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ABSTRACTS

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FOREWORD

"Days on Diffraction" is an annual conference taking place in May–June in St. Petersburg since 1968. The present jubilee event is organized by St. Petersburg State University, St. Petersburg Department of the Steklov Mathematical Institute, the Euler International Mathematical Institute and the ITMO University.

The abstracts of 200 talks to be presented at oral and poster sessions during 5 days of the conference form the contents of this booklet. The author index is located on the last pages.

Full-length texts of selected talks will be published in the Conference Proceedings. They must be prepared in LaTeX format and sent not later than 6 July 2017 to `diffraction17@gmail.com`. Format file and instructions can be found at `http://www.pdmi.ras.ru/~dd/proceedings.php`. The final judgement on accepting the paper for the Proceedings will be made by the Editorial Board after peer reviewing.

As always, it is our pleasure to see in St. Petersburg active researchers in the field of Diffraction Theory from all over the world.

Organizing Committee

List of talks

Effect of variation of the ice density caused by chemical reaction on a nonlinear ice rod-structure vibrations

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In this paper, we introduce a new nonlinear model for the moving ice rod-structure interaction. Nonlinear vibrations of that system is a complicated process, which involves an ice failure, as a result of the interaction between a moving ice and a structure and also chemical reaction. Here we propose a model, which develops the known models in this field, in particular, the Matlock–Sodhi model. Similar to the Matlock model, a structure is considered as a single oscillator, but we took into account the chemical reaction which cause the variation of the ice density. The deformations of the ice rod are described taking into account a contact between the oscillator and the rod and change in the ice density . A possible water input into the pressure affecting the oscillator is considered. For calculation of the oscillator-ice rod interaction, an extrusion effect is taken into account. The aforementioned effects make the problem more complicated: partial differential equations (PDE's) for the ice rod and ordinary differential equations, (ODE's) for the structure are involved. The main difficulty of the problem is that these PDE and ODE's are coupled via boundary conditions for ice rod deformations. Nonetheless, we a capable to resolve this problem using a new asymptotic approach. This approach allows us to find the ODE for the oscillator, where ice deformations are excluded. This equation describes (for a single mode approximation) nonlinear oscillations of the oscillator. The terms in that equation admit transparent physical interpretations and relate to: 1) the effect of water extrusion under an ice rod pressure, which leads to a particular type of a friction force and nonlinear effects; 2) the contact interaction between ice rod and the oscillator, which leads to additional nonlinearities and to an oscillator frequency shift. The main result of the asymptotic investigation and numerical simulations is the origin of a resonance for some ice rod velocities is the result of an ice-structure contact and variation of the density caused by the chemical reaction in the ice. This effect is a negative friction effect. For multi-oscillator models the resonance zone increases. Moreover, the resonance peak shape may be complex due to many resonances and a nonlinear interaction between modes. The vibrations for small velocities of ice, "resonance" velocities, and large velocities have quite a different character. The Fourier analysis of the structure response shows that for large ice velocities the response is essentially more stochastic and noisier than it is for a case of small ice velocities.

The solutions of equations for nonlinear model of deformation of the crystal media allowing martensitic transformations

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Mathematical methods of the solution of the nonlinear equations of deformation of the crystal media with a complex lattice allowing martensitic transformations are developed. In nonlinear model [1, 2] deformation of the medium is described by a vector of acoustic mode $\mathbf{U}(t,x,y,z)$ and by a vector of optical mode $\mathbf{u}(t,x,y,z)$. They can be found from system of six connected nonlinear equations. Vector of the acoustic mode $\mathbf{U}(t,x,y,z)$ is sought in the Papkovish–Neuber form. The system of six connected nonlinear equations is transformated to a system of separate equations. Three equations for the optical mode were divided into one sine-Gordon (SG) equation with variable coefficient (amplitude) before the sine and two Poisson equations. The finding of the acoustic mode $\mathbf{U}(t, x, y, z)$ is reduced to the solution of scalar and vector Poisson equations. The nonlinear equation for the optical mode can be transformed in the classical SG equation and double SG equation if to impose some restrictions for the model or the field of microdeformations. The some simple solutions are constructed for these approximations [3]. The analysis of these solutions has shown that the nonlinear model describes such specific features of deformation as formation of a superlattice, phase transformations, appearance of defects, and other processes, which are implemented in the field of high external stresses and which can not be described by classical mechanics of the continuous media.

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Thermoeffect influence on band-gaps and pass-bands in layered phononic crystals

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Periodic composite materials get more and more widely used in various fields and applications, for example in engineering, aircraft manufacturing, shipbuilding, etc. Such periodic structures are called phononic crystals and acoustic/elastic metamaterials. Phononic crystals are the acoustic/elastic equivalent of photonic crystals, where a periodic organization of components in a homogeneous host material causes certain ranges of frequencies where incident waves are completely reflected by the structure. Successful application and commercial exploitation of new devices based on phononic crystals depends to a large degree in increasing the material's figure-of-merit. This in turn is closely dependent upon the formulation of an adequate theoretical model and its practical realization. Thermal and thermoelectric effects should be considered for phononic crystals in order to estimate their influence on the mechanical, electrical and thermal wave-fields.

Influence of temperature on wave propagation in layered phononic crystals is considered and numerical results showing thermal effects are discussed. The study presented is an extension of previous investigations [1, 2]. In order to describe plane wave propagation the transfer matrix method is used, while the analysis of eigenvectors and eigenvalues of the transfer matrix gives the localization factor for Bloch waves propagating through infinite phononic crystal. Detailed analysis of thermal effects influence on pass-bands, low transmission pass-bands and band-gaps as well as on wave-fields and transmission/reflection coefficients is provided.

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Generalized function method in boundary value problems of elastodynamics at supersonic transport loads

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Investigations of the rock massif dynamics at mobile transport loads action is among scientific and technical problems which relevance increases every year in connection with increased construction of thoroughfares, tunnels, subways and growth of vehicles speeds. Research of such processes on a basis of mathematical simulation leads to boundary value problems (BVPs) of mathematical physics by using various models of continuous media mechanics. Transport BVPs are most studied for isotropic elastic media, when the methods of complete and incomplete separation variable and integral Fourier's transformations can be successfully used.

The mathematical theory of construction of solution of such problems for a class of stationary loads, moving with constant speed (transport loads), depends on the relation of its speed to the speeds of bulk (dilatational) and shift waves (sound elastic waves). In works [1, 2] the author shown that the type of motion equations (in to mobile system of coordinates) changes from elliptic to hyperbolic with growth of a velocity of loads motion. In article [3] the transport BVP were solved for isotropic elastic medium, limited cylindrical surface, on which a transport load moves with a subsonic speed. This class of BVPs simulates dynamics of underground constructions, such as transport tunnels. It gives the elliptic BVP in the mobile coordinates system, connected with transport load.

Here we consider the similar task, but in supersonic case, when the speed of transport load is more than the speeds of sound elastic waves. In this case the shock elastic waves appear in the medium. The most convenient device for creation of solutions of this hyperbolic tasks is the method of generalized functions (GFM). The uniqueness of the decision of this BVP with accounting of shock waves has been proved in [4]. With use of GFM the decision in space of the generalized vector functions has been received, its regularization is carried out and its regular integral representation is built. On the basis of asymptotic properties of kernels the singular boundary integral equations has been constructed which resolve the BVP.

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A Γ-convergence result for variational problems with respect to non equicoercive measures

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A one-dimensional variational problem is considered with respect to a sequence of measures that weakly converge to a binomial measure, singular with respect to the Lebesgue measure. The convergence of the sequence of solutions is proven in the space of functions of Bounded Variation and the distributional derivative of the limit function is absolutely continuous with respect to a different binomial measure.

Delta-type solutions for the non-Hermitian system of induction equations

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The asymptotic behavior of solutions of the Cauchy problem for the linearized system of magnetohydrodynamic equations with initial conditions localized near a two-dimensional surface was obtained by the authors earlier. Here, this asymptotic behavior is refined.

Wavelets, exact renormalization group, fixed points

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Continuous wavelet transform is applied to the vertex functions calculation in quantum field theory. It is shown that the renormalization group turns to be a symmetry group in a field theory initially formulated in a space of scale-dependent functions — wavelet coefficients, — earlier described in [1, 2], is finite by construction. The effective action $\Gamma_{(A)}$ of our theory is complementary to the exact renormalization group effective action. The role of the regulator is played by the basic wavelet — an "aperture function" of a measuring device used to produce the snapshot of a field ϕ at the point x with the resolution a. The standard RG results for ϕ^4 model are reproduced [3]. The fixed points of the scale-dependent ϕ^4 are studied in one loop approximation. The application of the same method to the hydrodynamic turbulence theory is discussed.

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Elements of quantum simulators in arrays of quantum dots with dipole-dipole interaction

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We consider the arrays of InGaAs/GaAs quantum dots as perspective possible implementation of quantum neural networks. The systems of quantum dots interacting to each other by dipole-dipole interaction and linearly coupled to the common phonon bath of the GaAs substrate were studied numerically by means of the solution of von Neumann evolution equation for the density matrix. The systems of 2 and 3 quantum dots were simulated in a wide region of temperature 50–300 K. The presence of entangled states have been observed up to $\sim 10^2$ K. The methods of controlling 3 qubit systems and the stability of final states are discussed.

Amplitude and phase modulation applied to the mask to achieve the superresolution

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A novel approach to improve the axial and lateral resolution of the point spread function (PSF) of the optical system is investigated. The approach is based on the amplitude and phase masking for modifying the focused light fields. The degree of masking in the annulus and the pupil function of the central part, modify the transmittance and the intensity distribution at the focus significantly which can be tailored in the achieved PSF. The results after determining suitable parameters for the mask can be useful to resolve two closely situated point sources under different considerations.

Earlier studies were anxious about uniform distribution of the light radiation across the pupil function of the imaging system. However, in those studies the axial resolution of the optical system is obtained at the price of deteriorating its lateral resolution. Amplitude and phase modulation can be used as a process that suppress simultaneously side-lobes and sharpen the central peak of the point spread function (PSF). It is the process of controlling both amplitude and phase of the transmitted light in the output plane of the optical system. In this process of light manipulation, a two-dimensional pupil mask is deliberately transformed into a complex mask of three different zones, in which the two narrow annulus zones of equal width, having uniform amplitude transmittance, but anti-phase relative to each other, whereas the central zone performs the amplitude modulation. The optical systems with this complex mask yields the point spread function (PSF) which is not rotationally symmetric. The resulting diffraction pattern of the optical system is divided into the two fragments: a part of the pattern with the suppressed sidelobes and narrower central peak is achieved at the cost of worsening its counterpart with the enhanced sidelobes and broadened central peak. Normally, apodization is not applicable to the processing of astronomical images since the diffraction spots of the distant objects exceed the diffraction limits in the presence of atmospheric turbulence. The side-lobes of the PSF of the high intensity object can conceal the low intensity object so it is important to suppress the side-lobes of the diffraction pattern of the high intensity object in order to detect the image of the faint object. However, this method has merit to resolve the two images of same or different intensities are separated by a small distance in the astronomical and the spectroscopy observations.

Structural and optical properties of chrysotile with gold nanoparticles in channels

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One possible method of obtaining set of parallel nanowires is based on their synthesis in templates with parallel channels. These materials, once their pores are infilled with appropriate materials, comprise a parallel array of nanowires made of these fillers.

An interesting object in this respect is offered by the natural mineral chrysotile (asbestos), since this matrix can reach macroscopic dimensions. Chrisotyle consists essentially of nanotubes with an inner diameter of about 5 nm and an outer diameter of about 30 nm. These nanotubes can be about 1 cm long and ordered in a nearly hexagonal packing. Previously, chrysotile has already been successfully used to form semiconductor, ferroelectric nanowires. The structure and electrical, ferroelectric, magnetic, thermal and other properties of which have been studied.

The present work was aimed at studying the structure and optical properties of chrysotile samples, the channels of which is embedded with gold nanoparticles.

The channels were impregnated with aqueous solution of a metal salt, hydrogen tetrachloroaurate (III). Then, the samples were dried in air and the metals were reduced by heating in hydrogen. The completion of metal reduction was checked by the analysis of gaseous product.

The presence of gold in the pores was confirmed by X-ray diffraction (XRD) measurements.

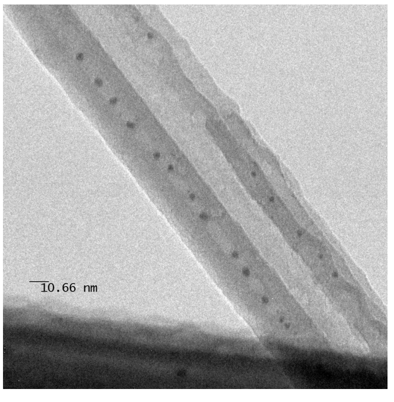

Figure shows nanotubes of chrysotile with gold as imaged in a JEM-2100F transmission electron microscope. As can be seen, the chrysotile nanotubes are filled with gold nanoclusters, whose diameter is less than 5 nm. They are located approximately at the same distance.

The samples are quite transparent in the visible and near-IR range because the empty chrysotile produces no large attenuation and the nanotube diameter is small. Nevertheless, some scattering of light takes place at blocks of fibers. The study of transmission and reflection spectra of chrysotile samples with gold in channels was continued. Earlier it was shown that optical properties have features in the 600 and 1300 nm regions associated with plasmon effects. In this study, features related to plasmons were found in the region of 300 nm.

Thus, it is shown that gold clusters can be embedded in chrysotile nanotubes. Moreover, it is possible to observe optical features associated with the plasmon properties of gold nanoparticles. Possible applications of macroscopic matrices of chrysotile or individual nanotubes with gold particles in plasmonic devices are also discussed.

Electromagnetic forces in negatively refracting photonic crystals

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Electromagnetic waves can exert forces on particles. These forces can be expressed as a sum of two terms corresponding to conservative (gradient) and nonconservative (scattering force) parts [1]. Conservative forces are related to gradients of intensity and are responsible for the trapping of particles, whereas the scattering forces follows the direction of the gradient of the phase (the direction of the local wavevector) and allows long range transport of particles.

It has been previously hypothesized that electromagnetic waves in left-handed materials can produce a wave propagating in a direction that is opposite of the Poynting vector that can result in negative radiation pressure, i. e. pressure in a direction opposite to the general power flow. While left-handed materials, as hypothesized by Veselago [2], have negative electric permittivity and magnetic permeability, we also can create a material with left-handed properties by means of a periodic system of optical nanostructures with positive permittivity and permeability. These structures are called left-handed photonic crystals [3]. On the contrary to the quasi-homogeneous left-handed materials, photonic crystals are composed of structures that are large in comparison to the wavelength, and the distance between these structures is also big enough to allow study of electromagnetic forces in the gaps with a probe particle, as shown in Fig. 1a. In this paper, we present a numerical examination of the optical forces within photonic crystals that demonstrate negative refraction and reversed phase propagation. To deal with nonuniform intensity distribution inside photonic crystals, we modified the polarizability of the probe particle to make it insensitive to gradient forces. We show that even though the scattering force, shown in Fig. 1b, does not follow the direction of the power flow, shown in Fig. 1c, positive radiation pressure is preserved inside the photonic crystal structure.

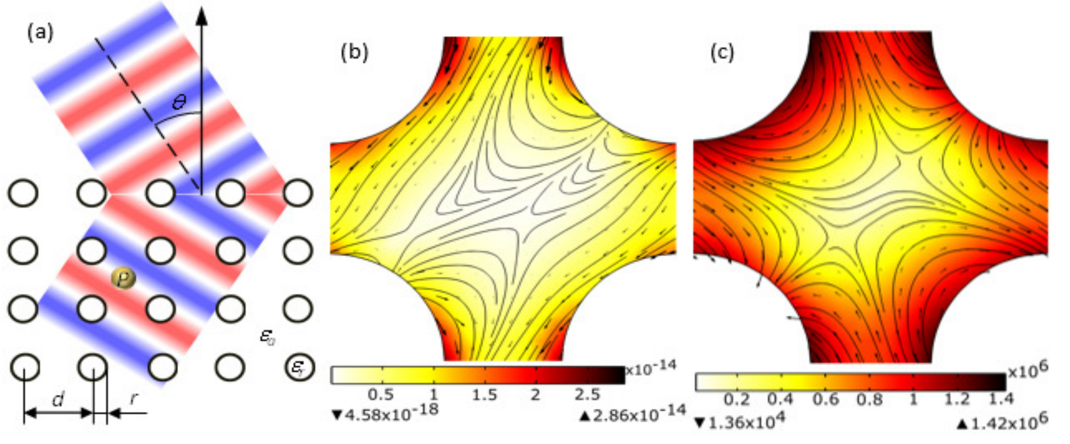

Fig. 1: (a) A left-handed photonic crystal with incident electric field θ creates a negatively-refracting field. The particle P is modeled as a dipolar particle. (b) The forces within a unit cell of a left-handed photonic crystal is shown using the streamlines and arrowfield; the magnitude is shown using the color map. (c) The powerflow within the same unit cell as in 1b. is shown using lines and arrows; the color map indicates the norm of the electric field.

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Scalarization of semiclassical stationary problems for vector systems and application in plasma physics

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We propose a method for finding asymptotic solutions of stationary problems for differential (and pseudo-differential) operator pencils with symbol being a self-adjoint matrix. It is shown that the construction of asymptotic solutions associated with a selected eigenvalue of the matrix symbol (called effective Hamiltonian, or term, or mode) can be reduced to the study of objects associated with the determinant of the matrix symbol. The benefit of such a reduction is that in higher dimensions the determinant can be evaluated analytically as opposed to eigenvalues. We discuss the application of the method to the linearized system of cold plasma equations [1–4].

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Asymptotics of Green function for the linear waves equations in a domain with a non-uniform bottom

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We consider the linear problem for gravity waves created by sources on the bottom and the free surface in a 3-D basin having slowly varying bottom $z = -D(x)$, $x = (x_1, x_2)$. The fluid is assumed to be incompressible and irrotational, so to verify Euler–Poisson equations, with potential $\Phi(x, z, t)$, subject to Neumann boundary condition at $z = -D(x)$. These equations have been given a Hamiltonian form in [1], involving canonical variables $(\xi(x, t), \eta(x, t))$ describing the dynamics of the free surface; variables (ξ, η) are related by the free surface Dirichlet-to-Neumann operator. Here we neglect quadratic terms in Zhakharov equations, and consider the linear response to a time-depending disturbance of $D(x)$, in the semi-classical approximation where the wave-length is assumed to be small with respect to the characteristic size of the basin. After properly scaling, the equations take the form

$$\begin{aligned} &h^2\Delta_x\Phi + \frac{\partial^2\Phi}{\partial z^2} = 0, \quad -D(x) < z < 0 \\ &h^2\frac{\partial^2\Phi}{\partial t^2} + \frac{\partial\Phi}{\partial z} = f^+(x,t), \quad \text{for } z = 0 \\ &\frac{\partial\Phi}{\partial z} + h^2\langle\nabla_x D, \nabla_x\Phi\rangle = f^-(x,t), \quad \text{for } z = -D(x) \end{aligned} \tag{1}$$

with Cauchy data

$$\Phi|_{t=0} = \phi(t,x)|_{t=0}, \quad h\frac{\partial\Phi}{\partial t}\Big|_{t=0} = -\eta|_{t=0} \quad \text{for } z = 0.$$

Taking Laplace transform $t \mapsto s$, we can recast (1) in terms of a boundary value problem of elliptic type, locally in s. Standard variational methods then ensure existence and uniqueness of the solution, provided the data enjoy suitable Sobolev regularity.

We may as well consider a time-harmonic disturbance with frequency ω and look for (stationary) solutions of the same type; this approach is suitable for asymptotic methods, provided the boundary data are Lagrangian distributions. Our main concern is actually to construct a parametrix for the stationary problem, as in [2–4]. By parametrix, or "asymptotic Green function with limiting absorption condition" of a inhomogeneous PDE of the form $\mathcal{H}(x, hD_x; h)u = f(x; h)$, where $f(x; h)$ is a suitable Lagrangian distribution, we mean here a family of distributions u_ε such that $(\mathcal{H}(x, hD_x; h) - i\varepsilon)u_\varepsilon = f(x; h) \bmod \mathcal{O}(h^\infty)$ in some Sobolev norm, uniformly as $\varepsilon \to 0+$.

The first step in this program consists in determining the linearized Dirichlet-to-Neumann operator L such that

$$\frac{\partial\Phi}{\partial z}(x, z, t)|_{z=0} = L\phi(x, t)$$

when Φ satisfies (1) with $f^-(x, t) = 0$. Parametrix of L essentially determines the parametrix for (1).

We show that $L = L(x, hD_x)$ is a self-adjoint h-Pseudo-differential operator whose principal symbol $L_0(x,p) = |p|\tanh(D(x)|p|)$ gives the usual dispersion relation $L_0(x,p) = E$. The study of L is further simplified by considering Maupertuis–Jacobi correspondence as in [5], mapping $L(x, hD_x)$ at energy E onto a Helmholtz-type operator whose coefficients depend on E. Such a Green function has been described “geometrically” in [6] using a suitable modification of Maslov canonical operator.

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Modeling of influence of microwave sintering geometric effects on the effective electrodynamic parameters of powdered metals

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In this work, we are going to investigate the influence of geometric consolidation of metal particles during microwave sintering on the effective electrodynamic parameters of powdered metals. So, the purpose of this work is to simulate the transition from spherical separate periodically arranged particles to the model of porous material consisting of the spherical periodically arranged particles that are connected to each other by conductive bridges of the formed bonds.

The suggested model will be considered as an effective medium that can be described in terms of effective electromagnetic properties μ_{eff}, ε_{eff}, σ_{eff}. Therefore, speaking precisely, the dependencies of μ_{eff}, ε_{eff}, σ_{eff} on the radius r of conductive cylindrical bridges between spherical particles will be calculated, investigated and analysed.

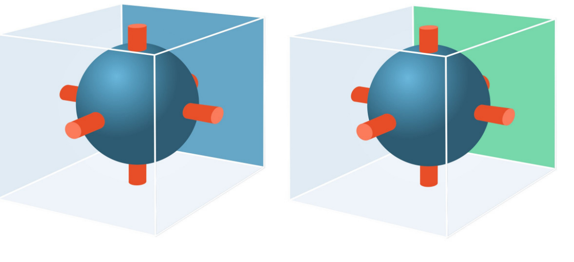

Fig. 1: The dual model of unit cell of spherical conductive particles connected to each other by conductive cylindrical bridges of radius r.

To calculate these dependencies of effective electrodynamic parameters, the dual model of unit cell of periodic broadcasting (Fig. 1) will be simulated using Comsol electromagnetic software and server computer with 75 GB RAM and 25 cores of two Intel Xeon E5-2630 processors.

The dual model is needed to simulate both perfect electric conductor (PEC) and perfect magnetic conductor (PMC) substrates on the bottom of these unit cells to get the closed system of impedance equations for μ_{eff} and ε_{eff} that allows deriving of μ_{eff} and ε_{eff} values unequivocally [1].

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Generation and control of unipolar light pulses in nonlinear media excited by ultra-short pulses

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Generation of few-cycle light pulses is an area of principle interest in modern optics [1]. Such pulses are used for many practical, for example to the ultrafast control of quantum systems on the atomic and molecular time scales. In addition, generation of unipolar optical pulses, i. e. pulses with constant sign of the electric field, attract various researches for the past decades, see review [2]. Due to their unidirectionality such pulses can effectively delivers mechanical momentum to charged particles and hence, can find be effectively used for control the electrons dynamics. Generation of unipolar half-cycle pulses at the first sight seems to be unphysical because of acceleration of bounded system of charges is bipolar. However, approximately unipolar pulses were obtained experimentally in terahertz range and predicted theoretically by several authors in diferent situations, see review [2] and references therein.

In the present work, we demonstrate theoretically a new possibilities of unipolar pulse generation when nonlinear media (for example, Raman-active medium, RAM) is excited by a train of extremely short light pulses [3, 4]. Furthermore, we demonstrate in this talk the possibility of unipolar pulse shape control via considering string of nonlinear oscillators excited by few-cycle pulses [5–7]. By appropriate choosing of the medium geometry and excitation conditions it is possible to generate rectangular pulses with top flat, curve flat etc.

Finally, we predict a novel possibility of unipolar half-cycle generation via unusual reflection of single-cycle bipolar pulse from thin metallic or dielectric layer [8]. This generation is based on peculiarities of Green function of 1D wave equation. Namely, in 1D case generated field is proportional not to charges acceleration as it is usually the case, but to charge velocity. When electric charge is illuminated by single-cycle pulse the velocity of charge can keep its sign constant. This lead to the formation of half-cycle unipolar pulse.

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Spectral study of the Laplace–Beltrami operator arising in the problem of wave scattering by a quarter-plane

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The Laplace–Beltrami operator on a sphere with a cut arises when considering the problem of wave scattering by a quarter-plane. Recent methods developed for sound-soft (Dirichlet) and sound-hard (Neumann) quarter-planes rely on an a priori knowledge of the spectrum of the Laplace–Beltrami operator. In this presentation we consider this spectral problem for more general boundary conditions, including Dirichlet, Neumann, real and complex impedance, where the value of the impedance varies like α/r, r being the distance from the vertex of the quarter-plane and α being constant, and any combination of these. We analyse the corresponding eigenvalues of the Laplace–Beltrami operator, both theoretically and numerically. We show in particular that when the operator stops being self-adjoint, its eigenvalues are complex and are contained within a sector of the complex plane, for which we provide analytical bounds. Moreover, for impedance of small enough modulus $|\alpha|$, the complex eigenvalues approach the real eigenvalues of the Neumann case.

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Positional characteristics of generalized decentered elliptical Gaussian beams on extended atmospheric paths

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The results of an experimental and theoretical investigation of the topography of regular and random aberrations for a collimated Gaussian beam under distortions on paths with non-centric refractive inhomogeneities are presented. Experimental samples were obtained for stationary regimes on a lab trace up to 100 meters long and for non-stationary regimes on an extended atmospheric path with a length of more than 1000 meters [1].

A theoretical model is proposed for a solitary coherent Gaussian beam and for a group of coherent and partially coherent Gaussian beams. Typical distortions of the initial beam profile at distances up to 10 Rayleigh lengths from the waist are described. It is these characteristic distances that are critical for applications using a collimated beam — optical sensing, power transmission and data exchange [2]. The invariants of the wave-front curvature tensor, the beam quality parameter, and methods for correcting refractive aberrations are discussed. The regularities of defocusing and astigmatic transformations of generalized decentralized elliptical Gaussian beams based on the apparatus of matrix optics are described [3]. The translational and rotational transformations of the intensity profiles of the beam, its phase, the curvature of the wave-front as a function of the parameters of the decentralising and the shift of the wave front are studied.

The structure of the beam profile is described in various sections corresponding to the direction of the initial propagation, the direction of the decentralising shift and the direction of propagation of the maximum of the intensity distribution. The patterns of variation of the third and fourth spatial moments along the propagation axis of the beam are established, which make it possible to determine the quantitative parameters of the refractive elements. The mechanisms of vortex profile transformations in the general cases of decentralising displacements and beam shifts are considered. The passage of a long trace with a decentralized elliptic Gaussian beam under various turbulence development regimes has been experimentally studied. The beam profile invariants constructed on the basis of differentiable mappings of the first and second orders for the intensity distribution profile are proposed.

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Resonances of 4th order differential operators

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We consider fourth order ordinary differential operator with compactly supported coefficients on the line. We determine asymptotics of the number of resonances in complex discs at large radius. We consider resonances of an Euler–Bernoulli operator on the real line with the positive coefficients which are constants outside some finite interval. We show that the Euler–Bernoulli operator has no eigenvalues and resonances iff the positive coefficients are constants on the whole axis.

Criteria for the absence of bounded solutions at the threshold frequency in a junction of quantum waveguides

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In a junction Ω of several semi-infinite cylindrical waveguides, we consider the Dirichlet Laplacian whose continuous spectrum is the ray $[\Lambda_\dagger, +\infty)$ with a positive cut-off value $\Lambda_\dagger$. We give two different criteria for the threshold resonance which is generated by bounded solutions to the Dirichlet problem for the Helmholtz equation $-\Delta u = \Lambda_\dagger u$ in Ω. The first criterion is quite simple and convenient to disprove the existence of nontrivial bounded solutions. The second criterion is rather involved but can be used, for example, in numerical schemes to detect concrete shapes supporting the resonance. Our study is incited by an eminent dimension reduction procedure for lattices of thin quantum waveguides which also requires to distinguish between stabilizing, i. e. bounded but non-descending solutions, and trapped modes with the exponential decay at infinity. The second criterion discriminates trapped modes in a natural way, however the first one does not and we only find a simple sufficient condition to reject a trapped mode initiating the threshold resonance.

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Electron scattering in a resonator system with finite work function

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Ballistic electron transport in quantum waveguides with constrictions (forming a resonator in the case of two or more constrictions along the waveguide) is studied. We present the results of numerical simulations of electron transport in waveguides having a finite work function. In addition we provide the results for the corresponding infinite work function waveguides. The comparison shows that infinite work function approximation is only suitable for studying the qualitative picture of one channel electron scattering in a waveguide. For the wave number above the second threshold

(the third threshold, if the waveguide is symmetric about its longitudinal axis) the difference in the pictures grows rapidly with respect to the decreasing work function, so that the infinite work function approach becomes inappropriate even for qualitative description of resonant tunneling.

Femtosecond photochemical and photomechanical activation of chemical reactions

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Traditionally, chemical reactions are mainly controlled by temperature and concentration, but the operating range and number of handling control parameters can be greatly expanded by using induced laser light and tuning the laser wavelength, power, polarisation, pulse width, repetition rate and numerical aperture NA of the focusing element. In this contribution, in order to improve the efficiency of activation of chemical reactions we consider femtosecond pulse width and kilohertz repetition rate, which first excite electrons and subsequently produce photochemical and photomechanical effects without changing temperature of the system. We show that the chemical activation energy can be overcome by laser-induced photochemical activation. The photon polarization also provides additional control for inter-molecular orientation in the case of photochemical activation.

For a chemical reaction $A + B \longrightarrow C$, its reaction rate can be described by an Arrhenius-type equation:

$$\frac{d[C]}{dt} = -\frac{d[A]}{dt} = k[A][B]\exp\Big(-\frac{E_a}{RT}\Big),$$

where $[A]$, $[B]$, and $[C]$ are the reactant and product concentrations, t is time, k is the frequency factor, R is the universal gas constant, T is the absolute temperature, and E_a is the activation energy.

The activation energy or energy barrier that separates the reactants and product bound states can be presented on an atomic scale as:

$$E_a = \frac{N_A hc}{\lambda},$$

where N_A is the Avogadro's constant, h is the Planck's constant, c is the velocity of light, and λ is the wavelength.

For typical values of the activation energy of 20 to 300 kJ/mol the wavelength of 0.4–6 micron can be used for photochemical activation.

The photomechanical activation effect, demonstrated by multiphoton absorption induced by focusing a femtosecond laser pulse into solution, leads to formation of a short-term bubble with a concentration increase at the bubble boundary. The high-concentration on the newly formed multiphase bubble boundary accelerates the chemical reaction, the rate of which can be precisely controlled by the number of laser pulses. The bubble surface life time of tens of microseconds is well-suited for chemical reactions with the time-scale in a nano- to micro-second range.

An overview of the recent research on femtosecond laser crystallisation will be given.

Solitary strain waves in nanostructured rod

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Composite materials (composites) are of a great interest nowadays because of their advantageous mechanical properties, which differ from those of homogeneous materials [2]. Composites are classified into few types, e. g., nanocomposites representing multicomponent materials with a polymer base (matrix) and a filler, and layered composites which consist of few homogeneous isotropic layers.

A comprehensive study of dynamical behaviour of composites is the challenging problem and can be performed through the non-destructive testing (NDT), based on an evolution of a propagating strain wave [1] in a bulk nanostructured waveguide. The method explores wave parameters for investigation of elastic properties of the waveguide material. Although the nanoparticle inclusions even in small concentration often lead to substantial decrease of transparency of material, the optical methods still can be used for investigation of strain waves. For example, the specimen can be done by bonding of an opaque waveguide under study between two transparent waveguides having parameters given. Nonlinear strain waves can be detected in transparent waveguides by means of digital holography, since a refractive index gradient occurs in the area of strain wave propagation. In order to generate nonlinear strain wave in solid waveguide a shock wave in water was produced by means of laser induced metallic evaporation. Energy transfer from the shock wave in water to the solid waveguide resulted in generation of strain wave. We report the results of comparison of nonlinear strain wave detected before and after the studied nanocomposite of cylindrical shape. The presented method allows one to estimate nonlinear strain wave velocity in the described waveguide configuration as well.

For mathematical description of the wave processes under study two initial-boundary value problems were considered. The doubly dispersive equation (DDE), governing propagation of bulk nonlinear longitudinal strain solitary waves in a cylindrical rod with nanoinclusions, was derived for the first one, and the Mindlin theory of elasticity of microstructured solids was explored for this purpose. The second problem was devoted to the nonlinear bulk longitudinal strain solitary waves propagation in a three-layered rod with rectangular cross-section. Two different subproblems were considered. In the first one all layers were isotropic, homogeneous, whereas external layers were made of the same material different from that of the internal layer. In the second one we suppose both external layers were made of nanocomposite, while the internal layer was made of a isotropic homogeneous material, and this structure was suitable for physical experiments.

Results of numerical simulation and experiments of bulk strain soliton propagation in nanocomposite waveguide are presented.

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Wave models of Sturm–Liouville operators

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For a semi-bounded symmetric operator the notion of the wave spectrum was introduced in [1]. The wave spectrum is a topological space determined by the operator in a canonical way. The definition uses a dynamical system associated with the operator: the wave spectrum is constructed from its reachable sets. We illustrate this notion by two examples: we give a description of the wave spectrum of two symmetric Sturm–Liouville operators defined by the differential expression $-\frac{d^2}{dx^2}+q$ considered on the half-line $(0,\infty)$ and on the interval $(0,l)$, $0<l<\infty$. They act in the spaces $L_2(0,\infty)$ and $L_2(0,l)$ and have defect indices $(1,1)$ and $(2,2)$, respectively. We construct functional wave models of these operators. In these models the elements of the original spaces are realized as functions on the wave spectra. These constructions can be used for solving inverse problems.

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Experimental estimating frequency dependence of reflection coefficient for a flat layer under oblique incidence

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A method is presented for measuring frequency dependence of reflection coefficient for a flat surface with non-ideal boundary conditions irradiated from oblique incidence direction. We use MLS method [1] and a monopole source for measuring impulse responses of a the surface for a set of angles of incidence. Then, to convert the results into the flat wave reflection coefficients, the Fourier–Bessel integral is inverted, i. e. an integral equation is solved numerically.

The presented method is supposed to be more informative than the reverberation room method that works for diffuse field. The new method is competitive with the impedance tube method, which works only for normal incidence.

The frequency dependencies found by the new method are compared with the ones theoretically calculated using Biot theory [2, 3] for appropriate surfaces.

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Benchmark of approaches to dynamics of axially impacted rod

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The problem to report is the axial elastic impact by a body on a thin rod. Three approaches are considered and the results of theoretical, finite element and experimental approaches to the solution of the problem are compared. The theoretical approach takes into account both the propagation of longitudinal waves in the rod and the local deformation due to the Hertz model. The result of this approach is a differential equation with a delayed argument. The results of the first approach are compared with the results of three-dimensional dynamic problem in the frame of finite-element approach, in which the wave propagation and local deformation are automatically taken into account. A benchmark of these two approaches showed a complete qualitative and satisfactory quantitative agreement of the results in the contact force and the impact time.

The experiment carried out at the National Taiwan University delivered only the impact time. Comparison of the measured impact time with the theoretical and finite-element results was satisfactory. However the tested rod was relatively short and it forced us to develop additional approximate model with two degrees of freedom. It was proved that the latter model was also appropriate for calculation of the impact time.

The problem of excitation of transverse oscillations after the impactor rebound off the rod will be touched upon, too. Parametric resonances appear which lead to that the motion has the character of beats at which the energy of longitudinal oscillation is transferred into the energy of transverse oscillation and vice versa. An estimate for the maximum possible amplitude of transverse oscillation is obtained.

On the calculation of matrix exponential of a large order

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Matrix approach to the theory of n-wave diffraction in layered media is based on computation of so-called *transfer* (or *scattered*) matrix $T = \exp(Ad)$, where d is the layer thickness and matrix $A = \|a_{ij}\|$ of order n consists from coefficients of corresponding differential equations. Direct application of polynomial representations [1] $\exp(Ad) = \sum_{l=0}^{n-1} c_l (Ad)^l$ for calculating the matrix exponential for large values n is complicated due to the growth of round-off errors in computations of coordinates c_l and high powers of matrix Ad. Approximations of the form

$$\exp(Ad) \approx E(L,K) = \sum_{l=0}^{L} \kappa_l (Ad)^l, \text{ where } L \leq n-1 \text{ and } \max |a_{ij} d| < \frac{1}{2n-1}, \tag{1}$$

are considered. Here $\kappa_l = \sum_{j=0}^{K+l} (\mathscr{B}_{j+n-1-l}(n) - \sum_{g=1}^{K} p_g \mathscr{B}_{j+n-g-1-l}(n))/(j!)$, $K < L$ and polynomials $\mathscr{B}_g(n)$ are expressed [2] through coefficients p_j of characteristic equation of the matrix Ad. If the latter condition in the formula (1) is not satisfied, the proposed approximation is used in combination with scaling and squaring: $\exp(Ad) = (\exp(Ad/2^j))^{2^j}$.

Evaluation of the accuracy of the formula (1) is done. For example, approximations $E(9,3)$, $E(12,4)$, $E(16,4)$, $E(20,5)$ use 4, 5, 6, 7 matrix multiplications and give a matrix exponential with relative error less than 10^{-13}, 10^{-20}, 10^{-25}, 10^{-33}, respectively. Comparison of the method (1) with Padé approximation [3] is given.

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Hörmander's solution singular at a running point and the Bateman solution

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In his famous monograph [1], Hörmander presented a somewhat implicit construction of a solution of the wave equation having a singularity at a running point and otherwise smooth. We are concerned with a detailed analytical investigation of this solution for the case of three spacial variables. In particular, we established that Hörmander's solution of the equation

$$u_{xx} + u_{yy} + u_{zz} = c^{-2} u_{tt}, \quad c = \text{const} > 0$$

is the classical Bateman's solution (see [2, 3])

$$u = (z - ct)^{-1} f(z + ct - (x^2 + y^2)/(z - ct)),$$

where the arbitrary function $f(\cdot)$ is assumed smooth and compactly supported. Complexified versions of the Bateman solution play an important role in the theory of localized waves (see, e.g., [4]), but its classical form did not find earlier any application.

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Resonance states completeness for Schrödinger and Dirac quantum graphs

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The problem of eigenstates (bound states) completeness is well-studied. Scattering problem leads to a system of quasi-eigenstates (resonance states) corresponding to quasi-eigenvalues (resonances). Resonances were actively investigated. As for completeness of the systems of resonance states, there are only few results [1–3]. We consider this problem for different quantum graph models. Namely, we deal with the Schrödinger and Dirac operators on quantum graphs. Star-like graphs and graphs with loops are considered. The Sz.-Nagy functional model is used. In the framework of this approach the completeness problem reduces to the problem of the characteristic function factorization.

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Local well-posedness in the problem of flow about infinite plane wedge with inviscous non-heat-conducting gas

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As is well-known [1], on stationary supersonic gas flow over infinite plane wedge (angle σ at the point of the wedge is small enough, $\sigma < \sigma_{\text{lim}}$) theoretically there are two possible stationary solutions: one of them corresponds to strong shock wave when gas speed beyond the shock is less than the speed of sound, i. e. $u_0^2 + v_0^2 < c_0^2$ (u_0, v_0 are components of the speed vector, c_0 is the speed of sound), and the other corresponds to the weak shock wave, when, generally speaking, $u_0^2 + v_0^2 > c_0^2$ (figure illustrates the position of the shocks, θ_s, θ_w are angular coordinates of the strong and the weak shocks accordingly).

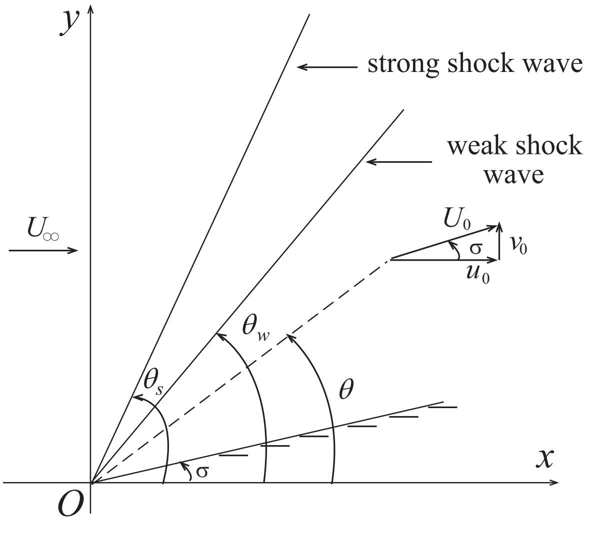

Fig. 1: Sketch for weak and strong shock waves.

However, in numerous physical and computational experiments if there is no additional information, for example about the value of the pressure down the flow, the case of weak shock wave is realized. As of today there is no strict mathematical explanation why this is happening. R. Courant and K. O. Friedrichs noticed in their monograph [1], that there is an opinion that strong shock wave is unstable by Lyapunov while weak shock wave is on the contrary stable.

In previous works of this talk authors [2–4], this assumption was grounded on the linear level. In this work we prove local-in-time existence of solution of mixed problem for initial quasilinear problem.

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Numerical simulation of wave scattering in a plane channel with sharp corners

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Lately a lot of electromagnetic waveguides with complicated structures have appeared. Some of them (e. g., in optics) necessarily contain sharp corners. For waveguides with sharp corners it is very difficult to find any numerical method since the electromagnetic field may be singular at the corners. The most methods and commonly used numerical codes (packages) are limited in the accuracy. The method of discrete sources (MDS) seems to be the most promising one in such the case. MDS is based on the anzats as follows: an unknown solution is sought as a linear combination of the Green (source) functions of the infinite strip domain. Substitution of such an anzats into the boundary conditions leads to a set of linear algebraic equations (SLAE). In the case of sharp boundaries the resulting matrices of SLAE are often very large, that is why the most numerical algorithms are time consuming and the accuracy may be limited. Effectiveness of the MDS depends essentially on the way of source placement. Our initial scheme of source allocation for sharp-pointed domains were suggested in [1, 2]. In this paper a new "dipole" source allocation in the neighbourhood of the sharp points is suggested. It requires essentially smaller matrices and leads to an essentially higher accuracy in comparison with other methods.

As a test the model plane scattering problem in a strip region with a sharp ledge is considered. The problem is governed by the Helmholtz equation together with the Neumann condition on the boundary and the radiation condition in infinity. Numerical results demonstrate the effectiveness of the idea proposed.

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Boundary triplets for point-like perturbation of Rashba Hamiltonian

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We consider a model of point-like perturbation of Rashba Hamiltonian using the boundary triplet approach. Boundary triplet and the Weyl function are obtained, properties of the Weyl function are studied.

The criterion of absence of bound states in two-dimensional perturbation of the discrete Schrödinger equation

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We consider a semi-infinite Jacobi matrix corresponding to the discrete Schrödinger operator with two-dimensional perturbation. Let a, b be non-zero diagonal elements of the Jacobi matrix. Our goal is to find the exact condition on the coupling constants a, b for which there are no bound states of the Schrödinger equation. In addition it is proved that the lines bounding the area of allowable values a, b are the "critical lines" for the coupling constants a, b.

Uniform asymptotics of far internal gravity waves fields in stratified rotating medium

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An important mechanism of internal gravity wave fields excitation in ocean, Earth's atmosphere, and artificial stratified media is their generation by various perturbation sources whose nature can be natural (traveling typhoon, flow past roughness of the ocean bottom relief, leeward mountains) and anthropogenic (marine technological structures, collapse of the zone of turbulent mixing, underwater explosions) [1, 2]. One of the basic factors that determines the characteristics of exited wave fields is rotation of the entire stratified medium. The aim of the present study is to construct asymptotic solutions describing the amplitude-phase characteristics of the far internal gravity waves fields exited by a traveling perturbation source in a finite-depth stratified medium which rotates as a whole. The problem of constructing uniform asymptotics for the far internal gravity waves fields generated by a moving source of perturbations in flow of a finite-depth stratified rotating medium is considered. The solutions obtained describe the wave perturbations both inside and outside the wave fronts and can be expressed in terms of the Airy function and its derivatives [3–6]. Uniform asymptotic solutions constructed in the study make it possible to describe the amplitude-phase characteristics of far fields of internal gravity waves generated by a local perturbation source moving in flow of a finite-depth stratified medium, which rotates as a whole, both outside and inside the corresponding wave fronts. The asymptotics of far internal gravity waves fields obtained make it possible not only to calculate efficiently the basic characteristics of wave fields but also to analyze qualitatively the solutions obtained. We can see that taking rotation of the stratified medium as a whole into account leads to a appreciable complication of both amplitude and phase characteristics of the generated far wave fields, namely, to appearance of not only the longitudinal but also the transverse wave packets which are absent in media without rotation. It is shown that the far field asymptotics make it possible to efficiently calculate the main characteristics of the wave fields and to qualitatively analyze the solutions obtained. This opens wide opportunities for studying wave patterns as a whole, which is important for correctly constructing the mathematical models of gas dynamics, including for express-estimating in natural measurements of wave fields. Note that such wave patterns can be observed in the remote probing and observation of the internal gravity waves excited by various sources of perturbations in both the ocean and Earth's atmosphere [1, 2, 5, 6].

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Bound states in the continuum and light localization in dielectric arrays

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We report on light trapping by structural resonances in linear periodic arrays of identical high-index dielectric elements. As the basic elements both subwavelength spheres and rods with circular cross section have been considered. The optical properties of the arrays are studied by solving linear Maxwell's equations with a Mie multiscattering approach taking into account electric and magnetic multipoles of all orders [1]. When the array is infinite it is known to support bound states in the radiation continuum (BSCs) [2], i. e. structural resonances with infinite life-time embedded into the continuous spectrum of scattering states. Two classes of the BSCs can be identified, namely, topologically and symmetry protected states. In case of arrays of dielectric spheres we show that there is a Bloch guided BSC mode which is stabilized by a topological singularity in space of the resonance coupling constant. We demonstrate numerically that this Bloch BSC can be employed for guiding light pulses above the line of light [3]. If the infinite array is terminated at both ends to form a finite chain of dielectric elements the BSCs become high-Q resonances. We evaluate the asymptotic behavior of the Q-factor of such resonances against the number of elements in the array. We demonstrate numerically that under illumination by a plane wave finite arrays of 10–15 silicon nanospheres can be used to enhance the amplitude of the impinging light at least by order of magnitude in the visible-to-near infrared range when the material and geometrical parameters of the systems are tuned to the structural resonance associated with a BSC [4].

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Effective permittivity and permeability of metamaterial from rectilinear thin wires array

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A composite structure consisting of rectilinear thin wires array forming a 2D square lattice within the non-conductive host media was theoretically investigated in this paper. The effective permittivity and permeability of the composite structure were calculated using numerical solution of Maxwell equations in differential form. It was determined that in order to find effective characteristics it is

enough to simulate the electromagnetic field distribution in the area of the two unit cells of a structure translation. It was found that in the first photonic transparency range ($0.3 < a/\lambda_m < 0.5$, where a is the period of wire array, λ_m is the wavelength of electromagnetic wave in the non-conductive host media) the investigated structure with a large number of cells (more than 10 in each direction) can be considered as a metamaterial, and its electromagnetic properties can be described by effective permeability and permittivity. In the second photonic transparency range ($0.65 < a/\lambda_m < 1$) the investigated structure demonstrates the properties of both the left-handed metamaterial and the photonic crystal allowing Bragg diffraction of electromagnetic waves. In this range, the propagation of oscillations of the effective field can be described using effective parameters. Wherein, it is necessary to take into account the formation of additional oscillations on the effective field background arising due to the diffraction on the scatterers lattice. The criteria for the transition between metamaterial and photonic crystal were defined.

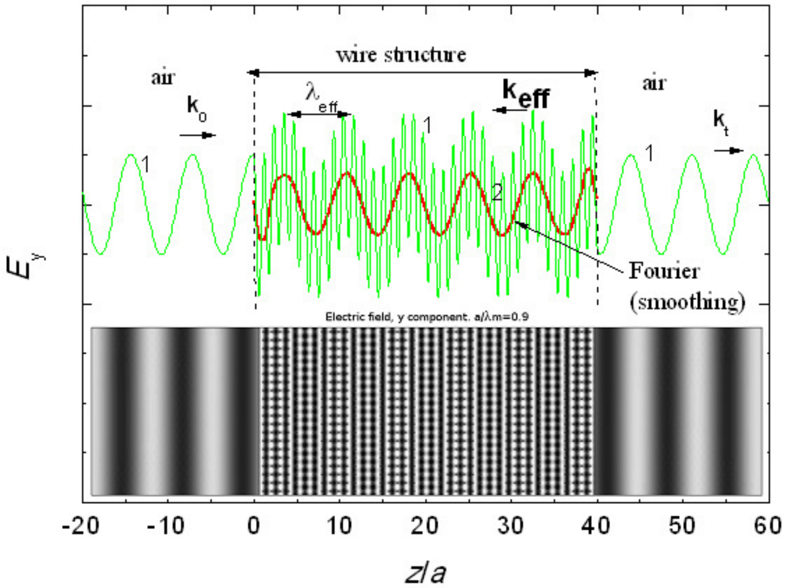

Fig. 1: Curve 1 — the dependence of an alternating electric field on coordinate z along a line passing in between the wires, in sizes of period a. $\lambda_m = 2\pi c/(\omega\sqrt{\epsilon_m\mu_m})$, $\epsilon_{\text{eff}} = \mu_{\text{eff}} = -1$. Curve 2 — a signal inside the wired structure, smoothed with Fourier filter. Bottom electric field distribution, dark area corresponds to maximum values of E_y, the light one depicts minimal values of E_y. The intensity of incident wave is taken to be equal to 1.

Gaps in the spectrum of Neumann problems in domains with strongly corrugated boundary

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The talk concerns periodic second order self-adjoint differential operator posed on periodic domains with waveguide geometry. It is well-known that the spectrum of such operators has a band structure, i. e. it is a locally finite union of compact intervals called bands. In general, the neighboring bands may overlap, otherwise we have a gap in the spectrum, that is an open interval having an empty intersection with the spectrum but with ends belonging to it. In general, the presence of gaps in the spectrum is not guaranteed, for example the (Neumann, Dirichlet) Laplacian on a straight strip has no gaps. In the talk we discuss spectral properties of a periodic waveguide Ω_ε ($\varepsilon > 0$ is small parameter) obtained from a straight strip Ω by attaching a family of small identical protuberances $T_{i,\varepsilon}$ ($i \in \mathbb{Z}$), ε-periodically along Ω. Each protuberance $T_{i,\varepsilon}$ consists of two parts — a small square (“rooms”) and a narrow rectangle (“passage”) connecting the “room” with Ω. The diameter of $T_{i,\varepsilon}$ is of order $\mathcal{O}(\varepsilon)$. We prove that under a suitable choice of sizes of “rooms” and “passages” the gaps open up in the spectrum of a certain Neumann problem in Ω_ε, moreover this choice allows to control the location of gaps as $\varepsilon \to 0$.

We also discuss some previous results about the case when $\Omega \in \mathbb{R}^n$ is a bounded domain and $\Omega \in \mathbb{R}^2$ is a straight strip of fixed width.

The results are part of joint works [1–3] with Andrii Khrabustovskyi (Karlsruhe Institute of Technology, Germany).

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Analysis of equations of complex heat transfer with moving source in context of endovenous laser ablation

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Endovenous laser ablation is a very effective minimally invasive therapy to manage leg varicosities. The corresponding mathematical model is based on conversion of absorbed laser energy into heat. Considering the features of the process, accounting for the emission of black-body radiation is appropriate. Therefore, to describe the process of endovenous laser ablation, the radiative-conductive-convective heat transfer model with moving source is chosen.

The conventional non-stationary normalized P_1 approximation of the complex heat transfer model describing radiative, conductive, and convective contributions is considered in a bounded domain $\Omega \subset \mathbb{R}^3$

$$\partial\theta/\partial t - \operatorname{div}(a\nabla\theta) + \mathbf{v}\cdot\nabla\theta + b\kappa_a(|\theta|\theta^3 - \varphi) = u_1,\ -\alpha\Delta\varphi + \kappa_a(\varphi - |\theta|\theta^3) = u_2,\ x \in \Omega,\ t \in (0,T). \quad (1)$$

Here, θ is the normalized temperature, φ the normalized radiation intensity averaged over all directions, $\mathbf{v}$ a given divergence free velocity field, and $u_{1,2} = u_{1,2}(x,t)$ describe the intensities of heat and radiation sources. Parameters a, b, κ_a, and α describe the radiation and thermal properties of the medium (see [1]). The following boundary conditions on $\Gamma := \partial\Omega$ and the initial condition at $t = 0$ are assumed:

$$a\partial_n\theta + \beta(\theta - \theta_b)|_\Gamma = 0, \quad \alpha\partial_n\varphi + \gamma(\varphi - \theta_b^4)|_\Gamma = 0, \quad \theta|_{t=0} = \theta_0. \tag{2}$$

Here, the boundary functions, θ_b, β, γ, and the initial function, θ_0, are given.

We consider the right-hand sides of equations (1) having the structure $u_{1,2} = P_{1,2}(t)f_\varepsilon(x, r(t))$, $x \in \Omega$, $t \in (0, T)$. The functions $P_{1,2}$ describe the powers of heat and radiation sources, $f_\varepsilon(x, r(t)) = 1$, if $|x - r(t)| < \varepsilon$, and $f_\varepsilon(x, r(t)) = 0$ otherwise; $r(t)$ is a given trajectory of a moving source.

In the current work, a priori estimates of temperature and radiation intensity in the space L^∞ ensuring the unique solvability of the problem (1), (2) are presented. The theoretical analysis is illustrated by numerical examples simulated the process of endovenous laser ablation.

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Dissipative Kerr solitons and Cherenkov radiation in optical microresonators

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The discovery of optical frequency combs in whispering gallery mode microresonators suggests a possibility of the development of novel types of frequency comb sources with characteristics unachievable for the systems based on conventional mode-locked lasers. Kerr frequency combs are generated by coupling a cw laser into a high-Q microresonator that converts the initial frequency into a broadband frequency comb by cascaded four-wave mixing processes. They are widely employed in various applications such as precision frequency metrology, highly multiplexed spectroscopy, low noise microwave generation, fiber telecommunications and many others [1]. Nonlinear process of comb formation leads to the arbitrary phase relations between individual spectral lines, that is quite different from conventional laser-based frequency combs, and results in the emergence of significant phase noise of RF beatnote. The generation of dissipative Kerr solitons allows to solve this problem and opens a way to coherent, broadband optical frequency combs [2]. It was shown that the dispersion of the microresonator plays an important role in the possibility of comb generation. The phase-matching condition for four-wave mixing processes remains easier to satisfy in anomalous group velocity dispersion regime. It was predicted via numerical simulations that the process of the dispersive wave formation (optical analog of Cherenkov radiation) caused by the higher order dispersion terms may expand the comb generation bandwidth into the normal dispersion regime [2]. The formation of the dispersive wave may be explained as follows: if a microresonator is pumped at a laser pump wavelength characterized by the anomalous mocroresonator GVD, temporal solitons can be generated. If the duration of the soliton is short enough so that its bandwidth extends to the normal dispersion regime, near the point where the GVD is close to zero, the wavelength matching becomes especially favorable for four-wave mixing processes producing sharp spectral peak, corresponding in time domain to oscillating soliton tails — dispersive waves. Dynamics of such process in space-time representation can be described by the Lugiato–Lefever equation with higher order dispersion terms. Previous analysis was limited, however, as only the position of the dispersive wave was investigated. We performed a complete closed form analytical asymptotic analysis of dissipative Kerr solitons in microresonators with third dispersion. Combining direct perturbation method with the method of moments we find expressions for the frequency, strength, spectral width of the dispersive wave and soliton parameters. The formation of the dispersive wave leads to a shift of the soliton spectrum maximum from the pump frequency (spectral recoil), while the soliton displaces the dispersive wave spectral peak from the zero dispersion point. It has been demonstrated that the dispersive wave formation in the presence of a soliton contributes to expansion of the bandwidth of the generated comb to the normal dispersion frequency range [3].

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Mobility of the charged clusters in polarized liquid

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The problem of a correct definition of the charged carrier effective mass in superfluid helium is revised. It is shown that the effective mass of such a quasiparticle can be introduced without Atkins’s idea about the solidification of liquid He^4 in the close vicinity of an ion (the so-called “snowball” model). Moreover, in addition to the generalization of Atkins’s model, the charged carrier effective mass formation is considered within the framework of the two-fluid scenario. The physical reasons of the normal-fluid contribution divergence and the way of the corresponding regularization procedure are discussed. Agreement between the theory and the available experimental data is found in a wide range of temperatures.

Special discontinuities in nonlinear elastic media

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Solutions to nonlinear hyperbolic systems describing weakly nonlinear quasitransverse waves in weakly anisotropic elastic media are studied. The influence of small-scale effects of dissipation and dispersion are analyzed. Small-scale processes determine a discontinuity structure and a set of discontinuities with stationary structures. Among discontinuities with stationary structures there are special ones on which (in addition to relations following from conservation laws) some additional relations should be satisfied which follow from the requirement for the discontinuity structure to exist. On the phase plane the structure of such discontinuity is represented by an integral curve connecting to saddle points.

In [1, 2] stationary structures of discontinuities in nonlinear elastic weakly anisotropic media are studied for a positive non-linearity parameter and in [3] — for a negative one. It is shown (in the framework of the model chosen) that the number of special discontinuities depends not only on small-scale processes of dissipation and dispersion, but on a sign of non-linearity parameter as well.

The existence of special discontinuities leads to non-unique way to construct self-similar solutions to the problem of arbitrary discontinuity disintegration. In [3] asymptotics of non-self-similar problems of equations with dissipation and dispersion taken into account were numerically obtained. These asymptotics of non-self-similar problems correspond to those for self-similar ones.

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Violation of the classical Laue–Bragg–Wulff law of scattering due to amplitude form-factor of the periodical lattice of discontinuities

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The problem of the surface acoustic Rayleigh wave scattering on the deterministic (not statistical) cylindrically symmetrical solid surface roughness, having periodical lattice of discontinuities, is solved in the Born (the Rayleigh–Born) approximation of the perturbation theory in the roughness amplitude. The classical Laue–Bragg–Wulff [1–9] scattering limit, when the wavelength of the incident Rayleigh wave is more less than the size of the Laue–Bragg–Wulff lattice elementary cell, is considered. Expression for the dimensionless indicatrix of scattering, which is proportional to the intensity of the scattered wave, in dependence on the ratio of the lattice elementary cell size to wavelength, angle of scattering, configuration of the lattice and the Poisson coefficient of an isotropic solid is obtained in this limit.

The new dependencies of the scattering indicatrix on the ratio of the lattice elementary cell size to the wavelength, on the number of lattice elementary cells and lattice configuration are obtained. These new laws of the diffuse, i. e. short-wavelength, scattering modulate the amplitudes of the classical Laue–Bragg–Wulff resonances. The amplitudes of these resonances can increase or decrease with increasing of the lattice elementary cells number depending on the lattice configuration.

It is obtained that cylindrical symmetry influences the positions of the classical Laue–Bragg–Wulff maxima. They dance around values of the lattice elementary cell to wavelength ratio defined by the classical Laue–Bragg–Wulff law in dependence of the roughness discontinuities lattice configuration. When the number of the lattice elementary cells tends to infinity the positions of the Laue–Bragg–Wulff maxima tend to the positions defined by the Laue–Bragg–Wulff law of scattering.

It is obtained that indicatrix of scattering oscillates in the region between the Laue–Bragg–Wulff maxima as well. The frequency of these oscillations as a function of the transmitted wave-vector depends on the lattice elementary cells number and the lattice configuration contrary to the Laue–Bragg–Wulff resonances.

W. L. Bragg [3], contrary to the M. Laue, W. Friedrich, P. Knipping [1, 2], takes into account amplitude form-factor of the wave scatterers, that is the amount of atoms in the scattering planes of atoms [6, 7]. But according to the Bragg hypothesis and law this form-factor influences only the intensity of the scattered wave but not the positions of the Laue–Bragg–Wulff resonances.

The phenomenon of the scattering indicatrix strong dependence on the amplitude form-factor of the discontinuities lattice is obtained in the present work. This phenomenon leads to violation of the classical Laue–Bragg–Wulff law of scattering. It means, for example, that at a definite form of discontinuities amplitude dependence on the number of discontinuity in the periodical lattice the values of all the Laue–Bragg–Wulff maxima are equal to zero, the scattering, as a whole, is suppressed, and the new positions of scattering indicatrix maxima are defined by the amplitude form-factor of the discontinuities lattice.

This phenomenon can be used for the modelling of the indicatrix of scattering on the periodical Laue–Bragg–Wulff lattice, that is for the metamaterials construction.

This phenomenon differs from the one considered in the [8, 9], which violates the generalized Laue–Bragg–Wulff law on the single discontinuity as well.

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Transmission spectra of one-dimensional bi-periodic photonic crystals

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The bi-periodic one-dimensional (1D) photonic crystals (PCs) recently attracted attention as a new promising class of photonic structures, such as photonic-magnonic crystals [1, 2] and photonic hypercrystals [3]. In this communication, we present the theoretical and numerical study of the spectra of electromagnetic waves propagating through 1D dielectric bi-periodic PCs $[(A/B)^N C]^M$ with the layers A, B and C (of thicknesses d_A, d_B and d_C) chosen to be the dielectric oxides Al_2O_3, SiO_2 and TiO_2, respectively, which are transparent within the visible and near infrared regimes. Two periods of the structure, $D_0 = d_A + d_B$ and $D_1 = N(d_A + d_B) + d_C$, are formed by the bilayers (A/B) and repeating layers C [see Fig. 1(a)]. We examine modifications of the TE- and TM-modes spectra within the first photonic band gap (PBG) and its vicinity with the change of the period numbers N and M, layer thickness d_C and incidence angle θ. The PBG spectra of both TE- and TM-modes exhibit the set of defect modes (DMs). The number of DMs for both TE- and TM-polarized electromagnetic waves is correlated with M and strongly depends on d_C [Figs. 1(b) and 1(c)]. The change of θ allows to govern the positions of both PBG edges and DMs and to control the PBG width. Modifying d_C, one can reach the significant shift of the DMs towards the PBG edges, as well it is possible to increase the DMs number and to obtain the satellite PBGs.

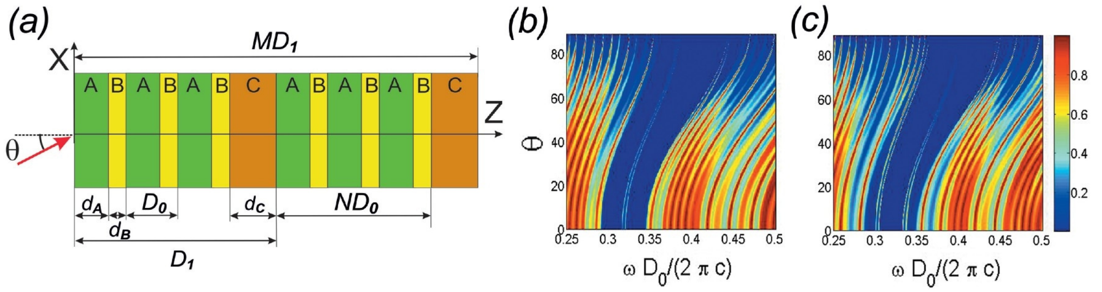

Fig. 1: Schematic of the bi-periodic 1D PC (a). Transmittivity of TE-mode as function of the normalized frequency $\omega D_0/(2\pi c)$ and θ for $[(A/B)^{10}C]^2$ with $d_A = 0.281$ µm, $d_B = 0.341$ µm and $d_C = 0.796$ µm (b), and $d_C = 1.592$ µm (c).

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Voltage-tunable vapour detector using optical beam shifts in a magneto-electric multilayered structure

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The Goos–Hänchen shift (GHS) is the longitudinal shift that an electromagnetic wavepacket experiences in its incidence plane when it is reflected from a surface. It is usually accompanied by a small angular shift in the same plane. Measuring the GHS has been suggested as the principle of, among others, chemical and biological sensors [1, 2]. In this Communication, we investigate the spatial and angular GHSs of a Gaussian light beam reflected from a structure consisting of a nematic liquid crystal (LC) cell sandwiched between indium-tin oxide electrodes and deposited on a bilayer made of a magneto-electric yttrium-iron garnet film and a non-magnetic dielectric substrate of gadolinium-gallium garnet [Fig. 1(a)]. We show that the external voltage applied to the LC cell allows an efficient tuning of the GHS. Reversing the polar magnetization in the magneto-electric layer leads to an increase of the GHS in the $s \to p$ or $p \to s$ polarization configurations of the incident and reflected beams. We propose a scheme for a vapour detection technique based on the measurement of the GHS variations induced by tiny changes of the relative permittivity $\varepsilon^{(\mathrm{air})}$ of the air surrounding the system. For well-chosen values of the applied voltage and incidence angle, even vapour-induced relative variations of $\varepsilon^{(\mathrm{air})}$ as small as 0.1% can lead to variations of the spatial and angular GHS of the order of 25% and 200%, respectively. The estimated sensitivity of that technique for spatial and angular GHS is $4{\cdot}10^3$ µm and $1.8{\cdot}10^3$ degrees per unit of $\varepsilon^{(\mathrm{air})}$, respectively.

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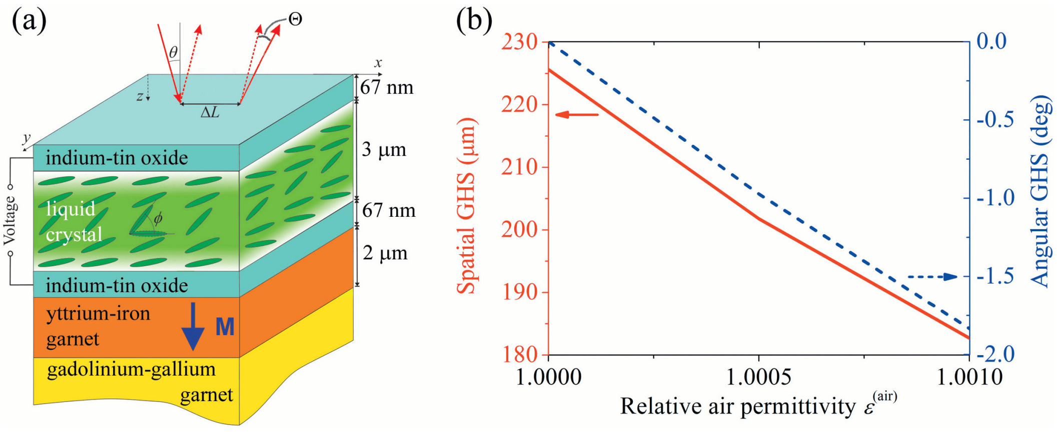

Fig. 1: (a) Schematic of the structure. θ is the incidence angle, ϕ is the tilt angle of the LC molecules, **M** is the saturation magnetization in magnetic film. The spatial and angular GHSs are ΔL and Θ. (b) Variation of the GHSs for $\varepsilon^{(\mathrm{air})}$ varying from 1.000 to 1.001.

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Electric current induced amplification of slow surface plasmon polaritons in semiconductor-graphene-dielectric structure

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Surface waves propagating along the boundary between two media, one of which has a negative dielectric permittivity, are called surface plasmon polaritons (SPPs). The use of materials with negative permittivity unavoidably leads to ohmic loss. The loss compensation techniques, based on the optical-pumping-generated induction of population inversion in the active medium located near the surface of the metal are characterized by low power efficiency, require an external laser, and only work in pulsed mode, which does not allow to count on their wide practical use. The alternative approach is based on mechanism of energy transfer from plasma oscillations sustained by direct current, to SSP electromagnetic waves. The evanescent SPP wave amplification in the structure with d.c. current can be observed when the SPP phase velocity and the charge drift velocity are comparable. To obtain this phase matching condition, in this communication we suggest to use the graphene layer placed on the planar interface where the SPP propagates, i. e. between a semiconductor film and a dielectric substrate (Fig. 1(a)). We show that under the phase matching condition, a slow SPP wave can be substantially enhanced by the drift current in graphene [1]. The amplification coefficient α can reach huge values which are orders of magnitude larger the ohmic loss coefficient β'' (Fig. 1(b)).

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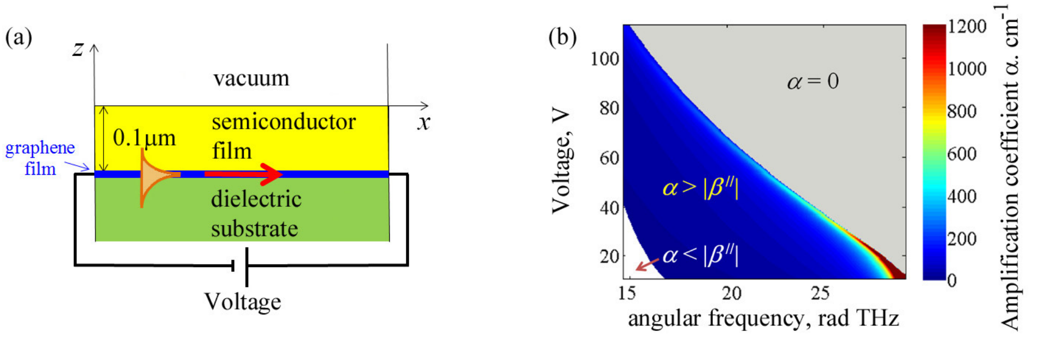

Fig. 1: (a) Schematic of the structure: a semiconductor film with deposited graphene is placed between semi-infinite dielectric substrate and vacuum. The arrow shows the directions of SSP wave propagation and electrons flux in graphene under an applied voltage. (b) Amplification coefficient $\alpha > |\beta''|$ of SPP as function of the wave angular frequency and applied voltage. The grey and white areas correspond to $\alpha = 0$ and $\alpha < |\beta''|$, respectively.

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Reconstruction of acoustic wave field from boundary measurements

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Consider the following initial boundary value problem for the wave equation

$$u_{tt} - u_{xx} - u_{yy} = 0, \quad (x, y) \in \mathbb{R}^2_+, \ t \in \mathbb{R}, \tag{1}$$

$$u|_{y=0} = 0, \quad u|_{t=0} = u_0, \quad u_t|_{t=0} = 0, \tag{2}$$

where $\mathbb{R}^2_+ := \{(x, y) | \, x \in \mathbb{R}, y \geq 0\}$, and $u_0 \in C^3(\mathbb{R}^2_+)$ satisfies $u_0\big|_{y=0} = (u_0)_{yy}\big|_{y=0} = 0$. We deal with the observation problem, in which the function u_0 is to be determined from the function $v := u_y\big|_{y=0}$ restricted to a bounded subset U of the plane $\{y = 0\}$. Taking into account the Dirichlet boundary condition, the function v determines the Cauchy data on U. Reconstruction of the solution u (which is equivalent to reconstruction of the corresponding initial data u_0) from the Cauchy data given on the part of the plane $\{y = 0\}$ is known to be an ill-posed problem. We propose an algorithm, which allows determining $u_0(x_0, y_0)$, $y_0 \geq 0$, provided that the function v is given on the set

$$U := \Big\{(x, t) \, \Big| \, |x - x_0| \leq D\Big(\sqrt{y_0^2 - t^2}\Big), \, 0 \leq t \leq y_0\Big\}, \quad D(z) := \frac{z}{\sqrt{1 + 2\alpha z}}, \quad \alpha \geq 0.$$

Namely, the following relation holds true

$$u_0(x_0, y_0) = \lim_{h \to 0+} \int_U K_h(x - x_0, t) \, v(x, t) \, dx dt, \tag{3}$$

where the kernel K_h is defined as follows

$$K_h(x, t) := \frac{1}{\pi^{3/2}} \operatorname{Re} \left[\frac{1}{\sqrt{h(1 + i\alpha x)}} \int_0^{\pi/2} \exp\left(-\frac{1}{4h(1 + i\alpha x)} \Big(x + i\sqrt{y_0^2 - t^2} \cdot \sin s \Big)^2 \right) ds \right]$$

(we assume $\mathrm{Re}\sqrt{1+i\alpha x} > 0$). The set U depends on the coordinates x_0, y_0, and the parameter α, which means that one can use the Cauchy data given on various sets to determine $u_0(x_0, y_0)$. In this respect, the problem in consideration is close to the the analytic continuation problem.

A constructive solution of the observation problem for the wave equation was first proposed by R. Courant. In contrast to (3), the scheme of R. Courant involves the derivatives of all orders of the data. The paper by A. S. Blagoveshchensky and F. N. Podymaka [1] deals with the observation problem for a more general initial boundary value problem than (1)–(2). Though three-dimensional case was considered in [1], we point out that the inversion formula obtained there provides a reconstruction of the solution, which is more stable than reconstruction by relation (3). However, this inversion formula requires the data on the entire space-time boundary rather than on a compact set.

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The method of integral equations to analyze carbon nanotubes of arbitrary geometry

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Significant amount of studies is dedicated to investigating electrodynamic characteristics of carbon nanotubes (CN) (see [1–3] for example). And in the majority of the studies the analysis of electrodynamic characteristics is done using Hallen integral equation or Leontovich–Levin integral equation. Such an approach allows to consider either single rectilinear CN as vibrators, or the structures that consist of such vibrators.

However, when investigating the structures composed of arbitrary oriented CN, which can also have complex configuration, and when solving the tasks of electromagnetic wave scattering by CN, it would be more appropriate to use Pocklington integral equation (PIE) [3] to build a mathematical model. At the same time, and for the case when PIE is used, the authors [3] limited their investigation to analyzing rectilinear structures based on CN.

In some studies it is noted, that different ways of synthesizing CN cause the significant amount of CN with the morphology other than rectilinear to appear. For instance, in the process of catalytic cracking of acetylene the significant amount of twisted CN and CN in form of helices with different diameter and pitch distance are present besides rectilinear ones [4].

To calculate phase-amplitude current distribution of a CN, PIE is suggested to be used for arbitrary geometry [5]. Test calculations were carried out and compared to the data in [1, 2, 6]. As the result, the influence of CN geometry on current distribution nature and resonance properties were estimated.

The suggested algorithm within the frames of antenna approximation allows to carry out the calculations of current distribution for a CN of arbitrary geometry, and, correspondingly, calculate constitutive parameters of a composite material, that contains similar nanotubes.

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Investigation of asymmetrical metasurface based on photonic nanojet

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Optically asymmetric metasurfaces are structures that pass light in one direction, but block it in another. These structures are remarkable for practical application and, for example, can be used as light-trapping layers for thin film solar cells [1].

This paper presents a numerical and experimental study of asymmetric metasurface based on photonic nanojet effect. Forward transmission in optical band for numerical calculations is about 90% [2], while in experiments forward transmission is 80∼85%. Photonic nanojet effect was observed on SiO_2 spheres with diameter 1.5 µm. The spheres were deposed by spin coating on 300 nm gold layer with hexagonally arranged cylindrical holes, which are made by electron-beam lithography. The holes diameter is about 500 nm.

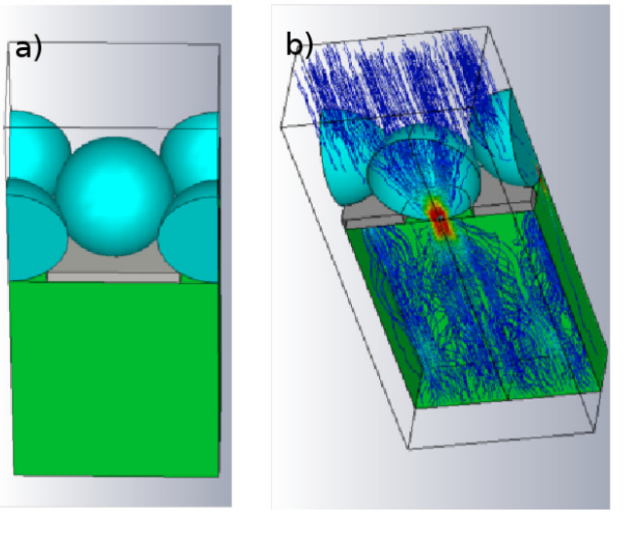

Fig. 1: (a) A unit cell of the structure and (b) Pointing vector in the case of forward transmission [2].

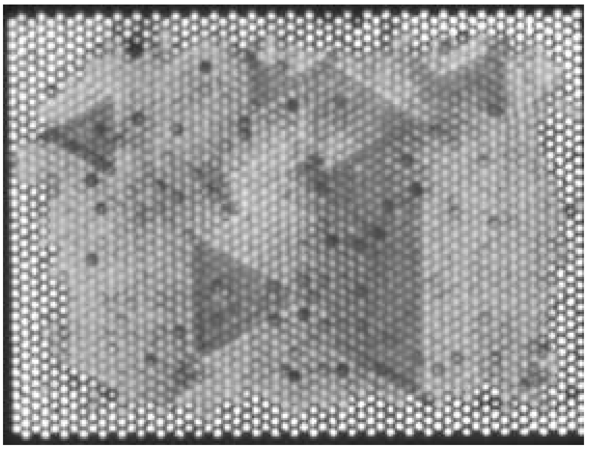

Fig. 2: Image of the metasurface in transmitted laser light (405 nm). The transmittance measurements and imaging were made using LSM710 (Zeiss) laser scanning.

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The Whitham equation with capillarity

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The validity of the Whitham equation as a model for surface gravity waves of an inviscid incompressible fluid is under consideration. Making use of the Hamiltonian structure of the free-surface water-wave problem and the analyticity of the Dirichlet–Neumann operator, we derive a nonlocal Hamiltonian system. Obtained equations describe H long surface waves and meanwhile are fully dispersive in the linear part. They can combine gravitational and capillary or elastic effects. By restricting to one-way propagation, the system reduces to the modified Whitham equation taking into account capillarity or elasticity. The elasticity Whitham equation can be used for describing displacement of an ice sheet covering liquid, for example. The accuracy of unsteady solutions of the capillary Whitham equation and other equations like the KdV and Kawahara is compared with the Euler system solutions in different scaling regimes [1]. Performance of the elasticity Whitham model is carried out for steady solutions [2].

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Nonstandard characteristics in asymptotics of linear problems with localized initial data, and applications to crystal lattices and water waves

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Characteristics in linear problems describe the propagation of singularities of hyperbolic systems. They are defined as solutions of Hamiltonian systems with Hamiltonians homogeneous in the momenta. There are quite a few interesting physical problems with small parameters having rapidly oscillating solutions, which are described by other type of characteristics appearing in the framework of ray expansions, WKB method, and their generalizations like the Maslov canonical operator and Fourier integral operators. From the viewpoint of applications, it is of interest to consider problems whose solutions include both singularities and rapid oscillations. In this case, there arise *nonstandard characteristics* (the term is due to Maslov [1]). My talk discusses nonstandard characteristics arising in the linear equations of crystal lattices and water waves and their applications to the construction of asymptotic solutions of Cauchy problems with localized initial data. We also discuss the possibility of using van Vleck type formulas in these cases, some ideas of V. A. Fock and V. A. Borovikov, and recent results [2] giving an efficient description of waves near leading edge front, the appearance of the linearized Boussinesq equation, geometric objects associated with this asymptotics [3, 4], relationship with the previous results [5–7], etc.

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Uniform asymptotics of boundary values of the solution of a linear problem on the run-up of waves on a shallow beach

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We consider the Cauchy problem with a spatially localized source for a two-dimensional wave equation with variable velocity degenerating as the square root of the distance from the boundary. This problem arises in the theory of tsunami wave run-up on a shallow beach.

S. Yu. Dobrokhotov, V. E. Nazaikinskii, B. Tirozzi in [1] developed a method for constructing asymptotic solutions of this problem. The method is based on a modified Maslov canonical operator on (nonstandart) characteristics unbounded in the momentum variables. It was shown in [2] that the restriction of the asymptotic solutions to the boundary is determined by the standard canonical operator.

We show that the boundary values of the asymptotic solution constructed by Dobrokhotov and Nazaikinskii in [2] approximate the boundary values of the exact solution in the uniform norm.

This work was supported by the Russian Science Foundation (project no. 16-11-10282).

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Density of photonic states and dispersion of light in one-dimensional liquid-crystalline photonic crystals

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Liquid crystals form many one-dimensional and three-dimensional photonic-crystalline structures composed by orientationally ordered chiral molecules. Unique optical properties, potential for applications of photonic liquid crystals, ability to controllably modify their spectral characteristics explain growing interest in these structures from experimentalists and theorists. Among most interesting characteristics of chiral photonic liquid crystals are unusual properties of eigenmodes of light propagating in the photonic crystal and their strong dependence on polarization of light.

For description of photonic crystals a fruitful approach is usage of universal Kramers–Kronig relations and the notion of density of photonic states [1, 2]. Analysis of the photonic structure is performed on the basis of complex measurements: transmission, reflection spectra, rotation of the plane of polarization of light, measurements of polarized fluorescence spectra in the vicinity of the photonic stop band. Fluorescent dopants with different orientation of the dipole moment in transition with respect to the local director of liquid crystal were used. Dispersion of light in the photonic crystal is reconstructed from measurements. Comparison of the density of photonic states with experimental fluorescence spectra shows that positions of maxima in the fluorescence correlate with the maxima in density of states but their relative intensities are governed by the orientational ordering of the dopant.

Fluorescence spectra are modeled on the basis of approach taking into account the structure of eigenmodes of light and degree of orientational ordering of dye molecules. Calculated spectra show good agreement with experiment both in the region of the photonic band and in its vicinity. We discuss the modification of the photonic structure and photonic density of states related to finite thickness of the sample and the transformation of the photonic band gap into the pseudogap. The results of our studies demonstrate that universal approaches can be effectively applied for the description of optical properties of liquid-crystalline photonic crystals.

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Long-period polar structures in smectic liquid crystals

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Liquid crystals are formed by orientationally ordered anisotropic organic molecules. Polar smectic liquid crystals have layered structure and can be characterized by a two-component order parameter. Modulus of the order parameter corresponds to the tilt of long molecular axes; phase of the order parameter is the azimuthal angle of orientation of molecules in the layers [1]. Recently it has been shown that smectic liquid crystals form a number of polar phases with multilayer periodicity [2–4]. The primary tool used to study these polar phases has been resonant X-ray diffraction. In spite of essential progress in experiment and theory [1–5], many questions related to polar smectic structures, in particular the nature of interactions responsible for their appearance still remain open.

We employ Landau theory of phase transitions with two-component order parameter to describe the manifold of polar smectic phases [6–9]. It is known that short-range interactions between nearest and next nearest layers induce phases with period $n \leq 4$ layers. An essential feature of the dependence of phase diagrams on short-range interactions is the existence of degeneracy points, where energies of a large number of phases are equal. In such a situation small changes can lift the degeneracy and lead to appearance of new structures. One of the ways of lifting the degeneracy is change of the order parameter modulus from layer to layer, which decreases the energy of a number of phases. Degeneracy can also be lifted by interactions of longer range with a constant order parameter modulus. We considered both these possibilities in our calculations. Structures with long periods were obtained. Analysis of the experimental data enables to indicate which physical reasons and interactions are responsible for appearance of structures with long periods.

We discuss peculiarities of X-ray diffraction from different long-period phases depending on the employed interactions and spatial variation of the two-component order parameter. Possibility to observe diffraction peaks with resonant and non-resonant character is analyzed. We propose experiments aimed at establishing the details of structure of polar phases and elucidating the physics behind it.

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Optoelectronic system for fingerprint recognition

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In the modern world biometric recognition acquires the increasing relevance. Identification by fingerprint recognision appears to be the most handy. Despite advantages (simplicity, compactness, ample opportunities on integration) this method has a major problem which is a facility of falsification [1]. For example the commercial fingerprint reader could be easily spoofed by silicone dummy model [2]. The distinguishing capability and falsification resistance of dactyloscopic scanners are possible to be increased by combined optoelectronic methods.

In this work the opportunities of simple optical scanner combination with second channel for high confident biomedical identification is considered. Different techniques can be applied for second biometric channel scheme, the most perspective ones are presented to be:

- raman spectroscopy;
- speckle-field analysis [3];
- polarization image visualization or polarimetry [4];
- optical coherence tomography.

The polarimetry channel was tested in this work. This technique is based on the analysis of backscattered polarised radiation spatial distribution. Scattered light is estimated for various polarisations and positions of a light source. Images detected in various polarizations contain information about internal structure of the disseminating object. As the internal structure cannot be forged, the polarimetry technique provides high falsification safety of the fingerprint detector. In the experiments samples of real tissues and model sites were examined. Images of internal structure in different polarizations were received. By applying a formalism of Mueller–Stokes matrixes information about biometric parameters, for example, about the size, form and orientation of the disseminating inhomogenuity can be studied to differ complex organic compounds from real tissue. The results received in this work argue for efficiency of polarimetry technique as the second biometric channel in the scheme for fingerprint recognition.

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Optical properties of nematic liquid crystals confined to nano(micro)pores of polymer films

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Recently, complex systems based on porous polymeric films filled with liquid crystals (LCs), have been proposed to control the propagation of THz electromagnetic radiation frequency range [1, 2]. Optical properties of such complex systems depend on orientational structure (director configuration) of LCs within the pores which defines by a chemical interaction of liquid crystal molecules with the aligning surface, the anchoring strength and elastic parameters of LC material. Here we report the results of experimental investigation of optical properties of nematic 5CB confined to the pores of thin polyethylene terephthalate (PET) films with (without) surface treatment. In our experiments we used PET films [3, 4] of a thickness $h = 23$ µm and a pore diameter $d = 0.1 \ldots 5$ µm, characterized by cylindrical open-end and randomly allocated pores, oriented normally to the film's surface. The porous films were preliminary treated by a chromium stearyl chloride for homeotropic alignment of LC molecules on the pore walls. Both treated and non-treated films were studied. The PET films were filled with nematic LC 5CB in the isotropic phase after degassing and then slowly cooled to a room temperature. Polarized optical microscopy with 1000x magnification was performed to study optical textures of 5CB within highly anisotropic porous films ($\Delta n = 0.31$) with large enough diameter of the pores ($d = 5$ µm). Analysis of 5CB texture within the films treated by chromium stearyl chloride revealed a formation of the escaped-radial configuration with point defects (ERPD), whereas non-treated films showed a non-uniform distribution of the director within such pores.

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Interaction of elastic guided waves with localized three-dimensional thickness changes in plate-like metallic structures

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The detection and characterization of abrupt localized thickness changes in metallic plate-like structures caused by material degradation through corrosion and errosion are among essential tasks for Structural Health Monitoring (SHM) systems [1]. Ultrasonic elastic guided waves (GW), being already a recognized tool in active SHM due to the their long-range propagation ability and sufficient sensitivity to localized changes in waveguide geometry or material properties, are among the potential solutions of this problem. Though the GW diffraction by a variety of damage types is extensively studied both numerically and experimentally, multiple boundary and structural element reflections, as well as internal or external noise sources, sufficiently complicate damage evaluation. Therefore, it is of great demand to investigate any specific damage characteristics that could enhance the quantification opportunity with SHM systems. Among such features mode conversion phenomena and GW resonance interaction with localized obstacles are of particular interest for the considered type of damage. While the former might indicate the presence of one-side material thickness loss, the latter makes possible the precise characterization of defect severity [2].

In this talk the results of theoretical and experimental investigation into GW propagation and diffraction by localized three-dimensional thickness changes simulating pitting corrosion in a metallic plate-like structure are going to be presented and discussed. Numerical evaluation of the considered problem relies on semi-analytical models of various complexity ranging from first-order plate theories [3] to the general equations of three-dimensional linear elastodynamics solved with laminate element method [4]. Experimental investigations are carried out for metallic specimens with artificial obstacles. Contactless laser Doppler vibrometry is utilized for the wave propagation sensing and visualization, while GW are excited by surface mounted piezoelectric wafer active sensors. The obtained experimental data confirm the intensity of the simulated GW mode conversion phenomena and verify the predicted values of the resonance frequencies, reveling their strong dependence on the obstacle severity. For example, a slight change of the notch depth results in a remarkable shift of the corresponding resonance frequencies, making them useful for the precise evaluation of the corrosion intensity.

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Analysis of generalized separation of variables method in electrostatics

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For the problem of light scattering by small axisymmetric particles, we construct the Rayleigh approximation in which the polarizability of particles is determined by a generalized separation of variables method. In this case, the electric-field strengths are the gradient of some scalar potentials, which are represented by expansions in term of the eigenfunctions of the Laplace operator in the spherical coordinates. As the separation of variables in the boundary conditions is incomplete, the problem is reduced to an infinite system of linear algebraic equations relative to the unknown expansion coefficients. We have examined the asymptotic behavior of the system elements at large values of their indexes. It has been shown that the necessary condition of the solvability of the system coincides with the condition of correct application of the extended boundary conditions method (T-matrix method). We have performed numerical calculations for Chebyshev particles with one maximum (Pascal's snails). The obtained results for the asymptotic expressions for the system elements and for the T-matrix support our theoretical prediction. We have shown that the scattering and absorption cross sections of the examined particles can be calculated in a wide range of parameter values with an error of about 1–2% using a simple spheroidal model. Such a model is also applicable when the condition of solvability of the system for non-convex particles is violated; in this case, the SVM should be considered as an approximate method, which frequently ensures obtaining results with an error less than 0.1–0.5%.

Quasiclassical asymptotics of solutions to difference equations with meromorphic coefficients

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Consider the difference Schrödinger equation

$$\psi(z+h) + \psi(z-h) + v(z)\psi(z) = E\psi(z), \tag{1}$$

where z is the complex variable, v is a given function, and $h > 0$ is a constant translation parameter. One encounters such equations, for example, in solid state physics. Note that the equation with $v(z) = \cos(z)$ is the famous Harper equation, see, e. g. [1], and the equation with $v(z) = \tan(z)$ is a close relative of the well-known Maryland equation introduced by D. Grempel, S. Fishman and R. Prange in [2].

When studying an electron in a crystal in a weak magnetic field, one arrives to a difference equation with the translation parameter h proportional to the magnetic flux through the periodicity cell, and thus, this h is small. In applications, there are other problems, where h is small, and we study the asymptotics of solutions to (1) as $h \to 0$. As formally $\exp\left(h\frac{d}{dz}\right)\Psi(z) = \Psi(z+h)$ the parameter h in (1) can be regarded as a small parameter in front of the derivative and, thus, appears to be a standard quasiclassical parameter.

To study the one dimensional differential Schrödinger equations in the quasiclassical limit, one uses the classical complex WKB method, see [3]. In [4, 5] the authors developed an analog of the complex WKB method to study one-dimensional difference Schrödinger equations with analytic coefficients. Now, we consider the case, when v has (simple) poles.

Assume that v has a pole at a point z_0. Then, even if ψ is a solution to (1) analytic at z_0 and to the left of z_0, the equation implies that ψ can have poles at the points $z_0 + nh$, $n = 1, 2, 3\dots$ If h is small these poles become close one to another. In the talk, we describe the asymptotics of meromorphic solutions to (1) as to the right of z_0 so in its neighborhood.

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Berry phase for difference equations

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We study solutions to the difference equations of the form

$$\Psi(z+h) = M(z)\Psi(z), \quad z \in \mathbb{C}, \tag{1}$$

where $M : \mathbb{C} \to SL(2, \mathbb{C})$ is a given matrix valued function, and $h > 0$ is a small constant parameter.

It is worth comparing (1) with the differential equation

$$h\frac{d}{dz}\Psi(z) = M(z)\Psi(z), \quad z \in \mathbb{C}. \tag{2}$$

As $h \to 0$, its solutions admit quasiclassical asymptotics described in the framework of the complex WKB method, see [1].

Since formally $\exp\left(h\frac{d}{dz}\right)\Psi(z) = \Psi(z+h)$ the parameter h in (1) can be regarded as a small parameter in front of the derivative and, thus, appears to be a standard quasiclassical parameter.

In [2, 3] the authors developed an analog of the complex WKB method for one-dimensional difference Schrödinger equations. Now, we extend this method to equation (1) for vector functions. As for (2), the asymptotics of solutions to (1) appear to contain a sort of Berry phase that we discuss in this talk.

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Asymptotic analysis of the waves with the negative group velocity in cylindrical shell

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The problem of joint oscillations of infinite thin cylindrical shell of Kirchhoff–Love type filled with acoustical liquid is considered [1]. The free harmonic vibrations of the system are founded. The propagating waves are analyzed. Much attention is given to exploration of waves with negative group velocity in the neighborhood of bifurcation point of dispersion curves. The asymptotic of dispersion curves are used in the neighborhood of bifurcation point for this case. The diapason of frequencies and wavenumbers where this effect observed is also estimated. The difference in kind of asymptotic for the regular case and for the case of bifurcation is discussed. The dependence of processes on the relative thickness of the shell and other parameters of system is viewed. The possible fields of applicability of the gained effects are established.

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Wave motion in functionally graded piezoelectric layered phononic crystals with electroded surfaces

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Over the past decades, considerable research efforts have been made in the field of the mechanics of piezoelectric composites with their increasing usage in various applications, especially for smart structures or intelligent systems. Acoustic metamaterials and phononic crystals (PnCs) are artificial composites of periodic structure engineered to have advanced properties arising from thorough matching of components and design of the internal structure. Phononic crystals can be used in order to manipulate wave propagation in piezoelectric and elastic structures due to their unique properties like the effect of the signal absorption or the refraction in certain frequency ranges known as band-gaps or stop-bands. Wave propagation characteristics of piezoelectric materials can be conveniently controlled by using the electric field, e. g. with a system of electrodes.

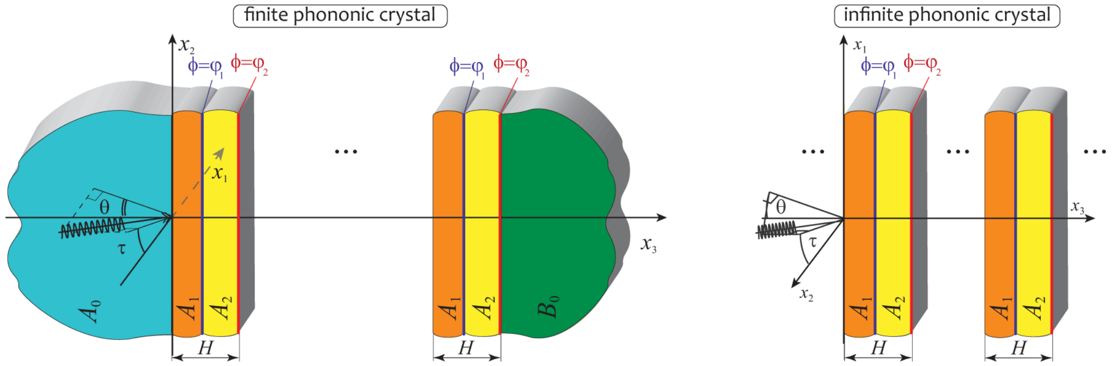

Fig. 1: Layered piezoelectric phononic crystal with electroded interfaces.

Plane wave excitation and propagation in layered piezoelectric phononic crystals with electric potentials at the metallic interfaces and with functionally graded interlayers is studied in the present work. Two kinds of piezoelectric layered phononic crystals are considered: an infinite periodic structure [1] and periodic structure surrounded by two half-spaces [2] (Fig. 1). The elastic plane wave transmission caused by the electric potentials in the PnCs as well as wave excitation by the electrodes are investigated using Bloch theorem and the transfer matrix method. The analysis of energy transmission coefficient, dispersion curves of Bloch waves and parametric study of band structure are presented.

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Radiation of a charged particle bunch exiting an open-ended circular waveguide within regular vacuum waveguide

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In this report, we deal with the case when a point charge or charged particle bunch exits the open-ended cylindrical waveguide placed inside collinear regular waveguide with larger radius. We consider both vacuum case and the case with dielectric loading of the inner waveguide. Open-ended waveguide structures with dielectric loading excited by specially prepared electron bunches are considered as promising candidates for development of contemporary sources of Terahertz (THz) electromagnetic waves [1]. Although open-ended waveguide structures were actively investigated [2, 3], the cylindrical geometry with dielectric filling in the case of excitation by a moving charge was rarely analyzed, excluding recent papers of the authors of this report [4–7]. Rigorous solution of these problems in the frequency domain is constructed using residue-calculus technique and analytical expressions for coefficients of mode excitation in each subarea are obtained. In the vacuum case, the time domain dependencies of the total field are calculated using numerical integration. We note that "the image" of the bunch is generated in the coaxial area of the structure. In the case with dielectric, the portion of the total field connected with Cherenkov radiation (excited in the inner waveguide) is analytically extracted and corresponding time dependencies are presented. We also compare the obtained results with the results of direct 3D simulation.

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Numerical solution of eigenproblems for linear Hamiltonian systems and their application to non-uniform rod-like systems

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The natural frequencies identification problem for transverse oscillations of one-dimensional rod-like systems (e. g. Euler–Bernoulli, Rayleigh and Timoshenko beam models, pipeline models) can be represented as an eigenvalue problem for a fourth order differential equation. For mechanical systems with distributed parameters, the solution procedure can be rather sophisticated and the accuracy of the found eigenvalues is not always easy to guarantee [1, 2]. In addition, for hybrid systems (e. g. pipelines) the problem becomes nonlinear in the spectral parameter.

Here, a method for numerical solution of eigenproblems for linear self-adjoint Hamiltonian systems with nonlinear dependence on the spectral parameter is presented. It can be applied to foregoing self-adjoint boundary value problems for fourth order differential equations for natural frequencies finding. The method is based on a Newton-type iterative procedure with a spectral correction on each step. It provides a two-side estimate for the eigenvalue and has a quadratic convergence rate. The method is a generalization of the method of accelerated convergence for scalar [1] and vector [3] Sturm–Liouville problems with a nonlinear dependence on the spectral parameter.

Figure 1 shows an example of computations of the first eigenvalue $\lambda = \frac{\rho\omega^2 l^2}{E}$ for the Timoshenko beam model for a circle rod with hinged ends. Here, the rod has a variable cross-section $r = r_0(1 - \gamma x^2/l^2)$, $r_0/l = 50$, and constant physical parameters E, G, ρ, κ; l is the rod's length, r is the radius, E is the Young's modulus, G is the shear modulus, ρ is the density, $\kappa = \frac{6(1+\nu)^2}{7+12\nu+5\nu^2}$, $\nu = 0.35$ is the Poisson's ratio, and ω is the natural frequency. In the figure, the scaled eigenvalue $\lambda_1 = \frac{4\lambda l^2}{\pi^4 r_0^2}$ is represented as a function of γ coefficient, $0 \le \gamma \le 0.97$.

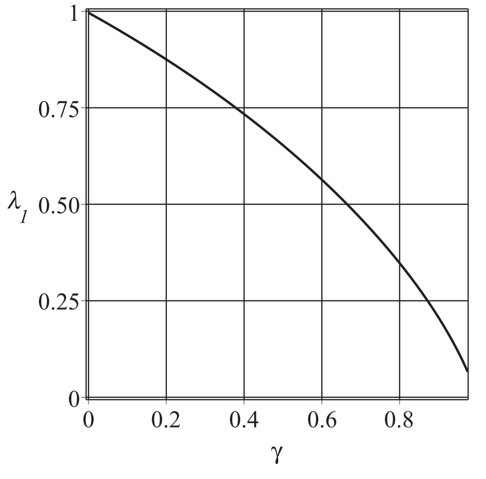

Fig. 1: First eigenvalue behaviour.

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Evolution of a trapped mode of oscillation in a string on the Winkler foundation with point inhomogeneity

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We consider a mechanical system with mixed spectrum of natural oscillations. This is an infinite string on the Winkler elastic foundation with point inhomogeneity (the spring with negative stiffness). The governing equation is the Klein–Gordon equation

$$T_0 u_{xx} - \rho u_{tt} - k_0 u = -P_0(t)\,\delta(x), \tag{1}$$

$$P_0(t) = K_0 u(0,t) + p_0(t). \tag{2}$$

Here x and t are the position and time, respectively; u is the displacement, T_0 is the string tension, $P_0(t)$ is the force on the string from the point spring, $p_0(t)$ such that $p_0(t) \equiv 0$ for $t > t_0 > 0$ is the external force on the inclusion, $-K_0$ (where $K_0 > 0$) is the spring stiffness.

It easy to show that for $K_0 = \text{const}$, $p_0 = 0$ this system in a certain domain of the problem parameters can have a mixed spectrum of natural frequencies of oscillation. Frequencies above the waveguide cutoff frequency $\Omega_* = \sqrt{k_0/\rho}$ are associated with sinusoidal running waves. Such frequencies form a continuous spectrum. A specific feature of systems with discrete inclusions is that there can exist a discrete spectrum of natural frequencies of oscillation below the cutoff frequency. The localized (or trapped) modes of oscillation corresponding to them possess a finite energy, and forms are localized in a neighborhood of inclusions. In the system under consideration there exists one trapped mode of oscillation. The corresponding frequency is $\Omega_0 > 0$.

In the paper, we use asymptotic procedure analogous to the one suggested in [1, 2] for different problems and based on the method of multiple scales [3], to construct the solution of (1), (2) together with zero initial conditions $u\big|_{t<0} \equiv 0$ in two different cases. The first case is the spring with slowly varying stiffness $K_0 = K_0(\epsilon t)$, the second one is the string with slowly varying tension $T_0 = T_0(\epsilon t)$. Here ϵ is a formal small parameter. We show that for the weakly non-stationary system the amplitude of localized oscillation is proportional to $\frac{(\Omega_*^2-\Omega_0^2)^{1/4}}{\Omega_0^{1/2}}$ in the first problem, and $\frac{(\Omega_*^2-\Omega_0^2)^{1/4}}{(c\Omega_0)^{1/2}}$ in the second problem. Here $\Omega_0 = \Omega_0(\epsilon t)$, $c = c(\epsilon t) = \sqrt{T_0(\epsilon t)/\rho}$.

In the first case, when we consider the point spring with slowly varying stiffness, equation (1) is a PDE with constant coefficients. This allows one to verify the constructed solution by independent numerical calculations. The comparison demonstrates a good mutual agreement.

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Radiation illusion at the arbitrary location

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The emission illusion device based on metamaterials with radial and angular dependent tensors of permittivity and permeability is analyzed and numerically simulated in this work. The illusion effect was numerically simulated using finite element method for a point source.

Nowadays, many investigations have been directed to develop variety devices with extraordinary properties such as beam splitters, exotic lenses, cloaks, illusion devices, etc. using transformation optics [1, 2]. The mirage effect creating [3, 4] is of particular interest for communication systems or in order to cheat radar detector.

This work main idea is aimed to virtually modify the spatial location of a emission source. An observer will have the impression that the emitter is located elsewhere instead of its real physical location. The principle of the proposed transformation allows modifying the location of the source as shown in Fig. 1. Indeed, when the emission source with coordinates (ρ_1, φ_1) is placed in the transformed material shell, an observer will see a virtual image of source to appear at another predetermined position (ρ_2, φ_2).

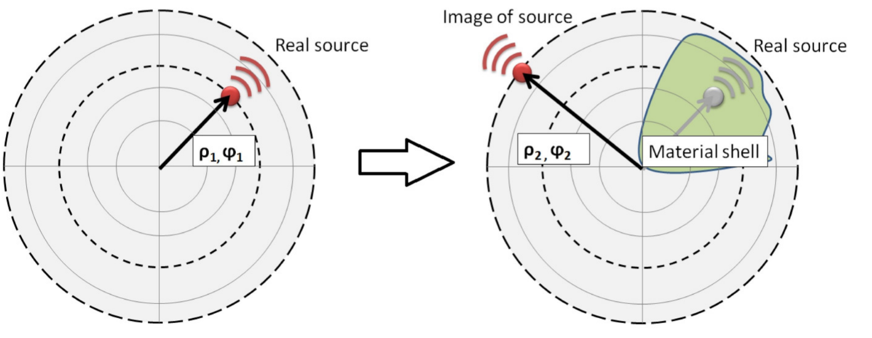

Fig. 1: Principle of transformation with virtual delocalization of emission source at the another radial and angular position.

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Backward leaky wave phenomena and energy fluxes in the coupled system: ultrasound transducer – acoustic fluid – immersed plate

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Interaction of acoustic wave beams generated by an ultrasound transducer with a fluid-immersed plate is considered. The investigation of wave energy transfer from the source to the plate and further to infinity is carried out within the analytically based computer model presented at the preceding DD conferences [1]. The model is based on the explicit integral and asymptotic representations derived for the bulk, traveling, and leaky waves excited by a point source (Green's function), which is convoluted with a distributed load simulating the transducer's radiating element. The source energy partition among the generated waves (incident, transmitted, reflected and guided) is evaluated using closed-form representations for time-averaged wave energy transfer through arbitrary plane-horizontal, spherical and cylindrical surfaces. In addition, to visualize the trajectories of energy fluxes, energy streamlines specified by the power density vector field are traced from the source into the environment including the lines passing through the immersed plate.

The present research is focused on the backward leaky mode phenomenon [2] and the effect of increasing energy transmission through the plate at those frequencies [3]. Due to the fluid environment, the real wavenumbers of classical free-plate Lamb modes shift into the complex plane and traveling waves become guided leaky waves with the logarithmic decrements proportional to their imaginary parts. The backward mode range is featured by inverse energy fluxes coming from infinity along the plate. In the near field, they are turned away by the more powerful energy flux radiated from the source. The numerical results for wave transmission through the plate obtained within the low-cost semi-analytical model developed coincide with those calculated by FEM and experimentally measured [3]. In addition, energy streamlines and plots of energy density show specific ways of increased energy transfer through the plate at the resonance frequencies.

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Resonance and regular elastic wave diffraction by planar delaminations in multilayered structures

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Nowadays composite materials are very widely exploited in aerospace, aircraft, building construction as well as in high-performance products due to their enhanced properties. Crack analysis becomes a natural task for engineering applications as soon as composite structures may sustain defects and flaws known as delaminations. In the 90s of the last century the potential of the methods based on guided waves for damage detection in composite materials has been demonstrated. Nowadays application of guided waves has become very common: many publications can be found in literature on this topic. The aim of this work is to study resonance and regular elastic wave diffraction by planar delaminations in multilayered structures.

The total wave field in a multilayered structure is a sum of an incident field and a scattered field. The latter is a superposition of the wave fields scattered by all the delaminations. The incident wave field is calculated using the integral approach [1]. The boundary integral equation method and Galerkin scheme are used in order to describe wave fields scattered by the delaminations [2]. Wave resonances are usually related to the wave energy capturing and localization, which also shows resonance blocking or, in some cases, wave transmission. The wave energy flow vector and the corresponding energy streamlines are visualized to obtain a better and deeper understanding of the wave propagation, resonance, localization and focusing phenomena in a layered structure with interior or interface cracks [3]. Numerical examples are presented and discussed to show the effects of frequency, crack size, crack number, crack spacing and location, as well as other material and structural parameters on the wave propagation characteristics and resonances in a layered waveguide.

Examples of typical wave resonances are presented. If the location or the size of the delaminations is shifted from the resonant one, then a usual wave scattering is observed. The energy vortices then do not close the waveguide for the energy flow and bend around them, so the energy flow of the elastic waves can thus pass all the vortices and the delaminations [4].

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Plane wave propagation in multilayered elastic structures with doubly periodic array of planar delaminations

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The scattering of elastic waves by damages is of great importance for non-destructive evaluation and structural health monitoring, where ultrasound is used to detect defects. At the same time cracks or slits can be artificially introduced into structure in order to change scattering properties or focus elastic wave energy [1]. A delamination model based on the spring boundary conditions [2] is a more general model than an open crack model and it can be used for simulation of imperfect contact zone between media. Imperfections can be also modelled as a periodically [3] or randomly distributed delaminations [2].

In the present work planar doubly periodic array of delaminations situated at the interface between two dissimilar elastic half-spaces is considered. The damaged interface is artificially separated into unit-cells with a reference unit-cell that is centered in the origin of coordinates. Time-harmonic plane elastic wave propagation is considered. The displacement vector satisfies the governing Lame equation. The boundary conditions at the delamination are the spring boundary conditions. The solution is obtained using the boundary integral equation method and the integral approach [4]. The total wavefield is a superposition of the incident field and the scattered field expressed in terms of the two-dimensional Fourier transform [2]. According to the Floquet–Lyapunov theory, the crack opening displacements (CODs) for each delamination is expressed by the COD at the reference delamination. Substitution of the integral representation for the traction vector into the boundary conditions leads to the boundary integral equation at the delamination situated at the reference unit-cell. The kernel of the integral equation is evaluated using relation of an exponential series and a sum of Dirac delta function and the equation is solved via Bubnov–Galerkin method [1].

The damaged interface is also modelled as a stochastic (random) distribution of cracks using the approach described in Ref. [2]. It assumes that the spring model and the random distribution of cracks give the same transmission coefficient. The equalities of the transmission coefficients for both models give the normal and transverse effective spring stiffnesses. The spring stiffnesses estimated for stochastic distribution of circular cracks are compared with corresponding stiffnesses for periodic hexagonal distribution of cracks [3]. Influence of the kind of crack distribution and their shapes on spectral properties, resonance and regular transmission is carefully analyzed.

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Numerical analysis of propagation of x-ray whispering gallery waves along liquid meniscus

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Whispering gallery is one of the most interesting wave phenomena. It has been known from antique architectural acoustics and means wave propagation along curved surfaces with high intensities [1]. We analyze, for the first time, diffraction of x-ray whispering gallery waves which propagate along the large-radius meniscus of deionized water or silicic hydrosols. Using exact solution of the Helmholtz equation [2], we define the reflection coefficient and intensity distribution of x-ray radiation turned on a rather big angle, in respect to the incident wave direction, due to the concave liquid meniscus and multiple reflections. An exit of grazing incidence x-ray fluorescence of ions Cs^+ levitating near the hydrosol surface has been measured before. Under numerical consideration, we take into account, of course rigorously, absorption and non-uniformity of a wet layer where reflection exists, as well as both polarization states for the incident radiation. Our results are compared qualitatively with those obtained using the experiment and high-frequency analytical technique.

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Relativistic wavelets for seismics and their connection to the Gaussian beams

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In the recent work [1] the techniques based on the relativistic Poincaré wavelet analysis was elaborated for seismic problems. The core of the techniques is the representation of the initial-boundary value problem for the wave equation in the inhomogeneous medium. The representation is based on the decomposition of the boundary data into wavelets in such a way that allows natural compression of the boundary data and corresponding reduction of the neccecary calculations. We found that conventional method based on the decomposition of the field in the Gaussian beams [2] can be obtained as particular case of the our approach by the appropriate choice of the bases.

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Waves in the reduced elastic Cosserat medium with transversal anisotropy of the coupling between linear rotational and translational deformations or of the nonlinear prestressed state

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Reduced Cosserat medium is a continuum consisting of infinitesimal rigid bodies where each point body may perform independent translational and rotational motion, but the medium does not react to the gradient of turn.

For the linear isotropic case such a medium was suggested for the first time in [1] for description of granular materials. In a classical linear elastic medium strain energy depends on the strain tensor $\nabla\mathbf{u}$, where $\mathbf{u}$ is an infinitesimal vector of the linear displacement, in the full linear elastic Cosserat medium it depends on tensors $\nabla\mathbf{u} + \boldsymbol{\theta} \times \mathbf{E}$ and $\nabla\boldsymbol{\theta}$. In the linear reduced elastic Cosserat medium the strain energy depends only on $\nabla\mathbf{u} + \boldsymbol{\theta} \times \mathbf{E}$. This leads to the existence of a forbidden band of frequencies for certain waves, i. e. this medium is a single negative acoustic metamaterial.

We shall consider two specific kinds of the reduced Cosserat continuum. The first one is a linear medium which is isotropic on the large scale but has a transversally isotropic coupling between translational and rotational strains, something typical for a granular material. Another kind of the medium is a nonlinear isotropic reduced Cosserat medium near its (transversally isotropic) prestressed state. The intuitional idea behind this model is a gravity that may lead to a packing with a "local anisotropy" coupling rotations and translations on the grain level, or to a nonlinear prestressed state in the material. We consider two specific directions of wave propagation (parallel and perpendicular to the axis of the isotropy, i. e. gravity). This could be useful also to model an experiment on the wave propagation or instabilities in the triaxial test.

Waves in a bulk of the linear isotropic reduced Cosserat continuum were considered in [2]. A special case of anisotropic coupling between translational and rotational strains is investigated in [3]. In this work we consider a general kind of transversally isotropic coupling between $\nabla\mathbf{u}^S$ and $\nabla\mathbf{u}^A + \boldsymbol{\theta} \times \mathbf{E}$ (i. e. there exists a coupling between volumetric/shear and rotational strains). We consider only special directions of the wave propagation: axis of symmetry either is parallel or orthogonal to the direction of the wave propagation. For these specific directions of the wave vector we obtain results similar to the case considered in [3].

A nonlinear elastic reduced Cosserat continuum is a medium whose strain energy depends on the $\mathring{\nabla}\mathbf{R} \cdot \mathbf{P}$, where $\mathring{\nabla}$ is nabla operator with respect to the reference configuration, $\mathbf{R}$ is the position vector, $\mathbf{P}$ is the turn tensor. We have considered the wave propagation and material instabilities in the spherical prestressed state of such a medium [4]. Here we consider the wave propagation in a transversally isotropic prestressed state (parallel and perpendicular to the axis of symmetry).

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Newton–Puiseux series for non-holonomic D-modules and factoring

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It is well-known that any polynomial equation $t(x, y) = 0$ has $\deg_y(t)$ (counting with multiplicities) zeroes being Newton–Puiseux series

$$y(x) = \sum_{i_0 \le i < \infty} y_i x^{-i/q} \tag{1}$$

for suitable integers $q \ge 1, i_0$ and the coefficients y_i from an algebraically closed field.

In this paper an analogue of Newton–Puiseux series for linear partial differential equations $T = 0$ is proposed. Whereas a Newton–Puiseux series is developed for a (plane) curve, we restrict ourselves with linear partial differential operators $T = \sum_{0 \le j+l \le n} b_{j,l} d_x^j d_y^l$ in two derivatives d_x, d_y.

One of the principal features of Newton–Puiseux series is the appearance of fractional exponents. Thus, a question arises, what could be an analogue of fractional powers, so to say "fractional derivatives"? An evident observation shows that in the derivative $y'(x) = \sum_i (-i/q + 1) y_{i-q} x^{-i/q}$, the i-th coefficient depends on the $(i - q)$-th coefficient of $y(x)$ itself.

That is why as a differential analogue of Newton–Puiseux series we suggest a *fractional-derivatives series* of the form

$$\sum_{0 \le i < \infty} h_i G^{(-i/q)}, \tag{2}$$

where h_i being elements of a differentially closed field F and $G^{(-i/q)}$ is called $(-i/q)$*-th fractional derivative* of G. The symbol $G = G^{(0)} = G_{(s_2,\dots,s_k)}(f_1, f_2, \dots, f_k)$ is defined by rational numbers $1 > s_2 > \dots > s_k > 0$ and $f_1, \dots, f_k \in F$ (if to continue the analogy with curves, G plays a role of a uniformizing element). For any rational s the s-th fractional derivative $G^{(s)}$ fulfills the identity

$$dG^{(s)} = (df_1)G^{(1+s)} + (df_2)G^{(s_2+s)} + \dots + (df_k)G^{(s_k+s)},$$

where either a derivative $d = d_x$ or $d = d_y$. The common denominator q of $s_2, \dots, s_k$ plays a role similar to one of the common denominator of the exponents in a Newton–Puiseux series (1). The inequality $k \le q$ holds.

In a particular case $k = 1$, we have $q = 1$ and as G one can take $g(f_1)$ for any univariate ("undetermined") function g, provided that the composition makes sense, the fractional derivatives $G^{(s)} = g^{(s)}(f_1)$ for integers s. We note that finite sums

$$\sum_{i_0 \le i \le i_1} h_i G^{(-i)}$$

(so, for $k = q = 1$) appear in the Laplace method as solutions of some second-order equations $T = 0$.

Theorem 1. *A linear partial differential equation $T = 0$ for operator T of order n has a Newton–Puiseux series solution of the form (2) with denominator $q \leq n$.*

As a consequence we design an **algorithm** which finds all first-order $d_x + ad_y + c$ right divisors of T.

Corollary. *One can factor an operator T of order at most 3.*

A D-nodule $M \subset (F[d_x, d_y])^m$ is called *holonomic* if the degree of the Hilbert–Kolchin polynomial of M equals 0. The latter is equivalent to that the space of solutions of M is finite-dimensional over the subfield of constants of F. The next theorem generalizes the previous one.

Theorem 2. *A non-holonomic D-module M has a Newton-Puiseux series solution.*

Association between the parameter of the acoustic anisotropy and measures of the stress-deformed condition for a sample with a stress concentrator

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Based on measurement of the acoustic anisotropy (AA), it is one of perspective non-destructive testing method. This method allows us to define the current stress-strain state of the loaded structural elements without preliminary unloading. The method consists in measurement of the relative difference between the speeds the waves polarized in parallel and perpendicularly to the direction to action of loading. Proportionality of change of AA and change of axial tension is for the first time experimentally confirmed in [1] and theoretically proved in [2]. However, in these works only elastic deformation while plastic deformation, damage accumulation and other factors has significant effect on AA is considered [3, 4].

The purpose of this study is to establish a connection between the parameter AA and the measures of the stress-strain state at elastic-plastic deformation on the basis of comparing the experimental data on the samples with the central circular hole with the finite-element calculations results.

It is as a result established that for the points which are in elastic area of value of coefficients of correlation of between AA and measures of tension, and also between AA and measures of deformation almost coincide, while for the points which are in elasto-plastic area they there are different. The best correlation is shown by the second invariant of the strain tensor deviator, which indicates the nonlinearity of the connection between AA and deformations.

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Simulation of circular dichroism enhancement in gold nanocubes array filled by chiral medium for optical frequency range

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Since the discovery of negative refraction in metamaterials the great attention was paid to the development of novel effects in such artificial, composite media [1]. However very promising metamaterials properties are at the same time strictly limit their applicability due to high losses and resonant nature of numerous composites. Hence for for investigation of high quality biosensors these disadvantages should be virtue.

Focusing on these facts we have considered the effect of enhancement of circular dichroism (CD) for chiral medium in the golden particles array. To obtain necessary information about electromagnetic properties of chiral medium the bi-isotropical optical coefficient was extracted from the experimental CD spectra of HgS nanocrystals [2].

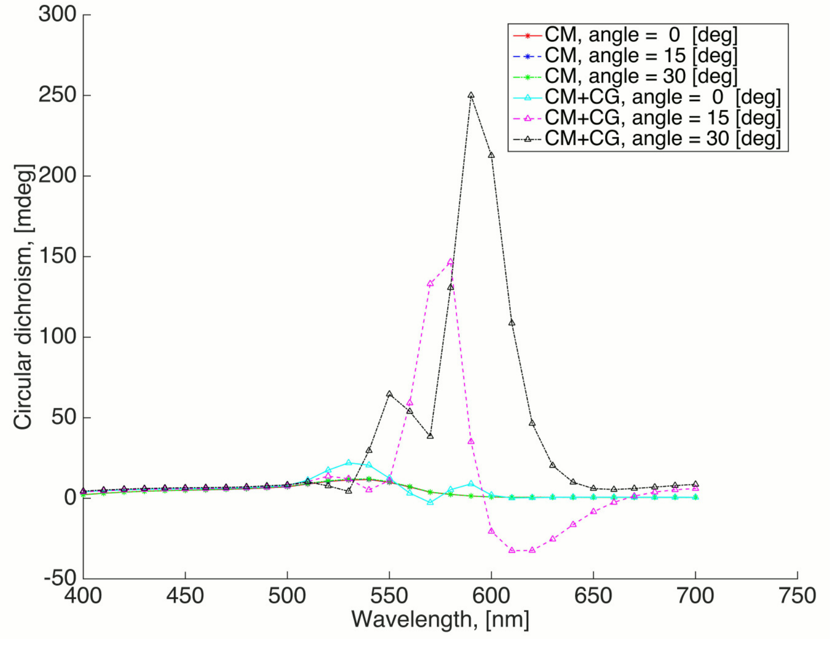

Fig. 1: The enhancement of circular dichroism in dependence of incident angle of electromagnetic circularly polarized plane wave for cubes with height $h = 300$ [nm] and side $a = 60$ [nm] in rectangular periodic grid with $T = 110$ [nm].

The effect of enhancement of CD was observed for rectangular grid of golden nanocubes filled by continuous medium with chiral HgS nanocrystals CD properies. The effect of CD enhancement grows from the resonant amplification of absorbance via golden array in the normal incedence regime and hyperbolic enhancement of the CD factor for the angle incident of electromagnetic circularly polorized plane wave.

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Compact Er-fiber:laser frequency comb: development and evaluation

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An Er-fiber frequency comb with highly stable repetition rate and carrier envelope offset (CEO) frequency has been realized. The seed mode-locked laser is based on the nonlinear polarization evolution (NPE) mechanism, which has several advantages such as fast saturation and good repeatability. A temperature fluctuation within 0.1° of the laser cavity has been obtained through precision electronic control. By now, the mode-lock laser is able to continually operate for several months. The CEO frequency has been detected right after the supercontinuum generation by a compact single-beam f-2f self-referencing interferometer. By using a combination of digital charge-pump phase-locked loop and a temperature stabilizer, both the repetition rate and the CEO frequency are referenced to a hydrogen maser so that the frequency stability of 5E-13 has been achieved. Currently, the repetition rate f_r is 132 MHz with a residual fluctuation of 0.6 mHz measured with a 1 s gate time, and the CEO frequency can be tuned in the range of $[0, f_r]$. The entire frequency comb system is small, compact and easy to use, which pave the solid foundation for further innovation and development of precision metrology instrument based the fiber frequency comb.

Solutions of the bi-confluent Heun equation in terms of the Hermite functions

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We construct an expansion of the solutions of the bi-confluent Heun equation in terms of the Hermite functions. The series is governed by a three-term recurrence relation between successive coefficients of the expansion. We examine the restrictions that are imposed on the involved parameters in order that the series terminates thus resulting in closed-form finite-sum solutions of the bi-confluent Heun equation. A physical application of the closed-form solutions is discussed. We present the five six-parametric Lamieux–Bose potentials for which the general solution of the one-dimensional Schrödinger equation is written in terms of the bi-confluent Heun functions [1, 2] and further identify a particular conditionally integrable potential for which the involved bi-confluent Heun function admits a four-term finite-sum expansion in terms of the Hermite functions [3]. This is an infinite well defined on a half-axis. We present the explicit solution of the one-dimensional Schrödinger equation for this potential and discuss the bound states supported by the potential. We derive the exact equation for the energy spectrum and construct a highly accurate approximation for the bound-state energy levels.

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Multichannel acousto-optic switch for fiber-optics telecommunication system

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Currently a number of researchers work on a design of all-optical switches (photonic switches) [1, 2]. The essential features required of these devices are low switching delay and low power consumption. Acousto-optic (AO) devices may be considered as they have low switching delay (500 ns – 10 µs), low power consumption and have no moving parts [3]. Some acousto-optic switches have two orthogonal acousto-optic cells or one crystal with two orthogonal piezoelectric transducers [4].

We consider AO switch 1×N to transmit signal from one optical fiber to a number of optical fibers (fig. 1). At the same time this device can transmit multiplex signals from a number of optical fibers (5) to one optical fiber (1). Space-multichannel AO cell (deflector) (3) and multi-sectional piezoelectric transducer (8) commutate signals from output optical fibers. Each channel is switched to an amplifier (6) via service power supply (7). Frequency of control signals specifies the value of frequency of acoustic wave in corresponded channel and changes output optical fibers in set (5).

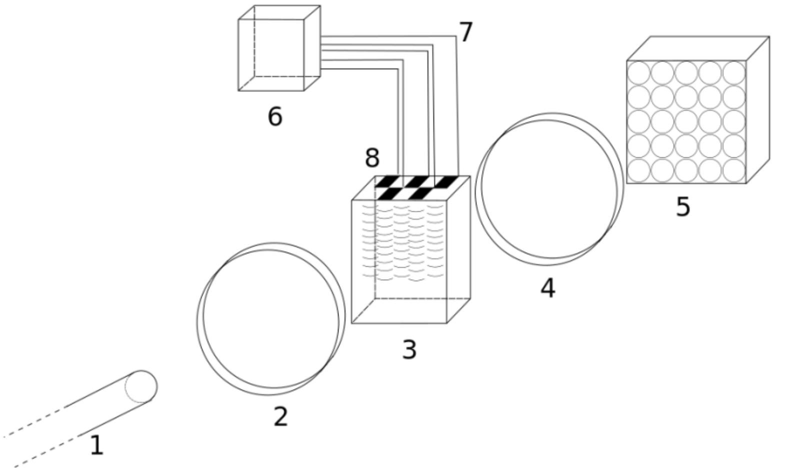

Fig. 1: Scheme of 1×N commutator based on multichannel acousto-optic deflector: 1 – optical fiber; 2, 4 – lenses; 3 – acousto-optic cell, 5 – set of output optical fibers; 6 – amplifier; 7 – service power supply; 8 – multi-sectional piezoelectric transducer.

Set of output optical fibers (5) consists of ten columns corresponded to a number of channels and ten rows corresponded to a bandwidth of control signals. So, a switch 1×100 is realized. We used standard single-mode fibers with fiber core diameter 9 µm. The material of AO cell crystal is lead molybdate $PbMO_4$. Efficiency of diffraction of this material is weakly related to polarization of optical wave signal. Therefore the AO cell is considered as polarization independent element and additional elements for polarization change are not necessary. Lenses (2) and (4) are used to increase diffraction efficiency and efficiency of signals passing from input fiber (fibers) to output optical fibers (fiber).

The results of our studies allowed us to create an inexpensive small-size functional device with low power consumption. Switch 1×100 based on presented scheme is performed. It is also possible to create a two-dimensional AO deflector by modification of control electronics.

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Optical antitrapping of nanoparticles in gaussian beam due to surface modes of a substrate

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Optical tweezers performance is investigated when the Gaussian beam is focused on the metal substrate with the subwavelength particle. Green's function is used to build analytical formalism underlining surface plasmon contribution to the optical force. Excitation of surface plasmon affects optical forces with significant impact at the surface plasmon resonance condition. The field and force are reconfigured by defocusing beam relative to the substrate. When the beam is focused above the substrate optical force increases about an order of magnitude due to evanescent field of surface plasmon. Novel effect of repulsion from Gaussian beam ("anti-trapping") is obtained when the beam waist is moved below the substrate which is confirmed by both the analytical approach and finite element simulation. Anti-trapping of subwavelength dielectric particle can be applied for sorting of different-sized particles identical by shape and composition [1].

In order to study tweezer performance 2D Gaussian focused on the metal substrate with a dielectric particle on top is considered. As the beam is defocused and the waist is moved below the metal-air interface (focus at $f = -100$ µm), the particle experiences repulsion from the beam center, Figure 2(a). This new phenomenon of anti-trapping is caused by excitation of plasmon, the force without the evanescent field of surface wave (Fx_0) being almost the same magnitude but of opposite direction. Anti-trapping is several times stronger than trapping if the particle placed at the metal instead of glass substrate. Elimination of evanescent waves from the Green's function decomposition shows that plasmon inverts the direction of the optical force. Finite-element simulations, Comsol Inc., for beam focused at $f = -100$ µm also predict particle repulsion, Figure 2(b). In the numerical case study final size of the particle is taken into account and stronger antitrapping is achieved compared to the analytical model.

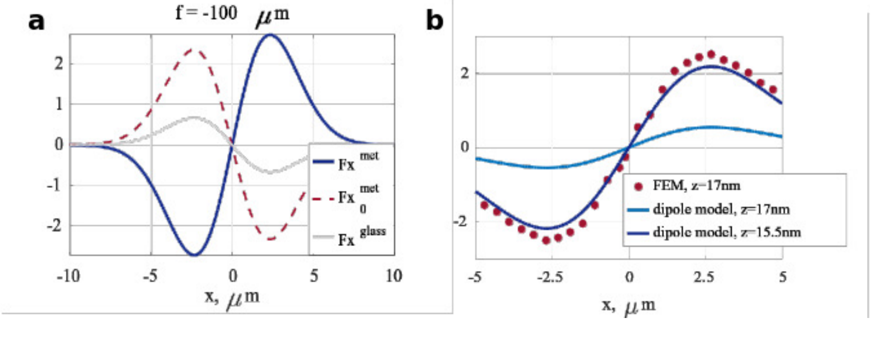

Fig. 1: Optical anti-trapping of a bead in Gaussian beam of waist 10 µm focused below ($f = -100$ µm) the air-silver interface. Dielectric particle of radius $R = 15$ nm and dielectric permittivity $\varepsilon = 3$ is lying on metal ($z = 15$ nm). The wavelength $\lambda = 342$ nm corresponds to resonance of surface plasmon.

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Model of the GTEM-cell for EMC testing

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For the purpose of researching electrical and electronic devices for the emission of radio-frequency interference and resistance to them, are used various solutions based on shielded chambers, anechoic chambers and open measuring areas. The test solution with use the gigahertz TEM-chambers differs from others with accessibility. Such a design excludes the appearance of resonances, external electromagnetic waves, and make possibility the measurements in the all band by the single frequency sweep. Compact GTEM cameras can be used to achieve acceptable EMC performance with minimal additional costs at the stage of designing personal computers, mobile phones and other equipments.

In this work, a numerical model of the GTEM measuring cell was created. The modeling domain was a polyhedron formed by a quadrangular pyramid and a section passing through its height. The plane of the cross section at the same time is represented by the mirror symmetry of the GTEM cell, which helped to reduce the calculated complexity and increase the accessibility. A detailed distribution of the electric and magnetic field components and the dimensions of the working region was obtained by using the finite element method, taking into account the boundary conditions, appropriate to the cell geometry and the transverse type of the electromagnetic waves. Maxwell's equations was used in the following form: (1)

$$\mathrm{rot}\left(\mu^{-1}\,\mathrm{rot}\left(\vec{E}\right)\right) - k_0^2\left(\varepsilon - \frac{i\sigma}{\omega\varepsilon_0}\right)\vec{E} = 0 \qquad (1)$$

The partition of the model was carried out in the calculation of inhomogeneity of the field. The modeling area was extended to quarter of the wavelength [1]. The possibility of using metamaterials for manufacturing the absorbing wall of the GTEM cell and the septum was also investigated. Using the data obtained during the work, a working prototype of the GTEM-measuring chamber with a working area of 60×200×200 mm was produced, used in conjunction with the Aeroflex 2399C spectrum analyzer.

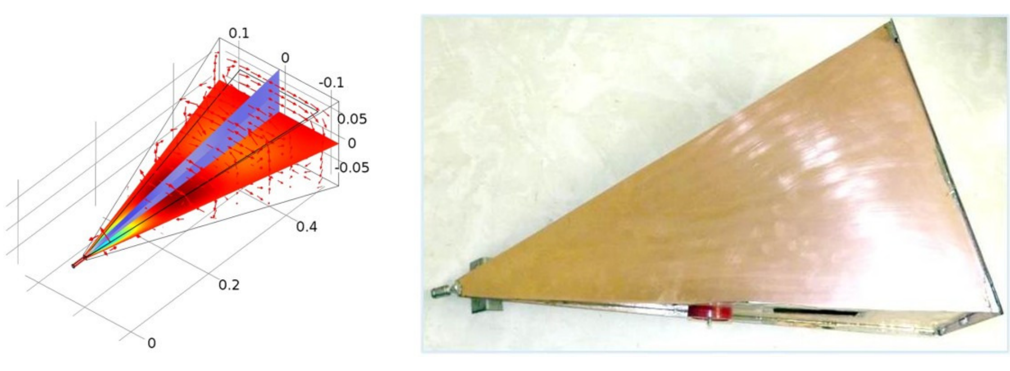

Fig. 1: Numerical model of GTEM-cell and its prototype.

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Microwave resonators filled with the dielectric and magnetoelectric materials

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Split ring resonators are widely used to create metamaterials — structures with unusual macroscopic electromagnetic properties, unattainable with the use of natural materials. The propagation of electromagnetic waves in such structures is determined by the strong interaction of the elements near the resonant frequencies and can be described using the effective values of the material constants μ_{eff} and ε_{eff} [1].

In this work dynamic properties of microwave resonator when filling capacitive gap dielectric ceramics or magnetoelectric composite material is investigated. A numerical model is created and experimentally investigated single resonators and the planar structures basis on this resonators. Possibility of adjustment of the resonance frequency and changes the Q of such devices is demonstrated. For planar structures the transmission in limit waveguide is studied. To calculate the resonance frequency of a plane resonator located on a substrate with a dielectric permittivity ε_n, the following formula was used:

$$f_0 = \frac{1}{2\pi}\left(\frac{1+\varepsilon_n}{2}\varepsilon_0\mu_0\frac{\pi d}{\ln\frac{2l}{h}}\left(\ln\frac{8R}{d+h}-\frac{1}{2}\right)\right)^{\frac{1}{2}} \tag{1}$$

When a substance with a dielectric constant different from air is introduced into the region of the capacitive gap, the resonant frequency changes by Δf_i and is determined by the non-uniformity of the electric field (edge fields):

$$\varepsilon_x = (1+\varepsilon_n)\left(1-\frac{\Delta f_i}{f_0}\right)^{-2} - \varepsilon_n \tag{2}$$

To investigate the frequency characteristics of resonators and planar structures, we used a VSWR and reflection meter R2M40 of the company "Micran". Measurements were made from the curves of the reflection and transmission coefficients.

This work was supported by the Russian Foundation for Basic Research according to the grants № 15-07-08111A.

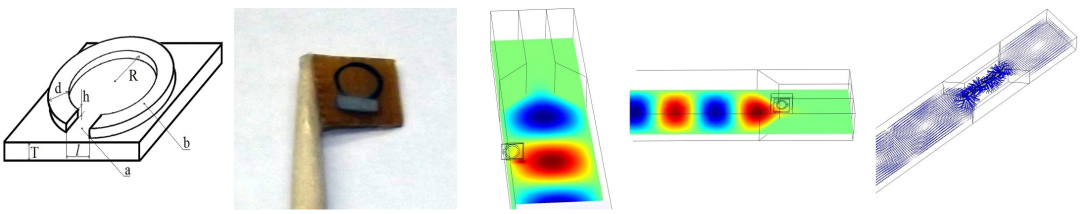

Fig. 1: Inductive-gap resonator: photo, numerical models of its and of planar sructure in limit waveguide.

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Resonance capture in nonlinear oscillators

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A system of differential equations, which describes interaction of two weakly connected nonlinear oscillators, is considered. Initial data is such that under absence of connection the first oscillator is far from equilibrium while the second oscillator is near equilibrium and the proper frequencies are closed. The capture in resonance is investigated, when the frequencies of the connected oscillators remain closed and amplitudes of the oscillations are varied significantly, in particular the second oscillator moves far from equilibrium. It is discovered that the initial stage of the resonance capture is described by the second Painlevé equation. Such description is obtained in asymptotical approach with respect to small parameter, which corresponds to connection coefficient.

Sea bottom topography using by model based on the radiative transfer equation

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Propagation of acoustic signal on the frequencies of tens kHz is described by the radiation transfer equation [1]:

$$\frac{1}{c}\frac{\partial I}{\partial t} + \boldsymbol{k}\cdot\nabla_r I + \mu I(\boldsymbol{r},\boldsymbol{k},t) = \frac{\sigma}{4\pi}\int_\Omega I(\boldsymbol{r},\boldsymbol{k}',t)d\boldsymbol{k}' + J(\boldsymbol{r},\boldsymbol{k},t). \tag{1}$$

Here, $\boldsymbol{r}\in\mathbb{R}^3, t\in[0,T]$ and wave vector $\boldsymbol{k}$ belongs to the unique sphere $\Omega=\{\boldsymbol{k}\in\mathbb{R}^3:|\boldsymbol{k}|=1\}$. The function $I(\boldsymbol{r},\boldsymbol{k},t)$ is interpreted as radiation intensity of wave in moment t in point $\boldsymbol{r}$, propagated in the direction $\boldsymbol{k}$ with constant velocity c. The coefficients μ and σ denote the attenuation and the scattering, correspondingly. J describes the density of inner sources.

The signal propagation occurs in $G=\{\boldsymbol{r}\in\mathbb{R}^3: r_3>-l+u(r_1,r_2)\}$ with boundary $\partial G=\{\boldsymbol{y}\in\mathbb{R}^3: y_3>-l+u(y_1,y_2)\}$. Thus, the function $u(y_1,y_2)$ describes small deviation of sea bottom relief from the middle altitude l. Eq. (1) is complemented by initial and boundary conditions:

$$I|_{t=0}=0,\quad I|_{\partial G}=\frac{\sigma_d}{\pi}\int_{\Omega((\boldsymbol{k}\cdot\boldsymbol{n})<0)}|(\boldsymbol{n}(\boldsymbol{y})\cdot\boldsymbol{k'})|I(\boldsymbol{y},\boldsymbol{k}',t)d\boldsymbol{k}', \tag{2}$$

where σ_d is the constant sea bottom reflection coefficient. The source and the receiver locate at the same vehicle which moving along path $\boldsymbol{r}=\boldsymbol{V}t$ with constant velocity V, where $\boldsymbol{V}=(0,V,0)$. The source is point-view and emits parcels during time-intervals $[t_i+\Delta, t_i-\Delta]$, where $\{t_i\}$ is uniform grid on total sounding interval $[0,T]$. Received signal of the receiver satisfy to

$$I(t)=\int_\Omega S(\boldsymbol{k})I(\boldsymbol{V}t,\boldsymbol{k},t)d\boldsymbol{k}, \tag{3}$$

where $S(\boldsymbol{k})$ sets the directivity pattern of receiving antenna which concentrated in the Azimuth angle interval $(-\beta,\beta)$.

Authors study an inverse problem for determination the function $u(y_1,y_2)$, described a sea bottom relief, from eqs. (1)–(3). Using single-scattering approximation they deduced an integral equation for $u(y_1,y_2)$. Further, authors assume the source is impulse and $\beta\to 0$ [2]. Thus, authors obtained the explicit formula for $u(y_1,y_2)$. Such assumptions leads to defocusing relief objects on the boundary ∂G. Authors carry out a numerical analysis for determination of influence of impulse length and width of directivity pattern on the sea bottom relief reconstruction.

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Whispering gallery acoustic waves propagating in a shallow sea along the circular coastline

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The problem of sound propagation in a cylindrically-symmetric shallow-water waveguide is considered. The bottom relief in cylindrical coordinates (r, θ, z) is described by a function $h(r)$, that is monotonically decreasing with respect to r (i.e. we deal with a cylindrical lake that is shallower for greater r). The waveguide consists of the water column $0 \le z \le h(r)$ and the penetrable bottom $z > h(r)$. At the surface $z = 0$ the pressure-release boundary condition for the acoustic pressure $P(r, \theta, z)$ is imposed [1].

In the framework of the mode theory, the sound pressure in the water column due to a time-harmonic point source writes in the form of a modal expansion [1] $P(r, \theta, z) \approx \sum_{j=1}^{N_m} A_j(r, \theta)\phi_j(z, r)$, where $\phi_j(z, r)$ are mode functions computed at given range r, and A_j are modal amplitudes. The latter are obtained by solving the so-called horizontal refraction equations [1].

In our work the condition for the media parameters and the bottom relief surface $z = h(r)$ ensuring the formation of a whispering gallery wave is studied. This condition has the form of a simple relation between the horizontal wavenumbers rate of change with respect to r and the waveguide geometry. The relation is validated via the semi-analytical solution of horizontal refraction equation.

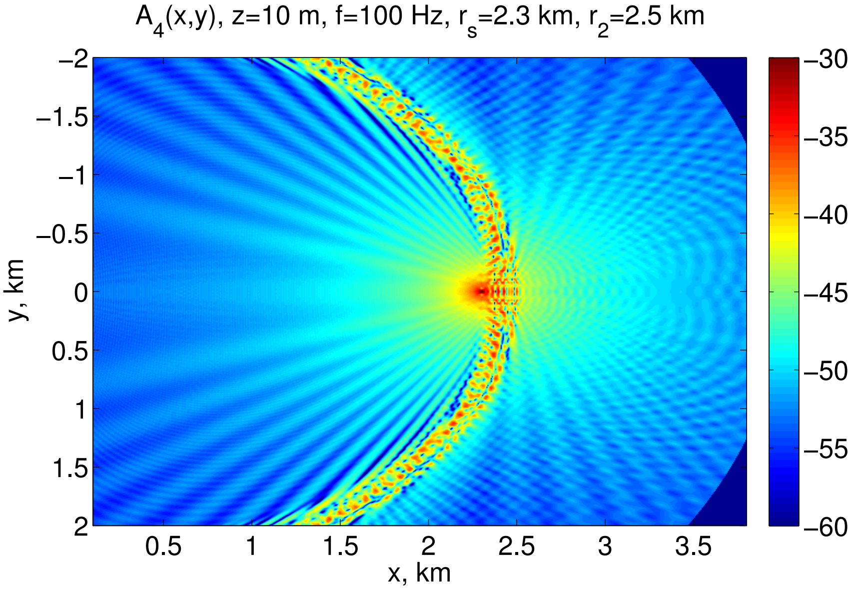

Fig. 1: A typical whispering gallery interference pattern in a cylindrically-symmetric lake.

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Integral symmetries of the Heun equations with added apparent singularities

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Different integral symmetries of the Heun equations with added apparent singularities are presented and discussed. These symmetries generate corresponding relations for the monodromy group of equations.

Branching Monte Carlo methods for the Cauchy problem of the radiative transfer equation

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The paper deals with the initial boundary value problem for the radiative transfer equation [1–4]

$$\left(\frac{1}{v(r)}\frac{\partial}{\partial t}+\omega\cdot\nabla_r+\mu(r)\right)I(r,\omega,t)=\sigma(r)\int_\Omega p(r,\omega\cdot\omega')I(r,\omega',t)d\omega',\quad (r,\omega,t)\in G\times\Omega\times[0,\infty), \tag{1}$$

$$I^-(r,\omega,0)=h_0(r,\omega), \tag{2}$$

$$I^-(z,\omega,t)=h_{ext}(z,\omega,t),\quad z\in\partial G, \tag{3}$$

$$I^-(z,\omega,t)=R(z,\nu)I^+(z,\omega_{re},t)+T(z,\nu)I^+(z,\omega_{tr},t),\quad z\in\gamma. \tag{4}$$

This model describes the nonstationary process of radiation propagating in an inhomogeneous medium $G\subset\mathbb{R}^3$, where $I(r,\omega,t)$ is interpreted as the flow density at the time t and at the point r of the particles moving with the velocity v, in the direction of the unit vector ω, the function μ is the attenuation factor, σ is the scattering coefficient, p is the phase function of scattering. Initial and boundary conditions (2), (3) define the initial flux density distribution in the medium, for the time moment $t=0$ and external radiation sources on the border ∂G. Fresnel matching conditions (4) describe refraction, according to Snell's law, and specular reflection on the material interface γ [2, 3].

We study several modifications of the Monte Carlo scheme [2, 4] for numerical solution of the initial boundary value problem (1)–(4). Our methods are based on different approach to Markov's chain branching. We evaluate a theoretical estimation for some statistical properties of the methods, to prove its correctness and get limits of application effectiveness for each method. The article presents the results of method comparison, through numerical testing by evaluating the laboriousness as a main measure of algorithm effectiveness. A set of test input data, on which one method beats all others, are found for each method. We also tested the algorithms on a visualization problem. The obtained results are consistent with the natural concepts of the light propagation.

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Fock–Leontovich parabolic equation method on prolongated bodies with Neumann boundary conditions

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We consider a diffraction problem where the scatterer is a strictly convex and prolongated body of revolution. The incident plane wave radiates along the revolution axis. The wave field is constructed in the vicinity of the shadow-light (penumbra, Fock's region) zone and in the shadow zone. The problem formulation is similar to the one in [1], however a different approach to the solution is used.

Fock's zone plays a crucial role in calculating a diffraction field, because it works as an origin of the field for the shadow region and in the vicinity of the creeping rays cone. Moreover the diffracted field can be described in terms of the Ray Method in the lighted part of a scatterer in case of the both curvature radii of the surface are much greater then the incident wavelength, [2].

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Photonic nanojets in near-field of metallic and metamaterial scatterers in Laguerre–Gaussian beams

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We study the light scattering problem for metallic spherical scatterers illuminated with Laguerre–Gaussian (LG) beams by using the T-matrix approach in the form presented in [1, 2]. Our method uses the remodelling procedure in which the far-field matching method is combined with the results for nonparaxial propagation of LG beams [2, 3].

The theoretical analysis of the optical field in the near-field region of dielectric particles [2] is extended to the cases of metallic and negative index metamaterial Mie scatterers. It is shown that the characteristics of photonic nanojets near the metallic and metamaterial particles profoundly differ from those for the dielectric scatterers and may significantly vary depending the mode number and the polarization state of the illuminating LG beam.

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Simulating diffraction of plane wave on periodic layer with the use of the method of projections

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A problem of incidence of a plane wave on a periodic layer having finite or infinite width is considered. The layer has periodic surface as well as periodic permittivity inside the layer. The reflected, absorbed and passed radiation is calculated using the method of parallel projections [1, 2]. This method allows us to reduce the problem to the system of ordinary differential equations. The computer realization of the algorithm was implemented using the C++ programming language. Source code of the program could be found at `https://bitbucket.org/Jclash/dpproj`. The web-interface (`http://ipmnet.ru/~knyaz/diffraction.html`) was used to remotely run computations on a supercomputer. The accuracy was verified on several cases that allow analytical solutions. The method could be used in solving direct and inverse problems of sea surface radiometry [3], lithography [4], acoustics [5]. The work was done with the financial support of the Russian Fund for Basic Research, project code 16-31-60096.

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Optical binding of two nanoparticles near interface

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Nowadays, studying of optical forces became a promising field due to widespread usage of single and organized molecular and atomic structures in chemical and biological research, nanophotonics, different miniaturized electronic devices [1–3]. Optical binding is an optical force appearing in the ensemble of particles that controls their motion and can lead to self-organization of structural elements. If an array of nanoparticles is illuminated by light, each element of the array scatters incident wave and a set of potential wells is created which defines stable positions of particles.

In this work we investigate optical binding of two nanoparticles in the presence of metallic substrate. Our system consists of two dipole nanoparticles in vicinity of planar interface. The system is illuminated by plane wave incident along OZ and polarized along OX. Both of particles are placed at the same distance z from the surface. One of the particles is localized at the origin of lateral coordinates $x = 0$, $y = 0$, while the other one changes its position in XY plane. Force acting on a second particle is considered.

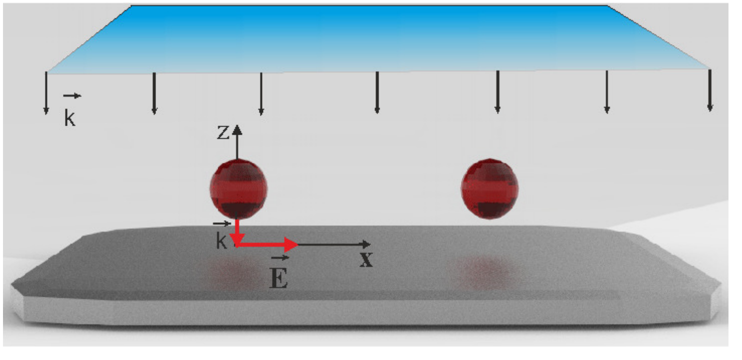

Fig. 1: The scheme of experiment: two nanoparticles above silver substrate are illuminated by a normally incident plane wave.

The force acting on a system could be described by equation:

$$\boldsymbol{F} = \frac{1}{2}\sum_i \mathrm{Re}\left(P_i^* \nabla E_i^{\mathrm{loc}}\right),$$

where p_i is the ith component of particle induced dipole moment, E_i^{loc} local field at the particle location, $i = x, y, z$. Both $\boldsymbol{E}^{\mathrm{loc}}$ and $\boldsymbol{p}$ can be obtained with dyadic Green's function for particle positioned at a planar substrate.

Presence of the substrate introduces an additional degree of freedom in the interaction. Interference of incident and reflected waves and intense evanescent field of the surface wave at the substrate can significantly change characteristics of the optical binding. Strong dependence on z coordinate shows that stable positions are strongly shifted towards the origin of coordinates. Thus stable dipole position could be tuned by varying distance between the substrate and the particles.

We demonstrated analytically the amplification of binding force for dipole particles located near the metal interface. Improvement of localization accuracy takes place in a wide range of frequencies where excitation of surface plasmon is possible. Lateral components of the forces were analyzed and equilibrium positions for nanoparticles were obtained. Spatial arrangement as well as the distance between the equilibrium positions are defined by excitation wavelength, particles' height above the substrate and dielectric parameters of medium surrounding the particles.

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Reconstruction of the sea bottom reflection coefficient

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Authors study a problem of reconstruction seabed surface based on signal of side-scan-sonar (SSS). The propagation of acoustic waves occurs in the semispace $G = \{\boldsymbol{r} \in \mathbb{R}^3 : r_3 > -l\}$ with inhomogeneous boundary $\partial G = \{\boldsymbol{y} \in \mathbb{R}^3 : y_3 = -l\}$. Signal propagation in a fluctuating ocean is described by the radiative transfer equation with corresponding initial and boundary conditions [1]. The last one specifies of diffusion reflection on ∂G with the coefficient σ_d. The source and the receiver are located at the same vehicle which moving along path $\boldsymbol{r} = \boldsymbol{V}t$ with constant velocity V, where $\boldsymbol{V} = (0, V, 0)$. The source emits parcels during time-intervals $[t_i + \Delta, t_i - \Delta]$, where $\{t_i\}$ is uniform grid on total sounding interval $[0, T]$. Thus, authors formulated an inverse problem for determination σ_d based on the signal emitted and received by SSS. Using assumptions of single-scattering and point-view source authors deduce a formula for the function σ_d which described inhomogeneous boundary ∂G[2]:

$$I(t) = \frac{1}{\pi}\frac{1}{2\Delta}\frac{1}{2\sin\beta}\int_{t_i-\Delta}^{t_i+\Delta}\int_{-\epsilon}^{\epsilon}\sigma_d(y_1,y_2)\frac{cl^2 J_i \exp(-\mu c(t-t'))}{y_1(t-t')|\boldsymbol{V}t'-\boldsymbol{y}|^2|\boldsymbol{y}-\boldsymbol{V}t||k_2V-c|}dk_2dt' + I_G(t), \qquad (1)$$

where y_1, y_2 satisfy $\boldsymbol{k} = \dfrac{\boldsymbol{V}t-\boldsymbol{y}}{|\boldsymbol{V}t-\boldsymbol{y}|}$, and $\epsilon = \sin\beta\sqrt{1-k_3^2}$, $k_3 = \dfrac{2lc(t-t')}{(c^2-V^2)}$. Here, $I(t)$ denotes received signal of SSS, 2β is angle of width of directivity pattern, l is altitude, the coefficients μ and c corresponds to the attenuation and the sound speed. $I_G(t)$ is signal caused by the volume scattering in G. $\boldsymbol{V}t$ and $\boldsymbol{V}t'$ denote points of the receiver and the source, correspondingly.

The first integral corresponds to non-impulse source and integrated over pulse time-interval whereas the second one to the width of directivity pattern of the receiving antenna. In paper [3] for solving the inverse problem authors use assumptions such as impulse source and narrow-beam antenna. That approach leads to defocusing objects on the reconstructing boundary ∂G. In this paper, for determination the function σ_d authors solve the integral eq. (1) using by discretization method. On the first stage, they make a rectangular grid on ∂G. In each rectangular σ_d is set as constant. Hence, the inverse problem is reduced to SLE with sparse matrix which solving by iterative method. Thus, authors construct the algorithm for focusing objects on the inhomogeneous boundary based on the signal of SSS.

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Analysis of thermal processes in a multilayer biotissue exposed to optical radiation

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The interest in studying problems of complex heat transfer where the radiative and conductive effects are simultaneously taken into account (see e. g. [1]) is motivated by their importance for many engineering applications such as glass manufacturing, design of cooling systems, laser thermotherapy, etc. It is worth to mention an important application related to optical imaging of biological tissues. The propagation of optical radiation in biological tissues leads to internal heating caused by the absorption of radiative energy with its transformation into heat (see [2]). Thus, although optical radiation is relatively harmless for human tissues, there is a danger of overheating sensitive structures. In particular, it is important to estimate heat distribution in the cerebral vessel network, e. g., under Transcranial Optical Vascular Imaging (see [3]). This requires the development, theoretical analysis, and numerical implementation of physically relevant and enhanced models that describe the complex heat transfer in multicomponent and layered structures.

In the current work, a one-dimensional steady-state model of radiative-conductive heat transfer is considered in a multilayer medium. The nonlinear model takes into account the effects of reflection and refraction of radiation at the interfaces, scattering and absorbing phenomena, black-body radiation, and blood perfusion. The P_1 approximation of the model is constructed and studied. The theorems of existence and uniqueness are proven without any a priory requirements on the regularity, boundedness, etc. of solutions. The numerical experiments simulated passing an optical radiation through a human skin is fulfilled. The influence of various model parameters on the heat and radiation distribution is discussed.

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Small-amplitude steady water waves with critical layers: non-symmetric waves

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The problem for two-dimensional steady water waves with vorticity is considered. Using methods of spatial dynamics, we reduce the problem to a finite dimensional Hamiltonian system. As an application, we prove the existence of non-symmetric steady water waves when the number of roots of the dispersion equation is greater than 1. This is a joint work with E. Lokharu.

Wave structure identification by the method of instantaneous phases

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Unsteady axially symmetric flows of a viscous incompressible liquid in a spherical layer, which are formed by modulation of the rotation rate by one of the spherical boundaries, are considered. The wave structures of such flows are investigated by the method based on determination of the instantaneous difference in phases between the sphere velocity and the azimuthal velocity at each flow point. The steady-state character of the distribution of the instantaneous phase difference in the meridional plane of flows is established. The possibility of applying the method at very low vibrational amplitudes is shown.

Theory of a strip antenna located at the plane interface between isotropic and uniaxial anisotropic media

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Electrodynamic characteristics of strip antennas excited by a time-harmonic given voltage and located at the interface of isotropic and anisotropic media have been considered for some geometries in [1–3]. For a straight strip, the previous analysis has been focused on the case where the anisotropic medium on the one side of the interface at which the antenna is placed is resonant [3]. Recall that in a resonant anisotropic medium, one of its normal waves has an unbounded branch of the refractive index surface. The described behavior is inherent in such media as, e.g., magnetized plasmas or anisotropic dielectric metamaterials if their permittivity tensors have diagonal elements with different signs. However, of no less interest is the case where the anisotropic medium is nonresonant such that the corresponding elements of its permittivity tensor have identical signs and the refractive index surfaces of the normal waves are bounded.

In this work, we consider an antenna having the form of an infinitesimally thin, perfectly conducting narrow strip located at the plane interface between an isotropic medium and a nonresonant uniaxial anisotropic dielectric. The antenna is perpendicular to the anisotropy axis of the uniaxial medium and is excited by a given voltage that is applied to a small gap of the strip conductor. Singular integral equations for the surface-current density of an infinitely long antenna are obtained, on the basis of which its current distribution and input impedance are found in closed form. The essential differences between the electrodynamic characteristics of such an antenna are revealed in the cases where the diagonal elements of the permittivity tensor of the anisotropic dielectric are simultaneously positive or negative. Then the limits of applicability of an approximate method based on the transmission line theory for determining the current distribution and the input impedance of the considered antenna are established. It is shown that the electrodynamic characteristics obtained within the framework of this method coincide with those yielded by the integral equation approach if the strip is sufficiently narrow. Using the transmission-line method, the results for an infinitely long strip are generalized to the case of a finite-length antenna. It is also shown that the model of a uniaxial anisotropic material for the medium on the one side of the interface containing the strip may be used as a reasonable approximation if this material ceases to be uniaxial and acquires gyrotropic properties.

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Babenko's equation for periodic gravity waves on water of finite depth

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For the nonlinear two-dimensional problem, describing periodic steady waves on water of finite depth in the absence of surface tension, a single pseudo-differential operator equation (Babenko's equation) is derived. This equation has the same form as the equation for waves on infinitely deep water; the latter had been proposed by Babenko in 1987 and studied in detail by Buffoni, Dancer and Toland in 2000. Unlike the equation for deep water involving just the 2π-periodic Hilbert transform $\mathcal{C}$, the equation obtained in this paper contains an operator which is the sum of $\mathcal{C}$ and a compact operator depending on the depth of water.

Trapping of time-harmonic waves by bodies in a two-layer fluid covered by brash ice

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We consider linearized problems of radiation and scattering of time-harmonic waves by bodies floating in a fluid. The latter is assumed to be inviscid and incompressible and its motion is of small amplitude. In [1] we studied the case of infinitely deep water covered by brash ice and proved the existence of trapped modes for surface-piercing bodies (the modes are finite-energy solutions of the unforced problem, representing fluid oscillations localized near the structure). This work extends the results to the case of a fluid covered by brash ice, but consisting of two layers of different density. We write the statement of the problem and derive an explicit form of Green's function. By using derivatives of the source potential and a modification of the semi-inverse procedure, we find examples of geometries for which trapped modes occur. The new effects by stratification are discussed, in particular, we present examples of totally submerged trapping bodies (intersecting the interface).

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Electromagnetic metamaterials: the new world of outstanding opportunities

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The popular research area of metamaterials has been flourishing with intriguing ideas and promising designs for some 17 years of its controversial but exciting progress [1]. While metamaterials are now seeking their way towards applications, fundamental ideas for their assembly are far from being exhausted. One of the major highlights of metamaterials is a collective response of specially engineered elements (meta-atoms), so that their individual features evolve via mutual interaction into unusual macroscopic properties (which were not available with the meta-atoms alone).

Metamaterial ideas offer rich opportunities for exotic material properties, however typically their response is limited to a specific frequency range, and a narrow combination of physical features. Indeed, even though a good range of electromagnetic, acoustic or even thermal metamaterials is known [2–4], relatively few implementations combine such distinct properties. However, interlinking different types of response provides a new degree of freedom to metamaterial design, resulting in a growing variety of fruitful nonlinearities [5], with the prominent examples of magnetoelastic metamaterials [6], conformational flexibility [7], or exotic mechanical links such as intrinsic rotation [8]. Furthermore, unusual interactions can be realised between incomparable frequencies: for example, coupling between low-frequency resonators via an optical bridge, built by including light-emitting and photo-sensitive materials within resonators [9].

Overall, opening new degrees of freedom leads to interesting new physics, and offers a tool to design nonlinear feedback or impose interaction possibilities. Our ability to choose the structure and components of a meta-atom, as well as direct access to the way meta-atoms are assembled in an array, serves not only to enhance the initial material properties, but also to interlock them in a highly controllable manner, unthinkable in natural materials. Potential impact of this concept on the development of metamaterials is envisaged to be immense [10], paving the way from the fantasy of sophisticated designs towards a solid practice of artificial materials.

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Experimenatal demonstration of reconfigurable magnetic Fano resonance in hybrid oligomers

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Plasmonic oligomers consisting of nanoparticles of nobel metals (e. g. silver and gold) are the cornerstone of modern nanophotonics due to a sharp effect of resonant scattering originating from destructive interference between super-radiant and sub-radiant modes, which can be described in terms of the Fano resonances [1]. In addition to a strong local field enhancement, the asymmetric profile of the Fano resonance in such structures allows to control the radiative damping of the localized surface plasmon resonance. Recently, all-dielectric oligomers based on high-index dielectric nanoparticles (e. g. silicon) have been proposed theoretically and realized experimentally as a more efficient counterpart to the plasmonic ones. It has been shown that the all-dielectric oligomers can exhibit not only an electric type of Fano resonance, but also a magnetic one, which is associated with the optically induced magnetic dipole mode of individual nanoparticle. However, for practical applications such as biosensing or nonlinear nanophotonics, it is necessary to have a possibility for a fine-tuning of the spectral features of the Fano resonances in the fabricated nanoparticle oligomer structures. Here, we present the results of our recent studies [2] of reconfigurable hybrid metal-dielectric oligomers with the magnetic Fano resonance in visible range. We show that the Fano resonance wavelength can be precisely tuned via femtosecond laser modification of gold nanoparticles of the oligomer.

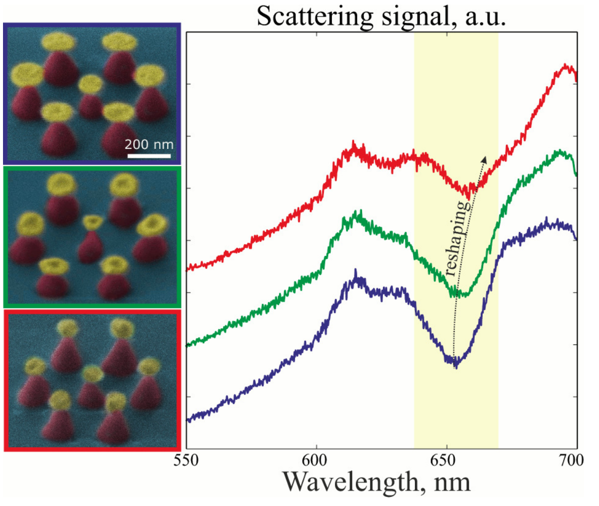

Fig. 1: Dark-field spectroscopy signals of the hybrid oligomer with gold nanodisks (blue curve), nanocups (green curve) and nanospheres (red curve).

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Dielectric chain driven by electron-hole plasma photoexcitation

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Nowadays, all-dielectric nanophotonics based on high-index dielectric nanoparticles [1] became a powerful platform for modern light science, providing many fascinating applications, including high-efficient nanoantennas and metamaterials, enhanced nonlinear optical response, nonradiative sources, sensing, and all-optical data processing. High-index dielectric nanostructures are of a special interest for nonlinear nanophotonics, where they demonstrate an inherent magnetic resonance-enhanced frequency conversion processes, and special types of optical nonlinearity, such as electron-hole plasma photoexcitation [2], which are not inherent to plasmonic nanostructures.

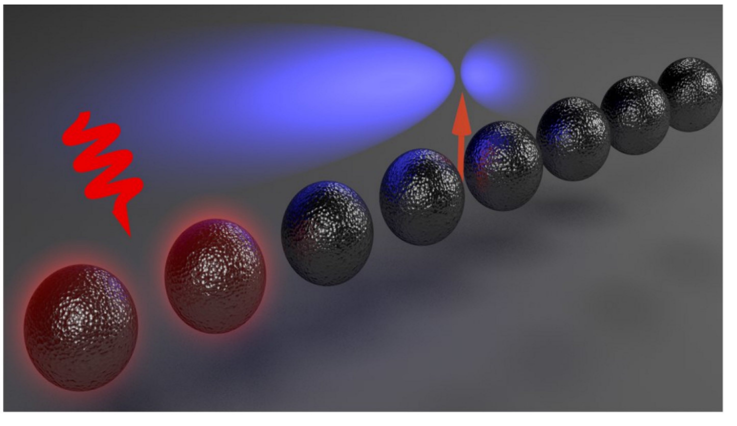

Fig. 1: General view of the considered all-dielectric chain and the dipole source (red arrow).

In this work, we propose a novel type of highly tunable all-dielectric Yagi–Uda nanoantennas, consisting of a chain of Si nanoparticles exciting by an electric dipole source, which allow tuning of their radiating properties via electron-hole plasma photoexcitation. We study a chain of N spherical dielectric nanoparticles excited by an electric dipole placed in the centre of chain, Fig. 1. We have theoretically and numerically demonstrated the tuning of radiation power patterns and the Purcell effect by additional pumping of several boundary nanoparticles with relatively low peak intensities of fs-laser. The electron-hole plasma photoexcitation leads to decrease of the real part of dielectric permittivity from $\varepsilon_1 = 16$ up to $\varepsilon_2 = 14$ and results in changing of Purcell factor and power pattern. Moreover, we have calculated the power patterns of the nanoantenna located on the SiO_2 substrate. When the particles are not-affected, the radiation power pattern is symmetric with respect to the dipole axis and has 2 main lobes (and few of side-lobes). The modification of three border particles dramatically changes the power pattern of the nanoantenna. We have shown, that this reconfiguration of power patterns via excitation of electron-hole plasma can be used for unidirectional launching of waveguide modes.

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Silicon-based metamaterials: phase transitions in periodic structures

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The concept of optical phase transition in periodic photonic structures was introduced in the paper [1] considering a square lattice of dielectric rods. The structure acts as a metamaterial with negative magnetic permeability $\mu < 0$, when Mie resonance becomes the fundamental (lowest-frequency) gap rather than Bragg gap related to the photonic crystal properties. The phase diagram shows that the transition between photonic crystal and metamaterial regimes can be observed when the value of dielectric permittivity ε exceeds 19. In the case of most popular high-ε dielectric material crystalline silicon, its permittivity does satisfy the conditions $\varepsilon > 19$ in the short wavelengths range of visible spectrum $\lambda < 490$ nm. However, in this range the permittivity of silicon is complex and it has essential frequency dependence $\varepsilon(\omega)$. Thus, we cannot analyze optical phase transitions by exploiting standard computational methods as in the Ref. [1].

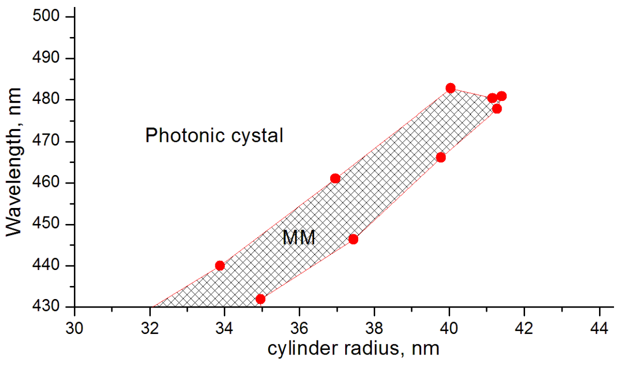

Fig. 1: The phase diagram of structure composed from the square lattice of silicon nanorods. The white area corresponds to photon crystal regime and the shaded area corresponds to metamaterial regime.

Here we use inverse dispersion method [2] to construct phase diagram of silicon-based periodic photonic structures. The inverse dispersion method allows us to calculate the complex band diagram by evaluating wave vector for a given frequency. First, we consider the structure with complex dielectric permittivity with a weak imaginary part $(1 + 0.002i)\varepsilon$. The dispersion diagram changes dramatically its properties at certain parameters of structure. Namely, we find that for each radius-to-period r/a ratio it exists a critical value of permittivity ε_c. For $\varepsilon < \varepsilon_c$ the plot of the real part of dispersion diagram (real part of wave vector k vs. ω) demonstrates a crossing behavior and for $\varepsilon > \varepsilon_c$, the regime is changed to anti-crossing. The critical permittivity ε_c appears to be in excellent agreement with the phase diagram constructed by standard method [1]. Therefore, the inverse dispersion method is applicable for construction of phase diagram. Next, we study the square lattice of silicon nanorods. Since silicon has frequency-dependent $\varepsilon(\omega)$, the Maxwell equations are not scalable as in constant ε approximation used in the paper [1]. We fix the cylinder radius and vary the lattice period to find the change between the crossing to anti-crossing regimes. Also we obtain the corresponding wavelength. The phase diagram in axes wavelength and cylinder radius is shown in Fig. 1. The domain of parameters for metamaterial regime are shown by shading.

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Full-wave analysis of a periodic two-layer strip dipole array near-field energy

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Integral equation based on Floquet theorem using spectral Green's tensor functions for surface current of a periodic two-layer array thin strip dipole in free space is formulated. Expression for calculation stored electric and magnetic energies, quality factor of array element is obtained as a result of the integral equation solution by the Galerkin method and a subsequent calculation of reactive and active energy. The current is expanded in a set of arbitrary piecewise-sinusoidal basis functions. The array elements can be excited both by an external field and by lumped sources.

Simulated result shows that it is possible to achieve a significant decrease in narrowband infinity array elements quality factor for two-layer array with different types of excitation. The possibility of increasing the array operating frequency band by optimizing the mutual coupling, when the array period is decreased, as it is considered in [1, 2], is confirmed by results obtained for two-layer array with external incident field excitation. Thus, close-packed antenna arrays is possible to be considered to be promising in terms of construction reflector array and feedthrough array.

Finite feedthrough array simulation on the basis of printed and slotted elements, obtained using techniques described, shows that packing density of vibrating printed elements increase in turn increases the operating frequency range by 2 times. In addition, for slotted multilayer finite feedthrough array it is also possible to achieve an expansion of the operating frequency band with increasing packing density. The obtained gain frequency dependences for geometries predetermined on the basis of a compromise between the value of the reflection coefficient, the range of phase adjustment, and the minimization of the of the element quality factor in Floquet cell confirms the expediency of using the proposed method for the rapid feedthrough array characteristics evaluation.

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Finite element modeling of ultrasonic waves propagation in the material with considering different mechanical properties in the thin surface layer

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Acoustoelasticity is modern method of NDT, based on proportionality of acoustic anisotropy and elastic stresses [1, 2]. Acoustic anisotropy is the relative difference between the speeds of two perpendicularly polarized transverse ultrasonic waves. In case of the inelastic stress-strain state there is a lot of different factors are affecting on value of acoustic anisotropy: plastic deformation, damage of material, plastic stresses, etc. According our studies, one of the most significant influence on the value of acoustic anisotropy is exerted by structural changes of material in thin surface layer [3, 4]. The speed of propagation of transverse ultrasonic waves depends of two mechanical properties — shear modulus and density. Modeling of this effect is the main goal of current work.

Direct numeric simulation of propagation of transverse ultrasonic waves in system "elastic layer – piezoelement" was performed. Thin surface layer with different mechanical properties (density and shear modulus) was considered in elastic layer. Simultaneous consideration of piezoelectric equations and dynamic equations was performed. Full stress-strain state of system was performed. Time delay of transverse ultrasonic waves' propagation was measured. Dependencies of time delay on deviation of surface layer properties were defined. The model correctly describes presented experimental data.

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Small-amplitude steady water waves with vorticity

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We consider the nonlinear problem of steady gravity-driven waves on the free surface of a two-dimensional flow of an incompressible fluid. The surface tension is neglected and we assume a flow to be rotational and also allow interior stagnation points. We will discuss a recent progress in the theory of small-amplitude steady water waves with vorticity. Such waves are small perturbations of uniform streams with a several counter-currents. In contrast to the irrotational case of zero vorticity, when only Stokes and Solitary waves exist, the class of small-amplitude waves with vorticity includes multi-modal and non-symmetric waves and possibly waves with even more complicated geometry.

Empiric investigations on bio-chemical networks of the growth of complexity parameters for the tropical equilibration problem

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Bio-chemical networks are bi-partitite graphs involving the reacting species and the reactions as vertices. These can range from a few for so called network motifs up to several thousands (e. g. for networks reconstructing yeast metabolism or human metabolism). Assuming well-mixing and mass action kinetics the dynamics of the networks is given by a system of ordinary differential equations with polynomial vector fields. Whereas a priori only little special structure is known to reduce the complexity of several computational tasks on the the given systems, it has recently been shown that some complexity parameters are only growing slowly.

In this talk we will focus on computing tropical equilibrations, which are important for several purposes, e. g. for model reduction, but is NP-complete in general. Performing computations on the BIOMODELS database we found that the number of maximal solution polytopes is much smaller than had to be expected. We will discuss computational heuristics to limit the growth of intermediate solution polytopes.

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Scattering of acoustic waves by an impedance sector

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In this work we developed a motivated procedure to study the problem of diffraction by a semi-infinite sector with impedance boundary conditions. Such kind of the boundary conditions thwarts complete separation of the variables. The separation of the radial variable in the boundary conditions leads to a condition which is non-local with respect to the parameter of separation. Nevertheless, after separation of the radial variable the problem on the unit sphere with the non-local condition on the cut AB admits an efficient study. To that end, the traditional theory of extension of sectorial sesquilinear forms has been exploited. The Watson–Bessel integral representation for the solution is not efficient for the derivation of the far field asymptotics that is why reduction to the Sommerfeld integral representations was used. Analytic properties of the Sommerfeld transformants were considered. In particular, domains, where singularities of the transformants are localized, were indicated. Although a complete study of the singularities and of their contributions to the far field asymptotics is postponed to a forthcoming publication, we obtained a practically useful simple formula for the diffraction coefficient of the spherical wave from the vertex in 'oasis' in the case of scattering by a narrow impedance sector.

Multi-spectral amplitude and phase measurement by means of acousto-optic diffraction in interference schemes

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Interferometric techniques are one of the main tools for the analysis of the wavefront amplitude and phase structure. Such methods as quantitative phase microscopy, optical coherence tomography, digital holography are now widely spread in biomedicine, deformation analysis, non-destructive testing. Tunable spectral filtration in interference schemes is used as for contrast visualization of particular object properties (multiple-wavelengths narrow-band phase mapping), so for the collection of hyperspectral data in order to calculate the dependance of reflectance signal on the depth (spectral-domain optical coherence tomography). In this paper, the advantages and possible applications of acousto-optic (AO) light filtration in interference schemes are discussed. An applicability of AO filters to Michelson, Mach–Zehnder, tau- and other types of interferometers is shown. The results of multiple original experiments are presented. It is demostrated that AO filtration of interference signals provide non-contact and fast registration of information about the spatial distribution of amplitude and phase of the wave reflected from or transmitted through the object in arbitrary spectral intervals.

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Modeling of sound propagation in the ocean by means of random matrices

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Problem of sound propagation in the randomly-inhomogeneous oceanic waveguide is considered. We consider a recently developed promising approach [1, 2] for modeling of sound propagation. The approach is based on the usage of the random matrix theory (RMT). Using spectral analysis of a wavefield propagator [3], we examine validity of the RMT-based approach. It is shown that spectral properties of the propagator constructed by means of the RMT-based approach are basically consistent with direct solutions of the standard parabolic equation. We have detected discrepancy only for low frequencies and relatively short distances, when long-lasting cross-mode correlations are significant [4].

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Hybrid perovskite nanoparticles

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Film formation of Organic-inorganic perovskites of methylammonium lead trihalides ($MAPbI_3$) and the optimized perovskite photovoltaic architectures allowed conversion efficiency increase to 22.1%. New applications of this materials have been investigated, including light emitting diodes (LED) and semiconductor optical amplifiers and lasers [1].

Also, metal halide perovskite solar cells can rival with traditional crystalline solar cells. There are instability issues due to electrical, atmospheric, heat, and light stresses is provided. That is why one of main problem connect with concern the long term operational stability this materials [2].

In this work we demonstrate the possibility of deposition perovskite nanoparticles (NPs) (Fig. 1a and 1b) by laser printing method [3] for revival perovskite. For comparing photoluminescence efficiency, we choose NPs with different diameter(Fig. 1f). The basis for proving the revival of perovskite was comparison average life time from "old" $MAPbI_3$ film (Fig. 1c) and printed perovskite NPs (Fig. 1d). Experimental measurements show the increase average lifetime of perovskite NPs.

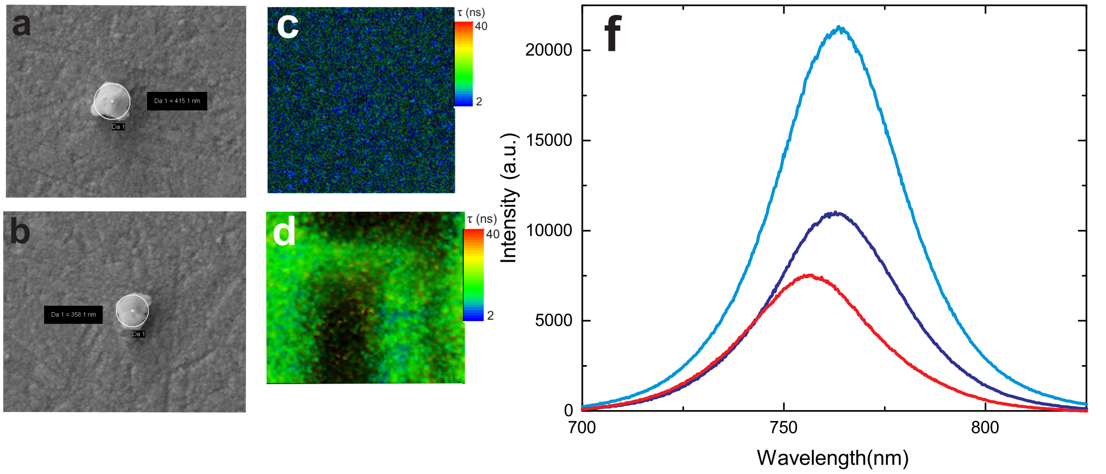

Fig. 1: SEM image (a, b) of resonant perovskite nanoparticles. Maps of average lifetime surface "old" perovskite sample (c) and perovskite nanoparticles on gold film surface (d). Photoluminescence spectra from different diameter perovskite nanoparticles.

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Kerr waveguide and microstructures formation under middle IR femtosecond laser irradiation of germanium

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Formation of the micro- and nanogratings under femtosecond laser irradiation of semiconductors is of current subjects of laser-matter interaction. Among the not fully understood problems is the gratings (**g**) formation of abnormal orientation $\mathbf{g} \perp \mathbf{E}$, where $\mathbf{E}$ is the tangent component of electric field strength vector of laser radiation. Recently the production of microstructures of abnormal orientation have been observed under linear polarized series of laser pulses irradiation of germanium surface with $\tau = 150$ fs and $\lambda \sim 3$ µm [1, 2]. The essential peculiarities of produced gratings were the following: the periods of the structures were incident angle independent; the one-pulse threshold laser power density was lower than the melting laser threshold; two distinct periods were produced:

with $d_1 = \lambda/n$ and $d_2 = \lambda/2n$, where n is germanium's refractive index. The set of the observed peculiarities allow us to suggest a new model of microstructures formation based on the optical Kerr effect waveguide formation and interference process with mutual waveguide modes (WM) interference (d_2) and WM and incident laser radiation interference (d_1). Abnormal grating orientation is due to fact that most lossless WM are the TE type ones. Really the rough estimate for $q \approx 0.4 \cdot 10^{13}$ W/cm^2, $\lambda = 3$ µm gives $\Delta n = n_2 q \approx 1.6$, where the Kerr coefficient value $n_2(\lambda = 3\ \mu\text{m}) \approx 4 \cdot 10^{-13}$ cm^2/W. Taking the nonequilibrium free carriers concentration $n_{el} = 4 \cdot 10^{20}$ cm^{-3} $< n_{rc} \approx 5 \cdot 10^{20}$ cm^{-3}, one can obtain the typical attenuation length of laser radiation into germanium, $l \sim 5$ µm. The zero TE mode existence condition has the form $\Delta n \cdot l \geq 9\lambda/16$ and is fulfilled for our case ($\Delta n = 1.6$, $\lambda = 3$ µm, $l = 5$ µm). The produced asymmetric gradient waveguide is multimode one. So the periods of produced main gratings are in the range:

$$\frac{1}{n + \Delta n} \leq \frac{d_1}{\lambda} \leq \frac{1}{n}, \tag{1}$$

where n is the refractive index of germanium at laser radiation wavelength. This theoretical result is well coincides with experimental data [1, 2].

In conclusion, the new model of field waveguide formation is suggested which well explains the experimental results for germanium microgratings formation under femtosecond laser irradiation of mid-IR spectral range.

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Focusing and defocusing of reflected light beams from chirped dielectric layered structure

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Reflective properties of a volume chirped Bragg gratings make it possible to compress and stretch short light pulses effectively [1]. Although a mechanism of compressing or stretching light pulses is the same for both normal and oblique ways of incidence on a chirped grating, when we deal with spatial beams, a lateral shift [2] and the more complicated change of the phase front of the reflected beam at oblique incidence occur [3]. Along with that, an efficiency of the compressing and stretching of light beams in such systems essentially depends on diffraction.

In the present paper we study in detail some peculiarities of Gaussian beam reflection from the chirped dielectric two-component layered structure. The period of such chirped structure linearly deviates along the propagation direction. In turn, deviation leads to the appearance of local band gaps not only in frequency and in angular spectra but also in the quasiperiodic structure itself along the propagation direction — so called photonic barriers.

With the help of numerical simulation we calculated general lateral shifts of reflected light beams with narrow angular spectra and demonstrate that the shifts depends on the positions of photonic barrier in the quasiperiodic stricture. Focusing and defocusing of light beams with wide angular spectra after reflection from the chirped grating are shown. We discuss also the efficiency of chirped grating application for compressing and stretching of light beams with small angular divergence.

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Asymptotic behaviour of the singular values of variable Toeplitz matrices

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We describe asymptotic spectral properties of sequences of variable-coefficient Toeplitz matrices. These sequences, $A_N(a)$, with a being in a Wiener type algebra and defined on an annular cylinder, widely generalizes the sequences of finite sections of a Toeplitz operator. We prove that if the function a does not vanish then the singular values of $A_N(a)$ have the k-splliting property, which means that, there exist an integer k such that, for N large enough, the first k-singular values of $A_N(a)$ converge to zero as $N \to \infty$, while the others are bounded away from zero, with k being the sum of the kernel dimensions of two Toeplitz operators.

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Terahertz rectification by hybrid plasmon modes in periodic graphene structure

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Unique properties of graphene such as high mobility and drift velocity of charge carriers allows for utilizing graphene plasmons for rectification terahertz (THz) radiation in graphene micro- and nanostructures [1]. Due to nonlinear properties of plasma waves in graphene [2] there are two different physical mechanisms of THz plasmonic rectification which caused by the effects of differential plasmonic drag and electron-hole plasmonic ratchet [3].

The structure under consideration is consist of a graphene monolayer screened by an interdigitated metal dual grating-gate (DGG) (Fig. 1(a)). Two sub-gratings of the DGG are laterally shifted in respect to each other in order to introduce an asymmetry into the unit cell of the periodic DGG graphene structure.

The plasmonic rectification of THz radiation in graphene structure is defined by both geometrical asymmetry of the structure and asymmetry of the plasmon electric field. The spatially asymmetric electric field of plasmons is achieved in the weak-coupled anticrossing regime of plasmon modes originating from different plasmon cavities [4]. Such hybrid plasmon modes can be exited in DGG graphene structure. Figure 1(b) demonstrates the calculated THz responsivity of DGG graphene structure. The resonant peaks at Fig. 1(b) correspond to excitation of hybrid plasmon modes under gate fingers. In such a case, the current responsivity can achieve the value of 20 A/W for small s_1/s_2 which is more than an order of magnitude greater than the responsivity of the plasmonic detector based on GaAs/AlGaAs field-effect-transistor array [5].

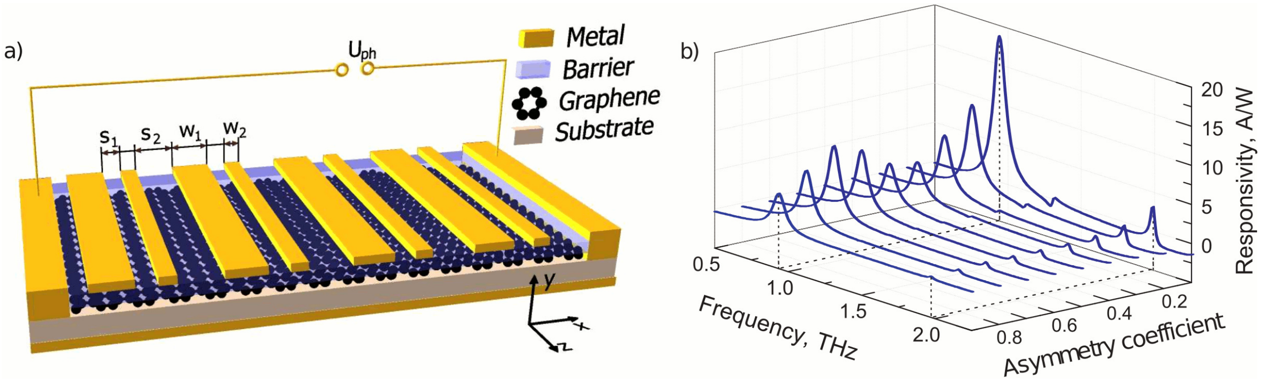

Fig. 1: (a) Schematic view of the DGG graphene structure; (b) Current responsivity vs. frequency and asymmetry coefficient s_1/s_2. Parameters of the calculations are $w_1 = 1$ µm, $w_2 = 0.25$ µm, $s_1 + s_2 = 0.375$ µm, and $\gamma = 5 \cdot 10^{12}\ s^{-1}$.

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Thermoelastic waves propagation in gas media considering heat relaxation

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This paper presents numerical approach in solution of laser radiation on thin layer of gas medium. Aim of article is to investigate coupled thermal and acoustic waves propagation accounting heat flux time relaxation and therefore damping and finite velocity of thermal wave due to hyperbolicity in heat conduction's equations. Spatial description is used in order to describe continuum fields. Density, temperature, heat flux and velocity are evaluated through the system of balance equations: mass, energy and momentum balances are taken in integral form. High speed thermal impact is modeled by defining the distribution of heat sources in the volume for the semitransparent medium. The power of the laser pulse depends on time as the Dirac delta function or as the Heaviside function do.

Number of problems with different speed of air is considered which leads to different boundary and initial conditions. For numerical calculation both explicit and implicit techniques' scheme are applied; in attempt to verify method similar issue of laser interaction on solid continuum was chosen that have analytical solution to compare with.

Plasmonic nanolaser based on a hybrid mode of plasmonic crystal

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The important field of nanoplasmonics is a development of nanoscopic coherent source of light. Such devices can be build through the compensation of SPP losses by amplification of SPP by stimulated emission of radiation [1]. Plasmonic nanolasers were realized in different experimental schemes. The important one is based on the use plasmonics crystal to create a coherent source of SPP in the most challenging spectral region in nanoplasmonics — visible spectral range [2].

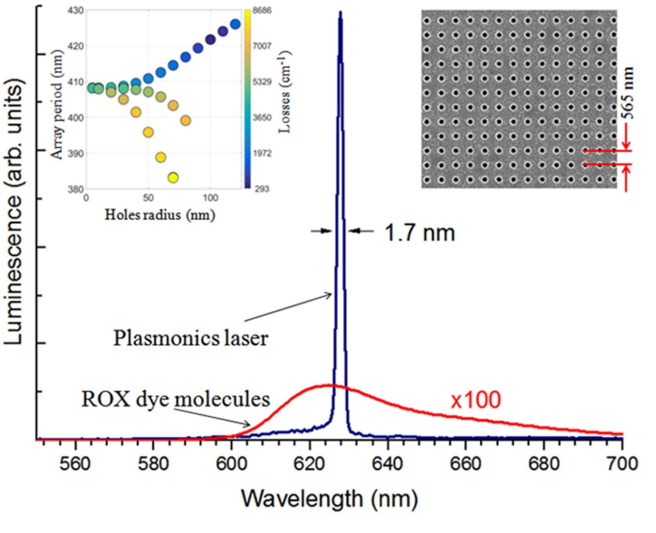

Fig. 1: Emission spectrum of plasmonic nanolaser radiated perpendicular to the surface of SPP crystal: above threshold (blue line), below threshold (red line). Inserts: (Right) SEM image of the plasmonic crystal; (Left) plasmonic crystal losses for brigh and dark plasmonic modes.

The implementation of plasmonic nanolaser, which allows the generation of coherent single spatial hybrid SPP mode in the visible spectral range at room temperature remains a challenge. We have demonstrated the realization of the nanolaser using plasmonic crystal and ROX dye molecules as an active medium. The crystal is based on an array of nanoholes made in the silver film. Ultra low threshold of the plasmonic nanolaser is implemented using a careful design of plasmon-photon hybrid mode.

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Asymptotic analysis of viscous plate model

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There are many fields of engineering and natural sciences where slender viscous objects occur. In particular, they are essential in glass and polymer manufacturing, arise in geological science (lithosphere flow) and biology (cell membrane).

The presented work demonstrates asymptotic approach for deriving simplified equations of dynamics of a nearly planar thin viscous sheet — viscous plate. In contrast to elastic plates, which has been studied for the last two centuries, the viscous analog attracted attention much later. Historically, elasticity models exploit some *a priori* form of the plate displacements and use variational principles to obtain governing equations, while viscous problems are mostly based on formal asymptotic expansions of Navier–Stokes equations. All asymptotic methods start with scaling analysis of the problem. There are two the most used scalings: Trouton [1], which is appropriate for stretching of a viscous sheet, and BNT [2], which arises in a bending problem. There were some successful attempts to incorporate these scalings into one model [3–5].

We present a formal procedure of matching two asymptotic solutions for stretching and bending problem. In addition, the developed model includes surface tension forces and inertia terms. The obtained nonlinear governing equations represent incompressibility condition, in-plane stress balance, and bending moment balance. Furthermore, they incorporate coupling of in-plane stresses with out-of-plane velocity. The developed model appear equivalent to von Karman model of elastic plate with large deflections, which is based on nonlinear elasticity laws [6].

The solution of the governing equations was calculated numerically by FEM for a few plate configurations.

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Spectral problem for Dirac operator for a bent chain of nanospheres

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Modern nanotechnologies allow constructing new structures such as nanotube filled with fullerene molecules [1–3]. These structures can be considered as nanospheres chains. In order to apply them in nanoelectronics, spectral properties of an electron inside have to be investigated. We suggest a simplified model which helps to carry out spectral analysis of a relativistic electron placed into chain structure. The model is based on the theory of self-adjoint extensions of symmetric operators (see, e. g., [4]). An analogous approach has been already used for non-relativistic electron [5]. The system's periodicity is taken into account by the Bloch approach or transfer-matrix approach.

We consider a bent chain which consists of nanospheres connected through one-dimensional straight wires. The contact points on spheres are assumed to be at opposite positions, except one sphere where bending occurs (one contact point is supposed to be rotated by angle γ). A sphere with a connected wire are assumed as the lattice cell. This system generates a hybrid manifold (2D spheres and 1D segments). The Dirac operator on segment has the form

$$H_l = ic\frac{d}{dx} \otimes \sigma_1 + \frac{c^2}{2} \otimes \sigma_3,$$

while on sphere we have the following operator

$$H_s = -i\hbar\sigma_1 \left(\partial_\theta + \cot\theta/2\right) - i\hbar c\sigma_2\partial_\phi/\sin\theta + Mc^2\sigma_3,$$

where σ_i $(i = 1, 2, 3)$ are the Pauli matrices, c is the speed of light, $\hbar$ is Plank's constant, M is the particle mass, $\partial_\theta = \partial/\partial_\theta$, $\partial_\phi = \partial/\partial_\phi$. The coupling between sphere and segment is described by the operator extensions theory method [5–7]. First, restriction of the initial operator on the set of functions vanishing at the contact point gives a symmetric operator. Then, its self-adjoint extension generates the model of coupled sphere and segment. The Bloch condition at the endpoints of the cell are considered in order to describe periodic case.

The solution on the segment is represented as linear combination of exponents and on the sphere is thought as a linear combination of Green's functions for Dirac operator [7]. The coupling conditions imply linear system on the solution's coefficients. The solvability condition of this system gives spectral equation. It allows investigation of the both continuum spectrum and discrete spectrum. The latter is caused by bending of the chain.

This work was partially financially supported by the Government of the Russian Federation (grant 074-U01), by grant MK-5161.2016.1 of the President of the Russian Federation, by grant 16-11-10330 of Russian Science Foundation.

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Homogenization of nonstationary periodic systems in a bounded domain: L_2-operator error estimates

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Let $\mathcal{O} \subset \mathbb{R}^d$ be a bounded domain of class $C^{1,1}$. In $L_2(\mathcal{O}; \mathbb{C}^n)$, we consider matrix selfadjoint second order differential operator $B_{D,\varepsilon} = b(\mathbf{D})^* g(\mathbf{x}/\varepsilon) b(\mathbf{D}) + \sum_{j=1}^{d} \left(a_j(\mathbf{x}/\varepsilon) D_j + D_j a_j(\mathbf{x}/\varepsilon)^*\right) + Q(\mathbf{x}/\varepsilon)$, $\varepsilon > 0$, with the Dirichlet boundary condition. The coefficients of $B_{D,\varepsilon}$ are periodic with respect to some lattice in $\mathbb{R}^d$ and depend on $\mathbf{x}/\varepsilon$. So, they oscillate rapidly, as $\varepsilon \to 0$. The matrix-valued function $g(\mathbf{x})$ is assumed to be bounded and uniformly positive definite. Next, $b(\mathbf{D})$ is a first order differential operator with constant coefficients. The symbol $b(\boldsymbol{\xi})$ is subject to some condition which ensures strong ellipticity of the operator $B_{D,\varepsilon}$. The coefficients a_j and Q belong to suitable

L_p-spaces on the cell of periodicity. Adding an appropriate constant to the potential Q, we assume that $B_{D,\varepsilon} > 0$.

Homogenization of the resolvent $(B_{D,\varepsilon} - \zeta I)^{-1}$ was obtained in [3, Theorem 10.1].

Theorem 1 [3]. *Let $\zeta \in \mathbb{C}\backslash\mathbb{R}_+$, $\zeta = |\zeta|e^{i\phi}$, $|\zeta| \geqslant 1$. Let $c(\phi) = |\sin\phi|^{-1}$ for $\phi \in (0,\pi/2)\cup(3\pi/2,2\pi)$ and $c(\phi) = 1$ for $\phi \in [\pi/2, 3\pi/2]$. Then, for sufficiently small $\varepsilon > 0$, we have*

$$\|(B_{D,\varepsilon} - \zeta I)^{-1} - (B_D^0 - \zeta I)^{-1}\|_{L_2(\mathcal{O})\to L_2(\mathcal{O})} \leqslant C_1 c(\phi)^2 \varepsilon |\zeta|^{-1/2}. \tag{1}$$

Here B_D^0 is the effective operator with constant coefficients. The constant C_1 is controlled in terms of the problem data. For a fixed ζ, estimate (1) is of the precise order $O(\varepsilon)$.

Approximation valid in a larger domain of the spectral parameter ζ is obtained in [3, Theorem 9.1]. The error estimate in this approximation has another behavior with respect to ζ.

Our goal is to deduce the results for nonstationary problems from the elliptic results.

Theorem 2. *For $t \in \mathbb{R}$ and sufficiently small $\varepsilon > 0$, we have*

$$\left\|\left(\cos(tB_{D,\varepsilon}^{1/2}) - \cos(t(B_D^0)^{1/2})\right)(B_D^0)^{-2}\right\|_{L_2(\mathcal{O})\to L_2(\mathcal{O})} \leqslant C_2\varepsilon\left(1 + |t|^5\right), \tag{2}$$

$$\left\|\left(e^{-iB_{D,\varepsilon}t} - e^{-iB_D^0 t}\right)(B_D^0)^{-2}\right\|_{L_2(\mathcal{O})\to L_2(\mathcal{O})} \leqslant C_3\varepsilon(1 + |t|). \tag{3}$$

The constants C_2 and C_3 are controlled in terms of the problem data. For a fixed t, estimates (2) and (3) are of the precise order $O(\varepsilon)$.

Results of this type are called operator error estimates in homogenization theory. To prove estimate (2), we apply the inverse Laplace transform and the results on homogenization of the resolvent with two-parametric (with respect to ε and ζ) error estimates. Inequality (3) follows immediately from (1) and [1, Lemma 19.1].

In $L_2(\mathbb{R}^d;\mathbb{C}^n)$, homogenization of nonstationary systems for the operator $A_\varepsilon = b(\mathbf{D})^* g(\mathbf{x}/\varepsilon) b(\mathbf{D})$ was studied in [1, 2, 4]. In [1], estimates of the order $O(\varepsilon)$ for the norms $\|e^{-itA_\varepsilon} - e^{-itA^0}\|_{H^3(\mathbb{R}^d)\to L_2(\mathbb{R}^d)}$ and $\|\cos(tA_\varepsilon^{1/2}) - \cos(t(A^0)^{1/2})\|_{H^2(\mathbb{R}^d)\to L_2(\mathbb{R}^d)}$ were obtained. In [2, 4], it was shown that these results cannot be improved with respect to the type of the operator norm. So, it seems that inequalities (2), (3) are not sharp with respect to the type of the norm. But, up to the author's knowledge, there were no results at all on operator error estimates for nonstationary systems in a bounded domain.

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Inverse data for the acoustical and quantum scattering problems for the Schrödinger operator on the half line

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We consider inverse quantum and acoustical scattering problems for the Schrödinger operator on the half line. The goal will be to establish the connections between inverse data for these problems. The central objects which serve as a source for all formulas are the kernels of so-called connecting operators and Krein equations.

Some explicit asymptotic formulas for 2D run up problem

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The Cauchy problem for 2D wave equation degenerating on the boundary of the domain is considered. Near the boundary characteristics (trajectories) are unbounded in the momentum variables. Recently the modified Maslov canonical operator was defined for this case and asymptotic formulas was obtained (see [1]). Based on these results an explicit asymptotic formulas are obtained near the velocity degeneration line. Using these formulas run up problem is studied for a special family of localized initial perturbations.

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Amplification of THz plasmons in graphene pumped by optical plasmons

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Creation of detectors, amplifiers and generators in terahertz (THz) frequency range is an actual problem of nanophotonics and nanoplasmonics [1]. Graphene, being a two-dimensional material with zero band gap, can be a basic material for amplifiers and generators of THz radiation [2–4]. An inverted energy distribution of charge carriers in graphene can be created by external pump. The direct optical pumping of graphene has been considered in [2–4]. More effective diffusion pumping of graphene was proposed in [5].

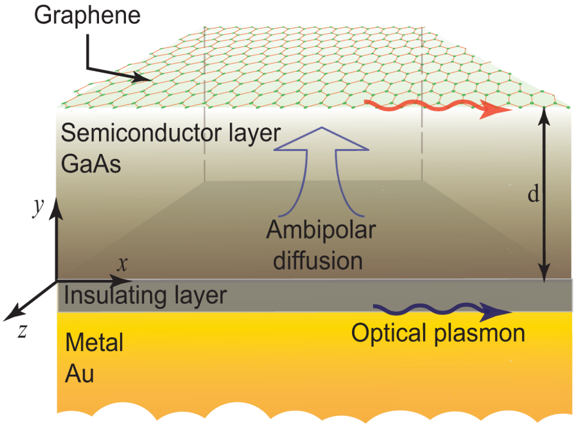

Fig. 1: Schematic view of the structure.

In this work, the amplification of THz plasmons in graphene with diffusion pumping by optical plasmons is investigated theoretically. Charge carriers exited in semiconductor layer (GaAs) by optical plasmons propagating on a metal (gold) surface diffuse to graphene creating the inverted carrier distribution in graphene (Fig. 1). An insulating layer prevents the transfer of charge carriers from the semiconductor to metal. The maximum value of the THz plasmon gain for diffusion pumping of graphene by optical radiation is achieved when the thickness of the semiconductor layer is about 2 microns, which is comparable to the diffusion length in the semiconductor layer [5]. When graphene pumped by optical plasmons, the maximum value of THz plasmons gain in graphene is achieved for a semiconductor layer thickness 180 nm, which is an order of magnitude less than the diffusion length in the semiconductor. This occurs due to strong localization of the optical plasmon near metal. Hence the charge carriers are exited in a thin layer of a semiconductor and almost all charge carriers reach graphene. As a result pumping of graphene by optical plasmons can reduce the pump power required to achieve the maximum value of THz plasmon gain in graphene by 30% as compared to diffusion pumping by optical radiation.

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Boundary value problems on gratings

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The present talk deals with boundary value problems for the Helmholtz equation issued from wave diffraction by periodic gratings with equal and different spacing widths. We briefly describe the formulation of the boundary value problems as convolution operators acting on Bessel potential periodic spaces and the equivalence to Toeplitz operators acting on spaces of matrix functions defined on composed contours. We analyse the Fredholm properties of the operators under consideration.

Application of perturbation theory to the problem of existence of backward Lamb waves

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The so-called backward waves, i. e. waves with energy transport in opposite direction to that of wave front movement, have attracted considerable attention in recent years in various fields of wave physics. A formal mathematical criterion for the existence of backward Lamb waves, found in isotropic plates in [1], was established by Mindlin [2]. According to that criterion, the existence of backward waves is determined by the competition between two factors represented by two terms in the asymptotic solutions of the secular equations [3]. One of them is the curvature of the slowness for bulk waves along a plate normal. The second factor is the interference field of longitudinal and transverse partial bulk waves which constitute Lamb modes. However there still remain unresolved questions about in-depth physical interpretation and understanding of the reasons for the occurrence of such waves in isotropic plates.

To clarify these issues, we use two approaches that have a novelty of application to this particular problem. First, the usual expansion of the secular equations for symmetric and antisymmetric Lamb modes in isotropic plates near the points of thickness resonances is used. However, unlike previous studies, these equations are represented in the form which includes explicitly the coefficients of mutual transformations of bulk longitudinal and transverse waves at oblique incidence on the free surface. The second approach used is the perturbation theory based on the divergence relation. The necessity of taking into account the quadratic term in the expansion is required from us to modify this known calculation procedure in the case under consideration.

The first mentioned approach makes it possible to understand that the significance of interference-field contribution is determined by (i) both two coefficients of mutual transformation of bulk waves (the transformation of longitudinal waves into transverse ones and vice versa), namely, their geometrical mean; (ii) the final result is given not simply by the value of this geometrical mean, but by its derivative with respect to the angle of deviation of the wave vector of partial bulk wave from a normal to the middle plane of the plate.

The second and more general approach, applicable both to isotropic and anisotropic plates, gives the final results of expansion of the secular equations near the points of thickness resonances in terms of energy fluxes. This allows us (i) to give a simple geometric interpretation of the appearance of the factor proportional to the curvature of the bulk wave slowness, (ii) to explain the importance of the interference term by polarization peculiarities that can make a contribution of the transformed mode into the secondary component of particle displacement vector to be dominant one.

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Plasmonic aluminium nanoparticles for enhancing the light absorption in silicon solar cells

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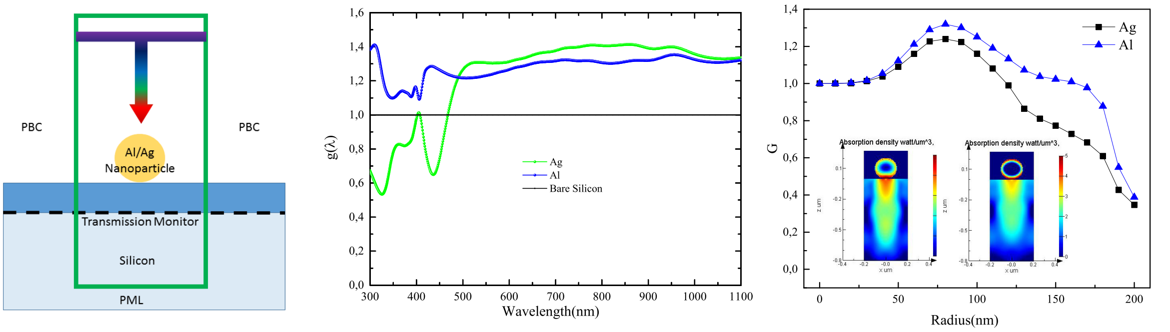
Plasmonics metal nanoparticles supporting localized surface plasmons (LSP) have great attention in research interest to improve the absorption by light trapping and energy conversion efficiency in solar cells. Traditionally the noble metals Ag and Au nanoparticles were widely studied and demonstrated materials due to their LSPR located in the visible range and also strongly interact with the solar intensity in this region. However, such noble metal nanoparticles on the front surface of solar cells always introduce attenuated light absorption in silicon at the short wavelengths due to the Fano effect, i. e., the destructive interference between the incident light and the scattered modes at the wavelengths below LSPR. Furthermore, the noble metals Ag and Au are impractical to utilize for large scale thin film solar cell due to the limitation of their high cost, limiting their real life applications. Alternatively, low cost and earth abundant material aluminum (Al) would avoid all these drawbacks due to their LSPR lying in the ultraviolet wavelength region, which potentially lead to very less negative impact to the solar cells performance caused by the Fano effect. In this work, we numerically investigated the light trapping behavior of varying Al nanoparticles diameter on the front surface of silicon wafer solar cells and compared with Ag nanoparticles by using FDTD method. The insights in this work Al nanoparticles proven a low cost and highly efficient enhancement of absorption in thin film silicon solar cell.



(a) (b) (c)

Fig. 1: Schematic diagram of simulation setup — (a), absorption enhancement vs wavelength for 100 nm diameter of different materials silver (Ag) and aluminium (Al) — (b), enhancement factor vs diameter for the same materials and absorption distribution in silicon (inset) — (c).

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On the spectral flow of a 2D discrete Dirac type operator and creation of electron–hole pairs in graphene

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The talk is based on ongoing joint work with M. I. Katsnelson (Radboud University, Nijmegen) and J. Brüning (Humboldt University, Berlin).

In topological terms, we compute the spectral flow of a family of Dirac type self-adjoint operators with classical (local) boundary conditions on a compact Riemannian manifold with boundary under the condition that the initial and final operators of the family are conjugate by a vector bundle automorphism. This result is applied to a family of such operators in a two-dimensional domain. The physical interpretation of nonzero spectral flow is discussed. The main part of the talk deals with establishing a relationship between this spectral flow and the spectral flows “near the Dirac points” of the lattice Hamiltonian describing electronic states in graphene.

The results have been partly published in [1]. The computation of the spectral flow is based on an application of the index locality principle (see [2]).

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Localized waves traveling along open waveguides in double-periodic junctions

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We consider a double-periodic lattice composed from massive bodies connected by thin long straight ligaments and make an asymptotic analysis of the Neumann problem for the Laplace operator to describe the geometrical characteristics of the band-gap structure of its spectrum. Furthermore, we examine changes in the spectrum when one or several semi-infinite rows of bodies and/or ligaments get perturbations. It is shown that these perturbations, called open waveguides, can produce new spectral bands in the spectrum of the perturbed Neumann problem. We also detect and classify waves localized near the open waveguides and decay exponentially in transverse directions.

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Superradiant enhancement from states with nonzero dipole moment

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Superradiance (SR) is an enhancement of collective spontaneous radiation of an array of emitters interacting with a common light field. For a subwavelength array of quantum emitters, this phenomenon was predicted by Dicke [1]. The Dicke model assumes that identical quantum emitters interact via the common field of their radiative response. Once excited, the emitters form a special Dicke state [2] and radiate a pulse. The dipole moment of a system of N excited identical two-level atoms is zero [3]. The duration of the SR burst is smaller than the radiation time of a single emitter by a factor of $1/N$. The intensity of the radiation is proportional to N^2 and the delay time of the SR peak $\log N/N$.

The Dicke explanation of SR is based on a strong assumption about the time evolution from one Dicke state to another (only in this case can one use the Fermi's golden rule) and on an ability of a quantum system with zero dipole moment to radiate a photon. There is no rigorous proof that these conditions are either necessary or sufficient for SR.

First, a more rigorous description of SR in terms of the master equation

$$\dot{\rho} = \frac{\gamma_0}{2}\left(2\hat{J}^-\rho\hat{J}^+ - \hat{J}^+\hat{J}^-\rho - \rho\hat{J}^+\hat{J}^-\right) \tag{1}$$

shows that the system evolution does not go through pure Dicke states but rather through mixed states with a density matrix which is a linear combination of density matrices of various pure Dicke states $\rho_{\text{mixed}} = \sum_n c_n |\psi_D\rangle\langle\psi_D|$ [2]. These mixed states still have a zero dipole moment. Second, SR is not the sole prerogative of quantum systems. It can also occur in an ensemble of nonlinear classical oscillators, which surely has a nonzero dipole moment [4]. In such a system, SR results from the constructive interference of long-period envelopes of rapidly oscillating dipoles [4].

We study the possibility of SR in an ensemble of two-level atoms in the general case, in which the system is not initially in a Dicke state, modeling the master equation (1). For a quantum system, there is a unified mechanism of SR for both Dicke states with zero dipole moment and non-Dicke states for which the total dipole moment is not zero. Moreover, there is an analogy between SR in quantum and nonlinear classical systems [4]. This analogy can be recognized by considering SR from non-Dicke states.

To conclude, we have studied the dynamics of quantum emitters interacting via their radiation field. In contrast to the Dicke model, in which all emitters are assumed to be in a state with zero dipole moment, the new SR regime arises in a more realistic system in which the initial state may have a nonzero dipole moment. We demonstrate that the Dicke state is not necessary for SR.

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Deep generative models for molecule development in oncology

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Recent advances in deep learning and specifically in generative adversarial networks have demonstrated surprising results in generating new images and videos upon request even using natural language as input. In this work we present an application of generative adversarial autoencoders (AAE) and variational autoencoders (VAE) for generating novel molecular fingerprints.

Optical phase transitions of multi-band periodic photonic structures

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Recently, our team introduced the concept of photonic phase transitions to describe the transformations between the regimes of metamaterials and photonic crystals [1]. We studied a square lattice of dielectric rods with the parameters changing continuously, and found that the photonic crystal regime is associated with the Bragg scattering. Each structural element (a dielectric rod) supports the Mie resonances associated with modes TE_{01}, TE_{11}, TE_{21} and other modes, and the frequencies of the Mie resonances depend on rod's parameters only. The TE modes are related to the magnetic response, and an all-dielectric metamaterial composed of rods exhibits negative magnetic permeability $\mu < 0$ [2]. Such a periodic structure turns into a metamaterial when the Mie resonance associated with the TE_{01} mode lays below the lowest-frequency Bragg resonance. In the paper [1], we constructed a phase diagram demonstrating the domains of the structure parameters corresponding to the case of the TE_{01} mode being the lowest resonant mode in the structure.

Here we study, both theoretically and experimentally, new effects in such photonic structures described by multi-band phase transitions related to the second-order Mie resonance and the TE_{11} mode. Our structure consists of identical dielectric rods with large dielectric permittivity ($\varepsilon = 80$) arranged in a two-dimensional periodic square lattice. In experiment, as material we use distilled water having high-index dielectric properties in microwave frequency range (1–3 GHz). Measurements are performed in an anechoic chamber with a pair of TE-polarized horn antennas. The geometry and a photo of our experimental setup and a sample are shown in Figs. 1(a, b), respectively. The radius-to-period ratio lays in the interval $r/a = 0.07$ to 0.3 that corresponds to a change of that lattice period a from 194 mm to 45 mm. These parameters are suitable to observe multi-band photonic phase transitions, and we report an excellent agreement between the theoretical results and measured transmission spectra.

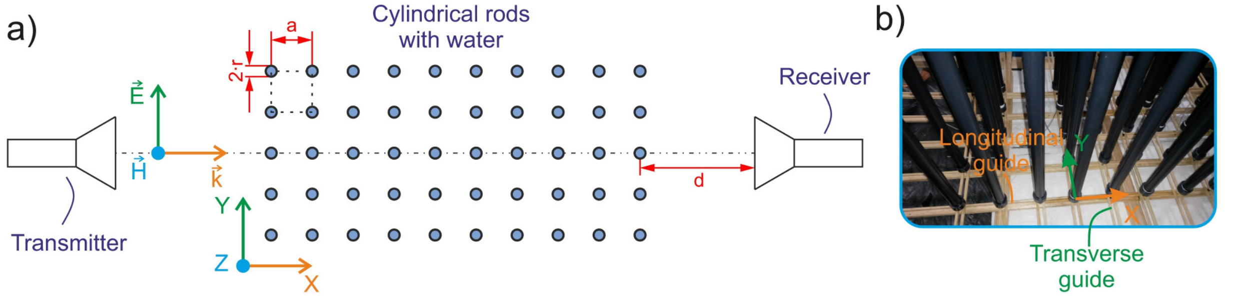

Fig. 1: (a) Schematic of the experimental setup. We arrange 5 by 10 rods in a square lattice placing the structure between a pair of horn antennas. (b) Photo of the experimental structure composed of a lattice of cylinders filled with water.

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Accounting of the influence of sphericity of real cavity mirrors on laser beam parameters

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In the literature, the optical elements of the cavity with spherical surfaces are considered practically indistinguishable from the parabolic surfaces [1, 2]. Since spherical mirrors are usually used [1, 2], the deviation of the shape of the surface of the optical elements of the cavity affects the characteristics of the formed beam. In our work, the wave aberration function of the mirror of the cavity with radius of curvature R_j at the point (x_j, y_j) is represented by

$$W_j(x_j, y_j) = -\frac{h_j^2}{R_j}\left(1 + \frac{1}{4}\frac{h_j^4}{R_j^2} + \frac{1}{8}\frac{h_j^6}{R_j^4} + \frac{5}{64}\frac{h_j^8}{R_j^6} + \dots\right), \quad h_j^2 = x_j^2 + y_j^2. \tag{1}$$

Here the first term corresponds to the paraxial approximation, and the rest to the aberrations of the mirrors of the 3rd, 5th and 7th orders, respectively.

This problem is solved by using the Fresnel–Kirchhoff integral [1, 2]. As can be seen from expression (1) the exponent of the degree of the integrand is a polynomial of the eighth degree with respect to the transverse coordinates (x_j, y_j). Such an integral is not expressed in elementary functions and can be calculated only by numerical methods. In order to carry out the analytical integration of expression, we cut down the polynomial (1). We use the Chebyshev polynomials that deviate least from zero on the canonical section $[0, +1]$ [3]. If we now neglect the second, third, and fourth order Chebyshev polynomials, then it is possible to perform the analytic integration of expression (including in finite limits). At the same time, the aberrations of mirrors and free space are taken into account in the quadratic term [3, 4].

Calculations show that for cavities that are close to confocal, concentric and plane configurations the parameters of the laser beam differ significantly from the beam formed by the ideal cavity due to aberration of spherical mirrors. This conclusion agrees with the results of [5]. Analysis of the G-diagram for cavities with aberrations allows us to make the following conclusions: 1) there are configurations of stable concentric (in the first quarter) and confocal (in the third quarter of the diagram) cavities; 2) small distortion in the field of the formed beam has non-confocal and non-concentric cavities far from the boundary of the curves characteristic for these cavities.

The developed method makes it possible to calculate the field distortion functions and the deviation of the parameters of a beam of an ideal cavity and with aberrations taken into account.

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Selective excitation of nanoparticles with vector complex source vortex beams

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Among the active fields of research in nanosciences are nanoparticles and their properties. The Mie theory was the very first description of light interaction with a particle and it was extended for describing the interaction with highly focused beams. The polarization properties of highly focused electromagnetic beams strongly influence the size and shape of the focal spot of the beams [1]. In recent publications, the interaction between such beams and nanoparticles has been investigated [2, 3]. The optical response of the nanoparticle is strongly dependent both on the particle location in the focal plane and on the polarization state of the beam, and it differs notably from that of the classical Mie theory [3]. An accurate analytical description of highly focused linearly, radially and azimuthally polarized light beams can be obtained via an extension of the so-called complex source beam (CSB) model [4]. Furthermore, the CSB model can be also employed for an accurate and unified vectorial description of highly focused vortex beams of various polarizations that are rigorous solutions of Maxwell's equations [5].

In this work, we theoretically investigate the interaction of vector complex-source vortices (CSV) with a spherical particle placed in the focal plane. We start by expanding CSVs analytically into vector spherical harmonics (VSHs). Such an expansion is essential for understanding the interaction of light with nano-objects such as atoms, molecules or particles. Those nano-objects locally respond to the various multipole components of the incident field. Normally, the dipole components are dominant, but nano-structures, such as (meta-) atoms are also capable of sensing quadrupole and even higher order excitations [6]. By knowing the expansion of optical beams into multipoles, it is straightforward to controllably excite various plasmonic resonanses inside a particle or in a cluster of particles. Moreover, this multipole approach also provides an efficient method for the exact nanointerferometric amplitude and phase reconstruction of tightly focused vector beams [7].

We report on the defocusing of highly focused CSVs as they interact with a spherical particle placed in the focal plane. The generalized Mie theory is used to investigate the scattering off a spherical gold particle in detail.

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Selective control of chiral response in clustered nanoparticles via material selection

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Optical chirality is a property of light that has been under studies in the past and which still attracts attention in the field of modern optics. Chirality is also property of certain natural or artificial materials, which enables their interaction with the spin angular momentum of the electromagnetic field [1]. A chiral material senses the handedness of the light and this allows for a different interaction with left- and right-handed circularly polarized light. In this fashion such optical phenomena as polarization rotation or circular dichroism (CD) do appear.

Creation of artificial structures, where chirality is controlled via shape and geometry, is enabled by modern micro- and nano-fabrication techniques [2]. Relatively strong chiral response can be achieved in single or densely packed micro or nanoscopic helices [3]. Optical response of single and clustered structures are controlled here via the size of a single helix, or with pitches and twists around their axis. A similar optical behavior can result also from purely geometrical properties of a three-dimensional arrangement of nanoobjects without chirality, such as nanospheres [4].

The optical response of the nanoparticle is strongly dependent both on the particle location in the focal plane and on the polarization state of the beam, and it differs notably from that of the classical Mie theory [5]. Moreover, the optical response of a clustered nanospheres should be considered using the so-called T-matrix [6].

Here, we analyze a novel approach to optical chirality, which is observed in a clustered two-dimensional nanostructure, made from single nanospheres of different sizes and materials. The resulting chirality is induced here by the choice of heterogeneous material composition of a particle assembly, where individual properties of the cluster's constituents breaks the symmetry of the cluster. We report here on investigation of such planar clustered structures using numerical approaches.

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Asymptotic analysis of the periodic in time non-steady Navier–Stokes equations in thin structures

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Thin structures are some finite unions of thin rectangles (in 2D settings) or cylinders (in 3D settings) depending on small parameter epsilon $\ll 1$ that is, the ratio of the thickness of the rectangle (cylinder) to its length. We consider the non-steady Navier–Stokes equations in thin structures with the no-slip boundary condition at the lateral boundary and with the inflow and outflow conditions with the given velocity of order one. The steady state Navier–Stokes equations in thin structures were considered in [1–3]. In the present paper we consider the non-stationary high frequency time periodic Navier–Stokes equations. The asymptotic expansion of the solution is constructed. The error estimates for high order asymptotic approximations are proved. The dimension reduction leads to a non-local in time Reynolds' type equation on the graph. This problem is analogous to the problem on the graph studied in [4, 5]. The present work is supported by the grant number 14-11-00306 of Russian Scientific Foundation executed in Moscow Power Energy Institute (Technical University).

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Beamforming network using waveguides for 5G FD-MIMO

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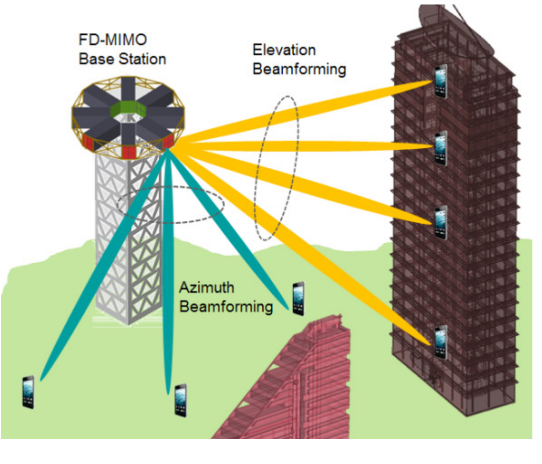
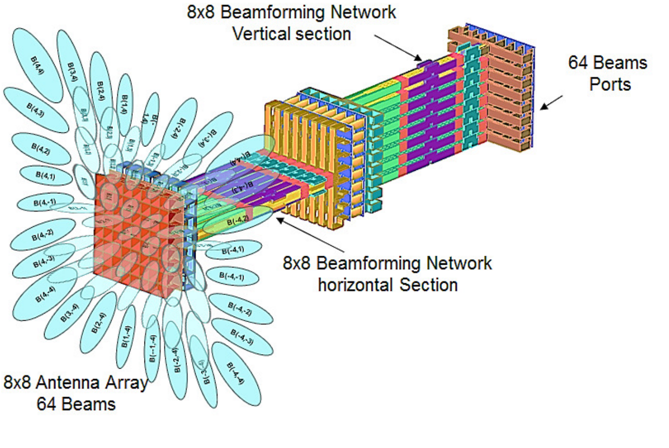
Fifth generation (5G) wireless communication is next major phase of mobile communications standards beyond the current LTE-Advanced. In order to meet 5G faster speed (1–10 Gbps), millimeter wave frequency band has been recommended for 5G wireless communication. Ka band has been targeted for short-range, high data rate communication. One major challenge in implementation of 29 GHz wireless communication is high path loss and to overcome this limitation, a high gain network is needed. Microstrip based networks are lossy at such high frequency and cannot handle high power. Waveguide based networks are appropriate in Ka band because of low loss, high power handling capability, and reliability. Due to the 5G demand for high data rate communication and treasure frequency bandwidth, phased array systems are found significant in wireless communications. Depending on the operating environment, phased arrays can adapt their radiation patterns as well as cancel out information in unwanted directions. Mobile communications networks use MIMO technology to achieve high data rates. In an FD-MIMO (Full Dimension MIMO) system, a base station with an active phased array antenna supports multi-user joint elevation and azimuth volumetric beamforming (3D beamforming), which results in much higher cell capacity compared to conventional systems. Beamforming network (BFN) is main component of phased array antenna. In the beamforming network, the Butler matrix has been used extensively over the years. The Butler matrix has N input ports and N output port and it consists of 90 degree hybrid couplers and fixed phase shifters. It is used to drive an array of N antenna elements. This talk discusses the current status of research in Beamforming network for Phased Array Antenna used in 5G communication system. A new design concept of a beam steerable high gain phased array antenna based on WR28 waveguide at 29 GHz frequency for fifth generation (5G) full dimension multiple input multiple output (FD-MIMO) system is discussed. The 8×8 planar phased array is fed by a three dimensional beamformer to obtain volumetric beam scanning ranging from -60 to $+60$ degrees both in azimuth and elevation direction. Beamforming network (BFN) is designed using 16 set of 8x8 Butler matrix beamformer to get 64 beam states, which control the horizontal and vertical angle. This is a new concept to design waveguide based high power three-dimensional beamformer for volumetric multibeam in Ka band for 5G application. The maximum gain of phased array is 28.5 dBi that covers 28.9 GHz to 29.4 GHz frequency band.





Multiple mechanisms of cardiac arrhythmias studied using anatomically accurate modeling

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Cardiac arrhythmias account for about 1 death in 10 in industrialized countries. Although cardiac arrhythmias has been studied for well over a century, their underlying mechanisms remain largely unknown. Over the years, several factors that favor arrhythmias initiation were established. Among them are ionic and dynamical heterogeneity, remodeling and fibrosis of cardiac tissue. In addition a lot of attention was given to the arrhythmias which occur due to channelopathies which result in the long QT syndrome. In my talk I will present our recent studies on role of these arrhythmogenic factors performed using anatomically accurate model of ventricles of the human heart developed in our group.

In particular, I will address possible role of small size ionic heterogeneities found the human heart on the onset and dynamics ventricular arrhythmias. We show that they can attract and anchor the rotors. Further I discuss role of heterogeneous fibrosis on the onset of cardiac arrhythmias. Our results show that spatial heterogeneity of fibrosis increases the probability of arrhythmia induction. We also show that properties of arrhythmias in such conditions are mostly determined by the maximal local fibrosis level. Based on that we hypothesize that it may be explained by the attraction of the rotors to the fibrotic scar. We further study this process and demonstrate that rotors indeed can be attracted by fibrotic scars via the process of dynamical reorganization of the excitation pattern which we call dynamical anchoring. We illustrate this process using patient specific model of the heart. Finally we have also studied arrhythmias which occur as a result of early after depolarizations (EADs). EADs occur in many forms the long QT syndrome or under the action of pharmacological agents and as a result of cardiotoxicity. In clinical situation they frequently induce the Torsade de Pointes (TdP) arrhythmia. We show possible spatial patterns of excitation which can occur in such systems and discuss their possible relation to the TdP mechanisms. We also present our analysis of recent experimental data on 3D excitation patterns during TdP obtained in the chronic AV block dog model.

3 wells. An example of 2-dimensional tunneling

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I consider a 2-dimensional Schrödiger equation with a potential having three nondegenerated minimae situated in the apices of an equilateral triangle. In this assumption I obtain splitting formulae for lower energy levels.

Large-time asymptotics of the fundamental solution to the diffusion equation in a periodic medium and its applications

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A linear parabolic equation with 1-periodic measurable coefficients is studied in $\mathbb{R}^d$, $d \geq 2$. We are interested in a large-time behaviour of the fundamental solution. Its approximations are found with point-wise and integral error estimates of order $O(t^{-\frac{d+j+1}{2}})$ and $O(t^{-\frac{j+1}{2}})$, $j = 0, 1, \ldots$, respectively, as the time t tends to $+\infty$. Zero and first approximations corresponding to $j = 0$ and $j = 1$ were written in [1]. For $j \geq 2$, we demand additionally that coefficients of the equation are of Lipschitz class. To construct these approximations we use

(i) the known fundamental solution to the homogenized diffusion equation (having constant coefficients) and, likewise, its spatial derivatives;

(ii) solutions to series of auxiliary problems on a periodicity cell which are formulated in a recurrent way.

These results on asymptotic behaviour of the fundamental solution are applied to obtain operator-type estimates of homogenization in operator L^p-norms, $1 \leq p \leq \infty$. There are also some corollaries to probabilistic estimates which can be considered as analogues of the well-known Berry–Esseen estimate concerning central limit theorem in probability theory.

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Acoustical and dynamic light scattering investigations of polymer porous films filled with a liquid crystal

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In this report we present the results of investigations of phase state and dynamic properties of composite LC material, which consists of a polymer polyethylene terephthalate (PET) porous films filled with a liquid crystal pentyl-cyano-bephenyl (5CB). Previous study of such composite media revealed a strong optical response on the applied electric field, which can be used in fiber optics and THz applications [1, 2]. The new results, described in this report, were obtained by usage of ultrasonic and dynamic light scattering spectroscopy.

The samples of porous films of thickness 23 µm with normally oriented cylindrical pores of a radius R ranging from 100 nm to 5000 nm were prepared by filling the track-etched PET membranes with a liquid crystal preliminary heated up to isotropic phase.

In the ultrasonic experiments the pieces of thin films were stacked in a multilayer (about 13–130 layers) sample of total thickness about 0.3–3.0 mm. It provided measurements of relative changes of ultrasonic attenuation (in the frequency range 3–10 MHz), induced by temperature variation. The analysis of the results of ultrasonic measurements made possible to estimate the typical values of the relaxation time in the system under investigation and compare them with the corresponding values, obtained for bulk samples of 5CB. It also made possible to get the value of the nematic-isotropic phase transition temperature T_{NI} for LC, confined by cylindrical pores.

The dynamic light scattering spectroscopy was used to study the nematic orientational fluctuations of confined LC samples. The correlation function and the corresponding relaxation time were obtained as functions of R and temperature T at slow enough cooling rates (0.3–0.6 K/h). It made possible to determine the dependence $T_{NI}(R)$ and to estimate the anchoring strength and the effective shear viscosity of LC confined in cylindrical pores.

This work was supported by Ministry of Education and Science of Russian Federation (project number RFMEFI58316X0058).

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Reflection of electromagnetic wave on a wired material prism

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In this work, we are going to investigate the reflective properties of prisms from wire media known as electromagnetic crystals. In paper [1] it was shows that a material consisting of a dielectric and periodic metallic inclusions, this material has a band structure of the reflection and transmission coefficients.

Samples for research were tetrahedral prisms of a dielectric and the same dielectric, with conductive rods placed in them. The cut off angle for the two types of samples was 45°. We measured the amplitude of the reflected electromagnetic radiation.

It has been shown experimentally that a sample with periodic wire inclusions has a significantly lower reflection coefficient in certain frequency regions than a sample of the same dielectric without wire inclusions.

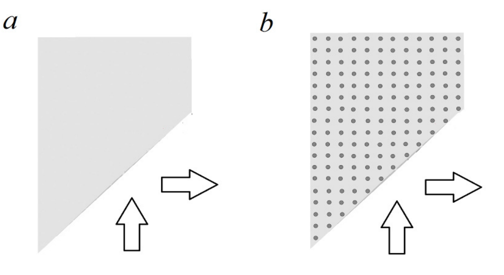

Fig. 1: Reflection coefficient measurement. a) Dielectric, b) Wire medium.

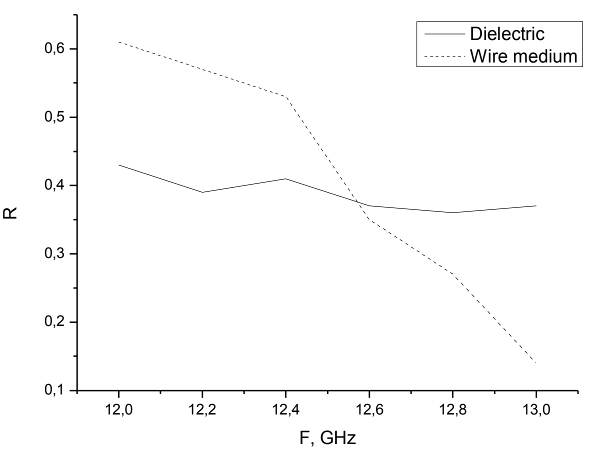

Fig. 2: Frequency dependencies of the coefficient of reflection of wire material and dielectric, angle of 45°

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Shifted and tilted Bessel–Gauss and Helmholtz–Gauss beams

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We consider the solutions of the paraxial parabolic equation

$$u_{xx} + u_{yy} + 2iku_z = 0, \tag{1}$$

having the form

$$u = A(x, y, z)G, \tag{2}$$

where

$$G = \frac{1}{q(z)} \exp\left\{\frac{ik}{2}\frac{(x-a_x)^2 + (y-a_y)^2}{q(z)}\right\} \tag{3}$$

with $q(z) = z - ib$, $b > 0$, some constants $a_{x,y}$, is the shifted fundamental mode, and the function A is known as the complex amplitude [1]. It was established in [2] (in the case $a_x = a_y = 0$) that functions (2) with

$$A(x, y, z) = \exp\left\{\frac{iK^2}{2kq(z)}\right\}\Psi(X, Y)\,, \tag{4}$$

satisfies to (1) if the function $\Psi(X, Y)$ is the solution of the Helmholtz equation

$$\Psi_{XX} + \Psi_{YY} + K^2\Psi = 0 \tag{5}$$

with respect to $X = (x - a_x)/q(z)$, $Y = (y - a_y)/q(z)$, here K is an arbitrary complex constant. Such solutions were named Helmholtz–Gauss beams, see [1–3]. If $K = 0$, we come to Laplace–Gauss beams [1, 2]. It is easy to see [4] that if constants $a_{x,y}$ are not real, then (3) becomes tilted Gaussian beam, and in this case the function (2) with (3),(4),(5) can be named tilted Helmholtz–Gauss beam.

In the case $a_{x,y} = 0$ and

$$\Psi = J_m(KR)\exp(\pm im\varphi)\,, \tag{6}$$

where φ is a polar angle, $R = \sqrt{X^2 + Y^2} = \sqrt{x^2 + y^2}/q(z)$ and J_m is the Bessel function of the first kind, we come to the Bessel–Gauss beams firstly obtained in [5]. Here we consider the generalization of this solution having a form (2)–(5) with shifted amplitudes

$$\Psi = J_m\left(K\sqrt{(X - X_0)^2 + (Y - Y_0)^2}\right)\cdot\left[\frac{(X - X_0) + i(Y - Y_0)}{(X - X_0) - i(Y - Y_0)}\right]^{\pm m/2}, \tag{7}$$

with arbitrary complex constants X_0, Y_0, where nonzero $a_{x,y}$ are also allowed. This formula presents a new family of exact localized solutions of (1), which includes asymmetric Bessel–Gaussian beams [6] (with $Y_0 = iX_0$, $a_{x,y} = 0$) and noncoaxial Bessel–Gaussian beams [7] (with $X_0 = -ia_x/b$, $Y_0 = -ia_y/b$, where $a_{x,y}$ are real) as subfamilies. The solution (2)–(4), (7) with zero-order Bessel function and $a_{x,y} = 0$ was considered in [8] for a case of media with quadratic refraction index.

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Paraxial Gaussian modes with simple astigmatic phases and nonpolynomial amplitudes

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We are concerned with the solutions of the paraxial parabolic equation $u_{xx} + u_{yy} + 2iku_z = 0$, having the form

$$u = A(x, y, z)G, \tag{1}$$

where

$$G = \frac{1}{\sqrt{q_1 q_2}} \exp\left\{\frac{ik}{2}\left(\frac{x^2}{q_1} + \frac{y^2}{q_2}\right)\right\}$$

with $q_j = z - z_j - ib_j$, $j = 1, 2$, is the fundamental simple astigmatic mode. Well known are solutions of the form (1) with A polynomial in x and y, such as Laguerre and Hermite modes. We present a family of more general solutions for the amplitudes A, involving an arbitrary function. In the axisymmetric case of $q_1 = q_2$, they reduce to generalized Bessel–Gauss modes found in [1].

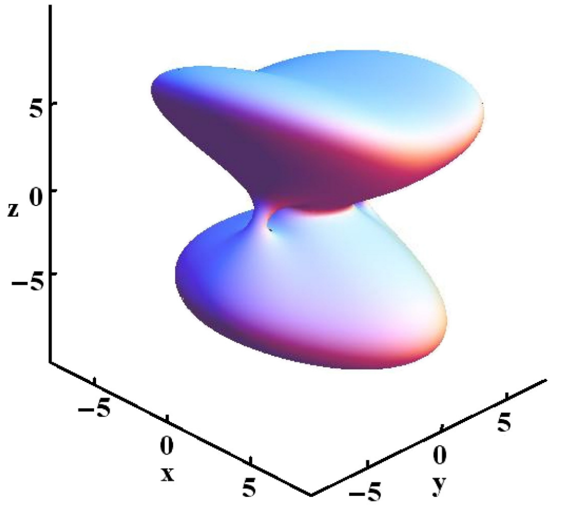

Fig. 1: Absolute values of one of novel solutions.

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A bundle-to-bundle mapping problem in geometric optics

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We consider a problem of transformation of light rays via mirror reflections in the framework of geometric optics. A parallel bundle of light rays is transformed into another (and codirectional) parallel bundle by several reflections from curved mirrors. This transformation induces a smooth map of two plane domains representing the cross sections of the original and the final bundles. The question is: which maps can be realized this way, and how many reflections are needed? We prove that (a) a gradient map can be realized by 2 reflections, (b) an orientation reversing map can be realized by 4 reflections, and (c) a general map can be realized by (at most) 6 reflections. This is a joint work with S. Tabachnikov (Penn State, USA) and D. Treschev (Academy of Science, Russia).

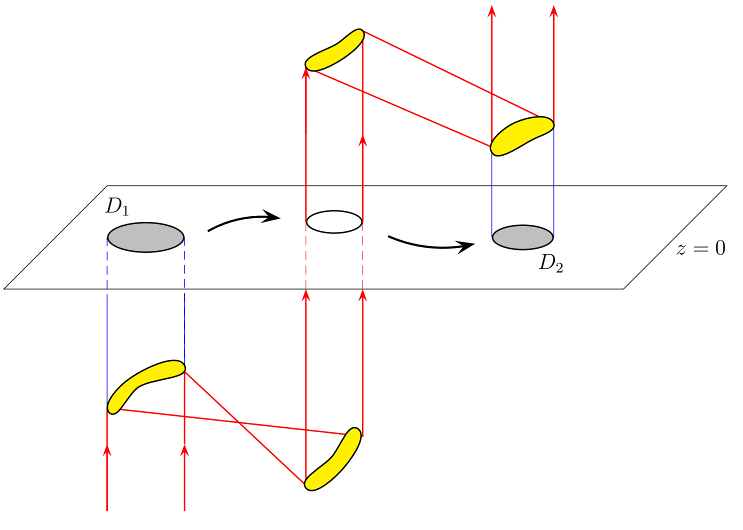

Fig. 1: A 4-mirror transformation of parallel bundles of rays induces a map from D_1 to D_2.

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On generation of an electromagnetic field in waveguides with metal walls by a source cross-sectional orientation

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It is shown that the longitudinal components of the electromagnetic field in the waveguide of an arbitrary section with metal walls can be expanded into series of the functions of the special form. These functions are the products of eigenfunctions for the Laplace operator with homogeneous conditions on the boundary of the section and the solutions to the telegraph equation.

The solution of the mixed boundary value problem for the telegraph equations is construed by two different methods. Firstly, the boundary value problem is reduced to the Volterra integral equation, and secondly, the solution to this problem is obtained in the explicit form.

The problem of generation of an electromagnetic field in the waveguide with metal walls by a source placed to the cross-section is transformed into two infinite sets for mixed boundary value problems for the telegraph equation with various coefficients.

Surface effect of the waves of plastic deformation and hydrogen distribution in metals

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The process of wave propagation in elastic-plastic media is accompanied by an oscillatory exchange between the potential and kinetic energy. The wave process decays rapidly if there is a large dissipation of energy in a continuous medium. In this connection, the paradox of the formation and propagation of plastic deformation waves under uniform deformation has not yet been unambiguously described. Meanwhile, the phenomenon itself has fundamental importance, since before the destruction of most structural materials, their plastic deformation practically always occurs. For the description of plastic deformation a lot of phenomenological models are used.

For the first time the effect of non-uniform plastic flow under uniform strain was discovered in the 19th century [1]. But it was systematically investigated by Portevin – Le Chatelier. The main mechanism leading to the appearance of the Portevin – Le Chatelier effect is the falling section on the stress-strain characteristic observed for some metals at the beginning of the plastic flow region. A lot of works are devoted to modeling the effect, which describe the formation of Luders bands. These bands, are sites of localization of plastic deformation on the surface of deformable metal as a result of the Portevin – Le Chatelier effect.

In the 80s of the 20th century [2] it was established that the process of forming the Luders bands has a wave nature and the term "plastic wave" was formulated. The available models do not explain the effect of propagation of a plastic wave.

In the samples studied by us, several periods of the plastic deformation wave were observed. The distribution of hydrogen is completely correlated with plastic deformation, where it is larger — there is increased hydrogen content. This effect is explained by structural changes in the metal accompanying the plastic deformation. An important feature is the first observed localization of changes in a thin surface layer of the studied samples. The thickness of this layer is estimated. The thickness of the critical layer depends on the magnitude of the plastic deformation. It is of the order of the characteristic grain size.

This suggests that the leading role in the formation of the Luders bands is played by the surface tension forces of the grains. These forces compete with the elastic forces of a continuous medium at the beginning of plastic deformation. A mechanism for the formation of a surface plastic wave is proposed.

Thus, new wave processes have been discovered that require model description and research.

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Modelling of modified Fabry–Perot cavities

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Negative effect of parametric oscillatory instability takes place in laser gravitational wave detectors by increasing of the stored power value in the Fabry–Perot (FP) cavity optical mode along with a huge number of optical modes with great Q-factor and small diffraction losses in spherical mirror's cavity [1, 2]. It is planning level of stored energy in interferometer arms of 0.8 MWt, but recently this effect was observed at 50 kWt [3].

For suppression parametric oscillatory instability it was suggested to use practically single mode FP cavity with little changed mirror's shape describing by following function [4, 5]:

$$A(r) = x_0 e^{-\eta(1+\alpha\eta+\beta\eta^2)}, \quad \text{where} \quad \eta = \frac{r^2}{2R_c x_0}, \tag{1}$$

which turning into spherical profile at $\alpha = \beta = 0$ and $x_0 \to \infty$.

For modelling it was chosen the method based on Hankel transform. This numerical method of FP cavity modelling for axial symmetry modes was developed in [6] for arbitrary shape of mirrors with axial symmetry. This method was further generalized for non-axial symmetric high order optical modes with dependence $e^{i\ell\varphi}$ on azimuthal angle φ (ℓ is integer). This method was used for calculation of eigenmodes of FP cavity with non-spheric mirrors and its diffraction losses to find cavity with thinned out spectrum.

During the investigation of stability such systems under influence of different factors we obtained and formulated permissible ranges of parameters and recommendations for minimization of calculation errors:

- conformity to gaussian beams;
- the most suitable values of parameters in simulations of FP cavity;
- allowable small variations of parameters x_0, α, β;
- estimation of value of permissible tilt angle of mirror;
- influence of ordinary tiny roughness of mirror surface after polishing.

In this way this method may be applied for modeling of huge class of the FP cavity and we used it for FP cavity with non-spherical mirrors in gravitational wave detectors such as aLIGO [5].

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Frequency mixing of guided electromagnetic waves in hyperbolic metamaterials

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We show a possibility of nanoengineering of the metamaterials which support phase-matched guided ordinary and backward electromegnetic waves (BEMWs). Contra-directed phase velocity and energy flux in BEMWs give rise to extraordinary greatly enhanced frequency and propagation direction changing coherent nonlinear optical processes [1–3]. The key point of this work is to show the possibility of satisfying a whole set of requirements of a paramount importance: i) the possibility to ensure the coexistence of the guided EMWs with positive and negative spatial dispersion $\partial\omega/\partial k$ [4–7]; ii) the frequencies of the combining waves should satisfy the photon energy conservation law; iii) the phase velocities of the combining waves must be equal, whereas the group velocities to be contra-directed. Phase matching of the ordinary and BEMWs is a challenging problem [8]. The outlined possibility is proven through numerical simulation of the dispersion and attenuation of the guided modes and of the unusual transient nonlinear optical frequency mixing processes, including frequency-shifting reflectivity, in the proposed model of the metamaterial in the THz frequency ranges.

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Scattering amplitudes in the direction of limit rays

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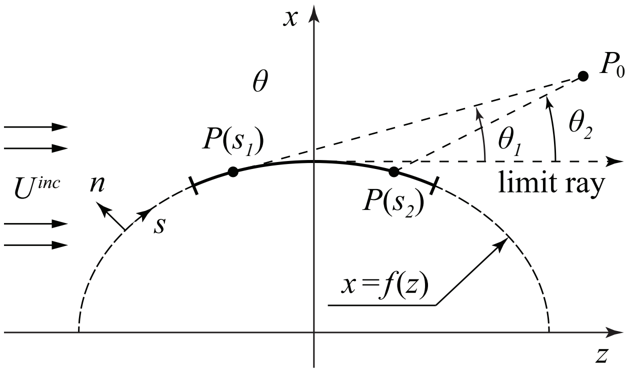
In the report we consider diffraction of a plane wave by smooth prolate bodies of revolution (axisymmetric cases) in the short-wave approximation and calculate scattering amplitudes in the direction of limit rays, i. e. the rays which touch the surfaces of the bodies at light-shadow boundary (equator). The wave field is supposed to satisfy Helmholtz equation and Dirichlet or Neumann boundary conditions on the surface of the body.

In our previous papers [3, 4] we proposed and developed an approach to those diffraction problems based on the classical method of Leontovich–Fock parabolic equation [1, 2]. The approach makes it possible for compute the current on the surface of the scatterer in a boundary layer around light-shadow zone (see Fig. 1 (left)). Expressions for the scattering amplitudes were deduced from the Green formula applied to the wave field under conditions of short-wave approximation and large distance from the scatterer. They contain integrals of the current in the light-shadow zone. Numerical experiments are focused on influence of prolateness of the bodies on the scattering amplitudes and on their dependents of physical parameters involved in the asymptotics (see Fig. 1 (right)).



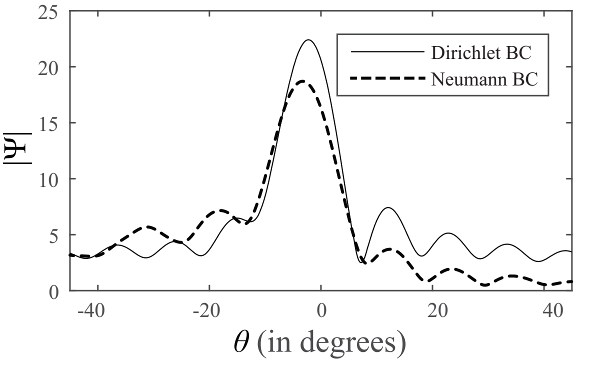


Fig. 1: Geometry of the problem (left); one from the numerical results for the modules of scattering amplitudes $\Psi(\theta)$ (right).

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Fast methods for numerical evaluation of the continuous wavelet transform

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The talk will review recent results [1, 2] in the development of fast numerical methods for the calculation of the direct continuous wavelet transform with the Morlet wavelet [1, 2]

$$w(a,b) = \int_{-\infty}^{+\infty} f(t) e^{-i\omega_0 \frac{t-b}{a}} e^{-\frac{(t-b)^2}{2a^2}} \frac{dt}{\sqrt{2\pi a^2}},$$

and its inversion defined as [3]

$$f(t) = \frac{1}{\sqrt{2\pi}} \operatorname{Im}\left[\int_0^{\infty} \frac{\partial w(a,b)}{\partial b} da\right],$$

being concentrated on the practical computational issues aimed to speed up calculations using possibilities of matrix data structures and operations provided by languages of technical computing (such as MATLAB/OCTAVE), as well as memory saving based on the effective spline approximation of the Gaussian sliding window function.

There will be considered several illustrative examples of pattern analysis for structures generated by non-linear dynamical systems and obtained in physical experiments. Some open problems and perspective research challenges in this area will be announced and discussed.

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Modelling low-voltage defibrillation in 2D medium

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Spiral waves of electrical excitation arise in many physical, chemical and biological media. Studying spiral waves in myocardium is especially important because such waves originate dangerous arrhythmias and therefore have to be terminated. It is known that spiral waves disappear when they touch the medium boundary. Induction of drift of spiral waves toward the medium boundary is a perspective strategy of treatment of cardiac arrhythmias [1, 2]. Here we simulate the dynamics of spiral waves on an isotropic square using phenomenological and biophysical models of myocardium. External stimulation is implemented by a stimulation current from point and linear electrodes. We find trajectories of spiral wave drift. We observe two mechanisms of the elimination of self-sustaining waves, drift toward the boundary and annihilation with a new spiral wave.

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Effects of dispersion and delay in mathematical models of mechanics

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The paper presents an analysis of mathematical approximations underlying description of different environments, and suggests new models. For rarefied gas the effects of self-diffusion and thermo-diffusion which were foretold by S. V. Vallander were obtained from kinetic theory. Normal and tangent components of the velocity are investigated for continuum environments and for rarefied gas.

The equation for density is

$$\frac{\partial}{\partial t}\int_{\tau}\rho\delta\tau+\int_{\sigma}\rho V_n\delta\sigma=\int_{\tau}\dot{M}\delta\tau.$$

In our case

$$\frac{\partial\rho}{\partial t}+\frac{\partial[(\rho u)\cdot\mathbf{n}+(\rho u)\cdot\boldsymbol{\tau}]}{\partial x_i}=0.$$

It is so for the classic Boltzmann equation. But for large gradients of physical values (density, temperature, pressure, velocity) model shall include the angular momentum, delay, new position inertia centre. Previous studies [1–3] have examined the effect of the angular momentum and the associated with it rotation of an elementary volume at writing of the conservation laws of continuum mechanics and kinetic theory.

Nonsymmetric stress tensor was obtained and set method of accounting of the this tensor. It was found that the stress tensor is nonsymmetric for structureless particles. Examples were given to demonstrate the contribution of the nonsymmetric part of the stress tensor of the simplest problems on the theory elasticity and the boundary layer.

Definition of modified equations based on the kinetic theory, for which it was suggested that the angular momentum is as an additional force. A more accurate asymptotic approximation was suggested with a resolution of Hilbert's paradox. Elementary volume can be rotated around the axis of inertia or to be involved in the rotational movement. In both cases, the density of the flow across the border changes by $(d(\rho u))/dr\cdot(r'-r)+\cdots$ by rotation of the elementary volume.

The contribution of other components is small, taking into account the smallness of the volume and the absence of rotation on axis. Additional issues arise when writing communications stress tensors with tensors of strain rate. The analysis of the recording of the Lagrangian function for the collective interaction of the particles with the change of the center of inertia of the moving particles and the effect influence angular momentum are made.

In general case for closed system $\frac{dL}{dt}=\sum_i\left[\frac{\partial L}{\partial q_i}\dot{q}_l+\frac{\partial L}{\partial\dot{q}_i}\ddot{q}_l\right]+\sum_i\left[\frac{\partial L}{\partial(q_i-a)}(\dot{q}_l-\dot{a})+\frac{\partial L}{\partial(\dot{q}_i-\dot{a})}(\ddot{q}_l-\ddot{a})\right]$, where $a=\sum_i\frac{m_i\mathbf{r}_i}{m_i}$, for electric interaction $a=\sum_i\frac{e_i\mathbf{r_i}}{e_i}$. In view of the angular momentum, we have $F=F_0+\nabla\left((R-a)\times\frac{\partial U}{\partial R}\right)$, where R is the current range. This formula is transformed with the derivatives that can be permutability

$$F=F_0+\nabla\left((R-a)\cdot\frac{\partial U}{\partial R}\right).$$

So we can obtain the force for collective interaction.

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New representations for square-integrable spheroidal functions

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We discuss the solution of boundary value problems that arise after the separation of variables in the Schrödinger equation in oblate spheroidal coordinates [1]. The specificity of these boundary value problems is that the singular points of the differential equation are outside the region in which the eigenfunctions are considered. This prevents the construction of eigenfunctions as a convergent series. To solve this problem, we generalized and applied the Jaffe transform [2]. We found the solution of the problem as trigonometric and power series in the particular case when the charge parameter is zero ($a = 0$). Also, the solutions of the boundary value problem are obtained in the case of the degeneracy of the singular points of the differential equation. The application of the obtained results to the solution of the spectral problem for the model of a quantum ring in the form of a potential well of a spheroidal shape [3] is discussed with introducing a potential well of finite depth.

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Essential spectrum of Schrödinger operators on graphs

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Let $\Gamma \subset \mathbb{R}^n$ be a graph periodic with respect to the action of the group $\mathbb{G}$ isomorphic to $\mathbb{Z}^m$, $1 \le m \le n$. We consider a 1-dimensional Schrödinger operator

$$S_q u(x) = \left(-\frac{d^2}{dx^2} + q(x)\right) u(x), \quad u \in C_0^\infty(\Gamma \backslash \mathcal{V}), \quad q \in L^\infty(\Gamma)$$

defined on the edges of the graph Γ ($\mathcal{V}$ is the set of the vertices of Γ). The operator S_q is extended to a closed unbounded operator $\mathcal{H}_q$ in $L^2(\Gamma)$ with domain $\tilde{H}^2(\Gamma)$ consisting of functions u belonging to the Sobolev spaces $H^2(\Gamma)$ and satisfying the Kirchhoff–Neumann conditions at the vertices of Γ.

For the unbounded operator $\mathcal{H}_q$ we introduce a family $Lim(\mathcal{H}_q)$ of limit operators $\mathcal{H}_q^g$ defined by the sequences $\mathbb{G} \supset g_m \to \infty$ and prove that

$$sp_{\text{ess}}\,\mathcal{H}_q = \bigcup_{\mathcal{H}_q^g \in Lim(\mathcal{H}_q)} sp\,\mathcal{H}_q^g.$$

We apply this result to the calculation of the essential spectra of self-adjoint Schrödinger operators with periodic potentials perturbed by slowly oscillating at infinity terms. We show that such perturbations significantly change the structure of the spectrum of Schrödinger operators with periodic potentials.

Model order reduction and hybridization of biochemical networks by tropical geometry methods

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Biochemical networks are used as models of cellular physiology with diverse applications in biology and medicine. In the absence of objective criteria to detect essential features and prune negligible details, networks generated from data are too big and therefore out of the applicability of many mathematical tools for studying their dynamics and behavior under perturbations. However, under circumstances that we can generically denote by multi-scaleness, large biochemical networks can be approximated by smaller and simpler networks, called dominant subsystems. Model reduction is a way to find these simpler models that can be more easily simulated and analyzed. Biochemical networks with multiple timescales can have several dominant subsystems. Different reductions are valid in different domains of parameter and phase spaces and can change along a trajectory. I will discuss methods based on tropical geometry allowing, given a biochemical network, to compute its set of dominant subsystems with their domains of validity and to find the possible transitions between different dominant subsystems. As a matter of fact, dynamics of biochemical networks with multiple timescales is intrinsically hybrid and tropical geometry methods can be used to hybridize such networks. I will also discuss potential applications of these techniques to virtual cell and organ models for biomedical research.

An approximate analytical model for the refracted field at reflection of a Gaussian light beam

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One knows a whole class of phenomena, for which the spatial inhomogeneity effects take an important place. These are the surface optical phenomena, which are caused by the total internal reflection and by excitation of various surface field resonances: waveguide modes, polaritons and plasmons (see, for example, [1–3]). At the present time, they are widely used for investigation of various surface physical, chemical and biological processes and structures. However, up to now the most works on the given topic in the literature devoted to the study of several extremely specific effects for reflected field.

We have presented an approximate theoretical model, well-grounded physically and consistent logically. On the bases of simple analytical expressions, it provides the opportunity to simulate spatial structure of the refracted field under Gaussian beam incidence on a plane interface at any angles different from grazing one, assuming the presence of absorption in a medium and the possibility for initial beam to propagate on long distances before incidence on a refracting boundary. At all angles of incidence, excepting the critical one or close to that, the spatial structure of refracted field is perfectly predictable and replicates the cross structure of the incident beam. However, under conditions of total internal reflection, the situation becomes complicated, because the refracted field is determined by the incident spectrum, including as propagating plane-wave components, whose angle of incidence does not yet reach the critical value, as decaying ones, whose incident angle exceeds this value. That is why the imaginary addend to the dielectric permittivity of a refracting medium arising naturally as a consequence of complexity of the normal propagation parameter, averaged over the spectrum, and its value depends on the effective width of a Gaussian beam spectrum. Further, since the effective width is inversely proportional to its effective cross spatial width, the refracted field parameters of propagation and decay are dependent on the beam width. Apart from the effective parameters of phase propagation and amplitude decay, explicit dependence of the incident beam width displays also for the coefficients, which determine the square coordinate function for the exponential refracted field function. Thereby, we have proved that initial spatial inhomogeneity of a Gaussian light beam affects significantly on the spatial structure of a refracted field in the directions of a boundary and in the depth of a refracted medium, under the incidence of this beam at the angle of total internal reflection.

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Laplacians on periodic graphs with guides

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We consider Laplace operators on periodic discrete graphs perturbed by guides, i. e., graphs which are periodic in some directions and finite in other ones. The spectrum of the Laplacian on the unperturbed graph is a union of a finite number of non-degenerate bands and eigenvalues of infinite multiplicity. We show that the spectrum of the perturbed Laplacian consists of the unperturbed one plus the additional so-called guided spectrum which is a union of a finite number of bands. We estimate the position of the guided bands and their length in terms of geometric parameters of the graph. We also determine the asymptotics of the guided bands for guides with large multiplicity of edges. Moreover, we show that the possible number of guided bands, their length and position can be rather arbitrary for some specific periodic graphs with guides.

Planar elements of photonics for controlling Bloch surface waves in one-dimensional photonic crystal

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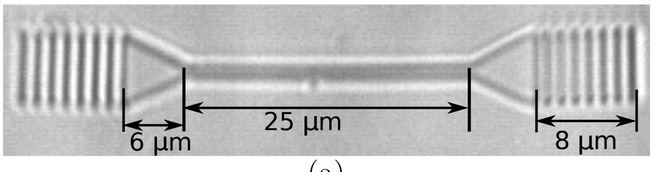
In past decades, a lot of effort were addressed on development of planar integrated circuits and active devices based on surface plasmon polaritons (SPPs) [1]. SPPs are surface electromagnetic waves on the interface of a dielectric and a metal. However, SPPs possess several drawbacks such as short propagation length due to Ohmic losses and spectral illiberality, because excitation wavelength is defined by the metal. Surface electromagnetic waves on the interface of one-dimensional photonic crystal and dielectric, named Bloch surface waves (BSW) are potential candidate on substitution of the SPPs [2]. Due to the fact that structure supporting the BSW is all-dielectric, this waves has long propagation length (order of 1 mm). Moreover, the BSW excitation wavelength could be tailored by the selection of photonic crystal layers thicknesses and depends strongly on the thickness of the layer on the interface with dielectric. Recently, the fabrication waveguides and other two-dimensional structures supporting the BSW has been demonstrated [3].

In this work we develop elements of photonics based on the BSW such as waveguides, splitters and Mach–Zender interferometer. We use two-photon polymerization — a lithography technique based on the phenomena of two-photon absorption in the volume of photosensitive material [4]. The main advantage of this technique is the ability to fabricate planar and three-dimensional structures of any complexity and geometry with a 100-nm resolution during one session of exposure. Samples were created on a photonic crystal consisting of 10-period stack of SiO_2 and Ta_2O_5 layers whose thicknesses are 140 and 98 nm, respectively. Waveguide is a stripe of photopolymer with two triangles and two diffraction gratings (Fig. 1a). Gratings are used to couple radiation into waveguide, while triangles allow one to increase coupling efficiency. To visualize propagation of the BSW inside the structure (Fig. 1b) we use leakage radiation microscopy [5] setup. We estimate effective refractive indices of the modes propagating in the waveguides by analyzing back focal plane images. Waveguide structures created in the work support TE_{00} and TE_{01} BSW modes with the effective refractive indices of 1.15 and 1.07, respectively.

The properties of the guided BSW modes were studied by numerical simulations with the help of the finite-difference time-domain method. We obtain mode effective refractive index from geometrical proportions received by atomic force microscope. Numerical result is in a good agreement with experimental.



(a)

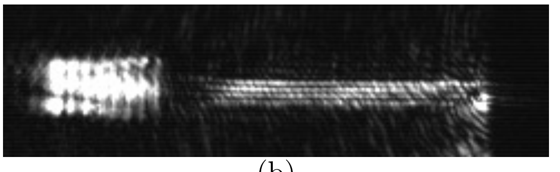

(b)

Fig. 1: a) Waveguide image obtained by optical microscopy. b) Waveguide image via leakage radiation microscopy. Light coupled through right grating, which were shut to prevent overexposure of a camera.

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On homogenization for locally periodic strongly elliptic operators

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In homogenization theory, one is interested in studying asymptotic properties of solutions to differential equations with rapidly oscillating coefficients. Here we consider such a problem for a matrix strongly elliptic operator $\mathcal{A}^\varepsilon = -\operatorname{div} A(x, x/\varepsilon)\nabla$, where A is Lipschitz in the first variable and periodic in the second. We do not require that $A^* = A$, so $\mathcal{A}^\varepsilon$ need not be self-adjoint. It is well known that the resolvent $(\mathcal{A}^\varepsilon - \mu)^{-1}$ converges, in some sense, as $\varepsilon \to 0$. In this talk, we will discuss results regarding convergence in the uniform operator topology on $L_2(\mathbb{R}^d)^n$, the strongest type of operator convergence. We present two terms in the approximation for $(\mathcal{A}^\varepsilon - \mu)^{-1}$ and a first term in the approximation for $\nabla(\mathcal{A}^\varepsilon - \mu)^{-1}$. Particular attention will be paid to the rates of approximation.

Simulation of propagation for acoustics signals in the deep ocean with Maslov's canonical operator

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The problem of sound propagting in a deep sea is very interesting and important in acoustics. In mathematical language this problem can be formulated as the Cauchy problem for the inhomogeneous wave equation with zero initial functions. There are many methods for solving it [1], in this talk we want to present another method based on the modified Maslov's canonical operator and provide the asymptotic solution for such problem in analytical form. The 2D case was considered in [2] and here we use the same ideas as in [2, 3] and constructing the asymptotic solution for the 3D wave equation

$$\frac{1}{c^2(x)} p_{tt} - \Delta_x p = Q(x,t), \quad x = (x_1, x_2, x_3) \tag{1}$$

with the initial conditions

$$p|_{t=0} = p_t|_{t=0} = 0.$$

Here $Q(x,t)$ depends on parameters λ, l:

$$Q(x,t) = \lambda^2 g_0'(\lambda t) \frac{1}{l^3} V\left(\frac{x - x^0}{l}\right)$$

with some fast decaying at infinity real functions $V(x)$, $g_0(\tau)$ and parameters $l \ll 1$, $\omega = \frac{c_0}{\lambda l} < \omega_0$. The solution of (1) splits into the sum of the propagating p_{prop} and transient p_{trans} parts. We are interested only in the propagating part.

Using work [3] we describe the procedure based on the modified Maslov's canonical operator for the asymptotic solution of the wave equation (1). We simulate with this procedure the propagation of the acoustic waves in the deep ocean. As an example we use the Munk profile to desribe the sound velocity. The asymptotic solution is built in the point where located the reciever. This method is compared with method of normal modes [1].

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Transient phenomena in a three-layer waveguide

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A waveguide composed of three acoustic layers having different wave velocities and different densities is studied. The waveguide is excited by a point source with a pulse time profile. The receiver is put at some distance from the source, and the dependence of the acoustic pressure on time is recorded. It is well known that if the receiver is very far from the source, the received signal can be expressed through the excitation spectrum and the dispersion diagram of the waveguide. The velocities of propagation of wave components are equal to group velocities of the waveguide modes. However, not far from the source one can observe transient phenomena (such as a precursor or the first arriving signal, FAS). Such phenomena can be described by using the analytic continuation of the dispersion diagram. Namely, in the domain of complex frequency one can find a branch corresponding to the precursor. Such a technique has been developed in [1] for a planar homogeneous elastic waveguide and in [2] for a two-layer waveguide.

In the proposed talk we are developing this method for the example of a three-layer waveguide. Such an example is chosen since there can exist some new physical phenomena comparatively to the cases of [1, 2]. Namely, there can be an hierarchy of transient waves, which can be described by different contours on the Riemann surface of the dispersion diagram.

In the talk we bring some order into the structure of the Riemann surface of a dispersion diagram of a layered waveguide. The main tool of this study is the presentation of the waveguide as a set of layers connected through the surfaces of variable rigidity. Zero rigidity corresponds to disjoint layers, and the infinite rigidity corresponds to the layered waveguide under consideration. The position of the branch points of the Riemann surface can be easily found for asymptotically small rigidity, and then analytically continued with respect to rigidity to its infinitely large values.

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Diffraction problem with rotational symmetry in parabolic approximation

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The problem of diffraction by a body having a rotational symmetry is studied (examples of such problems are diffraction by a cone or diffraction by a junction of several cones). The incidence is assumed to be axial. The problem is considered in the parabolic approximation from the very beginning, i.e. the governing equation for the waves is the parabolic equation of diffraction theory. The boundary conditions are of Neumann type [1].

The problem is solved using the boundary integral equation of the Hong's type [2]. Such an equation is derived in the Cartesian coordinates. This equation is of the Volterra type, so it can be solved by iterations. Besides, in the general case the equation can be solved numerically and for some particular cases (e. g. for diffraction by a cone) it can be solved analytically.

In the talk we demonstrate the capabilities of the boundary integral equation method in the parabolic approximation. Some known formulae are re-derived using this method. Numerical results are presented.

The work is supported by the RSF grant 14-22-00042.

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Homogenization problems with nonlinear boundary conditions

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We consider the homogenization of the boundary-value problems for the p-Laplace operator in a perforated domains. On the boundary of perforations we have a nonlinear boundary conditions or restrictions for the solutions and their flux. Supposing that $1 < p$ we obtained the homogenized problems for all relations between different parameters of the problem.

Application of normal modes approximation for acoustic logging inversion in boreholes with non-circular cross section

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Conventional processing techniques for acoustic logging data based on analytical models have limitations in case of anisotropic formation and/or non-circular borehole cross-section. A classical inverse problem can be formulated as the minimization of an error functional, for instance the misfit between synthetic and observed data. Standard non-linear optimization methods require either expensive calculation of Frechet derivatives or adjoint-state method to compute gradient of error functional [1, 2]. In this work, continuing development [3] of acoustic logging inversion, we demonstrate how numerically simulated spectrum of normal modes can be used for approximation of the wave field in a borehole and calculation of a functional gradient value.

Assuming the homogeneity of formation and borehole geometry along axial direction z, the direct problem of elastic waves propagation splits into a set of two-dimensional problems in frequency-wavenumber (ω, k) domain. In plane of the borehole cross-section the problem is discretized by a finite element procedure and reduced to a generalized eigenvalue problem which is solved numerically. The considered approach is referred as semi-analytical finite element method [4]. The solution can be represented as an expansion in eigenfunctions corresponding to different normal modes of the borehole.

It was shown [3] that for the considered problem the solution can be approximated with just a few number (3 to 7) of normal modes, which can be chosen separately for each frequency. The coefficients of the expansion are calculated by convolution of eigenvectors with a source function that is reproduced by error minimization for mode with highest energy and a priori knowledge of the design of the measuring tool. The high-accurate 3D simulations by spectral element method [5] are taken as reference logging data.

Using the obtained numerical approximation we formulate complete expressions for the gradient in terms of Frechet derivatives and adjoint-state problem in (ω, k) and (ω, z) domains. We use these expressions for numerical estimation of gradient values for several formations in elliptical boreholes, and verify them against direct method.

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Optical diffraction from 'photonic graphene' structures fabricated by two-photon polymerization

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In this study, we fabricated by the two-photon polymerization method 2D photonic structures of limited size with the symmetry of a square, triangle, and hexagon. Particular attention was given to theoretical and experimental studies of the Laue diffraction on 'photonic graphene' structures constituted by honeycomb cells. By specially choosing the lattice parameters and laser wavelength, we visualized the diffraction patterns on a flat screen positioned behind the sample. A detailed interpretation of the complex diffraction patterns was enabled by a step-by-step analysis of the structural factor $S(\mathbf{q})$ in the Born approximation, beginning with a 1D linear chain of scatterers, then using a 2D structure with a shape of a parallelogram, then a hexagonal structure formed by triangles with one scatterer per unit cell, and, finally, a graphene structure formed by honeycombs with two scatterers per unit cell.

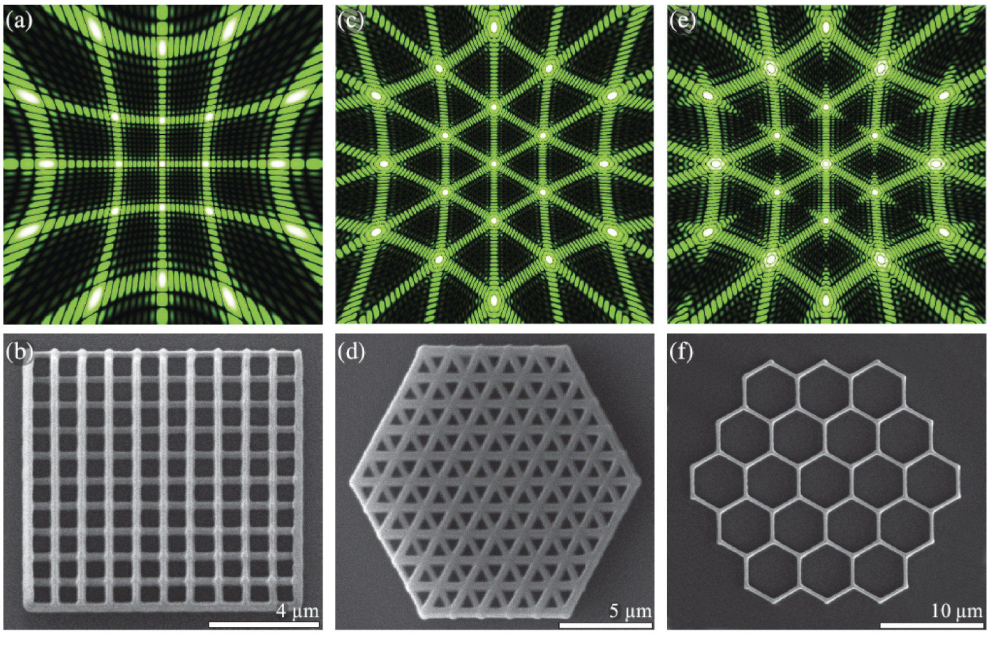

Fig. 1: Calculated diffraction patterns for square (a), triangular (c) and graphene (e) photonic structures on a flat screen positioned behind the sample. SEM images of the square (b), triangular (d) and graphene (f) photonic structures fabricated by the two-photon polymerization method.

As a result, three types of specific features were identified in 2D diffraction patterns. Two kinds of these, the principal and additional maxima, are observed for structures with one scatterer per unit cell, and the third, the dropout of diffraction reflections and the break of strips and arcs after every three principal maxima, is only observed in the diffraction patterns of graphene. An excellent agreement between the calculated and experimental results was observed. It should be noted that the optical diffraction patterns observed for the structure with a small number of scatterers can be regarded as radiation patterns of an optical antenna with spatially resolved lobes.

The study was supported by the Russian Science Foundation (Grant 15-12-00040).

On the attenuation coefficients in polycrystalline ferroelectric materials obtained on the basis of hysteresis models of polarization

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An algorithm is proposed for finding the attenuation coefficients in polycrystalline ferroelectric continuum on the basis of the dielectric and deformation hysteresis model for harmonic processes under the assumption that the damping does not depend on the oscillation frequency. The developed algorithm for calculating tangents of the angles of losses is a further generalization of models Korchinskii, Davidenkov, Panovko, Pisarenko [1] to the case of ferroelectric ceramics. The proposed method is based on the mathematical model of polarization of polycrystalline ferroelectric materials for quasistatic processes in the three-dimensional case. That is why the proposed method refers to the case when the work of internal friction forces per cycle of oscillations per unit volume of material does not depend on the frequency of oscillations. The main idea of this approach is to determine the area of loops of strain and dielectric hysteresis, which we can build on the basis of the developed three-dimensional model of polarization of polycrystalline ferroelectric continuum [2]. In turn, this model contains a number of parameters that are preliminarily found for each specific material from the condition of coincidence of calculated and experimental data. Once the model parameters have been determined, we assume that the model is tuned and we can describe the dielectric and strain response also for low-intensity fields. Therefore, we use it for the case of small mechanical and electric fields, for which the hysteresis dependences are very small. It is known that ceramics, polarized to saturation in a homogeneous electric field, belongs to the class of a transversely isotropic body. In the proposed approach, it is suggested that partially polarized or polarized in a nonuniform way ceramics refers to the class of a locally transversely isotropic body with an isotropy axis coinciding with the direction of the residual polarization vector. The same assumptions apply also to the tensor of the attenuation coefficients. Taking into account the obtained attenuation coefficients, a number of problems were solved for band, bar and disk converters, in which numerically some physical characteristics were obtained. It is shown that such an approach includes a special case when attenuation is taken into account only by introducing tangents of the angles of losses for elastic and dielectric coefficients.

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Generation and removal of apparent singularities in linear ordinary differential equations with polynomial coefficients

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We discuss several examples of generating apparent singular points as a result of differentiating particular homogeneous linear ordinary differential equations with polynomial coefficients and formulate two general conjectures on the generation and removal of apparent singularities in arbitrary Fuchsian differential equations with polynomial coefficients. As particular examples of application of the equations involving apparent singularities, we consider the reduction of the one-dimensional Schrödinger equation to the deformed double-, bi- and tri-confluent Heun equations. We show that this leads to the solution for the inverse square root [1], the Lambert-W step [2] and the Lambert-W singular [3] potentials in terms of the confluent hypergeometric functions.

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On a homogenization of a thin corrugated beam

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We consider a homogenization problem for a thin elastic corrugated beam clamped at the ends (see Fig. 1). This construction depends on two small parameters δ and h.

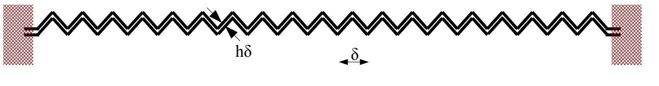

Fig. 1: A thin corrugated beam.

A cell of periodicity has the size of order δ. It consists of two beams $Q^+(h)$, $Q^-(h)$ with the thickness $h\delta$. The beams are connected in two knots $G_h^\bullet$, G_h° (see Fig. 2).

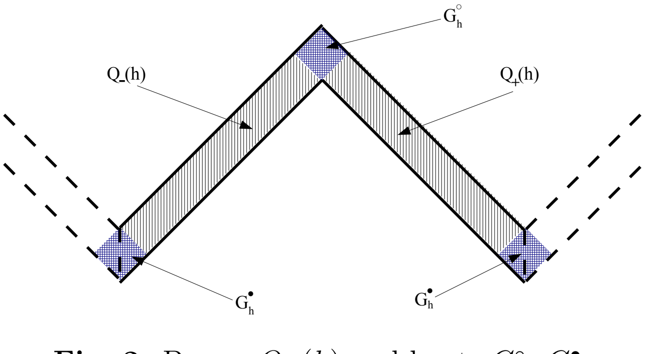

Fig. 2: Beams $Q_\pm(h)$ and knots G_h°, $G_h^\bullet$.

The homogenized operator essentially depends on the energetics characteristic of boundary layers near the knots $G_h^\bullet$ and G_h°. It has a form diag$\{M_1\partial^2/\partial x^2, M_2\partial^4/\partial x^4\}$. Here x is a longitudinal coordinate, a constant M_2 depends only on the elastic properties of the beams material and a constant M_1 equals to the sum of elastic energies of the limit problems for the boundary layers near the knots $G_h^\bullet$ and G_h°.

Persistent current in a chain of two Holstein–Hubbard rings in the presence of Rashba spin-orbit interaction

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In this work we consider the magnetic flux going through the quantum ring in such a way that the magnetic field is zero at the radius of the ring. It is a quantum mechanical phenomenon which can be observed in small metallic rings whose size is comparable to the electron coherence length. The energy spectrum is periodic in flux and consequently, the persistent current which is the change in ground state energy with respect to the magnetic flux is also periodic in flux.

The existence of persistent current in a normal metal ring was first proposed by Büttiker, Imry and Landauer [1]. Cheung [2] have studied the effects of temperature, chemical potential and randomness on persistent current in strictly one-dimensional (1D) normal rings. With the advent of nanofabrication techniques, several experimental investigations have been made to confirm the existence and the periodicity of persistent current in semiconductor quantum rings. The most useful model to study persistent current is the Hubbard model in which the ring consists of discrete lattice sites and the electrons can hop from one site to another. Several works have been carried out on the Hubbard ring to understand the magnetic response and the behaviour of persistent current. But most of them have neglected the electron-phonon interaction which can actually play quite an important role in the low-dimensional systems. The effect of electron-phonon interaction on persistent current can be captured by considering the Holstein–Hubbard model [3].

The effects of spin-orbit interaction [4] are found to be pronounced in quantum rings. There are two microscopic origins for the spin-orbit interactions. One originates due to the structural inversion asymmetry which is known as the Rashba spin-orbit interaction and the other is due to the bulk inversion asymmetry which is known as the Dresselhaus spin-orbit interaction.

The Rashba effect makes it possible to control the electron spin by the external electric field which is precisely the idea behind spintronics. The persistent current in Holstein–Hubbard ring in the presence of Rashba spin-orbit interaction was observed in [5]. In the present work we use the method of [5] and study the effects of Rashba spin-orbit interaction on persistent current in a 1D chain of two Holstein–Hubbard rings threaded by an Aharonov–Bohm flux. The effective electronic Hamiltonian was also finally diagonalized by using the Hartree–Fock approximation and persistent current was calculated by differentiating the GS energy. These results allow investigating the effects of Aharonov–Bohm flux, temperature, chemical potential spin-orbit interaction and electron-phonon interaction on the persistent current.

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On the image source method for sound propagation in a penetrable wedge: some corrections and appendices

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In this study the image source solution for sound propagation in a penetrable wedge [1, 2] is revisited. This solution is a very important 3D benchmark in computational underwater acoustics, since a wedge bounded from above by a sea surface and overlying a sloping penetrable bottom (see Fig. 1(a)) is the simplest model of a shallow-sea waveguide in coastal areas. The basic idea of the image source method is a decomposition of the total sound field into a sum of contributions from a series of image sources. Each image source corresponds to the waves that were emitted by the real source and underwent a certain number of reflections at the wedge boundaries, i. e., the surface and the sea bottom. The corrected formulae for positions of the image sources and image bottoms are presented together with the explanation of their derivations. Most importantly, the problem of branch choice in calculation of reflection coefficients is thoroughly discussed (which was not presented in the original paper [1]). The intuitive branch choice scheme [3] for reflection of spherical waves at a flat bottom is proved to be not rigorous for that at a series of sloping image bottoms in the framework of the image source method. A proper branch choice scheme for the reflection coefficients is essential in obtaining correct results of acoustic field associated with each image source. The modified branch choice scheme proposed in this study is proved to be able to offer accurate and robust results of acoustic field. Finally, the solution for wedges with elastic bottom (referred to as elastic wedge) is also studied and its comparison with the solution obtained using the finite elements method [4] is presented. The 3D solution for an elastic wedge is believed to be a useful benchmark for 3D seismo-acoustic sound propagation modelling in ocean waveguides.

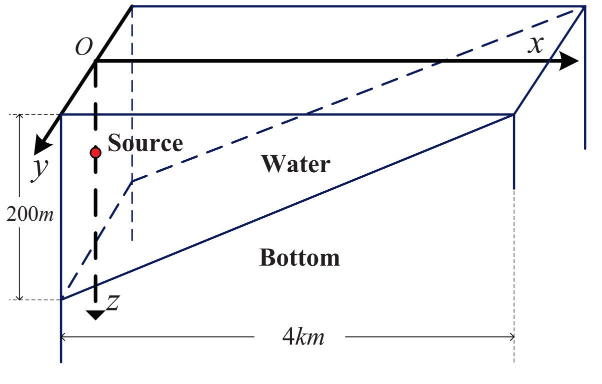

Fig. 1: Geometry of a 3D penetrable wedge.

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Non-contact sound vibration measurement by phase shifting self-mixing interferometer with nanometer resolution

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A compact phase shifting laser interferometer using self-mixing configuration was implemented to non-contact mirror the micron-amplitude vibration of a sound speaker with frequency from 20 Hz to 20 KHz. The single-mode dual-output laser worked as a microwave-photonic resonator optically-injected by back-scattered lights from external sound vibration and promised a moderate distance (1–5 m) measurement. Relationship between sound vibration and laser oscillation was mathematically expounded on a half-external cavity theory. High slewing-rate electro-optic crystal transferred interferometric phase induced by sound vibration to amplitudes of harmonics and predicted an angstrom level resolution by phase extraction. The amplitude-to-frequency response characteristics, hysteresis curve and ratio of voltage-to-amplitude of the sound speaker were summarized. Experimental results shown this optical system obtained one magnitude accuracy improvement than general electromagnetic sensors in recording sound vibration. Since the adopted method featured with broad bandwidth, low speckle noise, nanometer resolution at poor reflection environment, potentially, the ultrasonic and infrasound vibration can be precisely measured by proposed system as well.

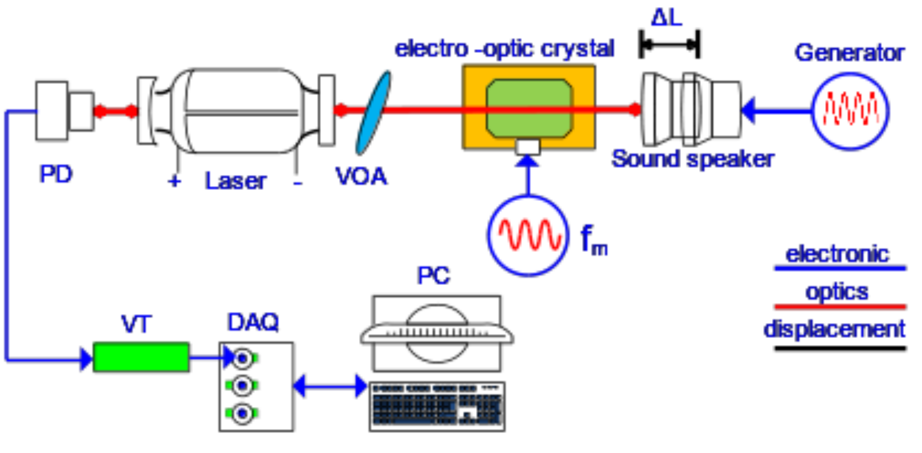

Fig. 1: The straightforward optical configuration of non-contact sound vibration measurement by phase-shifting self-mixing interferometer established on a dual-output He-Ne laser and an electro-optic modulator. VT denotes an adjustable resistor for photo current-to-voltage conversion, PC denotes computer for signal processing. DAQ denotes the analog-to-digital card. PD denotes a photo electric detector. VOA denotes the variable optical attenuator.

Modeling of second-harmonic generation in all-dielectric nanodimers

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Modern approaches to dielectric nanophotonics enables the visualization and detection of bio-objects [1]. Despite the rapid development of this field of science, there are a number of limitations for observing single bioobjects. One of such limitations is the strong scattering of light in the visible range in biological tissues, which reduces both the permissible thickness of the tissue under investigation and the resolution of the microscopy. Since biological tissues in the infrared (IR) band have minimal losses for scattering [2], nonlinear particles based on BaTiO3 can be used to solve this problem, allowing locally to convert light ("pumping") from infrared to visible light due the effect of the second harmonic generation [3]. However, the efficiency of nonlinear conversion turns out to be quite low because of the small absorption cross section of nonlinear nanoparticles. To overcome this problem, one can use dielectric nanostructures as nanoantennes [4] to amplify the generation of a signal from nonlinear nanoparticles.

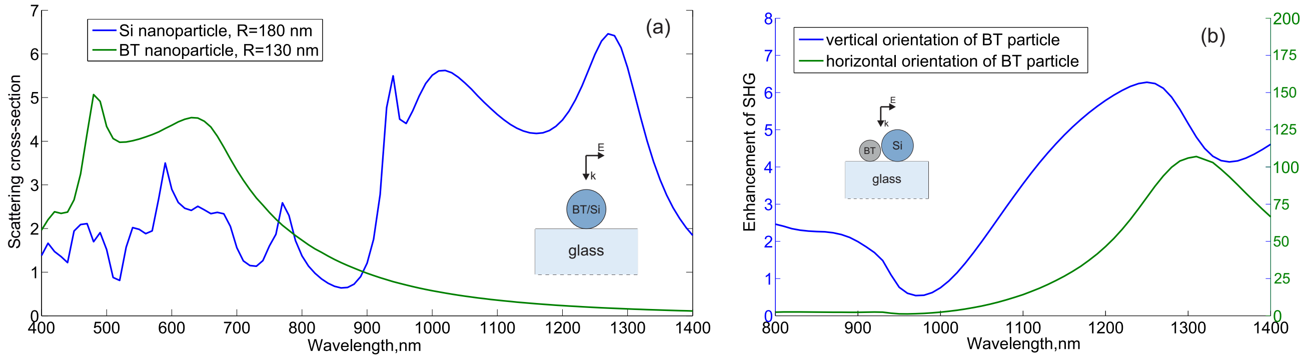

Fig. 1: (a) Elastic light scattering spectrum of a single $BaTiO_3$ nanoparticle (260 nm diameter) and single Si nanoparticle (360 nm diameter). The wavelength of the magnetic dipole resonance of the silicon nanoparticle is twice longer than for $BaTiO_3$ particle $\lambda_{MD}^{\mathrm{Si}} = 2\lambda_{MD}^{\mathrm{BaTiO_3}}$. (b) Enhancement of the second harmonic generation from $BaTiO_3$-Si nanodimer comparing to a single $BaTiO_3$ particle.

In this paper, we theoretically studied the second harmonic generation (SHG) from a nanodimer consisting of a single barium titanate $BaTiO_3$ nanoparticle and a silicon nanoparticle ($BaTiO_3$ (260 nm) / Si (360 nm)). The dimer acts as a nano-antenna efficiently collecting light at the fundamental wavelength due to silicon nanoparticle resonance and emitting second harmonic at the resonance of $BaTiO_3$ particle (see Fig. 1 (a)). We considered two different orientations of the crystalline lattice of the $BaTiO_3$ particle. The simulation results showed that use of silicon nanoparticle as collecting antenna leads to a significant increase in SHG from $BaTiO_3$ nanoparticle (see Fig. 1 (b)), and in a change in the mode structure of a nanodimer. The study was carried out by numerical simulation using software the Comsol Multiphysics.

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Multipole optical response of silicon nanoparticles of different shape

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Optical properties of nanoparticles attract great scientific interest [1, 2]. Subwavelength scatterers can support the excitation of multipole resonances which enhance the light-matter interaction in a controlable manner by changing the nanoparticles size, shape and material [3]. It can be used for different applications, including nanoantennas [4] and nanolenses [5], cloaking [6], chemiluminescence microdevices and composite plasmonic waveguide sensors [7]. While methods of multipole analysis of nanostructures optical properties are permanently improving, phenomenological aspects of nanoparticles multipole optical response are not carefuly studied.

Here, we explore the optical multipole resonances in silicon nanoparticles of cubic, pyramidal and cone-like shapes. We use the Finite Element Method (FEM) for solving the Maxwell equations for different nanoparticles shape. Harnessing the multipole decomposition technique we study excited optical resonances in silicon nanoparticles and the influence of high-order multipoles to scattering patterns of considered nanoparticles. Non-symmetrical combination of multipole contributions due to illumination from top and bottom sides of cones and pyramides is also considered.

Our work provides important information about the role of high order multipoles in the light scattering by non-spherical nanoparticles, incuding non-symmetrical cases. Our results could be applicated, for example, for development of metasurfaces and metamaterials in optical range.

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Travelling waves in quasiperiodic media

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We study speeds of fronts in bistable, spatially inhomogeneous media at parameter regimes where speeds approach zero. We provide a set of conceptual assumptions under which we can prove power-law asymptotics for the speed, with exponent depending a local dimension of the ergodic measure near extremal values. We also show that our conceptual assumptions are satisfied in a context of weak inhomogeneity of the medium and almost balanced kinetics, and compare asymptotics with numerical simulations.

Acoustic anisotropy and dissolved hydrogen as an indicators of waves of plastic deformation

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The method of acoustoelasticity consists in measuring the relative difference $\Delta a = 2(v_1 - v_2)/(v_1 + v_2)$ in the propagation velocities of transverse ultrasonic waves polarized in mutually perpendicular directions. Δa is called acoustic anisotropy.

In the fundamental paper [1], in the case of purely elastic deformations, it was obtained that $\Delta a = a_0 + C_A(\sigma_1 - \sigma_2)$. This formula is based on the nonlinearly elastic Murnaghan model with allowance for third-order terms in the deformation components. Later, it was extended to the area of plastic deformation under the assumption of the uniform nature of the plastic deformation of metals in the form [2]:

$$\Delta a = a_0 + a_1(\varepsilon_1^P - \varepsilon_2^P) + C_A(\sigma_1 - \sigma_2).$$

We have experimentally found that in the case of uniaxial plastic deformation of samples cut from aluminum rolled products, a wave-like character of the change in the acoustic anisotropy with monotonic increase in strain is observed. This effect can not be described by the generally accepted formula for acoustic anisotropy [2]. The wave-like character of the change in the acoustic anisotropy can be explained by the effect of the Portevin – Le Chatelier [3]. This effect leads to spatial inhomogeneity of plastic deformations and to the movement of plastic deformation waves in a plastically deformed medium.

The experimental fact that we discovered is not described by the Murnaghan model and the elastic-plastic model. The effects predicted by the elastic-plastic model are too small.

In connection with this, we propose a new empirical formula relating the anisotropy with principal stresses σ_1, σ_2, plastic strains ε_1^P, ε_2^P, and the state of the surface (parameter a_2):

$$\Delta a = a_0 + a_1(\varepsilon_1^P - \varepsilon_2^P) + C_A(\sigma_1 - \sigma_2) + a_2.$$

This formula takes into account the strong influence of microcracks in the region of localization of plastic deformation waves, and will allow us to give a more adequate description of the results of measuring acoustic anisotropy than it was done in [2].

The functional dependence of the parameter a_2 on the stresses, strains and material damage should be investigated.

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Quantum dynamics of polarization states in linear electro-optic effect

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Based on full quantum model and effective Hamiltonian approach [1] to parametric frequency modes interaction we investigate a dynamics of quantum polarization state for each frequency mode under phase modulation process. A simple model describes an evolution of noninteracting orthogonal linear polarization states H and V oriented along the principal axes of noncentrosymmetric nonlinear crystal and governed by the Hamiltonian of the form $H_{\text{eff}} = H_H \otimes I_V + I_H \otimes H_V$. For each frequency mode the polarization state transformation which is realized through the nonzero difference between phase velocities of different polarization components is determined by a rotation of Stokes vectors obtained from mean values and dispersions of Stokes operators. All results were obtained for the case of finite number of interacting frequency modes and compared with that which were obtained from the semi-classical model of phase modulation process describing in [2].

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Competition between Anderson and nonlinear localization of chirped laser pulse in 1D disordered photonic crystal

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Light localization is of great interest for all-optical devices and attracts strong attention during past twenty years. One of the well-known physical mechanisms which leads to light localization is the Anderson localization in a linear periodic structure [1, 2], for example, in 1D photonic crystal (PC). It takes place when quite strong random fluctuations of PC layers lengths create local band gaps for various frequencies. For this reason the effect occurs for a PC with large number of layers (more than 30) in contrast to a nonlinear localization which takes place even for PC with several layers [3]. It should be also stressed that the Anderson localization appears if a carrier frequency of an incident pulse is near the band gap. Under such conditions the light energy localizes in a part of PC.

A nonlinear light localization is observed at the pulse frequencies belonging ether to transmission band or to forbidden band. In this case, it is necessary that the pulse intensity should be greater than a certain value. It should be expected that the random fluctuations of PC parameters influence a nonlinear light localization.

In [3] we have investigated a nonlinear light localization in 1D layered disordered structure in which the layers with cubic nonlinear response alternate with linear layers. An incident pulse did not possess a chirp. However, it is well-known that a nonlinear propagation of a chirped pulse differs essentially from the corresponding propagation of a non-chirped pulse. Therefore, it is very important for practice to consider a soliton formation in some layers of the disordered nonlinear 1D photonic crystal if the incident femtosecond pulse is a chirped one.

Our aim is to investigate an interplay between Anderson and nonlinear localization of the chirped pulse. We expect that an increasing of layers lengths fluctuations leads to the decreasing of the incident pulse intensity at which a soliton appears in nonlinear layers of the photonic crystal. As we believe, a pulse chirp causes the increasing of the incident pulse energy localized in the nonlinear photonic crystal.

The investigation was made using the support of the Russian Science Foundation (Grant No. 14-21-00081).

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Optical wave trapping by induced free-electron high concentration front at laser pulse propagation in semiconductor

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We study laser pulse propagation in a medium with non-instantaneous TPA. This process is described by the following dimensionless Schrödinger equation

$$\frac{\partial A}{\partial z} + iD_2\frac{\partial^2 A}{\partial t^2} + i\alpha\rho A + \delta(1-\rho)|A|^2 A = 0,$$
$$\frac{\partial \rho}{\partial t} + \frac{\rho}{\tau_r} = q(1-\rho)|A|^4,$$

where $A(z,t)$ is the dimensionless complex amplitude, $\rho(z,t)$ is a normalized free electron concentration with its zero-value initial value: $\rho(z, t = 0) = 0$, D_2 characterizes second order dispersion, τ_r is the relaxation time of free electrons, α relates to the medium refractive grating due to free electron generation, q characterizes a free electron generation level and depends, in particular, on the incident laser pulse maximal intensity, and δ is the maximal coefficient of the nonlinear absorption.

We provide a computation simulation with incident pulse shape as Gaussian one: $A(z = 0, t) = E\exp\big(-(t-t_0)^2/\tau_0^2\big)$. Due to nonlinear interaction with a medium the incident shape is transformed into a pulse with the self-similar shape $A(z = 0, t) = E_p\big(1-\tanh\big(-(t-t_p)^2/\tau_p^2\big)\big)\cosh^{-1}\big(-(t-t_p)^2/\tau_p^2\big)$. The physical mechanism of this consists in the optical wave trapping by the front of free-electron high concentration [1]. Some results of our computer simulation are depicted in Fig. 1. It is seen that the computed pulse shape is in a good agreement with the expected form. While the pulse propagates, its maximal intensity shifts in a domain of time increasing. We see simultaneous motion of the maximal intensity and the free electron concentration front.

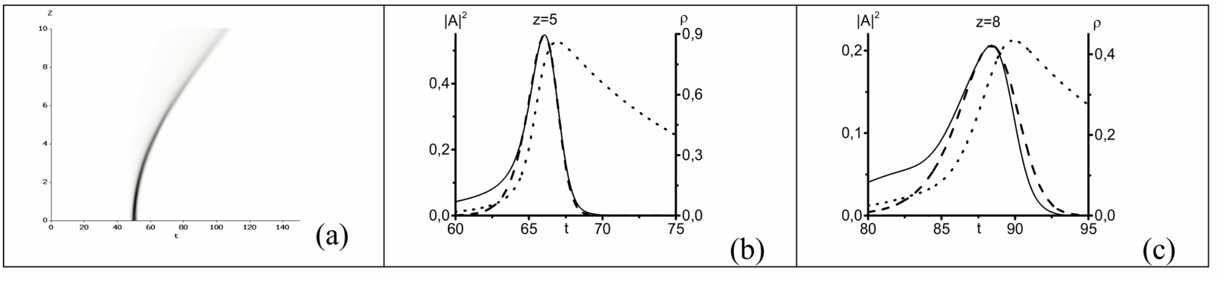

Fig. 1: Intensity distribution (a) and free-electron concentration profile together with pulse shape (b, c). Solid line — calculated pulse shape at $z = 5$ (b) and $z = 8$ (c), dashed line — theoretical self-similar pulse shape, dotted line — free electron concentration. Parameters: $D_2 = 1$, $\alpha = 5$, $\delta = 0.5$, $q = 5$, $\tau_p = 10$, $E = 1$, $\tau_0 = 2$, $t_0 = 50$.

The investigation was made using support of the Russian Science Foundation (Grant № 14-21-00081).

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Numerical method for solving the nonstationary radiation transfer equation in a layered medium

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An approximate method for solving the nonstationary radiation transfer equation in a layered medium, $G = G_1 \cup G_2 \cup \ldots \cup G_n$, $G_i = (z_{i-1}, z_i)$, is proposed. The radiation transfer equation has the form [1]

$$\left(\frac{1}{v(z)}\frac{\partial}{\partial t} + \nu\frac{\partial}{\partial z} + \mu(z)\right) I(z,\nu,t) = \mu_s(z)\int_{-1}^{1} p(z,\nu,\nu')I(z,\nu',t)d\nu', \quad z \in G,\ t \in [0,T]. \tag{1}$$

Here, $I(z,\nu,t)$ is the radiation flux density at the point $z \in G$ and in the direction which angle cosine with the positive direction of the axis z is $\nu \in [-1,1]$, μ is the attenuation factor, μ_s the scattering coefficient, v the velocity of radiation, p the phase function of scattering.

The radiation transfer equation is supplemented by matching conditions at the interfaces [1],

$$I^-(z_i,\nu,t) = BI^+(z_i,\nu,t), \quad i = 2,\ldots,n-1, \quad t \in [0,T], \tag{2}$$

conditions at the external boundaries,

$$I^-(z_i,\nu,t) = h_{ext}(z_i,\nu,t), \quad i = 1,n, \quad t \in [0,T], \tag{3}$$

and an initial condition,

$$I^-(z,\nu,0) = h_0(z,\nu), \quad z \in G. \tag{4}$$

In each subdomain G_i, the functions $v(z)$, $\mu(z)$, $\mu_s(z)$, and $p(z,\nu,\nu')$ do not depend on depth of layer z. The Fresnel matching operator B describes the dependence between limit values $I^{\pm}$ of the function $I(z,\nu,t)$ at the boundary $z = z_i$. It is defined by the refractive indices of the layers G_i [1,2].

The proposed approximate method for solving the problem consists of several steps. At the beginning, using the Laplace transform, the initial-boundary value problem (1)–(4) is reduced to steady-state boundary-value problem with parameter. Further, to solve the steady-state problem, the diffusion approximation is used. And finally, applying the inversion of the Laplace transform, a solution of the problem (1)–(4) is found.

A numerical implementation of the algorithm and its comparison with Monte Carlo method are performed.

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Complex large time behaviour of reaction diffusion systems

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We consider semiflows generated by initial boundary value problems for reaction-diffusion systems. In these systems, reaction terms satisfy general conditions, which admit a transparent chemical interpretation.

It is shown that the semiflows generated by these initial boundary value problems exhibit a complicated large time behavior. Any structurally stable finite dimensional dynamics (up to an orbital topological equivalence) can be realized by these semiflows by a choice of appropriate external sources and diffusion coefficients (nonlinear terms are fixed).

The results can be applied to the morphogenesis and pattern formation problems. They allow us to obtain a complete analytic description of chaotic in time spatial patterns. The method uses some asymptotic approaches, fundamental ideas of A. Turing for morphogenesis and realization of vector fields (a special approach proposed by P. Polacik and, in an asymptotical form, by the author). The realization of vector fields also allows us to construct algorithms for symbolic description of chemical kinetics (these results are conjoint with A. Weber and D. Grigoriev).

Features of electromagnetic field generated by interaction of high energy electrons with solid medium

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As part of the quasi-classical approach to describe the characteristics of the electromagnetic fields of the electrons produced by the interaction of ionizing radiation with the environment [1], the changes of the electromagnetic field components at different angles of observation in the space time representation are calculated.

In the model experiment, describing the interaction of electrons with an energy of 1.9 MeV with glass, the distribution of electrons in the phase space in a substance in the solid state (glass) is obtained using GEANT4 package [2]. The calculation of electromagnetic fields formed by electrons passing through matter is performed.

We applied the technique previously used for calculation of fields produced by ionizing particles in gas and liquid medium [3, 4]. The principle of superposition and model of linear current were used. It is shown that the calculated angular distribution of energy changes is in a good correspondence with the experiment [5].

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Fluctuations of the spectrum of commutator of unitary invariant random matrix ensembles

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We consider the ensemble of the commutator of $n \times n$ random matrices in the following form:

$$H_n = i(A_n U_n^* B_n U_n - U_n^* B_n U_n A_n),$$

where A_n and B_n are hermitian random (or non-random), having the limiting Normalized Counting Measure (NCM) of eigenvalues for $n \to \infty$, U_n is unitary, uniformly distributed over $U(n)$, and A_n, B_n and U_n are mutually independent. By using the technique introduced in [1] and [2], we prove the Central Limit Theorem for the fluctuations of linear eigenvalue statistics of ensemble. Besides, we calculate in the explicit form the variance of limiting Gaussian distribution of fluctuations in terms of the Stieltjes transform of the limiting NCM of ensemble using the functional equations for this transform studied in [3].

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Self-testing optical quantum random number generators

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The generation of truly random numbers plays a crucial role in many important applications. High-quality random numbers are absolutely essential in the field of cryptography, numerical simulations, in the gaming industry and some other areas. Software pseudo-random generators are of deterministic nature even though they can have a very long period (e. g., $2^{19937} - 1$ for the Mersenne Twister algorithm). Popular hardware random number generators (RNG) make use of a Zener diode operated in the reverse breakdown region. In this scheme, voltage fluctuations are amplified and compared to a threshold to generate random bits. Noise in such a system comes fundamentally from two sources: shot noise from quantum effects and classical thermal noise. In practice, both noises tend to appear side by side and are difficult to isolate. Critical applications, such as the secret key generation in cryptography, require truly random bits that can be obtained only from a quantum source.

Many physical processes can serve as a source of quantum randomness: radioactive decay, quantum tunneling, orientation of spin in the magnetic field, etc. Optical quantum RNG are very attractive due to their relative simplicity and a rich choice of implementations. Light from lasers, light emitting diodes or single photon sources is a convenient and affordable source of truly random numbers. Optical methods for quantum random number generation are varying from simple statistics of arrival time or photon number counting to the advanced types based on spontaneous parametric down-conversion, Raman scattering, amplified spontaneous emission, laser phase noise and others.

In the case of Raman scattering, a spontaneous event can be subject to optical amplification by the stimulated Raman scattering process. A strong pump without any input signal is usually used for this purpose. The field is amplified to a macroscopic level transforming quantum fluctuations in uncertainty of the optical phase. The random phases can be converted into interference patterns by combining the Stokes field and the reference pulse in the beam splitter.

A randomness source in the optical parametric oscillator is the phase of the macroscopic field inside the cavity, which is inherited from the vacuum fluctuations. An interesting approach was proposed to use two independent cavities in order to convert the phase variations into a binary random numbers by interference at a beam splitter [1].

All random number generators face a problem of trust. An attacker having access to the device can influence the produced numbers by inserting hardware trojans. For physical RNGs there are also such problems as imperfection of components, degradation of physical properties, the possibility of spontaneous failure, which can be hard to detect. Quantum mechanics (QM) provides a new way to deal with untrusted devices. The Kochen–Specker theorem guarantees that no classical system can simulate certain QM properties. The entanglement of space-like separated subsystems and the use of Bell-like inequalities can serve as a device independent test for randomness. Optics is the optimal choice for constructing this new generation of quantum RNGs.

We discuss several approaches to construct self-testing optical quantum random number generators and suggest some modifications.

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Diffraction on thin dielectric bodies

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In this research we study the scattering of electromagnetic waves by the dielectric impediment in 2D geometry. The impediment is determined through the inhomogeneous component of the refractive index in the Helmholtz equation. It is supposed that the characteristic gauge of one of the two impediment sizes is much less in comparison with the length of the waves generated by the monochromatic point source. Nevertheless, we do not neglect the impediment width considering this parameter to be infinitely small. The proposed model elaborated in the present study gives us the opportunity to compute the electromagnetic field outside the impediment rather than in its body. The made assumption enables us to represent the impediment as a set of electromagnetic waves sources distributed along an axis with some unknown density. Integral equation with respect to this density is derived. The obtained equation is subject to rigorous mathematical analysis to make sure in the model's correctness. In spite of the fact that the analogous problem is tackled in a number of publications (see for instance [1]), the details of its formulation and the methods of study differ significantly from our approach. In essence, the latter remark concerns the method of introducing the source of the electromagnetic field: in the most of the mentioned works the process is launched by an incident wave what provides the Lippmann–Schwinger formulation [2] of the scattering problem. Thus, the mathematical modelling for the scattered waves under special additional conditions constitutes one of the goals of our study. It is worth noting that the proposed model is especially convenient in the process of studying the scattering on the impediments with rough surfaces of different arbitrary profiles. Except this, scattering of electromagnetic waves on the plane with slots and on the knife-edge type impediments located on radiopaque plane along with a number of other problems representing a significant interest for practice may be studied within the proposed model. A comparative analysis with the previous approaches is given. The incentive to study this problem is partially caused by the necessity of the objects identification by the methods based on the analysis of the parameters of waves scattering from their surfaces.

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Generation of orbital-angular-momentum entangled biphotons in twisted nonlinear waveguides

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Quantum biphoton states with orbital-angular-momentum (OAM) entanglement offer important benefits for a variety of quantum applications [1, 2]. Recently, a new type of coupled waveguides with a twisted geometry were demonstrated in a photonic-crystal fiber [3]. Integration of generation and transmission of photon states can offer further benefits in terms of compactness and stability of quantum devices [4]. In this work, we study the effect of twist on quantum walks of photons. In this paper, we investigate theoretically the generation of photon pairs through SFWM and their propagation in twisted waveguide arrays with cubic nonlinearity, as shown schematically in Fig. 1.

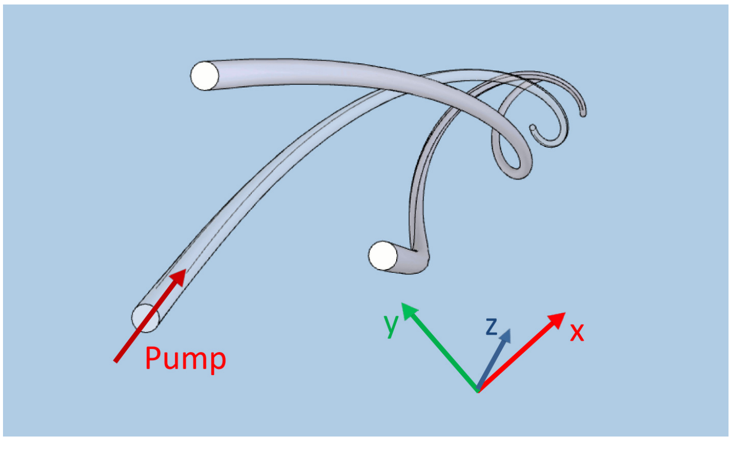

Fig. 1: Scheme of twisted circular array of coupled waveguides. Pairs of signal and idler photons are generated through spontaneous four-wave mixing from the pump.

We consider an array of closely spaced optical waveguides, which are twisted around a central axis along the propagation direction. Such structure composed of three waveguides is schematically shown in Fig. 1. We derive Schrödinger-type equation of the biphoton wavefunction, taking into account the waveguide bending through the appearance of additional phase in the coupling coefficients [5].

A pump beam with a particular OAM will generate only pairs of signal and idler photons which satisfy the angular momentum conservation expressed through the Kronecker delta function. We illustrate this relation by summarizing in Table 1 the possible states of generated biphotons for different OAM in a three-waveguide structure. In Table 1, the first column represents the OAM of a pump beam coupled to the array, and the second column shows the corresponding pump propagation constants. The third column lists the possible OAM states of the generated biphotons, and the fourth column presents the corresponding biphoton propagation constants. For clarity, we explicitly write down the equivalence of pump states due to the discrete periodic symmetry. It then becomes easy to see how the selection rules work based on the OAM conservation for the generated biphotons: $m_s + m_i = 2m_p$ up to multiples of N (number of waveguides). We see that two different momentum states of biphotons can be generated for each pump OAM. Further selectivity in excitation of particular photon states can be achieved by adjusting the phase mismatch.

Pump state $\lvert m_p\rangle$	Propagation const β_{m_p}	State $\lvert m_s, m_i\rangle$	Propagation const β_{m_s,m_i}
$\lvert 0\rangle$	$2C\cos(\phi_0)$	$\lvert 0,0\rangle$	$4C\cos(\phi_0)$
		$\lvert -1,1\rangle + \lvert 1,-1\rangle$	$-2C\cos(\phi_0)$
$\lvert -2\rangle \equiv \lvert 1\rangle$	$-C[\cos(\phi_0) - \sqrt{3}\sin(\phi_0)]$	$\lvert 0,-1\rangle + \lvert -1,0\rangle$	$C[\cos(\phi_0) - \sqrt{3}\sin(\phi_0)]$
		$\lvert 1,1\rangle$	$-2C[\cos(\phi_0) - \sqrt{3}\sin(\phi_0)]$
$\lvert 2\rangle \equiv \lvert -1\rangle$	$-C[\cos(\phi_0) + \sqrt{3}\sin(\phi_0)]$	$\lvert 0,1\rangle + \lvert 1,0\rangle$	$C[\cos(\phi_0) + \sqrt{3}\sin(\phi_0)]$
		$\lvert -1,-1\rangle$	$-2C[\cos(\phi_0) + \sqrt{3}\sin(\phi_0)]$

Table 1: Summary of biphoton states which can be generated by pump beams with different OAM in a three-waveguide structure.

Building on a recent experimental demonstration of classical OAM state transmission through twisted multi-core photonic-crystal fibers [3], our results suggest a potential for developing quantum communications for cryptography and other applications by combining robust photon-pair generation in twisted structures and their subsequent transmission through twisted optical fibers.

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Topological 3D-dissipative optical solitons

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Twenty years ago, the seminal paper by L. D. Faddeev and A. J. Niemi [1] generated considerable interest to nontrivial topology of nonlinear classical fields. However, search for physical objects with predicted in [1] stable knot-like localized structures encountered considerable obstacles. In the talk, we present a natural way to form stable 3D-soliton-like structures with complex topology of energy flows.

Using lasers as a source of high-power and coherent radiation, as well as new efficient nonlinear-optical materials, it is possible to generate complex field structures, including different types of optical solitons. In dissipative optical systems with energy input and output, there are extremely stable dissipative solitons presenting attractors in the corresponding phase space [2], whereas in conservative systems with only negligible dissipative factors, there are no such attractors. Correspondingly, conservative solitons are not so stable as dissipative ones.

Especially important is this difference for topological solitons, and a proper object can be a laser or homogeneous laser medium with saturable absorption [3]. The model is described by the generalized complex Ginzburg–Landau equation for the electric field envelope [3]. Earlier, 2D-topological laser solitons were studied adequately [2, 4]. Recently we have predicted 3D-topological vortex dissipative optical solitons with curved vortex line (where the field amplitude turns to zero) and with rotation and precession of the solitons [5]. In the talk, we report a much more wide class of 3D-topological optical solitons, i. e. stable localized structures of electromagnetic field in a homogeneous medium with nonlinear (saturable) laser gain and absorption and linear frequency dispersion and angular anisotropy (dichroism). We demonstrate their origin from 2D-topological dissipative soliton structures, increasing their dimensionality by their rotations and twisting around some axes in 3D-space. The solitons' vortex lines are closed and unclosed, knotted and unknotted, single and multiple, and their intensity distribution can be rigid or deformable during propagation. We report the dynamics of their formation, the topology of energy fluxes [4, 5], the overlapping domains of stability of different types of the topological solitons, and scenarios of decay of these solitons outside these domains.

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Resonant properties of 3D electromagnetic diffraction by a flat polygon

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Recently published book [1] contains many new formulas which make it possible to formulate a new approach — the method of basic components (MBC) [2], relating to the physical theory of diffraction (FTD). On the basis of MBC, it is possible to construct various heuristic (based on physical principles) formulas in accordance with the requirements of a practical problem. The formulas can significantly improve the efficiency of solving the practical problem. Since heuristic formulas do not have mathematical rigor, they need to be verified with a rigorous solution.

A numerical solution for the problem of scattering by a polygon was obtained by V. I. Kalinichev with use of finite differences method. The polarizations of the incident field were set in the same way as in [3]. The amplitude of the scattered field vector was investigated.

A rigorous solution obtained by the finite differences method was compared with heuristic solutions obtained with the help of the physical optics method (PO) and the method of equivalent contour currents (MEC) with modification of the solution at the vertex (MMEC) [1]. The MEC and MMEC approaches refine the solution, taking in account the field perturbation at the edge of the scatterer. The problem of electromagnetic wave scattering by a flat 3D polygon was investigated. It is shown that the signal scattered by a planar 3D polygon has resonant properties associated with the characteristic size of the scatterer.

In the case when the scattering has a resonance character, it is possible to achieve a more exact coincidence, for example, in the GTE approximation by taking into account multiple re-reflections of the waves at the edges on the scatterer surface. In this paper, we did not do this procedure. Comparison of strict and heuristic solutions has shown that only qualitative coincidence takes place. In the presence of this methodical error (not taking into account the re-reflections), we investigated the ratio between the amplitudes of the two polarizations in the same solution instead of investigating the relationship between the strict and heuristic solution. For a rigorous solution the ratio has a pronounced resonance character, for heuristic solutions the ratio is constant. Due to the complexity of the algorithm for solving a practical problem, the decision to apply refinements should be made taking in account the specific benefits that can be obtained.

According to the same procedure, scattering diagrams of a plane angular sector [3] (which does not have a dimensional parameter), were investigated. It is shown that the heuristic and strict solutions for the given geometry coincide better, and the change in the shape of the scatterer does not affect the relationship between the polarizations in the heuristic solution.

Author wishes to thank V. I. Kalinichev for providing the numerical solution.

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Semicircular nanostructured plasmonic interferometers

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Surface plasmon interferometry has recently been suggested as a promising new technique for plasmonic biosensing. In this paper, we report the design, simulation and characterization of novel plasmonic interferometric sensors that consist of semicircular grooves and apertures patterned on a gold film. This structure is proposed to form two-arm, three-beam, planar plasmonic interferometers, as shown in Fig. 1. By combining the proposed plasmonic interferometers with a finite-difference time-domain numerical algorithm, we found that the transmission spectrum of the designed plasmonic structures can be readily tailored by changing arm lengths and number of grooves. Based on the principle, the characteristics of refractive index sensing are also demonstrated by simulation. Agreement between theory and simulation results demonstrates the operating principle of the device. These results show that this simple, efficient and controllable scheme possesses unique features of high contrast, narrow linewidth, and large amplitude. Meanwhile, these results show that the proposed structure is promising for portable, efficient and sensitive biosensing applications.

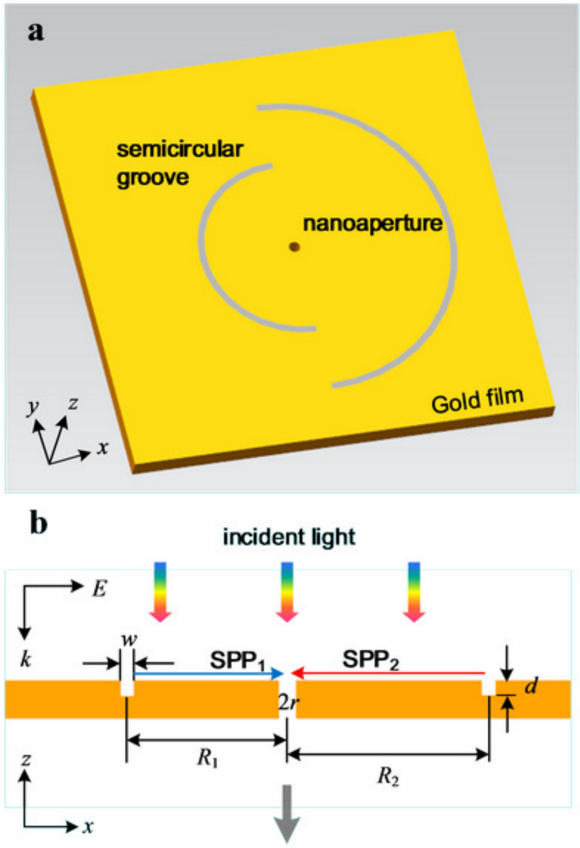

Fig. 1: a) Schematic of the proposed plasmonic interferometer. b) Side view and the working principle of the interferometer structure.

Analysis of dispersion relations for viscoelastic hollow circular cylinder in the case of Rabotnov's rheological model

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In this talk we discuss the dispersion of time-harmonic waves in an infinite viscoelastic hollow circular cylinder whose material properties are modeled by Rabotnov's hereditarily elastic rheological model. The dynamic stress-strain state of the cylinder is described by the three-dimensional equations of motion in stresses and displacements. The constitutive equations are written in the integral form. The integral operators are defined by formulae

$$\tilde{E} = E(1 - \Gamma^*), \quad \tilde{\nu} = \nu + \frac{1-2\nu}{2}\Gamma^*, \quad \Gamma^* f(t) = k\int_{-\infty}^{t} \unicode{x042D}_\alpha(-\beta, t-\tau) f(\tau) d\tau,$$

were E, ν are instantaneous values of the Young's modulus and Poisson's ratio, k, β are parameters of the material, α is a rational singularity parameter, $-1 < \alpha \le 0$, ($\alpha = 0$ corresponds of the standard viscoelastic body).

As the kernel of integral operator we use the fractional exponential Rabotnov's function [1]

$$\unicode{x042D}_\alpha(-\beta, t) = t^\alpha \sum_{n=0}^{\infty} \frac{(-\beta)^n t^{n(1+\alpha)}}{\Gamma\left((n+1)(1+\alpha)\right)},$$

where $\Gamma\left((n+1)(1+\alpha)\right)$ is the gamma-function.

The case of stress-free faces is considered. By deriving the dispersion equations the special fundamental system [2] for Bessel equation is used instead of the traditional system of Bessel and Neumann functions. For the axisymmetric problem the dispersion equations are derived and solved numerically both for the longitudinal and torsion waves. The influence of Rabotnov's material parameters k, β and α on the behavior of the dispersion curves is investigated.

The asymptotic behavior of the dispersion equations roots is of interest for the analysis of transient waves fronts and quasi-fronts in shells on the basis of 3D equations. To obtain asymptotic expansions for the small and large frequencies we assume the singularity parameter α to be rational and apply the generalized power series expansion. By using of computer symbolic calculation tools several terms of asymptotics for each of the roots under consideration are derived which allow us to make following conclusions: the first coefficient do not depended upon α; an asymptotic expansion of less than four terms is applicable only for very small values of frequencies; comparison of asymptotic expansions shows that with increasing $|\alpha|$ the number of terms must be increased to maintain the same accuracy; the behavior of imaginary part of the complex wavenumber depends essentially upon singularity parameter α.

The developed method of asymptotic analysis can be easily generalized for the viscoelastic models involving fractional derivatives [3].

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Three-dimensional investigation of transient edge waves on an elastic semi-infinite plate subject to tangential edge loads

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Edge waves localized near the edge of an elastic plate are thoroughly investigated in the cases of homogeneous problems and non-homogeneous ones with time-harmonic excitations (see, e. g., review [1]). The number of contributions concerning the transient edge waves exited by a space-time limited or by a shock loading is considerably less and restricted to the investigations on the basis of two-dimensional plate theories. However, the two-dimensional plate theories are not valid in the vicinities of front and quasi-fronts of transient waves [2]. From a practical point of view the edge waves exited by time-limited loads are of interest because such a loads are usually applied in experimental investigations. In the case of a relatively thick plate the accurate describing of such waves requires the applying of 3D theory.

In the present work edge waves exited by edge tangential point load are considered. The dynamic stress-strain state of an elastic plate with free faces is described by 3D equations of elasticity. Two cases of loads are considered: the impulse load and time-limited load which depends on time as two sine cycles multiplied by sine-window function. The Laplace transform with respect to the time and the Fourier transform with respect to the spatial coordinate along the edge are applied. The solution of the two-dimensional problem for the transformed displacements is written as modal expansion. The inverse transform are evaluated for the poles corresponding to the fundamental symmetrical and antisymmetrical 3D edge waves [3]. The results are compared with the solutions obtained by using the explicit asymptotic models for the extensional [4] and bending [5] waves in the theories of plates. The influence of 3D edge waves dispersion which is not taken into account by plate theories is discussed.

It is shown in [3, 6] that besides the extensional and bending edge waves described by plate theories an infinite spectrum of higher order waves exists. To reveal the contribution of the latter in transient wave field we consider the simplified problem for a plate with mixed boundary conditions on its faces. We assume that faces are free in tangential direction an fixed in normal direction. In this case an analytical solution describing higher order edge waves can be written. On the basis of this solution the contributions of higher order waves in transient wave field is derived and analyzed.

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Direct frequency comb spectroscopy of ^{87}Rb 5S-5D two-photon transitions by using Er-fiber femtosecond frequency comb

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The femtosecond optical frequency comb (FOFC) has proved to be a powerful tool in studying the nonlinear optical interactions. As the simplest and most instructive cases, the two-photon transitions of alkali atoms have been widely researched based on CW laser and Ti:sapphire FOFC. As an alternative, an Er-fiber FOFC working at the optical communication bands can be used to explore the $5S_{1/2}$-$5D_{5/2}$ two-photon transition of thermal ^{87}Rb atom as well. The infrared Er-fiber comb (1556 nm) is frequency doubled to a visible frequency comb (778 nm) through a PPLN crystal. By precision control of the PPLN temperature, the frequency conversion efficiency is up to 20% and the power of the 778 nm comb reaches 30 mW. In our experiment, the direct two-photon absorption spectroscopy is obtained by using counter-propagating pulses to suppress the Doppler background. The repetition rate (near 132 MHz) of the comb has been scanned in range of 22 Hz so that the comb line in the optical range can cover all the $5S_{1/2}$-$5D_{5/2}$ transitions of ^{87}Rb. The two-photon transition is monitored by detecting the 420 nm fluorescence derived from the cascade 5D-6P-5S transitions. Experimental results agree well with previous measurements and calculations, indicating that the Er-fiber frequency comb is capable of performing fast direct spectroscopy with high precision, which has significant importance to precision metrology and corresponding scientific research.

A boundary problem of modulated wave reflection from a stress-free surface of an anisotropic elastic body

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This work can be regarded as generalization of the known results [1, 2] on the reflection of plane monochromatic waves from a flat boundary of an anisotropic elastic medium. For the problem under study, the boundary is assumed to be a smooth and stress-free surface, and the anisotropic elastic medium to be inhomogeneous. We define an incident wave $\vec{U}_0$ as a space-time ray series [3] with phase $\varphi = \rho\varphi_0$, where ρ is a large parameter. The waves arising from the reflection of wave $\vec{U}_0$ off the boundary, are also represented as space-time ray expansions.

Each of the propagating waves, assumed here to be high-frequency modulated waves, can be treated, with allowable error, as a locally plane wave in the vicinity of a point in four-dimensional space-time at time t. Taking this into account, we assume that at point M_0 on surface S, the wave $\vec{U}_0$ angle of incidence ψ lies within the range $\pi/2 > \psi > \psi_0$, where ψ_0 exceeds the critical angles of this wave. In this case, according to Snell's law, one of the reflected waves is excited as an elastic wave with a real phase, while the other two elastic waves are generated as waves with complex phases, i. e., the so called inhomogeneous waves.

The principal terms of the space time (ST) series for all reflected waves meet homogeneous equations and form eigenvectors of a 3-row symmetrical matrix A (real or complex). The solvability conditions for these equations, reduced to algebraic equations of the third degree for eigenvalues of matrix A, are valid both on and outside surface S. On surface S, all these equations, in turn, can be reformulated as a single algebraic equation of the sixth degree in normal components of vectors $\operatorname{grad}\tau$ (see [1]), where τ denotes the phases of the reflected waves (the other two components of these vectors coincide with analogous components of $\operatorname{grad}\varphi_0$). In our case, the three roots of this algebraic equation roots with their corresponding radiation conditions make it possible to construct an asymptotic solution to this problem.

The real phase of the reflected wave is found by the method of characteristics applied to the modified dispersion equation. Here, curvilinear coordinates (q_1, q_2) of point M_0 on surface S and time $t = t_0$ of the reflection event are chosen as parameters to define a specific curve from a family of characteristics (here, space-time rays). The amplitude of this wave is usually found using the space-time ray method [3], with an arbitrary factor that is a function of the above parameters.

The amplitudes and complex phases of inhomogeneous waves that decay exponentially with depth, are constructed as expansions in powers of n (here, n denotes the distance of point M to surface S). In these cases, we deal with complex eigenvalues of matrix A, which can be represented as roots of a third-degree algebraic equation using Cardano's formulas. By differentiating these relations and equations for amplitudes successively with respect to n, power expansions both for the complex phases and normalized polarization vectors of these waves are found.

The boundary conditions for the absence of stress on surface S lead to a linear inhomogeneous system for the coefficients of the ST asymptotic rays of the reflected waves, in fact, for the arbitrary factors contained in the asymptotical formulas for these coefficients. An asymptotic solution to the boundary problem is unique if the determinant of this system is non-zero. The fact of the determinant's non-triviality bears out the proof of the linear independence of the polarization vectors of the reflected waves.

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Interaction of electromagnetic waves guided by a plane channel with enhanced density in magnetoplasma

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Electromagnetic waves guided by cylindrical plasma structures aligned with an external static magnetic field have received much careful study (see [1] and references therein). Nonlinear interaction of waves guided by an anisotropic cylinder in free space have been considered in [2, 3]. This interest has been motivated by the many applications, such as propagation of whistler-mode waves through the ionosphere, plasma diagnostic and so on. In this report we study the interaction of electromagnetic waves guided by plane plasma slab with enhanced density in homogeneous background magnetized plasma. The anisotropic media are described by dielectric tensor with nonzero off-diagonal elements. We consider the interaction of guided waves in the cases of a nonresonant as well as resonant magnetized plasma. Recall that the cold collisionless magnetoplasma is nonresonant if the diagonal elements of a dielectric tensor have identical signs and is resonant otherwise [1]. The above stated condition is satisfied for example when the frequency of the waves belongs to the interval between the lower hybrid frequency and the electron-cyclotron frequency.

The dispersion characteristics and structures of the fields for the even and the odd modes in each of the above-mentioned cases are analyzed for parameters typical of modeling laboratory experiments [4]. We show that under certain conditions the presence of a time-harmonic external electromagnetic field in magnetoplasma can lead to the interaction of waves guided by enhanced-density plasma slab. In the approximation of a week nonlinearity the equations for the amplitudes of interacted waves and the expressions for their instability increment are found. Detailed numerical results will be presented for various sets of the slab parameters.

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On hyperbolized nonlinear Schrödinger type equations

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The hyperbolization method [1, 2] is intensively developed nowadays (in hydro and gas dynamics especially) due to its good adaptation to parallel high-performance computing systems.

Hyperbolization of equations is provided by an additional term with a small parameter as the coefficient before the second derivative in time. Such a term makes it possible to construct three-layer explicit schemes possessing the better stability condition in comparison with traditional explicit schemes both for parabolic equations and for non-stationary Schrödinger-type ones [3].

The key question is: "What is the difference between the solution of the hyperbolized AKNS system (Schrödinger type equation particularly) and the original one". To verify possibilities of the approach proposed a lot of tests (both numerical and analytical) is performed. A comparison between the hyperbolization method and some standard ones for non-stationary Schrödinger equation is performed.

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Lacunas in anisotropic elastic media under the action of concentrated forces

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A investigation of the waves propagation in continuous media under the influence of various external and internal sources of natural or synthetic origin refers to the actual problems of mechanics and mathematical physics, and is associated with the solution of boundary value problems for systems of equations of hyperbolic and mixed types. Solutions of these equations can have characteristic surface on which themselves solutions, or their derivatives are discontinuous [1]. A special place in the study of wave processes takes the cases of wave propagation from point source.

Here a anisotropic mediums are considered. Such mediums have the characteristics closest to the real environment, in particular rocks. Wave propagation in such medium is subject to more complex laws than in an isotropic medium, and the stress-strain state of the medium depends strongly on the degree of anisotropy. For example, in medium with strong anisotropy of the elastic properties we have the Lacunas (moving unperturbed regions bounded by the wave fronts and expanding over time), and the front of the wave is very different from the classic, has a complex non-smooth shape.

The motion equations of anisotropic media are described by the strictly hyperbolic system of equations with derivatives of the second order. By using the Fourier transform of generalized functions fundamental solutions are constructed and are obtained the conditions on the wave front [2]. The obtained fundamental solutions are important, because they can be used to receive the solution in the medium under the influence of various mass forces.

The results of the numerical calculations, illustrating the pictures of wave fronts and displacements for elastic orthotropic medium under the influence of different types of sources are presented in [3, 4].

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Thermal effects in optical plasmonic structures

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Electromagnetic interactions in metal nanoparticles (NPs) are among the most intriguing phenomena in plasmonics which are used in numerous breakthrough applications. Interaction of NPs with high intensity pulsed laser fields, plasmonic heating [1, 2] and near-field heat transfer [3] are among them. All of these processes deal with strong heating and even melting of NPs. Moreover, high intensity light may cause mechanical effects due to interparticle forces. Therefore, the development of physical model that will be able to describe interrelation between electromagnetic, thermodynamic and mechanical interactions in systems of NPs is of topical interest.

In our work we have considered the linear chain of 11 spherical silver nanoparticles located on the substrate. It is obvious that thermal effects in OPW should strongly depend on the intensity of incident laser radiation.

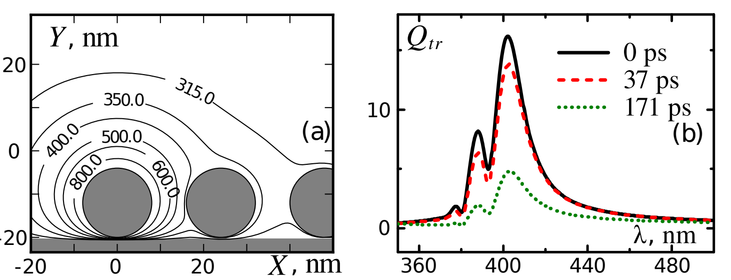

Fig. 1: The temperature T distribution at $t = 1$ ns for first three OPW Ag NPs. Wavelength $\lambda = 402$ nm and polarization along the X axis — (a). Transmission spectra of the OPW at different moments of time: the initial moment of time (solid line); the beginning of first NP melting (dashed line); the end of first NP melting (dotted line) — (b).

In fig. 1(a) one can see that only first three particles are significantly heated, and the temperature of the first particle reaches the melting point. We take into account heat exchange between NPs through environment and between a NP chain and thermally stable substrate.The OPW transmission spectrum in fig. 1(b) slightly changes when the first NP reaches the melting temperature ($T \approx 1080$ K for NP radius 8 nm, dash line). This change is associated with the change of the dielectric constant of the particle material. It is clearly seen that OPW transmission efficiency drops threefold in this case (dotted line). The further increase of NP temperature in a chain occurs due to heat exchange between them, between NPs and host medium including substrate.

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Dispersion properties of plasmonic waveguides with core-shell nanoparticles

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The signal propagability in the periodic structure of metallic nanoparticles, such as the plasmon waveguide, is interesting from the point of view of numerical modeling and the possibilities of experimental application. Classical materials for such waveguides are silver and gold. High waveguide properties of such materials are shown in many papers. It was shown that metal [1] or dielectric [2] substrates can significantly impair transmission properties of the waveguide. In this paper we consider the possibility of using a core-shell structure with silver as shell material and silicon dioxide as a core material for plasmon waveguides plased on technological substrate.

To evaluate the waveguide properties, we use the Eigendecomposition method [3]. The advantage of this method is the simultaneous calculation of dispersion relation and corresponding modes quality for various types of plasmonic chains.

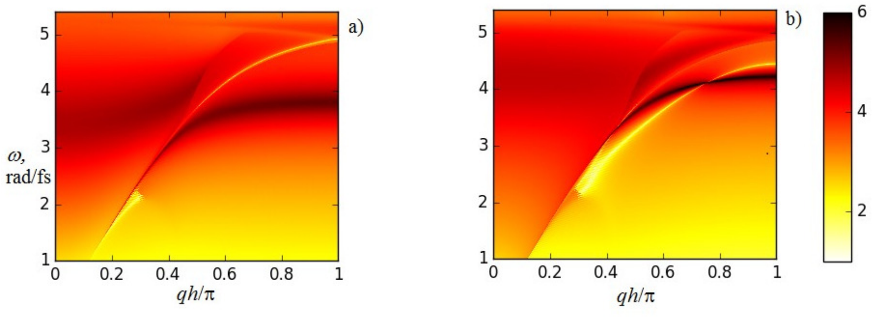

Fig. 1: Dispersion properties for various components of polarization of wave that propagate in a chain of nanoparticles that locate on the Silver substrate in the environment of silicon dioxide.

Dispersion properties of chain of spherical core-shell nanoparticles with silver as a shell material and silicon dioxide as a core material in the environment of silicon dioxide with metalic (Silver) substrate are shown on fig. 1. Dispersion curve for polarization that perpendicular to the chain fig. 1a is similar to the dispersion curve for plasmon waveguide with solid spherical nanoparticles. In the fig. 1b we can see the gap on the dispersion curve. Summarizing the results, we can say that the chains of the core-shell plasmonic nanoparticles have good waveguide properties and can be investigated experimentally.

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The wave field of a point source that acts on the impermeable stress free boundary of a Biot half-plane

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Initial boundary value problem of wave propagation in half-plane filled with fluid-saturated porous solid is considered. Biot's medium is isotropic homogeneous and pores are closed on the boundary. Using complex analysis techniques, explicit formulae for displacements in elastic and fluid phases are obtained.

Peculiarities of electromagnetic waves propagation in hybrid liquid metamaterials

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The control of optical fields at microscale and nanoscale levels is usually the primary motivation for developing, designing, and manufacturing metamaterials. Another important characteristic of the metamaterials is their tunability and multifunctionality. In this context, easily reconfigurable liquid metamaterials [1] may become a basis for the new generation of structural elements of the photonic circuits. Recently, the tunable metamaterial called liquid metacrystal (LMC) was suggested in [2]. It was supposed in [2] that such a material may be realized as a collection of elongated resonant metallic nanoparticles (meta-atoms) dispersed in viscous medium (liquid). The external dc electric field being applied to LMC aligns meta-atoms along one axis that imparts the anisotropic properties to the metamaterial. The characteristics of electromagnetic waves propagating in LMC strongly depend on the relative orientation of anisotropy axis and wavevector that makes possible to change the effective refraction index of LMC simply changing the reorientation of dc electric field. The meta-atoms can also be reoriented by means of high-frequency electromagnetic field that results in strong nonlinearity of LMC. These properties of LMC were predicted in [2] and basically demonstrated in experiment [3].

The linear and nonlinear electromagnetic response of LMC studied in [2, 3] has electric character due to considered electro-dipole meta-atoms. In this report, we suggest LMC containing structural elements of two types, electric and magnetic or meta-atom possessing both electric and magnetic dipole moments. We call the metamaterials of these kinds hybrid LMC (HLMC). We show that such *double anisotropic* structures expand the possibilities for electromagnetic wave control. HLMC can be the medium with elliptic, electro-hyperbolic, magneto-hyperbolic, and left-handed types of dispersion. Furthermore, HLMC may exhibit some combined types of dispersion which cannot be attributed to particular class of propagation conditions. In HLMC in addition to electric orientational nonlinearity the magnetic orientational nonlinearity emerges. Also, HLMC exhibits more wide possibilities for thermal topological transitions in contrast to LMC [4].

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All-dielectric liquid metacrystals

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Several current research directions aiming to control optical fields at microscale and nanoscale are driving development, design, and fabrication of novel types of metamaterials. Tunable structures are of particular interest as they can increase functionality and/or bandwidth of metamaterial devices, e. g. when modulation of the wave properties is required. There are several approaches for making tunable metamaterials. For example, electromagnetic properties of composite media change when the geometry of their structural elements (meta-atoms) and/or their mutual arrangement changes. In this context, reconfigurable *liquid metamaterials* can provide a basis for the design of highly tunable nanophotonic devices. Recently, tunable liquid metamaterials were experimentally [1] and theoretically [2] studied. In this first works, material was made of elongated metallic (e. g. plasmonic) nanoparticles suspended in viscous liquid, and they were called liquid metacrystals (LMC). The external dc electric field aligns meta-atoms along one axis that induces the anisotropy of metamaterial. The number of unique properties of LMC such as high tunability, strong nonlinearity, the ability to support surface and guided waves of new kinds, as well as existence of thermal topological transitions, etc. were theoretically studied [2–5]. Further advances in designing liquid metamaterials with elements of complex shape were demonstrated in Ref. [6].

In this work we present the first theoretical study of a new type of liquid metamaterial based on all-dielectric structures. One of the important advantages of all-dielectric metamaterials as compared to the metal-based ones is relatively low losses. Furthermore, all-dielectric metamaterials allow some scaling which enables using the same calculations for different frequency domains, where dielectric materials with similar properties can be found, whereas metals behave vastly different in microwave, terahertz and optical frequency ranges. We show that all-dielectric LMC can exhibit electric as well as magnetic, and also simultaneous electric and magnetic responses determined by the design of meta-atoms and structure of excited eigenmodes. We numerically calculate the electric and magnetic polarizabilities of meta-atoms of different shapes (sticks, discs, rings, spheroids, etc.). Using the simplest Lorenz–Lorentz approach, we further calculate effective dielectric permittivity and magnetic permeability tensors of LMC. We also estimate the typical strength of static electric field required for rotating and aligning elements in the LMC for different shapes and sizes of meta-atoms.

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Nonlinear interaction of waves at torsional oscillations in a liquid

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Flows of a viscous incompressible fluid in a spherical layer that are due to rotational oscillations of its inner boundary at two frequencies with respect to the state of rest are numerically studied. It is found that an increase in the amplitude of oscillations of the boundary at the higher frequency can result in a significant enhancement of the low-frequency mode in a flow near the outer boundary. The direction of propagation of the low-frequency wave changes from radial to meridional, whereas the high-frequency wave propagates in the radial direction in a limited inner region of the spherical layer. The role of the meridional circulation in the energy exchange between spaced waves is demonstrated.

Reduction of wave-induced vibration by resonant periodic obstacles

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The paper aims to gain knowledge on how resonant periodic obstacles can mitigate low-frequency vibration induced by gravity waves in a floating elastic plate. An analytical technique is used assuming that the obstacles are straight-line, situated as a periodic array in the plate. The fluid is homogeneous, incompressible and inviscid. The incident waves are regular. Advantages and disadvantages of single obstacles and periodic obstacles are shown. Exact analytical solutions for the plate deflection and the velocity potential in the fluid are derived. Several quantities of practical importance (e. g., shear force and bending moment) are also calculated.

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Waves in elastic half-space caused by inclined center of rotation source

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Center of rotation point source is a natural model for a seismic source with torque. A response of smooth inhomogeneous and weakly anisotropic media to a center of rotation excitation is studied in [1, 2]. Placed in an infinite homogeneous lossless and linear elastic medium it generates a pure shear wave polarized as SH wave with respect to the plane of incidence containing the torque vector (rotation axis). In the presence of a free plane surface orthogonal to the torque direction it causes a trivial SH reflection. We present our research on a problem with an arbitrarily oriented center of rotation in elastic half-space. A buried in elastic half-space point force exerted in an arbitrarily direction is considered in [3]. An inclined center of rotation source (and correspondent wavefield) decomposes into two components: torque orthogonal and parallel to the free surface. The latter problem has no cylindrical symmetry. It is solved in Cartesian coordinate system setting the displacement and stresses jumps with Weyl identity and subsequent transition to cylindrical coordinates by special integrals transformations. The azimuthal dependencies, as usual, appear as trigonometric multipliers (the azimuthal patterns for bulk waves were derived in [4]). The incident wavefield is a composition of SH and SV polarized waves. We provide classical asymptotic estimates for the wavefield components: reflected SH, SV and SP bulk waves; head wave; Rayleigh wave; inhomogeneous wave due to a secondary saddle point contribution (we follow the approach given in [5] for S^* wave). The wavefield components polarization patterns presented in Fig. 1 lead to vanishing of displacement signal at certain seismic traces and presence of a headwave (P-wave velocity arrivals) with SH polarization at certain direction due to additional component (in the next terms of ray series expansion). The full waveform modeling is performed by Chebyshev spectral method given in [6] and compared with the theoretical results.

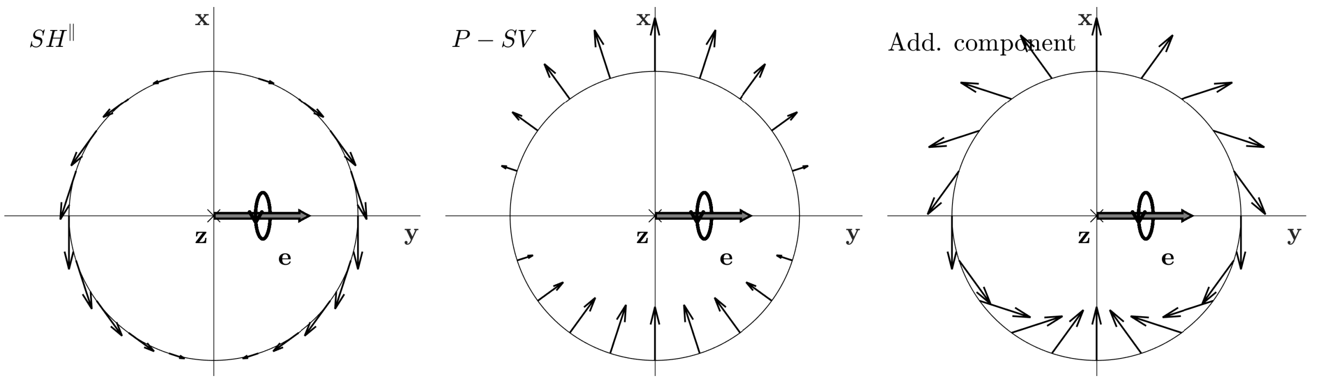

Fig. 1: Reflected wavefield components polarization patterns for a source with **e** torque.

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Dielectric-based nanoscale heat source and thermometer

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Metal nanoparticles are characterized by a strong interaction with light due to excitation of the localized surface plasmon resonance. At the resonant optical and near-IR frequencies, this interaction leads to the excitation of free electrons with subsequent conversion of their energy into phonons. These losses, related to the phonon excitation, support Joule heating of nanoparticles. In modern applications, such as photochemistry, photocatalysis, photothermal therapy and biosensing, plasmonic nanoparticles are used as nanoscale heat sources. Effective optical heating is achieved through the use of small nanoparticles, nanorods, nanoshells etc, but their melting can easily occur under the influence of moderate laser intensities, which will change the shape of the nanoparticles, and dramatically reduce the efficiency of conversion of light into heat. On the other hand, the non-active phonons in Raman scattering of plasmons metals do not allow to measure the temperature and, therefore, precisely controlled heating during photothermal process.

To overcome these problems, and to expand the applicability of nanoscale heaters, we propose novel non-plasmonic heating strategy. It uses dielectric crystalline nanoparticles, for example silicon, having a melting point above the threshold of 1500 K and active phonon modes Raman, which are suitable for nanoscale thermometry, due to the thermal shift of Raman frequency.

In this paper, we experimentally and theoretically demonstrated that the localization of field also provides the optically effective heating of nanoparticles, in spite of the low level of optical losses [1]. As it turned out, the optical resonant heating silicon nanoparticles may be more effective in some cases than the heating plasmonic nanoparticles. Indeed, unlike plasmonics, the interaction of light and dielectric nanoparticles leads to strong accumulation of the electromagnetic field inside the nanoparticles with low losses due to a small imaginary part of the permittivity, resulting in more effective light-to-heat conversion. Moreover, we have experimentally demonstrated the possibility of precise optical monitoring during the optical heating of the silicon nanoparticles.

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