of silicon nanodisk arrays are two orders of magnitude larger than those of an unstructured bulk silicon slab [4]. Arranging nanoparticles in an oligomer (trimer) geometry provides for another degree of freedom in tailoring their nonlinear-optical response [5]. Finally, pump-probe measurement results of all-optical switching in silicon-based metasurfaces will be presented.

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Optical forces induced by Bloch surface waves on a one-dimensional photonic crystal

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Bloch surface waves (BSW) are surface electromagnetic modes that propagate in all-dielectric structures and provide large local field enhancement [1]. In our work, BSWs are shown to be a promising tool for optical manipulation of dielectric microparticles. Momentum of the BSW at a one-dimensional photonic crystal/water interface is experimentally demonstrated to be transferred to a 1-µm polystyrene microsphere located in the vicinity of the photonic crystal surface. As it is observed by means of optical microscopy, the interaction force generated by the BSW is large enough for particle localization near the surface and propulsion along the BSW propagating direction. To measure the force quantitatively, photonic force microscopy is used.

Photonic force microscopy technique is based on the determination of the particle displacement in an optical tweezers trap at an external force influence [2]. The experiment scheme and results of photonic force microscopy of the BSW are shown in Fig. 1. The measured force decays exponentially with moving off the surface at large distances, but there is a surprising diminution of the force at surface/particle gaps less than 150 nm. The maximum value of 0.25 fN at the exciting radiation intensity of 1.6 kW/cm^2 is observed at the BSW excitation resonance.

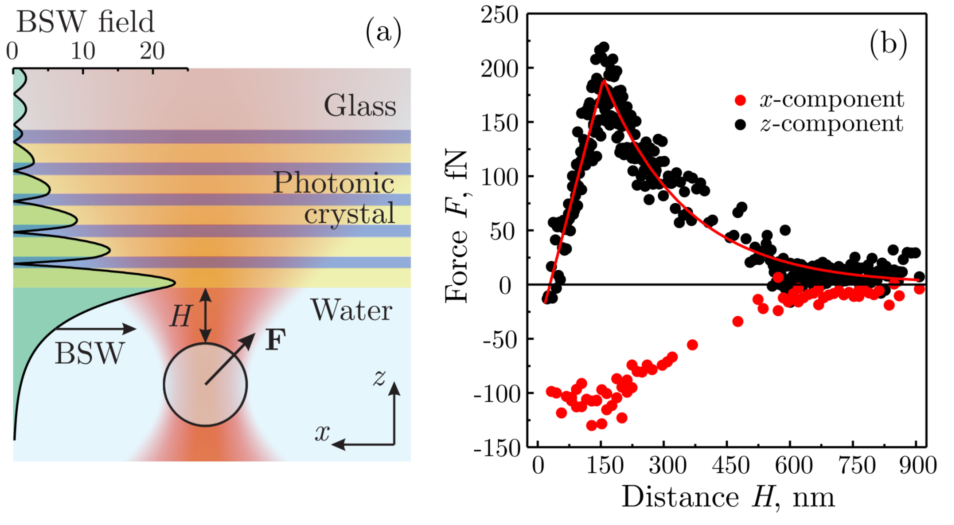

Fig. 1: (a) The experiment scheme. The calculated distribution of the incident BSW electric field amplitude is shown in green. (b) The x- (red dots) and z-coordinate (black dots) projection value of the measured force depending on the distance between the particle and the photonic crystal surface.

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Distributed feedback laser

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We consider lasing in one dimensional photonic crystal, which primitive cell consists of two layers: Maxwell–Bloch-like active layer and passive layer. It is shown that when lasing starts the effective dielectric structure loses its periodicity because the Bloch mode is not periodic. However, at the band-edge where the Bloch mode is periodic and its period is equal to the photonic crystal cell size, the distribution of electromagnetic field in photonic crystal becomes also spatial periodic. Besides, due to Borrmann effect electromagnetic field tends to concentrate in the passive or active layers of the photonic crystal. Moreover the electromagnetic field in the passive layers oscillates in phase at the second band-edge. If we assume that the dissipation in passive layers refers to emitting the light to the outer space than the lighting beam have to be highly directional. The effect might explain the recent experiments on lasing in active plasmonic systems [1].

We also consider DFB laser when frequency of active medium is inside the band gap. Putting active medium frequency inside the band gap creates detuning in the laser, so this might lead to non-monotonic behavior of the threshold. The effect is not followed by any changes in spatial distribution of the electromagnetic field above the threshold. Unlike in work of Stone [2] such threshold behavior appears because of the greater overlapping between the lasing mode and pumping in frequency space.

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Third harmonic generation spectroscopy of plasmonic meta-atoms in the vicinity of the magnetic dipolar resonance

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Over the last decade many kinds of metamaterials with remarkable optical properties have been proposed [1]. Modifying substance on the nanoscale one can achieve desired optical properties; therefore metamaterials exhibit a variety of novel peculiar features which can be used in many different applications [2].

The linear optical properties of metamaterials have been under intense investigation over the last years [3]. Chirality, optical magnetism and many others optical properties can be imposed on bulk media by nanostructuring. Moreover, metamaterials exhibit distinct nonlinear optical response manifested as new components in the nonlinear susceptibility tensor as a result of the intrinsic nanoscale symmetry [4]. Metamaterials with optical magnetism — like the well-known fishnet metamaterial — also demonstrate strong increase of their nonlinear response in the vicinity of the magnetic resonance due to the enhancement of the local field inside the structure [5] and a special angular dependence of the third harmonic generation (THG) due to the retardation effects [6].

In this work we perform the THG spectroscopy measurements of plasmonic meta-atoms in the vicinity of their magnetic dipolar resonance. The sample represents an array of three-layer $Au/Al_2O_3/Au$ nanodisks with different diameters and layer thicknesses. We demonstrated the two orders resonant THG enhancement in the vicinity of the magnetic resonance as well as THG resonance blue-shift from the linear absorption resonance, which can be connected with the far-field interference of the local nonlinear sources inside nanodisks. These results give us new understanding of the optical magnetism contribution to the nonlinear optical response of plasmonic metamaterials.

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Circuit model of plasmon-enhanced fluorescence

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Plasmon-enhanced fluorescence has a rather long history and the amount of papers dedicated to this phenomenon is huge. Hybridized decaying oscillations in a nanosystem of two coupled elements — a quantum emitter and a plasmonic nanoantenna are considered as a classical effect. The circuit model of the nanosystem extends beyond the assumption of inductive or elastic coupling implying the near-field dipole-dipole interaction. In the simplest and most practical case this fluorescence is obtained in a structure comprising two optically small elements coupled by the dipole-dipole interaction. Element 1 is the fluorescent quantum emitter (QE) and element 2 is the optically small plasmonic nanoantenna (NA) also called nanocavity or nanoresonator. Often, the radiated power of plasmon-enhanced fluorescence can be calculated via the so-called Purcell factor F_P. The Purcell factor describes the decay rate of the spontaneous emission in presence of the NA. Its increase is equivalent to the gain in the radiated power at the emission frequency. The cases when the spectral line of the fluorescence nearly keeps its shape and only the level is modified is usually called weak coupling regime. This regime has been recently analyzed in our paper [1] where a simple classical model of the Purcell effect for the nanosystem realizing the plasmon-enhanced fluorescence has been built. This model was successfully validated and allows a good accuracy in the case when the emission shape is not distorted and the presence of element 2 only modifies the emission level for object 1.

In the present work we expand the same model beyond the regime of the weak coupling. We analyze how the regimes of weak and strong coupling of QE with the field described by the ratio of the coupling constant to decay rates of elements 1 and 2 correspond to the regimes of weak and strong *mutual* coupling of these elements. The mutual coupling is described by the so-called Rabi constant, and we show that our model being purely classical surprisingly allows its correct calculation. In the regime of strong mutual coupling the emission is modulated by beating frequency (doubled Rabi constant). In fact, it was noticed long ago that this effect in spite of the quantum nature of the emitter can be interpreted as a purely classical effect. The classical model is deterministic and rather simple, whereas the semiclassical model results in the system of Langevin–Schrödinger equations of motion and requires difficult numerical simulations. Therefore, it is not surprising that there were several attempts in the literature to describe the strong mutual coupling and Rabi oscillations using the model of two coupled classical oscillators. These attempts were partially successful as they properly delivered the bounds of strong mutual coupling and allowed proper predictions of the absorption spectra for some special cases (when the NA is replaced by a small optical cavity). However, these models [2, 3, 4, 5] being too simplistic, do not allow one to obtain correct formulas for the plasmon-enhanced fluorescence spectra.

Our model is more elaborated — instead of the elastic coupling we introduce the dipole-dipole interaction. Both elements 1 and 2 are described by equivalent circuits whose elements are found through Lorentzian parameters of their individual polarizabilities. Their dipole interaction results in the hybridization of the eigenmode and this hybridization results in the modification of the frequency dependence of the total dipole moment of the nanosystem, i.e. in the distortion of the fluorescence spectrum. We obtained the closed-form solution for this spectrum. Analyzing this solution we found the quenching which obviously occurs in the regime of strong mutual coupling. So, strong Rabi splitting is not compatible with Purcell enhancement. It is the non-radiative regime and corresponds to the fluorescence quenching.

Our model allowed us to reveal in between the weak and strong coupling regimes several intermediate regimes when the emission rate and spectrum are strongly distorted but the quenching does not occur. In these regimes the Rabi splitting occurs without quenching and in one of them it is possible even together with the enhancement. For another intermediate regime we found the Fano resonance. This is the case when the Lorentzian line is strongly distorted without birefringence of the spectrum. In this case, as one can see in Fig. 1 the emission is strongly enhanced.

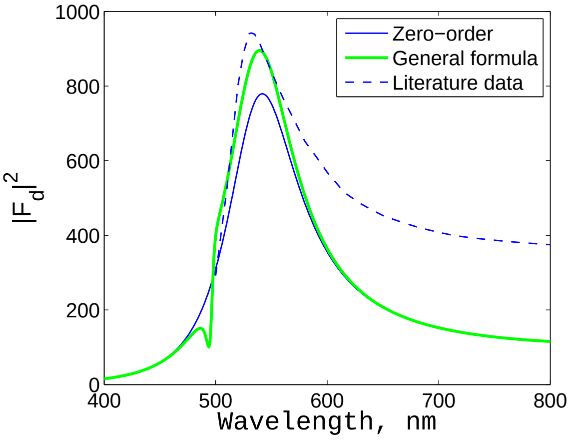

Fig. 1: Fluorescence spectrum of a quantum dot coupled to a golden nanosphere normalized to that of the single quantum dot. Thin line — zero-order model [1]. Dashed line — exact simulations of the semiclassical model (literature data). Thick line — our general formula.

Our results fit those of the semiclassical modeling and known experimental observations. Our original and simple classical model can be used for the express-analysis of the plasmon-enhanced fluorescence through the design parameters of the nanosystem.

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Usage of meta-resonators for improvement of magnetic resonance imaging

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Despite the fact, that metamaterials, artificial electromagnetic media, are vivid and rapidly developed area, the number of realistic metamaterial devices implementations is still scarce. In this talk, we will review our recent activities with metamaterials application for Magnetic Resonance Imaging. The general recepty how to efficiently use metamaterial based resonators in order to improve the magnetic resonance imaging will be explained. In particular, we will use metamaterials to manipulate, enhance and redistribute electromagnetic fields inside MRI tube. Theoretical and experimental results for several different prototypes will be presented (Fig. 1a). Finally, we will mention several ways how our approach can be used for real applications in hospitals (Fig. 1b).

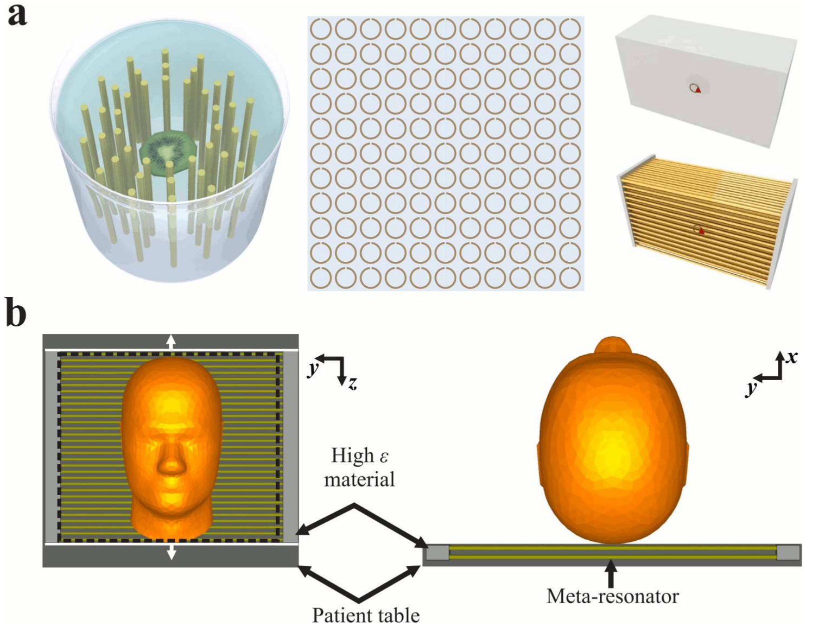

Fig. 1: (a) Metamaterial based resonators. (b) Possible implementations of the meta-resonator.

Nanophotonics in material-systems of large sizes

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Nanophotonics provides superb opportunities for tailoring the flow of light. This way, many novel physical phenomena can be enabled, as well as many important functionalities for novel energy applications (e.g. energy conversion, displays, lighting). In order to make these phenomena useful for large systems, large-area nano-fabrication techniques have to be successfully implemented. In this talk, I will present some of our recent theoretical and experimental progress in exploring these opportunities.

High-Q dielectric resonators for wireless power transfer system

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Nowadays, wireless power transfer (WPT) is an interesting research topic. WPT is not a new concept since Nikola Tesla invented the Tesla coils in the early 20th century which could transport energy wirelessly [1]. WPT can be achieved by radiative or nonradiative ways depending on different working mechanisms. Radiative power can be emitted and propagate over a long distance in the form of electromagnetic wave. Nonradiative WPT depends on the near-field magnetic coupling between the transmitter and the receiver [2]. By working distance, WPT can be classified into short-range induction power transfer (IPT) and mid-range via strong magnetic coupled resonance power transfer (MRPT). In 2007, an MIT's group proposed the first scheme of MRPT, in which 60 W of power was transferred over a distance of 2 m with 45% efficiency [3]. The proposed WPT system utilized traditional wire coils resonators with Q-factor of 950. Nevertheless, the resonators play an important role in the development of WPT systems. To achieve high power transfer efficiency, the Q-factor of the resonators should be as high as possible.

Dielectric resonators have shown many advantages over metallic resonators due to its ultra low loss. Under proper excitation, dielectric sphere shows a significant magnetic resonance, which makes it possible to improve WPT system by replacing traditional copper coil resonators by all-dielectric resonators.

In this talk, we propose a way to increase the WPT efficiency by using high-Q dielectric resonators instead of traditional copper coil resonators. We consider WPT systems based on dielectric resonators of different shapes (sphere and cube) and numerically analyse power transfer efficiency in CST Microwave Studio. In our system the resonators operate on the magnetic dipole (MD) mode as well as on magnetic quadrupole (MQ) mode. It turns out that the WPT system operating at MQ mode has higher efficiency than the traditional copper coil system in the sense of normalized distance. The obtained WPT efficiency can exceed 90% in the mid-range (4 times of radii). Meanwhile we analyse the angular stability of the proposed WPT system, i.e. the influence of the rotation angle of receiver on the WPT efficiency. Operating in MQ mode, the WPT efficiency does not suffer a great drop caused by the rotation of the resonator.

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Low-threshold nonlinear optical metamaterials. Transmission, reflection, absorption

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In the last decade has been observed experimentally that some dielectric nanoparticles embedded in a dielectric matrix make the resulting nanostructure optical nonlinearity.

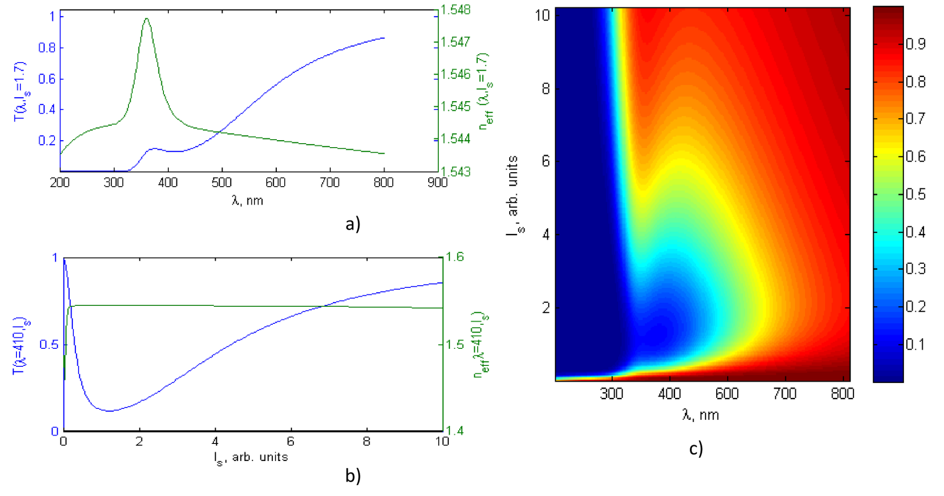

Fig. 1: Optical characteristics of the metamaterial: T and n_{eff} a) the wavelength dependence, b) dependence of external radiation, c) transmission spectrum $T(\lambda, I_s)$.

Low-threshold optical nonlinearity and its atypical dependence on the intensity of energy is due to the structure of the charge carriers of nanoparticles and giant electric dipole moment induced in the nanoparticles by external radiation [1, 2]. The report presents the results of mathematical modeling of the optical characteristics of metamaterials, such as an effective complex refractive index $n_{\rm eff}$, the transmission spectrum T, reflection and absorption.

It was found that under the influence of external radiation is possible to control the refractive index and transmittance metamaterial at a wavelength that is outside the absorption band of the substrate material, wherein the control effect is possible at a relatively low intensity of radiation (less than 1000 W/cm^2).

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Optical forces induced at the metal surface

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The optical manipulation with evanescent fields and surface waves is compatible with co-planar integrations needed for miniaturization of lab-on-a-chip platforms and micro-fluidic devices [1]. The optical forces acting in a system can be significantly amplified if Surface Plasmon Polaritons (SPP) on metal-dielectric interfaces are used [2]. Here we discuss the unusual situation when nonconservative force from SPP wave creates attractive force acting on a small particles towards the source of light (plasmonic tractor beam) (see Fig. 1(a)). The nanoparticle excites directional [3] SPP that results in appearance of the attractive reaction force which spectral dependence is shown in Fig. 1(b). The role of the SPP is crucial in the observed effect. The attractive force originates from the mechanical momentum exchange between the particle and SPP.

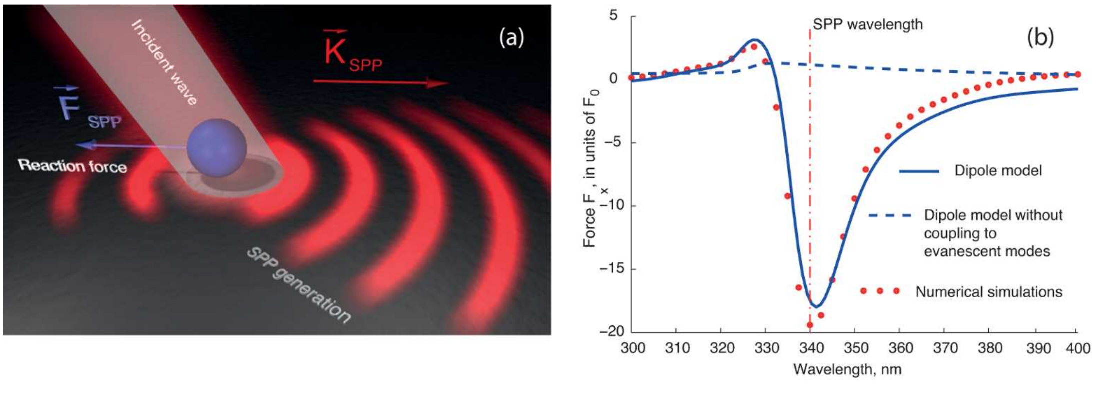

Fig. 1: (a) The geometry of nanoparticle attraction with SPP induced pulling force. (b) Spectral dependence of pulling force acting on a dielectric nanoparticle of radius $R = 15$ nm placed above the silver substrate at the distance $z = 27$ nm. F_0 is the radiation pressure force in vacuum. Angle of incidence is 35°. The wavelength of SPP excitation is shown with dash-dot line.

We also discuss the optical binding effects related to SPP generation at air-metal interface. We show that one can achieve optical binding of two nanoparticles through their coupling via SPP interference. Contrary to conventional optical binding, SPP induced binding distances is well below the wavelength because of large in-plane wave vector of SPP.

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Nano-absorbers

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In this review talk we will present and discuss some new results on understanding, design, and realizations of thin (nano-scale) layers for full absorption of light. We will first briefly introduce the fundamental principles of operation of subwavelength-thickness absorbers and review the classification of these devices, based on the physical mechanisms of absorption (a detailed review is available in [1], and the problem of maximization of absorption in dipole particles and in periodical arrays of dipoles is discussed in tutorial review [2]).

Next we will discuss an interesting possibility to fully absorb incident light just in a single-layer array of coated metal (silver or gold) nanoparticles [3]. This possibility follows from an analogy with the recently conceptualized and experimentally tested microwave single-array absorbers [4], where the necessary electric and magnetic polarizations were created in so called balanced spirals. These new absorbers do not contain any back mirror reflectors and are transparent outside of the absorption band. Another interesting feature of absorbers [3] is that most of the incident power is absorbed not in metal particles but in the dielectric coating or in the dielectric or semiconductor substrate, which is very desirable in energy harvesting applications.

Finally, we will present recent results on full absorption of light in a semiconductor nanocoating of metal substrates. This is probably the simplest possible perfect absorber topology, since no nanostructuring of the absorbing layer is required: it is simply a thin homogeneous layer. In the microwave frequency range, full absorption in a dielectric layer on a metal ground plane requires quarter-wavelength thickness of the layer, but in the optical range the thickness can be dramatically reduces if the absorption resonance is organized as a quasi-static resonance of the inductive surface impedance of the metal substrate and the capacitive reactance of the semiconductor coating.

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2-dimensional hyperbolic medium for electrons and photons based on the array of tunnel-coupled graphene nanoribbons

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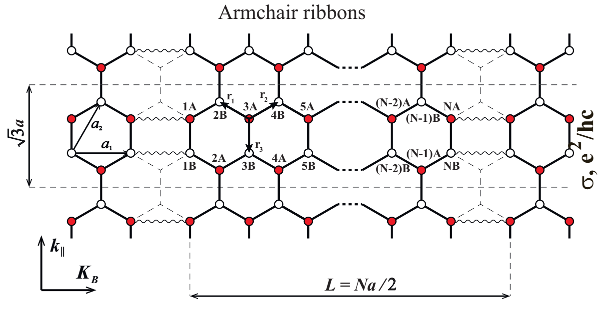
Graphene, two-dimensional lattice of carbon atoms, exhibits a wide range of unique electronic and optical properties. Graphene plasmonics [1] became a rapidly growing research field, both because plasmons in graphene exist in widely demanded THz field and because they can be effectively tuned with an external gate voltage. In [2] it has been shown that a wide range of metasurfaces can be construced based on graphene sheets. In our work we show that nanostructuring the graphene sheet we can tailor the electronic properties directly and as consequence control the conductivity of these metasurfaces.

Conductivity and localized surface waves of the metasurface. We study the system shown in Fig. 1. We calculate the electronic band structure and eigenfunctions for the array of tunnel coupled armchair and zigzag nanoribbons. We show that the coupled edge states of zigzag nanoribbons form the additional electronic band characterized by the hyperbolic Fermi surface and the band structure of coupled armchair ribbons is characterized by the overlap of electron and hole bands which results in the Fermi surfaces which simultaneously possess electron and hole pockets. Then, using the Kubo formula we calculate the AC conductivity tensors of these structures. We show that for the case of armchair nanoribbons (see Fig. 1) the system can be described by an uniaxial conductivity tensor with principal components of different sign and low loss. We obtain the dispersion equation for the Dyakonov-like surface modes at the anisotropic metasurface. We also study the polarization properties of the localized waves: these modes are linear combination of TM and TE surface plasmon modes appearing in 2 dimensional graphene. Directivity of these plane waves can be controlled with the external voltage which can be used for the creation of graphene-based photonic integrated circuits.



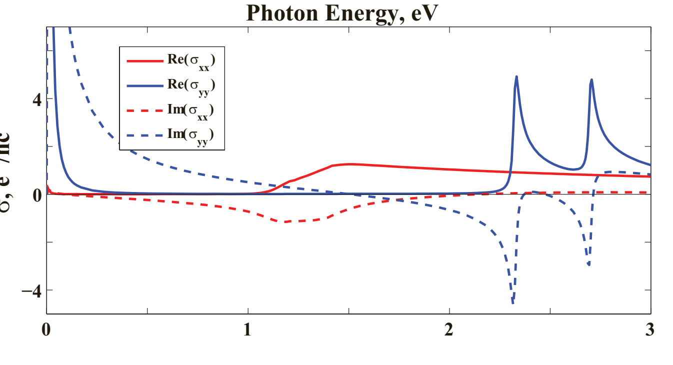


Fig. 1: Atomic structure (left column) and conductivity (right panel) of the array of armchair nanoribbons.

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Concentrated radiation of particle bunches in presence of wire structures and dielectric objects

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Concentration of charged particle radiation is an important problem for bunch diagnostics, detection of particles, and new methods for generation of powerful electromagnetic radiation. As a rule, this problem is solved with help of lenses and mirrors. These devices do not participate in generation of radiation, they only focalize the radiation into some small volume. However, the effect of radiation concentration can be reached without such additional devices.

We consider two methods for obtaining of concentrated radiation of charged particle bunches here. The first method consists in using of periodic wire structures with the spacing which is much less than the wavelengths in question. We study both 3-dimentional (wire metamaterials) [1, 2] and planar [3] structures, which can be as infinite so as semi-infinite. In all considered cases we assume that the bunch moves perpendicularly to the conductors. The 3-dimentional wire structure is described by an effective permittivity tensor and exhibits both spatial and frequency dispersion. It is shown that the moving charge generates non-divergent Cherenkov radiation in this structure (the radiation is concentrated near a certain lines behind the charge). It is interesting as well that the radiation is generated at any velocity of the charge. We demonstrate images with typical spatial distributions of the wave field (field "snapshots"). They show that the radiation can be used to determine the bunch size and shape.

Further, we consider the 2-dimensional wire structure (planar grid) as a target for generation of radiation for two variants: infinite grid [3] and semi-infinite grid. This structure is simpler for practical implementation in comparison with the 3-dimensional metamaterial. We describe the grid properties using the averaged boundary conditions method that allows taking into account both thickness of the wires and their finite conductivity. The most interesting part of the total radiation is the surface wave. The properties of this wave are similar to the properties of Cherenkov radiation in the 3-dimentional wire structure.

We also consider a different method to obtain concentrated radiation. This technique consists in using of a "dielectric concentrator of radiation" [4]. In contrast to traditional devices, this target can focalize almost the whole radiation without using additional lenses or mirrors. We analyze the case where the radiation is produced by a point charge moving along the axis of a cylindrical channel inside an axially symmetrical target. The specific form of the target is determined using the laws of ray optics. The field is calculated using the aperture integration method that can determine the field near the focus. Typical spatial distributions of the field outside the target are presented. They demonstrate large amplification of the field in the focus neighborhood in comparison with the field on the surface of the target.

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Flexible high-impedance surfaces and miniature antennas for on-body system applications

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Development of flexible and wearable wireless systems is highly challenged due to their applicability in a wide spectrum of applications such as personal communication, medicine, firefighting, military, and radio frequency identification (RFID) [1]. Their light weight, low fabrication cost, easy manufacturing, and the availability of inexpensive flexible substrates (plastics, papers, and textiles) make flexible electronics very attractive. A new branch of wireless system technology is connected with the Wireless Body Area Network (WBAN). Such systems in combination with personal area networks provide monitoring of the state of biological systems (human body) in real time. The efficiency of these systems depends on the characteristics of the integrated antennas. The nature of flexible wireless technologies requires the integration of flexible, low profile, light weight, and compact antennas. These antennas should be mechanically robust, stable to bending, and at the same time provide desirable radiation and matching characteristics.

The flexible or conformal antennas are designed in the same way as planar antennas using flexible substrates. The most of investigations contains results of simulation and experimental investigations of radiation patterns and input impedance obtained for different types of bended and crumpled flexible antennas. Very popular are textile antennas for body-worn applications. Interesting results have been obtained for stretchable and reversibly deformable antennas based on silver nanowires [2]. Embroidered antennas on paper or fabric substrate are used on clothes of people involved in monitoring problems. Fractal antennas are also of interest. For example, the antenna consisting of a Sierpinski carpet radiator realized on the top side of the flexible substrate with a Hilbert slot on the bottom side was analyzed [3]. Various types of bending the antenna were simulated: the antenna was folded along one-third and two-thirds of its length in one direction and was folded twice along the one and two thirds of its length. The simulation results were confirmed by measurements.

Wearable antennas are in close proximity of a human body, which influences the antenna performance. The effect of human body on wearable antennas is studied by many authors. In general, the body influence can be smoothed by using screening structures. For this purpose, artificial structures known as perfect magnetic conductors may be recommended. The structures can be engineered to have desired features within a necessary frequency range. The integration of these artificial structures with antennas and microwave circuits are extremely beneficial in on-body antenna applications [4]. The artificial magnetic conductor can be designed as a high-impedance surface (HIS). HISs are successfully applied in many antenna systems for performance enhancement utilizing their unique features. Their advantages include surface wave suppression and reduction of mutual coupling among radiating elements in antenna arrays. The application of such structures in flexible and wearable technologies is limited by their relatively high profile and their size and bandwidth dependence on the substrate permittivity and thickness. There is a need for flexible, conformal, and compact HIS to be integrated within the targeted technology.

The HIS behaves as a perfect magnetic conductor in a specific frequency range and provides a reflection phase of 0° for the electric field of a normally incident plane wave. HIS structures can be artificially engineered and are typically realized as periodic metalization patterns printed on a grounded dielectric material.

A HIS is used to improve the wearable antenna performance. Both input impedance and radiation pattern were investigated using numerical and experimental methods. The textile antennas under bending and crumpling conditions have been modeled and measured. Textile antennas based on the conventional square-patch based HIS in terms of Specific Absorption Rate (SAR) were analyzed. SAR was assessed using a human body model (for example, HUGO) to verify the feasibility of the proposed designs. The HIS ground plane is utilized to isolate the human body from undesired electromagnetic radiation in addition to minimizing the antenna impedance mismatch caused by the proximity to human tissues. A significant reduction in SAR achieved when the HIS structure is included. Flexible, compact antenna system integrated with a HIS have been intended for telemedicine applications. Antennas operating in the industrial, scientific, and medical frequency band (2.45 GHz) integrated with a HIS ground plane have been designed with slotted Jerusalem Cross, dog-bon, 2 LC structure etc. The antennas express 7–18% impedance bandwidth. The integration of the HIS ground plane increases the front to back ratio by 6–8 dB, provides 3–5 dB increase in gain, and 50–70% reduction in SAR. The low profile antennas integrated with HIS are of high interest. Different types of flexible wearable antennas based on flexible HISs have been simulated and verified experimentally under flat and bent conditions.

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Is frustrated total reflection really caused by the surface plasmon excitation?

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We show that the generally accepted explanation of the phenomenon of frustrated total reflection as a result of the resonant excitation of the surface plasmons is not quite correct. Even though the wave numbers involved in both phenomena are close, surface plasmons can only be excited for a finite beam aperture only. Moreover, the incident beam should include evanescent components. For a plane wave, frustrated total reflection is caused not by surface plasmons excitation but by a zero of the reflection coefficient that arises due to anomalously large Goos–Hänchen effect.

Formally the Kretschmann effect may be described as excitation of virtual plasmon. The latter is a leaky wave, whose position lies on the nonphysical Riemann-surface sheet.

A simple analytical model describing a substrate influence on the dipole resonance wavelength of spherical particle

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Simple analytical quasi-static model was proposed to estimate the dependence of a spectral shift of the dipole resonance (DR) of a spherical nanoparticle placed near the transparent dielectric substrate on the refractive index (RI) and inter-distance of the substrate. Good agreement between the analytical quasi-static estimations and supporting finite-difference time-domain simulations was found for sufficiently small nanoparticle radii. In addition, we demonstrated analytically and numerically that the detection of the DR spectral shift allows one to map very small (down to 0.0005 refractive index units) RI variations of the transparent substrate. Pointed scanning probe based on a fiber microaxicon with the spherical nanoparticle atop realizing the RI mapping method was proposed and numerically investigated.

All-dielectric nanostructures for enhanced antireflection and light trapping in thin-film silicon solar cells

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Many suggestions for increasing absorption in thin-film solar cells are based on the use of light-trapping structures (LTS). In recent papers one can find LTS performed as arrays of densely packed dielectric nanospheres [1, 2]. The effect of LTS appears in structures due to the resonant photonic nanojet regime, due to whispering gallery modes excited at several frequencies and due to photonic crystal trapping that arises for the oblique incidence. In our other works we suggested a broadband antireflection coating — an array of nano-voids in the dielectric covering [3, 4, 5]. The spectral range of the reflection damping is broader than that achievable for a standard homogeneous ARC.

We compare the enhancement of the photovoltaic absorption offered by a square array of nano-pillar shaped voids in the dielectric covering of the cell with that granted by a flat blooming layer, and a densely packed array of dielectric nanospheres. We optimize these coatings and show that the newly proposed nanostructure allows a significant increase of the photovoltaic absorption. The dependence of anti-reflection and light-trapping properties on the angle of incidence is numerically investigated, and it is shown that the array of voids keeps optimal also after averaging over the incidence angles. Our slightly tapered voids can be fabricated using the nanoimprint lithography. The application of our LTS seems to be promising for TFSC of different configurations.

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The influence of field enhancement and Purcell effect on third harmonic generation

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The main goal of the paper is to consider how the interplay of two effects, the enhancement of the pump field and Purcell effect, affect the process of third harmonic generation. To consider this problem we use a very simple physical system consisting of a dielectric Fabry–Perrot resonator with a nonlinear delta-layer placed at the first interface of the resonator. The schematic view of the resonator is shown in panel (a) of Fig. 1. The pump beam excites the resonator at normal incidence. In this work we consider the case of cubic (Kerr) nonlinearity. Because of the nonlinearity the incident radiation excites the current at the frequency triple of that of the pumping wave and this current causes the emission of the third harmonic.

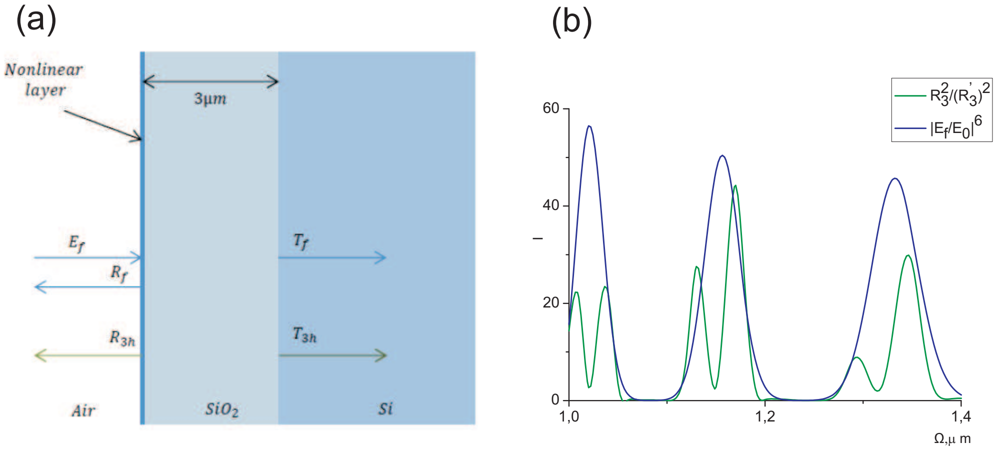

Fig. 1: Panel (a) shows the geometry of the our problem. The dependency of the third harmonic radiation intensity vs the wavelength of the fundamental frequency is shown in panel (b) for the case when the nonlinear layer is situated at the air-glass interface. The curve is normalised on the third harmonic intensity from the nonlinear layer placed on the silica substrate in the absence ot glass resonator. The blue line shows the intensity of the oscillations of the nonlinear polarization. $d = 3$ μm, $h = 0.01$ μm.

It was found that the enhancement of the field due to Fabry–Perrot resonance increases the third harmonic current which in its turn leads to the increase of the third harmonic radiation. However it happens that the resonator also affects the imaginary part of the Green function governing the efficiency of the radiation. This phenomenon known as Purcell effect makes the radiation intensity to be frequency dependent with pronounced maxima at the certain frequencies. The dependencies of the field intensity on the fundamental harmonic and the normalized third harmonic radiation intensity are shown in panel (b) of Fig. 1. One can see that within each maximum of the pump field intensity there are two well-resolved maxima of the third harmonic intensity. This can be explained by the fact that the maximum of the third harmonic current is not a sufficient condition of the maximum of the radiation. The Purcell effect must also be taken into account to explain the observed dependencies.

More details on the processes involved in the generation of the third harmonic in the resonators will be shown in the presentation.

New types of surface waves on hyperbolic metasurface

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Development and study of metamaterials bring a lot of features in fundamental physics and applied problems due to their extraordinary properties [1, 2]. Last years the splash of interest to the 2D-metamaterials (which are called metasurfaces) is observed. Metasurfaces can be very useful in the modern planar fabrication technology and integration into the on-chip optical devices.

In this work, we study a special class of metasurfaces characterized by a local diagonal anisotropic conductivity tensor. We have presented a comprehensive analysis of surface waves propagating along hyperbolic metasurface. The dispersion equation is solved analytically and it is similar to the dispersion of magnetoplasmons [3]. It was shown that the spectrum of the hyperbolic metasurface waves consists of two branches corresponding to hybrid TE-TM waves (Fig. 1). Simultaneous propagation of both surface waves at the same frequency is possible in the certain range of angles like Dyakonov surface waves [4].

The contours of equal frequency for surface plasmons were investigated. Also we analyze the losses and derive the corresponding analytical asymptotic expressions. The polarization properties of these modes depend on the wave frequency and propagation angle and can be linear, elliptic and circular. Listed properties allow to apply discovered waves in multiple disciplines.

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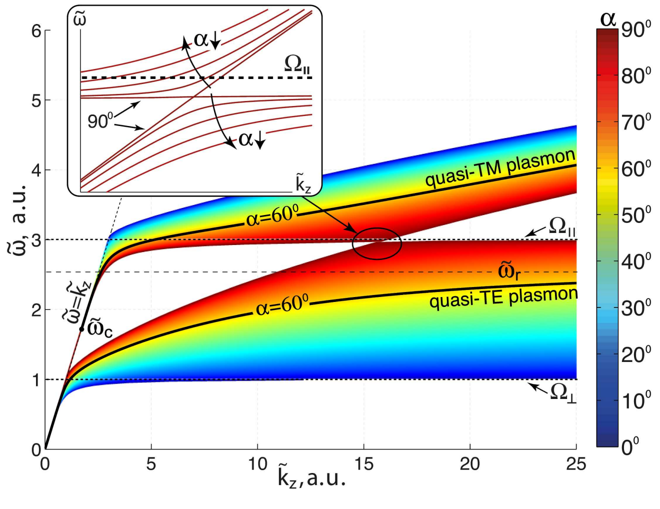

Fig. 1: Dispersion of wave vector $\widetilde{k}_z$ for the surface waves on hyperbolic metasurface for different propagation directions α.

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Modal analysis, fabrication and characterization of silicon Mie particles

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Dielectric and metallic nanoparticles feature the ability to resonantly interact with light thanks to the excitation of electric and magnetic like resonances and modal or singular analysis of photonic resonators is a very efficient tool to optimize their optical properties. Lastly, we introduced in photonics the so-called Weiestrass factorization to analyze the scattering properties of nanoparticles. This factorization permits to express the scattering matrix in terms of spectral singularities. We show that the location of poles and zeros in the frequency complex plane of the scattering fully determines all scattering properties and is able to accurately reproduce the Fano anomalies of core-shell nanoparticles [1]. This technique is also very interesting in the framework of the multipolar theory since it allows to express the scattering matrix with respect to the zeros of the Bessel function. This formulation was successfully applied to provide the analytical expression of the particle diameter that satisifies the ideal absorption condition of metallic particles [2].

Modal analysis was also applied in the context of silicon Mie resonators that are expected to play an important role in the design of novel light nanocavities [3]. By taking advantage of the latest progress in modal analysis of optical nano-cavities, we derived the analytical expression of the effective volume of silicon Mie particles featuring electric and magnetic resonances [4]. These effective volumes permit to calculate the decay rates of quantum emitters coupled to the cavity through the Purcell factor proportionnal to the ratio between the quality factor and the effective volume. This study shows that the effective volume is a well-defined physical quantity, even in nano-optics, and that it is related to the translation-addition formulas, or multipolar coupling [4].

Besides theoretical investigations, it is important to develop orginal strategies to fabricate high quality silicon particles. We will present a fabrication technique that combine electron beam lithography to define with a high versatility well defined maks with an alcaline etching. Wet alcaline etching is very interesting to fabricate silicon particles since it avoids the use of reactive ion etching that can lead to high local temperatures and that is time consuming. We will study the influence of the alcaline solution on the shape of the particles, and will see that this techniqe can be applied either to monocrystalline or amorphous silicon [5].

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Opto-mechanical Hall effect for nanoparticles

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In this work we present the study of the behavior of subwavelength neutral nanoparticles in the field of TM-polarized inhomogeneous plane electromagnetic wave and find conditions for existence of the average non-zero scattering force in the direction perpendicular to the wave propagation. This is an optical analogue of the classical Hall effect for electrons, and we call it *an opto-mechanical Hall effect*. We assume that the wave incident on nanoparticles is described by magnetic field in the form $H_y = A(z)\exp(-ikx)$. We consider nanoparticles with the shape of the ellipsoid of revolution and their response to electromagnetic waves is described within the dipole approximation. We assume that the orientation of each particle is controlled by electrophoretic force of additional constant electric field. Such particles experience the action of averaged ponderomotive optical force which can be written as

$$\begin{aligned} F_x &= -\frac{k}{(k_0\varepsilon)^2}\left\{\mathrm{Im}(\alpha_{xx})\left(\frac{dA}{dz}\right)^2 + k^2\,\mathrm{Im}(\alpha_{zz})A^2\right\}, \\ F_z &= \frac{1}{(k_0\varepsilon)^2}\left\{\frac{1}{2}\left(k^2(\mathrm{Re}\,\alpha_{xx}+\mathrm{Re}\,\alpha_{zz}) - k_0^2\varepsilon\,\mathrm{Re}\,\alpha_{xx}\right)\frac{dA^2}{dz}\right\} \\ &\quad + \frac{k}{(k_0\varepsilon)^2}\,\mathrm{Im}(\alpha_{xz})\left\{(k_0^2\varepsilon - k^2)A^2 + \left(\frac{dA}{dz}\right)^2\right\}, \end{aligned}$$

where α_{ik} are the particle polarizability tensor components and ε is the dielectric permittivity of the surrounding medium. One might expect that scattering part of the force which is proportional to imaginary part of polarizability has to be directed purely along the wavevector of light (x-direction in our case). However, there is a non-vanishing component of this force which is perpendicular to the direction of electromagnetic wave propagation. In the field composed of two identical plane waves converging at some angle such seeming symmetry breaking has simple explanation. Indeed, in the case when principle axes of the ellipsoid do not coincide with the wavevector direction two plane waves act differently on the particle causing constant drift normal to the field momentum. The magnitude of this force depends on the angle of convergence of the plane waves as well as on the orientation of particles. Such a behavior resembles classical Hall effect for electrons. The drift of nanoparticles, however, takes place not because of the magnetic field that leads to the emergence of non-diagonal component of conductivity tensor but because of the asymmetry of the nanoparticle orientation producing non-diagonal part of polarizability tensor.

Sorting of plasmonic nanoparticles with light

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We introduce a new approach for fine spatial sorting of shaped plasmonic nanoparticles by two crossing TE-polarized light beams. We consider electrically neutral subwavelength metal nanoparticles whose response is described within the dipole approximation. To be more specific, we assume that the shape of the nanoparticles is an ellipsoid of revolution. Such nanoparticle is characterized by the depolarization factor n which depends on the ratio of ellipsoid semi-axes. The tensor of the particle polarizability is strongly dispersive due to the existence of two orthogonal dipole surface plasmon modes supported by the nanoparticle. These modes can be resonantly excited by light, and the optical force experienced by particles strongly depends on n. This strong dependence allows to spatially separate particles with nearly identical shapes, i.e. with very small difference in n. The eigen frequencies of plasmons strongly depend on the depolarization factor as well as on the permittivity of the environment and may fall into wide frequency range covering near IR, visible and UV.

We study frequency dependence of time-averaged ponderomotive force acting on particles from electromagnetic field. Such force consists of two parts — electric and magnetic. Electric part appears due to the polarization of the particle placed in an inhomogeneous electric field, while magnetic part is the Lorentz force acting on the polarization current. These forces can be represented as a superposition of the gradient force which is proportional to the gradient of light intensity, and a scattering force induced by the momentum transfer from photons to particles. If we fix the orientation of the elliptic particles by applying constant electric field, and then illuminate these particles by two interfering light beams, we can create frequency domains where gradient ponderomotive force changes sign for certain range of n. Thus, the particles with n in this range move in one direction, while all other particles move in the opposite direction. This effect allows us to select metal nanoparticles of a particular shape from the mix, and separate them. As an example we present the conditions for the selection of almost spherical silver nanoparticles with $n \sim 1/3$.

Controllable coupling of the surface and volume electromagnetic waves in a liquid metacrystal

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Liquid metacrystal (LMC), that is resonant elongated particles (meta-atoms) dispersed in a viscous liquid has recently been suggested as a new kind of resonant metamaterial possessing a number of unique properties, such as, for example, a high tunability and a strong nonlinearity [1, 2]. These properties arise from meta-atoms reorientations in both constant and high-frequency electric fields resulting in the change of direction of anisotropy axis of LMC. In this report we study linear electromagnetic surface waves supported by the dielectric/LMC interface as well as by the slab of LMC within a dielectric medium. We show that in the case of single interface the surface waves are always leaky, radiating from an interface inward LMC. This is caused by the presence of two different types of normal polarization-orthogonal (ordinary and extraordinary) eigenmodes when LMC is transparent for the ordinary wave while for the extraordinary wave LMC is opaque. These eigenmodes are coupled at the interface that determines the energy transfer between localized and volume field components which depends on meta-atoms orientation. Therefore, intensity of surface wave radiation can be managed by a choice of the anisotropy axis direction. Such a coupling of surface and volume waves leads to the novel type of nonradiating modes guided by the slab of LMC which contain simultaneously both surface and volume components of different polarizations.

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Dark-field hyperlens: superresolution microscopy for weak scatterers

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Metamaterials representing materials absent in nature have been proposed as a means to beat the Abbe resolution limit in optical microscopy (Fig. 1a). Particularly promising is the idea of the hyperlens (HL) made of extremely anisotropic media with indefinite permittivity tensor. In a HL, information about subwavelength features of imaged objects is transferred from the near field to the far field [1]. However, propagating waves in the object area also get transmitted, creating strong background and limiting the use of the HL to cases when background can be eliminated [2].

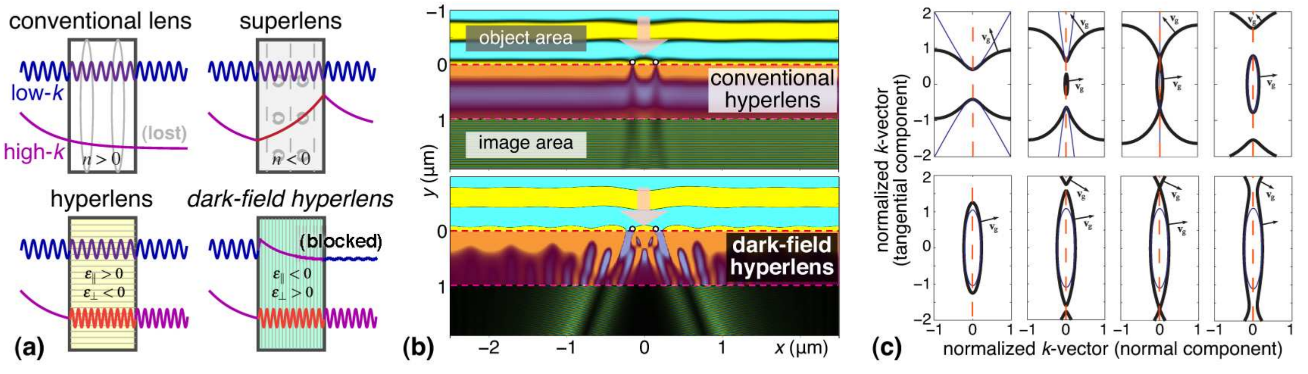

Fig. 1: (a) Illustration of metamaterial-based superresolution imaging approaches. (b) Comparison between the conventional [2] and dark-field [3] HL for externally illuminated dielectric nanospheres. (c) A variety of dispersion relations for periodic plasmonic multilayers [4].

In this paper, we put forth a modified concept of the HL (the *dark-field* HL), where the dispersion relation of the underlying metamaterial is modified so as to filter out low-k, propagating waves and only transmit high-k waves scattered from the objects (Fig. 1a), similar to the dark-field microscope [3]. As a result, the subwavelength resolution of the image is retained, but its contrast is greatly enhanced. Thus the dark-field HL becomes suitable for the imaging of externally illuminated weakly scattering objects (Fig. 1b), which is relevant to nearly all label-free biological imaging scenarios.

We further explore the use of multilayer plasmonic metamaterials [4] to bring the image processing or metalens [1] functionality of the dark-field HL. Examples include selectivity with respect to the k-vector of image-forming waves by using plasmonic multilayers with engineered dispersion relation in the high-k domain (Fig. 1c). The resulting HL can be used to selectively highlight certain subwavelength image features based on the size and/or shape of the objects being imaged.

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Laser with *PT*-symmetry breaking in a polarization space

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In the last decade, there has been rising interest in optics of artificial materials. Among such materials, the systems with balanced loss and gain regions have attracted particular attention [1–3]. The concept of these systems stems from the idea of the extension of quantum mechanics to non-Hermitian Hamiltonians possessing PT-symmetry [4]. PT-symmetric systems are invariant with respect to the simultaneous spatial inversion and time inversion. The former performed by the linear operator which inverts coordinates and momenta, while the time inversion is performed by the antilinear operator, which inverts momenta [5]; the simultaneous application of these operators inverts coordinate, but does not change momenta. In optics condition of PT-symmetric system reduces to conditions on dielectric permittivity $\varepsilon(x) = \varepsilon^*(-x)$. Due to anti-linearity of the PT-operator, the eigensolutions of PT-symmetrical system may or may not be PT-symmetric depending on the values of permittivity [4]. When parameters of the systems are varied, one or multiple transitions between phases with PT-symmetric and non-symmetric eigensolutions may happen [1–4]. The symmetry breaking may leads to suppression of lasing in a laser with nonuniform pumped when pump is increased [6].

In the presence paper, we investigate the lasing of the laser that contains gyrotropic and anisotropic layers and the active medium with linear anisotropy (GALA laser). We demonstrate that equations for the modes of GALA laser with the linear polarization are identical to the equations for the optical modes in the PT-symmetrical systems. As a result, in the GALA laser may observed PT-symmetry breaking in a polarization space. PT-symmetry breaking in the GALA laser leads to suppression of lasing.

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