INTERNATIONAL CONFERENCE

DAYS ON DIFFRACTION 2023

ABSTRACTS

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FOREWORD

"Days on Diffraction" is an annual conference taking place in May–June in St. Petersburg since 1968. The present event is organized by St. Petersburg Department of the Steklov Mathematical Institute, St. Petersburg State University, and the Euler International Mathematical Institute.

The conference is supported by the Ministry of Science and Higher Education of the Russian Federation (the grant to the Leonhard Euler International Mathematical Institute in Saint Petersburg, agreement № 075-15-2022-287, the grant to the Steklov International Mathematical Center, agreement № 075-15-2022-265).

The abstracts of 151 talks, presented during 5 days of the conference, form the contents of this booklet. The author index is located on the last pages.

Full-length texts of selected talks will be published in the Conference Proceedings. Format file and instructions can be found at `http://www.pdmi.ras.ru/~dd/proceedings.php`. The final judgement on accepting the paper for the Proceedings will be made by editorial board after peer reviewing.

Organizing Committee

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In memoriam of
Natalia Yakovlevna Kirpichnikova
1939–2023

Natalya Yakovlevna Kirpichnikova, an applied mathematician of high international reputation, a leading researcher at the St. Petersburg Deparment of the Steklov Mathematical Institute, passed away on Sunday, April 30, 2023, aged 83. Natalya Yakovlevna devoted her life to science: from the end of the 60s and until recently, she was actively involved in studies of problems of diffraction and wave propagation. The main area of her interest and the lifelong topic of her studies concerned high-frequency asymptotic theory of wave propagation, where she was an eminent expert. In co-authorship with Vassily M. Babich, she published a significant monograph *The Boundary Layer Method in Diffraction Problems* (Springer, Berlin, 1979). Also, in collaboration with V. M. Babich, she obtained important results on the theory of propagation of elastic waves. For the last 30 years, Natalya Yakovlevna has been a member of the organizing committee of the Days of Diffraction, and her contribution to the successful work of the conference can hardly be overestimated. Combining a bright mind, the highest level of expertize and organizational skills with modesty, warmth, goodwill, and charm, Natalya Yakovlevna was not only a talented scientist and organizer, but also a wonderful person. She will be deeply missed by her family, numerous friends and colleagues.

75 years to A. P. Kiselev

The Organizing Committee of the “Days on Diffraction 2023” is pleased to congratulate its permanent member since 1997, Aleksei Prokhorovich Kiselev, on the occasion of his 75th birthday.

A. P. is a Chief Research Scientist at the St. Petersburg Department of the V. A. Steklov Mathematical Institute and a part-time Principal Research Scientist at the Institute for Problems in Mechanical Engineering, Russian Academy of Sciences. Also, he is a Professor at the Faculty of Physics, St. Petersburg State University. His research concerns localized solutions of hyperbolic equations and high-frequency elastic waves, and the expertize based on deep knowledge of various aspects in diffraction theory is very helpful in organizing “Days on Diffraction”, whose Programme Committee he currently heads, and in editing of the Conference Proceedings.

Aleksei Prokhorovich, the Organizing Committee wishes you to stay in good health, to maintain your high research productivity and to achieve further success in organizing “Days on Diffraction”.

90 years to V. A. Solonnikov

The Organizing Committee of "Days on Diffraction 2023" congratulates Vsevolod Alekseevich Solonnikov on the occasion of his 90th birthday. The program of the conference includes a symposium dedicated to the anniversary.

Vsevolod Alekseevich is a brilliant representative of St. Petersburg mathematical school. His results in the field of partial differential equations and their applications to hydrodynamics are well-known throughout the world. The scientific methods proposed by V. A. Solonnikov have enabled rapid development of free boundary problems for viscous fluids, in which Vsevolod Alekseevich is undoubtedly the recognized leader nowadays. For more than 65 years, V. A. Solonnikov's affiliation has been St. Petersburg Department of the Steklov Mathematical Institute; V. A. also taught at the Faculty of Mathematics and Mechanics of St. Petersburg State University. He is the author and coauthor of almost three hundred papers. Research activity of V. A. Solonnikov is highly appreciated by the mathematical community, he was awarded the Humboldt Foundation Prize in 2003, the Lavrentiev Prize of the Russian Academy of Sciences in 2009 and the Chebyshev Prize of St. Petersburg Government in 2013.

The Organizing committee wishes Vsevolod Alekseevich good health and scientific longevity.

List of talks

Workshop on wave phenomena in complex structures

Asymptotic solutions or the wave equation with abruptly varying velocity

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We study short-wave asymptotic solutions of the Cauchy problem for wave equation with rapidly-varying velocity. Namely, we assume that the velocity has a smoothed jump near a smooth hypersurface. We describe propagation of short-wave packets as well as localized beams; for this cases we find the rules of changing of the geometric objects which define the solution. As a result we obtain complete asymptotic serie for the solution of the Cauchy problem.

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Inverse problems for a Boussinesq system for incompressible viscoelastic fluids

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In this talk we study two inverse problems for the nonlinear Boussinesq system for incompressible viscoelastic non-isothermal Kelvin–Voigt fluids. The studying inverse problems consist of determining an intensities of density of external forces and heat source under given integral overdetermination conditions. Two types of boundary conditions for the velocity $\mathbf{v}$ are considered: sticking and sliding conditions on boundary. In both cases of these boundary conditions, the local and global in time existence and uniqueness of weak and strong solutions are established under suitable assumptions on the data. The large time behavior of weak solutions is also studied.

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On symplectic filtration of (co)adjoint orbit of complex orthogonal group in nilpotent case

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Any orbit of the orthogonal group is the symplectic space. I consider the nilpotent case now, it is the most interesting and complicated one.

I will construct some procedure that induce a structure of the symplectic filtration on the algebraic charts of the orbit. That is the sequence of the fibrations, where the total space of the next fibration is the base of the previous one. The bases and the fibers are the symplectic spaces and the symplectic forms on the total space is the sum of the pull-backs of the forms from the base and the fiber.

A nilpotent matrix from the Lie algebra of the orthogonal group can be considered as a matrix of a nilpotent linear transformation of some auxiliary linear space. Each step of the procedure consists of two parts. Consider a subspace that is the intersection of the kernel and the image of the transformation. We project a part of the basis on this subspace, and project another part of basis on the orthogonal complement to this subspace simultaneously. It is a first part of the step. A second part is some orthogonal transformation that moves a given coordinate subspace to the complement of the intersection of the kernel and the image to the kernel. This transformation will be presented explicitly.

The constructed filtration is not rational but irrational of the second degree. That means we have to calculate not the scalar products $\langle X, Y\rangle$ only, but the lengths of vectors $\sqrt{\langle X, X\rangle}$ too.

Radiation characteristics of a rotating electric dipole in a magnetoplasma

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There is currently a strong interest in the radiation of twisted electromagnetic waves in a magnetoplasma. Such waves have a helical phase front and carry orbital angular momentum along with the spin angular momentum, which can be used for some promising applications [1]. Most works on the subject deal with arrays of appropriately phased loop antennas, i.e., sources of the magnetic type [2–5]. At the same time, the features of radiation from phased sources of the electric type in a magetoplasma still need additional consideration.

In this work, we consider the radiation characteristics of a source in the form of two quadrature-phased orthogonal electric dipoles immersed in a cold homogeneous magnetoplasma. Each dipole is perpendicular to the static magnetic field superimposed on the plasma. As is known, such a source is equivalent to the rotating electric dipole which, depending on its rotation direction, is capable of exciting waves with different helicity types of the phase fronts. Based on an eigenfunction expansion representation of the source field [4, 5], we calculate the field components of the rotating dipole as well its total radiated power and the partial powers going to different azimuthal harmonics of the excited field. The emphasis is placed on the nonresonant part of the whistler frequency range and the plasma parameters typical of the Earth's ionosphere. It is established that the rotating elementary electric dipole can excite only the 1st or (-1)st azimuthal field harmonic, which is determined by the rotation direction of the total dipole moment, i.e., the sign of the phase shift, $\pi/2$ or $-\pi/2$, between the currents in the two crossed dipoles. It is also shown that allowance for a finite length of these dipoles results in excitation of the higher odd harmonics. However, their amplitudes turn out to be very small for an electrically short sources in the considered part of the whistler range. The results obtained can be useful in understanding the basic features of exciting twisted whistlers by phased antennas of the electric type in a magnetoplasma.

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Two canonical forms of the metric graph eikonal algebra and the graph geometry

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The algebra of eikonals $\mathfrak{E}$ of a metric graph Ω is an operator C^*-algebra determined by the dynamical system with boundary control, that describes wave propagation on the graph. This algebra was introduced in [1] and later studied in [2–4]. The newest results, which are described below, were published in [5].

In this paper, two canonical block forms (*algebraic* and *geometric*) of the algebra, $\mathfrak{E}$ are provided for an arbitrarily connected locally compact graph. These forms determine some metric graphs (*frames*) $\mathfrak{F}^{\rm a}$ and $\mathfrak{F}^{\rm g}$. Frame $\mathfrak{F}^{\rm a}$ can be determined by the boundary inverse data. Frame $\mathfrak{F}^{\rm g}$ is related to graph geometry. On these graphs coordinates are introduced and, in the case of so-called *ordinary graphs*, frames are identical $\mathfrak{F}^{\rm a} \equiv \mathfrak{F}^{\rm g}$ (points of two graphs are connected via same coordinates). The results are supposed to be used in the inverse problem that consists of determination of the graph from its boundary inverse data.

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Two-dimensional EIT problem for Riemannian surfaces

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Let (M, g) be a Riemannian surface with the boundary Γ and metric tensor g. The Dirichlet-to-Neumann map of (M, g) is given by $\Lambda: \ f \mapsto \partial_\nu u^f|_\Gamma$, where u^f is a harmonic function in M with the trace f on Γ and ν is the outward normal to Γ. It is well-known that the DN map Λ determines not the surface (M, g) but only its conformal class $[(M, g)]$.

In the talk, we discuss the algebraic version of the BC-method for determining $[(M, g)]$ via Λ. The generalizations of this method for the cases of non-orientable surfaces and surfaces with internal holes are described. Also, the characterization of DN maps is provided. Finally, we establish a continuous dependence (in a relevant sense) of the conformal class $[(M, g)]$ on its DN map Λ.

A functional model of a class of symmetric semi-bounded operators

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Let $\{\mathscr{H}, \mathscr{K}; L_0; \Gamma_1, \Gamma_2\}$ be the Space of Boundary Values (by M. I. Vishik) of a semi-bounded (positive definite) symmetric operator L_0, so that

$$(L_0^* u, v)_{\mathscr{H}} - (u, L_0^* v)_{\mathscr{H}} = (\Gamma_1 u, \Gamma_2 v)_{\mathscr{K}} - (\Gamma_2 u, \Gamma_1 v)_{\mathscr{K}}, \qquad u, v \in \operatorname{Dom} L_0^*$$

holds, where

$$\mathscr{K} := \operatorname{Ker} L_0^*, \quad \Gamma_1 := \mathbb{I} - L^{-1} L_0^*, \quad \Gamma_2 = P L_0^*,$$

L is the Friedrichs extension of L_0, P projects in $\mathscr{H}$ on $\mathscr{K}$. Let the deficiency indices of L_0 be $(1, 1)$.

In the talk, a functional model of L_0 is proposed. The model is constructed using operators and spaces of the dynamical system

$$\begin{aligned} & u_{tt} + L_0^* u = 0 && \text{in } \mathscr{H}, \ \ 0 < t < T \leqslant \infty; \\ & u|_{t=0} = u_t|_{t=0} && \text{in } \mathscr{H}; \\ & \Gamma_1 u(t) = h(t) \in \mathscr{K}, && 0 \leqslant t \leqslant T \end{aligned}$$

associated with the SBV of L_0. Let $u = u^h(t)$, $0 \leqslant t \leqslant T$ be its trajectory. The basic element of the construction is the triangular factorization by M. G. Krein of the operator $C^T := (W^T)^* W^T$ in the space $L_2([0, T]; \mathscr{K})$, where $W^T : h \mapsto u^h(T)$ is the control operator.

This result contributes to a program of studying the *wave models* of symmetric semi-bounded operators.

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Investigation of the problems for the parabolic equations with the incompatible initial and boundary data

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When we study boundary value problems in the Hölder space $C_{x\ \ t}^{2+k+\alpha, 1+\frac{k+\alpha}{2}}(\overline{\Omega}_T)$, $k = 0, 1, \ldots, \alpha \in (0, 1)$, we require the fulfillment of the compatibility conditions of the initial and boundary data of the $0, \dots, 1 + [k/2]$ orders, if the smoothness of the boundary of the domain, given functions and coefficients of the problem corresponds to the smoothness of the solution. These conditions guarantee the continuity of the solution and all its admissible derivatives in the closure of domain. Let the considered problem is the mathematical model of some physical process. If the process goes continuously, then the compatibility conditions of the initial and boundary functions are fulfilled. But if we consider the problem describing the process since its beginning or the moment, when the functions and the coefficients of the problem have jumps, the compatibility conditions are not fulfilled, but the process can go.

The first problem with the incompatible initial and boundary data is two-phase classical Stefan free boundary problem for the heat equations [1], p. 260. The solution of this problem is express via the integral of probability $\operatorname{erfs}\frac{x}{2\sqrt{a_i t}}$, $x > 0$, $\operatorname{erfs} z := \frac{2}{\sqrt{\pi}}\int_z^\infty e^{-\zeta^2}\,d\zeta$, here $a_i > 0$, $i = 1, 2$, are the coefficients from the heat equations. We have $\partial_x \operatorname{erfs}\frac{x}{2\sqrt{a_i t}} = -\frac{1}{\sqrt{\pi a_i t}}e^{-\frac{x^2}{4a_i t}} = -2\Gamma_i(x,t)$ and $\partial_t \operatorname{erfs}\frac{x}{2\sqrt{a_i t}} = a_i\partial_x^2 \operatorname{erfs}\frac{x}{2\sqrt{a_i t}} = -2a_i\partial_x\Gamma_i(x,t)$, where $\Gamma_i(x,t)$ is a fundamental solution of the heat equation. We can see that the incompatibility of data leads to appearance of the singular solution,

There are considered the following one-dimensional problems for the parabolic equations with the incompatible initial and boundary data: two-phase free boundary problems [2], linear the first, second boundary value problems [3], the problem with the time derivative in the boundary condition [4] and conjugation problem [5]. The singular solutions are obtained in the explicit form, for the linear problems it is proved that the solution of each of them is represented as a sum of a Hölder function and singular functions, the number of which equal the number of incompatible conditions. In [6] there is determined the weighted Hölder space permitting to find the singular solutions of the boundary value problems for the parabolic equation with the variable coefficients.

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Lagrangian intersections and typical transitions of phase in semi-classical approximations

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The problem of semi-classical Lagrangian intersections arises naturally when constructing "semi-classical Green functions", e.g. the solution of an equation of Helmholtz type with Hamiltonian

$H(x,p)$ and a right hand side microlocalized (supported) on some Lagrangian manifold Λ, namely

$$H(x, hD_x)u_h(x) = f_h(x).$$

The standard, or generic situation, was studied initially by Melrose and Uhlmann [1] for high energy asymptotics of Cauchy problem for the wave equation, and extended to the semi-classical setting in [2, 3]. It refers to the case when the Hamilton vector field in $\{H(x,p) = E\}$ is transverse to Λ. However, in some physical application such as the propagation of Bessel beams in Optical fibers, the intersection of Λ with its flow-out $\Lambda_+(E)$ by the Hamiltonian flow, can fail to be transverse for some E. The Lagrangian manifolds Λ and $\Lambda_+(E)$ may then intersect along some "glancing" points: when isolated they were called "kisses" by Y. Eliashberg and M. Gromov; otherwise they can give raise to "triads". Classification of such glancing points in phase-space was performed in [4]. The simplest situation is the 2-D case where $H|_\Lambda$ has a non-degenerate maximum (or minimum) at a point $\rho_0 \in \Lambda$, $H(\rho_0) = E_0$, and no other critical points with critical value close to E_0. Then in some Darboux coordinates (y, η) where $H - E_0 = \eta_1$, Λ has a generating function of the form $\phi(y) = \pm\frac{y_1^3}{3} + y_1 y_2^2$.

First we describe the unfolding (metamorphosis) of the level sets $L_E = \Lambda \cap \{H(x,p) = E\}$ for E close to E_0. This is exactly the situation of propagation of Bessel beams, namely $H(x,p)$ can be identified with a conformal metric with a non-degenerate maximum or minimum at the origin, and $f_h(x_1, x_2) = J_0\big(\sqrt{(x_1 - a)^2 + x_2^2}/h\big)$, $a \neq 0$, where J_0 is Bessel function, in which case Λ is a Lagrangian cylinder.

Our next result consists in constructing the semi-classical approximation of $u_h(x; E)$ for E near E_0, as an oscillatory integral with boundary, using a phase function related to $\phi(y)$ and an amplitude verifying the transport equations. In other words, we construct the associated semi-classical distributions describing the "diffracted Bessel beams".

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Method of spectral mappings for differential operators with distribution coefficients

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In this talk, we will consider the solution of the inverse spectral problem for the Sturm–Liouville equation

$$-y'' + q(x)y = \lambda y, \quad x \in (0, \pi), \quad q \in W_2^{-1}(0, \pi), \tag{1}$$

subject to the boundary conditions

$$y^{[1]}(0) = 0, \quad y^{[1]}(\pi) + Hy(\pi) = 0 \tag{2}$$

by the method of spectral mappings. Here $q = \sigma'$ in the sense of distributions (generalized functions), $\sigma(x)$ is a real-valued function of $L_2(0,\pi)$, $y^{[1]} := y' - \sigma y$ is the quasi-derivative, λ is the spectral parameter, H is a real constant. The inverse problem consists in the recovery of $\sigma(x)$ and H from the eigenvalues $\{\lambda_n\}_{n\ge 0}$ of (1)–(2) and the weight numbers $\{\alpha_n\}_{n\ge 0}$, where $\alpha_n := \int_0^\pi y_n^2(x)\,dx$ and $y_n(x)$ are the eigenfunctions normalized by the condition $y_n(0) = 1$.

The method of spectral mappings was developed by Yurko [1] for higher-order differential operators with regular coefficients. This method consists in the reduction of a nonlinear inverse problem to a linear equation in a suitable Banach space. In this talk, we will discuss the generalization of this method to differential operators with distribution coefficients. As a simple example, we will consider the application of the method of spectral mappings to the Sturm–Liouville problem (1)–(2) (see [2]). By developing the ideas of this method, the spectral data characterization has been obtained for the matrix Sturm–Liouville operator with general self-adjoint boundary conditions (see [3]) and a constructive approach to inverse spectral problems for higher-order differential operators with distribution coefficients has been developed (see [4] and references therein).

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On resolvent of Schrödinger operator on graph with small loops

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We consider a Schrödinger operator on a model graph with several small loops incident to the same vertex and assume that such operator does not obey a typical non-resonance condition. In previous works such condition ensured that the resolvent of general elliptic operators on general graphs with small edges was holomorphic in the small parameter governing the lengths of the small edges. For our model we study the behavior of the parts of the resolvents corresponding in certain sense to finite and small edges. We show that despite the nonresonance condition is violated, the parts of the resolvents keep their holomorphy property. At the same time, in the leading term of the Taylor series associated with small edges there arise an additional qualitatively new term describing a certain localization of the resolvent on the small edges.

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Inverse problem with unknown sources for a quasi-linear complex heat transfer model

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The process of complex heat transfer in a bounded domain $\Omega \subset \mathbb{R}^3$ with the boundary Γ in the time interval $(0, T)$ is described by the following quasi-linear system of PDEs:

$$\sigma\partial\theta/\partial t - \operatorname{div}(k(\theta)\nabla\theta) - \mu_a\varphi = \sum_{j=1}^{m} q_j(t) f_j(x), \quad -\operatorname{div}(\alpha\nabla\varphi) + \mu_a\varphi = 0, \quad x \in \Omega, \quad 0 < t < T, \tag{1}$$

$$\theta|_\Gamma = \theta_b, \quad \alpha\partial_n\varphi + \beta(\varphi - \theta_b^4)|_\Gamma = 0, \quad \theta|_{t=0} = \theta_0. \tag{2}$$

Here, θ is the temperature, φ is the intensity of radiation averaged over all directions, α is the photon diffusion coefficient, μ_a is the absorption coefficient, $k(\theta)$ is the coefficient of thermal conductivity, σ is the product of the specific heat capacity and the volume density, the positive parameter β describes the reflection properties of the boundary.

Unknown functions q_j model the intensities of heat sources. The inverse problem is to find the intensities q_j and the corresponding solution θ, φ of system (1), (2) under additional integral conditions:

$$\int_\Omega f_j(x)\theta(x,t)dx = r_j(t), \quad 0 < t < T, \quad j = 1, 2, \ldots, m. \tag{3}$$

Let $\Omega_j \subset \Omega$, $j = 1, \ldots, m$, be disjoint subdomains; $f_j(x) = 1$ if $x \in \Omega_j$, and $f_j(x) = 0$ if $x \in \Omega \setminus \Omega_j$. In this case, the inverse problem consists in finding the coefficients $q_1(t)$, $q_2(t), \ldots, q_m(t)$ and solutions θ and φ of (1), (2) such that

$$\int_{\Omega_j} \theta(x,t)dx = r_j(t), \quad j = 1, 2, \ldots, m, \quad t \in (0, T).$$

Obviously, the values $r_j(t)$, $j = 1, 2, \ldots, m$, are the prescribed mean temperatures on the Ω_j.

The solvability of the inverse problem is proven without any smallness assumptions on the model parameters. In the class of bounded temperature fields, the uniqueness of the solution of the inverse problem is proven. Also, we study the Tikhonov regularization in the framework of a PDE constrained optimization problem and show that the approximating sequence contains a convergent subsequence.

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The analogue of advanced and retarded fundamental solutions for ultrahyperbolic equation

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We consider the inhomogeneous ultrahyperbolic equation of the following form

$$(\Delta_y - \Delta_x)\, u = f, \tag{1}$$

where $u = u(x, y)$, $(x, y) \in \mathbb{R}^d \times \mathbb{R}^n$, $d, n \geq 1$. Properties of ultrahyperbolic equations are considerably different from that of hyperbolic ones. Few examples of well-posed problems for equation (1) are known. One of them is the characteristic problem, in which $f = 0$ and the solution u is to be found in the region $\{|x| < |y|\}$ from its values on the characteristic cone $\{|x| = |y|\}$. Well-posedness of this problem was established by A. S. Blagoveshchensky.

We address the issue of constructing a fundamental solution for equation (1). The relevant results were obtained first by G. de Rham, I. M. Gelfand, G. E. Shilov, and by a number of authors afterwards. Fundamental solutions are non-unique, which is similar to the case of hyperbolic equations. Most of known results concerning ultrahyperbolic equations provide fundamental solutions invariant with respect to $\mathrm{O}(\mathbb{R}^d) \times \mathrm{O}(\mathbb{R}^n)$-rotations. In the theory of hyperbolic equations, however, the fundamental solutions of different type play crucial role. These are advanced and retarded fundamental solutions. We study their analogue for equation (1), which are solutions supported in the half-space $(x+y)\cdot\gamma \geq 0$, where γ is a given unit vector in $\mathbb{R}^k$, $k = \min(d,n)$. Such fundamental solutions were constructed by A. Melin in the case $d = n = 3, 5, \ldots$. We consider the case of arbitrary d, n, and, besides, we find an explicit representation for the fundamental solutions in the case $d = n = 3$.

Stability theory for the problem on rotation of a two-phase drop

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We state the nonlinear problem with a free surface outside the fluid Γ_t^- and an unknown interface Γ_t^+ with respect to the perturbations of velocity vector field $\boldsymbol{v} = (v_1, v_2, v_3)$ and pressure function p

$$\begin{aligned}
&\rho^\pm\big(D_t\boldsymbol{v} + (\boldsymbol{v}\cdot\nabla)\boldsymbol{v}\big) - \mu^\pm\nabla^2\boldsymbol{v} + \nabla p = \rho^\pm(\boldsymbol{f} + \varkappa\nabla U),\\
&\nabla\cdot\boldsymbol{v} = 0 \ \text{ in } \cup\,\Omega_t^\pm = \Omega_t^+ \cup \Omega_t^- \subset \mathbb{R}^3,\ t>0, \qquad \boldsymbol{v}(x,0) = \boldsymbol{v}_0(x) \ \text{ in } \cup\,\Omega_0^\pm,\\
&[\boldsymbol{v}]\big|_{\Gamma_t^+} \equiv \boldsymbol{v}^+(x,t) - \boldsymbol{v}^-(x,t) = 0, \qquad [\mathbb{T}(\boldsymbol{v},p)\boldsymbol{n}]\big|_{\Gamma_t^+} = \sigma^+ H^+\boldsymbol{n} \quad \text{on } \Gamma_t^+,\\
&\mathbb{T}(\boldsymbol{v},p)\boldsymbol{n}\big|_{\Gamma_t^-} = \sigma^- H^-\boldsymbol{n} \ \text{ on } \Gamma_t^-, \qquad V_{\boldsymbol{n}} = \boldsymbol{v}\cdot\boldsymbol{n} \quad \text{on } \Gamma_t = \Gamma_t^+ \cup \Gamma_t^-.
\end{aligned}$$

The condition $V_n = \boldsymbol{v}\cdot\boldsymbol{n}$ means the fact that boundary speed is equal to the normal velocity on Γ_t. Here $U(x,t) = \int_{\Omega_t} \frac{\rho^\pm \mathrm{d}z}{|x-z|}$, $\varkappa \geq 0$ is the gravitational constant, $[u]|_{\Gamma_t^+}$ denotes the jump of a function u across Γ_t^+; $\nu^\pm$, $\rho^\pm > 0$ are the step-functions of viscosity and density, respectively, $\boldsymbol{v}_o$ is initial velocity distribution, $\mathbb{T}$ is stress tensor with the elements $T_{ik} = -\delta_i^k p + \mu^\pm(\partial v_i/\partial x_k + \partial v_k/\partial x_i)$, $i,k = 1,2,3$; $\mu^\pm = \nu^\pm\rho^\pm$, δ_i^k is the Kronecker delta, $\sigma^\pm \geq 0$ are surface tension coefficients, $\boldsymbol{n}^\pm$ are the external normals to $\Gamma_t^\pm$; $H^\pm$ are the doubled mean curvatures of these surfaces, ω is the angular velocity of rotation about the axis x_3, $x' = (x_1, x_2, 0)$. The dot means the Cartesian scalar product. At initial instant $t = 0$, the surfaces $\Gamma_0^\pm$ are given.

We suppose to comply with conservation law of volume: $|\Omega_t| = |\mathcal{F}|$, $|\Omega_t^+| = |\mathcal{F}^+|$, $\Omega_t \equiv \overline{\Omega_t^+} \cup \Omega_t^-$, that of the center of gravity of both fluids, momentum and angular momentum. Here $\mathcal{F}$, $\mathcal{F}^+$ are axisymmetric equilibrium figures, symmetric also with respect of x_3.

A study of the problem is made in the Hölder spaces [1]. The stability of a rotating two-phase drop with self-gravity is proved for sufficiently small initial data, angular velocity and exponentially decreasing body forces, as well as the positiveness of the second variation of energy functional. The proof is based on the analysis of small perturbations of an equilibrium state of a rotating two-layer fluid. As a result, it is concluded that the perturbation of the axisymmetric equilibrium figure exponentially tends to zero as $t \to \infty$, while the motion of the drop turns into rotation of the liquid mass as a solid body. A similar result in the Sobolev–Slobodetskiĭ spaces was obtained by us in [2, 3].

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Possibilities of using the correlation-iterative approach to acoustic tomography of incoherent sources

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The using of active sources, which emit signals of a certain shape, is difficult or impossible in a number of acoustic tomography problems. In this case the antenna array record the values of the acoustic field produced by incoherent noise sources distributed in the medium. If their spatial power density is isotropic, the coherence function of the signals received at each pair of points is proportional to the difference between the retarded and advanced Green's functions with arguments at these points. The noise interferometry method is based on this observation [1]. However, the requirement of an isotropic arrangement of noise sources is a significant limitation, which is often not met in practice. At the same time, the problem of estimating the parameters of the medium with completely unknown field sources seems to be strongly underdetermined.

An iterative tomography algorithm is proposed for a situation where some of the noise sources are partially controlled. This means that for each of them the power and coordinates are known, and it is possible to make measurements in both its presence and absence. The set of Q-matrices is introduced, which are the differences of the field coherence functions in the presence and in the absence of each controlled noise source, normalized to its power. This set acts as the input data of the algorithm. If the scattering amplitude is known, these matrices can also be derived from it. At the zero iteration of the algorithm, the background medium is assumed to be homogeneous, and an a priori estimate of the scatterer is given. If there is no such information, then this estimate is assumed to be zero. Further, each subsequent iteration is carried out in two steps. At the first step, the direct scattering problem is solved and the Green's function of the inhomogeneous medium is calculate, taking into account the current estimate of the scattererd. At the second step, this estimate is considered as a background medium. The values of the Q-matrices obtained on the basis of the experimental data are substituted into the Lippmann–Schwinger type equation for the coherence functions of the noise field in the medium. Finally, this equation is solved with respect to the scatterer function, resulting in an improved estimate. The iterations are repeated until the scatterer estimate does not change significantly.

Numerical simulations were carried out in a two-dimensional case for scatterers of different strengths with a size of several wavelengths. The results of the reconstruction by the proposed algorithm, the "classical" iterative algorithm based on the scattering amplitude processing [2], and Novikov's functional-analytical algorithm [3, 4] were compared. It was shown that the proposed algorithm provides the reconstruction of scatterers of greater strength compared to the "classical" one, although it loses somewhat to the Novikov algorithm in this respect.

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Effective asymptotics of stationary (pseudo)differential equations with spatially localized right-hand sides

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The talk is devoted to the method of constructing semi-classical asymptotic solutions of multidimensional stationary linear inhomogeneous partial differential (and pseudo-differential) equations with localized right-hand sides. These problems are close to the problems on the asymptotics of the Green function for the corresponding operators, in particular, the problems on the asymptotics of the Green function for the Helmholtz equation studied in numerous papers and arise in various fields of physics. The method is based on ideas dating back to J. Keller, V. M. Babich, V. P. Maslov, V. V. Kucherenko, R. Melrose, G. A. Uhlmann, and allows one to describe asymptotic solutions using constructively defined families of trajectories in the form of WKB functions or the canonical Maslov operator in the presence of caustics and focal points. The constructed asymptotics contain information about the form of the wave-generating source. The method is illustrated by examples for the Helmholtz equation.

The results were obtained jointly with A. Yu. Anikin, V. E. Nazaikinskii, M. Rouleux and A. A. Tolchennikov were supported by the Russian Science Foundation (project № 21-11-00341).

Asymptotics of the fundamental solution of 2D Helmholtz equation with a linear refractive index and asymptotics of solution of this equation with a localized right hand side

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The talk will be devoted to the 2-dimensional model Helmholtz equation with linear refractive index and localized right side, and also the fundamental solution of this equation. In similar tasks according to the work [1], the construction of asymptotic solution is based on the Lagrangian surface, composed of semi-trajectories of the Hamilton system, released from circle (the intersection of the vertical plane with the zero surface Hamiltonian level). However, the direct result of this article cannot be applied, since there is one trajectory, which comes to a singular point with infinite time. This leads to the asymptotic solution will be localized not only in the neighborhood projections of a 2-dimensional Lagrangian manifold into physical space, but also in the vicinity of the projection of a special ray, which “breaks” from Lagrangian surface.

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Long nonlinear coastal waves and billiards with semi-rigid walls

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Long non-breaking waves of various nature are quite well described by a nonlinear system of shallow water equations [1]

$$\eta_t + \langle \nabla, (D(x) + \eta)\mathbf{u} \rangle = 0, \quad \mathbf{u}_t + \langle \mathbf{u}, \nabla \rangle \mathbf{u} + g\nabla\eta = 0, \tag{1}$$

$x = (x_1, x_2)$, where $\eta = \eta(x,t)$ (the elevation of the free surface) and $\mathbf{u} = \mathbf{u}(x,t) = (\mathbf{u}_1(x,t), \mathbf{u}_2(x,t))$ (depth-averaged horizontal fluid velocity) are unknown functions. The basin depth $D(x)$ is given function and g is acceleration of gravity. We assume that the function $D(x)$ is smooth in some neighbourhood of the closure of the domain $\Omega^0 = \{x \in \mathbb{R}^2 \mid D(x) > 0\}$ and $\nabla D(x) \neq 0$ on the boundary $\Gamma^0 = \partial\Omega^0 = \{x \in \mathbb{R}^2 \mid D(x) = 0\}$.

Water line $\Gamma(t)$ at any time t is defined by the equation ("boundary condition")

$$(\eta(x,t) + D(x))|_{\Gamma(t)} = 0, \tag{2}$$

and water occupies the area $\Omega_t = \{x \mid \eta(x,t) + D(x) > 0\}$. Thus, we consider the system (1) in the domain Ω_t with free boundary $\Gamma(t)$. We are interested in solutions that are periodic in time.

We consider solutions small enough to prevent wave breaking and to treat the nonlinearity in the equations as a perturbation. The main result of the work [2] is the construction of a change of variables, which reduces the considered system to a system with the same linear part (but more cumbersome nonlinearity), which already needs to be solved in a fixed region Ω^0. The asymptotic solution can then be constructed using an iterative procedure, at each step of which a linear equation is solved, and the reverse change of variables gives a parametric representation of the asymptotic solution of the nonlinear problem.

In the present talk, we implement this approach with two examples. First example is a bounded basin with a parabolic bottom. The other example is a billiard with semi-rigid walls. Namely, we consider the problem corresponding to the description of waves localized in the vicinity of the coastline of an island in the ocean.

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Adiabatic normal modes in an irregular elastic waveguide resting on a Winkler foundation

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Plane elastic homogeneous waveguides resting on a Winkler foundations are investigated in [1, 2]. Even in the homogeneous case, the problem is rather complicated to obtain explicit analytical results.

We study normal modes in the adiabatic approximation in case of a smoothly irregular waveguide in the frequency domain. To solve the problem, we should study the second order differential equations in partial derivatives with different boundary conditions on the upper and lower faces of the waveguide. The main feature of our paper is in the reduction of the problem to a system of differential equations of the first order, which are written as a Schrödinger-type equation with a small parameter near the derivative with respect to the variable along the axis of the waveguide by analogy with [3–5]. To construct adiabatic modes we choose the ansatz for adiabatic solution as in the Quantum Mechanics problems. The terms in the expansion satisfy the system of recurrent equations, which should be solved recursively. At each step, we should solve the inhomogeneous equation for elastic equations, where the inhomogeneity arises even in the relation between the strain and deformation. This fact causes additional difficulties in obtaining the solution. We give a procedure for constructing all terms of the expansion.

Formulas obtained show that the adiabatic approximation fails near the points of mode degeneration, i.e., near the section of the waveguide, where the slownesses of two modes coincide. We estimate the width of the zone near the degeneration points beyond which adiabatic expansion has an asymptotic nature and can be applied.

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Resonance method for monitoring osteoporosis changes in bone tissue

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Quantitative Ultrasound (QUS) assessment of the state of compact bone tissue is significant for the osteoporosis diagnosis and prevention. Osteoporosis is a chronic bone disease characterized by a decrease in bone mass and a violation of its microstructure, which increases the risk of atraumatic fractures. The QUS is based on the excitation and registration of ultrasonic guided waves (GWs) by piezoelectric transducers applied to the waveguide's surface, i.e., to the soft tissue covering the cortical bone layer, which is underlain by porous tissue and quasi-liquid bone marrow. The parameters of ultrasonic GWs are sensitive to changes in the bone mechanical and geometric properties; therefore, quantitative solutions to the corresponding wave problems create a basis for revealing osteoporosis diagnostic signs in wave measurement data. Fitting the calculated GW characteristics to experimental data (i.e., solving the inverse problem) makes it possible to estimate the thickness and elastic properties of the cortical layer, the degree of porosity of the inner layers, etc.

The paper is devoted to the mathematical and computer simulation of wave processes in layered samples (phantoms) mimicking waveguide properties of tubular bones. The mathematical models are based on the explicit representations for GWs generated in such phantoms of various structure by a surface source. These wave asymptotics are derived from the path integrals of the waveguide's Green matrix and source load using the residue technique.

The performed analysis [1,2] has shown that a layer of soft tissue creates a serious difficulty in extracting useful information about the weakened internal layers from the surface received signals. The main part of the GW energy is concentrated in the soft upper layer, and, as a result, the changes in the wave characteristics associated with the weakening inner layers are difficult to detect against the powerful wave interference in the upper layers. Therefore, the frequency spectra, time-frequency wavelet patterns, and amplitude frequency-response characteristics of excited traveling waves do not look sufficiently promising as diagnostic indicators. On the other hand, the parametric analysis carried out on the basis of the developed computer model has revealed a noticeable sensitivity of the GW resonance frequencies to the variation of elastic properties of deepened inner layers. These frequencies manifest themselves as prolonged oscillations in the received signals; thus, they can be easily detected through the spectral analysis of signals' tails, which was experimentally confirmed.

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Light scattering calculations for multilayered spheroids using spherical and spheroidal bases

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We consider the light scattering by multilayered spheroidal particles with confocal and non-confocal boundary layers. In the first case, the spheroidal layer surfaces are the coordinate ones in the same system, in the second case, they are in different systems. When applying a spherical basis to solve such light scattering problems, all fields are expanded in terms of the vector spherical functions $\vec{M}^{\vec{a}}$, $\vec{N}^{\vec{a}}$ that contain spherical harmonics being solutions to the scalar wave equation written in spherical coordinates. For spheroidal particles, this approach leads to numerical troubles when the layers boundaries are essentially non-spherical. In the electrostatic case, it has been proved that solvability of the problem takes place under the condition that for each spheroidal layer, except for the core, the aspect ratio of the external boundary $a/b < \sqrt{2}+1$ [1].

Applying spheroidal bases to solve the problem better involves its geometry and allows one to find highly accurate solution in a wide range of parameter values. Analytical relations between spheroidal and spherical functions make it possible to transfer the spheroidal T-matrix to the standard spherical one. The latter is interesting as it allows one to analytically calculate different cross-sections for ensembles of randomly oriented spheroidal particles, which is useful in various applications.

In this work the earlier obtained solution for a spheroid with two layers [2] is generalised to the case of an arbitrary number of both confocal and non-confocal layers. An essential improvement of the solution is provided by "zero exclusion" being a procedure that uses some symmetry properties to reduce the problem to two systems of four times smaller dimension. We also discuss the features of the transformation of the spheroidal T-matrix in to the spherical one. Surprisingly, even for highly aspherical spheroids ($a/b > 30$), the numbers of the expansion terms required for the spherical and spheroidal bases to reach a given accuracy of results differ by just about 30%.

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3D-topological vector solitons in laser media

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Within the framework of the three-dimensional complex Ginzburg–Landau equation for the electric strength envelope and the Bloch equations for a medium with laser amplification and nonlinear absorption, vector solitons with wavefront and polarization structure singularities are found and studied. In the inertialess limit, these solitons are a generalization of two-dimensional vector laser solitons [1]. An example is given in Fig. 1. The effect of relaxation on the stability region of solitons has been studied.

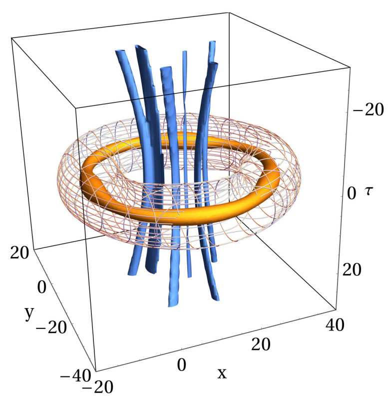

Fig. 1: Stable vector soliton with topological charges of circular components E_+ and E_- equal to 6 and -1, correspondingly. Two isosurfaces of the total intensity at level 1 (mesh) and 8.5 (solid, yellow) are shown. All six vortex lines of E_+ (blue lines) are in constant motion within the ring of high intensities. The x-axis and y-axis represent the transverse Cartesian coordinates. The third dimension τ is time in the moving coordinate system, $\tau = t - z/v$, where t is time in the laboratory coordinate system, z is the longitudinal coordinate, and v is the group velocity.

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Complex WKB method and topological phase

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The version of the WKB method developed to study semiclassical asymptotics of solutions of the one dimensional differential and difference equations to complex plane is what we call in the talk the complex WKB method. When studying systems of differential and difference equations on the complex plane, one encounters the topological (or geometrical phase or even Berry phase). It appears that, in the framework of the complex WKB method, it is very natural to consider this phase as an integral of a meromorphic differential on a Riemann surface. In this talk, we discuss properties of this differential and some applications.

On expression of the field in the 3D space through its values on the outer surface of a rectangular prism in parabolic approximation

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Numerical solution of the 3D parabolic equation is quite often done using some finite-difference scheme, which is necessary limited to a finite computational domain. It should be noted that the most practically convenient domain is rectangular, which in the 3D space has a shape of a rectangular prism extended in the direction of radiation propagation. After a finite-difference solution is found, one may need to recover the field amplitude in the whole free 3D space based on the newly obtained values in the computational domain but also taking into account the initial conditions. How to do this is well known in case of the 2D parabolic equation [1], while for the 3D case none such a method has been developed so far.

In this presentation a method to express the field amplitude in the 3D space in the parabolic approximation based on the amplitude's values on the outer surface of a rectangular prism is proposed. The connection of the proposed method to the 3D exact transparent boundary condition [2] in a rectangular computational domain is considered. The numerical experiments with Gaussian beams demonstrate the validity of the proposed method. Possible extensions of the developed method to spaces with more than three dimensions are discussed.

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Guided waves propagation in multilayered piezoelectric waveguide with electrodes connected via electrical circuit

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The paper presents an extended boundary integral equation method for simulating the elastic guided wave excitation and propagation in a bimorph as a multi-layered piezoelectric laminate with electrodes connected via electrical circuit or connected to capacitors. The method is based on the integral representation in terms of the Fourier transform of the Green's matrix for the whole piezoelectric laminate [1]. The obtained boundary integral equations are solved numerically using the Bubnov–Galerkin method with Chebyshev polynomials of the second kind. The present method is compared with the standard finite element method (FEM), and the efficiency and convergence of the present method are also demonstrated by several representative numerical examples. The numerical parametric investigation of electrical impedance influence on wave propagation is presented and discussed. The main advantages of the present extended boundary integral equation method provides an efficient tool for a comprehensive parametric analysis of the influence of the electrical impedance on guided wave propagation through the zone between connected electrodes. Besides, the developed semi-analytical methods allows us to investigate spectral properties of the connected electrodes, and it can be further extended for the analysis of periodic arrays of connected electrodes, which can be employed to construct acoustic/elastic metamaterials with active control [2].

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The influence of porosity and gradient of water saturation of soils on surface acoustic waves

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Investigation of soil processes and sub-surface water flow dynamics is critical for agriculture (forecasting and improving of yields, for instance) and forestry, flood prediction and natural resources conservation, etc. The objective of this investigation is to develop a methodology for estimation of soil moisture level, porosity and other effective parameters including hydraulic parameters of the porous elastic fluid-saturated medium estimations based on the measurements of surface acoustical waves excited by dynamic loads.

The theoretical investigation of water infiltration process in pores based on the finite element solution of Richards equation demonstrates the complex nonlinear distributions of water contents via soil depth which dependent on the soil and water hydraulic parameters, initial water saturation and time of the infiltration [1]. The simulation of elastic wave propagation in the considered composite porous medium leads to the problem for functionally graded poroelastic half-space with water saturation dependent on the depth and a slow-time coordinates.

The theoretical study of elastic wave propagation in fluid-saturated porous media is based on the Biot–Frenkel model. In this investigation the elements structure-phenomenological approach that uncovered the relation between effective Biot's parameters of poroelastic medium and microstructure contents of soils are presented and discussed. The time-harmonic and transient solutions are obtained in terms of the Fourier transform of Green's matrix and external load, exciting the wave-field. In this investigation the algorithm for Green's matrix of multilayered porous solid, published in [2], was improved in terms of numerical stable iterations for the considered vertically functionally graded medium [3]. The mechanical and mathematical models as well as numerical analysis of the porosity and input water saturation profile influence on the phase velocities of surface waves are discussed in details.

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On the method of functionally invariant solutions for nonlinear wave equations

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The equation of the form

$$\frac{\partial^2 U}{\partial x^2}+\frac{\partial^2 U}{\partial y^2}+\frac{\partial^2 U}{\partial z^2}-\frac{1}{v^2}\frac{\partial^2 U}{\partial t^2}+\lambda_1\frac{\partial U}{\partial x}+\lambda_2\frac{\partial U}{\partial y}+\lambda_3\frac{\partial U}{\partial z}+\gamma\frac{\partial U}{\partial t}=V(U), \qquad (1)$$

where v, λ_1, λ_2, λ_3, γ are constants, $V(U)$ is the given function, is considered in the work. If $\lambda_1=\lambda_2=\lambda_3=\gamma=0$ and $V(U)=\sin U$, then equation (1) is the known sine-Gordon equation, for $V(U)=\exp U$ is the Liouville equation, for $V(U)=\sinh U$ is the sinh-Gordon equation. In works [1, 2] it was shown that some solutions of the sine-Gordon equation can be found by a method similar to the search for functionally invariant solutions of the wave equation [3]. The solution $u(x,y,z,t)$ is called functionally invariant if for any twice differentiable function $F(u)$ of the argument u, the function $F(u(x,y,z,t))$ is a solution to the considered equation too. A description of all functionally invariant solutions of the $2+1$-dimensional wave equation was given in the work [3], and of the $3+1$-dimensional equation — in the work [4]. The functionally invariant solutions to equation (1) cannot exist if there is a nonlinear function $V(U)$ on the right side. However, after after series of transformations and the introduction of a new unknown function, the search for some solutions to the sine-Gordon equation has been reduced in [1, 2] to solving the auxiliary system of partial differential equations, which corresponds to the system of equations for functionally invariant solutions of the wave equation.

This paper discusses a similar solution method for the equation of type (1), new exact solutions are obtained, a set of solutions, which can be found in this way, is analyzed.

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Radiation of a charged particle bunch exiting an open-end of a dielectric-loaded circular waveguide

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We continue developing a rigorous method for solving circular open-ended dielectric-loaded waveguide diffraction problems based on Wiener–Hopf–Fock technique. In previous papers, we dealt with case of internal excitation by a waveguide mode of a waveguide uniformly loaded with dielectric or having a layered filling [1, 2]. In this report, we present rigorous solution for the case of iniform dielectric filling and excitation by an ideally thin charged particle bunch moving along the structure axis. This topic is closely related to recent applications of dielectric-loaded waveguides for development of high-power narrow-band beam-driven THz sources [3].

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An example of the anti-localization of non-stationary quasi-waves in a 1D semi-infinite harmonic chain

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We demonstrate a new example of the mechanical system where the phenomenon of the anti-localization of non-stationary (quasi)-waves can be observed. This is a 1D semi-infinite harmonic chain subjected to an impulse loading at the free end. The anti-localization of non-stationary waves [1] is zeroing of the non-localized propagating component of the wave-field in a neighbourhood of an inclusion or defect. The known examples of systems, where this wave phenomenon is detected, are a 1D infinite harmonic chain with an isotopic defect [2], and an infinite string on the Winkler foundation with a discrete oscillator [1].

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On the Helmholtz decomposition of BMO spaces of vector fields

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The Helmholtz decomposition of vector fields is a fundamental tool for analysis of vector fields especially to analyze the Navier–Stokes equations in a domain. It gives a unique decomposition of a (tangential) vector field defined in a domain of an Euclidean space (or a riemannian manifold) into a sum of a gradient field and a solenoidal field with supplemental condition like a boundary condition. It is well-known that such decomposition gives an orthogonal decomposition of the space of L^2 vector fields in an arbitrary domain and known as the Weyl decomposition. It is also well-studied that in various domains including the half space, smooth bounded and exterior domain, it gives a topological direct sum decomposition of the space of L^p vector fields for $1 < p < \infty$. The extension to the case $p = \infty$ (or $p = 1$) is impossible because otherwise it would imply the boundedness of the Riesz type operator in L^∞ (or L^1) which is absurd.

In this talk, we extend the Helmholtz decomposition in a space of vector fields with bounded mean oscillations (BMO) when the domain of the vector field is a smooth bounded domain in an Euclidean space. There are several possible definitions of a BMO space of vector fields. However, to have a topological direct sum decomposition, it turns out that components of normal and tangential to the boundary should be handled separately. This decomposition problem is equivalent to solving the Poisson equation with the divergence of the original vector field v as a data with the Neumann data with the normal trace of v. The desired gradient field is the gradient of the solution of this Poisson equation. To solve this problem, we construct a kind of volume potential so that the problem is reduced to the Neumann problem for the Laplace equation. Unfortunately, taking the usual Newton potential causes a problem to estimate the necessary norm so we construct another volume potential based on normal coordinate. We need a trace theorem to control L^∞ norm of the normal trace. This is of independent interest. Finally, we solve the Neumann problem with L^∞ data in a necessary space. The Helmholtz decomposition for BMO vector fields is previously known only in the whole Euclidean space or the half space so this seems to be the first result for a domain with a curved boundary. This is a joint work with my student Z. Gu (University of Tokyo).

Self-induced transparency in gradient waveguides

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The effect of self-induced transparency (SIT) provides minimization of energy loss for propagation through the medium of optical pulses with definite shape [1]. The model behind this effect is usually simplified by using plane-wave approximation. Some transverse phenomena connected with SIT were studied in [2, 3]. Here we consider SIT in a gradient waveguide with quadratic transverse dependence of medium refractive index. We present effect of waveguide parameter variation on the characteristics of the wave packet in the parabolic approximation (the method of slowly varying envelope).

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Body wave asymptotics for an anisotropic elastic half-space with a surface source

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The problems of modeling wave processes in elastic anisotropic materials arise in various applications, such as ultrasonic non-destructive testing, geophysics, acoustic electronics and acoustic optics. Nowadays, to calculate the total wavefield generated in such media by a source, software packages implementing direct mesh-based numerical methods, e.g., finite element method (FEM), are commonly used. However, the FEM is usually too computationally expensive for wave simulation, and an additional post-processing is required to extract specific wave modes (e.g., quasi-longitudinal or quasi-shear waves) from the total numerical solution.

Explicit integral representation via the Green's matrix of the structure considered and the source parameters provides a quantitative solution, the same as FEM simulation. Moreover, the asymptotics of those path integrals yields closed physically clear representations for the source-generated waves of various types. The stationary-phase asymptotics of body waves excited by a surface source in a stratified isotropic half-space was derived from such integral representation in the 1980s [1]. But at that time, it was not realized for the anisotropic case due to the more complex form of the Green's matrix and phase functions, which are expressed via the roots of the characteristic Christoffel equation. Unlike the isotropic case, there are no analytical representations for these roots and stationary points, and the search for stationary points requires additional numerical differentiation.

At present, such a semi-analytical scheme of asymptotics deriving has been implemented for materials with arbitrary anisotropy; Green's matrix and Christoffel's roots are calculated using the algorithms [2]. This gives physically evident representations for each specific body wave excited by the source, the same as the modal analysis can provide. But the latter gives them as plane-wave eigensolutions up to constant factors accuracy, while the derived representations give unambiguous amplitude-frequency characteristics for the generated quasi-spherical body waves, strictly accounting for the source parameters. This makes it possible to address such problems as the source energy distribution between the excited wave modes, dependence on the propagation direction (patterns of amplitude angular diagrams), source grouping and phased source arrays, etc.

In the report, we plan to briefly discuss the derivation and computer implementation of this scheme and its verification by FEM calculations and known analytical results [3], illustrating them with numerical examples for various kinds of anisotropy. The specific anisotropy-induced bending of some stationary point curves, which leads to the appearance of several waves, associated with the same curve, that propagate in the same direction, is also subject to discussion.

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The advanced boundary integral equation method for modelling wave motion in laminates with doubly periodic arrays of cracks

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The three-dimensional problem of the modelling of elastic wave propagation in a multi-layered laminate with doubly periodic arrays of cracks of an arbitrary shape is considered. The boundary integral equation method [1, 2] is extended for the case of multiple doubly periodic arrays of cracks. The introduced unknown crack opening displacement vectors for each array are related using the Floquet theorem and solved using the Galerkin method at the reference delaminations for each array. The developed advanced boundary integral equation method provides an efficient tool for fast parametric analysis of the influence of the characteristics of the periodic arrays of cracks on the transmission and diffraction of elastic waves. Two modifications of the boundary integral equation method are proposed and compared for rectangular cracks. The convergence of the arising double series is proved and an algorithm for optimal calculation is proposed based on the preliminary analytical evaluation of the arising integral representations in terms of the Fourier transform of Green's matrices and the crack opening displacements. The employment of the proposed method for experimental and theoretical studies of the wave propagation in acoustic metamaterials [3] is discussed as well.

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X-ray echelles: rigorous account of groove profile deformations in diffraction efficiency electromagnetic calculus

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An attempt to compare the measured and calculated efficiencies of X-ray gratings for diffraction orders higher than the first was made in 1987 [1]. However, the quality of blaze (with triangular groove profiles) holographic gratings produced at that time and the impossibility of taking into account realistic border profiles in computations led to large discrepancies in the efficiency of diffraction orders higher than the first — on hundreds and even thousands of percent. The idea of the need to calculate the efficiency of X-ray gratings using rigorous methods of diffraction and the groove profiles measured using scanning tunneling (probe) microscopy was proposed in 1989 [2]; at about the same time a computer program appeared that allows calculating multilayer gratings with real boundary profiles and random roughness on based on the modified method of boundary integral equations [3].

The efficiency of the W/C multilayer soft X-ray (SXR) and extreme ultraviolet (EUV) echelle with an ideal triangular groove profile operating at a grazing incident angle of $\sim 15^\circ$ and blaze angle of $\sim 10^\circ$ in diffraction orders of 10 and higher, was calculated using the rigorous differential method and the comparative phenomenological approach [4]. The influence of the parameters of the reflecting facet, primarily its 'camber', on the diffraction efficiency of realistic silicon gratings [5] with blaze angles of 1°–4° and a period of 400–4000 nm, intended for operation in the SXR–EUV radiation in high orders, is studied in the present report. The absolute and relative (grating) efficiencies were determined by simulation using the PCGrate™ code based on the method of boundary integral equations and groove profiles measured by atomic-force microscopy. The possibility of achieving high efficiencies of high diffraction orders (with numbers ~ 10 and more) is discussed. An estimate of the influence of the introduced camber parameter of the reflected facet on the maximum achievable order number of highly efficient diffraction is obtained. It demonstrates the possibility of unique X-ray echelles fabrication.

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Development of a computation technique for solving the inverse heavy numerical problems

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A new combined computation technique for inverse heavy computational problems is proposed. The suggested technique uses a range of optimization methods to obtain the solution to inverse problems in the most efficient way. The proposed method application is illustrated on several computational problems: the direct diffraction grating and simulation of the quantum cascade structure problem. In the first case, the boundary integral equation method is applied ([1], ch. 12). Oblique-incident (off-plane) radiation scattering from one-periodical gratings (2D structures) with arbitrary conductivity and various functions of the border profile is considered. For the quantum cascade structures, InGaAs/AlInAs active region materials are used. The solution to the direct problem is obtained through the symmetrical transfer matrix approach [2].

The implemented technique could be applied in solving diffraction problems, which find their application from the creation of basic optical filters to the advanced space exploration systems [3]. The proposed technology was also relevant in direct and inverse the problems of reflectometry, which are of great use in determining the parameters of the structures and gratings under study, e.g., layered semiconductor structures like quantum-cascade lasers which find applications in areas such as medical diagnostics, high-speed communication, etc. [2].

For the testing of the proposed method a number of numerical experiments were computed. The objects of study were mainly diffraction gratings in the extreme ultraviolet wavelength ranges. The optimization were performed for a number of gratings with trapezoidal profiles with variation of the parameters of the profile of the grating and computation time. The method showed significant computation time reduction without crucial decrease in accuracy. The described approach was also tested for performance optimization of quantum cascade lasers and showed improvement in convergence time. The applied optimization techniques were based on the methods proposed by authors earlier [3]. This work was supported by Russian Scientific Foundation (project № 23-29-00216).

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Interfering circle combs and uniform hyperbolicity of cocycles

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Let $f : \mathbb{T}^1 \times J \to \mathbb{R}$ be a smooth function defined on a product of the circle $\mathbb{T}^1$ and an interval $J = [\alpha, \beta] \subset \mathbb{R}$. Suppose also that 0 is a regular value of f. Denote $X_0 = \pi_1(f^{-1}(0))$, where π_1 is the projection on the first component of $\mathbb{T}^1 \times J$. We define a *circle comb*, $\mathcal{C}$, with jags at X_0 as a product $X_0 \times I_0$, where $I_0 = [0, 1]$.

If one takes another identical copy of $\mathcal{C}$, rotate it by an angle $2\pi\omega$ and superposes it over the first copy of $\mathcal{C}$, one may see the interfering picture ([2]).

Assuming that number, N, of jags is large whereas the distance between neighbour jags is small we construct a simple asymptotic theory of this phenomenon. We apply this theory to establish the uniform hyperbolicity of a family of skew-product maps

$$F_A : (x, v) \mapsto (\sigma_\omega(x), A_t(x)v), \quad (x, v) \in \mathbb{T}^1 \times \mathbb{R}^2,$$

over irrational rotation $\sigma_\omega(x) = x + \omega \mod 1$, $\omega \in \mathbb{R} \setminus \mathbb{Q}$, which depends on a real parameter $t \in J$. Under assumption that transformation $A_t : \mathbb{T}^1 \to SL(2, \mathbb{R})$ admits a representation

$$A_t(x) = R(\varphi(x,t)) \cdot Z(\lambda(x,t)), \quad R(\varphi) = \begin{pmatrix} \cos\varphi & \sin\varphi \\ -\sin\varphi & \cos\varphi \end{pmatrix}, \quad Z(\lambda) = \begin{pmatrix} \lambda & 0 \\ 0 & \lambda^{-1} \end{pmatrix}.$$

with smooth functions $\lambda : \mathbb{T}^1 \times J \to \mathbb{R}$ and $\varphi : \mathbb{T}^1 \times J \to \mathbb{T}^1$ and the critical set (see [1]) of such skew-product is non-trivial we describe a subset of parameter values which correspond to the uniform hyperbolicity of the skew-product.

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Numerical analysis of the scattering of plane waves by two-dimensional diffraction gratings

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A two-dimensional diffraction grating with a periodic boundary is considered. By means of the augmented scattering matrix (ASM) we study the scattering in an appropriate waveguide. The experiments show that the use of the ASM instead of the conventional scattering matrix both allows to detect surface waves and improves the convergence of the approximations.

Perturbation of a simple wave: from simulation to asymptotics

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The equation, which determines the dynamics of the domain bounds in a weak ferromagnet [1]

$$\frac{\partial^2\phi}{\partial t^2} - c^2\frac{\partial^2\phi}{\partial x^2} + \Omega^2\sin\phi\cos\phi + \omega^2\,\sin\phi + \alpha\frac{\partial\phi}{\partial t} = 0 \tag{1}$$

is considered. Coefficients $c^2, \Omega^2, \omega^2, \alpha$ depend on the slow time $\tau = \varepsilon t$, [2]. Here $0 < \varepsilon \ll 1$ is a small parameter. In the case of constant coefficients the simple (traveling) wave solution $\phi = \Phi(x - v\,t)$, $v = \mathrm{const}$ is determined by the ordinary differential equation. In the case of variable coefficients an asymptotic solution as $\varepsilon \to 0$ is constructed. Main problem is the velocity of the perturbed wave. The concept of stability of the leading edge of the wave is proposed to find the velocity. This idea was suggested by computer experiments.

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On the application of the generalized WKB method for acoustic field modelling in underwater acoustics

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Coupled mode theories are a variety of techniques that can be used for modeling of waves propagation in inhomogeneous waveg-guides. Among the main issues that often make the mode coupling equations challenging for practical computations are in-stabilities and the need to resolve fast oscillations. The most natural way to separate fast oscillations consists in the use of the WKB theory. WKB approximation is success-fully used in many fields of physics, including quantum mechan-ics [2], optics, acoustics [1], internal waves theory.

In this study a generalization of the WKB method for the case of coupled modes propagation in problems of underwater acoustics is proposed. The generalized WKB ansatz has the form of a product of a matrix of phase factors and vector of amplitudes. The transport equation in the matrix form is derived, and the phase factors are computed exactly. The similarities and differences with other forms of mode coupling equations are discussed. A numerical example demonstrating the accuracy of the generalized WKB solution is presented.

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Solitons in model systems with the same dispersion law as for Krauklis or Konenkov waves

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A remarkable correlation between the delay time and the latitude of occurrence of various earthquakes was found in Ref. [1]. This correlation allowed the soviet mathematician Guberman to assume the existence of very slow trigger waves that are generated at the Earth's poles and move between them, initiating destructive seismic events. The generation occurs at those moments of time when the speed of the Earth's rotation reaches a minimum. The physical nature of these dynamic perturbations, called D-waves by Guberman, is still not clear. The D-wave velocity, according to the estimates of various authors, falls in the range of 10–100 km/year.

In layered elastic structures, there are two types of waves, which can have an abnormally low propagation velocity and strong dispersion. These are a flexural wave in thin plates and a symmetrical elastic wave (Krauklis mode) in a thin, liquid-filled gap between two solids. For both of them, the phase velocity is proportional to $(\omega h)^{1/n}$, where ω is the angular frequency, h is the thickness of plate or gap, $n = 1/3$ for gaps or $1/2$ for plates. To be earthquake triggers, these waves must have a large amplitude, which implies a strong influence on them of the nonlinearity of the medium. The simultaneous influence of nonlinearity and dispersion can lead to the formation of solitons. However, the velocity of motion of solitons depends on their amplitude which seems to contradict estimates of the properties of D-waves. This contradiction is resolvable if the condition of invariance of the soliton amplitude is provided. To do this, it is necessary to exclude the geometric divergence of wave fronts when waves move from pole to pole. This is possible in the case of waveguide propagation along the faults of the Earth's crust. On the other hand, the invariability of the soliton generation amplitude can be provided by such an excitation mechanism as the loss of stability of the shape of the Earth's shell. This mechanism is associated with the formation of an additional convexity or concavity at the poles and their mutual switching at the moment of violation of the balance of elastic, gravitational and centrifugal forces during the deceleration of the Earth's rotation. The D-wave pulse length of about 10 km and its low velocity imply that the switching process occurs very slowly and so it does not produce observable shock waves or tsunamis.

We model nonlinear flexural waves (Konenkov mode) and nonlinear Krauklis mode propagating along the the Earth's faults using simplified differential equations for their description. In the linear limit these equations give the same dispersion laws that are known for modes under consideration. The nonlinearity for the symmetric Krauklis mode is considered to be quadratic, and for the antisymmetric Konenkov mode it is considered to be cubic. The found soliton solution for the Konenkov mode is described by the sech-function, while for the Krauklis mode it is described by the sech^2-function. From a comparison of these solutions with seismic data, the numerical values of the coefficients included in the model equations are estimated.

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Saddle point method interpretation of forerunners in car tires

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We solve a non-stationary problem of wave propagation in a waveguide excited by a δ-shaped pulse in time and space. The waveguide is assumed to have complex multi-component structure, but be homogeneous in some direction. As it is well-known [1, 2], solution of this problem can be represented as a double integral over frequency ω and wavenumber k. There is a classical problem of asymptotical estimations of this integral to describe physically conditioned wave pulses propagating in the waveguide.

If there are no energy losses in the waveguide, all special points of the formal solution integrand are poles, which give residues. The result of the residue theorem application is the solution in a form of an integral of an analytical function $f(k)$ (or $f'(\omega)$) with a complex integration contour laying on several sheets of the Riemann surface of $f(k)$ [3,4]. We assume that $|f(k)|$ is defined mainly by an exponential factor. This allows one to apply the saddle point method.

In the current work we apply a muti-contour saddle point method [5] to describe a forerunner (a fast exponentially decaying wave) in a car tire. To describe non-stationary processes in a tire, it is useful to consider it as a waveguide homogeneous along the azimuthal angle θ [6]. Since a tire is a complex structure composed of rubber and cords, a set of its saddle points is not trivial. We build this set and classify the saddle points into contributing and non-contributing to the integral asymptotic.

The tire pulse response was both numerically calculated by the multi-contour saddle point method and measured experimentally. A good agreement between the experimentally observed forerunner and its theoretical prediction is shown.

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A monochromatic coupled TE-TE wave propagation in a plane shielded waveguide filled with anisotropic nonlinear medium

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We consider a monochromatic electromagnetic wave $(\mathbf{E},\mathbf{H})e^{-i\omega t}$, where $\omega > 0$ is a circular frequency,

$$\begin{aligned} \mathbf{E} &= \left(0, \mathrm{E}_y(x)e^{i\gamma z}, \mathrm{E}_z(x)e^{i\gamma y}\right)^\top, \\ \mathbf{H} &= \left(\mathrm{H}_{x1}(x)e^{i\gamma z} + \mathrm{H}_{x2}(x)e^{i\gamma y}, \mathrm{H}_y(x)e^{i\gamma y}, \mathrm{H}_z(x)e^{i\gamma z}\right)^\top, \end{aligned} \tag{1}$$

are complex amplitudes and γ is a real parameter, propagating in a plane shielded dielectric waveguide $\Sigma := \{(x,y,z) : 0 < x < h, (y,z) \in \mathbb{R}^2\}$. The field (1) is a sum of two TE waves characterized by a common wave number γ; so, we call field (1) a monochromatic coupled TE-TE wave.

The permittivity within the waveguide is described by diagonal tensor

$$\varepsilon = \begin{pmatrix} \star & 0 & 0 \\ 0 & \varepsilon_1 + \alpha_1\mathrm{E}_y^2 + \alpha_3\mathrm{E}_z^2 & 0 \\ 0 & 0 & \varepsilon_2 + \alpha_2\mathrm{E}_z^2 + \alpha_4\mathrm{E}_y^2 \end{pmatrix},$$

where $\varepsilon_l > 1$, $\alpha_1, \alpha_2, \alpha_3, \alpha_4 > 0$ are constants and element $\star$ does not affect the wave propagation. Such tensor corresponds to the tensor Kerr effect [1]. Scalar case of the Kerr effect is a main nonlinear law in nonlinear optics [2] and also arise in other fields of nonlinear mathematical physics.

Field (1) must satisfy Maxwell's equations

$$\operatorname{rot}\mathbf{H} = -i\omega\varepsilon_0\varepsilon\mathbf{E}, \quad \operatorname{rot}\mathbf{E} = i\omega\mu_0\mathbf{H},$$

where ε_0 and μ_0 are dielectric and magnetic constants, respectively. Tangential components E_y and E_z of the electric field must vanish at the absolutely conducted walls; note that H_{x1} and H_{x2} are connected with E_y and E_z by relation $\mathrm{H}_{x1} = -\gamma(\omega\mu_0)^{-1}\mathrm{E}_y$ and $\mathrm{H}_{x2} = \gamma(\omega\mu_0)^{-1}\mathrm{E}_z$ (one can get it substituting (1) into Maxwell's equations) and thus vanish at the boundaries too. We also suppose that E_z' and E_y' have fixed values at boundary $x = 0$ that is $\mathrm{E}_y'(0) = A_1$, $\mathrm{E}_z'(0) = A_2$; note that E_z' and E_y' are connected with H_y and H_z by relation $\mathrm{H}_y = -(i\omega\mu_0)^{-1}\mathrm{E}_z'$ and $\mathrm{H}_z = (i\omega\mu_0)^{-1}\mathrm{E}_y'$ (one can get it substituting (1) into Maxwell's equations). Let A be some real constant; we assume that

$$|\mathbf{H}|^2|_{x=0} = \left(\mathrm{H}_y^2 + \mathrm{H}_z^2\right)\big|_{x=0} = -\left.\frac{(\mathrm{E}_y')^2 + (\mathrm{E}_z')^2}{\omega^2\mu_0^2}\right|_{x=0} = -\frac{A_1^2 + A_2^2}{\omega^2\mu_0^2} = -\frac{A^2}{\omega^2\mu_0^2}.$$

So, the problem — we call it problem $\mathcal{P}$ — is to find a couple $(\gamma, A_1) = (\widehat{\gamma}, \widehat{A}_1)$ such that there exists field $\mathbf{E}$, $\mathbf{H}$ satisfying to all above conditions.

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Inverse extremum problem with phase constraints for a quasi-linear complex heat transfer model

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The extremum inverse problem for quasi-linear equations of radiation-conductive heat transfer simulating the process of complex heat transfer in the bounded domain G with the boundary $\Gamma = \partial G$ during the time interval $(0, T)$ consists in minimizing the target functional

$$J(\theta, \varphi, u_1, u_2) \to \inf \tag{1}$$

on the solutions of the initial-boundary value problem:

$$\sigma\partial\theta/\partial t + \operatorname{div}(k(\theta)\nabla\theta) + b(|\theta|\theta^3 - \varphi) = u_1\chi, \quad x \in G, \quad 0 < t < T, \tag{2}$$

$$-\operatorname{div}(\alpha\nabla\varphi) + \beta(\varphi - |\theta|\theta^3) = u_2\chi, \quad x \in G, \quad 0 < t < T, \tag{3}$$

$$\theta|_\Gamma = \theta_b, \quad \alpha\partial_n\varphi + 2^{-1}\varphi|_\Gamma = 0, \quad \theta|_{t=0} = \theta_0. \tag{4}$$

Moreover, the following restrictions take into account:

$$\{u_1, u_2\} \in U_{ad}, \quad \theta|_{G_b} \le \widetilde{\theta}. \tag{5}$$

Here, θ is the temperature, φ is the intensity of radiation averaged over all directions, α is the photon diffusion coefficient, $k(\theta)$ is the coefficient of thermal conductivity, σ is the product of the specific heat capacity and the volume density, positive parameters b and β describe the radiation properties of the medium, u_1 describes the power of the source spending on heating, u_2 is the power of the source spending on radiation, χ is the characteristic function of the part of the medium in which the source is located divided by the volume of the source. The set U_{ad} describes restrictions on the unknown right-hand sides u_1, u_2 and the temperature in the subdomain G_b cannot exceed the critical value $\widetilde{\theta}$.

Estimates for the solution of the initial-boundary value problem (2)–(4) are obtained. On the basis of these estimates the solvability of the control problem (1)–(4) is proved and an algorithm for solving its solution is proposed. The efficiency of the algorithm is illustrated by numerical examples.

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Whistler modes of a radially nonuniform cylindrical duct with enhanced density in a nonresonant magnetoplasma

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A great number of works have been devoted to studying the propagation of whistler-mode waves in magnetic-field-aligned cylindrical plasma structures, known as density ducts, in a magnetoplasma (see, e.g., [1, 2] and references therein). This is explained, on the one hand, by an important role played by whistler waves, which are guided by such plasma structures, in many physical processes occurring in the Earth's ionosphere and magnetosphere [1]. On the other hand, use of the guiding properties of ducts opens up new possibilities for solving a number of applied problems of space plasma diagnostics and developing novel methods of the efficient excitation of whistler waves in a magnetoplasma [2, 3]. Most theoretical works on the subject deal with specific features of ducted propagation of whistler waves in a resonant magnetoplasma. Recall that in the whistler range, the magnetoplasma is resonant at the frequencies lying above the lower hybrid resonance frequency, when the refractive index surface of the extraordinary (whistler mode) wave is open. Of no less interest is the whistler wave propagation below this frequency, i.e., under conditions of a nonresonant magnetoplasma where the refractive index surface of the extraordinary wave is closed.

In this work, the propagation of axisymmetric whistler modes in radially nonuniform cylindrical ducts with enhanced density in a nonresonant magnetoplasma is studied. It is demonstrated that such ducts can support the guided propagation of eigenmodes whose dispersion characteristics and field structures allow an approximate analytical description. It is based on the fact that the fields of these modes comprise both large- and small-scale constituents, which contribute differently to the components of the mode fields. This makes it possible to employ a perturbation theory technique and obtain at first the components dominated by the large-scale constituent. Using them in the next order of perturbation theory, the field components determined by the small-scale constituent, which is localized in the vicinity of the duct wall, are found. The analytical results are shown to be in good agreement with the results of numerical calculations of the rigorous field equations. The implications of the obtained results in relation to whistler wave excitation in smooth-walled density ducts will also be discussed in some detail.

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2-dimensional sloshing: examples of domains with interior ‘high spots’

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Sloshing in containers of various geometries has numerous practical applications and also is of considerable mathematical interest. The following boundary value problem

$$u_{xx} + u_{yy} = 0 \text{ in } W, \quad u_y = \nu u \text{ on } F, \quad \partial u/\partial n = 0 \text{ on } B \tag{1}$$

describes the sloshing velocity potential $u(x, y)$ in two dimensions. Here W is the domain of the water’s cross-section, the bounded interval F on the x-axis is the cross-section of the water’s mean free surface and B is the water’s bottom.

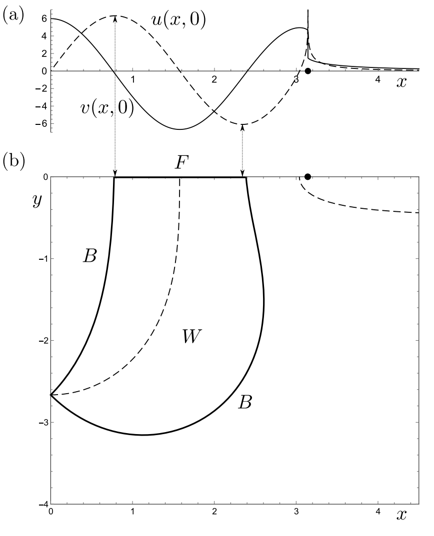

Fig. 1: Plotted for $\nu = 2$: (a) the traces of $u(x, 0)$ (dashed line) and of $v(x, 0)$ (solid line); (b) the nodal lines of u (dashed lines) and the level line $v \approx -0.185125$ (solid line). Interior high spots on F are marked by arrows connecting them with extrema of the velocity potential trace.

The notion of ‘high spots’—points, where extremal values are attained by the free surface of the fundamental sloshing mode—was introduced in [1], where a characterization of 2D domains without interior high spots was also given. The question whether there exist 2D domains with interior high spots remained open for 14 years. It is answered in the positive here; see Fig. 1, where u, satisfying problem (1) with $\nu = 2$, and the conjugate stream function v are given by some explicit formulae.

A characteristic feature of domains with interior high spots is that these domains are bulbous on each side, where such a spot is close to the endpoint of F.

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Dispersion of optical vortices in twisted elliptical optical fibers with torsional mechanical stresses

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In this paper, the dispersion of optical vortices (OVs) with a topological charge $\ell \geq 1$ in twisted elliptical fibers (TEF) with torsional mechanical stresses (TMS) is studied. Analytical expressions for polarization, topological, and hybrid dispersion are established on the basis of spectra of vortex modes in step and gradient TEFs with TMS. It is shown that in intensely twisted fibers with a gradient refractive index profile, new types of dispersion appear in comparison with the case of a step-index profile. The dependence of the dispersion of OVs on the material and geometrical parameters of the fiber is studied numerically. It has been established that in step-index TEFs with TMS, there is a spectral region, near which all types of dispersion for OVs with $\ell \geq 1$ take a near zero value. It has also been demonstrated that for TEFs with a gradient profile, this regime is realized for highly twisted highly elliptical fibers.

On the global stability of large Fourier mode for 3-D Navier–Stokes equations

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In this paper, we first prove the global existence of strong solutions to 3-D incompressible Navier–Stokes equations with solenoidal initial data, which writes in the cylindrical coordinates in the form: $\mathrm{A}(r,z)\cos N\theta + \mathrm{B}(r,z)\sin N\theta$, provided that N is large enough. In particular, we prove that the corresponding solution has almost the same frequency N for any positive time. The main idea of the proof is first to write the solution in trigonometrical series in θ variable and estimate the coefficients separately in some scale-invariant spaces, then we handle a sort of weighted sum of these norms of the coefficients in order to close the *a priori* estimate of the solution. Furthermore, we shall extend the above well-posedness result for initial data which is a linear combination of axisymmetric data without swirl and infinitely many large mode trigonometric series in the angular variable.

An edge wave propagating along an angular junction of two semi-infinite membranes bounding an acoustic medium

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We study existence of a localized wave that can propagate along the angular contact of two semi-infinite membranes in an acoustic medium. The acoustic medium fills the space between the membranes. The acoustic pressure satisfies the Helmholtz equation and the third-order boundary conditions on the bounding membranes and other appropriate conditions.

We make use of integral representations (of the Sommerfeld or Kontorovich–Lebedev type) for the solutions and reduce the problem to the functional equations. Their non-trivial solutions from a relevant class of functions exist only for some particular values of the wave numbers. The asymptotics of the solutions is also addressed.

Trapped modes of nonlinear dispersive waves

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We consider two related problems [1, 2] on the nonlinear interaction of wave modes with dispersion, which are studied using asymptotic methods of bifurcation theory. The first of them investigates the branching of periodic solutions of a system of weakly coupled equations that describes strong interaction of internal wave modes in a stratified fluid. The second problem relates to trapped solitary waves in a stationary fluid flow over complex topography.

Coupled KdV-type equations arise in multimodal dispersive models such as the Gear–Grimshaw system which describes weakly nonlinear internal waves in neighboring pycnoclines. Coupling occurs when two or more phase speeds of different modes are close together. We obtain analytic conditions which provide the existence of synchronized cnoidal-type wave trains. Application of the Lyapunov–Schmidt method reduces this problem to the nonlinear system of implicit bifurcation equations for unknown phase shift and wave amplitude. Asymptotic analysis of these equations results sufficient condition of synchronization, which involves the Poincare–Pontryagin function depending on coupling nonlinear terms.

The problem on solitary waves, forced by a chain of gently sloped obstacles of small height, is studied in the case when the far-upstream flow is slightly supercritical. Fully non-linear stationary 2D Euler equations are formulated via the von Mises transformation. We construct and analyze approximate solitary wave solutions by using long-wave expansion procedure with two small parameters. The Lyapunov–Schmidt method ensures an analytical condition of the wave trapping formulated in terms of the Melnikov function. A specific class of multi-bumped topographies is considered numerically in order to demonstrate multiplicity of forced waves.

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Navier–Stokes equations: a new estimate of a possible gap related to the energy equality of a suitable weak solution

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The talk is concerned with the IBVP of the Navier–Stokes equations. The result of the paper is in the wake of analogous results obtained by the authors in previous articles Crispo et al. (Ricerche Mat., 70, 235–249, 2021). The goal is to estimate the possible gap between the energy equality and the energy inequality deduced for a weak solution.

Computer-assisted time-fractional modeling of diffusion-wave polarization processes in ferroelectrics

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In recent years, the time-dependent Ginzburg–Landau equation (TDGL) has attracted much attention from various research fields. Successfully applied in the framework of the theory of superconductivity, the TDGL equation has been developed to this day and its different variations have been introduced in diverse branches of physics. In addition, substantial benefits can be provided by the use of fractional generalization of the TDGL approach [1]. The experience of the last decades has shown that fractional partial differential equations can be applied to model numerous phenomena and processes with non-locality and long-range interaction [2]. A time-fractional derivative is used to formalize the memory effects while a space-fractional derivative is applied to describe the flow of processes in objects with a complex structure or multiphase composition.

Previously, we proposed a time-fractional modification of the Landau–Khalatnikov model to describe the dynamics of ferroelectric polarization switching [3]. A finite difference scheme has been derived and numerically analyzed for the subdiffusion regime. At the same time, to describe polarization fluctuations in low-dimensional ferroelectric structures, the wave analogue of the Landau–Khalatnikov equation becomes more relevant. The present study was designed to construct the computational techniques to implement the extended time-fractional Landau–Khalatnikov model applicable for the generalized case when varying regimes from subdiffusion to superdiffusion.

The time-fractional Landau–Khalatnikov model is governed by the following semi-linear time-fractional partial differential equation complemented by the initial and boundary conditions:

$$\xi \frac{\partial^\alpha P}{\partial t^\alpha} = d\frac{\partial^2 P}{\partial x^2} + aP + bP^3 - cP^5, \quad 0 < x < L, \quad 0 < t < \theta, \tag{1}$$

$$P|_{t=0} = P_0(x), \quad 0 \le x \le L, \tag{2}$$

$$P|_{x=0} = \phi_1(t), \quad P|_{x=L} = \phi_2(t), \quad 0 \le t \le \theta, \tag{3}$$

where $P(x,t)$ is related to the space-time distribution of polarization in ferroelectric material, a, b, c, d, ξ are the positive constants, α is the order of the time-fractional Grünwald–Letnikov derivative, $0 < \alpha < 2$, $\alpha \neq 1$.

In order to solve the problem (1)–(3) numerically, we derive a computational scheme based on an implicit finite-difference method in combination with an iterative procedure due to presence of nonlinear reaction terms in equation (1). The program application was designed in Matlab to perform a series of computational experiments.

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Investigation of the analytical properties of solutions to dispersion equations for waveguide acoustic media with linear variation of the squared refractive index

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A class of problems of wave propagation in waveguides consisting of one or several layers that are characterized by linear variation of the squared refractive index along the normal to the interfaces between them is considered. In various problems arising in practical applications, it is necessary to efficiently solve the dispersion relations for such waveguides in order to compute horizontal wavenumbers for different frequencies. Such relations are transcendental equations written in terms of Airy functions, and their numerical solutions may require significant computational effort. A procedure that allows one to reduce a dispersion relation to an ordinary differential equation written in terms of elementary functions exclusively is proposed. The proposed technique is illustrated on two cases of waveguides with both compact and non-compact cross-sections. The developed reduction method can be used in applications such as geoacoustic inversion.

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Sommerfeld-type solution of a BVP for the Helmholtz equation in a nonconvex angle with periodic boundary data

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We consider the following model boundary value problem in a plane angle Q of magnitude $\Phi > \pi$ with a complex wave number $\omega \in \mathbb{C}^+$:

$$\begin{cases} (-\Delta - \omega^2)u(x) = 0, x \in Q, \\ u(x)|_{\Gamma_1} = f_1(x), u(x)|_{\Gamma_2} = f_2(x). \end{cases}$$

Here Γ_l for $l = 1, 2$ are the sides of the angle Q, $f_l = e^{ik_l|x|}$, $k_l > 0$, $l = 1, 2$.

Problems of this type arise in many areas of mathematical physics, for example in diffusion of desintegration gas. Besides, some problems of the diffraction theory are reduced to it as a model problem. The problem differs from numerous similar problems in which the boundary data are summable functions. Using the method of complex characteristics [1, 2], we reduce the problem to the Riemann–Hilbert problem for a Neumann data on the Riemann surface of zeros of the symbol of the Helmholtz operator. We solve this problem in quadratures and we give the solution u in the Sommerfeld type form. We find the asymptotics of the solution at the vertex and we describe its uniqueness class. This paper was supported by UMICH and CONACYT (México).

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Discrete dynamic inverse problem for complex Jacobi matrices

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We consider the inverse dynamic problem for a dynamical system with discrete time associated with a finite complex Jacobi matrix. We derive discrete analogs of Krein equations and answer a question on the characterization of dynamic inverse data.

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Green's function of the two-dimensional water wave problem with an ice plate floating on the surface

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We consider the two-dimensional linear problem of time-harmonic (with circular frequency ω) oscillations of an ideal incompressible fluid filling in the lower half-plane $\mathbb{R}^2_- = \{(x, y) : x \in \mathbb{R}, y < 0\}$ in the presence of ice covering a finite-size part of the free surface, whereas the other part is clean. The ice plate is assumed to behave like a thin elastic plate. The motion of fluid and ice is described in terms of a velocity potential.

We are interested in Green's function $G(x, y; \xi, \eta)$ of the problem (potential of the source located at the point $(\xi, \eta) \in \mathbb{R}^2_-$), submitted to the following set of conditions:

$$\begin{aligned}
&-\nabla^2_{x,y} G(x, y; \xi, \eta) = \delta(x - \xi)\delta(y - \eta), \quad (x, y) \in \mathbb{R}^2_-,\\
&\partial_y G - \nu G = 0 \quad y = 0,\ x < 0,\ x > L,\\
&\alpha\partial_x^4\partial_y G + \beta\partial_y G - \nu G = 0 \quad y = 0,\ 0 < x < L,
\end{aligned}$$

where $\delta(\cdot)$ is Dirac's delta-function, $\nu = \omega^2/g$, g is the acceleration due to gravity. The second condition is the standard free-surface condition, the third condition (on the ice-covered part of boundary) includes the parameters $\alpha = \frac{Ed^3}{12\rho g(1-\mu^2)}$ and $\beta = 1 - \frac{\rho_+}{\rho}\nu d$, where E is Young's modulus,

μ is Poisson's ratio, ρ is the density of fluid, ρ_+ is the density of the plate, d is its thickness (see [3]). Besides, the free-edge conditions (vanishing the bending moment and the crosscutting force)

$$\partial_x^n \partial_y G\big|_{y=0} \to 0 \quad \text{as} \quad x \to +0, \quad x \to L-0, \quad n = 2, 3,$$

must be satisfied by a solution at the points separating the free and the ice-covered parts. We also need the radiation condition guaranteeing that the wave field at infinity consists of outgoing waves:

$$\partial_{|x|} G - \mathrm{i}\nu G \to 0 \quad \text{as} \quad |x| \to \infty.$$

The problems of water wave scattering by thin elastic plates have been investigated by many researchers using various mathematical techniques, which, however, usually are limited to the case of finite depth (see, e.g., [1]). Besides, it is worth noting that interest to Green's functions is due to their applications in several directions (see [2]): many numerical models use Green's function to formulate an integral equation over the surface of bodies; it is also well-known that Green's function can be used for construction of non-uniqueness examples via inverse procedure.

In the present note, we propose an approach to finding the Green's function satisfying the condition above. The technique starts from the Fourier transform of the potential in the variable x, which leads to the general representation

$$G(x, y; \xi, \eta) = \mathring{G}(x, y; \xi, \eta) + \int_0^L f(k; \xi, \eta) \mathring{G}(x, y; k, 0) \mathrm{d}k,$$

where $\mathring{G}(x, y; \xi, \eta)$ is the well-known Green's function of the water wave problem in the absence of elastic cover, and f is an unknown function. Taking into account the third condition and the conditions at the edge points, we succeed to derive a Fredholm integral equation of the second kind for definition of the unknown density f. This equation (with the kernel including logarithmic part) can be easily approximated and reduced to a system of linear equations. It is to note that this equation is to be solved once for one given source point (ξ, η). Moreover, the matrix of the system can only be inverted once when L, α, β, and ν are fixed. Results of computations illustrating the theoretical findings will be presented.

Finally we note the method is applicable for the case of multiple plates of ice, and it can be used for defining and evaluating x- and y-dipoles as well as the multipoles of the subsequent orders.

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Resolvent estimates for viscoelastic systems and their applications

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In the theory of viscoelasticity, especially in mechanics, there are several dashpot models. We will focus on the Maxwell model and its extended version. In this talk, we are concern about the resolvent estimates for the system of equations for the extended Maxwell model and its reduced system. The reduced system is obtained as a closed system with respect to the velocity of displacement and the difference between the elastic strain and viscous strain. In this paper, the resolvent estimates for the system and the reduced system are given which yield a generation of a strongly continuous semigroup for the system and a generation of holomorphic semigroup for the reduced system, respectively. Also, for the reduced system, we show the exponential decay of solutions and the limiting amplitude principle.

Localization of natural oscillation of thin elastic gaskets

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A thin homogeneous isotropic plate $\Omega^\varepsilon = \omega \times (0,h) \ni x$ is inserted into the cavity $K^\varepsilon = \omega \times (0, \ell h)$ and glued to its absolutely rigid surface while the other part of the surface $\partial\Omega^\varepsilon$ is free of external action. Here, ω is a planar domain bounded by a simple smooth contour $\gamma = \partial\omega$ whose characteristic size is reduced to one by rescaling, $h > 0$ is a small dimensionless parameter, that is, the relative thickness of the plate, and $\ell \in [0,1]$ indicates the relative depth of the cavity. Natural oscillations of the elastic solid are described by the problem

$$-\mu\Delta_x u^\varepsilon(x) - (\lambda+\mu)\nabla_x\nabla_x \cdot u^\varepsilon(x) = \kappa_\varepsilon^2 u^\varepsilon(x),\ x \in \Omega^\varepsilon,$$

$$u^\varepsilon(x) = 0,\ x \in \partial\Omega^\varepsilon \cap \partial K^\varepsilon, \qquad \sigma^{(n)}(u^\varepsilon; x) = 0,\ x \in \partial\Omega^\varepsilon \setminus \partial K^\varepsilon,$$

where $\lambda \geq 0$ and $\mu > 0$ are the Lamé constants, u^ε is the displacement vector and $\sigma^{(n)}(u^\varepsilon)$ the vector of normal stresses, and $\kappa_\varepsilon > 0$ is the oscillation frequency.

The result of the asymptotic analysis as $h \to +0$ of the formulated problem is as follows: there exists $\ell_* \in (0,1)$ such that, for $\ell \in (\ell_*, 1]$, the asymtotics of eigenfrequencies of Ω^ε is described by the classical two-dimensional elasticity problem on the cross-section ω but, for $\ell \in [0, \ell_*)$, by a second-order ordinary differential equation on the contour γ. Hence, in the first case the elastic eigenmodes are distributed through the whole plate while in the second one they are concentrated in the vicinity of its lateral side $\gamma \times (0,\varepsilon)$. The divergence of the asymptotic behaviour of eigenmodes is based on different properties of the spectrum of the elasticity problem in the semi-infinite elastic strip describing the effect of boundary layers, namely, for $\ell \in [0, \ell_*)$, this problem has an isolated eigenfrequency and the corresponding eigenmode of exponential decay at infinity but, for $\ell \in (\ell_*, 1]$, the discrete spectrum of this model problem is empty.

The discovered effect of concentration of eigenmodes is also observed [1] for a thin elastic gasket between two straight absolutely rigid stamps. In the case of non-circular gaskets the effect enhances so that oscillations concentrate near points of maximal curvature. Some other abnormal behaviour of elastic eigenmodes will be described.

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Meliorated one-dimensional models of flow in a system of thin canals

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Flows of fluids in junctions of thin canals can be observed overall in dead and living nature, for example, in engineering water-supplying systems or blood circulation vessel joints in human bodies. Absolute majority of mathematical models of such "thin" flows are constructed in the following way:

edges and vertices of a one-dimensional graph are "thickened" to obtain a three-dimensional domain, then by means of an asymptotic analysis a family of ordinary differential equations on the edges with certain transmission conditions at vertices are imposed and finally error estimates are derived. At the same time, in reality the primary object is nothing but a three-dimensional junction while the choice of lengths of its ligaments and sizes of its nodes, of course, is not prescribed a priori. The main goal of the talk is to demonstrate that an appropriate fixation of the knots and therefore the ligaments provides a one-dimensional model of the sufficiently improved accuracy.

First of all, a result in the paper [1] shows that a proper choice of lengths of one-dimensional images of thin blood vessels in the arterial tree described by the Stokes equations with no-slip boundary conditions, gives a graph with the Reynolds' equations on its edges and the Kirchhoff transmission conditions at its bifurcation points which provides the accuracy of the incredibly small, exponential order $e^{-d/h}$ where h is a relative thickness of the vessels, a small parameter, and d is a positive exponent. It is worth to mention that traditional models give the power-law precision of approximation of spacial velocity field and pressure.

The second problem under consideration in [2] is the spectral Neumann problem for the Laplace operator in a junction of thin tubes (accepted as models of acoustic waveguide or ideal liquid). Again a proper choice of lengths of edges of the one-dimensional graph and statement of Kirchhof–Robin–Steklov transmission conditions at the vertices leads to the approximation accuracy of order h^2 while the traditional model gives only order h where h is a small relative diameter of tubes.

The results are based on elaborated analysis of boundary layers occurring in the vicinity of small knots of three-dimensional junctions.

Results of the paper [2] are obtained under financial support of the Russian Science Foundation (Project № 22–11–00046).

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On unitarity of the scattering operator in non-Hermitian quantum mechanics

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We consider the Schrödinger operator with regular short range complex-valued potential in dimension $d \geq 1$. We show that, for $d \geq 2$, the unitarity of scattering operator for this Hamiltonian at high energies implies the reality of potential (that is Hermiticity of Hamiltonian). In contrast, for $d = 1$, we present complex-valued exponentially localized soliton potentials with unitary scattering operator for all positive energies and with unbroken PT symmetry. We also present examples of complex-valued regular short range potentials with real spectrum for $d = 3$. Some directions for further research are formulated.

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Visualization of waves by the Boundary Control Method, numerical experiment

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We present the results of the numerical experiment about visualization of acoustic waves in 2d by the inverse data: waves at the boundary. We use the Boundary Control Method, and more precisely, the approach, proposed by M. I. Belishev, based on the Cholesky decomposition of the connecting operator. We present the scattering waves and the image of the unknown acoustic medium in semi-geodesic coordinates.

Pseudodifferential mode parabolic equation with mode coupling

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In many problems of underwater acoustics sound field $P(x, y, z)$ for any fixed (x, y) can be represented in the form of an expansion

$$P(x, y, z) = \sum_{j=1}^{J} A_j(x, y)\, \phi_j(z, x, y) \tag{1}$$

over orthogonal basis formed by normal modes $\phi_j(z, x, y)$ (that are functions of depth z parametrically depending on x, y). Mode amplitudes $A_j(x, y)$ in this expansion satisfy a coupled system of horizontal refraction equations [1]

$$\frac{\partial^2 A_j}{\partial x^2} + \frac{\partial^2 A_j}{\partial y^2} + k_j^2(x, y)A_j + \sum_{\ell} \mathcal{U}_{j\ell} A_\ell + 2\sum_{\ell} \mathcal{V}_{j\ell} \frac{\partial A_\ell}{\partial x} + 2\sum_{\ell} \mathcal{W}_{j\ell} \frac{\partial A_\ell}{\partial y} = -\delta(x)\delta(y)\phi_j(z_s, 0, 0), \tag{2}$$

where $\mathcal{W}_{j\ell}, \mathcal{V}_{j\ell}, \mathcal{U}_{j\ell}$ are elements of the so-called coupling matrices that allow acoustic energy exchange between different modes.

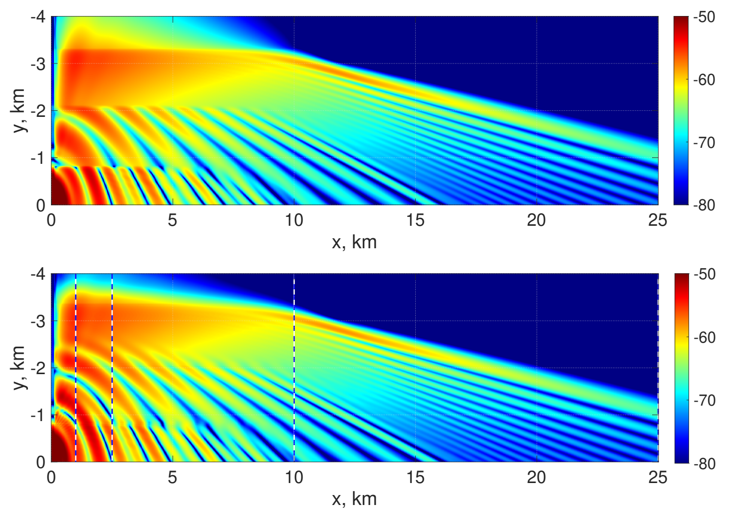

Fig. 1: Acoustic field $P(x, y, z_r)$ (in dB re 1 m) at fixed depth $z = z_r$ as a function of horizontal coordinates x, y computed using adiabatic PDMPE (upper subplot) and one-way counterpart of Eq. (2) with mode coupling taken into account (lower subplot).

In previous work we developed an approach to the computation of mode amplitudes within the so-called adiabatic approximation, when the aforementioned coupling coefficients are neglected, and the equations (2) can be uncoupled. Our approach is based on the one-way counterpart of (2), that contains a square root of a differential operator, called pseudodifferential mode parabolic equation (PDMPE). In the reported study our approach is generalized to the case of of the coupled system (2). Its one-way counterpart is an equation containing a square root of a matrix whose elements are differential operators. An efficient method for numerical solution of such equation is proposed.

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Spatial behavior of solutions to the time periodic Stokes system in a three dimensional layer

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In this talk we present results about existence and spatial asymptotics for solutions to the time-periodic Stokes and Navier–Stokes problem with Dirichlet conditions in a layer. This includes also the case of nonhomogeneous boundary conditions with nonzero flux which is of particular interest if one wants to model real life problems. Moreover, in unbounded domains the knowledge about the general spatial asymptotic behaviour is usually helpful to construct optimal so called artificial boundary conditions. The latter are needed if one wants to approximate the solutions by numerical calculations.

Electromagnetic pulses in vacuum and in vacuum with charges

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The effectiveness of the impact of extremely short electromagnetic pulses on micro-objects is determined by their “electric area” — the integral of the electrical field strength over time. The shortest pulses contain only one half-wave of electrical strength, that is, they are unipolar. Although the possibility of a nonzero electric area of pulses was noted in [1], discussions continue in the literature about the reality and possibility of propagation of such pulses.

In the present communication, we show that in an unbound vacuum, in which there were never any charges and no medium anywhere, pulses with a non-zero electric area would have infinite energy. But such pulses can be formed in a vacuum with moving charges, and it is even possible to create field structures completely localized in a finite region of space and in a finite time interval [2].

The study was carried out according to the plan of the Russian Science Foundation grant № 23-12-00012.

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Interaction of water waves with a viscoelastic ice sheet

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The equations for the water waves on the surface of the ocean covered by a viscoelastic sheet in the framework of general theory of nonlinear hyper-elastic shells with large deformations are derived. The modeling of the viscosity effects is based on the Maxwell and Antman hypotheses. The energy dissipation principle is established. The local existence theorem is proved.

Fixed angle inverse scattering for velocity

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An inhomogeneous acoustic medium is probed by a plane wave and the resultant time dependent wave is measured on the boundary of a ball enclosing the inhomogeneous part of the medium. We describe our partial results about the recovery of the velocity of the medium from the boundary measurement. This is a formally determined inverse problem for the wave equation, consisting of the recovery of a non-constant coefficient of the principal part of the operator from the boundary measurement.

Asymptotically isospectral periodic graphs

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We consider discrete Laplace operators on periodic graphs. Their spectrum consists of a finite number of bands. We perturb a periodic graph by adding long edges, i.e., edges connecting distant vertices (by periodically way) and show that if the length of the added edges tends to infinity, then the perturbed graph is asymptotically isospectral to some periodic graph of a larger dimension but without long edges. This periodic graph is an infinite-fold covering graph over the perturbed one. One of the simplest examples of such asymptotically isospectral periodic graphs is the square lattice perturbed by long edges and the cubic lattice. We also obtain asymptotics of the endpoints of the spectral bands for the Laplacian on the perturbed graph as the length of the added edges tends to infinity.

Localized asymptotic eigenfunctions for the Laplacian inside the torus

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Localized eigenfunctions of the Laplacian are very important from the different points of view. In [1–3] the asymptotic eigenfuctions for such operators were studied inside a 2D area with smooth and convex boundary. Such eigenfunctions were presented in terms of the Airy functions. In the [4] the localized asymptotic solutions for the Helmholz equations were constructed in 3D case along geodesic for the strictly convex areas.

In the present talk, we consider the following problem for the Laplacian eigenfunctions

$$-\Delta u = \lambda^2 u,\ (x,\ y,\ z) \in T, \quad u_{|\partial T} = 0.$$

Here T is the torus, $\Delta u = u_{xx} + u_{yy} + u_{zz}$, and $(x,\ y,\ z)$ are the 3D Cartesian coordinates.

We construct localized near ∂T asymptotic eigenfunctions (quasimodes) while λ is sufficiently large. Following the operator separation of variables [5], we are able to reduce this problem to the 1D problem for the Schrödinger equation with potential well. As the result of such reduction, we present localized asymptotic eigenfunctions in terms of Airy functions.

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Maslov complex germ in the Cauchy problem for the Schrödinger equation with delta potential

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We consider the Schrödinger equation with a delta potential localized on a surface of codimension 1. The initial data are selected as a Gaussian wave packet. The Schrödinger operator with a delta-potential is defined as self-adjoint extension of the Schrödinger operator with a smooth potential restricted to the functions vanishing on the surface of the carrier of the delta-function. The domain of a such an operator requires the fulfillment of certain boundary conditions on this surface.

In this work describe the asymptotics as $h \to 0+$ of the solution of the Cachy problem discribed above. The solution of the problem is reduced to solving the Hamilton–Jacobi and transfer equations with given the boundary conditions. A Gaussian wave with a center moving along the trajectories of the Hamiltonian system propagates in space and interacts with the delta potential. After the interaction it splits in two parts, one of which propagates further, while the other is reflected.

We provide the solution in terms of the Maslov complex germ theory. The approach leads us to the consideration of complex vector bundles over isotopic surfaces and their rebuilding at the points of the surface of the carrier of the delta-function.

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Taking into account thermoviscous boundary conditions in computational problems of acoustics

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Viscous and thermal effects in air under normal conditions are known to be small [1] so acoustical waves propagation can be described using wave equation. However, these effects cannot be neglected in thin tubes and grooves. Accurate description can be provided by compressible Navier–Stokes equations. Near the wall these equations can be approximated by a wall boundary layer condition. Unfortunately, the conditions are complicated and usually introduced in the frequency domain ([2]):

$$\delta_V \frac{i-1}{2} \Delta_\tau \tilde{p} - \delta_T k_0^2 \frac{(i-1)(\gamma-1)}{2} \tilde{p} + \frac{\partial \tilde{p}}{\partial n} = 0, \tag{1}$$

where

$$\delta_V = \sqrt{\frac{2\nu}{\omega}}, \quad \delta_T = \sqrt{\frac{2\kappa}{\omega \rho_0 c_p}},$$

$\tilde{p}$ is the acoustical pressure in the frequency domain, i is the imaginary unity, Δ_τ is the tangential part of the laplacian, $k_0 = \omega^2/c^2$ is the wave number, ω is the angular frequency, ν is the kinematic viscosity coefficient, c_p is the specific heat capacity at the constant pressure, γ is the heat ratio, and κ is the thermal conductivity. The time dependence $\exp\{i\omega t\}$ is used.

In some cases the wave equation needs to be solved in the time domain. As it can be seen in (1) that leads to fractional derivatives. The initial equations are reduced to the system as follows:

$$\frac{1}{c^2} \frac{\partial^2 p}{\partial t^2} - \Delta p = 0, \quad \mathbf{x} \in \Omega \tag{2}$$

$$\sqrt{\nu} \delta_\tau {}_0^C D_t^{-1/2} \Delta_\nu p + \frac{(\gamma-1)}{c^2} \sqrt{\frac{\kappa}{\rho_0 c_p}} {}_0^C D_t^{3/2} p + \frac{\partial p}{\partial n} = 0, \quad \mathbf{x} \in \Gamma_v, \tag{3}$$

where p is the acoustical pressure, $\mathbf{x}$ is a coordinate vector, Ω is some domain, Γ_v is the boundary of parts of domain in which thermal and viscous effects cannot be neglected, ${}_a^C D_t^\beta p$ is the Caputo derivative [3].

In this work the weak formulation of (2)–(3) is introduced and reduced to Volterra integro-differential equation system using FEM:

$$\mathbf{M}\frac{d}{dt}\mathbf{y} = \mathbf{K}\mathbf{y} + \mathbf{V}\int_0^t \frac{1}{\sqrt{\pi(t-\tau)}}\mathbf{y}(\tau)d\tau + \mathbf{T}\int_0^t \frac{1}{\sqrt{\pi(t-\tau)}}\frac{d}{d\tau}\mathbf{y}(\tau)d\tau + \mathbf{f},$$

where $\mathbf{M}$, $\mathbf{K}$, $\mathbf{V}$, $\mathbf{T}$ are some constant matrices, and $\mathbf{y}$ is the vector of unknowns.

Several numerical demonstrations are also provided.

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Generalization of Navier–Stokes equations for two-phase flows and applications

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The thermodynamics of a two-phase granular liquid is developed by the Khalatnikov–Landau method, which makes it possible to generalize the Navier–Stokes equations to the case of particle suspension flow. The new mathematical model explains a number of known experimental effects. These include the Boycott effect (Nature 1920), according to which erythrocytes in the blood in an inclined test tube are deposited more intensively than in a vertical one [1]. Fig. 1 depicts how rapidly the upper zone of a pure liquid without particles increases with time. This result was established by numerical methods.

Another phenomenon is known as the Segre–Silberberg effect (Nature, 1961). It consists in the fact that in a steady flow along a pipe, particles accumulate not on the axis and not near the walls, but in a concentric annular region with an average radius $r = 0.6R$, where R is the radius of the pipe. The developed theory explains this effect by the rotation of particles, the angular velocity of which is taken into account in the law of conservation of internal angular momentum by using the Cosserat continuum.

Experiments show that in the case of a channel branching into two branches, the particles "prefer" the branch in which the liquid flow rate is higher. This Zweifach–Fung (1968, 1973) effect can also be explained in terms of the formulated equations.

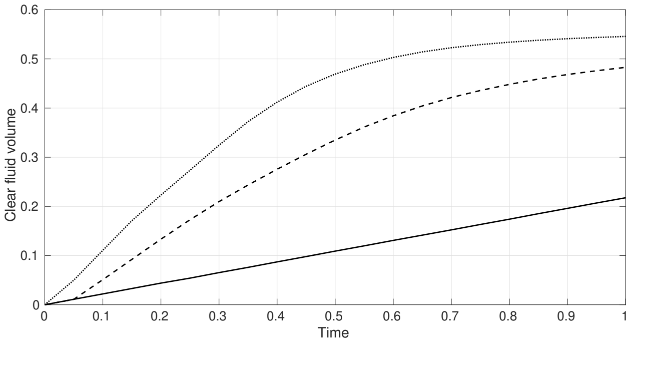

Fig. 1: Reduced volume $V(t)$ of the clear fluid zone without particles versus dimensionless time for the inclination angles $\alpha = 0°$, $\alpha = 20°$ and $\alpha = 35°$ from the bottom upwards.

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L_1-maximal regularity approach to the free boundary problem of the Navier–Stokes equations

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I will talk about the L_1-maximal regularity of the Stokes equations with free boundary conditions and its application to free boundary problem for the Navier–Stokes equations. We use the resolvent estimate approach to obtain the L_1-maximal regularity.

Free boundary problems of the incompressible Navier–Stokes equations in the critical Besov space

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We consider a free surface problem of the incompressible Navier–Stokes system with non-flat initial surface. To obtain global well-posedness, we establish end-point maximal L^1-regularity for the initial-boundary value problems of the Stokes equations. The proof depends on the explicit expression of the fundamental integral kernel of the linearized Stokes equations and almost orthogonal estimates with the space-time Littlewood–Paley dyadic decompositions. For nonlinear terms, we employ bilinear estimates both in the half-space and on the boundary. We also discuss our recent development for the generalization on the corresponding problem.

Diffraction of a symmetric TM mode at an open-ended deeply corrugated circular waveguide with a flange

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We investigate the diffraction of a slow TM symmetrical mode by an open-ended corrugated cylindrical waveguide with a flange (Fig. 1). Note that this mode can be generated by a charged particle bunch passing along the waveguide axis. We analyze the so-called "longwave" range when the wavelengths and the waveguide radius are much greater than the corrugation period. In this case, the corrugated waveguide wall can be replaced with a smooth one on which the so-called equivalent boundary conditions (EBC) are fulfilled [1]. Here, we also assume that the wall is deeply corrugated, i.e. the corrugation depth is of the same order as an inverse wavenumber [1]. It should be noted that the radiation under investigation is principally different from the Smith–Purcell radiation which is often considered in waveguides.

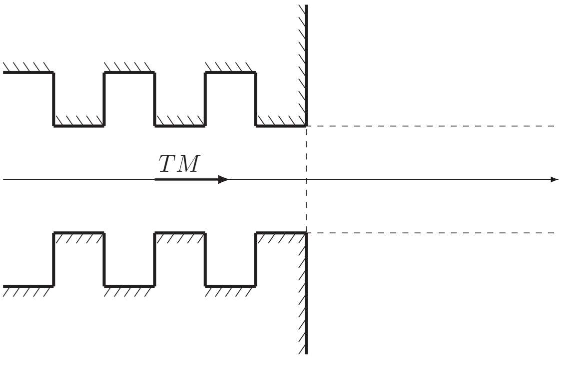

Fig.1: The open-ended deeply corrugated circular waveguide with a flange.

It is worth noting that we have effectively applied the EBC method to solve a series of the "longwave" problems. Particularly, in papers [2, 3], we analyzed the radiation from a charged particle bunch moving in the presence of a deeply corrugated planar structure and showed that the bunch generates the powerful surface radiation which is highly sensitive to the corrugation depth.

The approach presented uses the solution of the corresponding Wiener–Hopf–Fock equation. This solution is in turn utilized to construct an infinite linear system for reflection coefficients which can be solved numerically using the reduction technique. The dependences of the components of the electromagnetic field on the problem parameters are obtained and analyzed.

This work was supported by the Russian Science Foundation (Grant № 18-72-10137).

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Approximate Lipschitz stability for phaseless inverse scattering

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We continue the studies of monochromatic phaseless inverse scattering with background information in dimension $d \geq 2$. Approximate Lipschitz stability is proved. Moreover, these stability estimates are given in terms of non-overdetermined and incomplete data. Related results for reconstruction from phaseless Fourier transforms are also given. Results are provided in [1] . Prototypes of these estimates for the phased case were given in [2].

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Change in the velocity invariants of bulk acoustic waves in piezoelectric crystals

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The phase velocities V of three different bulk acoustic waves propagating in a given direction in an anisotropic media are found from the condition of a vanishing determinant of the coefficient matrix for the Green–Christoffel equation. This condition gives a cubic equation for V^2. According to Vieta's formulas, the coefficient in front of V^4 is equal, up to a sign, to the sum of the squared velocities for the three mentioned waves. This sum can be used to estimate the average velocity in a given propagation direction. On the other hand, the same coefficient is equal to the trace of the Christoffel tensor, which makes it possible to find its orientational dependence. It is known that the sum under discusion is invariant (i.e. it does not depend on the direction) in cubic crystals [1]. On the other hand, it takes constant values on second-order surfaces for the phase space in the general anisotropic case [2]. In hexagonal crystals, it depends only on the polar angle. We found that trigonal crystals have the same property. The last statement is not obvious, since each of the three velocities in a given direction in trigonal crystals also depends on the azimuthal angle. The universal invariant is the sum of the squared velocities of all nine bulk waves, which can propagate in three arbitrary but mutually orthogonal directions in crystals of any class [3]

$$\sum_i \sum_j v_{ij}^2 = c_{11} + c_{22} + c_{33} + 2(c_{44} + c_{55} + c_{66}). \tag{1}$$

The aim of the present work is to study the influence of piezoelectric effect on the specified invariants. It is found that the invariance of the velocity combination for cubic crystals is violated in this case. The piezoelectrically induced spatially dependent perturbation of this invariant can be used to find from experimental data such an important characteristic of piezoelectrics as the electromechanical coupling coefficient. It is shown that the piezoelectric effect does not change the considered invariant in hexagonal crystals, but this is not so in trigonal ones. In the general anisotropic piezoelectric case, constant values of the sum of the squared velocities correspond to complex surfaces of the fourth order. The possibility of a simplified study of the shape of these surfaces in weak piezoelectrics is considered in the approximation of the smallness of their deviation from the second-order surfaces.

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On acoustic imaging via the quasi-reversibility and boundary control methods

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Two methods, the Quasi-Reversibility (QRM) and Boundary Control (BCM), are applied to acoustic imaging. This is motivated by the fact that these methods provide, at least theoretically, a global convergence and reduce an originally nonlinear inverse problem to a linear one. Two mathematical models are considered within the frameworks of these methods. The Pestov's formalism [1] in a bounded and simply connected domain $\Omega \subset \mathbb{R}^2$ with the smooth boundary $\partial\Omega$ is utilized within the framework of the BCM while the Cauchy problem for the acoustic wave equation

$$q(x)u_{tt} - \Delta u = 0 \quad \text{in} \quad \mathbb{R}^2 \times (0, \infty), \tag{1}$$
$$u(x,0) = 0, \quad u_t(x,0) = \delta(x - x_0), \quad x, x_0 \in \mathbb{R}^2, \tag{2}$$

is exploited as a mathematical model within the framework of the QRM. All functions are supposed to be real valued. Here, $c = q^{-1/2}$ is the sound speed, which is supposed to satisfy the non-trapping condition in $\mathbb{R}^3$ and be positive and bounded away from zero and infinity. In this case, an inverse problem is formulated as follows. Suppose that the origin is placed inside $\mathrm{supp}(q) = \Omega$ and C_R is a circle with the center at the origin and with the radius R, such that $G_R = \{|x| < R\}$, $\overline{\Omega} \subset G_R$, $C_R = \partial G_R$. Let u be the solution of (1)–(2). Given the functions

$$p_0(x, x_0, t) = u(x, x_0, t), \tag{3}$$
$$p_1(x, x_0, t) = \nabla u(x, x_0, t) \cdot \nu(x), \tag{4}$$

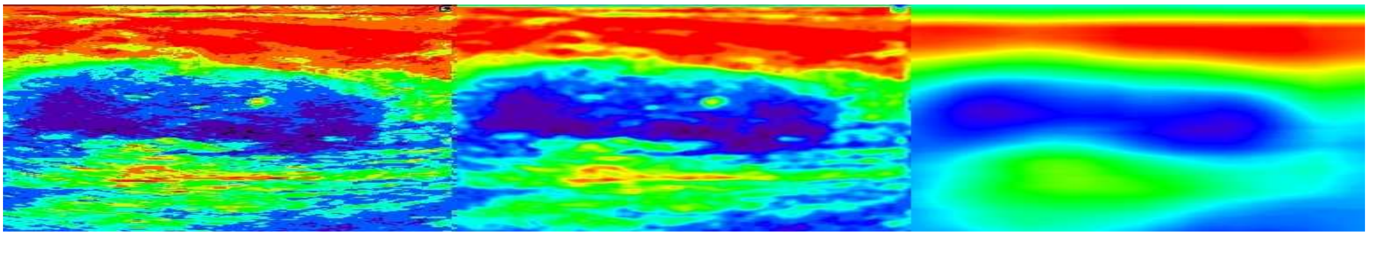
where $(x, x_0) \in C_R$, $x \neq x_0$, determine q in Ω. Following the Lavrentiev's approach, this inverse problems is reduced to the integral equation of the first kind, namely the Lavrentiev equation

$$\int_\Omega \ln \frac{|x - x'|}{2} \ln \frac{|x' - x_0|}{2} \xi(x') dx' = \psi(x, x_0), \quad x, x_0 \in C_R, \tag{5}$$

where $\xi = q - 1$ and ψ is a known function that depends on p_0 and p_1. Denote $w(x, x_0)$ the left-hand side of (5), introduce a new variable $v = w/|x - x_0|$ and transform (5) to a system of elliptic ODEs $L(V) = 0$, $V = s_0$, $\nabla V \cdot \nu = s_1$, $x \in G_R$, where V is a vector-function whose components are the coefficients in the expansion of the solution in a special basis. Then the application of the QRM results in minimizing the Tikhonov functional

$$J_\alpha(V) = \int_{G_R} (L(V))^2 dx + \alpha \|V\|^2_{H^2(G_R)} \tag{6}$$

Once a minimizer of this functional is found, the coefficient q is calculated using explicit formulae. To demonstrate the efficiency of computational algorithms implementing these methods, the series of numerical experiments were conducted to reconstruct q. As an example, some images of q reconstructed from the noiseless data are shown below. From left to right: the discontinuous ground truth q; the reconstructions by the QRM ane BCM. The ground truth q is represented by a real ultrasound scan of breast with a malignant tumor.



The results reported were obtained in the course of a collaborative research project with Professor Michael Klibanov.

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Propagation of a two-frequency electromagnetic wave in a plane waveguide with nonlinear permittivity: nonlinearizable guided waves

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Let $\Sigma = \{(x,y,z) \in \mathbb{R}^3 : 0 \leqslant x \leqslant h, (y,z) \in \mathbb{R}^2\}$ be a dielectric layer sandwiched between two half-spaces $x < 0$, $x > h$. The half-spaces are filled with nonmagnetic media having real constant permittivities $\epsilon = \epsilon_s \geqslant \epsilon_0 > 0$ and $\epsilon = \epsilon_c \geqslant \epsilon_0 > 0$, respectively, where ϵ_0 is the permittivity of free space. Everywhere the permeability μ is a positive constant. We consider a two-frequency field

$$\mathbf{E}_\omega = \mathbf{E}_1 e^{-i\omega_1 t} + \mathbf{E}_2 e^{-i\omega_2 t}, \quad \mathbf{H}_\omega = \mathbf{H}_1 e^{-i\omega_1 t} + \mathbf{H}_2 e^{-i\omega_2 t}, \tag{1}$$

where

$$\begin{aligned} \mathbf{E}_1 &= (0, e_y, 0)^\top e^{i\gamma_1 z}, \quad \mathbf{H}_1 = (h_x^{(1)}, 0, h_z)^\top e^{i\gamma_1 z}, \\ \mathbf{E}_2 &= (0, 0, e_z)^\top e^{i\gamma_2 y}, \quad \mathbf{H}_2 = (h_x^{(2)}, h_y, 0)^\top e^{i\gamma_2 y}; \end{aligned} \tag{2}$$

here e_y, e_z, $h_x^{(1)}$, $h_x^{(2)}$, h_y, h_z depend only on x; γ_j is a real unknown propagation constant [1, 2].

The permittivity ϵ_l of Σ is a diagonal (3×3)-tensor havinthe form

$$\epsilon_l(\tilde{\mathbf{E}}_\omega) \equiv \operatorname{diag}\{*, \epsilon_y + \beta_1(|\mathbf{E}_1|^2 + |\mathbf{E}_2|^2) + \beta_3 g_1, \epsilon_z + \beta_2(|\mathbf{E}_1|^2 + |\mathbf{E}_2|^2) + \beta_4 g_2\} \tag{3}$$

where the first entry $*$ does not affect the wave propagation process; $\epsilon_y, \epsilon_z, \beta_1, \ldots, \beta_4 > 0$ are real constants, g_1, g_2 are continuous functions with respect to $|\mathbf{E}_1|, |\mathbf{E}_2|$. The terms $\beta_3 g_1$ and $\beta_4 g_2$ are considered as small perturbations of the permittivity. It is assumed that $\min\{\epsilon_y, \epsilon_z\} > \epsilon_s \geqslant \epsilon_c > 0$.

Substituting (1) into Maxwell's equations, we obtain

$$\begin{aligned} e^{-i\omega_1 t}\operatorname{rot}\mathbf{H}_1 + e^{-i\omega_2 t}\operatorname{rot}\mathbf{H}_2 &= -i\epsilon\big(\omega_1 \mathbf{E}_1 e^{-i\omega_1 t} + \omega_2 \mathbf{E}_2 e^{-i\omega_2 t}\big), \\ e^{-i\omega_1 t}\operatorname{rot}\mathbf{E}_1 + e^{-i\omega_2 t}\operatorname{rot}\mathbf{E}_2 &= i\mu\big(\omega_1 \mathbf{H}_1 e^{-i\omega_1 t} + \omega_2 \mathbf{H}_2 e^{-i\omega_2 t}\big). \end{aligned} \tag{4}$$

Since the derived equations take place for all t, then we obtain

$$\operatorname{rot}\mathbf{H}_j = -i\epsilon\omega_j \mathbf{E}_j, \quad \operatorname{rot}\mathbf{E}_j = i\mu\omega_j \mathbf{H}_j, \quad j = 1, 2. \tag{5}$$

The fields satisfy the standard electromagnetic conditions at the interfaces of the waveguide (continuity of the tangential components of the fields), additional local conditions at $x = 0$ and we also assume that the fields decay as $|x| \to \infty$. We look for vector propagation constants $\bar{\boldsymbol{\gamma}} = (\bar{\gamma}_1, \bar{\gamma}_2)$ and corresponding guided waves $(\mathbf{E}_\omega, \mathbf{H}_\omega)$, where $\mathbf{E}_\omega \equiv \mathbf{E}_\omega(\bar{\boldsymbol{\gamma}})$, $\mathbf{H}_\omega \equiv \mathbf{H}_\omega(\bar{\boldsymbol{\gamma}})$ which solve the above formulated problem.

We develop a nonclassical perturbation approach to solve this problem [3]. In particular, we prove existence of solutions with and without linear counterparts if β_3, β_4 are sufficiently small (at the same time, it is not necessarily assumed that $\beta_1, \beta_2 > 0$ are small). We also present numerical simulations and comparison between similar problems.

This work was supported by the Russian Science Foundation grant № 18-71-10015.

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Generation of higher-order optical vortices in acoustically perturbed optical fibers

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In this paper, we have studied the acousto-optic interaction that is induced by circularly polarized higher order flexural acoustic vortices in circular optical fibers. We show that both the spin and the orbital angular momentums of phonons can be transformed into the orbital angular momentum of photons, and vice versa, during the fiber acousto-optic interaction. This results in the acoustically-controlled tunable generation of higher-order optical vortex beams directly from a Gauss-like mode. A counterintuitive interrelation between topological charge of the generated optical vortex and exited in the fiber acoustic one is unveiled and explained. Our results can be useful in different vortex-based photonics applications.

An evolution free boundary problem for the Navier–Stokes equations

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I would like to briefly present a work in collaboration with V. A. Solonnikov. The work is in progress and concerns an evolution free boundary problem for the Navier–Stokes equations. In particular, we consider 2-dimensional evolutionary free boundary problem for a viscous incompressible fluid partially filling a container V whose boundary consists of two straight lines connected by a smooth curve. The boundary of the domain Ω_t filled with the fluid consists of two curves: Σ_t and the free curve Γ_t that meets Σ_t at two contact points. The curve Γ_t is given for $t = 0$ and should be found for $t > 0$ together with the velocity vector field $v(x; t)$ and the pressure function $p(x; t)$ satisfying the Navier–Stokes system and "natural' boundary conditions. Our aim is to construct sufficiently regular solution in the finite (possibly, small) time interval $(0; T)$, which can be done if the contact angles (i.e., the angles between Σ_t and Γ_t at the contact points) are small.

Modeling of the retroreflective film influence on the dispersion curves and attenuation of guided waves in an aluminum plate

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This work represents, in a sense, the continuation of the work [1], where the approximate theory of coated plate, in which the coating layer is modeled via effective boundary condition [2], is presented and tested. This investigation was motivated by the necessity of evaluating the results of laser Doppler vibrometry with taking into account the retroreflective film (RRF) [3], glued to one of the faces of the plate in order to improve its reflection properties. But the comparison with the experimental data has shown that the single layer coating does not allow to describe the influence of the RRF properly. This fact could be explained by the peculiar composite structure of the RRF, which consists of an epoxy layer with semi-included glass spheres.

In this work, the 2D model of RRF is used instead of the single layer coating, which was specially constructed to take into account the peculiar composite structure. The equations of this model are analogous to those of the classical laminated plate theory [4], but the stiffnesses are determined in numerical experiments, modeling the RRF structure. To take into account the imperfect contact of the RRF and the plate, the spring-type boundary conditions are introduced between the coating and the surface of the plate. It is shown that by adjusting the parameters of the RRF-model, the discrepancies in the slownesses of the Lamb waves revealed in [3] can be reproduced in the theoretical dispersion curves with a good accuracy. By modeling the epoxy part of the RRF as a viscoelastic material, the attenuation caused by RRF can be also described. The influence of the RRT on the propagation of edge waves is also considered and compared with the experimental data.

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Resonance interaction of VLF waves guided by a cylindrical density trough in a magnetoplasma

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The interest in the nonlinear interaction of very low frequency (VLF) waves in a magnetoplasma is motivated by their numerous applications, such as the energy transfer by the corresponding waves over long distances, space plasma diagnostics, etc. The nonlinear interaction of symmetric eigenmodes guided by infinitely long cylindrical density ducts in the low ionosphere is studied in [1]. The parametric instability of whistler waves ducted by a planar density trough under conditions of the upper ionosphere is considered in [2]. Note that such field-aligned density depletions can be created in the upper ionosphere under the action of high-power ionospheric heaters such as the Sura and HAARP facilities (see, e.g., [3] and references therein). The present work is devoted to analysis of the nonlinear interaction of whistler waves belonging to the VLF range and supported by a cylindrical density trough in the upper ionosphere.

At first, the properties of the whistler modes guided by a sharp-walled cylindrical depletion with parameters typical of the upper ionosphere are analyzed. Then the nonlinear resonant interaction of these modes is studied. It is shown that under certain conditions, an external electromagnetic field may lead to the parametric instability of both axisymmetric and nonsymmetric guided modes. The emphasis is placed on the case of a resonant cold magnetoplasma whose dielectric tensor is characterized by the different signs of the diagonal elements. The hydrodynamic and Maxwell equations are used to calculate the characteristics of the three-wave interaction. The instability growth rate and the threshold of the parametric instability of the forward- and backward-propagating guided waves are determined. Numerical results will be reported for some practically interesting cases.

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Separable representation of three-dimensional wide-angle parabolic equation propagator using Fourier series

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Parabolic diffraction equations are extensively used to solve wave propagation problems in acoustics, geophysics, radiophysics, and optics. A key approximation in derivation of these equations is the assumption of one-way propagation of waves, which leads to significant simplification of the solution procedures in comparison to methods of solving the original Helmholtz equation [1]. Particularly, efficient marching algorithms can be constructed to solve the parabolic equations. A standard parabolic equation (SPE) known also as small-angle or paraxial approximation forms the basis of hierarchy for different parabolic equations. Generalizations introduced via operator formalism and factorization of the forward/backward propagating waves in the Helmholtz equation lead to different families of wide-angle parabolic equations (WAPEs), which significantly extend the domain of applicability of SPE to higher diffraction angles and stronger refraction index variations in inhomogeneous media.

A common way to construct a WAPE wave propagation model is to write at first the one-way pseudodifferential propagation equation:

$$\frac{\partial \varphi}{\partial z} = ik_0\left(\sqrt{1+\widehat{L}} - 1\right)\varphi, \tag{1}$$

where φ is a slowly varying amplitude of wave field (for example, pressure complex amplitude in acoustics), k_0 is a reference wavenumber, z is a coordinate along propagation direction, $\widehat{L} = \left(\partial^2/\partial x^2 + \partial^2/\partial y^2\right)/k_0^2 = \widehat{L}_x + \widehat{L}_y$ is the transversal Laplacian operator. In more general cases the operator $\widehat{L}$ may include refraction index and medium density inhomogeneities. Differential operators $\widehat{L}_x = \partial^2/\partial x^2/k_0^2$ and $\widehat{L}_y = \partial^2/\partial y^2/k_0^2$ correspond to transversal coordinates x and y of Cartesian system. The operator $\sqrt{1+\widehat{L}}$ is the pseudodifferential square-root operator. Secondly, a formal solution of the one-way equation is written as $\varphi(z+\Delta z) = \widehat{H}\varphi(z)$, where the operator

$$\widehat{H} = \exp\left[ik_0\Delta z\left(\sqrt{1+\widehat{L}} - 1\right)\right] \tag{2}$$

is a one-way propagator which is acting on a field at the distance z to generate the field at the distance $z + \Delta z$. Thirdly, the propagator is usually approximated using Padé approximations of different orders raising the following type of equations to solve

$$\left(1 + \nu\widehat{L}\right)\varphi = f, \tag{3}$$

where ν is some complex coefficient and f is a known right hand side function.

When this WAPE scheme is applied to two dimensional problems with one range propagation direction (z axis) and one transversal coordinate (x axis), the equation (3) is a one dimensional second order boundary value problem, which does not present computational difficulty for any suitable numerical methods such as finite differences or finite elements. However, in three dimensional case, it is totally different since the resulting two dimensional second order boundary value problems are difficult to solve. For example, due to the presence of the second order partial derivatives along both transversal coordinates (x and y), straightforward application of finite differences results in large block-tridiagonal linear systems which are computationally intensive to invert. Either iterative approaches to solve the linear systems [2] or different ways to deal with cross-terms in expansion of the square-root operator have been proposed.

In this study, an alternative approach of constructing a wide-angle approximation is considered, which mathematical form admits numerical schemes with separate solution procedures along each transversal coordinate. Note that in all formulae the operator $\widehat{L}$ can be replaced by its eigenvalue ξ, in that way transforming an operator to a complex valued function and vice versa.

Our method implies that the propagator can be decomposed to finite Fourier series over a specified range of eigenvalues ξ [3]:

$$H\left(ik_0\Delta z, \xi\right) = \exp\left[ik_0\Delta z\left(\sqrt{1+\xi}-1\right)\right] \approx \sum_{n=-N}^{N} A_n \exp\left(B_n\xi\right), \tag{4}$$

where A_n are amplitude coefficients, B_n are discrete frequencies, and N is an integer. However, when Fourier decomposition is applied directly to the function $H\left(ik_0\Delta z, \xi\right)$, the approximate propagator has spurious oscillations due to two basic facts. First, $H\left(ik_0\Delta z, \xi\right)$ is not a naturally periodical function and, as a consequence, after a cut in any finite segment of ξ to make it periodical, the periodized function becomes discontinuous. Second, it has a discontinuous derivative when $\xi \to -1$.

In order to remove these unwanted properties of the exact propagator the function has been modified with the help of high-order Hermit interpolation polynomials. As a result, an approximate propagator has three favorable properties: 1) it is periodic and continuous together with a prescribed number of derivatives; 2) it attenuates evanescent waves although with a rate different from attenuation rate of the exact propagator (2); 3) it is very close to the exact propagator in a segment of an operator spectrum, which corresponds to a selected angular range of the propagating waves. As a consequence, Fourier series (4) converge quickly and N may be not greater than 100 for a tolerated error of propagator of 10^{-7} within selected angular range.

A computational simplification of the proposed WAPE model comes from the fact that in order to evaluate the wave field at the next propagation step $\varphi\left(z+\Delta z\right)$, an action of a number of exponential propagators of a kind $\exp\left(B_n\widehat{L}\right)$ on the wave field $\varphi\left(z\right)$ must be calculated. If operators $\widehat{L}_x$ and $\widehat{L}_y$ commute, as in the case of a homogeneous propagation medium, then exponent of a sum of operators exactly breaks up to a product of two exponents

$$\exp\left[B_n\left(\widehat{L}_x+\widehat{L}_y\right)\right] = \exp\left(B_n\widehat{L}_x\right)\exp\left(B_n\widehat{L}_y\right), \tag{5}$$

each of which can be efficiently evaluated using Padé approximations of an exponent and finite difference representations of operators $\widehat{L}_x$ and $\widehat{L}_y$. If the operators do not commute, for example, if inhomogeneities of refraction index are present, then different splitting schemes of an exponential of a sum of non-commuting operators can be used, which finally result in a product of the same kind of individual exponents as in (5).

The proposed WAPE model was tested by evaluating the pressure field of a focused ultrasound transducer with characteristics typical for high intensity focused ultrasound systems. Simulations were performed in water. A numerical algorithm with finite-difference evaluation of individual exponential propagators was realized and computed results were compared with a reference solution obtained with the use if an angular spectrum method.

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Boundary layer approach to diffraction by contours with non-smooth curvature

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We address the construction of high-frequency asymptotic formulas for 2D diffraction by a contour whose curvature is smooth everywhere except one point. An incident wave comes to the singular point of the contour non-tangentially. The cases are considered where the curvature or its derivative of the j-th order ($j = 1, 2, \dots$) either jumps or has the so-called "Hölder singularity".

We present a review of our recent results [1–4] found in the framework of systematic boundary layer approach [5]. First, in a small neighborhood of the singular point an "inner" problem is formulated in stretched variables. Its explicit solution is found and matched outside the transition zone with the ray formula for the diffracted wave. Inside the transition zone, the solution is matched with the special ansatz expressed in terms of parabolic cylinder functions and intended to describe coalescence of two cylindrical waves. Thus, for every type of curvature singularity, an explicit formula for the diffracted wave is found as well as a description of the transition zone.

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Workshop on wave phenomena in complex structures

Dielectric Fourier metasurfaces for reconfigurable anomalous refraction at large angles

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Anomalous refraction — deflection of light into a single outgoing diffraction channel [1] — is crucial for lensing, holography and beam splitting. It can be performed by dielectric metasurfaces [2], however, achieving high efficiency for large deflection angles by conventional designs (based on arrays of cylinders and prisms) is problematic. Freeform optimization yields sophisticated structures still substantially limited in the near-grazing directions [3].

Dielectric Fourier metasurfaces are thin layers of high refractive index materials with periodic surface nanorelief [4, 5]. Parametrization of the relief in terms of a few Fourier harmonics allows optimizing them using moderate computational resources. Moreover, in many practically important cases, light interaction with Fourier metasurfaces can be accurately described by a semi-analytical theory in terms of the Rayleigh hypothesis. In particular, it allows fast optimization of silicon Fourier metasurfaces with the relief described by a superposition of only three Fourier harmonics for efficient anomalous refraction at large near-grazing angles. Normally incident green light can be deflected up to 85° from the surface normal with a corresponding diffraction efficiency of about 80% [6].

Metasurfaces, operating in the vicinity of a certain diffraction order cutoff, can drastically alter the direction of outgoing light controlled by subtle variations of the optical setup. Accordingly, we propose to employ Fourier metasurfaces as wide-angle optical Y-junction switches. We show that one can deflect 70%–80% of green light by more than 150° when the beam incident at a silicon metasurface is inclined by only 2°, or when the relative substrate permittivity is varied by 0.1 [7].

Finally, we analyze the prospects of anomalous refraction at large angles by Fourier metasurfaces made from materials with lower refractive index, such as titanium dioxide and lithium niobate. Optimizations are performed independently by full-scale numerical simulations in COMSOL Multiphysics and by semi-analytical modeling within the Rayleigh hypothesis. Comparing the results allowed us also to determine the limits of applicability of the Rayleigh hypothesis. For material permittivity $\varepsilon \geq 14$ both optimizations yield identical nanoreliefs. As we further decrease ε, the optimization results diverge: the full-scale simulations produce metasurfaces with higher diffraction efficiencies and sharper reliefs, while semi-analytical theory yields metasurfaces with moderate efficiencies and shallow reliefs. Importantly for applications, we undoubtedly verify that Fourier metasurfaces made of materials with $\varepsilon \geq 5$ can efficiently deflect optical beams in near-grazing directions.

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Shaping quasi-bound states in the continuum for maximum optical chirality

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Maximum chiral metasurfaces are transparent to the waves of one circular polarization and completely block the waves of the opposite polarization [1]. In recent years it was demonstrated that maximum chirality can be based on specific nonradiating eigenstates – symmetry protected bound states in the continuum (BICs). The true advantage of BICs is that they can be easily transformed into quasi-BICs of desired polarization and Q-factor by symmetry-breaking perturbations [2] and, in particular, can be shaped into chiral quasi-BICs uncoupled from the waves of specific circular polarization. We discuss how depending on the metasurface symmetry one can design rotationally symmetric maximum chiral absorbers or lossless asymmetric chiral mirrors. We perform numerical simulations of metasurfaces in COMSOL Multiphysics, analyze their optical characteristics and study polarization of quasi-BICs with both intrinsic and extrinsic chirality.

If a metasurface possesses rotational symmetry of the order $N \geq 3$, its optical chirality is determined by different absorption of left and right circularly polarized waves [3]. We present a dielectric metasurface with a C_4 symmetry axis, consisting of four pairs of rods. Such arrangement hosts symmetry protected BICs with antiparallel displacement currents flowing within the rods of each pair. To selectively couple such eigenstates to waves of a particular circular polarization incident on the corresponding metasurface side, one can open each pair of rods and offset them to transform the BICs into chiral quasi-BICs. Then, varying the extinction coefficient, we match the critical coupling regime ensuring full absorption of this wave, while the wave of opposite polarization is fully transmitted.

Generally, metasurfaces having no point symmetry elements can perform all possible chiral light transmissions and reflections. Coupled mode theory predicts that uncoupling a quasi-BIC from waves of a specific circular polarization suppresses all outgoing optical channels for the waves of the opposite polarization, except for its chiral reflection [4, 5]. To demonstrate this, we consider a pair of identical rectangular lossless rods hosting an antiparallel electric dipole BIC. One can transform this BIC into a quasi-BIC by opening the rods or by putting them on different faces to produce a height offset. We combine both symmetry-breaking perturbations to ensure uncoupling of the quasi-BIC from one circularly polarized wave. At the same time, waves of the opposite polarization are fully reflected, preserving the handedness. Next, we consider a single lossless cuboid hosting an electric vortex eigenstate which is a true BIC. Again, one can shape it into a chiral quasi-BIC and achieve the maximum optical chirality by tilting the prism and transforming its base to trapezoidal.

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Unconventional axion response in metamaterial

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We investigate the electromagnetic response of a metamaterial described by the constitutives relations given by

$$\mathbf{D} = \epsilon\mathbf{E} + \Psi\nabla^2\mathbf{B}, \tag{1}$$

$$\mathbf{H} = \mu^{-1}\mathbf{B} - \Psi\nabla^2\mathbf{E}, \tag{2}$$

where Ψ can vary in space. These equations reproduce a set of modified Maxwell's equations, resembling the standard axion electrodynamics [1], which optical and electromagnetic properties are known and of great interest in the fields of topological materials, metamaterials and dark matter. We show that the optical properties of our exotic material, for a Ψ field constant piecewise function, are similar to the axion case, including Kerr and Faraday rotation, Brewster's angle, and surface waves [2–4]. However, the response of this material to the external quasi-static sources is strongly different from that expected in axion electrodynamics case.

Some metamaterial designs have shown to exhibit an effective electromagnetic response similar to axion electrodynamics such as multilayered structures and split-ring resonators (SRR) [5, 6]. Here, we discuss the extension of these approaches to propose possible metamaterial designs that manifest this unconventional electromagnetic response. Specifically, we consider higher-order spatial dispersion corrections to the response of non-reciprocal multilayered structures. Furthermore we explore the consequences of the nonreciprocal coupling between higher-order multipole resonances of the meta-atoms, such as quadrupole moments.

The study of this exotic functionalities will provide a deeper understanding of axion insulators and unconventional phenomena attainable in this kind of structures.

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Resonant optical properties of metal-dielectric and all-dielectric nanophotonic structures integrated into a slab waveguide

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In recent few decades, the investigation of resonant optical properties of various photonic structures has been an active research direction [1]. This is due to both fundamental interest and numerous prospective applications, in particular, in filtering, sensing, and analog optical computing. One of the promising platforms of integrated nanophotonics is the platform of dielectric slab waveguides, the modes of which are often referred to as semi-guided waves. For semi-guided waves, various integrated elements including beam splitters, lenses, and digital planar holograms have been developed. In our previous works, we proposed all-dielectric structures consisting of a few ridges or grooves on the waveguide surface and demonstrated that despite the simplicity of their geometry, they support high-Q resonances and bound states in the continuum (BICs). We derived simple but very accurate coupled-wave models describing the resonant optical properties of such structures and predicting the positions of the resonances and BICs in the parameter space.

In this work, we mainly focus on integrated metal-dielectric-metal (MDM) structures consisting of two metal strips "buried" in the waveguide core layer. Using both theoretical coupled-wave models and rigorous numerical simulations, we show that such structures can provide total absorption or coherent perfect absorption (CPA) of the incident guided modes. As an example, Fig. 1 shows the rigorously calculated field distribution in an integrated MDM structure consisting of two identical silver strips integrated into a 100 nm thick gallium phosphide waveguide and providing CPA of two symmetrically incident transverse-electric-polarized modes with the free-space wavelength of 630 nm obliquely incident at an angle of 55°.

We believe that the proposed integrated MDM structures may find applications as absorbers or filters in novel "on-chip" optical circuits.

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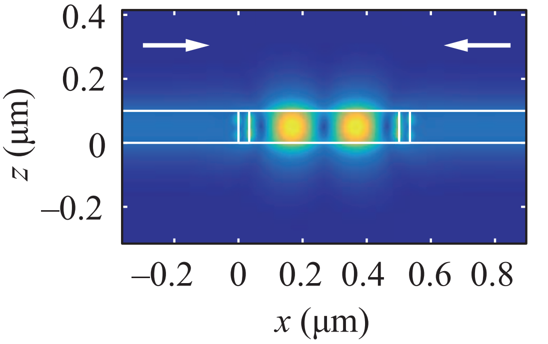

Fig. 1: Normalized magnetic field distribution in an integrated MDM structure providing CPA.

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Light scattering from dielectric split ring resonator

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Equidistant resonances are of great interest, since one can control their number in a fixed frequency range by changing the geometric parameters, dielectric constant, or scattering geometry. It is interesting to consider such resonances in split rings for which many fundamental results and practical applications in photonics have already been obtained [1, 2]. It was demonstrated that low-frequency scattering spectra from dielectric ring resonators with rectangular cross-section consist of separate galleries [3]. Each gallery starts with a broad Fabry–Perot resonance with the shape of a Fano or Lorentz contour and continues with an equidistant set of azimuthal high-Q resonances. However, the question remained open: what will happen to the galleries when we introduce a gap in the ring resonator?

This work is devoted to the study of the scattering spectra evolution of three types of structures during the topological transition *ring – split ring – cuboid* with the conservation of the resonator length. Scattering spectra were calculated and an experiment was performed on a resonator with a narrow gap ($\alpha = 0,62^o$, here α is the sector angle of the circle that determines the gap). The numerical results and experimental data are in complete agreement. It was found that when a narrow gap ($\alpha \leq 1^o$) is introduced, the degeneracy of the azimuthal resonances of the ring is lifted, with separation into two families of quasi-Fabry–Perot resonances with longitudinal even $2m$ and odd $2m+1$ indices. With an increase of the gap, the frequencies of the resonances practically do not shift, but their Q factor demonstrates a significantly nonlinear behavior, which is different for $2m$ and $2m+1$ families. It was shown that this behavior is associated with the interference of scattered waves in the gap. The main result is a periodic change in the maxima and minima of the Q factor depending on the number of half-wavelengths that fit in the linear distance between the ends of the split ring. It was also found that the intensity of resonances in the scattering spectra can be controlled by changing the orientation angle of the gap with respect to the direction of the incident wave. This allows us to efficiently excite families of $2m$ or $2m+1$ resonances in the scattering spectra.

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Laser-induced periodic structures on amorphous germanium thin films

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Structured germanium (Ge) thin films are a promising platform for photovoltaic, optoelectronic and sensing devices due to wide transmission window of Ge, its high carrier mobility, and high third-order non-linearity [1]. However, Ge films produced by common substrate-deposition techniques are often amorphous, which requires further crystallization for the mentioned properties to be exploited. Ultra-short laser pulses can be used for both crystallization and structuring of Ge films in a single laser treatment process [2]. Formation of laser-induced periodic surface structures (LIPSS) is an attractive approach for surface functionalization due to its flexibility and relative simplicity of the procedure of laser processing in ambient air with a single laser beam [3]. LIPSS usually represent a periodic relief appearing due to interference effects within the focal spot after laser irradiation of solids with a period, orientation, uniformity being dependent on the irradiation conditions. LIPSS formation on Ge has been studied since their first observation in the 1960s, with studies focusing on laser structuring of bulk crystalline Ge samples. However, to the best of our knowledge, there were no reports on the fabrication of LIPSS on Ge films.

Here, we present the results of LIPSS formation using femtosecond laser pulses with wavelength $\lambda = 1026$ nm, pulse duration $\tau = 230$ fs, and pulse repetition rate $f = 200$ kHz on amorphous germanium (a-Ge) films with thicknesses of 100 and 380 nm sputtered on glass substrates. Laser pulses were focused onto a film surface using a cylindrical lens to obtain an elliptically shaped focal spot with dimensions of ≈ 15 by 150 µm. During the experiment, the samples were scanned with a laser beam at different speeds with the short axis of the elliptic focal spot and the linear laser polarization being parallel to the scanning direction. Ordered LIPSS with different morphology, which depends on the applied laser fluence and scanning speed, are obtained. In general, periodic structures with periods around 650–950 nm (0.6–0.9 λ) are formed within the fluence range $F = 30$–$80\ \mathrm{mJ/cm^2}$ and at scanning rates $v = 1$–100 µm/s. Scanning speed dependent splitting of the LIPSS area into several individual tracks is observed. Raman spectroscopy reveals crystallization of the a-Ge film at high fluences and scanning rates. Germanium metallization under fs laser pulses and generation of surface plasmon-polaritons (SPP) have been used to explain the LIPSS formation.

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Simulation of wave propagation in weakly cohesive powders by DEM

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Description of cohesive powders is an interesting but complex problem, since granular media easily change their structure. We perform a numerical experiment on wave propagation in solid-like powders by the Discrete Element Method (DEM). A few reproducible real experiments on the propagation of waves in loosely packed granular media show a strong dispersion [1]. We question if they correspond to an "optical branch" related to the microstructure. We use DEM (adapting the code used in [2]) to simulate the propagation of a linear wave in a rectangular cell filled by a weakly cohesive 2D assembly of circular particles with cohesive elastic-frictional interactions. Numerical experiments comprise two steps.

1. Creation of random particle packings. Loose random packs are assembled by ballistic aggregation, and then subjected to isotropic compression, in the absence of gravity, as in Ref. [2]. We consider first relatively dense, weakly cohesive states, and obtain well equilibrated samples of about 2000 particles under isotropic pressure.

2. Small disturbance propagation. We send a sinusoidal impulse from the left wall. Its propagation is modeled to first order, linearizing the dynamical elastic response. The wave front and the peak velocities are measured by the time of flight method. The short-time motion of the front gives an approximation of phase velocity.

The longitudinal wave in such a sample is slightly dispersive and is not accompanied by a rotational wave. Its peak velocity is 15% higher than the classical P-wave velocity deduced from the static moduli (measured as in [2]). The transversal wave appears to be a shear–rotational wave. It is more sensitive to the packing than the longitudinal wave. It is essentially dispersive, and in some cases the perturbation changes sign. For some frequencies it is strongly evanescent. This behaviour might be characteristic of the reduced Cosserat model [3].

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3D printed optimized phased array with volumetric emitters

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In modern telecommunication systems, it is significantly important to develop such equipment that would allow establishing data exchange between two or more devices at high speed. One of the key features fitting this niche are antenna arrays [1]. It is known, that properly manufactured antenna array may produce either one highly directional beam or multiple beams towards selected by its drive directions, which is used in 5G telecommunications [2]. However, the majority of the phased arrays take a lot of space due to the large number of emitters. The main aim of this research was to create a phased array at low GHz frequency, which has both: high realized gain coefficient and relatively (in wavelengths) small aperture of the reflector. Since flat phased arrays with small aperture performed by patch antennas or mushroom antennas have relatively low realized gain [3], we decided to remove emitters from the surface into space. Such approach makes it possible to move from currents on the surface to volumetric currents in space, which may help to reduce the size. The result of this research is a functioning sample of a phased antenna array, whose characteristics are in Figure 1.

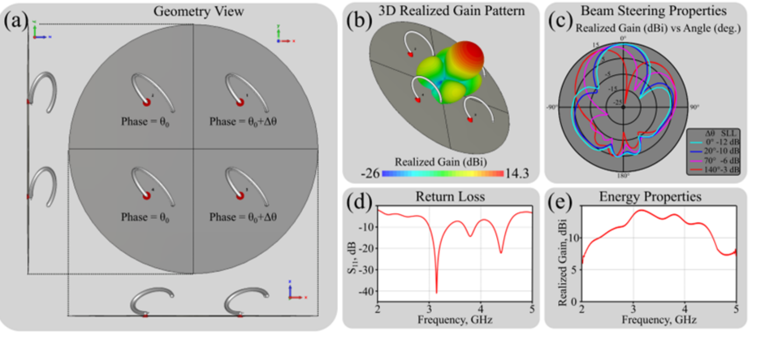

Fig. 1: The suggested phased array properties: (a) general view in three orthogonal projections, (b) radiation pattern at 3.14 GHz for in-phase feeding, (c) radiation plots for different emitters phase shifts, (d) return loss frequency dependence, and (e) realized gain frequency dependence.

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Resonances in guided-mode resonant gratings with linearly varying period

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Recently, there has been great interest in the study of resonant properties of subwavelength diffraction gratings. One of the most important applications of resonant gratings (RG) is in the problem of spectral filtering of light. When creating spectrometric systems, the problem arises of dividing the incident radiation into a large number of spectral channels. An effective approach to creating spectral filters for such systems is to use resonant diffractive structures with one or more parameters varying along the structure dimension. This allows one to filter different wavelengths in different areas of the filter. Such structures are usually referred to as linearly variable filters (LVFs). Along with the most well-known type LVFs, namely, Bragg gratings with a wedged central layer, RGs with one or more parameters varying in the direction of periodicity represent another promising type of LVFs [1, 2]. In addition to spectrometric applications, gratings with varying parameters are also promising for use as refractive index sensing [1].

In this work, we focus on a dielectric grating with the period d varying linearly as $d(x) = d_0 + \alpha x$, where d_0 is the "central" period and α is the period change rate (see Fig. 1). We assumed that α is about 10^{-3}, which, on the one hand, allows one to design compact filters but, on the other hand, makes the use of so-called local periodic approximation inapplicable [2]. Using rigorous electromagnetic simulations based of the Fourier modal method (FMM), we showed that in the considered case, the shape of the resonance differs from the conventional Fano and Lorentz line shapes and exhibits secondary peaks appearing near the main one. To describe this effect, we developed a coupled-mode theory (CMT) with varying parameters describing coupling of two counter-propageting eigenmodes in the grating. It was shown that the parameters of the CMT can be estimated by calculating the poles of the scattering matrix of a strictly periodic grating [2]. The CMT results are in perfect agreement with the FMM simulations.

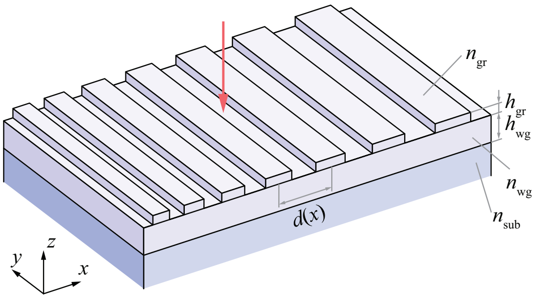

Fig. 1: Geometry of a guided-mode resonant grating with linearly varying period.

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Dyakonov surface states on confined interfaces

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We theoretically investigate Dyakonov-like surface waveguide modes propagating along the planar strip interfacial waveguide between two uniaxial dielectrics.

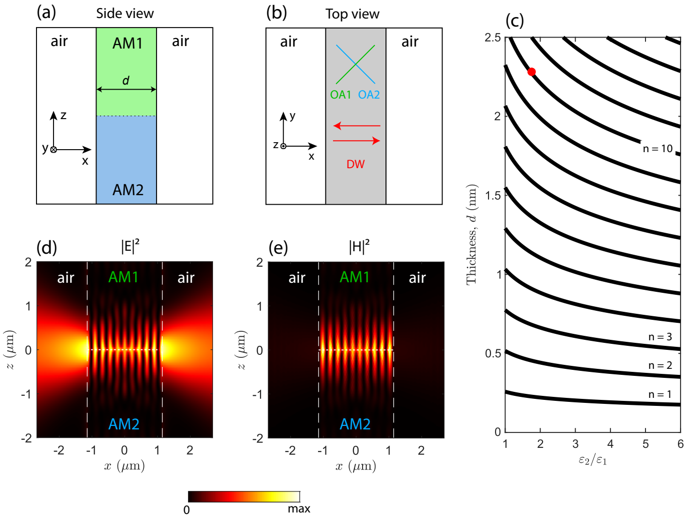

Fig. 1: (Color online) Side view (a) and top view (b) of the interface between two anisotropic materials bounded by air from left and right. Optical axes of anisotropic materials are perpendicular to each other and form an angle of 45° to the y-axis, as shown by green and blue lines in panel (b). The Dyakonov-like mode in such configuration is the superposition of Dyakonov waves reflecting from both sides of the boundary at the angle of $\alpha = 0°$ as is shown in panel (b) by red arrows. Corresponding Fabry–Perot resonances calculated by analytical model described in [1] are shown in panel (c). The intensity of electric (d) and magnetic (e) fields of Dyakonov-like mode in such configuration. Colorscale is shown on the right.

Due to the one-dimensional electromagnetic confinement, Dyakonov surface waveguide modes can propagate in the directions forbidden for the classical Dyakonov surface waves at the infinite interface. We show that this situation is similar to a waveguide effect and formulate the resonance conditions at which Dyakonov surface waveguide modes exist. In the one-dimensional confinement case, we also show the existence of Dyakonov Fabry–Pérot resonances. We also consider a case of two-dimensional confinement, where the interface between two anisotropic dielectrics is bounded in both orthogonal directions. We show that such a structure supports Dyakonov-like surface cavity modes. Analytical results are confirmed by comparing with full-wave solutions of Maxwell's equations. Our work paves the way for new insights into surface waves in anisotropic media.

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Quantum state transfer via topological pumping in a qubit array

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Superconducting circuits are known to be the leading candidate to realize quantum computing [1]. While decoherence leads to quantum information loss, quantum topological photonics predicts robust photon sources and protection of quantum correlations [2]. One of the simplest quantum algorithms is the transfer of excitations from one part of the superconducting circuit to another. Topological pumping [3] is a quantized excitation transport through the system during the cyclic Hamiltonian variation.

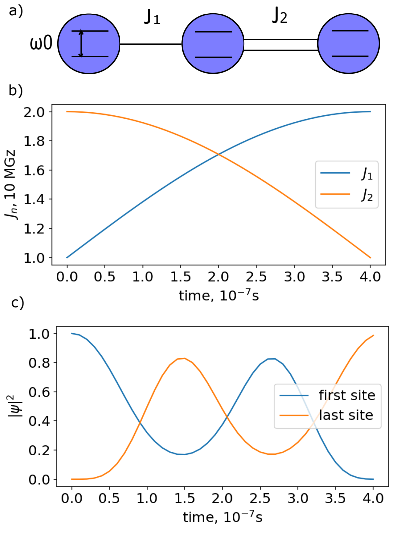

Fig. 1: a) Scheme of qubit array with $\omega_0/(2\pi) = 4$ GHz and $U = 0$ b) Coupling amplitudes variation c) Fock state evolution.

To assess the possibilities of topological pumping in the platform of superconducting circuits, we consider a chain of superconducting qubits (two level systems) connected by linear elements (Fig. 1 (a)) described by the standard Bose–Hubbard model [4]:

$$\hat{H}(J_1, J_2) = \omega_0 \sum_{i=1}^{3} a_i^\dagger a_i + \frac{U}{2} \sum_{i=1}^{3} (a_i^\dagger a_i - 1) a_i^\dagger a_i + J_1 a_1^\dagger a_2 + J_1 a_2^\dagger a_1 + J_2 a_2^\dagger a_3 + J_2 a_3^\dagger a_2 \,, \tag{1}$$

where a_i is an annihilation operator. In the case of single excitation the system can be reduced to the Su–Schrieffer–Heeger model with well-established topological order controlled by coupling ratio J_1/J_2.

As a first step, assuming the total photon number conservation, we neglect the incoherent and dissipative process and consider the transportation of quantum states through the system. The evolution of the wave function:

$$\psi(t) = \exp\left[-i\int_0^t \hat{H}(J_1, J_2)dt'\right]\psi_0 \tag{2}$$

We adiabatically vary the coupling amplitudes in the system reversing their values and causing a topological transition accompanied by the change of edge state localization.

We have considered a time evolution of the quantum state in the qubit chain. Varying the coupling amplitudes $J_{1,2}$ as shown in Fig. 1 (b), we have demonstrated a transfer of excitation from the left to the right edge. The initial state was prepared as the system's edge eigenstate or left-edge Fock state. The dynamics of the edge sites population are shown in Fig. 1 (c).

We anticipate that the concepts of topology may enable disorder-robust transfer of quantum states preserving both the number of photons as well as quantum correlations.

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Electromagnetic resonances of dielectric rings

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Resonators of various shapes and sizes are of interest both from a fundamental and applied point of view, and are used to create various devices, including antennas, microlasers, sensors [1]. The photonic properties of dielectric resonators depend on their shape, for example in axisymmetric structures, such as disks and cylinders, whispering gallery modes are observed [2]. Previously, we found that the scattering spectrum of dielectric rings is formed by separate photonic ring galleries [3], in addition, bound states in the continuum are observed in the spectra of cylindrical and ring resonators [4].

In this work, a study has been carried out that allows us to interpret in detail the changes in the low-frequency scattering spectra of a ring resonator with a rectangular cross section with a change in its height. In addition, the distribution of fields inside the ring was studied. Numerical simulations of the scattering spectra and field distribution were performed using COMSOL software. Azimuthal (m), radial (r) and axial (z) indices are determined from number of maxima and minima in the field distribution patterns, which play the role of photonic "fingerprints" of the eigenmodes of ring dielectric resonators. The main effect observed was that as the height of the ring increases, new axial galleries appear in the spectra with a successive increase in the axial index.

Scattering experiments in the far- and near- fields were carried out in an anechoic chamber for ring resonators with a height of 4, 5 and 6 mm. Images of the spatial distribution of the magnetic field in the microwave range were obtained. The experimental results are completely consistent with the calculation.

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Anomalous reflection at the boundary of hyperbolic medium

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Despite the apparent simplicity, the problem of refraction of electromagnetic waves at the planar interface between two media has an incredibly rich spectrum of unusual phenomena. An example is the paradox that occurs when an electromagnetic wave is incident on the interface between a hyperbolic medium and an isotropic dielectric. At certain orientations of the optical axis of the hyperbolic medium relative to the interface, the reflected wave is completely absent. In this paper, we formulate the aforementioned paradox and present its resolution by introduction of infinitesimal losses in a hyperbolic medium. We show that the reflected wave exists, but became extremely decaying as the loss parameter tends to zero. As a consequence, all the energy scattered into the reflected channel is absorbed at the interface. We support our reasoning with analytical calculations, numerical simulations, and an experiment with self-complementary metasurfaces in the microwave region.

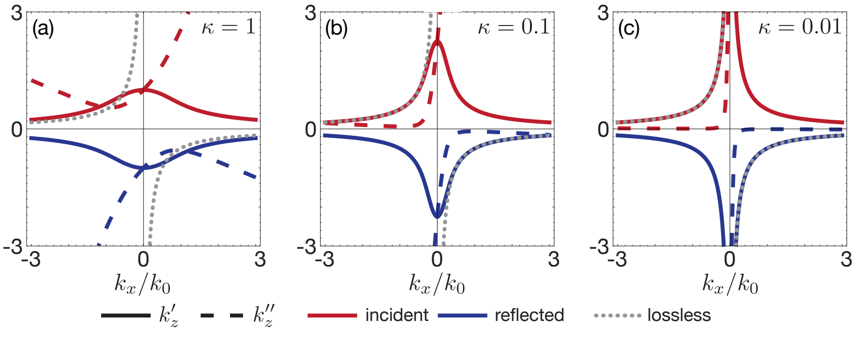

Fig. 1: Isofrequency contours for lossy hyperbolic medium for different values of loss parameter κ. Figures (a), (b), (c) correspond to the values of $\kappa = 1$, $\kappa = 0.1$, and $\kappa = 0.01$ respectively. Solid (dashed) line represents the real (imaginary part) of the z-component of the wavevector. Red (blue) color encodes the wavevector component for the wave propagating along (against) the z-axis. Additionally, the z-component of the wavevector for the lossless case is represented by the gray dotted line.

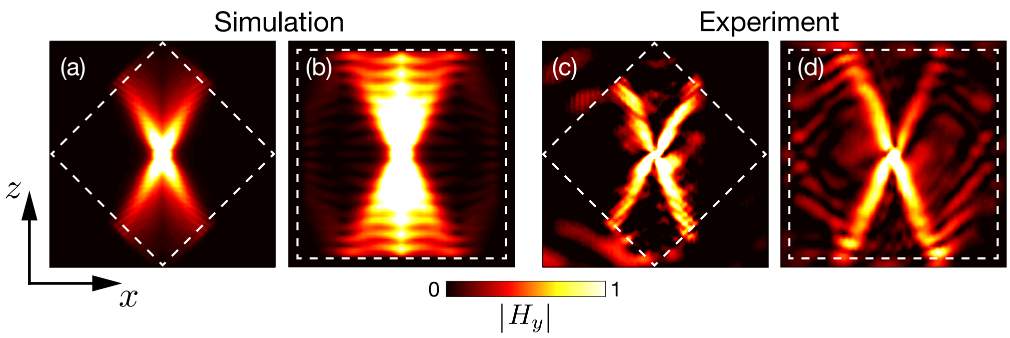

Fig. 2: Absolute value of the numerically calculated (a), (b) and experimentally measured (c), (d) magnetic field for rhombic (a), (c) and square (b), (d) metasurfaces. White dashed line shows boundaries of metasurfaces. At each figure, magnetic field is normalized by its maximal value.

Nonlinear scattering of classical and non-classical signals on single superconducting artificial atom

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Superconducting electrical circuits in quantum regime could be treated as single artificial atoms with arbitrarily engineered and controllable energies and transition rates. We place a single two-level superconducting atom into the coplanar waveguide, and a strong coupling between the circuit and electromagnetic modes of the continumm is easily achieved [1]. We study nonlinear mode mixing (intermodulation) on the single atom [2, 3], and find specific features of nonlinear spectra, which could be attributed to the quantum nature of the scatterer. Finally, we construct and study the cascaded quantum system of two artificial atoms, and observe non-classical features of nonlinear intermodulation within the system.

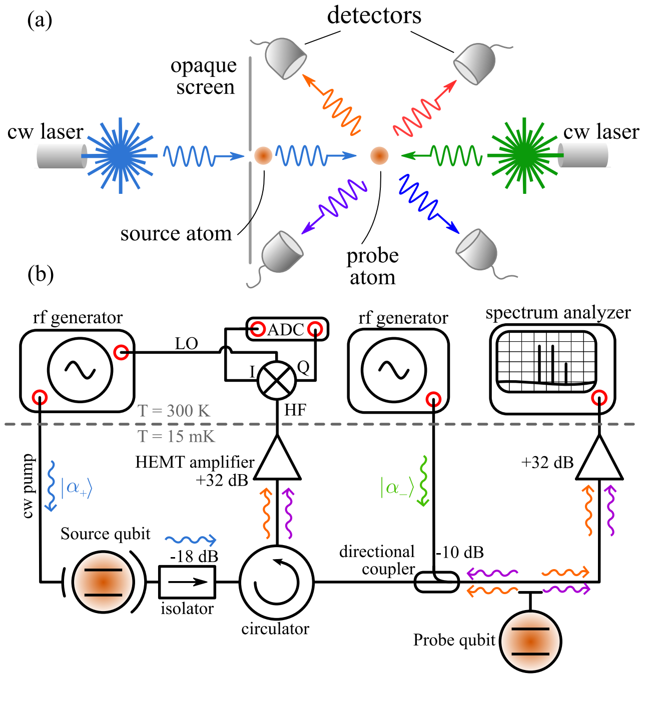

Fig. 1: (a) The optical concept of the experiment. The probe atom scatters two coherent fields: the non-classical one cominng from the source excited through small aperture in opaque screen, and the classical wave comes from external rf generator. In turn, the field from the probe is carefully detected and analyzed. (b) The simplified sketch of waveguide-QED microwave setup for the same type of experiment with two superconducting transmon qubits in the dilution refrigerator.

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Spatiotemporal transformation of optical signals with metal–dielectric–metal multilayers

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In recent years, there has been great interest in the investigation of photonic structures for all-optical information processing and analog optical computing. These structures are considered as promising building blocks that will help ensuring the further development of computing systems by supplementing the electronic components [1]. Among the most important operations of analog optical computing are the operations of the temporal and spatial differentiation of an incident optical signal. For the optical implementation of these operations, various resonant structures were proposed, including resonant diffraction gratings and layered structures. It is important to note that despite a huge number of published works on optical differentiation, there exist only a few works dedicated to the "spatiotemporal" optical differentiators implementing differential operators corresponding to a weighted sum of temporal and spatial derivatives.

Here, we consider the application of the three-layer MDM structure as a first-order spatiotemporal differentiator. We show theoretically and numerically that the differentiator based on the MDM structure enables generating an optical vortex (OV) in the spatiotemporal domain as well as implementing the "spatiotemporal" edge detection operation (Fig. 1). We also consider a "double" MDM structure consisting of two single MDM structures, which is shown to possess a reflection zero of the second order with respect to both angular and spatial frequencies. We demonstrate that this structure enables performing the operation of spatiotemporal differentiation of the second order corresponding to the sum of second-order spatial and temporal derivatives.

We believe that the proposed MDM structures may find application in analog optical computing and optical information processing systems.

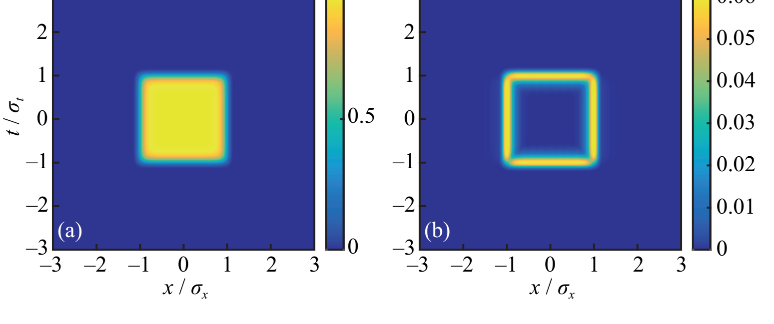

Fig. 1: Amplitude of the incident pulse with a "super-Gaussian" envelope ($\sigma_x = 6$ µm, $\sigma_t = 270$ fs) and (b) amplitude of the reflected pulse envelope.

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Fiber Bragg gratings inscription in multicore and multimode fibers by femtosecond laser radiation

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Fiber Bragg gratings (FBGs) is a key element in the modern fiber-optic sensors and fiber lasers [1]. Traditionally, periodic modulation of refractive index in the fiber core is produced by UV radiation during FBG inscription in photosensitive fibers through the mechanism of photo-induced refractive index change. FBGs inscription technique based on femtosecond (10^{-15} s) laser pulses has gained significant attention in the last decades due to capabilities of using non-photosensitive fibers, high-resolution 3D fiber core processing and a high temperature (up to 1000 °C) stability of modified fiber.

Here, we review our results of point-by-point FBGs inscription in multicore (MCF) and multimode (MM) fibers by IR femtosecond laser radiation. In the first case, the advanced 3D fiber shape sensors were demonstrated with an inscription of FBG arrays in the selected cores of MCF allowing high precision distributed measurements of strain in the given fiber cross-sections. Moreover, we proposed the new schemes of Raman fiber lasers with cavity formed by complex FBG reflectors inscribed in 7-core passive fiber with coupled cores [2]. In case of MM fibers, the inscription of FBG in the certain position within the fiber cross-section opens up the possibility for a transverse mode selection of the output beam. In particular, LP01 and LP11 transverse modes of Raman fiber laser were obtained in case of the FBGs located in the center and with certain shift from the center of the fiber core, respectively. In addition, the recent results on an inscription of various random structures (1D, 2D, 3D) with enhanced Rayleigh backscattering for advanced sensors and laser applications will be presented.

The work was supported by the Russian Science Foundation grant № 21-72-30024.

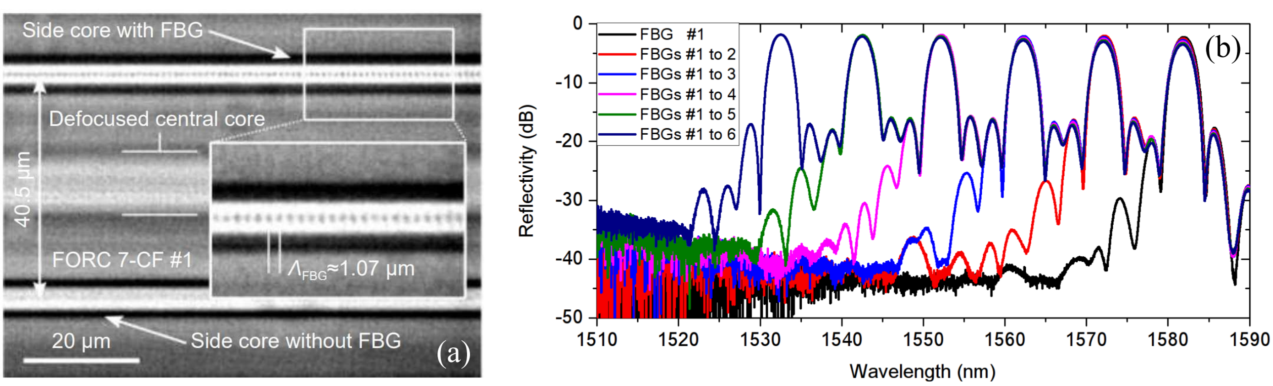

Fig. 1: Image of FBG inscribed in 7-core fiber (a), reflection spectra of FBGs array(b).

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Dyakonov surface waves in confined media

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Surface electromagnetic waves propagating along the interface between two heterogeneous media have been the subject of extensive research in recent decades, since they are one of the fundamental concepts of nanophotonics. Dyakonov surface waves (DSWs) are a family of surface waves that exist at the interface between two media, at least one of which is an anisotropic dielectric, as predicted in Refs. [1, 2] for the case of positively anisotropic materials (i.e. for materials with dielectric permittivity tensor $\hat{\varepsilon} = \mathrm{diag}\,(\varepsilon_1, \varepsilon_1, \varepsilon_2)$ such that $\varepsilon_1 < \varepsilon_2$). In Refs. [1, 2], Dyakonov surface waves were considered on a plane infinite interface.

In our previous works [3, 4], we theoretically study Dyakonov surface waveguide modes (DSWMs) that propagate along a flat stripe of the interface between two uniaxial dielectrics bounded on both sides by air or metal. We show that, due to the one-dimensional electromagnetic confinement, DSWMs can propagate in directions forbidden for classical DSWs at an infinite interface. We also theoretically demonstrate that, due to specific boundary conditions in such a waveguide, DSWMs can exist even in the case of dielectrics with negative anisotropy ($\varepsilon_1 > \varepsilon_2$), in contrast to classical DSWs. Finally, we provide experimental evidence of existence of DSWMs in a flat stripe of the interface between two negatively anisotropic metamaterials.

This work opens up new possibilities towards applications of DSWs in optics and photonics.

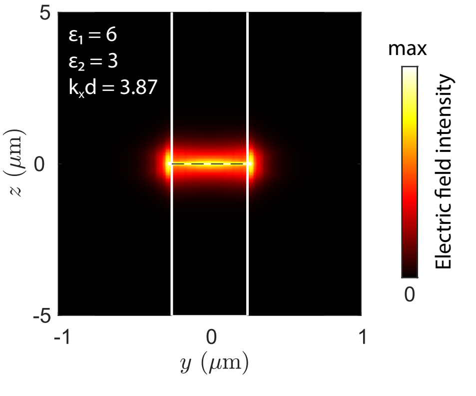

Fig. 1: Electric field intensity of the DSWM in a waveguide bounded by an ideal metal.

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Purcell effect in a bilayer of twisted dielectric photonic crystal slabs

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Twisted photonic metasurfaces are considered as a promising platform for nanophotonic devices. Their optical behavior is strongly affected by the moiré-pattern of superimposed periodic structures. In this work, using our recently developed moiré-adapted Fourier modal method [1], we theoretically study the Purcell effect of dipoles located in a bilayer of twisted one-dimensional dielectric photonic crystal slabs. First, we describe a theoretical model for calculating total and external Purcell factors in photonic crystal slabs in terms of dipole's emissivity to the near-field and the far-field. Then we apply this theory to twisted photonic crystal slabs to study how the Purcell factor depends on the wavelength and the rotation angle. We show that by placing the dipoles in the hot spots of modes with the zero group velocity, one can greatly enhance the Purcell factor in comparison to homogeneous slabs. We demonstrate that this effect is associated with Van Hove singularities. We also calculate partial contributions to the total Purcell factor from different energy dissipation channels.

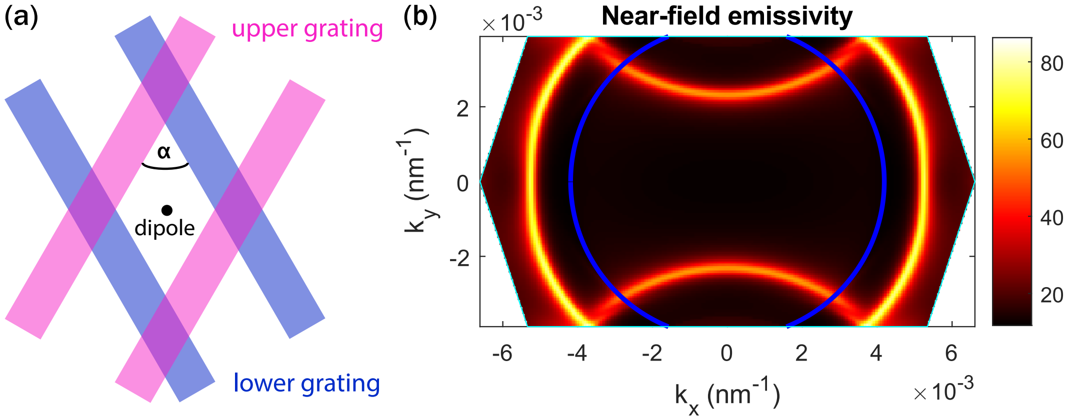

Fig. 1: (a) Top view of the bilayer of twisted photonic crystal slabs with parameters: $\varepsilon = 5 + 0.05i$, period $p = 500$ nm, width of stripes $w = 200$ nm, thickness $h = 350$ nm, gap between the slabs $g = 50$ nm, rotation angle $\alpha = \pi/5$. (b) In-plane wavevector dependence at $\lambda = 1500$ nm of the near-field emissivity of a vertical dipole located in the twisted bilayer, as shown in panel (a). The blue line shows the section light cone at $\lambda = 1500$ nm.

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Photonic topological insulator tight-binding model for Tamm plasmon-polariton based vertical-cavity laser array

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Partially disordered systems like one-, two- and three-dimensional crystalline solids with impurities can be described by Schrödinger's equation for electrons. Tight binding approximation and topological methods are also in demand in photonics. We apply the tight binding model to describe the behavior of Tamm plasmon-polaritons [1] inside the two-dimensional array of vertical-cavity lasers [2, 3].

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Experimental investigation of microwave light interaction with superconducting artificial atom arrays

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Superconducting artificial atoms (SAAs) used in the most prominent quantum computing experiments present also a promising tool for quantum simulation, i.e., modeling dynamics of otherwise intractable quantum systems by a controllable quantum device. We review some of our experimental efforts in modeling light-matter interaction processes using circuit QED devices in which we have observed non-conventional Autler–Townes effects in a artificial diatomic molecule [1] and coherent transport of electromagnetic radiation through a chain of five transmon SAAs [2].

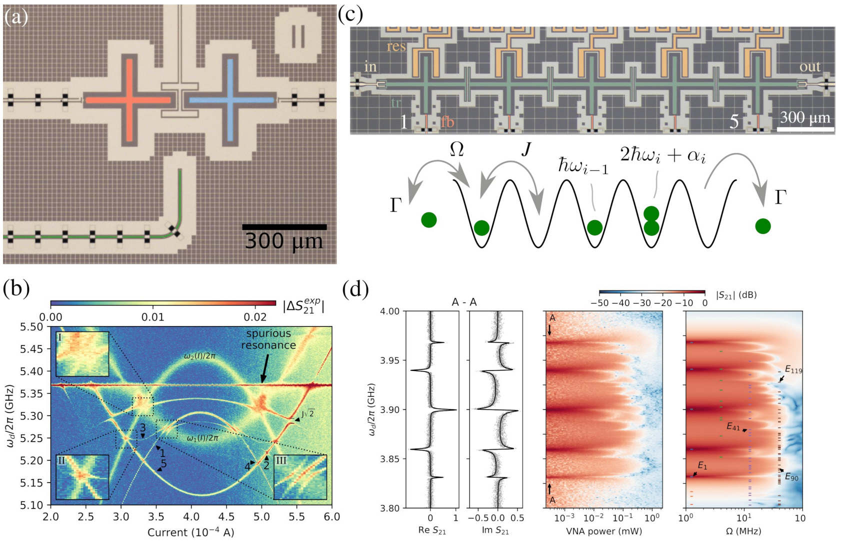

Fig. 1: (a) Diatomic molecule based on two capacitively-coupled transmon SAAs (orange and blue) irradiated by microwave coplanar antenna (green). (b) Light-dressed spectrum of the molecule, exhibiting bright multiphoton transitions. (c) Five-transmon chain connected to two waveguides at its ends and its transmission spectroscopy data (d), theory and experimental data.

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Homogenization of Mie-resonant metamaterials

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Optical metamaterials have attracted a lot of attention for over last twenty years. Indeed, the idea of development of artificial metamaterials based on dense photonic crystals implies realisation of the required optical properties on demand. The greatest expectations in this area were initially related to the development of metamaterials that provide properties unattainable in natural materials. Much effort was made to develop invisibility cloak , hyperbolic metamaterials [1, 2], metamaterials with negative refractive index [3], and many other interesting structures. Unfortunately, most of these ambitious projects require plasmonic materials, which due to Joule losses make the demonstration of fine effects extremely difficult. Nevertheless, there are still many promising structures that remain to be explored in detail.

One of the bright examples are fully dielectric metamaterials supporting Mie resonances. For instance, they demonstrate the remarkable phenomenon of negative effective magnetic permeability corresponding to an opening of the first Mie-resonant photonic stop-band below the first Bragg bandgap [4]. Metamaterial description of the other bandgaps still remain challenging. Indeed, they are associated with excitation of high multipole moments and therefore require introduction of some additional material parameters.

In this work, we develop a microscopic theoretical approach for the calculation of the effective parameters of photonic crystals. We formally consider the excitation of an infinite metamaterial in the same way as light scattering on single particles. This allows us to reformulate the infinite metamaterial excitation by a plane wave in terms of the problem of radiation of some current and, accordingly, to obtain the response of the considered structure as a function of polarization and, most importantly, the independent frequency ω and wave vector k. This approach allows us not only to describe the dispersion of the bulk waves, but also formulate the boundary conditions. In particular, we demonstrate that Mie-resonant excitation of high multipole moments affects the Fresnel reflection and transmission coefficients, which in turn enables some non-conventional physical effects.

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Rigorous methods of diffraction at the open-end of a dielectric-loaded circular waveguide

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We present a brief overview of rigorous diffraction methods used in our recent papers devoted mainly to radiation of a charged particle bunch (other forms of excitation were also considered) at the open end of a circular dielectric-loaded waveguide. We discuss both the obtained results and their possible applications in the area of beam driven radiation sources and bunch manipulation using strong THz fields.

First, an "embedded" structure is discussed: a semiinfinite open-ended circular waveguide is placed into a collinear infinite waveguide of larger radius. In paper [1], we have used the residue-calculus technique for analysis of the case where single TM mode propagates from inside the inner waveguide having uniform dielectric filling. In papers [2–4] we have adopted this approach for rigorous description of radiation produced by a charged particle bunch in the considered "embedded" structure. For vacuum case [2], it has been shown that the diffraction radiation mechanism results in excitation of electromagnetic pulse in the coaxial area that can "replicate" the temporal bunch profile [2]. For the case of dielectric filling of the inner waveguide [4], the main attention has been paid to analysis of penetration of Cherenkov radiation into vacuum regions of the outer waveguide.

Second, an open-ended circular waveguide with dielectric filling placed in the free space is discussed. In this case, so-called "generalized matching method" is used which leads to Wiener–Hopf–Fock equation and then to an infinite linear system for coefficients of reflected waveguide modes. In previous papers, we dealt with case of internal excitation by a waveguide mode of a waveguide uniformly loaded with dielectric [5] or having a layered filling [6]. Report [7] deals with the case of iniform dielectric filling and a thin charged particle bunch moving along the structure axis. External excitation in the form of a plane wave is also considered within the discussed framework.

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Light propagation in labyrinth-like photonic structures: numerical solution of Maxwell's equations

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Labyrinth spatial structures, which are typical for many nonequilibrium processes, such as the Belousov–Zhabotinsky reaction [1], were also found in nonlinear optical self-organization phenomena [2]. When light propagates in optically inhomogeneous or two-component media, it is possible to realize a situation where the structure of the medium allows for the presence of a labyrinth-like distribution of optical characteristics. Labyrinth-like photonic structures can be considered as transitional structures between photonic crystals and quasicrystals, systems of coupled waveguides, and disordered systems. At scales of structures comparable to the wavelength of optical radiation, it seems promising to carry out a direct numerical solution of Maxwell's equations in finite differences on a spatial and temporal grid (FDTD-method) [3].

In this work, based on the numerical solution of Maxwell's equations, the features of the propagation of electromagnetic waves in photonic structures with a labyrinth-like distribution of the refractive index are analyzed. It is shown that the distribution of the energy density of the electromagnetic field is characterized by a complex branching structure, and the time of passage of a light pulse depends nonlinearly on the size of the photon cell. The possibilities of developing new photonic devices (optical diode and light moderator) based on labyrinth-like structures are discussed. An example of the structure and the calculated typical spatial distribution of the electromagnetic field energy density are shown in Fig. 1.

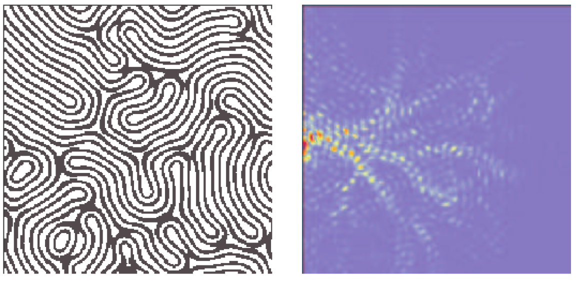
a *b*

Fig. 1: Spatial distribution of the refractive index (a) and energy density of the electromagnetic field (b).

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Electromagnetic properties of theranostic vaterite in drug delivery applications

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Mesoscopic photonic systems with tailored optical responses have great potential to open new frontiers in implantable biomedical devices. However, biocompatibility is typically a problem, as engineering of optical properties often calls for using toxic compounds and chemicals, unsuitable for in vivo applications. Here we will present optical properties of vaterite nanoparticles and discuss their appearance in a natural environment. At the next stage, a new approach to biofriendly delivery of optical resonances is demonstrated. We will show that the controllable infusion of gold nanoseeds into polycrystalline sub-micrometer vaterite spherulites gives rise to a variety of electric and magnetic Mie resonances, producing a tuneable mesoscopic optical metamaterial. Owing to the biocompatibility of the constitutive elements, "golden vaterite" paves the way to introduce designer-made Mie resonances to biophotonic applications

Maximum chiral light transformations by resonant dielectric metasurfaces

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Chirality is pivotal in chemistry, biology, and physics, as objects of omnifarious sizes and origin are non-identical to their mirror images. Weak optical chirality of natural materials is quantified by subtle difference of interaction with waves of opposite circular polarizations. Metasurfaces utilize the latest advances in optical theory and experiment to deliver extraordinary functionalities. Resonant dielectric metasurfaces are a game-changing platform for rationally designed optical chirality. Ultimate goals are set by the unique upper limit of *maximum chirality* [1]: transparency to the waves of one circular polarization and full block of those of the opposite polarization.

We show how the chiral metasurface design can be guided by phenomenological coupled mode theory (CMT) expressing optical chirality parameters by the properties of metasurface eigenstates and, especially, by their coupling with circularly polarized waves. Intriguingly, the maximum selectivity is compatible with the absence of true structural chirality: e.g., a planar mirror-symmetric metasurface can selectively reflect one polarization and transmit the other flipping its handedness. CMT provides a clear recipe based on a pair of eigenstates of opposite parity carefully tuned to a common resonant wavelength [2]. Metasurfaces lacking intrinsic chirality can exhibit strong resonant extrinsic optical chirality which allows designing ultracompact chiral light sources based on cavities filled with electroluminescent materials [3].

Chiral metasurfaces, depending on their symmetry, can be engineered to perform as maximum chiral absorbers [4] or reflectors [5, 6]. For exemplary designs, we utilize symmetry-protected photonic bound states in the continuum (BICs) turned by weak perturbations into chiral quasi-BICs coupled to specific circularly polarized waves. High quality factors of resonances underpinned by chiral quasi-BICs enhance numerous nonlinear-optical phenomena. Selectivity of excitation and emission of chiral quasi-BICs pave the way for various nonlinear-optical transformations of circularly polarized light: circularly polarized high-harmonic generation and lasing, giant high-harmonic circular dichroism, etc. Combining such flat components in chiral cavities expands the set of tools for engineering chiral light-matter interactions, and opens a door to chiral polaritonics and chiral electromagnetic vacuum prospectively facilitating chiral chemistry [2].

Support from the Russian Science Foundation (project № 23-42-00091) is gratefully acknowledged.

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Chiral molecular polaritons: cavities, analytical models, and applications

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Geometrical chirality is a universal phenomenon that surrounds us on many different length scales ranging from geometrical shapes of various living organisms to DNA and drug molecules. The majority of molecules involved in biological processes are chiral. Acting on a biological receptor, opposite enantiomers of the same molecule cause different response, perceived as different odor or taste. In pharmaceutics, the opposite enantiomer of a drug molecule can be useless at best, but often it is toxic for the chiral human body. In this regard, there is a great demand from the pharmaceutical industry to develop effective methods of separating chiral enantiomers.

Interaction of chiral matter with circularly polarized electromagnetic fields leads to the effect of circular dichroism, which underlies numerous methods for distinguishing molecular enantiomers. However, those interactions are usually weak and can be well understood without the need to consider a correlated motion between light and matter. If and how strong light-matter interaction can aid those challenging tasks remained largely unclear thus far

In this talk, I will overview the fundamentals of chirality of light and matter, present optical designs required for realization of chiral polaritonic states, discuss recently developed theoretical models, and speculate on the exciting phenomena that can be enabled by strong coupling between chiral light and matter. Recent theoretical efforts already indicate that chiral polaritonic systems may feature non-trivial optical phenomena, where the interplay of light and matter chirality plays the key role in determining the eigenfunctions of the system, as well as its response to external electromagnetic fields. Chiral polaritons – strongly coupled quasiparticles combining chiral excitations of light and matter – could become a new physical platform offering us more control over chirality of matter.

Quantum-optical description of strongly driven systems

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High harmonic generation (HHG) is an extremely nonlinear effect generating coherent broadband radiation and pulse durations reaching attosecond timescales. Conventional models of HHG treat the driving and radiated pulses classically and thus cannot capture the quantum-optical nature of the process. In a series of recent works [1–4], we developed the quantum-optical theory of extreme nonlinear optics and applied it to find the quantum nature of HHG in strongly driven single-body [1] and many-body [2] systems.

Our works unveil the conditions for HHG with quantum-optical features such as squeezing and entanglement. Specifically, we predict quantum effects that alter both the spectrum and photon correlations of HHG, showing when individual frequency components become squeezed [1]. Furthermore, we show that the defining spectral characteristics of HHG, such as the plateau and cutoff, are sensitive to the quantum state of the driving light [3]. Specifically, thermal and squeezed light drive substantially extends the cutoff compared to classical light of the same intensity — producing higher harmonics.

Our findings are a part of an emerging field combining attosecond science and quantum optics, showing new applications of quantum information science in extreme nonlinear optics. The prospects of this emerging field include the engineering of novel many-photon states of light over a broadband spectrum of frequencies, as well as using HHG as a diagnostic tool for characterizing correlations in many-body systems with attosecond temporal resolution.

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Reduced complex linear viscoelastic media as acoustic metamaterials

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We consider linear media of complex structure consisting of "bearing continuum" that reacts to all kinds of strain, enriched at each point by a "distributed dynamic absorber". For elastic continua it means that the strain energy U depends on two vectorial generalised co-ordinates: $\mathbf{q}_1$, corresponding to the bearing continuum, and $\mathbf{q}_0$, corresponding to the distributed dynamic absorber. The difference in the nature of these generalised co-ordinates is reflected in the structure of the strain energy: $U = U(\mathbf{q}_0, \mathbf{q}_1, \nabla\mathbf{q}_1)$, but U does not depend on $\nabla\mathbf{q}_0$. Such elastic linear continua without gyroscopic effects, under some symmetry restrictions, are single negative acoustic metamaterials: for certain harmonic free waves there is a band gap [1]. We consider as specific cases isotropic elastic continua and a gyrotropic elastic continuum, which demonstrates a very complex dispersive behaviour of free polarised waves, possessing not only band gaps, but also decreasing parts of the dispersion curves, thus being a double negative acoustic metamaterial in this domain. This behaviour is very sensible to the system parameters.

If we add linear viscosity to any of the existing elastic connections, we obtain a running evanescent solution at all frequencies. For small viscosity in most of cases a part of a band gap converts into a decreasing part of dispersion curve, and often the wave attenuation decreases in the domain of elastic band gap as small viscosity increases [2].

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The quantum and classical field scattered on a single two-level system

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In this work, we experimentally investigate an elastic scattering of freely propagating wave packets — equal vacuum-photon superposed one and the coherent one — on a single superconducting artificial atom strongly coupled to a coplanar waveguide. An emission line of the single photon emitter, giving on-demand arbitrary vacuum-photon superposed state, is connected through a cryogenic circulator to an input line of the scatterer atom, that allows one to explore the scattering of the single-photon state. Moreover, due to the presence of the separate input line, it is possible to independently excite the scatterer, so we compare the results obtained with the case of an incoming exponentially modulated coherent pulse, since the output line for the above-mentioned experiments is common. Source of the single photon superposed states [1] and an atom-scatterer are represented as tunable transmon qubits [2]. Using a standard heterodyne detection scheme with a fast ADC, the envelope of scattered fields is independently measured for both cases under consideration. Within typical open quantum systems theoretical framework: GKSL equation and input-output theory [3, 4], we obtain analytical expressions that completely describe the evolution of the envelope of scattered fields in the time domain. It is shown that the envelopes of the scattered mean field are different for both cases. Furthermore, we find the classical signal amplitude minimizing this difference experimentally and compare it with analytical results. Importantly, we prove the existence of a non-vanishing discrepancy between two scattering cases, due to the non-zero amplitude of the two-photon Fock state within the coherent pulse. The results of the experiments are theoretically extrapolated to the wide range of the radiative linewidths of both atoms.

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Bound states in continuum and lattice resonances in dipole lattices

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Surface lattice resonances (hybrid states obtained as a result of multiple scattering on individual particles forming a 2D lattice) and bound states in the continuum (non-interacting eigenstates with energy in the region of a continuous spectrum) both demonstrate an extremely high Q-factor, which is applied in the designing of metasurfaces [1]. Interestingly, surface lattice resonances (SLRs) and bound states in a continuum (BICs) can be observed in the same periodic structure with a certain selection of the angle of incidence and frequency of the exciting wave [2]. At the moment, both effects are quite well studied, but the connection between their simultaneous formation has not been established.

In this work we analyze a two-dimensional square lattice of small dielectric spheres in the point dipole approximation [Fig. 1(a)]. It has been shown that, taking into account the electric dipole moments, lattice resonance, Rayleigh anomaly and anapole (the turning of the Mi coefficient a_1 to 0 and the absence of scattered waves) can be observed at the same frequency at a certain angle of incidence. Next, we investigate the dispersion and Q-factor of the lattice resonance.

Fig. 1(b) shows the spectrum of the reflection coefficient of the structure at fixed angles. As the angle of incidence of light increases, the lattice resonance curve is clamped between the Rayleigh anomaly and the anapole until their frequencies coincide. In this case, there will be no radiation, and the Q-factor increases infinitely, which corresponds to BIC. This property can potentially be applied in biosensors, lasers and nonlinear optics.

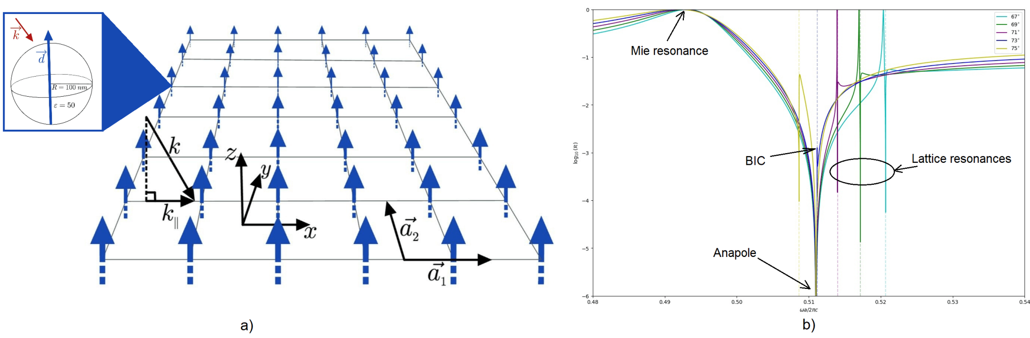

Fig. 1: (a) Investigated square lattice of dielectric dipoles (b) Reflection coefficient spectra near anapole frequency (log scale) at fixed angle of incident wave. Dashed lines correspond to diffraction channel openning.

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Perfect invisibility modes in dielectric nanoparticles

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With the help of the original mathematical method for solving sourceless Maxwell's equations, it is shown that in dielectric waveguides along with usual waveguide and quasi-normal modes, there are perfect invisibility modes or perfect non-scattering modes. In contrast to the usual waveguide modes, at eigenfrequencies of the perfect invisibility modes, light can propagate in free space. The properties of the invisibility modes in waveguides of circular and elliptical cross-sections are analyzed in detail [1]. Fig. 1 shows H_z as a function of the radius for usual quasi-normal (TM_{11}) and perfect invisibility (PTM_{11}) modes.

From Fig. 1 one can see that the perfect invisibility mode decreases at infinity. This fact radically distinguishes it from the usual quasi-normal mode, which increases without limit at infinity. It is shown that at the eigenfrequencies of the perfect invisibility modes, the power of the light scattered from the waveguide tends to zero and the optical fiber becomes invisible.

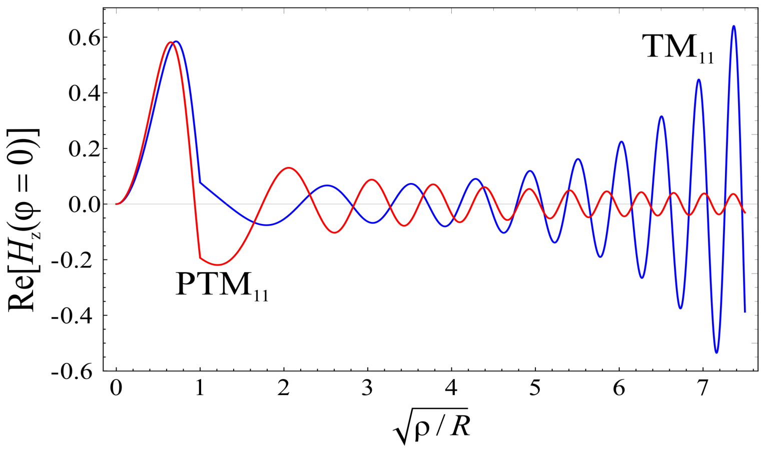

Fig. 1: Dependence of $\mathrm{Re}[H_z(\varphi = 0)]$ on $\sqrt{\rho/R}$ for the perfect invisibility (PTM_{11}, $H_z \neq 0$, $E_z = 0$) mode and for the usual quasi-normal (TM_{11}, $H_z \neq 0$, $E_z = 0$) mode in a circular cylinder with permittivity $\varepsilon = 12$.

Analogous perfect nonradiating modes were also found for nanoparticles of finite volume [2, 3]. The perfect invisibility modes have no analogues and are associated with the fundamentally new physics that is not based on the Sommerfeld radiation condition at infinity. Apparently, the perfect invisibility modes are closest to the Neumann–Wigner strange modes [4]. The found modes can be used to create highly sensitive nanosensors and other optical nanodevices, where radiation and scattering losses should be minimal.

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Electrostatic properties of finite slab of wire medium

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The focus of this study is on electrostatic screening in anisotropic artificial media composed of parallel metal wire arrays, which are investigated both analytically and experimentally. Recent theoretical proposal of spherically symmetrical electrostatic potential distribution [1] was expanded by boundary condition which allowed us to consider finite layer problem. The analytic result confirmed by experimental measurement demonstrate that, despite the finite dimensions of wires, the field distribution symmetry near the charge remains unchanged, but a local maximum is appeared at the boundary. Moreover, we propose an experimental method for plasma frequency extraction from static E-field measurement, since the screening depth of is determined by to distribution.

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Optical alignment of hot excitons in metal-semiconductor nanostructures

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Methods of optical orientation and alignment allow to investigate the fine structure of excitons in semiconductor nanostructures and to estimate exciton lifetime and spin relaxation time [1]. Normally one studies the excitons with small wave vectors $\boldsymbol{K} \approx 0$ by exciting them resonantly with polarized light at frequency ω_0, and measuring the luminescence polarization due to direct optical transitions. Orientation of hot excitons in bulk semiconductors is only attainable through indirect transitions accompanied by the optical phonon emission. However, in semiconductor nanostructures covered with metallic gratings, excitons acquire the wave vector and polarization (spin) of surface plasmon polaritons that propagate along the grating [2].

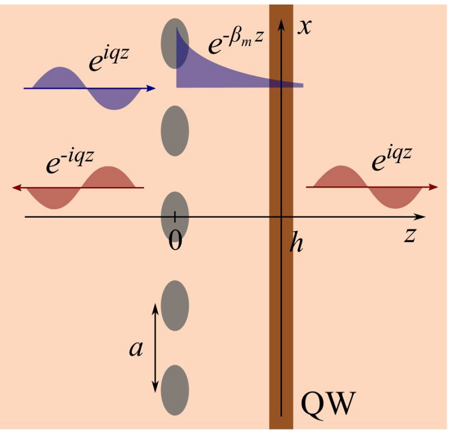

Fig. 1: Schematic of nanostructure including a semiconductor quantum well (QW) and a square short-period lattice of metal nanoparticles.

This paper considers the orientation of excitons in hybrid nanostructure shown in Fig. 1. Incident light propagating along the growth axis z generates in the QW excitons with wave vectors among $\boldsymbol{K} = \boldsymbol{b_m} = 2\pi a^{-1} (m_x \boldsymbol{e_x} + m_y \boldsymbol{e_y})$ which satisfy the resonance condition $\omega \approx \omega_0 + \hbar \boldsymbol{b}_{\boldsymbol{m}}^2/(2M_{exc})$. In case of the short-period lattice $qa < 2\pi$, hot excitons with $\boldsymbol{b}_{\boldsymbol{m}\neq 0}$ are generated by evanescent waves $\boldsymbol{E_m} \exp(i\boldsymbol{b_m}\boldsymbol{\rho} - \beta_{\boldsymbol{m}} z)$, $\beta_{\boldsymbol{m}} = \sqrt{\boldsymbol{b}_{\boldsymbol{m}}^2 - q^2}$. The heavy-hole-like excitons with the following properties are considered: exciton states with the spin $S_z = \pm 1$ are optically active in $\boldsymbol{e}_\pm = (\boldsymbol{e_x} \pm i\boldsymbol{e_y})/\sqrt{2}$ polarization, and split in external magnetic field B_z so that $\hbar\omega_\pm = \hbar\omega(K) \pm g_\parallel \mu_B B_z/2$.

In spite of the difference in polarization of the incident $\boldsymbol{E_0}$ and evanescent waves $\boldsymbol{E_m}$, the resulting luminescense polarization as a function of B_z is similar to the one obtained for the excitons with $\boldsymbol{K} \approx 0$. Thus we demonstrate the applicability of the optical orientation methods to study the excitons with large values of wave vector, generated via the metallic grating.

This research was supported by RSF (project № 22-12-00139).

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Control of light direction by rewritable structures on GST

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Control of the light direction is an actual task of modern nanophotonics [1–3]. Previously, the possibility of light direction control was shown by using liquid crystals [1], metasurfaces [2], and nanoantennas [3]. In this work to solve this problem we propose to use material with phase memory $Ge_2Sb_2Te_5$ (hereinafter GST). This material has non-volatile phase transition, a high optical contrast in the visible area, as well as a short switching time of about 10 ns [4].

In this paper one dimensional photonic crystals with various periods on GST were prepared by partial GST-switching to the crystalline phase. The switching of GST was made by direct laser writing technic. When using this method the region is irradiated by a focused laser beam, heats up to the required temperature and a local phase transition occurs. Thus, the resolution of the resulting structures is limited by the diffraction limit. Next, the reflection spectra with angular resolution of the obtained samples were measured. For this, the spectrometer fiber was attached to the rotating holder, the axis of rotation of which passed through the sample. The resulting spectra are presented in Fig. 1. As can be seen, in the presence of a lattice light spreads in addition angles, which allows us to talk about the control of the light direction. These angles can be controlled by changing the period of structures (see Fig. 1). Thus, when recording lattices with different periods one can directs light to any direction. In addition, the GST properties allow to do it many times and in a short time.

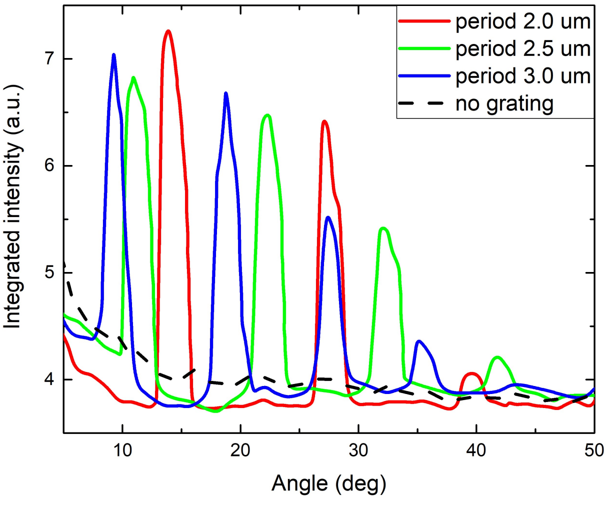

Fig. 1: Angularly resolved reflection spectra of structures with different periods

In this work we have shown the light direction control by periodic structures on a material with a phase transition. It has been shown that by creating lattices with different periods, it is possible to direct light to different angles. In addition, GST properties such as phase stability and a short switching time about 10 ns make it possible to change the direction of light quickly and stably.

The study is funded by Russian Science Foundation (Project № 21-79-10214).

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Anomalous Q-factor scaling laws of resonances related to bound states in the continuum

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We investigate the scaling behavior of resonances associated with bound states in the continuum (BIC) in metasurface consisting of circular rods with holes. We use numerical simulations and analytical calculations to study the dependence of the quality factor (Q-factor) of resonances, supported by different metasurface configurations on dimensionless asymmetry parameter α. The Q-factor scaling laws show both already known dependence $Q \propto \alpha^{-2}$ and completely new $Q \propto \alpha^{-4}$ and less than two power law $Q \propto \alpha^{-1.75}$. Overall, the article provides a comprehensive analysis of the scaling behavior of resonances related to BICs and demonstrates the usefulness of the effective Hamiltonian approach for studying these phenomena. The results have important implications for the design and optimization of various devices, including photonic and acoustic resonators.

On chip refractive index sensing with plasmonic structures

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The determination of refractive index is one of the most important optical methods of substance characterization [1]. The dependence of the position of the surface plasmon resonance peak on the environment refractive index can be observed with an optical fiber. Ease of manufacture, compactness, reliability, immune to electromagnetic radiation, chemically and biologically inertness makes optic fibers an excellent candidate for using them as SPR sensor [2]. Modeling of the signal transmission process of the SPR fiber optic sensor allows selecting optimal parameters and evaluating its sensitivity to the refractive index of the external environment [3–5]. Thus, the aim of the research is to study the sensitivity the optical fiber SPR sensor via modeling.

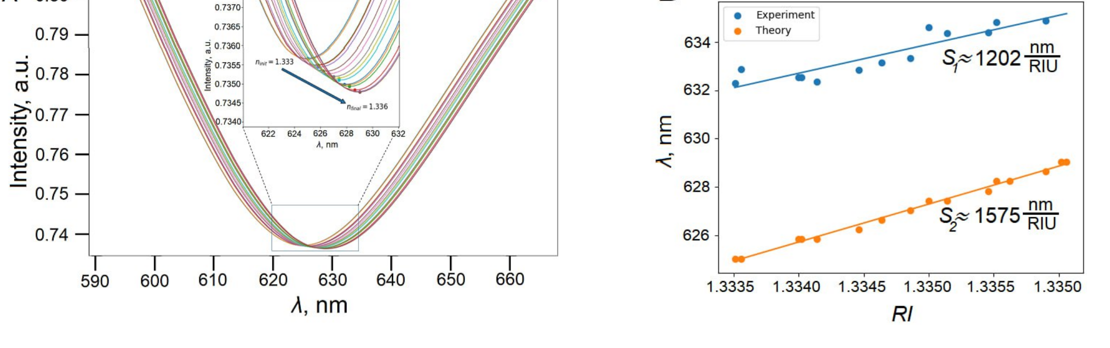

Fig. 1: A) Numerically calculated spectra of transmission through the sensitive element for different refractive indices of the environment (1.3335 – 1.336); B) Comparison of calibration directs of experimental data and theoretically calculated data.

Here, the sensing mechanism of the optical fiber, the influence the thickness of the metal film, as well as the length of sensing area upon the SPR spectrum were modeled. Numerically calculated spectra of transmission through the sensitive element for different refractive indices of the environment (1.3335 – 1.3360) are shown in Fig. 1A. In order to compare theoretical results with experimental ones, the optical fiber SPR sensor was fabricated to measure the effect of refractive index on the surface plasmon resonance peak. The sensing element of the optical fiber SPR sensor is the connection of the single-mode section of the optical fiber to the multi-mode optical fiber. Test solutions with NaCl of different concentrations corresponding to refractive indices from $n_{\mathrm{init}} = 1.3335$ to $n_{\mathrm{final}} = 1.3360$ were prepared and measured. Simulation and experimental results are shown in Fig. 1B. Sensitivity of the optical fiber SPR sensor modelled is 1575 nm/RIU. Sensitivity of the optical fiber SPR sensor fabricated is 1202 nm/RIU. Sum up, a good agreement is observed between the experimental results and the theoretical calculation results.

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The structure of the force field of a relativistic charged particle and its effect on the dynamics of a charged particle beam

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Calculation of the electromagnetic field is an important task of numerical simulations of beam dynamics [1] as well as for application in electron microscopy area [2]. In this paper, the structure of the Coulomb force field of a charged particle is studied in detail, including a three-dimensional directional digram and a vector representation of the Lorentz force.

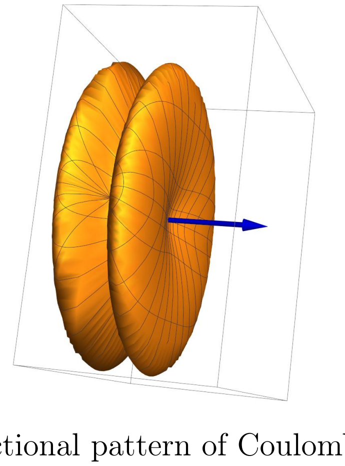

Fig. 1: Directional pattern of Coulomb force field.

Calculations were carried out for different directions of the source and the observation point. For the case of parallel motion of the test charge and the source, it is possible to show a complex structure of the force field, expressed in the presence of a minimum of the force modulus in the direction perpendicular to the velocity of the source. The examples of beam dynamics calculation show its connection with the graphical representation of the field.

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Application of tensor train decomposition to generalized source method for solving the grating diffraction problem

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One of the most efficient methods for modeling diffraction on a grating is Generalized Source Method (GSM). In this method, the unknown electric field in a given dielectric distribution in the presence of an arbitrary source distribution is obtained exactly from the known complete electric field solution in a trivial dielectric structure in the presence of the same source distribution. If we consider finite number of diffraction orders, formulated integral equation can be reduced to a system of linear equations in the form of a product of block-diagonal and block-Toeplitz matrices [1]. If however, the grating has a multi-scale pattern, matrices in such equations become too large to compute. This paper presents a method for modeling scattering on 1D grating with multi-scale patterns.

To simplify GSM computations we used a low-rank matrix approximation called Tensor-Train Decomposition (TT). A wide variety of regularly structured matrices (such as diagonal, block diagonal, Toeplitz, etc.) of size N can be represented in TT-format with $O(\log(N))$ elements [2]. But more importantly, TT-representation of any "smooth" matrix can be found by evaluating $O(\log(N))$ of its elements using TT-cross algorithm [3].

The resulting solution converges given a wide variety of grating patterns. The algorithm runtime grows logarithmically with respect to number of considered diffraction orders.

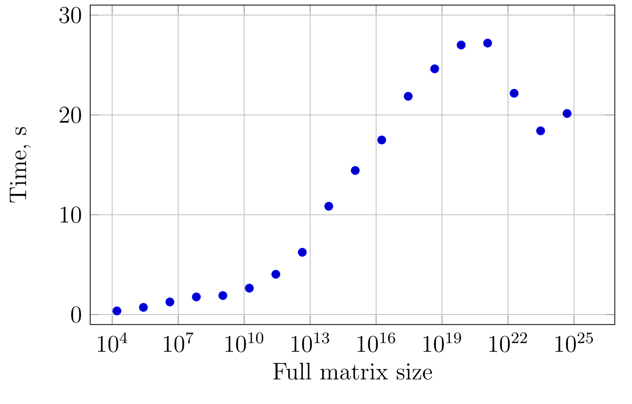

Fig. 1: Algorithm running time.

This method is to be optimized and generalized to 2D case. However, results obtained so far show their efficiency in solving diffraction on multi-scale pattern periodic structures.

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Topological coding metasurface by encircling an exceptional point

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There has been tremendous interest in non-Hermitian physical systems since Bender and Boettcher revealed that the Hamiltonian of non-Hermitian systems with parity-time (PT) symmetry may exhibit real eigenvalues [1]. Much of the interest has focused on exceptional points (EPs), where the eigenvalues and eigenvectors of such PT symmetric system coalesce, in contrast to the Hermitian system. Metasurfaces are artificial materials constructed with subwavelength arrays of spatially distributed nanostructures that enable versatile wavefront engineering [2].

Here, we show how to leverage the 2π-phase excursion that occurs on the coefficient value by encircling an EP. A topological coding metasurface is designed to realize the phase around EP and combine its unique properties with the Pancharatnam–Berry phase (PB phase) to realize the regulation of different polarized waves. The design of the structure and phase distribution of reflection with different polarization are shown in Fig. 1(a), the design and beam regulation of the coding metasurface combining EP topological phase with PB phase are shown in Fig. 1(b).

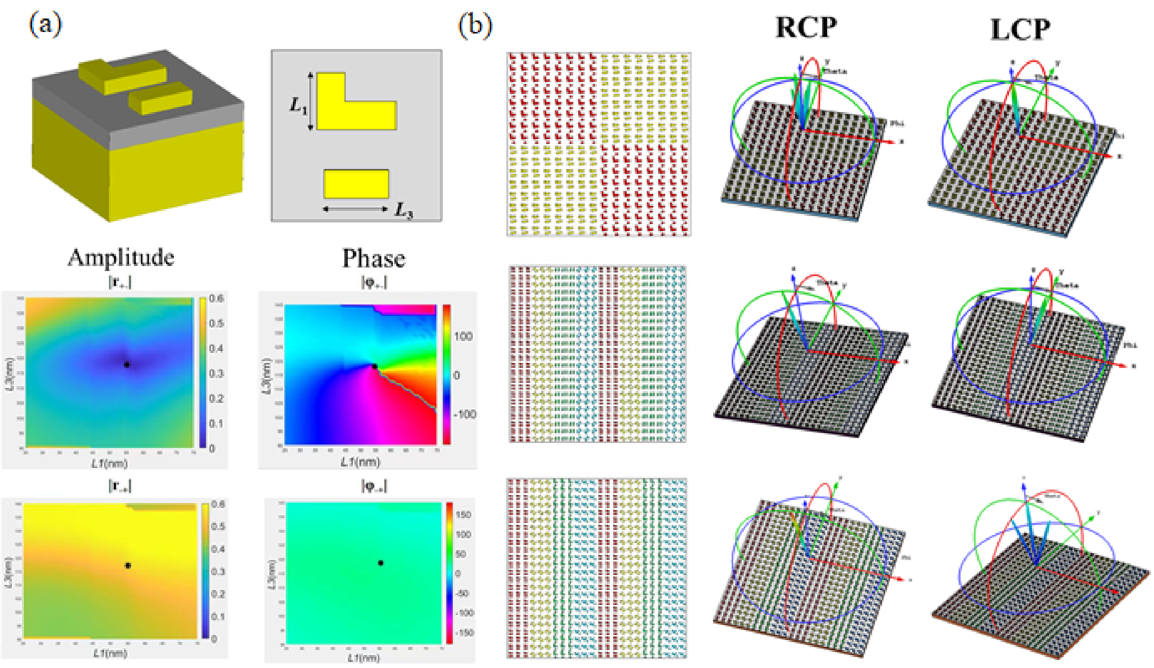

Fig. 1: (a) The design of the structure and phase distribution of reflection with different polarization. (b) The design and beam regulation of the coding metasurface combining EP topological phase with PB phase (EP phase, PB phase, EP and PB phase from top to bottom).

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Tuning of Purcell factor by $Ge_2Sb_2Te_5$

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Nanophotonics has become increasingly popular in recent years for its ability to manipulate light at a subwavelength level, allowing for various applications such as imaging and sensing. As a result, the trend in modern science and technology is towards creating devices that are both miniaturized and multifunctional. Active photonics is particularly noteworthy due to the emergence of halide perovskites as a material of interest [1]. These materials are attractive due to their low cost, tunability, high refractive index, and ease of integration with other platforms [2]. An important characteristic of perovskite photonic sources is their local density of states (LDOS). LDOS measures the Purcell factor, which enhances spontaneous emission in a photonic environment near metamaterials or metal surfaces. The modulation of the Purcell effect is a critical issue for many applications, such as quantum data storage [3]. To adjust the properties of emitted light, one can employ phase change materials (PCMs), which have undergone significant development in recent decades. By transitioning between the crystalline and amorphous phases of a PCM, one can modulate the optical characteristics of the entire structure. The most commonly utilized PCMs are chalcogenide compounds, such as $Ge_2Sb_2Te_5$ (GST), which support nonvolatile phase transitions [4]. This approach offers notable advantages, including fast crystallization, high optical and electrical contrast, and precise control over the transition between the two phases.

We consider a layered structure consisting of a glass substrate, GST, perovskite, and air. The perovskite layer produces spherical waves in both hemispheres as the emitting layer. It is important to note that these spherical waves undergo reflection not only at the air-perovskite and perovskite-GST interfaces but also at the GST-glass interface after transmitting to the GST domain. The air and glass domains are considered to be semi-infinite in this study. The thickness of the GST layer ranges from 10 to 100 nm, while the perovskite thickness ranges from 30 to 70 nm, both of which are experimentally achievable. LDOS is defined as follows:

$$\rho(r,\omega_0) = \frac{2\omega_0}{\pi c^2}\,\mathrm{Im}\left(Tr(G(r,r,\omega_0))\right). \tag{1}$$

Figure 1 shows the average Purcell factor for various perovskites, as a ratio between the crystalline and amorphous GST. Regions with Purcell factor values greater than 1 indicate an enhancement of spontaneous emission when transitioning from the amorphous to crystalline phase of GST. As the wavelength of the photoluminescence increases, the average value of the Purcell factor decreases. The specific behavior of the Purcell factor is due to constructive and destructive interference of Fabri–Perot modes within the structure.

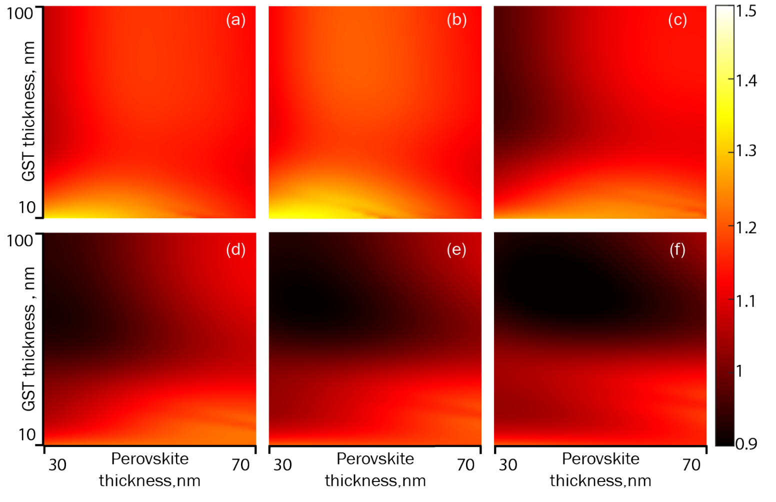

Fig. 1: Average Purcell factor ratio of c-GST to a-GST for different perovskites. (a) (470 nm) (b) (520 nm) (c) (575 nm) (d) (620 nm) (e) (690 nm) (f) (750 nm). Numbers in the brackets represent the central wavelength of photoluminescence.

To sum up, we have shown that it is possible to switch the Purcell effect for perovskite by depositing it on a GST. By controlling the GST's phase, we can significantly modulate the photoluminescence of perovskite. We have calculated the photonic LDOS and the Purcell factor for six different perovskites. Our findings indicate that GST performs well in the short wavelength range of spectra, making it a suitable candidate for developing devices in that range.

The study is funded by Russian Science Foundation (Project № 21-79-10214).

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Thermo-optic bistability with bound states in the continuum

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We consider thermo-optic bistability in a silicon structure supporting bound state in the continuum. Taking into account radiative heat transfer as the major cooling mechanism we constructed a non-linear model describing the optical response. It is shown that the thermo-optic hysteresis can be obtained with low intensities of incident light $I_0 \approx 1$ W/m^2 at the red edge of the visible under the critical coupling condition. This work received financial support through the grant of Russian Science Foundation and Krasnoyarsk Regional Fund of Science № 22-22-20056, `https://rscf.ru/project/22-22-20056/`.

Optimization of liquid crystal metasufaces for versatile diffraction

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We study liquid crystal (LC) metasurfaces self-assembled upon alignment layers treated with focused ion beam. The beam changes the anchoring action of the layers and establishes submicron-scale modulated LC director distributions according to predefined periodic and superperiodic patterns of stripes, rectangles, etc. Setting the width and position of different pattern elements allows controlling the output light phase profile. In this way, LC metasurfaces determined by specific patterns optimized for versatile diffraction of light can be created.

To study diffraction of light on such LC metasurfaces, we start with simple periodic stripe patterns which can be described within semi-analytical theory. The LC director distribution in the metasurface obtained within the one-constant LC elasticity approximation allows calculating the phase of outgoing light in the limit of geometrical optics and to evaluate the transmission and diffraction spectra [1]. To achieve anomalous refraction, when the light beam is redirected into a single diffraction order, the phase profile at the output should be sawtooth-shaped. We optimize the superperiodic pattern to produce such profile and to maximize the efficiency of +1 diffraction order on a wavelength of 450 nm. Numerical modeling in COMSOL Multyphysics of superperiodic LC metasurfaces shows deflection of up to 60% of incident blue light into +1 diffraction order [2]. Using two complementing superperiodic patterns doubles the available depth of optical path modulation and broadens the wavelength range of efficient diffraction. Optimization of such more complex double-sided metasurfaces requires precise solution of elastic and Maxwell's equations. Calculated spectra show 75% efficient +1 order diffraction of red light and up to 60% efficient +2 order diffraction of blue light [3]. To design LC metasurface analogs of Dammann gratings, in a similar way, we optimize the widths, positions and number of stripes in a periodic pattern to minimize the difference between diffraction efficiencies of many orders and obtain metasurfaces splitting blue light uniformly into 11 diffraction orders [4].

All such LC metasurfaces were fabricated and their transmission and diffraction spectra showed good agreement with the simulated. Applying alternating voltage of 3–8 V amplitudes across the metasurfaces switches them from diffracting to transmitting state within several milliseconds [5]. Applying lower voltage of a 1.5 V amplitude across the double-sided metasurfaces switches the diffraction of blue light between +2 and +1 orders and switches off the diffraction of red light into +1 order [6].

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Realizing multipolar higher-order topology in a multimode photonic waveguide lattice

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Topological phases of matter have attracted wide attention due to resilience of the associated topological boundary states to defects and fabrication imperfections. Recently, higher-order topological insulators (HOTIs) with corner and hinge states have been discovered in two- and three-dimensional systems. While most HOTIs could be classified via symmetry indicators of the Bloch states and are characterized by the effective bulk polarization associated with the filled bands [1], some insulators are characterized by the higher-order moments of the effective probability density. To date, the latter includes only two main types: the quadrupolar two-dimensional and octupolar three-dimensional HOTIs [2], based on square and cubic lattices, respectively. In this work, we uncover the new case of a multipolar HOTI realized in a C_3-symmetric undistorted photonic waveguide kagome lattice with two degenerate modes at each site. Having the distinct C_∞ and C_3 rotation symmetries, the two modes create a complex pattern of interactions which opens the topological bandgap. Via analytical, numerical and experimental studies, we find that the finite hexagonal sample of such lattice hosts topological corner modes which could be excited selectively. Importantly, the characteristic distribution of effective corner charges and expectation values of multipole moments suggest the multipolar origin of our system, characterized by quantized values of the two-dimensional octupole moment. Our findings give the first example of such an exotic topological phase beyond the conventional classification [1, 2], and show a new route to tailor topological phases by orbital hybridization in simple lattices without geometric distortions.

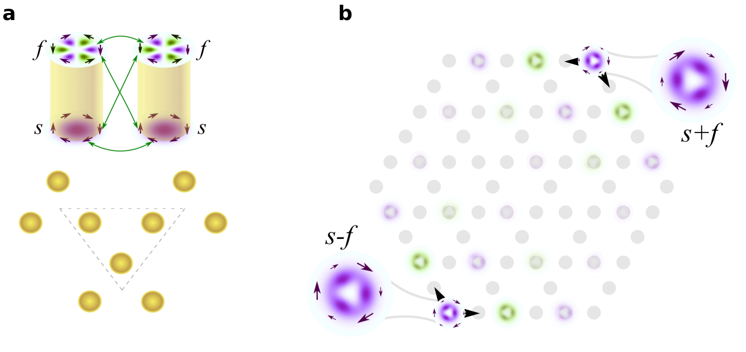

Fig. 1: (a) Model of the two-mode waveguide lattice highlighting the unit cell and mode interactions. (b) Sketch of the profiles of the hybrid corner states in a hexagonal sample of the lattice.

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High-Q band-edge resonances in one-dimensional arrays of dipolar scatterers

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In this work, we study dipolar oscillations in the finite one-dimensional arrays of scatterers supporting dipole resonances. Such model is relevant for description of the arrays of cold atoms as well as for the arrays of particles with purely dipole response. Due to the interference of radiation from individual dipoles, arrays of dipoles support collective resonant oscillations, characterized by suppressed radiative losses and consequently high quality factors, scaling as $Q \propto N^3$, where N is the number of the dipoles in the array [1, 2]. Such collective modes are interesting for their ability to store quantum information in the context of quantum optics [2], as well as for the possibility of significant increase of light-matter interaction strength in arrays of optical nanoresonators [3, 4].

Recently, it was shown theoretically that coupling between two or more collective modes via radiation continuum that occurs under variation of the parameters of such structures can significantly boost the quality factor of one of the modes [5]. The underlying physics is based on interference between the band-edge collective mode and a standing mode in the array, which occurs for a certain critical value of the period $a_{\mathrm{cr}} \approx 0.2\lambda_{\mathrm{r}}$ (where λ_{r} is the resonant wavelength of a single dipole), when the dispersion of an infinite array becomes non-monotonic. Such qualitative modification of the dispersion substantially affects the properties of the eigenmodes of the finite array, leading to the formation of spatially localized states with dramatically suppressed radiative losses with quality factors scaling as $Q \propto N^{\alpha}$, where $\alpha \approx 7$.

When considering an array of optical dielectric nanoresonators, the approximation of a single-dipole response usually fails, and one has to take into account at least two types of response of an individual scatterer simultaneously, which are electric dipole (ED) and magnetic dipole (MD) responses in the most simple and relevant case. At the same time, such a small critical period $a_{\mathrm{cr}} \approx 0.2\lambda_{\mathrm{r}}$ obtained for a chain of EDs is practically impossible to achieve due to technological restrictions on the precise fabrication of the chain of the nanoparticles. Hence, the single-type dipole approximation fails when considering optical dielectric nanostructures and it is important to study the effect of interference between different modes in a single scatterer and in a chain on the characteristics of the band-edge resonances. To this end, we have constructed a coupled-dipole model with both ED and MD responses in each scatterer. The MD polarizability was determined only by the resonant term, while the ED one also included quasistatic response.

Using the dipole model, we have revealed, that the presence of both electric and magnetic dipole responses in a particle leads to the increase of the critical period, i.e. when the dispersion changes its behaviour from monotonic to non-monotonic, to the realistic values $a_{\mathrm{cr}} \gtrsim 0.3\lambda_{\mathrm{r}}$. This can be achieved by proper adjustment of the spectral separation between ED and MD resonant frequencies. For realistic dielectric particles this can be done by changing the size and/or the shape of the particles [6]. Calculations for the finite chains showed that they possess eigenmodes with the quality factors also scaling as $Q \propto N^7$, as in the model with a single-dipole type response, which indicates that the responsible mechanism for such increase of the Q-factor is the interaction between two collective modes via radiation continuum. Finally, we have calculated and experimentally measured the properties of the ceramic particles in the microwave frequency range, which confirmed the main theoretical findings.

We believe that our findings will enrich the existing approaches for design of the compact optical nanostructures supporting high-Q states.

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About efficiency of piezoelectric transducers from porous piezoceramics when operating at electrical resonance and antiresonance frequencies

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As is well known, the efficiency of generating acoustic waves increases if the transducer operates in the vicinity of its resonant frequency. Due to the piezoelectric effect, piezoelectric transducers can emit acoustic waves into the external medium when electrical influences are applied to its electroded surfaces. In the mode of harmonic oscillations with a circular frequency ω, such influences can be either a potential difference $\Delta V \exp(j\omega t)$, or an electric current $I \exp(j\omega t)$ (charge $Q = \pm j\omega^{-1} I$), or both of these quantities, interconnected by a complex resistance or electrical impedance $Z = \Delta V / I$ (electrical admittance $Y = Z^{-1}$). In this regard, the resonance frequencies of a piezoelectric transducer can be divided into electrical resonance frequencies ω_{rk} and electrical antiresonance frequencies ω_{ak}. At electrical resonance frequencies in the absence of losses, classical resonant phenomena are observed for current and admittance

$$|I| \to \infty, \quad |Y(\omega)| \to \infty, \quad |Z(\omega)| \to 0, \quad \omega \to \omega_{rk},$$

and at electrical antiresonance frequencies, resonant phenomena are observed for potential difference and impedance

$$|\Delta V| \to \infty, \quad |Z(\omega)| \to \infty, \quad |Y(\omega)| \to 0, \quad \omega \to \omega_{ak}.$$

In turn, the resonant frequencies and the efficiency of electromechanical energy conversion as a whole depend on the geometry of the transducer, the values of material moduli, electromechanical coupling coefficients and quality factor (figures of merit). There are quite a lot of such parameters, but for canonical transducers operating on pronounced one-dimensional vibration modes, individual characteristics are important, expressed through the parameters listed above. Naturally, these parameters differ for the frequencies of electrical resonances and antiresonances.

This paper presents the results of previous ([1, 2], and others) and new studies of several specific piezoelectric transducers made of porous piezoceramics. The results were compared for transducers made of solid piezoceramics, conventional porous piezoceramics, and porous piezoceramics with metallized pore surfaces. The results showed significant differences in the efficiency of the considered types of piezoelectric transducers at the frequencies of electrical resonances and antiresonances when oscillations are excited by a potential difference or current, respectively. Note that vibrations of transducers at antiresonance frequencies when excited by an electric current are studied much less frequently than vibrations at resonance frequencies when excited by a potential difference. As a result, some types of efficient converters and their applications may be overlooked.

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Finite element investigation of disk transducers from porous piezoceramics of complex structure with cymbal end-caps in external medium

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Following [1], in this paper we consider piezoelectric transducers in the form of a disk with two metal cymbal end-caps, which are usually called cymbal transducers. Studies by other authors have shown that the efficiency of this device is determined both by its geometry and by the values of the piezoelectric charge coefficients d_{31} and d_{33} of the disc material. In this regard, various types of porous piezoceramics, which have unusual dependences of effective piezoelectric moduli on porosity, were used as the active material of the transducer. Namely, dense piezoceramics, ordinary porous piezoceramics, porous piezoceramics with a very thin metal coating of the pores, and porous piezoceramics with a relatively thick metallization layers on the surfaces of the pores at different porosities were considered. A similar investigation was presented earlier in [2] for a cymbal transducer, but only as a piezoelectric emitter. However, in [2], other material data with inaccuracies were used for porous piezoceramics with metallized pore surfaces.

Studies in the ANSYS finite element package were carried out in the steady state oscillation mode for two types of cymbal transducers. The first transducer was considered as a renewable energy piezoelectric generator and generated electric fields in a piezoceramic disk under low-frequency mechanical influences. The use of porous ceramics for this transducer showed lower electromechanical conversion coefficients compared to dense ones, but in cases of porous ceramics with metallization of the pore surfaces, better results were obtained, especially at a sufficiently high porosity and with a thin metal coating of pores.

The second type of cymbal transducer was considered as emitter of acoustic waves. It had more compliance metal cymbal end-caps and generated mechanical vibrations under electrical influences near the first resonant frequencies. In this case, when the transducer was excited by a potential difference at the first frequencies of electrical resonances, qualitatively similar results were obtained for the use of porous piezoceramics as for the cymbal piezoelectric generator. However, when the transducer was excited by an electric current at the first frequencies of electric antiresonances, the best results were obtained for ordinary porous piezoceramics.

In the development of these studies, the same transducers operating in external elastic and acoustic media were studied. As a result, the analysis showed the efficiency of using the materials of the considered porous piezoceramics both for “green energy” converters and for piezoelectric emitters of acoustic waves.

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Transformation of bound states in the continuum due to permittivity changing in one-dimensional periodic structure of dielectric rods

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Bound states in the continuum (BIC) can be considered as a resonant state with infinite large Q factor in an open system due to their huge radiative lifetime. As shown at [2] BICs in photonic slabs are vortex centers of the far-field polarization in parametric space. The vortex is described by a topological charge, which ensures the robustness of BICs against changes on optical parameters and geometry of the structure [1]. In [2] the authors investigated the BICs evolution in a 1D periodic structure of infinite rods due to the change in their radius. In our work, we studied the migration of BICs due to the change in permittivity of rods with a fixed radius $R = 0.439a$, where $a = 0.5$ µm is a period of structure. We also estimated the affect of Kerr nonlinearity on BICs migration. The simulations were implemented in the COMSOL Multiphysics Software.

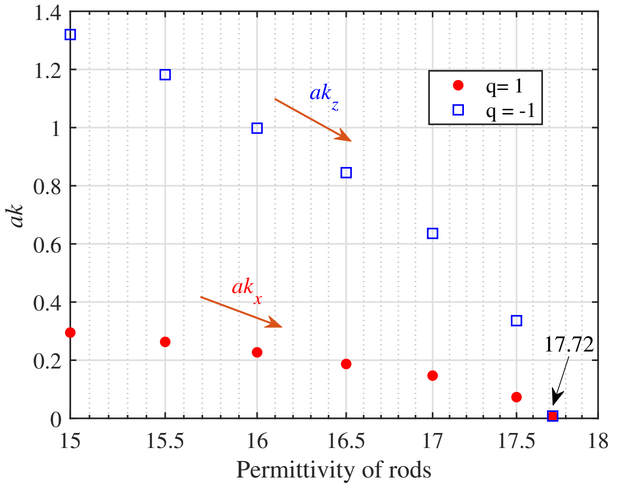

Fig. 1: Dependence of BIC's position at parametric space ak on permittivity of rods. Orange arrows show the direction of BICs migration with topological charge q.

We found that BICs migrate to the center of the Brillouin zone and annihilate when the rods permittivity increases from 15 to 17.72. After annihilation, a tunable (accidental) BIC with a zero topological charge appears at Γ point. Our results provide guidelines on experimental observation of the BICs transformation and appearance of merged BIC due to the nonlinear Kerr effect.

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Surface roughness simulations for 6G additive antennas

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The additive technologies show promising results in various applications, especially in manufacturing of antennas for multiple applications such as 6G antennas working in W-band (75–110 GHz) and higher. Additive technologies in a nutshell are based on 3d printing of an antenna frame and its further selective metallization. Additive manufacturing provides great flexibility and simplicity compared to conventional techniques such as CNC milling, whereas accuracy of these methods is comparable.

However, additive manufacturing is too sensitive and the resulting surface roughness, i.e., accuracy or quality of the manufacturing device strongly depends on many crucial factors. Among these factors are the quality of 3d printing itself, time of selective metallization, type of used metallic power and many other.

In this work we perform electromagnetic simulations to study an impact of surface roughness to the performance of some conventional antennas suitable for 6G applications. We investigated three devices such as rectangular waveguide, wide-band horn antenna and narrow-band antenna working in W-band. As a result, we show comparison of S-parameters, directivity patterns of the antennas with and without surface roughness. Finally, we could define the surface roughness requirements for the additive manufacturing process of 6G antennas.

Engineering the radiative lifetime of excitons in two-dimensional van der Waals heterostructures

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Engineering of spontaneous emission of excitons using resonant structures, based on changing the local density of optical states, has been demonstrated in various atomic and solid-state systems [1]. In recent years, researchers have actively studied the properties of excitons in ultra-thin semiconductors, monolayers of transition metal dichalcogenides (TMDs), in which optical transitions are dominated by spontaneous emission mechanisms [2]. Recent work demonstrates an increase in the speed of spontaneous emission of excitons by several orders of magnitude through the integration of TMD monolayers with resonant photonic structures, providing local enhancement of the electric field around excitonic centers [3]. The reverse effect, based on the suppression of spontaneous emission of excitons using nano-structured optical resonators, is of particular interest, especially for explaining the complex dynamics of relaxation and recombination of excitons in TMD monolayers [4]. However, this effect is currently not well studied.

In this work, we theoretically and numerically investigate the increase in radiative lifetime of excitons in two-dimensional TMD monolayers integrated with dielectric photonic structures, through changes in the relative position of the monolayer and modification of the local density of optical states. An analytical model of weak coupling between excitons in monolayers of DPM and leaky modes of a dielectric waveguide has been developed. An analytical expression in closed-form for the Purcell factor for excitons is obtained. It is shown that by placing an exciton in the local minima of the resonance mode field, it is possible to increase its radiative lifetime by a factor of n^2 compared to the radiative lifetime of an exciton in a TMD monolayer placed in air, where n is the refractive index of the waveguide material. The maximum lifetime is limited by the radiation losses of the waveguide leaky modes. It is also shown that, by extending the model to interactions with a periodic metasurface, the exciton lifetime can exceed the n^2 limit obtained for a for a waveguide by several orders of magnitude due to the complex structure of the local density of states of the metasurface. The developed theory can explain the results of recent experiments [4].

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Analytical calculation of SPP generation with structured substrates

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Plasmonics is a rapidly growing field with significant contributions to scientific areas such as biosensing [1] and Raman spectroscopy [2]. One of the challenges in plasmonics is the efficient generation of surface plasmon polaritons (SPPs). Methods used to generate SPPs on flat metal surfaces have limited ability to control and manipulate the properties of SPPs [3]. Structured substrates have emerged as a promising approach to overcome these limitations.

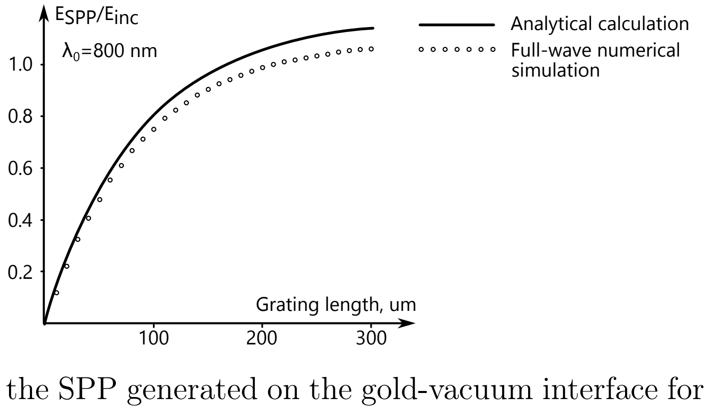

Fig. 1: The amplitude of the SPP generated on the gold-vacuum interface for different values of the grating length. The period matches the SPP wavelength, incident wavelength is 800 nm, and grating amplitude is 10 nm.

Several analytical and numerical methods have been proposed to model the excitation of SPPs on structured substrates [3]. However, these methods often involve solving complex equations that require significant computational resources or sophisticated analytical techniques, limiting their practical applicability. Therefore, there is a need for simpler methods to model SPP generation.

In this work, we propose a simple and efficient approach that utilizes mode decomposition and the Lorentz reciprocity theorem [4] to describe the generation of SPPs on structured substrates. The proposed technique has the potential to contribute to the development of new SPP-based devices.

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Temporal coupled mode theory for hybrid metal-dielectric resonator

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The three-channel scattering problem for the metal-dielectric resonator has been solved in the framework of the temporal coupled mode theory (TCMT) [1].

The obtained results qualitatively describe the resonant lines behavior in the experimental spectra of the photonic crystal/liquid crystal/metal layered microcavity.

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Functional magnonic interconnection: spin-wave coupling and broken translational symmetry in magnonic structures

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Recently magnetic micro- and nanostructures are extensively studied due to their potentials as candidates for future magnonic devices and beyond-CMOS technologies, as they are free of Joule heating and, respectively, free of power loss associated with traditional electronics. One of the interesting class of structures from applicational point of view is the magnonic structures with broken translation symmetry. Based on these structures the 3D magnonic junctions and meander shaped magnonic waveguides are considered as promising candidates to the functional magnonic units. Spin waves (SW) propagating in such structures are considered as signal carriers for information processing. In recent years much research has been directed towards the ways of the control of SW for signal processing at microwave and subterahertz frequencies. Here we report the micromagnetic study of magnonic structures with broken translation symmetry in the form of L-shaped magnonic block. The peculiar features of spin-wave transmission along the magnonic corner in 3D form manifest itself in the regimes of wavelength conversion and transformation of the type of magnetostatic spin wave from backward volume to surface and vice versa. Next, to model the magnonic band structure of the 3D magnonic crystals the micromagnetic simulations were performed and spin-wave dispersion was then obtained by performing a 2D Fast Discrete Fourier Transform. The comparison between measured and calculated dispersion relations for the CoFeB/Ta/NiFe and CoFeB meander structures, as well as the dispersion of an equivalent planar CoFeB/Ta/NiFe magnetic bilayer was provided. The work was supported by Russian Science Foundation (Project № 23-79-30027).

Longitudinal electromagnetic waves supported by interlaced wire medium in wide frequency range

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In current work, we study a particular metamaterial called an interlaced wire medium, which is composed of two cubic lattices of connected metallic wires. It is demonstrated that this metamaterial can support longitudinal waves with an exceptionally wide frequency band, ranging from low to Bragg resonance frequencies. These waves possess very short wavelengths comparable to the material's period. These findings emphasize the spatially dispersive response of the interlaced wire medium, which can generate electromagnetic fields with strong spatial variation. Moreover, we investigate the polarization of electromagnetic waves, which are typically transverse in most materials and in vacuum. Longitudinal electromagnetic waves can exist under specific conditions in certain media, such as plasma.

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Optical properties of twisted photonic crystal lattices

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In recent years, interest in the optical properties of twisted stacks of two-dimensional materials and metasurfaces growths fast [1, 2]. In studies, to predict properties it has been proposed either to apply effective medium approximation or to build a transfer/scattering matrix approach for the main

(0th) diffraction channel, which imposes serious restrictions on the geometry of structures [3]. Twisted photonic crystals lattices also demonstrate interesting properties and operation regimes [4, 5] and require more accurate computational methods. Here, we propose a careful adaptation of the Fourier modal method for the description of twisted 1D gratings' stacks that considerate also higher-order harmonics hybridization [6]. The developed method accelerates the computations from 1 up to 3 orders of magnitude. We use this method for the rigorous study of photonic crystals and demonstrate specific moiré-associated effects.

We show that moiré-periodicity in bilayer dielectric photonic crystal slabs leads to arise of extremely narrow optical resonances, which are very sensitive to the relative twist. The resonant frequency might be tuned in a wide wavelength range simultaneously with control of the quality factor [7]. Also, we demonstrate the change of plasmonic mode dispersion in a stack of gold lattices similar to photonic topological transition in two-dimensional hyperbolic materials [1]. Additionally, we demonstrate the chiral properties of twisted diamond photonic crystal slabs. And finally, we consider the twist effect on the near-field radiative heat transfer and Casimir force in stacks of the photonic crystal slabs.

Our results pave the wave for the utilization of moiré-assisted effects in multilayer photonic crystal structures.

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Electromagnetic theory of SERS effect of molecules encapsuled in spherical metal shell

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An electrodynamic model of the radiation of molecules placed in a metal shell has been developed. The model qualitatively describes the Surface-enhanced Raman Scattering (SERS) signal from protein globules coated with a thin silver film. The theory makes it possible to calculate the change in the SERS signal depending on the thickness of the metal nanolayer over the protein globule. The radiating molecular dipole interacts with the metal shell and excites surface plasmons. Plasmon oscillations reach a maximum when the dipole frequency is close to the plasmon resonance of the metal sheath, and the dipole itself is located near the plasmon sheath. The effective dipole of the collective electronic vibration can be much larger than the molecular dipole, which is amplified by itself due to the classical SERS effect. Therefore, the amplification of radiation due to the metal shell is multiplied by the SERS amplification factor, which is due to the direct interaction of the molecular dipole with the metal surface. Thus, the radiation is greatly enhanced. In calculations, the emitting dipole is located on the surface of a dielectric hemisphere with a refractive index of 1.4, located on a smooth optically opaque silver layer and covered with a thin silver layer. The distance from the dipole to the silver shell is chosen as a minimum of 1 nm. The dependences of the radiation gain of a dipole in a silver shell on the thickness of the shell at a wavelength of 852 nm were calculated, which correspond to the most characteristic spectral lines of amino acids with a Stokes shift of 1000 cm^{-1} (Fig. 1) (it is assumed that the pump wavelength is 785 nm).

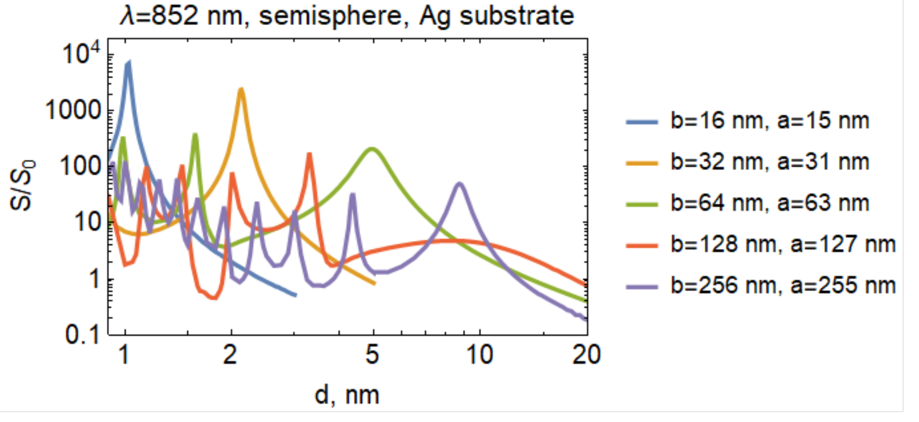

Fig. 1: Enhancement of the radiation S/S_0 of a dipole located near the silver shell at a distance of 1 nm depending on the thickness of the silver shell d for different hemisphere radii b.

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Planar SERS sensors for SARS-CoV-2 virus detection

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Surface enhanced Raman scattering (SERS) spectra from the receptor-binding domain (RBD) of the S-glycoprotein of the SARS-CoV-2 virus were recorded without the use of any molecular markers. The ability to measure the characteristic spectra of viral proteins at concentrations orders of magnitude lower than those required for virus detection in biological media makes it possible to develop a highly efficient optical method for a pathogen detection.

We have developed an analytical theory and carried out the experimental observation of plasmons that are excited in open resonators formed by a periodic metal film. Under resonance conditions, it is possible to achieve an increase in the electromagnetic field by many orders on magnitude, which increases the sensitivity of SERS method and other surface-enhanced spectroscopy. The modulated silver film is produced by four-beam photolithography. The technological process is similar to the production method of optical holograms. Holographic SERS substrates are easy to manufacture, have a low cost, and are suitable for mass production.

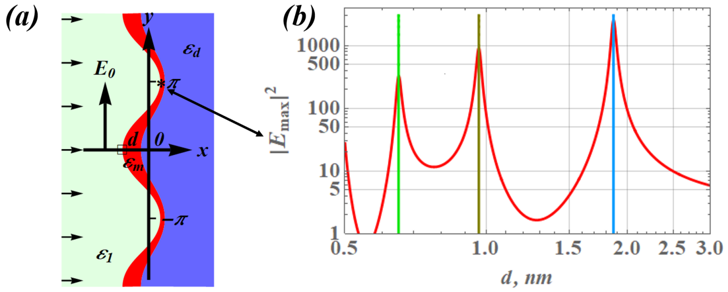

Fig. 1: (a) Modulated metal film (red) deposited on modulated photoresist; (b) local electric field at surface of silver film as function of film thickness d at wavelength $\lambda = 785$ nm, period $L = 50$ nm, modulation $h = 11$ nm, permittivity of photoresist $\varepsilon_d = 2$.

We investigate metal nanofilm modulated along y direction as it is shown in Fig. 1a. Electromagnetic wave is incident along x direction normal to the film. At the resonance surface plasmons are excited in the film and local electric field archives giant value. We have found exactly solvable model of the modulated film. It is convenient to introduce the complex variable $z = (x + iy)/L$, where L is the period of modulation. The front surface z_f of the metal film (left red surface in Fig. 1a) is determined by the parametric equation $z_f(\phi) = \ln[\exp(i\phi) - x_0]$ where the parameter x_0 is in the range $0 < x_0 < 1$; the variable ϕ changes within the limits $-\infty < \phi < \infty$. The rear surface of the metal film is determined by the equation: $z_b(\phi) = \ln[(1 + d_0)\exp(i\phi) - x_0]$. The film period L, modulation amplitude h and film thickness d determine the parameters $x_0 = \tanh(\pi h/L)$ and $d_0 = 2[\exp(2\pi d/L) - 1]/[\exp(2\pi h/L) + 1]$. Our model imitate the metal deposition process when metal is concentrated on the protruding parts of the dielectric substrate. To find the optical electric field we introduce conformal transformation from the complex variable z to $w = \exp(z) + x_0$. Under the conformal transformation the entire film is rolled up into a metal cylinder, which can be solved exactly [1].

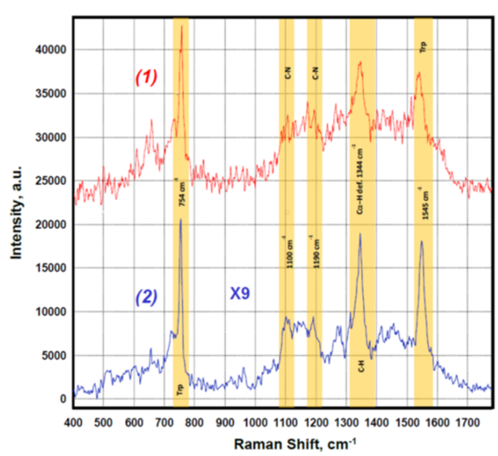

Fig. 2: (2) SERS spectrum of agglomerate RBD protein; (1) the same agglomerate but covered by additional 10 nm silver layer.

We demonstrated that the concentration of light in an agglomerate of SARS-CoV-2 virus S-glycoprotein receptor-binding domain (RBD) molecules on the silver surface makes it possible to obtain characteristic Raman spectra of proteins at concentrations sufficient for ultrasensitive detection of antigens of viral protein at physiologically relevant (sub picogram) levels. Agglomerates of RBD molecules are formed by the interaction of reduced sulfhydryl groups of RBD with silver. The increase in the quality factor of the resonator when coating RBD agglomerates with a silver shell of nanometer thickness leads to an additional amplification of the signal of the SERS protein by an order (orders) on magnitude, thereby providing sub-femptogram sensitivity of viral antigen detection. In Fig. 2 SERS spectrum of the RBD protein is shown. One can see the spectral lines of the amino acid tryptophan 761, 1330, and 1561 cm^{-1}, which is a component of RBR. The additional silver layer led to an almost tenfold increase in the SERS signal [2].

This work was supported by grant from Russian Science Foundation № 23-19-00788 (SERS measurements).

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Formation of super bound state in the continuum due to avoided crossing in grating

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Bound states in the radiation continuum (BICs) are featured by infinite quality Q-factor in photonic crystals (PhC) [1]. However, in practice, the Q of BICs is restricted by finite number of periods N of PhC. Thereby it is important to realize sufficiently large Q-factor with N before its saturation due to material losses and structural fluctuations [2]. There is a way to enhance Q-factor by avoided crossing of resonances (ACR) and merging multiple bound states in a continuum [3]. We show a significant boosting of the Q factor due to ACR of two symmetry-protected quasi-BICs in parametric space and merging BICs in momentum space simultaneously. The analytical theory predicts a transition of asymptotic behavior of the Q factor over the numbers of resonators from N^2 to exclusive N^3 for SP-BICs. The numerical experiment confirms the result of the theory.

Our consideration of Q factor of finite chain is based on separation of radiation leakages. The system under consideration is periodical array of silicon rods of rectangular cross-section with unaltered period L is leaking into radiation space owing radiation from the ends of array which has an universal dependence of $1/Q_{end} \sim 1/N^3$, and radiation from the surface $1/Q_{side} \sim 1/N^2$. Because of that the end losses do not appear because the surface contribution dominates over the $1/Q_{end}$ in relation $1/Q = 1/Q_{end} + 1/Q_{side}$. However the surface radiation can be suppressed by ACR for variation of cross-section of rods that alters to $1/Q_{aside} \sim 1/N^4$ or $1/N^6$. Then the end leakage plays main role in the radiation. The modal theory, like CMT, shows the change of the law from $imag(k0) \sim kx^2$ to $imag(k0) \sim kx^6$ at the merging point. Hence, we conclude that at merging we have extremely strong suppression of surface aside radiation to give rise prevailing of the end radiation with $1/Q_{end} \sim 1/N^3$.

The research was financially supported by the Russian Science Foundation (project № 22-12-00070).

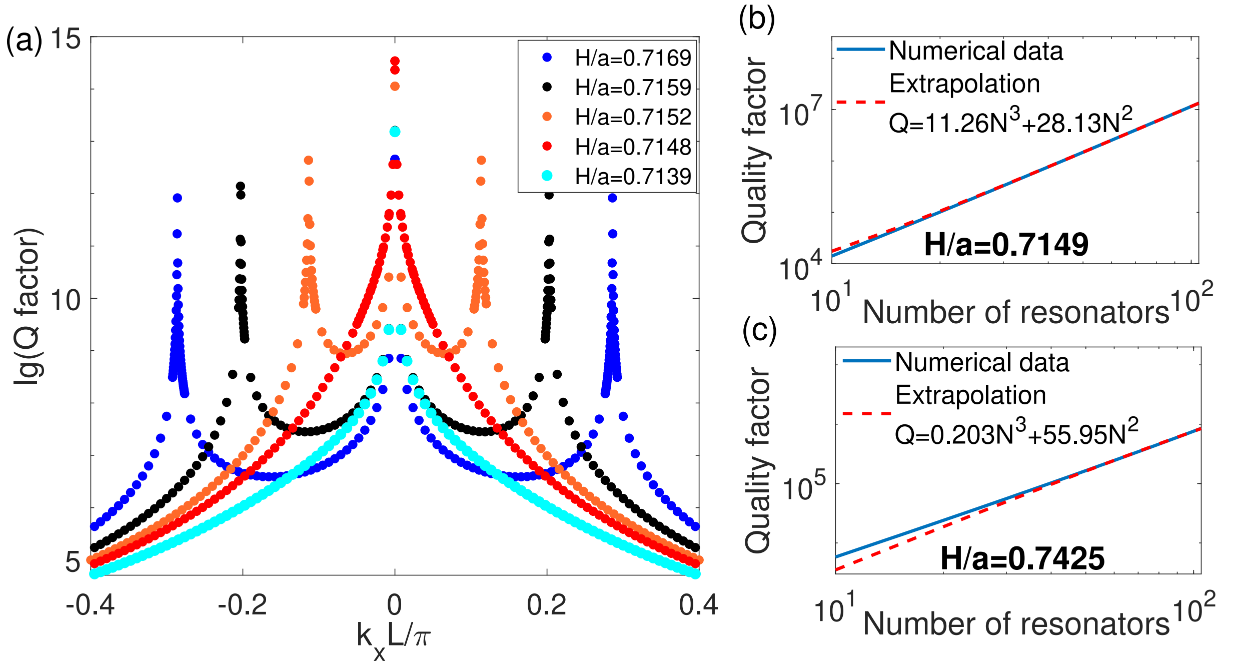

Fig. 1: (a) The Q factors of the merging BICs. The width of the cross section of the rod is D/L=0.75, the height (H/a) varies. (b), (c) Q factors as a function of the number (N) of resonators at merging and before merging.

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Implementing axion electrodynamics equations in metamaterials

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Axions are hypothetical particles that address several problems in modern physics at once. In field theory, they solve problems of CP invariance [1, 2]. At the same time, in cosmology, they are a promising candidate for the role of dark matter particles [3].

Until now, axions have not been detected, but there are real solid-state structures in which the propagation of light is described by the equations of axion electrodynamics. In the simplest case, the effective axion field in such structures is constant in time and space and they are called Tellegen media, while in condensed matter those materials are known as magnetoelectrics [4]. Structures with an effective axion response provide a convenient platform for studying the effects of axion electrodynamics.

However, the use of natural materials alone severely limits the research capabilities, while the use of artificially structured media can enable flexible control of the effective axion response. To achieve this functionality, we developed a composite photonic structure, a metamaterial with an effective isotropic Tellegen response, its schematic picture depicted in Fig. 1. The metamaterial is a one-dimensional photonic crystal made of gyrotropic material, in which the magnetization is periodically modulated so that the average magnetization of the entire structure equals zero. Hence, material parameters of the structure reads:

$$\hat{\varepsilon} = \begin{pmatrix} \varepsilon & i\,g(z) & 0 \\ -i\,g(z) & \varepsilon & 0 \\ 0 & 0 & \varepsilon \end{pmatrix}, \quad g(z) = \sum_{n\neq 0} g_n\, e^{inbz} \tag{1}$$

The structure of this metamaterial makes it possible to create gradients of the effective axion response, as well as to fine-tune its values.

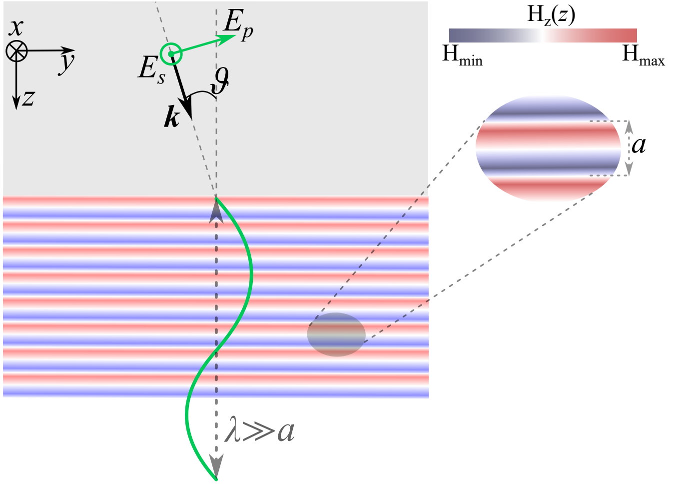

Fig. 1: Schematic of the designed multilayered structure composed of gyrotropic layers with spatially varying out-of-plane magnetization. The average magnetization of the structure vanishes.

In the theoretical analysis of the electromagnetic properties of the metamaterial [5, 6] the effective axion response was obtained:

$$\chi = -\frac{iq}{b} \sum_{n\neq 0} \frac{g_n}{n}\,. \tag{2}$$

We examine dependence of the effective axion response on the nature of the magnetization distribution within a unit cell. The magnetization distribution that maximizes the effective axion response was found.

We verify the correctness of the obtained results, by numerical simulations of the reflection of a planar electromagnetic medium from a metamaterial and an isotropic medium with an axion response. The results confirm the presence of an effective axion response of the structure.

Thus, an original photonic structure with a tunable axion response was developed. This structure also allows for spatial modulation of the effective axion response. A theoretical analysis of the structure was made and numerical simulations were performed, which confirmed the theoretical results. Details of this analysis can be found in our preprint [7].

In the future, the experimental realization of this structure is planned as well as the study of the possibility to create not only spatial gradients, but also temporal modulations. In this case, it will be possible to create an arbitrary distribution of the effective axion field in the proposed structure.

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Two approaches in defining topological charge of bound states in the continuum in multipolar lattices

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Bound states in the continuum (BICs) are the eigenstates of open physical system. Optical BICs refer to electromagnetic waves that are confined to a photonic structure, such as a waveguide or resonator, but have the same energy as waves that can propagate freely in space. Infinite Q-factor of these states are extremely valuable in engineering high-Q resonators and have potential applications in areas such as optical sensing, integrated photonics, nanophotonics and energy harvesting. In 2014 it was shown that both symmetry-protected and accidental BICs are vortex centers in the polarization direction of far-field radiation [1]. Later in 2017 it was suggested to associate BICs with topological singularity of the quasimode coupling strength [2]. In both approaches, the definition of the topological charge is reduced to calculation of the winding number of such vortexes.

In this work we numerically calculated polarization map in the far-field radiation and quasimode coupling strength for the lattice consisting of single resonant z-oriented magnetic octopoles. The topological charges of symmetry-protected BICs, obtained by the two approaches, are the same (Fig. 1). However, approach based on quasimode coupling strength shows additional singularities, which allows to get a complete view of scattered field.

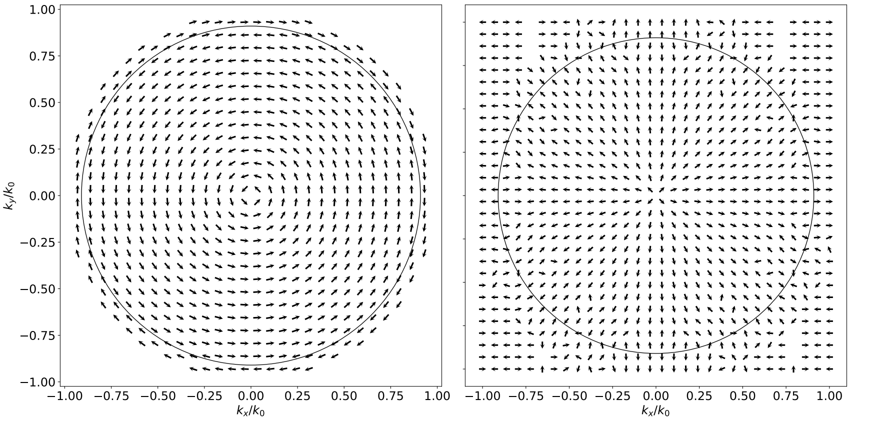

Fig. 1: a) Polarization map in the far-field radiation and b) quasimode coupling strength for the lattice consisting of single resonant z-oriented magnetic octopoles. Circles demonstrates the BIC supported on line which occurs for the multipolar lattice.

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Waveguides with dielectric filling: accelerating and undulator wakefield structures

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Waveguides with dielectric filling are intensively studied as accelerating structures excited by an electron beam. A relativistic electron beam moving in a vacuum channel near the dielectric filling of a waveguide generates Cherenkov radiation. The wakefield wave arising in the waveguide has both transverse and longitudinal components. In waveguides with symmetrical filling, a bunch of charged particles (typically electrons) moving near the axis generates a RF wave with a predominance of the longitudinal component. Positioning at a strictly defined distance from the generator bunch, followed by a small-sized bunch, leads to energy transfer from the generator bunch to the wake one and, as a result, to its acceleration. This principle is the basis of the wakefield method of acceleration. When the generator bunch deviates from the axis of the waveguide, significant transverse fields arise in it, the amplitude of which increases with increasing distance of the particles from the axis. This field leads to an exponentially increasing deflection of the particles of the generator bunch, which limits the possibilities of the wakefield acceleration method.

At the same time, wake fields can be used to create oscillations of beam electrons and generate undulator radiation, where the emphasis is on amplification of the transverse component of the wake field. As undulator structures, both cylindrical and rectangular waveguides bent in the longitudinal direction according to the harmonic law, and rectangular waveguides with asymmetric longitudinally periodic filling can be used.

Influence of structural disorder on the metamaterial regime

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Metamaterials are materials engineered artificially to have unique properties that could not be found in natural materials. The effects in metamaterials are due to resonances at individual structural elements. The optical properties of metamaterials can be described using material parameters (refractive index, dielectric permittivity, and magnetic permeability), which can take on the required values, be they positive or negative. Of particular interest is a structure where one or more parameters have a value close to zero. There has been a growing interest in metamaterial structures with parameters close to zero, mostly due to the way they could be used for various applications, such applications such as new types of beam steerers, modulators, band-pass filters, lenses, microwave couplers, and antenna radomes [1]. Also, it was discovered that changing the dielectric permittivity of structural elements or the filling factor in the structure is able to change the behavior of the structure from a photonic crystal to metamaterial and vice versa [2].

An important issue is the influence of structural disorder since defects in the position of structural elements are highly probable in real structures. Said defects can affect the structure in various ways. Strong structural defects can easily destroy the metamaterial and cancel its important features. However, the introduction of certain defects in metamaterials can lead to achieving specific useful optical properties, even going as far as obtaining total transmission and total reflection. The impact that defects have on metamaterial structures has been studied [3]; however, to the best of our knowledge, the effect that structural disorder has on the homogeneous mode in dielectric metamaterials has not yet been investigated.

In this work, we present the stability of the ε-near-zero regime in TM polarization (the magnetic field oscillates along the rod axis) in the metamaterial regime. We show that the homogeneous mode in the metamaterial regime is resistant not only to a change in the shape of the structure but also to the removal of the elements inside the structure. We investigated the influence of the disorder degree on the stability of the homogeneous mode. The ε-near-zero regime is less stable in a structure with parameters close to the boundary parameters of the "photonic crystal – metamaterial" transition. Although the homogeneous mode collapses at a fixed frequency, when the frequency is changed to a lower one, we can observe a homogeneous mode in a disordered structure. Since the change in frequency is insignificant, for real structures this can mean that the ε-near-zero regime is resistant to the introduction of structural disorder.

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Light scattering from a ring resonator: dependence on the $R_{\text{in}}/R_{\text{out}}$ parameter

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A dielectric disc resonator is characterized by an infinite set of equidistant (if dispersion is not taken into account) whispering gallery modes. The topological transition from the disc to a ring with a rectangular cross section leads to a fundamental restructuring of the spectrum due to an additional boundary condition on the inner boundary. The spectrum of the ring resonator consists of separate ring galleries, each of which begins with a wide transverse Fabry–Perot resonance, which corresponds to quantization between the inner and outer boundaries, characterized by the index r and continues with a series of azimuthal resonances with an exponentially increasing quality factor with increasing azimuthal index m [1]. Figure 1a shows the evolution of the low-frequency spectra of the ring resonator as a function of the relative width $R_{\text{in}}/R_{\text{out}}$ of the ring.

This work is devoted to a detailed analysis of the scattering spectra evolution of an electromagnetic wave from a dielectric ring resonator with a rectangular cross section depending on the parameter $R_{\text{in}}/R_{\text{out}}$. The parameters of the ring are the relative width $R_{\text{in}}/R_{\text{out}}$, the height $h = R_{\text{out}}/12.5$ and the dielectric permittivity $\varepsilon = 80$. The outer radius of the ring resonator R_{out} can be chosen in an arbitrary size range due to the scaling property of Maxwell's equations. Calculations were performed for three low-frequency photonic galleries. The main result: the frequency difference between the first maxima of different galleries, as well as the frequency difference between neighboring resonances in the same gallery, are determined by the $R_{\text{in}}/R_{\text{out}}$ parameter. It was also shown that modes with different radial index depend differently on changes in the $R_{\text{in}}/R_{\text{out}}$ parameter of the ring resonator.

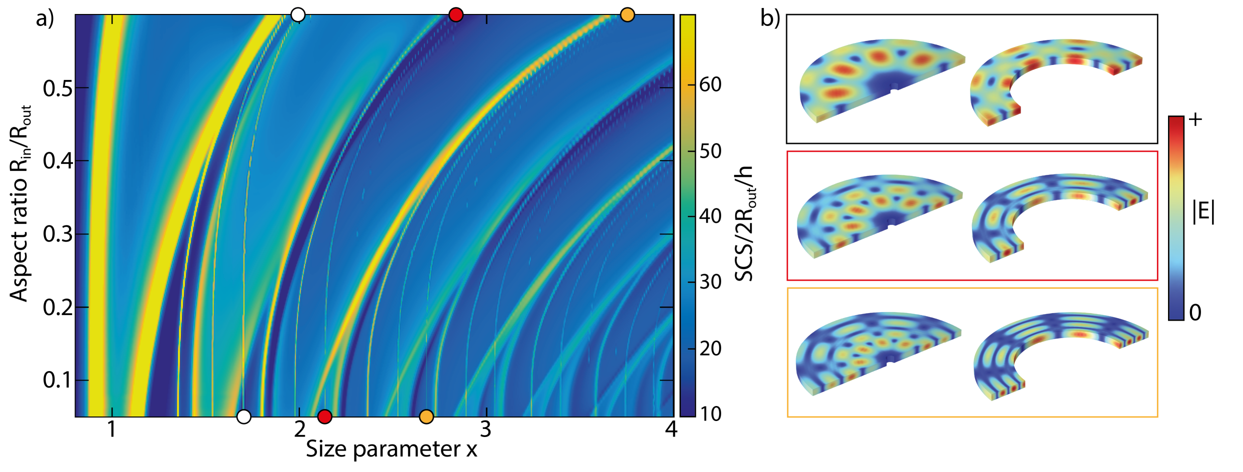

Fig. 1: The topological transition from a disc to a ring with a rectangular cross section. a) Evolution of the low-frequency scattering spectra as a function of $R_{\text{in}}/R_{\text{out}}$. b) Electric field distribution patterns $|\mathbf{E}|$ for rings with parameters $R_{\text{in}}/R_{\text{out}} = 0.05$ (left column) and $R_{\text{in}}/R_{\text{out}} = 0.6$ (right column) for the resonances marked on a) with the corresponding color. Size parameter $x = kR_{\text{out}}$.

The author is grateful to M. F. Limonov and K. B. Samusev for discussion of the results.

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Compact lumped superconducting resonators for topological quantum simulators

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Modern quantum computing is moving towards scaling up the number of qubits. To scale efficiently, all elements of the superconducting circuit must be compact. An important component of a quantum circuit is the superconducting resonator. Some of today's most compact ones are lumped-element resonators, which are discussed in this study.

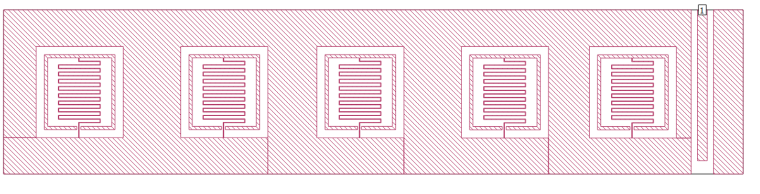

Fig. 1: Example of lumped-element resonators array.

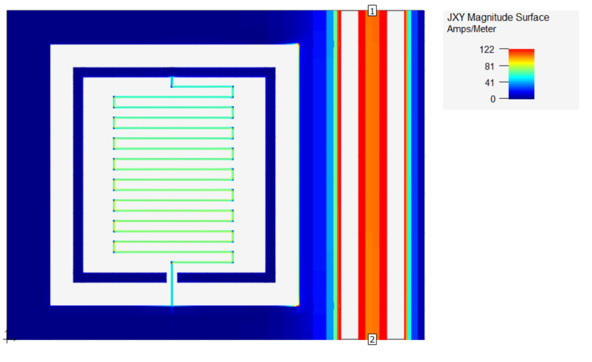

Fig. 2: Current density distribution on resonator.

Our work, based on numerical analysis, investigates properties of superconducting lumped-element resonators. We simulate losses in oxide films arising during fabrication, estimate quality factors and optimise resonators geometry. Furthermore, we plan to combine resonators into arrays for experimental observation of topological states.

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Controlling magnetic dipole resonances with a 2.5d metasurface

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Metasurfaces are artificial structures that allow engineer various types of effects. But there are two problems: postfabricaiton modification of properties and design potential due to 2D limitation. One of the promising way to solve the postfabrication modification is use the phase change materials (PCM), which have significant difference between two phases (coexisting under normal condition amorphous and crystalline phase) [1]. Nowadays, the most popular and well studied PCM is the Ge-Sb-Te alloy, which already found application in rewritable disks and PCRAM. Alloy $Ge_2Sb_2Te_5$ shows the highest difference of permittivities in alloy family during the phase transition from 15 to 47. Moreover, the amorphous phase demonstrate near zero losses in practically important range for telecommunication fiber lines. Problem of conferment in the 2D degree of freedom can be solved by partial extension into 3rd dimension (so-called 2.5D) providing additional degree of freedom.

In this work we combine switching ability of PCM and extension in the 3D dimension. We propose 2.5D metasurface containing 2 nanoparticles made of Ge-Sb-Te alloy in unit cell. This lattice itself represent by 2 sublatices with interplanar distance h.

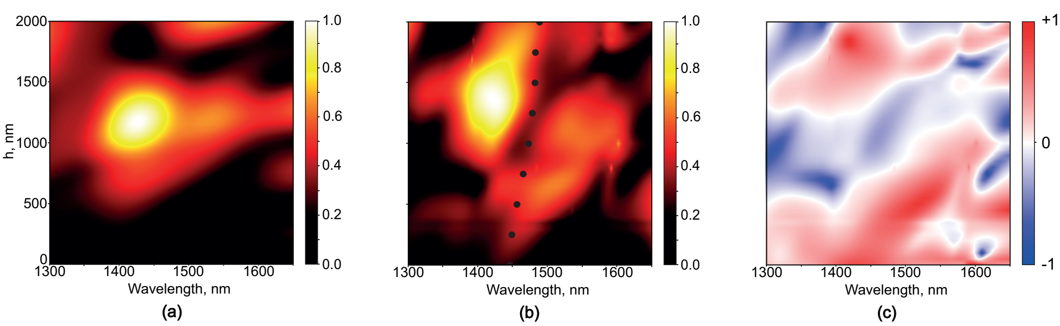

Fig. 1: Scattering spectra for crystalline (a) and amorphous phase (b) of $Ge_2Sb_2Te_5$. Magnetic dipole mode in single nanopartice shown as black dots on panel (b). Figure of merit for 2.5D metasurface on panel (c).

We carrying out a full-wave simulation of 2.5D switchable metasurfaces. We chose the single nanoparticle parameters in such a way that resonance scattering is not observed in the crystalline phase and it inherits the features of lattice resonances. At the same time, magnetic and electric dipole resonance observed in the amorphous phase [2]. In Fig. 1 shown integral scattering collected by an objective function with the numerical aperture NA = 0.55 as a function of interplanar distance. For crystalline phase in Fig. 1a we observe only 1 maximum, located at wavelength of 1405 nm and the interplanar spacing of 1100 nm. This feature is the contribution of the diffraction maxima with Muller indexes (1, 0), (2, 0) and (3, 0). For the amorphous phase, the peak is also observed at wavelength of 1405 nm and interplanar spacing of 1250 nm. Since there is a single particle resonance in this range, a minimum due to interference with the lattice resonance is observed. A dipole mode resonance is observed (marked with black dots in Fig. 1), which passes through the lattice resonance. The field of this mode has a similar magnetic dipole field profile as for a single particle. To analyze the modulation, a difference map $I_r = (I_{am} - I_{cr})/(I_{am} + I_{cr})$ was shown in Fig. 1c, where I_{am} and I_{cr} are scattering intensity for amorphous and crystalline phases respectively.

This representation allows us to show the effectiveness of the reflected signal from the metasurface.

The study is funded by Russian Science Foundation (Project № 21-79-10214).

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Ultrafast photoacoustics of carbon micro- and nanostructures: modelling and applications

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Irradiation of absorbing nanoparticles or nanostructures with electromagnetic pulses may cause photothermal effect or hyperthermia, photothermoacoustic and photoacoustic effects. Photothermal effect is observed when most of the absorbed energy transfers into heat resulting in considerable rising of the local temperature. This occurs when the sample is irradiated by a continuous wave for a significant time or by a laser pulse whose duration exceeds the acoustic wave propagation time in a nanoparticle. Photothermoacoustic and photoacoustic effects occur at the irradiation of nanoparticles by short and ultrashort laser pulses. In this case, acoustic waves generated due to heat expansion of nanoparticles can be used for nanoresolution sensing, photoacoustic visualization, theranostics [1, 2].

In the vast majority of applications the nanosecond and longer laser pulses are used. In this work the formation of acoustics waves under irradiation of different carbon micro- and nanostructures by picosecond laser pulses is theoretically studied. The considered model is based on numerical solution of equation of motion of continuum media in Lagrange form [3], and is able to calculate spatiotemporal distribution of main thermodynamic characteristics of media. Different aspects of photoacoustic response and its applications are discussed. For instance, it is theoretically shown and experimentally proved that irradiation of agglomerates of SWCNTs by picosecond laser pulses may cause cold photoacoustic destruction of tumor cells [4].

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Aperture method for calculation of bunch radiation in presence of dielectric objects

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Radiation of charged particle bunches moving in presence of dielectric targets is of significant interests for various applications in accelerator and beam physics. Typically, the size of the target is much larger than the wavelengths under consideration. Using this fact, we developed two approximate techniques: the "ray-optical method" and the "aperture method" (see [1–5] and references therein). The main topic of present work is developing and application of the aperture method.

Both methods assume a rigorous solution of some "key" problem, which takes into account only the interaction of the bunch field with the object's boundary closest to the bunch trajectory. The field at the outgoing boundary of the object ("aperture") is determined by the formulas of Snell and Fresnel. After that, the radiation outside the object is found either according to the laws of geometric optics with additional restrictions on the region under consideration ("ray-optical method"), or according to the Stratton–Chu formulas ("aperture integrals"). The latter method, which can be called the "aperture method", is more general (it allows the radiation calculation in the Fraunhofer diffraction region, in vicinities of focuses and caustics etc.). First, this report describes the basics of the aperture method. Examples of its application for a number of targets are given: a cone and a ball with a channel, a prism, as well as an object that we have called a "Cherenkov radiation concentrator". In some cases, a comparison is made with the simulation of the electromagnetic field using COMSOL package, which confirms the correctness of the developed method.

Further, a new version of the aperture method is developed. This variant uses only expansion in terms of plane waves in the analysis of radiation inside the object (in the previous version of the method, we found the field asymptotic inside the object). Such an approach is useful for objects that have only flat faces. We apply this technique to study the radiation of a charge moving along the face of a dielectric prism from the top to the base ("inverted" prism). Two waves propagating in the prism are taken into account: one falls directly on the base of the prism, and the second is preliminarily reflected from the inclined face. Analytical results are obtained for the components of the radiation field. On this base, we calculate some typical examples of the field distribution.

Also we consider another problem with a prism, when the charge moves at some small angle to the face of the prism from its base to the top. The degree of sensitivity of the radiation in relation to the deviation of the charge trajectory from the parallel direction is revealed.

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Three-dimensional topological structures in liquid crystals and magnets

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Topological magnetic and liquid crystal structures in chiral media are of great interest both from the point of view of fundamental science and their possible applications in the field of storage, processing, transmission and display of information. Three-dimensional topological structures — skyrmion tubes, hopfions [1], helicnotons [2] are of particular interest due to the possibility of encoding a large amount of information in them. The problem of the stability of such structures with respect to thermal fluctuations and random external perturbations, as well as their response to specific external impacts, are studied within the framework of the statistical transition state theory [3]. The results of modeling the effects of the formation of dissipative soliton structures in open thermodynamic systems far from equilibrium will be presented. The mechanisms of formation of rotating topological solitons in light-sensitive chiral nematics under the influence of laser radiation will be discussed [4].

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Exceptional point in a single dielectric ring resonator

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Exceptional point(EP), considered as the degeneracy of resonances, is at the frontier of fundamental research and has been observed for years in various photonic, acoustic, and electronic mechanisms [1]. The physical nature of EP is the resonance engineering by tailoring the gain-loss balance and mode coupling in coupled systems. Exploiting incident transversal electric wave as effective gain, EP can be achieved in passive material [2, 3].

In this work, we numerically study EP at radio frequency in a single dielectric ring resonator supporting Mie-like and Fabry–Pérot-like resonances. We show that, by mode destruction to manipulate the strong and weak mode coupling, both real [see Fig. 1(a)] and imaginary [see Fig. 1(a)] parts of eigenfrequencies ($\Omega = \omega + i\gamma$) of two modes can coalesce at the critical frequency where an EP is formed. Our approach to achieving EP can potentially contribute to the development of novel compact functional metadevices.

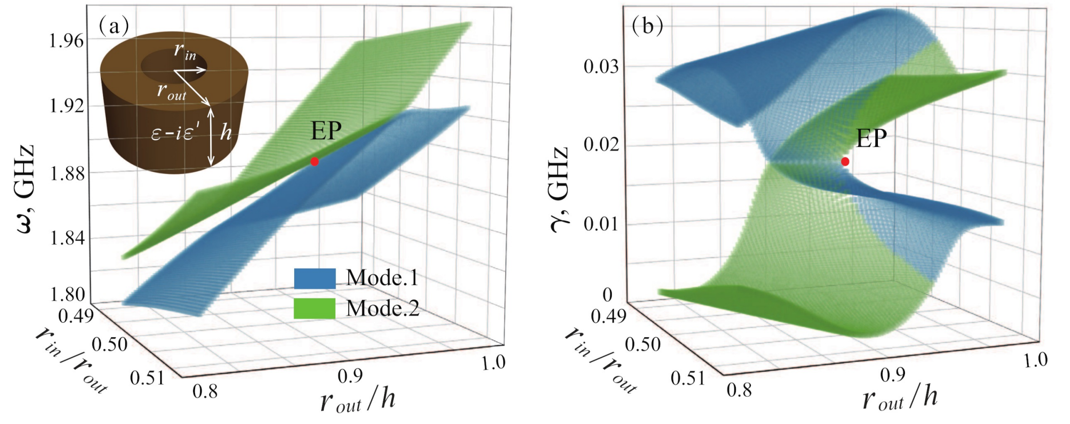

Fig. 1: Topology around an exceptional point singularity in non-Hermitian system: Evolution of the real (a) and imaginary (b) parts of eigenfrequencies, where the red dots indicate the position of EP. The inset indicates the dielectric ring resonator.

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