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DAYS ON DIFFRACTION 2026

ABSTRACTS

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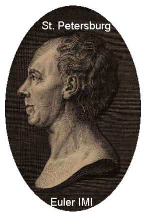

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FOREWORD

“Days on Diffraction” is an annual conference taking place in May–June in St. Petersburg since 1968. The present event is organized by St. Petersburg Department of the Steklov Mathematical Institute and St. Petersburg Leonhard Euler International Mathematical Institute.

This conference is supported by the Ministry of Science and Higher Education of the Russian Federation (agreement 075-15-2025-344 dated 29/04/2025 for Saint Petersburg Leonhard Euler International Mathematical Institute).

The abstracts of 73 talks, presented during 5 days of the conference, form the contents of this booklet. The author index is located on the last pages.

Full-length texts of selected talks will be published in the Conference Proceedings. Format file and instructions can be found at `http://www.pdmi.ras.ru/~dd/proceedings.php`. The final judgement on accepting the paper for the Proceedings will be made by editorial board after peer reviewing.

Organizing Committee

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In Memory of V. M. Babich

The Organizing Committee of “Days on Diffraction” notes with sorrow the passing of Vassily Mikhailovich Babich on 29 November 2025, a few months after his 95th birthday. He was a prominent representative of the Leningrad/St. Petersburg school of mathematical physics, known for his outstanding contributions to the mathematical theory of diffraction and wave propagation, and for his personal influence on the scientific community.

From 1954, Vassily Mikhailovich taught at the Department of Mathematics and Mechanics and at the Department of Physics of Leningrad State University. From 1967 he headed the Laboratory for Mathematical Problems of Geophysics at the Leningrad branch of the Steklov Mathematical Institute (PDMI). His weekly seminar at PDMI enjoyed a strong reputation, upheld by the exceptional quality of the talks and his masterful chairmanship.

V. M. Babich’s work laid the foundation for modern methods in mathematical physics, and his teaching shaped generations of scientists. For his work on the ray method applied to seismic wave propagation, Vassily Mikhailovich was awarded the USSR State Prize in 1982. In 1998, he received the V. A. Fock Prize for his groundbreaking advancements in asymptotic methods in diffraction theory.

The Organizing Committee honors the memory of V. M. Babich and extends its deepest condolences to his family, friends, and colleagues. He will be deeply missed and forever remembered for his wisdom, warmth, and enduring scientific legacy.

List of talks

Short-wave solutions of the wave equation with localized velocity perturbations

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This work studies a wave equation whose propagation speed has a localized perturbation on a certain surface M. The initial condition is given by a rapidly oscillating wave packet whose wavelength is not comparable to the scale of the inhomogeneity. Specifically, the initial wavelength is of order ε, while the width of the localized inhomogeneity is of order $\sqrt{\varepsilon}$, where ε is a small parameter. In contrast to the one-dimensional case, focal points of the phase with respect to the fast variable appear in this case, resulting in a reflected wave packet.

On a stability of time-optimal reconstruction by the Boundary Control method

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Let Ω be a Riemannian manifold with boundary. The time-optimal version of the BC-method [1] determines the parameters in the T-neighborhood Ω^T of $\partial\Omega$ from the boundary observations (response operator) R^{2T} on the time segment $[0, 2T]$. It visualizes the invisible waves supported in Ω^T, by reconstructing the operator W^T that creates these waves. The visualization is based on the triangular factorization of the operator $C^T := W^{T*}W^T$ in the form $C^T := F^{T*}F^T$ with a factor $F^T = U^T W^T$, where U^T is a unitary operator.

The factorization $C^T \mapsto F^T$ has certain continuity properties, due to which the time-optimal reconstruction $R^{2T} \mapsto C^T \mapsto F^T \mapsto W^T$ turns out to be continuous (stable) in the sense of relevant operator topologies (convergences) [2].

As an example, determination of the potential q in the wave equation $u_{tt} - \Delta u + qu = 0$ from R^{2T} is considered. We show that $R_j^{2T} \to R^{2T}$ implies $q_j \to q$ in $H^{-2}(\Omega^T)$. However, the question of quantitative estimates of stability (the rate of convergence) remains open.

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Self-modeling property of Schrödinger and Dirac operators on the half-line

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Self-modeling is a property of a class of operators in a function space which consists in a fact that an operator from the class is determined by any of its unitary copies up to an arbitrary transformation from a certain group (called shape equivalence). We will discuss the self-modeling properties of minimal positive-definite symmetric Schrödinger and Dirac operators on the half-line, which are proved using the construction of the so-called wave functional model of the Schrödinger operator with a matrix potential.

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Spectral singularities of Fano resonances in extinction cross-section

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Fano resonances arise in many areas of wave physics, including optical diffraction [1, 2], and manifest as asymmetric spectral line shapes originating from interference between resonant and non-resonant scattering channels. Owing to their sharp spectral features, they have found numerous applications in sensing, nonlinear optics, lasing, and switching [3].

Here we investigate Fabry–Perot–like modes in a dielectric split-ring resonator (insert in Fig. 1(c)) and analyze their spectral response using the exact quasinormal mode framework for electromagnetic Fano resonances [4]. We consider the parametric space defined by the angular gap size 2α and the incident angle φ (Fig. 1(a)), and calculate the corresponding resonance intensity σ and Fano asymmetry parameter q in this space (α, φ).

We reveal a previously unnoticed singular spectral feature — a singular point of the Fano asymmetry in the parametric space. When the system parameters trace a closed loop around the singular point, the spectral line shape continuously evolves, allowing any degree of q-asymmetry to be realized. Exactly in such points, the resonance intensity approaches zero. Microwave experiments confirm the theoretical predictions (Fig. 1(b–d)), demonstrating agreement between calculated and measured Fano parameters, resonance intensities, and extinction spectra.

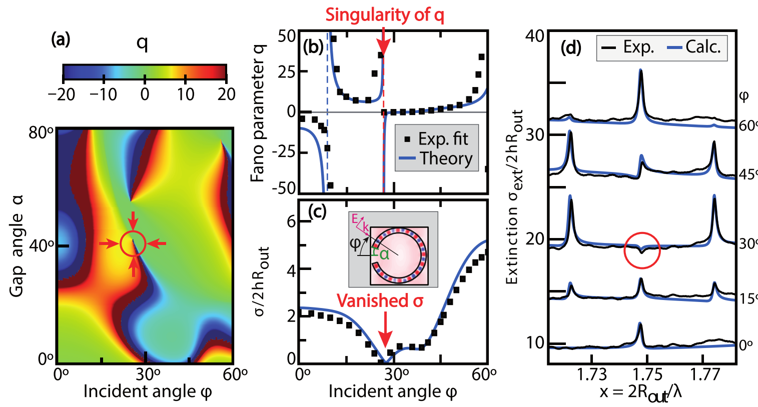

Fig. 1: (a) Calculated Fano parameter in the parametric space (α,φ). The red circle marks one of the singular points. (b) and (c) Comparison between calculated and experimental q and σ at fixed $\alpha = 40°$. (d) Experimental and calculated extinction spectra for different incident angels φ and fixed $\alpha = 40°$.

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Quasiperiodic inverse Sturm–Liouville problem

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Consider the Sturm–Liouville equation

$$-y'' + q(x)y = \rho^2 y, \quad x \in (0,1), \tag{1}$$

with the quasiperiodic boundary conditions

$$y(0) = ay(1), \quad y'(0) - hy(0) = a^{-1}y'(1),$$

where the potential q belongs to the class $L_{2,\mathbb{R}}(0,1)$ of the real-valued square-integrable functions, ρ^2 is the spectral parameter, $a \in \mathbb{R} \setminus \{-1, 0, 1\}$, and $h \in \mathbb{R}$.

Denote by $\varphi(x,\rho)$ and $S(x,\rho)$ the solutions of equation (1) under the initial conditions $\varphi(0,\rho) = 1$, $\varphi'(0,\rho) = h$, $S(0,\rho) = 0$, $S'(0,\rho) = 1$. Put

$$d(\rho) := a\varphi(1,\rho) + a^{-1}S'(1,\rho), \quad H(\rho) := a\varphi(1,\rho) - a^{-1}S'(1,\rho).$$

Let $\{\rho_n^2\}_{n\ge1}$ be the Dirichlet spectrum of (1) and $\sigma_n := \operatorname{sign} H(\rho_n)$, $n \ge 1$.

We study the following **inverse problem**: *given the Hill-type discriminant* $d(\rho)$, $\{\rho_n\}_{n\ge1}$, *and the signs* $\{\sigma_n\}_{n\ge1}$, *find* q, a, *and* h. A constructive algorithm for solving a more general quasi-periodic inverse Sturm–Liouville problem has been obtained in [1]. The talk will be mostly focused on the results of [2], which include necessary and sufficient conditions of solvability, local solvability and stability, as well as the uniform stability of the inverse problem. These results have been recently applied to an inverse Sturm–Liouville problem on a graph with a cycle in [3].

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Open source & AI/ML in device design: paradigm or exaggeration?

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AI has become an indispensable tool in modern research, supporting literature analysis, idea generation, writing, and code development, though its true scientific value remains to be validated in practice. In this paper, the utility of AI/ML tools is evaluated through a device-aware modeling task: the design and optimization of optical waveguide structures for mid-infrared quantum well infrared photodetectors. Such problems involve high-dimensional parameter spaces, coupled material and geometrical properties, and computationally expensive simulations. Previous studies show that they can be effectively addressed using state-of-the-art computational physics and machine learning methods [1, 2], making them a suitable benchmark for AI-assisted workflows.

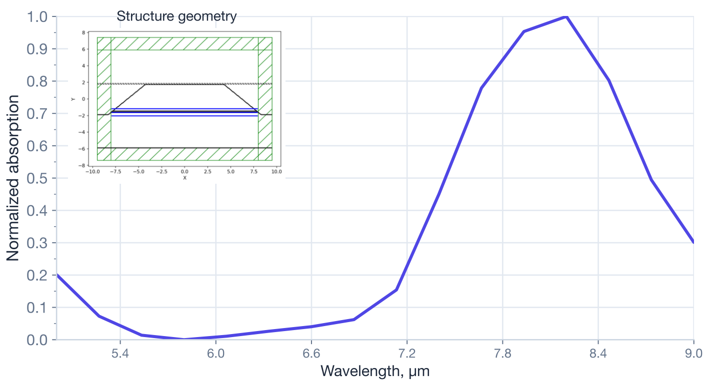

Fig. 1: Absorption coefficient of the simulated QWIP–waveguide structure.

The problem is primarily solved numerically using open-source Python packages such as Meep and FDTDX [3, 4], complemented by AI- and ML-based surrogate models [1]. The obtained results demonstrate that AI/ML methods not only accelerate the exploration of the design space but also enable the identification of optimal and non-trivial structural configurations. In particular, the optimized design achieves up to a 10% increase in absorption, showing the practical benefit of combining data-driven methods with traditional simulations. At the same time, the study shows that expert knowledge is still important for setting up the problem, checking the results, and guiding the optimization process. The work was supported by the Ministry of Education and Science of the Russian Federation (№ 075-00003-25-04, FSEE-2025-0011).

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On the ultrahyperbolic equation perturbed by the potential

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The subject of the talk is an ultrahyperbolic equation of a specific form. Although the equations of such type have very limited scope of possible applications, they do arise in some fields, such as the scattering theory for Schrödinger operator in $\mathbb{R}^d$:

$$H = H_0 + q(x), \quad H_0 = -\Delta.$$

One of the approaches to the corresponding inverse problem, proposed by L. D. Faddeev, is based on the intertwining operators U satisfying relation $HU = UH_0$. The Schwartz kernel $\Omega(x, y)$, $x, y \in \mathbb{R}^d$, of such an operator is a solution to the following ultrahyperbolic equation perturbed by the potential:

$$(\Delta_y - \Delta_x + q(x))\,\Omega(x, y) = 0. \tag{1}$$

In the present study, we seek for other forms of ultrahyperbolic equations that could be relevant to physics. To this end, we modify equation (1) as follows. First, we observe that in terms of the variables

$$u = \frac{x+y}{2}, \quad v = \frac{x-y}{2},$$

equation (1) reads

$$\left(\sum_k \partial^2_{u_k v_k} - q(u+v)\right)\Omega(u, v) = 0.$$

Next, we assume that the variables u, v are elements of Minkowski space $\mathbb{R}^{1,d}$ instead of $\mathbb{R}^d$:

$$u = (u^0, u^1, \ldots, u^d), \quad v = (v^0, v^1, \ldots, v^d),$$

and replace the principal term in the equation with $\partial_{u_\alpha}\partial_{v^\alpha}$. Here we adopt conventional covariant notation:

$$\partial_{u^\alpha} = (\partial_{u^0}, \partial_{u^1}, \ldots, \partial_{u^d}), \quad \partial_{u_\alpha} = (\partial_{u^0}, -\partial_{u^1}, \ldots, -\partial_{u^d}),$$

and the same for ∂_{v^α}, ∂_{v_α}. Finally, we replace the perturbation term with $iA^\alpha(u)\partial_{v^\alpha}$, which gives

$$(\partial_{u_\alpha} + iA^\alpha(u))\partial_{v^\alpha}\Omega(u,v) = 0. \tag{2}$$

Here $A(u)$ is an $\mathbb{R}^{1,d}$-valued function, which, in the case $d = 3$, can represent the electromagnetic four-potential in spacetime. Observe that both in (1) and (2) coefficients depend on one of two variables.

In the talk, some properties of equation (2) will be discussed. Presently, this equation is of purely mathematical interest for us. However, the solution $\Omega(u,v)$, in principle, can be related naturally (again, from the mathematical point of view) to operators acting in the state space of a scalar massive field in the context of relativistic physics.

Imbert–Fedorov shifts in the geometry of oblique incidence of a Gaussian beam on an isotropic gyrotropic medium

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Reflection and refraction of light beams at a planar interface between dielectric media are conventionally described by Snell's law and the Fresnel formulas. Nevertheless, the experimentally observed patterns of reflection and refraction of laser beams turn out to be considerably more intricate than those predicted on the basis of these formulas. In particular, the Fresnel formulas do not account for the optical response of the thin near-surface layer of the medium, whose proper description requires a modified formulation [1]. Besides, the reflection and refraction of elliptically polarized light beams are accompanied by longitudinal and transverse displacements of their centroids. Such displacements do not follow directly from the Fresnel formulas derived for plane waves. Known as Imbert–Fedorov shifts, these transverse displacements arise from the spin-orbit interaction of elliptically polarized beams (incident, reflected, and refracted) [2]. Currently, the occurrence of Imbert–Fedorov shifts is taken into account in the interaction of light with metamaterials and in the development of optical filters and switches. Consideration of Imbert–Fedorov shifts is also essential for the practical use of beams in optical communication, the construction of optical traps and tweezers [3], as well as various domains of nonlinear optics.

In this research, within the framework of classical electrodynamics, we have derived expressions for the transverse Imbert–Fedorov shifts of reflected and transmitted beams in an isotropic gyrotropic medium under oblique incidence of an elliptically polarized Gaussian beam. To this end, we employed modified Fresnel formulas that account for the influence of near-surface inhomogeneity of matter on reflection and refraction. It has been shown that, at small incidence angles, the gyrotropy of the medium is found to contribute to the shifts of both the reflected and transmitted beams, whereas the optical response of the near-surface layer influences only the transverse shift of the reflected beam. The analysis of the complex analytical dependencies of the Imbert–Fedorov shifts of the transmitted and reflected beams on the ellipticity of the polarization ellipse of the incident radiation and on the angle defining the orientation of its major axis reveals two main trends. First, the shifts induced by both mechanisms increase with the incidence angle of the Gaussian beam. Second, they exhibit a nonmonotonic dependence on both the orientation of the polarization ellipse and its ellipticity.

The obtained results are of critical importance for the correct description of the conversion of the total angular momentum of light beams incident on a medium. Indeed, the presence of shifts in the beam centroids gives rise to additional angular momentum and allows one to explain the nonconservation of the spin component of angular momentum, even in the simplest cases of oblique incidence of Gaussian beams on an isotropic medium.

This work has been supported by Foundation for the Advancement of Theoretical Physics and Mathematics “Basis”.

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Homogonization and dispersion effects in problems about short-wave asymptotics of the wave equation with an irregularly rapidly oscillating velocity

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The talk is devoted to the homogonization method for the two-dimensional wave equation with irregular spatial fast oscillations of the velocity in the case when its solutions are also rapidly changing and determine the propagation of short waves. One of the effects generated by rapidly changing coefficients consists in the appearance of dispersion effects in such problems, which are described by the linearized Boussinesq equation.

The work was supported by the Russian Science Foundation grant No. 24-11-00213.

Tilted unidirectional few-cycle waves

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In a recent time, in connection with the development of laser technologies, exact localized unidirectional solutions of the wave equation

$$u_{tt} - c^2 \Delta u = 0\,, \tag{1}$$

where $c > 0$ is the of wave speed, and $\Delta = \partial_x^2 + \partial_y^2 + \partial_z^2$ is 3D Laplacian, have attracted great interest [1–5]. The plane-wave decomposition of such solutions (which are also named causal or forward-propagating) contain only those components for which the projection of the velocity vector onto the chosen direction is nonnegative. Most of the known unidirectional solutions exhibit angular symmetry with respect to the specified direction.

The simplest known unidirectional solution [6] has a form

$$u = \frac{1}{S(S - z - i\zeta)}, \tag{2}$$

where

$$S = \sqrt{(ct + ib)^2 - x^2 - y^2}, \tag{3}$$

$b > 0$ and ζ are reak parameters, $\zeta < b$, the branch of the square root is chosen so that $S = ct + ib$ at $x = y = 0$. The function (2), (3) represents a low-cycle unidirectional wave, whose real and imaginary parts at any point take zero values not more then twice.

In this paper, an nonaxisymmetrical generalization of the function (2), (3) is given through two different but equivalent derivations, one is based on a complex shift along the transverse coordinate x, the other employs the Lorentz transformation with respect to this coordinate. Thus we come to tilted unidirectional waves, whose main direction of propagation makes a nonzero angle with the z-axis. This approach is similar to that used in [7] for construction of tilted Bateman-type solutions of (1).

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Analysis of axial intensity for aperture constraints beams

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Micro-optical components are widely used in modern technical systems, where miniaturization demands constantly increase [1]. Growing attention is directed toward understanding how light waves propagate beyond the paraxial approximation, as well as toward near-field diffraction effects [2].

We present a detailed analytical study on the axial intensity distribution formed when parabolic and ideal lenses focus light through a circular aperture [3]. We show that analyzing nonparaxial focusing requires accounting for the pupil size, particularly when the aperture is small. We derive expressions for each local diffraction maximum, the focal shift, and quantitative criteria for when aperture effects become significant. Numerical simulations based on the Fresnel and Rayleigh–Sommerfeld integrals confirm our theoretical results.

The work was partially supported by the Russian Science Foundation grant No. 25-12-20007 in terms of numerical modeling and the State Assignment of the National Research Center "Kurchatov Institute" in the theoretical part.

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Modal analysis of elastic guided wave scattering by localized obstacles in rods with complex-shaped cross-section

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Non-destructive inspection with elastic guided waves (EGWs) is a promising approach for monitoring damage occurrence in complex-shaped metallic structures such as load bearing rods and bars, pipes, or rails, since EGWs can propagate over long distances in them and show great potential for defect detection [1, 2]. However, due to the complex cross-section geometry typical for such waveguides, multiple EGWs with close group velocities exist simultaneously in them at any frequency, resulting in wave package overlapping and distortion. The latter sufficiently complicates the extraction of wave signals corresponding to damage-scattered wavefields from the sensor response. Therefore, prior studies of modal damage sensitivity with respect to its wave structure and obstacle location within the waveguide cross-section is an important step towards the tuning of such inspection systems.

In the current talk, considering an I-shaped beam as an example, numerical and experimental results revealing the peculiarities of EGWs scattering by an artificial surface mounted obstacle are discussed. Prior, dispersion curves and mode shapes of EGWs were evaluated for a broad frequency range employing the semi-analytical finite element method (SAFE) [3]. Matching the wave structure of each EGW mode in the waveguide cross-section and the obstacle location, such a SAFE analysis allows for the preliminary estimation of damage sensitivity for each specific normal mode. Moreover, preferable locations for actuator/sensor placement on the beam surface could be optimized to enhance excitability and acquisition of specific EGWs. Numerical simulation of EGW propagation in an elongated steel I-shaped beam sample, excited by a localized surface load and equipped with an added mass placed at various positions with respect to its cross-section, was performed using the standard finite element method (FEM). Both wavenumber-frequency analysis and matrix pencil technique were adopted to the obtained numerical data extracted from the model in the form of B-scans along the beam for the estimation of specific EGW contribution to the scattered wavefield and its damage sensitivity. Corresponding experiments carried out with a scanning laser Doppler vibrometer allowed to confirm the obtained numerical results qualitatively. In general, when the obstacle is located in different parts of the beam cross-section, each EGW among the analyzed set of normal modes has a different reflection intensity conditioned by its mode shape. Therefore, the damage location in the cross-section can be approximately estimated according to the presence of specific normal modes in wave packages reflected by it.

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Simulation of elastic guided wave scattering by a delamination in a layered anisotropic structure

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In engineering practice, 3D finite element method (FEM) is typically employed for the numerical simulation of elastic guided wave (GW) interaction with delaminations in anisotropic laminates [1]. However, standard FEM codes though providing remarkable level of flexibility with respect to the damage geometry are prone to high computational costs, especially when the number of lamina increases. At the same time, analytically-based methods are also capable to the simulation of linear wave phenomena in layered elastic structures with cracks and delaminations [2]. Their computer implementation does not require high performance computing systems and might allow for the comprehensive parametric analysis of corresponding GW-scattering processes.

In the current report, the application of the semi-analytical integral equation approach for the simulation of GW interaction with delamination-type defects treated as stress-free infinitesimally thin cracks in multilayered anisotropic structures is considered. Corresponding boundary value problem is reduced to the vector-form Wiener–Hopf integral equation over the unknown crack opening displacement (COD) with the hypersingular kernel which could be derived using the Green's matrix for the considered anisotropic structure. To solve the latter, Fourier transform technique together with the variational Galerkin scheme [2, 3] is adopted. Depending on the delamination geometry unknown COD is expanded either with axially symmetric (radial) δ-like basis functions or, in a special case of a rectangular-shaped damage, employing weighted Chebyshev polynomials of the second kind. Along with the numerical verification of the implemented analytically-based computer model against 3D FEM data, examples of the evaluation of GW scattering by circular and rectangular-shaped delaminations in anisotropic laminates arbitrarily located with respect to their thickness are provided and discussed.

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Gradient-based inverse lithography for EUV masks via the waveguide method and a physics-informed neural operator

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Extreme ultraviolet (EUV) lithography is the driving force behind the transition to the angstrom-era semiconductor technology nodes [1]. Unlike deep ultraviolet systems, which use transmissive masks and lenses, EUV systems utilize reflective photomasks composed of a multilayer Bragg mirror and a patterned layer consisting of a thick absorbing layer or a thin phase-shifting layer. The interplay between the patterned layer topography and obliquely incident electromagnetic waves causes mask 3D (M3D) effects, including shadowing, horizontal-vertical bias, phase errors, and best focus shift [2]. As the critical dimensions shrink, mitigating these complex diffraction and interference phenomena becomes paramount for preserving pattern fidelity on the silicon wafer.

Traditional rule-based and model-based optical proximity correction methods struggle with the changes in light-mask interactions at sub-5 nm nodes. Consequently, the industry is shifting towards inverse lithography technology (ILT) [3], which formulates mask optimization as an inverse problem to calculate the ideal mask topology based on the desired wafer image. While pixel-based ILT offers superior pattern fidelity and naturally generates complex curvilinear shapes, it is burdened by enormous computational costs due to a massive number of optimization variables. Recently, many attempts have been made to simulate the 3D effects of masks using deep neural networks to accelerate the diffracted field calculation [4, 5].

In this paper, we present an inverse lithography framework tailored for the wavelengths of current and promising industrial lithography systems (13.5 and 11.2 nm), which is based on using a waveguide neural operator [5] or a waveguide method as the field calculation module. The ILTs are presented in which either each pixel or the Fourier harmonics of the dielectric permittivity of the mask are optimized using gradient methods for a continuous pixel map of the mask for the given field pattern on the wafer. Numerical experiments have demonstrated that the considered ILT methods make it possible to obtain a mask structure that ensures the achievement of the desired field on the wafer. Furthermore, we demonstrate the possibility of implementing an ILT procedure that does not require expensive calculation of the field in the mask domain.

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Hybrid numerical-analytical scheme for the evaluation of elastic guided wave scattering by a localized obstacle in an isotropic layer

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The accurate localization and sizing of defects employing a distributed sensor network are among the key problems in ultrasonic-based nondestructive testing and structural health monitoring, particularly within the modern thin-walled constructions typical for high-tech industries. While the efficacy of elastic guided waves (EGWs) for this application is well-established, the interpretation of acquired signals and the subsequent extraction of damage characteristics is still a challenge. It is conditioned mainly by the dispersive nature of EGWs and complex wave phenomena occurring within the EGW scattering by real-case obstacles. While data-driven approaches are prone to be a promising solution of such problem, their training rises the need for huge datasets which could be evaluated within physically adequate numerical experiments. The latter, in turn, requires the development of reliable computer models for the simulation of EGW interaction with complex-shaped defects typical for the considered engineering structures.

In this report, a further development of a hybrid numerical-analytical scheme [1, 2] is presented which concentrates on the analysis of Lamb and SH-waves scattering by complex-shaped 3D obstacles in an elastic layer. Within the proposed scheme, the solution of the considered boundary-value problem is constructed in two stages. At first, a set of auxiliary numerical solutions is obtained in the vicinity of the inhomogeneity using appropriate finite element (FEM) software. Within the second stage, these FEM-based numerical solutions are coupled with a representation of the wave field in the remaining intact part of the waveguide in the form of a superposition of cylindrical guided waves. In cylindrical coordinates, each of such waves is represented as a product of three components: an amplitude coefficient depending on the distance from the region containing the inhomogeneity; a function describing the modal shape across the layer thickness; and an unknown angular intensity function characterizing the distribution of the radiated wave energy. The unknown functions are expanded into Fourier series with respect to the polar angle. The coefficients of the resulting series are determined from continuity conditions imposed on the displacement and stress fields at the interface between the cylindrical region containing the inhomogeneity and the surrounding waveguide. The advantage of this approach, as well as the original methodology on which it is based [1], is that it enables to handle complex-shaped inhomogeneities without the need to develop custom FEM solvers, allowing straightforward integration with the available FEM software. Moreover, such hybrid technique provides access to modal scattering coefficients, enabling a refined analysis of wave phenomena associated with EGW. The proposed approach is validated by comparison with results reported in the literature for problems involving cylindrical holes and deep flat-bottom holes. Some examples of its application to evaluate the interaction of fundamental Lamb waves with complex-shaped damage which is either symmetric or asymmetric with respect to the layer thickness

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On the Rayleigh approximation for non-confocal core-mantle nanospheroids with a silver core

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We formulate the Rayleigh approximation for core–mantle spheroids with a core whose shape may be non-confocal to that of the particle. We study the applicability of this approximation for nanospheroids with a silver core, the shape of which can vary from oblate spheroidal to spherical and prolate spheroidal. We present the solution to the corresponding electrostatic problem in the form of the T-matrix, and use its element T_{11} to derive the polarizability of the particle employed in the Rayleigh approximation.

We perform numerical calculations of the cross-sections for spheroidal particles of different sizes and shapes, with a silver core of various geometries, using the exact solution to the light scattering problem from [1, 2], and compare its results with those derived from the polarizability given by the Rayleigh approximation. The exact solution was obtained using the extended boundary condition method with field expansions in proper spheroidal bases and has been shown to be applicable to particles over a wide range of parameter values. For homogeneous and layered spheroids that are small compared with the wavelength of the incident radiation, this approach yields extremely accurate results in the visible spectral region (300–700 nm), where the refractive index of silver changes from $m = 1.6 + 0.6i$ to $0.1 + 4.9i$ [3], which complicates the use of the standard approaches for treating inhomogeneous scatterers, such as the discrete dipole approximation.

We find that the Rayleigh approximation developed for non-confocal layered spheroids provides sufficiently accurate results in the visible range for particles with a silver core, provided their size is less than 10–20 nm. The results are compared to those recently obtained for confocal layer spheroids in [4].

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Grid-characteristic method on Chimera meshes for sonar acoustic and seismic study of ice ridges

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To successfully develop the hard-to-reach natural resources of the Arctic Shelf and exploit the Northern Shipping Route, it is necessary to ensure the safe operation of stationary and mobile offshore structures such as vessels, including ice-class vessels, oil platforms, and so on. Therefore, the issue arises of developing new methods for processing data from seismic [1] and sonar-acoustic [2] studies of ice formations. Ice ridges are formed as a result of the collision of ice floes and are characterized by numerous internal inhomogeneities, both gas-saturated and water-saturated, which complicates their study due to multiple reflections of body and surface elastic and acoustic waves. This paper is devoted to solving a joint initial-boundary value problem for elastic and acoustic wave equations using overset Chimera meshes [3] and a grid-characteristic numerical method [4, 5]. Using the grid-characteristic method on overset Chimera grids to solve the system of equations under consideration requires the use of several background grids, at the nodes of which the solution of the elastic and acoustic wave equations is sought, respectively. To describe the complex shape of ice ridges, fine grids are used that precisely coincide with the considered boundaries and contact boundaries of the integration domain, called overset or Chimera grids. Interpolation is performed between these overset grids and the background Cartesian grids. This report will discuss in detail a multi-stage calculation algorithm taking into account the complex configuration of computational grids under consideration. The proposed approach allows for modeling the diffraction and interference of a large number of elastic and acoustic body and surface waves in the complex heterogeneous structure of an ice ridge (Fig. 1). The developed numerical method can be used to generate a training set for subsequent solution of the inverse problem using convolutional neural networks.

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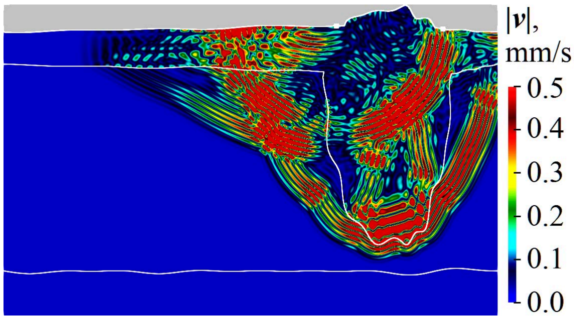

Fig. 1: Snapshot of the wave field of the velocity modulus in the cross-section of an ice ridge of arbitrary configuration. The calculation was performed using the grid-characteristic method on superimposed Chimera grids.

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Simulation of wave diffraction in damaged rails using grid-characteristic method on overset grids

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The development of rail transport and the increase in tonnage and throughput require careful provision of operational safety and, as a consequence, the development of modern methods for diagnosing the condition of communication tracks [1]. This report is devoted to the specifics of using the grid-characteristic method [2, 3] on overset Chimera grids [4] for solving direct problems of non-destructive testing of rails. Both two-dimensional and three-dimensional formulations are considered. The initial-boundary value problem of the elastic wave equation is solved. A background Cartesian grid, which is economical in terms of computational resources, is used in most of the integration domain. Thin curvilinear structured overset grids are constructed around the outer boundary of the rail and the contact boundaries of inhomogeneous inclusions. Interpolation is performed between the background and overset grids. Thus, it becomes possible to perform full-wave modeling of the non-destructive testing process of a rail with precise consideration of the geometric features. Saving computational resources allows for modeling, including in a three-dimensional setting (Fig. 1), while simultaneously simulating wave effects caused by the complex geometric shape of the rail and the various types of inhomogeneities under consideration. This poses challenges in constructing overset Chimera meshes that are optimal in terms of minimizing computational costs and decomposing them during parallelization in the presence of a large number of individual computational meshes. The proposed solutions will be discussed in the report. The proposed numerical method for solving the direct problem of non-destructive testing of rails can be used both for developing and testing new diagnostic methods and as a complement to and alternative to in-kind testing. The proposed numerical method can also be used to generate an accurate training set for subsequent solution of the inverse problem using deep learning and convolutional neural network.

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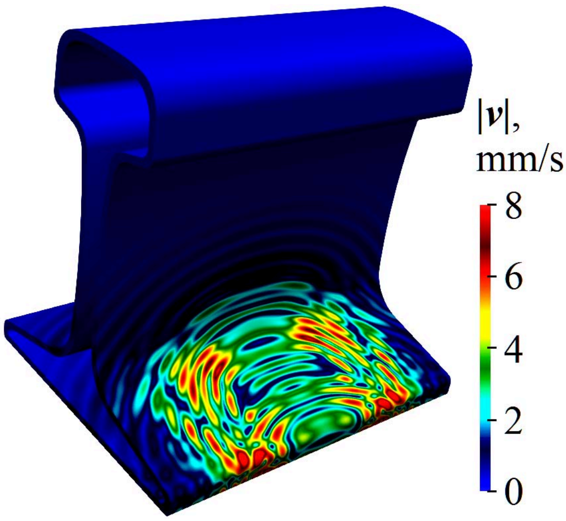

Fig. 1: Thin overset Chimera mesh accurately reproducing the rail shape, with the velocity field distribution from the source.

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Asymptotic solution of the problem of propagation of time-harmonic elastic waves from a localized source in an isotropic inhomogeneous medium

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We consider the differential elasticity equation in an isotropic inhomogeneous medium with a time-periodic external force localized in the vicinity of a certain point ξ. The general form of the elasticity equation is:

$$\rho\frac{\partial^2 s}{\partial t^2} = \operatorname{Div}\sigma + F, \tag{1}$$

where $\rho(x)$ is the density, s is the displacement vector, σ is the stress tensor, F is the external force. Under the assumption of medium isotropy, we have

$$\sigma = 2\mu\varepsilon + \lambda\operatorname{tr}\varepsilon E, \qquad \varepsilon_{ij} = \frac{\partial s_i}{\partial x_j}, \tag{2}$$

where $\lambda(x)$, $\mu(x)$ are the Lame parameters, ε is the strain tensor. We consider the external force of the form $F = F_0\left(\frac{x-\xi}{h}\right)\cos(\omega t) = \operatorname{Re}\left[F_0\left(\frac{x-\xi}{h}\right)e^{i\omega t}\right]$, where h is a small parameter, $\omega \sim \frac{1}{h}$. For convenience, we move on to complex values, $s = \operatorname{Re} U$. Using also relations (2), we transform (1)

$$\rho\frac{\partial^2 U}{\partial t^2} = (\lambda+\mu)\nabla(\nabla U) + \mu\nabla^2 U + \nabla\lambda(\nabla U) + [\nabla\mu\times[\nabla\times U]] + 2(\nabla\mu\nabla)U + F_0\left(\frac{x-\xi}{h}\right)e^{i\omega t}. \tag{3}$$

The solution is also sought in the form $U = U_0(x)e^{i\omega t}$. At $\omega = \frac{1}{h}$ we obtain:

$$\begin{gathered}\hat{\mathcal{H}}U_0 = F_0,\\ \mathcal{H}(x,p,h) = (\lambda+\mu)pp^T + (-\rho+\mu p^2)E - ih((p,\nabla\mu)E + p(\nabla\mu)^T + (\nabla\lambda)p^T),\\ \mathcal{H} = \mathcal{H}^0(x,p) + h\mathcal{H}^1(x,p).\end{gathered} \tag{4}$$

This is a system of differential equations with a localized right-hand side. For it, in our case, a solution can be written in the form of a linear combination of Maslov's canonical operators [1] on the manifolds Λ_+^j consisting of trajectories of the Hamiltonian system with the Hamiltonian H^j – the j-th eigenvalue of the matrix $\mathcal{H}_0$, emitted from L^j – its zero level at the point $x = \xi$ [2].

Theorem 1. *The time-harmonic asymptotic solution of equation* (3) *at* $\omega = \frac{1}{h}$ *is of the form*

$$\begin{aligned}U_{as}(x,t,h) = h\sqrt{2\pi}e^{\frac{\pi i}{4}+\frac{it}{h}}\Bigg(&K_{\Lambda_+^1}\left[\mathcal{P}^{t^1}_{\gamma^1_{\theta,\varphi}}\widetilde{F_0}(q^1(\theta,\varphi))\right]\\ &+K_{\Lambda_+^2}\left[\sqrt{\frac{\lambda(\xi)+2\mu(\xi)}{\rho(\xi)}}\frac{P^2(\theta,\varphi,t^2)}{|P^2(\theta,\varphi,t^2)|}q^{2T}(\theta,\varphi)\widetilde{F_0}(q^2(\theta,\varphi))\right]\Bigg),\end{aligned} \tag{5}$$

where θ,φ *are the coordinates on the spheres* L^j, $\gamma^j_{\theta,\varphi}$ *is the trajectory of the* j*th Hamiltonian system emitted from the point* (θ,φ) *on* L^j, $\mathcal{P}^t_\gamma$ *is the parallel transport operator along the curve* γ *for time* t, $q^j = p|_{L^j}$, $X^j(\theta,\varphi,t), P^j(\theta,\varphi,t)$ *are the solutions of the* j*th Hamiltonian system.*

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Propagation of the partially coherent X-ray radiation and exact transparent boundary conditions

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In practical radiation propagation problems the partially coherent radiation field is described by the complex two point correlation function $\varphi = \langle u(\boldsymbol{\rho}, z)u^*(\boldsymbol{\rho}', z)\rangle$ also known as the second order coherence function, where u is the field amplitude of the fully coherent radiation [1]. In the paraxial approximation function φ obeys a second order partial differential equation in traversal coordinates $\boldsymbol{\rho} = (x, y)$, which is also a first order equation in longitudinal coordinate z. In the 2D space (x, z) the equation for function $\varphi = \varphi(x, x', z)$ takes the following form

$$2ik\frac{\partial\varphi}{\partial z} + \frac{\partial^2\varphi}{\partial x^2} - \frac{\partial^2\varphi}{\partial x'^2} + k^2(\varepsilon(x) - \varepsilon^*(x'))\varphi = 0, \tag{1}$$

where k is the wavenumber, ε is the complex dielectric permittivity of the medium and ε^* is the complex conjugate of ε. Equation (1) can be obtained from the well known parabolic wave equation used to propagate the coherent wave field and is identical to the evolution equation for the density matrix in the quantum mechanics. Numerical solution of (1) can be achieved using a finite-difference scheme in a rectangular computational domain in the (x, x') plane, which requires appropriate boundary conditions at the border of the said domain. One of the possible options are exact transparent boundary conditions similar to those used in the solution of multidimentional parabolic equations [2] by the finite-difference method. In this work such a condition for equation (1) is proposed and applied to propagation of the coherence function of a partially coherent X-ray radiation beam in free space as well as in X-ray nano-waveguides [3]. The influence of the medium on the coherence properties of X-ray radiation is discussed and it is demonstrated that the beam becomes more coherent as it propagates in non-scattering media.

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Simulation of wave propagation in stratified porous elastic media based on multiscale modeling

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Porous materials are among the most common media. Natural materials such as soils and rocks are some examples of porous materials saturated with fluids (gas and liquid). Another examples of porous material are biological tissues, such as bones and lungs. There are many artificial materials which are porous too.

Porous media are complex multiphase composites. There are many theories for estimation of the static moduli of porous media, such as the Mori–Tanaka model, the differential theory of effective media, etc. However, it has been previously shown by experiments that the elastic properties of porous materials can differ significantly at high vibration frequencies compared to those in a static state [1].

In this paper,two-stage scheme for multiscale modeling of the dynamics of porous media is considered. Firstly, finite element modeling (FEM) is applied for mesoscale simulation of representative volume (unit-cell). The FEM-solutions are used to determine effective material properties for differential equations for macroscale motions. The mechanics underlying this simulation are discussed. In the second stage, the governing equations are solved by semi-analytical method [2, 3] and the phase velocities of waves in porous fluid-saturated media are studied.

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Quantum coherent superradiant states in hybrid perovskites: stability and fundamental solitons

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We investigate the stability of macroscopic coherent quantum states in quasi-2D hybrid perovskites [1–4]. The problem reduces to a 2D nonlocal nonlinear Schrödinger (NLS) equation governing the complex-valued polarization of Wannier excitons coupled to longitudinal optical (LO) and acoustic phonons.

The key feature of the model is its nonlocal nature, arising from exciton-phonon interactions. We perform a linear stability analysis of plane wave solutions for the nonlocal NLS equation. For exciton-LO phonon coupling, we establish a modulational stability criterion: the plane wave is stable provided the squared amplitude remains below a critical intensity determined by the interaction parameters. In the weakly nonlocal regime, the equation reduces to a purely nonlocal form, with stability results coinciding with the long-wave limit of the full model.

Exciton-acoustic phonon interactions narrow the stability window compared to the case without such coupling. Analytical results are confirmed by numerical simulations. Additionally, we numerically solve the 2D nonlocal NLS equation in polar coordinates and obtain a stable fundamental soliton solution.

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Controlled driving of a piezoelectric transducer array for selective guided wave excitation in elastic waveguides

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Layered elastic waveguides support a long-distant propagation of traveling guided waves (GWs). This property is used in various applications such as non-destructive structural health monitoring (SHM), micro electro-mechanical systems (MEMS), and surface acoustic wave (SAW) devices. In many cases only one pure mode is required, while their number increases with frequency, and other GWs, propagating at different speeds, only create interference. To selectively excite the desired waves in elastic plates, a set of ultrasonic transducers can be used. Their control parameters are calculated to suppress unwanted modes. The main difficulty in solving this problem stems from the mutual source influence through the substrate, which prevents the use of a simple superposition of their fields. Here, a strict solution to a coupled dynamic contact problem for the system of transducers as a whole is required.

A computational algorithm for obtaining proper driving fields has been developed within a mathematical framework that accounts for the coupling at the patch–structure interface, mutual interactions among the actuators, and the multimodal nature of guided wave propagation in the substructure. The proposed approach employs a semi-analytical Green's matrix formulation to model wave fields generated by surface-mounted piezoelectric actuators. This model is validated against finite element simulations and experimental measurements. Numerical examples demonstrate the feasibility of determining actuator drive parameters that enable targeted excitation of specific guided wave modes, including backward waves.

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Hidden mode A_0 in a submerged anisotropic plate

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It is well known that when an elastic plate is immersed in an acoustic medium (air or water), the traveling Lamb waves become leaky waves. Mathematically, the real roots ζ_n of the wavenumber characteristic equation shift into the complex plane [1]. In this case, the acoustic environment gives rise to two additional real roots, which are associated with the Scholte–Gogoladze (Scholte–Stoneley) waves [2], and a branch point, making the complex plane a two-sheet Riemann plane. Relatively recently, it was shown that in a certain low-frequency range $f < f_0$, the root ζ_0 of the fundamental antisymmetric mode A_0 shifts from the real axis to the unphysical Riemannian sheet, and therefore this wave cannot be excited in this range [3]. The absence of A_0 escaped attention due to the appearance of Sholte–Gogoladze waves with wavenumbers close to A_0. However, their eigenforms are completely different, and the absence of A_0 means the absence of a bending wave propagating in the plate. Instead, only waves in the acoustic environment travels along its surface at about the same speed. When f exceeds the threshold frequency f_0, the root ζ_0 crosses the cut, becoming less than the sound wavenumber κ_0 in the acoustic medium.

In the present work we study how this hidden-mode phenomenon manifests itself in the anisotropic case. Unlike the isotropic case, the wavenumbers ζ_n become dependent on the propagation direction, and the leaky guided waves generated by a local source become quasi-cylindrical. Accordingly, the threshold frequency f_0 also becomes angle dependent. This leads to the possibility of sectoral excitation of the fundamental flexural wave in a submerged anisotropic plate in a certain frequency range. In the talk, we analyze and discuss this phenomenon, illustrating it with examples of root trajectories in the two-sheet Riemannian complex plane and angular diagrams of the velocity, amplitude, and energy of quasi-cylindrical guided waves excited in layered samples with different types of anisotropy, in particular, in carbon fiber-reinforced polymer (CFRP) composite plates.

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Combined numerical-analytical evaluation of dominant guided waves generated at the edge of a composite plate by a focused high-frequency acoustic beam

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The velocity of ultrasound propagation along inner sublayers of composite laminate plates is determined by the group velocity of the most significant guided waves (GWs) generated by external dynamic loading. These waves are poorly excited from the surface of the plate, so experimental measurements are carried out by exposing a focused high-frequency acoustic beam onto its side face to generate pulses propagating in the desired sublayer [1]. Microacoustic high-frequency measurements are simulated using a semi-analytical Green's matrix based model for guided wave excitation and propagation in anisotropic laminate structures [2].

To select the dominant GW modes excited in the sublayer, dispersion characteristics and normal mode eigenforms are first calculated and analyzed. Then, the wave energy of each mode is evaluated using a combined numerical-analytical approach. It includes determining the plate-end displacements using the FEM package COMSOL, and calculating the modal amplitude coefficients using explicit representations obtained on the basis of formulas derived for the normal mode generalized orthogonality in an arbitrarily anisotropic layered waveguide. The developed approach is illustrated by numerical examples for typical plate samples. We also discuss the dependence of predicted ultrasound velocity on fiber orientation in composite sublayer and location of incident beam spot on plate edge.

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Characterization of elastic moduli of plastics in additive manufacturing using guided waves for simulation and design of prosthetic stump sleeves

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The widespread adoption of additive manufacturing in critical sectors necessitates reliable non-destructive methods for evaluating the mechanical properties of 3D-printed components. Since traditional static testing is often inapplicable to operational structures, dynamic methods based on elastic waves offer a viable solution for assessing local deformation characteristics without destruction. This study addresses an inverse coefficient problem to determine the elastic moduli of elastic isotropic waveguide. Experimental dispersion curves were extracted from B-scans of FDM-printed rectangular plates using laser Doppler vibrometry. An inverse problem was solved using the advanced unsupervised learning method proposed in [1] based on the calculation of the Fourier transform of Green's matrix [2] and fitting theoretical dispersion curves to experimental data to identify Young's modulus and Poisson's ratio. Validation against standard tensile tests demonstrated a good agreement confirming the reliability of the proposed ultrasonic approach. Investigations on pure thermoplastics and composites revealed that filler concentration significantly influences dispersion behavior, enabling accurate finite element modeling of prosthetic socket liners for enhanced patient comfort and durability.

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Two-stage method for identification of elastic moduli in FDM-printed polymers using guided wave dispersion

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Modern engineering structures require ultrasonic non-destructive testing methods, particularly based on the employment of Lamb waves, to assess integrity and mechanical characteristics. However, transforming measured signals into elastic properties remains challenging due to the complexity of multi-modal dispersion curve processing. This study compares two inverse problem approaches based on elastic guided wave characteristics for property identification. The first approach utilizes the Fourier transform of Green's matrix components to avoid computationally expensive theoretical slowness calculations [1], though it introduces potential bias. Alternatively, we propose a novel two-stage method, where the second stage minimizes slowness residuals within a local frequency window that eliminates the need for exact experimental and theoretical mode matching [2]. Our algorithm employs a nearest neighbor search across adjacent frequencies to account for dispersion curve shifts without penalizing frequency deviations. Validation was performed using synthetic data generated from theoretical slownesses with added noise and missing data points to imitate experimental conditions. Numerical analysis demonstrates that the proposed local frequency window method achieves higher reconstruction accuracy despite increased computational costs.

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An effect of grating groove geometry representations on diffraction efficiency

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It is known from mathematics that a discontinuous function cannot be approximated by a finite sum of continuous harmonic oscillations, an effect known as the Gibbs phenomenon. This and related effects can pose challenges when different approximation and smoothing techniques are applied to functions whose values, along with those of their first and second derivatives, determine the accuracy of the solution of the corresponding Helmholtz equation. In this work, various representations of diffraction grating groove geometry are analyzed — both ideal types (triangular, rectangular (lamellar), sinusoidal, etc.) and those with random roughness (polygonal). Using the developed computer program GratingAnalyzer, in addition to analytical and/or point-by-point specification of groove geometry, the Fourier series expansion, linear/quadratic/cubic interpolation, as well as various smoothing filters based on a Gaussian weighting function (standard, regression, robust) are investigated. Initially, the quality of the representation is evaluated by comparing the visual or geometrical difference between the original groove geometry and the converted geometry obtained using a given method. Finally, the quality of groove profile decompositions is evaluated by computing the diffraction efficiency in the PCGrate™ computer program [1], which is based on a rigorous boundary integral equation method [2].

The examples of the efficiency of gratings with original and high-precision converted profiles are presented in the present work. The comparing shows that the peak efficiency can be both significantly ($\sim$10%) higher or lower, depending on the groove profile type and the parameters of the incident radiation. Furthermore, in some cases, the scanning under a parameter leads to the distorted shape of the diffraction efficiency curve, as well as to a noticeable redistribution of energy between diffraction orders. Significant differences in the obtained diffraction efficiency are observed, first and foremost, for gratings designed for the short-wavelength range — from hard X-rays to the vacuum ultraviolet [3]. This is due to the small ratio of the wavelength to the groove period, but with the groove depth comparable to the wavelength.

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Hyperbolic properties of $SL(2,\mathbb{R})$-cocycles over irrational rotation

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We study a one-parameter family of skew-products, F_t, defined for any $(x, v) \in \mathbb{T}^1 \times \mathbb{R}^2$ by

$$(x, v) \mapsto (\sigma_\omega(x), A_t(x)v), \quad t \in [a, b],$$

where $\sigma_\omega(x) = x + \omega$ is the rotation of a circle $\mathbb{T}^1$, $\omega \in \mathbb{R}\backslash\mathbb{Q}$ and transformation $A_t \in C^2(\mathbb{T}^1, SL(2,\mathbb{R}))$ is assumed to be of the form:

$$A_t(x) = R(\varphi_t(x)) \cdot Z(\lambda_t(x)).$$

Here $R(\varphi)$ stands for the rotation in $\mathbb{R}^2$ over an angle φ, $Z(\lambda) = \mathrm{diag}\{\lambda, \lambda^{-1}\}$ is a diagonal matrix and families of C^2-functions $\varphi_t : \mathbb{T}^1 \to 2\pi\mathbb{T}^1$, $\lambda_t : \mathbb{T}^1 \to \mathbb{R}$ satisfy the following conditions

$$\begin{aligned}
&(H_1) \quad \mathcal{C}_0 \equiv \{x \in \mathbb{T}^1 :\ \cos(\varphi_t(x)) = 0\} = \{c_0\} \cup \{c_1(t)\},\\
&(H_2) \quad \left|\varphi_t'(x) - C_j\varepsilon^{-1}\right| \le C_2\varepsilon^{-1}|x - c_j|, \quad \forall\, x \in U_\varepsilon(c_j),\ j = 0,1,\\
&(H_3) \quad \left|\cos(\varphi_t(x))\right| \ge C_3, \quad |\varphi_t'(x)| \le C_4, \quad \forall\, x \in \mathbb{T}^1 \setminus \bigcup_{j=0}^{1} U_\varepsilon(c_j),\ j = 0,1,\\
&(H_4) \quad \mathrm{ind}(\varphi_t) = 0,\\
&(H_5) \quad \lambda_t(x) \ge \lambda_0 \gg 1, \quad |\lambda_t'(x)| \le C_5\lambda_0, \quad \forall\, x \in \mathbb{T}^1,\\
&(H_6) \quad c_1'(t) > C_6,\ \forall\, t \in [a,b].
\end{aligned}$$

where C_k are positive constants, $U_\varepsilon(x)$ is a ε-neighborhood of a point $x \in \mathbb{T}^1$ and $\mathrm{ind}(\varphi)$ stands for the index of a closed curve $\varphi(\mathbb{T}^1)$.

Such a skew product generates a cocycle $M(x,n) = A(\sigma_\omega^{n-1}(x)) \dots A(x), n \in \mathbb{N}$. We study the problem of constructive description of a set of those parameter values, which correspond to the uniform hyperbolicity (UH) of the cocycle. In the case of $SL(2,\mathbb{R})$-cocycles UH is equivalent to existence of positive constants C, Λ_0 such that

$$\|M(x,n)\| \ge C\Lambda_0^n, \quad \forall x \in \mathbb{T}^1,\ n \ge 0. \tag{1}$$

Despite all matrices $A(\sigma_\omega^k(x))$ are hyperbolic, estimate (1) may fail due to presence of the critical set $\mathcal{C}_0$. Each time a trajectory of a point $x \in \mathbb{T}^1$ under the rotation σ_ω falls into a small neighborhood of the set $\mathcal{C}_0$, the hyperbolic properties of the cocycle are weakened. Since σ_ω is ergodic, every point $x \in \mathbb{T}^1$ reaches a small neighbourhood of $\mathcal{C}_0$ after some iterations. Thus, hyperbolic properties of the cocycle are governed by the dynamics of the critical set.

In [1, 2] it was obtained sufficient conditions on ω and t, which guarantee UH due to secondary collisions of different parts of $\mathcal{C}_0$. Similar to [3], we count such collisions to estimate the Lebesgue measure of the set of correspondent parameter values. In addition, we describe a set of parameter values associated to sequences of primary collisions which lead to the loss of UH.

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Stability of a fast traveling wave in a hyperbolic equation

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A semilinear hyperbolic equation is considered, which is reduced to the form

$$\delta\frac{\partial^2\varphi}{\partial t^2} - \frac{\partial^2\varphi}{\partial x^2} + \frac{\partial\varphi}{\partial t} + f(\varphi) = 0, \quad x \in \mathbb{R},\ t > 0; \quad \delta = \mathrm{const} > 0. \tag{1}$$

It is assumed that

$$f(0) = f(1) = 0; \quad f'(0) > 0, \quad f'(1) < 0.$$

Zeros of the function $f(\varphi)$ are trivial solutions – the equilibria of the equation (1). One of them $\varphi \equiv 0$ will be dynamically stable, the other $\varphi \equiv 1$ is not stable. A traveling wave is a function $\Phi(s)$ that depends on a single variable of the type $s = x - S(t)$. The exact solution of the equation (1) in the form of a wave with constant velocity $V = S'(t) = \text{const} > 0$ is found from an ordinary differential equation

$$(\delta V^2 - 1)\frac{d^2\Phi}{ds^2} - V\frac{d\Phi}{ds} + f(\Phi) = 0. \tag{2}$$

Solution of an ordinary differential equation (2) with boundary conditions

$$\Phi(s) \to 0 \ \text{ at } s \to -\infty, \quad \Phi(s) \to 1 \ \text{ at } s \to +\infty$$

for the partial differential equation (1) describes the transition (wave) from stable equilibrium $\varphi \equiv 0$ to unstable $\varphi \equiv 1$. In contrast to the known results of [1], the case of a fast (supersonic) wave is considered when $V > 1/\sqrt{\delta}$. The conditions of the existence of such waves are revealed and their stability with respect to the disturbance of the initial data is analyzed. The analysis of a non-self-adjoint spectral problem forms the basis for identifying a class of initial functions relative to which the wave is stable.

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Solutions of the wave equation with arbitrary functions: A short review of relatively undistorted waves

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A review of important classes of solutions of the wave equation with three spatial variables,

$$u_{xx} + u_{yy} + u_{zz} - \frac{1}{c^2}u_{tt} = 0 \ , c = const > 0 \, ,$$

which involve arbitrary functions, is given. We address the relatively undistorted waves having the form

$$u = gf(\theta).$$

Here, the waveform f is an arbitrary function of one variable, and the phase $\theta = \theta(x, y, z, t)$ and the amplitude $g = g(x, y, z, t)$ are fixed functions. Our interest in such solutions is inspired primarily by the problem of constructing convenient expressions for describing localized wave beams and packets.

Four classes of relatively undistorted waves and some of their applications, particularly in Optics, are considered. We start with classical plane waves and their simple generalizations. Then we proceed to complexified spherical waves, known as “complex source wave fields”. Further, we consider the Bateman wave solution and its various generalizations. Finally, we address the recently introduced quasi-spherical waves. In connection with them, the rather new concept of unidirectional waves is discussed.

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Wave diagnostics of ionospheric plasma via spatial signal processing

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Satellite-based methods for ionospheric research provide a unique capability for global monitoring with high temporal and spatial resolution. These methods typically utilize a radio transmitter on an orbiting satellite that emits signals at regular intervals, effectively synthesizing an antenna array. A stationary ground-based receiver array (spatial grid) records the scattered radio waves after they propagate through the inhomogeneous plasma. By scanning the medium from multiple angles, a set of phase projections is obtained and used to reconstruct the plasma's physical properties. Satellite diagnostics are generally categorized into ray-tracing (beam) and diffraction methods. Ray-tracing techniques, based on the geometric optics approximation, are well-established and suitable for reconstructing structures larger than the Fresnel radius. However, to resolve plasma inhomogeneities smaller than the Fresnel radius, diffraction-based methods are required. Given the absence of a rigorous closed-form solution to the Helmholtz equation for inhomogeneous plasma, asymptotic wave field methods are employed. The Double-Weighted Fourier Transform (DWFT), based on a mixed coordinate-momentum representation, accounts for diffraction and multipath effects even under conditions of strong phase and amplitude fluctuations, without requiring prior knowledge of the inhomogeneity's location. This paper evaluates the potential of spatial field processing for satellite diagnostics of ionospheric plasma with super-Fresnel resolution. Specifically, it compares DWFT-based spatial processing of Low Earth Orbit (LEO) satellite signals against single-weighted processing based on Fresnel inversion. We present numerical simulations of satellite signal phase variations, calculated using various spatial processing techniques for radio waves propagating through multiple small-scale irregularities. The results demonstrate that when environmental parameters change slowly relative to the array synthesis time, DWFT-based processing achieves sub-Fresnel resolution without requiring information on the localization of the irregularities—a distinct advantage over single-step processing methods.

On non-homeomorphic surfaces with close DN maps

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It is known ([1]) that the topology of a surface (M, g) with boundary Γ is unstable under small perturbations of its DN map Λ. These perturbations can be realized by gluing in small handles or Möbius bands inside the surface.

In the talk, we discuss the following converse statement [2, 3]. Let (M', g') be a surface of genus $m' > \operatorname{gen}(M)$ with boundary Γ and DN map Λ' and let X' be its Schottky double obtained by gluing two copies of M' along the boundary. Denote by $L(X')$ the length of the shortest closed geodesic in the hyperbolic metric on X'. Then

$$\Lambda' \to \Lambda \text{ in } B(H^1(\Gamma); L_2(\Gamma)) \implies L(X') \to 0.$$

In other words, if Λ' is close to Λ in the operator norm, then X' contains closed curves whose tubular neighborhoods are conformally equivalent to thin cylinders. Thus M' contains thin domains that impede the flow of currents du^f between different parts of M', thereby effectively reducing its genus observed from the boundary.

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On stability for rotational Stokes waves with constant vorticity

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We investigate the stability properties of steady two-dimensional surface gravity waves in a channel bounded below by a flat rigid boundary, under the assumption of constant vorticity and negligible surface tension. We consider an analytic branch of Stokes waves bifurcating from a subcritical laminar uniform flow, following the framework developed for constant vorticity in [1]. In contrast to classical formulations that fix the spatial period, we treat the period as the primary bifurcation parameter while the Bernoulli constant is fixed (see [2]).

Our analysis centres on the spectral problem associated with the Fréchet derivative of the operator of the nonlinear boundary problem, reduced to a fixed domain via a domain-flattening transformation. We establish that the first eigenvalue remains strictly negative along the entire bifurcation branch. Consequently, the stability characteristics in a neighborhood of the bifurcation point are governed entirely by the second eigenvalue.

Through rigorous analysis, we derive a direct proportionality, with opposite sign, between the second eigenvalue and the leading-order coefficient in the expansion of the period. Thus, a positive second eigenvalue corresponds to a monotonically increasing period along the bifurcation curve, while a negative sign implies period decrease.

We analyze the dependence of the second eigenvalue on the constant vorticity and the laminar flow depth. We show the existence of a depth $d_0(a)$ for each fixed vorticity such that the eigenvalue is positive for $d < d_0(a)$ and negative above this threshold, thereby confirming the principle of exchange of stability in the former regime, as discussed in the context of general bifurcation theory [3].

Another finding concerns the geometric interplay between the depth $d_0(a)$ and the depth $d_s(a)$ at which a stagnation point appears in the laminar velocity profile. For positive vorticity, $d_0(a) < d_s(a)$. In contrast, for negative vorticity, we identify a parameter domain where the second eigenvalue remains strictly positive despite the presence of a counter-current near the channel bottom.

The sign of the second eigenvalue also plays a decisive role in the onset of subharmonic bifurcations, providing a precise spectral criterion for the emergence of secondary wave patterns (see [4] for related subharmonic analysis). Beyond spectral stability, we investigate the property of formal stability, defined by the positivity of a quadratic form generated by the Dirichlet–Neumann operator. By constructing asymptotics of the associated bilinear form, we find the domain in the parameter space where formal stability is preserved.

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Linear deep-water waves: from the bulk problem to local models

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A hierarchy of mathematical models describing linear surface gravity waves on deep water is analyzed. The transition from the classical three-dimensional boundary-value problem for the fluid velocity potential to the two-dimensional dynamics equations on the free surface is considered. Three main formulations for the two-dimensional equations are described: the bulk formulation (in terms of the potential), the nonlocal formulation (integro-differential, via pseudodifferential operators), and the strictly local formulation (a fourth-order differential equation in time). The corresponding energy conservation laws and expressions for the energy flux vector are derived. The advantages and disadvantages of each approach are discussed in the context of analytical and numerical modeling.

Radiation pattern of a system of concentric circular loop antennas in a magnetoplasma

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A desire to improve operational performance of electromagnetic sources used to launch whistler waves in a magnetoplasma has stimulated interest in various excitation systems consisting of loop antennas [1]. However, despite numerous works on the subject, very little discussion of the radiation patterns of even fairly simple antennas operating in a magnetoplasma at the whistler frequencies can be found in the literature. By the radiation pattern, we mean a quantity representing the angular distribution of the radiated power per unit solid angle. This quantity gives the power radiated from a transmitter to unit solid angle in a given direction and, thus, allows one to estimate the usefulness of a particular transmitter in performing experimental tasks. In addition, integrating the radiation pattern over the total solid angle yields the radiated power of the transmitting system. The available works dealing with the analysis of the radiation patterns of electromagnetic sources in a magnetoplasma present results only for elementary electric and magnetic dipoles [2] and for a single loop antenna of finite radius [3, 4].

In this work, we perform a rigorous analysis of the radiation pattern of a system consisting of concentric circular loop antennas immersed in a homogeneous cold magnetized plasma that is modeled upon the Earth's ionosphere. In the case where the loop axes are aligned with an external static magnetic field, an explicit expression for the radiation pattern of the described system is obtained. This expression is specialized to the radiation from such an antenna system in the whistler frequency range. It is shown that in the upper part of this range, which lies above half the electron gyrofrequency, the considered system of circular loop antennas with appropriately chosen radii and currents of the loops can be useful for controlling the directional properties of the whistler wave radiation in the ionospheric plasma.

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Energy radiation characteristics of a planar concentric circular array of magnetic loop antennas in a magnetoplasma

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There is currently a strong interest in the whistler wave radiation from phased antenna systems in a magnetoplasma. Most studies on the subject have addressed radiation characteristics of circular phased arrays that are capable of exciting waves carrying orbital angular momentum in a homogeneous magnetoplasma [1]. A majority of these works consider antenna arrays with the radiating elements located on a circle of given radius. To extend the possible applications of antenna systems with circular geometry, their elements can be placed along circles of different radii [2]. In this work, we consider the radiation characteristics of a two-dimensional planar array consisting of magnetic loop antennas placed uniformly along concentric circles in the case where the frequency of the loop-antenna currents lies in the resonant part of the whistler range. This frequency region is known to be of significant importance for some applications related to communication over long distances in the ionosphere as well as its diagnostics.

We assume that the symmetry axes of the loops in the array are aligned with an external static magnetic field superimposed on the plasma. Using the approach developed in [3, 4], we have derived a rigorous representation for the field of the described array and obtained its total radiated power, partial powers going to different azimuthal field harmonics, and distributions of these partial powers over the spectrum of transverse wave numbers of the excited whistler waves. Conditions have been determined under which the choice of the radial positions and currents of the array elements makes it possible to ensure desired forms of such distributions of the partial powers referring to the given azimuthal field harmonics that are selectively excited due to appropriate phasing of these elements. Numerical calculations of the above-mentioned energy radiation characteristics of the described antenna array have been performed for plasma parameters typical of the Earth's ionosphere and corresponding model laboratory experiments.

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Inter-channel interaction evaluation in a multichannel fiber optic communication line

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The current research considers the problem of the dependence of inter-channel interaction on the distance between channels in the frequency domain (channel spacing, $\Delta\omega$) based on the coupled nonlinear Schrödinger equation (NLSE), also known as the Manakov equations. The other channel influence on the channel of interest (COI) is observed after filtration on the receiver and further COI compensation. The Bit Error Rate (BER) is used as a measure of interaction. The effect of the residual level of interaction with the increase in channel spacing has been discovered. Further research is needed to examine the validity of the results for sufficiently larger channel number.

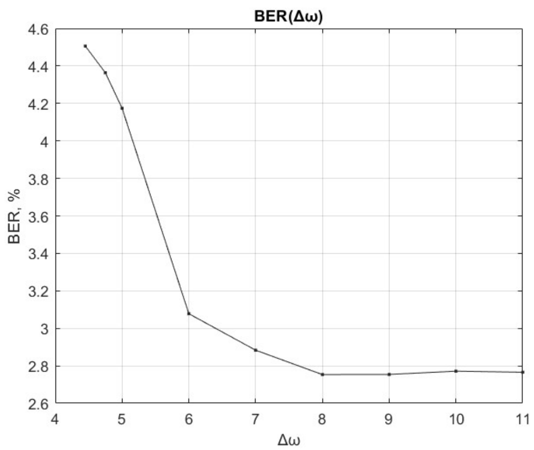

Fig. 1: Bit error rate of COI signal dependence on the channel spacing for two channels for 1600km-length fiber line.

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Controlling structured light in a chain of Sagnac resonators

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We have analytically studied the propagation of optical vortices (OVs) and Laguerre–Gaussian (LG) modes with azimuthal index $l > 1$ in a chain of sequentially connected Sagnac resonators (SRs) within the scalar approximation. We have demonstrated that in this system generation of frequency combs in the transmitted light is possible for both OVs and LG-modes. We have established that the chain of SRs can be used for the generation of frequency combs in the transmitted light for even and odd LG-modes, which are distinct from each other. Also, we have shown that the presence of a defect in the chain can induce high group delay at some wavelength.

Asymptotic evaluation of three-dimensional integral describing the sound radiation by moving source in the periodic medium

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The acoustic radiation of point source starting to move at time $t = 0$ in a two-dimensional medium (x_1, x_2) with parameters periodically dependent on the x_1 coordinate is considered. The wave equation for velocity potential ϕ is as follows:

$$\frac{\partial^2\phi}{\partial x_1^2} + \frac{\partial^2\phi}{\partial x_2^2} - \frac{1+\eta\sin(2\pi x_1/L)}{c_0^2}\frac{\partial^2\phi}{\partial t^2} = -Vv\delta'(x_1 - vt)\delta(x_2)\Theta(t), \tag{1}$$

where c_0 is the undisturbed speed of sound in the medium, η is relative amplitude of speed of sound change, L is the period of speed of sound change, V is the source volume, v is the source speed, δ, δ' and Θ are delta Dirac function, its derivative and Heavyside function respectively. The velocity potential ϕ can be represented as follows:

$$\phi(x_1, x_2, t) = \sum_{j=1}^{\infty} A \int_{-\pi}^{\pi}\int_{-\infty}^{+\infty}\int_{-\infty+i\epsilon}^{+\infty+i\epsilon} \frac{\varpi\, B_j(\xi_1,\xi_2)\,\Phi_j(\xi_1,\xi_2,\hat{x}_1)e^{i\Lambda(\xi_1+\xi_2 x_2/L-\varpi c_0 t/L)}}{C_j(\xi_1,\xi_2)\,\left(\varpi^2 - \varpi_j^2(\xi_1,\xi_2)\right)(1-e^{i(\mu\varpi-\xi_1)})} d\varpi\, d\xi_2\, d\xi_1, \tag{2}$$

where Λ and $\hat{x}_1$ are respectively integer and fractional part of x_1/L, $\mu = c_0/v$, $\varpi_j(\xi_1, \xi_2)$ and $\Phi_j(\xi_1, \xi_2, \hat{x}_1)$ are respectively eigenvalues and eigenfunctions of homogeneous version of Fourier transform of (1),

$$A = -\frac{iVc_0^2}{8\pi^3 L^3}, \qquad B_j(\xi_1,\xi_2) = \int_0^1 \exp\{i\mu\,\varpi_j(\xi_1,\xi_2)\,\hat{x}_1\}\,\Phi_j(-\xi_1,\xi_2,\hat{x}_1)\,d\hat{x}_1,$$

$$C_j(\xi_1,\xi_2) = \int_0^1 (1+\eta\sin(2\pi\hat{x}_1))\,\Phi_j(\xi_1,\xi_2,\hat{x}_1)\,\Phi_j(-\xi_1,\xi_2,\hat{x}_1)\,d\hat{x}_1.$$

Integral (2) is asymptotically analyzed using a method previously developed by the authors of this abstract [1]. Two types of waves are identified, corresponding to singular points of the integral: the first type corresponds to the source's motion in the medium, while the second type corresponds to the transient process occurring at the onset of motion. Topological conditions for the presence of the corresponding contributions to the field are obtained, the physical meaning of these conditions is discussed.

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Radiation pattern of a resistively loaded GPR antenna at the interface between two media

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Ground Penetrating Radar (GPR) is a pulsed radar that enables non-destructive subsurface surveying. The design of a deep-penetration GPR [1, 2] is based on a high-voltage gas discharger and dipole antennas with a resistive load implemented according to the Wu–King scheme [3] to prevent pulse reflections from the antenna ends. Due to the increased power of the probing pulse, GPR can be used in archaeological and geological tasks, reaching depths up to hundreds of meters. Currently, practical deployment schemes for large low-frequency antennas are being actively developed. In rough terrain conditions, an optimal solution is a tandem configuration of transmitting and receiving antennas (Fig. 1).

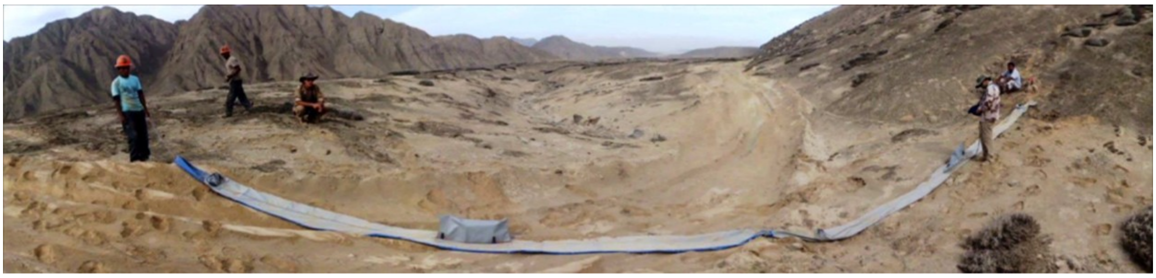

Fig. 1: Tandem configuration of low-frequency GPR antennas [1].

For successful implementation of a GPR set, it is necessary to study the transmitter and receiver directivity patterns in the subsurface medium.

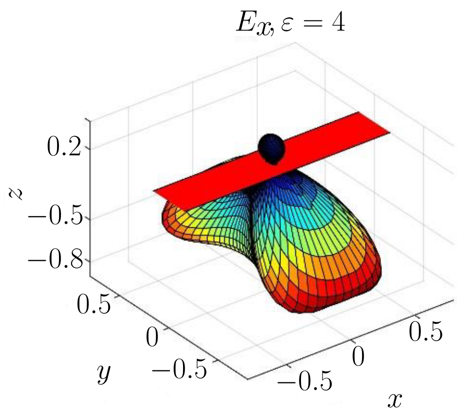

Fig. 2: Three-dimensional directivity pattern of a strip dipole antenna at the air-ground interface [4].

In [4], the directivity pattern of a conducting strip antenna lying on a dielectric half-space was analyzed. As shown in Fig. 2, presenting the result for a refractive index value ($n = 2$), the radiation pattern exhibits a minimum along the dipole axis, reducing the transmitter and receiver antennas coupling and GPR efficiency in real-world applications (Fig. 1). The first attempt to control the radiation directivity pattern through asymmetric excitation of the dipole arms has been reported in [5].

In this report, we investigate the space-time directivity pattern for an arbitrary resistance distribution along the arms of a GPR dipole antenna. This will enable an optimal sounding configuration being realized and corrections introduced into data processing algorithms.

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Study of the efficiency of multilayer spherical piezoelectric transducers from porous piezoceramics in an acoustic medium

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Spherical transducers made of porous piezoceramics were developed at the Research Institute of Physics of the Southern Federal University for ultrasound medical applications. These transducers had electrode-coated spherical surfaces and were made of porous piezoelectric ceramics, polarized by radius. Various configurations of these devices have been used for ultrasonic liposuction of adipose cells, for heating biological tissue, and as the active element of a lithotripter. Porous piezoceramics have a lower acoustic impedance than dense piezoceramics, and therefore better acoustic matching with the working acoustic medium, particularly soft biological tissue. Moreover, porous piezoceramics have relatively large longitudinal piezoelectric charge moduli, hydrostatic coefficients, and quality factors. Piezoelectric transducers made of dense piezoceramics typically use transition matching layers to reduce acoustic impedance, but for porous piezoceramics, it is possible to eliminate the matching layers, simplifying the piezoelectric transducer design.

In this study, we conducted a comparative analysis of single-layer, two-layer, and three-layer piezoelectric transducers with varying porosities of ferroelectric hardness piezoelectric ceramics. In the calculations, effective moduli of porous piezoceramics were used, found from the solution of individual homogenization problems considering various polarization models (uniform and non-uniform) [1]. Preliminary computations were performed using numerical and analytical one-dimensional models for plane and spherical piezoelectric transducers. For actual piezoelectric transducer designs in the form of spherical segments with a process hole in the center, finite element modeling in the ANSYS APDL package in an axisymmetric formulation was applied. In a series of calculations, the piezoelectric transducer thicknesses were varied depending on the ceramic porosity to ensure approximately equal main operating frequencies of thickness resonances.

An analysis of spherical transducers was also carried out when they were excited by various electrical influences at the frequencies of electrical resonances and antiresonances [2].

Simplified one-dimensional models allowed for optimization calculations, the results of which were used to select suitable configurations for real structures. In specific cases, the numerical results were compared with experimental data. A fairly good agreement with the experimental data was achieved, and a comparative analysis of a series of computational experiments concluded that porous piezoceramics are effective in spherical medical ultrasound transducers without matching layers.

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Development of a mathematical model for the analysis of resonant vibrations of a string of varying length on an elastic foundation, taking into account dissipative factors

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This paper examines the problem of transient processes in distributed systems with variable geometry, characterized by changing boundary conditions over time. The transverse vibrations of a string with a moving boundary are studied as a model example. The complexity of mathematical modeling of such systems stems from the need to account for a number of factors: boundary motion, the presence of an elastic foundation, dissipative forces, and bending stiffness. The goal of this paper is to conduct an analytical study of the transient dynamics of distributed systems with variable structure and to develop an effective method for their analysis, enabling highly accurate prediction of dynamic behavior, including the analysis of resonance properties.

To analyze the complex transient dynamics of a string with a moving boundary, the Kantorovich-Galerkin method is used, which allows for consideration of factors such as an elastic foundation, dissipative forces, and bending stiffness. As a result, a second-order numerical-analytical solution is obtained with respect to a small parameter characterizing the slow change in boundary conditions. Based on this solution, explicit analytical expressions were obtained for the key dynamic characteristics of the system, which is important for predicting the behavior of such complex systems.

Of particular practical value are the established quantitative estimates of oscillation amplitudes for two critical modes: steady-state resonance and the process of passing through resonance. Furthermore, the mechanism of oscillation excitation by harmonic action on a moving boundary—the most relevant case for engineering applications—was analyzed in detail.

The obtained results are of significant practical significance, as the developed methodological framework and analytical expressions enable quantitative evaluation of the dynamic behavior of a wide range of engineering structures and technical systems of variable length. This provides a basis for engineering calculations and design of objects such as flexible elements of modern robotics, transport, and lifting systems, where the variable geometry of the structure is a key factor. Analysis of resonant modes is of particular practical importance, as it enables the prediction and prevention of dangerous oscillations.

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Fredkin gate based on an add-drop Möbius resonator

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We have analytically studied an add-drop Möbius resonator based on a single-mode fiber. Using the transfer-matrix formalism we have built a theory for such a resonator and shown that this system can function as a Fredkin gate for fundamental modes with left and right circular polarizations.

Propagation of nonlinear coupled TE-TM waves in a planar waveguide filled with nonlinear media: analytic and numerical results

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We consider a problem of propagation of the so-called *nonlinear coupled TE-TM wave* [1] in a planar waveguide filled with spatially inhomogeneous nonlinear media and sandwiched between two half-spaces filled with linear media.

This physical problem is reduced to a nonlinear two-parameter eigenvalue problem for the system

$$\begin{aligned} \gamma_1^2 X(x) - \gamma_1 Z'(x) &= \varepsilon_1(x)X(x) + \alpha_1\left(X^2(x)+Z^2(x)\right)X(x) + \beta Y^2(x)X(x), \\ \gamma_2^2 Y(x) - Y''(x) &= \varepsilon_2(x)Y(x) + \alpha_2 Y^3(x) + \beta\left(X^2(x)+Z^2(x)\right)Y(x), \\ \gamma_1 X'(x) - Z''(x) &= \varepsilon_1(x)Z(x) + \alpha_1\left(X^2(x)+Z^2(x)\right)Z(x) + \beta Y^2(x)Z(x), \end{aligned} \tag{1}$$

where $x \in [0,h]$, h is the thickness of the waveguide, X, Y, Z are unknown real-valued functions, $\alpha_j > 0$, γ_j are real numbers (propagation constants), and $\varepsilon_j(x)$ are continuous real-valued functions such that $\varepsilon_j > 0, \varepsilon_j' \geqslant 0$, $\beta > 0$ is a small perturbation coefficient, with the transmission conditions

$$\begin{aligned} \varepsilon_1(0)\sqrt{\gamma_1^2-\varepsilon_s}X(0) - \varepsilon_s\gamma_1 Z(0) = 0, \quad Y'(0) - \sqrt{\gamma_2^2-\varepsilon_s}Y(0) = 0, \\ \varepsilon_1(h)\sqrt{\gamma_1^2-\varepsilon_c}X(h) - \varepsilon_c\gamma_1 Z(h) = 0, \quad Y'(h) - \sqrt{\gamma_2^2-\varepsilon_c}Y(h) = 0, \end{aligned}$$

and additional conditions

$$X(0-0) = X_0 > 0, \quad Y(0-0) = Y_0 > 0,$$

where ε_c and ε_s are dielectric permittivities in the half-spaces $x < 0$ and $x > h$, respectively, X_0, Y_0 are fixed constants.

We look for positive numbers $\gamma_1 = \hat{\gamma}_1$, $\gamma_2 = \hat{\gamma}_2$ such that there exist solutions $X(x) \in C^1(\bar{\mathrm{I}})$, $Y(x) \in C^2(\bar{\mathrm{I}})$, $Z(x) \in C^2(\bar{\mathrm{I}})$ of system (1) satisfying the above listed transmission and additional conditions.

To study this problem, we apply the non-standard perturbation method which was previously used in works [2, 3] to deal with similar problems for nonlinear non-autonomous differential equations. As any perturbation method, this approach assumes searching for solutions of the original problem near solutions of a simpler (nonperturbed) problem. This makes it possible to find so-called non-linearizable solutions of the studied nonlinear problem if the unperturbed nonlinear problem has such solutions. We prove existence of linearized as well as nonlinearized solutions in the main problem and present numerical results.

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On the dependence of acoustic modes on media parameters in the Pekeris–Airy waveguide

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A waveguide consisting of an isospeed water layer overlying a halfspace with a linear squared-refractive-index profile (Airy medium) is considered. It is shown that the dependences of the horizontal wavenumbers of normal modes on frequency, water depth, and bottom geoacoustic parameters are described by ordinary differential equations that can be solved numerically at very low computational cost. This provides a substantial increase in efficiency for broadband sound propagation modelling and dispersion-based geoacoustic inversion. Explicit formulas for the derivatives of horizontal wavenumbers with respect to bottom parameters are given. A new concise proof that the spectrum of the corresponding Sturm–Liouville problem is purely discrete is proposed.

This study was supported by the Russian Science Foundation No. 22-11-00171.

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Dynamic inverse problem for doubly infinite Jacobi matrices

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We set up and solve a dynamic inverse problem for a dynamical system associated with the discrete-time dynamical system associated with the doubly infinite Jacobi matrix. The system is controlled using a specially introduced boundary triplet. We consider the case corresponding to the discrete Schrödinger operator.

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Holographic uniqueness theorems in two dimensions

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We present holographic uniqueness theorems for two-dimensional Helmholtz equation in exterior region. The first of these theorems contributes to the classical holography and phaseless inverse scattering, whereas the second one contributes to passive imaging and the Gelfand–Krein–Levitan inverse spectral problem. Our proofs use, in particular, properties of Atkinson type expansion and Karp expansion for radiation solutions in two dimensions.

This talk is based on the recent works [1, 2].

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Geometrical optics approach to integrability of nearly circular or elliptical billiards

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Recently, we proposed a generalization of the ray transfer matrix method in axially symmetric optical systems [1] beyond the paraxial approximation [2]. In this formalism the optical map between the object plane and the image plane is constructed as a composition of optical transfer maps between support planes (located at the object plane, the image plane, and the mirror or lens vertices) together with the optical maps describing reflection or refraction at those support planes. Whereas optical transfer maps describing the ray propagation in vacuum are easily computable, the derivation of ray tracing formulas for the reflection at the support plane is more involved. For simplicity let us consider the ray tracing problem in the plane, see Fig. 1. A ray starting from the point $(x, 0)$ and going into the positive y-direction $(p, q) = (p, \sqrt{1-p^2})$ is reflected at the mirror, which we assume to be described by a curve $y = f(x)$, where $f(0) = 0$ and $f'(0) = 0$. Finding the optical map for the ray reflection at the mirror requires the computation of the direction of the reflected ray $\left(\hat{d}_x, -\sqrt{1-\hat{d}_x^2}\right)$ and the intersection point $(r_x, 0)$ of the reflected ray with the line $y = 0$.

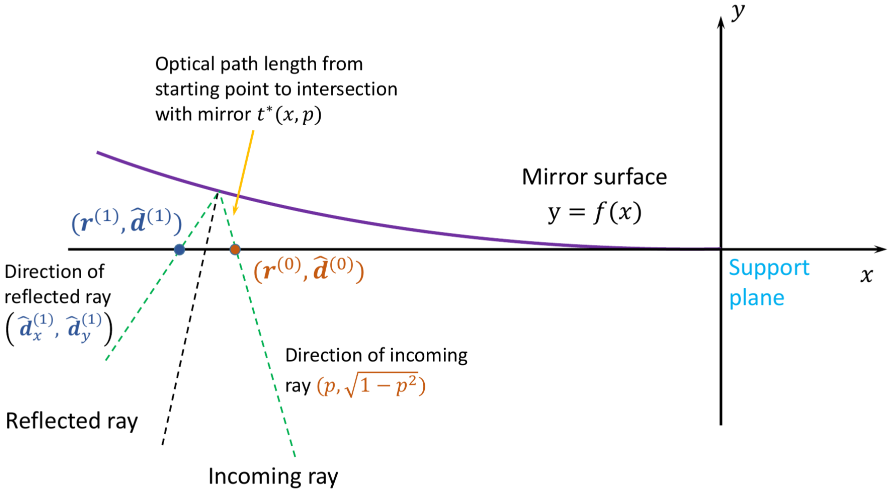

Fig. 1: Ray tracing configuration for the auxiliary problem.

Explicit reflection maps can be constructed only for a few cases: elliptic, parabolic, and hyperbolic mirrors, which have the surface profile given by

$$f_\kappa(x) = \frac{x^2}{R + R\sqrt{1-(1+\kappa)\frac{x^2}{R^2}}}, \tag{1}$$

see [3]. In all these cases there exist conservation laws, that is, quantities which are preserved after the reflection at the mirror, such as

$$\Phi_\kappa(p, q, L_z) = \frac{R^2}{(1+\kappa)^2} + \frac{R^2}{1+\kappa} - L_z^2, \tag{2}$$

as is well-known from the study of mathematical billiards [4]. The existence of such invariants is also closely connected with the Birkhoff conjecture. Note that conservation laws necessarily have to be of the form $\Phi(p, q, L_z)$, where L_z is the momentum with respect to $y^* = \frac{R}{1+\kappa}$ (see [5])

$$L_z = xq - \left(y - \frac{R}{1+\kappa}\right) p. \tag{3}$$

Using the reflection optical map, we derive an equation which gives a necessary condition for the existence of a conservation law $\Phi(p, q, L_z)$. We use this equation to show that there exist neither a conservation law $\Phi_{\kappa,\varepsilon}(p, q, L_z)$ nor a function $f_{\kappa,\varepsilon}(x)$ that depends smoothly on ε such that $\Phi_{\kappa,0}(p, q, L_z) = \Phi_\kappa(p, q, L_z)$ and $f_{\kappa,0}(x) = f_\kappa(x)$. In other words, conic sections (1) with conservation laws (2) are isolated.

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Fabry–Pérot resonances and above-barrier reflection in monolayer graphene under electrostatic and magnetic fields

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We study the scattering of electrons by an electrostatic barrier in monolayer graphene in the presence of an external magnetic field. Both the electrostatic potential and the magnetic field are assumed to be spatially inhomogeneous and localized within a finite interval, vanishing outside it. The corresponding scattering problem is solved, and the reflection and transmission coefficients, together with their phases, are derived. We assume that the fields vary sufficiently smoothly for the semiclassical approximation to be applicable.

Two mathematically distinct regimes are considered. First, for incident electron energies below the maximum of the electrostatic potential, we consider transmission through an n-p-n barrier. To determine the transmission coefficient, we separately analyze transmission through the increasing and decreasing parts of the electrostatic potential and then match the semiclassical solutions in the intermediate region. An equation for the Fabry–Pérot resonances is derived, and the qualitative features of the resulting interference pattern are discussed [1].

The second regime corresponds to electron energies close to the maximum of the electrostatic potential. Asymptotic formulas describing above-barrier reflection are derived. In the absence of a magnetic field, analogous formulas were previously obtained in [2]. We generalize the results of [2] to the case where an external magnetic field is present.

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An old/new path-integral tool for eigenvalue-eigenfunction computation in electromagnetics

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Path integrals have found limited applications in Classical Electromagnetics so far, and mostly used to model wave propagation in un-bounded (deterministic as well as random) inhomogeneous media. It turns out that an old path-integral based formula, first derived by (and credited to) M. D. Donsker and M. Kač can be used to compute EM Eigenvalues and Eigenfunctions in EM boundary-value problems with complex geometries and complex constitutive properties accurately and efficiently. In this communication we review the method, and its computational burden, including original work from the Author's Group.

Difference acoustic equation in discrete waveguides near thresholds

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The simplest example of a discrete waveguide is a discrete cylinder that is a graph periodic when shifted by a given vector and having a finite periodicity cell. Let us split a discrete cylinder into two connected parts by deleting a finite number of edges; the parts are called semi-cylinders. In this work we consider a *discrete waveguide* G consisting of several discrete semi-cylinders joined to a finite graph by a finite number of edges. The waveguide G is assumed to be embedded into the 3D Euclidean space with the norm $|x|$.

We denote the sets of graph vertices and edges by V and E, respectively; E_x stands for the set of the edges ending at x, and the second end of the edge $e \in E_x$ is denoted by $x + e$. On the graph G, we consider an equation of the form

$$-\sum_{e \in E_x} a(e)(u(x+e) - u(x)) + q(x)u(x) - \mu u(x) = f(x), \quad x \in V, \tag{1}$$

which is a discrete analogue of the acoustic differential equation $-\operatorname{div} a \operatorname{grad} u + q\, u - \mu\, u = f$. Here u and f are functions on V, and μ is a spectral parameter. The coefficients a and q are real functions on E and V, respectively. We assume that in each cylindrical outlet, the coefficients stabilize at infinity to periodic functions; the stabilization rate is exponential.

A number μ is a point of the continuous spectrum if there exists a solution $u(x)$ to the homogeneous equation (1), such that $u \notin \ell_2(V)$, but $|u(x)| \le \mathrm{const}(|x| + 1)^N$ with $N < \infty$ for all $x \in V$; such a solution is called a *continuous spectrum eigenfunction* (CSE). The CSEs are bounded ($N = 0$) for all μ in the continuous spectrum, excepting a finite number of values, which are called *thresholds*. When the spectral parameter varies on an interval between adjacent thresholds, there exists a basis of CSEs subject to the asymptotics at infinity:

$$Y_j^+(x,\mu) = u_j^+(x,\mu) + \sum_{k=1}^{\Upsilon} S_{j,k}(\mu) u_k^-(x,\mu) + o(1),\ |x| \to \infty, \quad j = 1, \dots \Upsilon. \tag{2}$$

Here, $u_1^+, \dots, u_\Upsilon^+$ stand for *incoming waves*, while $u_1^-, \dots, u_\Upsilon^-$ stand for *outgoing waves*. The matrix $S = \|S_{j,k}\|$ is called the *scattering matrix*. This form of the asymptotics is “unstable” at thresholds. The number Υ of incoming and outgoing waves remains constant on each interval between adjacent thresholds, but changes from interval to interval. Moreover, some of the waves $\mu \mapsto u_j^\pm(\cdot,\mu)$ in (2) have discontinuities at thresholds. In this work, in a neighbourhood of each threshold we construct new analytic incoming and outgoing waves $\mu \mapsto w_j^\pm(\cdot,\mu)$. Using these, CSEs are constructed that are analytic near the threshold and satisfy the “stable” asymptotics at infinity:

$$\mathcal{Y}_j^+(x,\mu) = w_j^+(x,\mu) + \sum_{l=1}^{\widetilde{\Upsilon}} \mathcal{S}_{j,l}(\mu) w_l^-(x,\mu) + o(1),\ |x| \to \infty, \quad j = 1, \dots, \widetilde{\Upsilon}, \tag{3}$$

where the error term is $o(1)$ uniformly with respect to μ. The matrix $\mathcal{S}(\mu)$ is analytic in the neighbourhood of the threshold. On the intervals adjacent to the threshold $\Upsilon \le \widetilde{\Upsilon}$, and $\Upsilon < \widetilde{\Upsilon}$ for at least one of the intervals. Near the threshold in each of the intervals, the original matrix $S(\mu)$ is expressed in terms of $\mathcal{S}(\mu)$, which allows us to prove the existence of finite one-sided limits of the matrix $S(\mu)$ at the threshold.

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The fixed angle inverse scattering problem for Riemannian metrics

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Wave propagation in an inhomogeneous acoustic medium may be modeled, for example, by the wave operators $\square + q$, $\rho\partial_t^2 - \Delta$ or $\partial_t^2 - \Delta_g$, for a real smooth function $q(x)$, a positive function $\rho(x)$, or a Riemannian metric $g(x)$ on $\mathbb{R}^n$, with $q, \rho - 1, g - I$ supported in a ball. The medium is probed by plane waves coming from a *finite* number (dimension dependent) of directions, and the resultant time dependent waves are measured on the boundary of the ball. We describe our partial results about the recovery of q, ρ, g from these boundary measurements, and the generalization to Lorentzian metrics. These are long standing formally determined open problems. These results were obtained in collaboration with Lauri Oksanen and Mikko Salo.

An inverse problem for a nonlinear acoustics equation

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A nonlinear equation that describes a process of acoustic waves propagation is considered. An inverse problem of recovering coefficient under nonlinearity depending from space variables is studied. It is supposed that the coefficient is a finite function and its support is contained in the ball B_R. The unknown coefficient is related with certain physical parameters of a medium that play very important role in the medical diagnostics. For solving the inverse problem, a direct problem related to a running wave going in a homogeneous media from infinity in the direction of the unite vector ν is considered for the nonlinear acoustics equation. The parameter ν is variable and it can run the unite sphere of the three-dimensional space. At the moment of time $t = 0$ the wave reaches the boundary of the ball B_R, then passes through B_R and registered at its boundary. An inverse problem of recovering the coefficient under nonlinearity is posed. It is assumed that the following information about the solution of the running plane wave is given: values of the derivative with respect to time of the sound pressure are known at moments arriving the wave to the boundary of the ball B_R. It is shown that the inverse problem is reduced to the well studied problem of X-ray tomography. It allows to effectively reconstruct the desired coefficient.

Topology of hodographs for electromagnetic and quantum waves

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Progress in producing increasingly short pulses of electromagnetic radiation poses new challenges to their theory and opens up new possibilities for detecting fast processes [1, 2]. The polarization structure of few- and subcycle electromagnetic pulses in a fixed spatial location r is conveniently characterized by the hodograph of electrical field $\mathbf{E}$ — the trajectory of the end of the vector $\mathbf{E}$, which is always closed for pulses [3]. We demonstrate hodographs in the form of topological knots and associated temporal polarization Möbius strips.

Within the framework of the quantum-mechanical nonrelativistic Schrödinger equation for an electron in the field of a polychromatic radiation, we present a hodograph of the average probability flux value with the shape of Lissajous knots [4]. Similar knots have been found for the Ehrenfest trajectories [5] — the time-depending mean coordinate values $\langle \mathbf{r}(t) \rangle$ for a quantum harmonic oscillator and for electron bound states, t is time. Diagnostics of such hodographs is fundamentally possible using extremely short electromagnetic pulses.

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Semi-classical resonances and adiabatic evolution

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We consider a linear adiabatic evolution problem for a semi-classical Schrödinger operator on $L^2(\mathbb{R}^n)$, with a time dependent potential, $H(t) = -h^2\Delta + V(t,x)$, where $V(t,x) = \omega(t)V(x)$ and $V(x)$ has non degenerate maximum (barrier top resonances). For an adiabatic parameter $\varepsilon \sim h^N$ we get adiabatic approximations of finite order over a time interval of polynomial size in ε, and their analytic extension to complex t. More information is available in the periodic case.

Coupled-mode theory for superscattering

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Maximizing the scattering cross-section of subwavelength nanostructures is one of the key challenges in nanophotonics [1]. Among the methods enabling the overcoming of fundamental limits and achieving the so-called superscattering regime are spectral mode overlap, mixing of scattering channels, and optical gain in the material [2–4], Fig. 1. We employ coupled-mode theory to analyze the scattering process in each case and to visualize the electromagnetic field profiles. We demonstrate the interference nature of the characteristic shadow [2, 3] behind the object in the superscattering regime, and establish the characteristic optical parameter values required to achieve it.

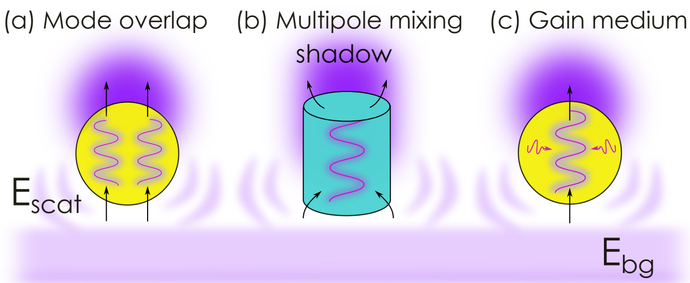

Fig. 1: Schematics of the systems supporting the three superscattering mechanisms analyzed in the work. **(a)** a spherical compact object with resonant modes coupled to separate scattering channels; **(b)** an axially symmetric compact object with a resonant mode coupled to several scattering channels; **(c)** a compact object supporting gain. The objects are illuminated by a plane wave decomposed into a series of spherical harmonics, producing scattered field and characteristic shadows.

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On isospectral periodic graphs

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We consider discrete Schrödinger operators with real periodic potentials on periodic graphs. The spectra of the operators consist of a finite number of bands. By "rolling up" a periodic graph along some appropriate directions we obtain periodic graphs of smaller dimensions called subcovering graphs. For example, rolling up a planar hexagonal lattice along different directions will lead to nanotubes with various chiralities. We describe connections between spectra of the Schrödinger operators on a periodic graph and its subcoverings. In particular, we provide a simple criterion for the subcovering graph to be isospectral to the original periodic graph. By isospectrality of periodic graphs we mean that the spectra of the Schrödinger operators on the graphs consist of the same number of bands and the corresponding bands coincide as sets. We also obtain asymptotics of the band edges of the Schrödinger operator on the subcovering graph as the "chiral" (roll up) vectors are long enough. This talk is based on the recent article [1].

The work is supported by the Russian Science Foundation (project No. 25-21-00157).

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Laplace–Beltrami equation in the framework of diffraction theory

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The Laplace–Beltrami equation on a unit sphere is considered with point sources and ideal Dirichlet scatterers that are geodesic arcs. The usual concepts of diffraction theory are applied to this problem, namely: the plane-wave decomposition, far-field directivity and the spectral equation. As the result, the set of results developed, say, for the strip problem in a plane [1, 2] becomes transferred to the spherical case.

This program seems puzzling since the sphere is compact, so there is no infinity on it, and thus a far-field directivity of a wave field cannot be defined. To overcome this circumstance, the problem is redefined on a complex projective sphere, and the solution is analytically continued to the complex domain from the real subdomain. After this trick, the asymptotical expansion of the field near the infinite points provides the directivity pattern.

A concept of a plane wave is generalized to the spherical case in the way that is also taken from the planar case. A Green's function for the spherical problem is known; it is expressed in terms of the Legendre's function and is easily complexified. The source of the Green's function is taken to infinity, and the resulting field on the real sphere is considered to be a "plane wave". There are two different plane wave for each direction of incidence. The dispersion diagram (the set of all plane waves) in this case is two copies of the 1D complex Riemann sphere. The complex structure on the dispersion diagram enables contour integration with Cauchy theorem.

The work is currently in progress. Some preliminary results are described in the preprint [3].

The work is performed in collaboration with R. C. Assier (The University of Manchester, UK) and V. D. Kunz (Ohio State University, USA).

The work is done in accordance with the state assignment of M. V. Lomonosov Moscow State University.

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On an exchange pulse associated with an avoided crossing

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It is well-known (see e. g. [1]) that the dispersion diagram of a waveguide may possess avoided crossings, i. e. patterns, when two branches of the dispersion diagram come closer and then repel from each other. Such branches exchange with the modal shapes.

As a simple waveguide with the avoided crossing pattern on its dispersion diagram, two weakly-coupled scalar Klein–Gordon equations are considered:

$$\left[\begin{pmatrix} c_1^2 & 0 \\ 0 & c_2^2 \end{pmatrix} \frac{\partial^2}{\partial x^2} + \begin{pmatrix} -\Omega_1^2 & \mu \\ \mu & -\,\Omega_2^2 \end{pmatrix} - \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \frac{\partial^2}{\partial t^2} \right] \begin{pmatrix} u_1(t,x) \\ u_2(t,x) \end{pmatrix} = \begin{pmatrix} f_1 \\ f_2 \end{pmatrix} \delta(t)\delta(x), \tag{1}$$

where t is time, x is longitudinal coordinate, $c_{1,2}$, $\Omega_{1,2}$, μ are the system's parameters, $u_{1,2}(t,x)$ describe the propagating field, the right-hand side of (1) describes excitation.

The non-diagonal element μ describes the coupling, μ is considered as a perturbation. The avoided crossing is formed near the phase synchronism of the unperturbed dispersion diagrams of the scalar Klein–Gordon equations.

The solution of (1) may be obtained with the Fourier transform with respect to x and the Laplace transform with respect to t. As a result, a double integral over wavenumbers k and frequencies ω is obtained. This integral may be estimated using the technique from [2]. Another approach is to apply the residue theorem to the inner integral of (1) to obtain the sum of integrals, which may be estimated with Cauchy's theorem [3] and the saddle point method [4].

There exists a special transient wave phenomenon, namely the exchange pulse [3] in the perturbed system. This pulse is highly monochromatic (the frequency is close to that of phase synchronism of unperturbed modes). Moreover, such an exchange pulse possesses the beginning and the end, namely, it is mainly localized in the domain $x/v_2 < t < x/v_1$, where v_1 and v_2 are the group velocities of the unperturbed modes at the point of phase synchronism ($v_2 > v_1$). Here we are going to describe the exchange pulse using all the listed approaches and compare the results.

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A method for constructing meromorphic function fields on coverings of higher-genus surfaces for solving diffraction problems

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As shown in [1, 2], the topology of the dispersion surface associated with a discrete diffraction problem can be used effectively to construct solutions. In particular, for a planar discrete lattice the dispersion relation can be written as

$$x + x^{-1} + y + y^{-1} + K^2 - 4 = 0. \tag{1}$$

This equation defines a dispersion surface $\mathbf{D}$ with the topology of a torus (genus $g = 1$). In [2], the diffraction by a Dirichlet right angle on a discrete planar lattice was solved using a Sommerfeld-type integral representation, in which the Sommerfeld transformant is a meromorphic function on a three-sheeted covering of $\mathbf{D}$, denoted as $\mathbf{D}_3$. Constructing this transformant requires an explicit basis for the field of meromorphic functions on the covering surface $\mathbf{D}_3$.

For other diffraction problems the dispersion surface may have a more complicated topology (genus $g > 1$). One example is a model anisotropic coupled system

$$\alpha\partial_m^2 U_1(m,n) + \partial_n^2 U_1(m,n) + \beta U_1(m,n) + \gamma U_2(m,n) = 0, \tag{2}$$

$$\partial_m^2 U_2(m,n) + \alpha\partial_n^2 U_2(m,n) + \beta U_2(m,n) + \gamma U_1(m,n) = 0, \tag{3}$$

where

$$\partial_m^2 U_i(m,n) = U_i(m+1,n) + U_i(m-1,n) - 2U_i(m,n),$$

$$\partial_n^2 U_i(m,n) = U_i(m,n+1) + U_i(m,n-1) - 2U_i(m,n)$$

are standard second-order discrete difference operators. Here $m, n \in \mathbb{Z}$ are discrete coordinates, $U_1(m,n)$, $U_2(m,n)$ are two modes of the system, α is an anisotropy parameter, β is a spectral parameter and γ is a coupling coefficient. One can show that the associated dispersion surface has genus $g = 5$.

These examples motivate the need for a general constructive procedure for constructing a basis for the field of meromorphic functions on an n-sheeted covering of a compact Riemann surface of genus $g > 1$, as required in Sommerfeld-type approaches to diffraction problems. In this work, we present such an algorithm and discuss some of its applications to diffraction problems with higher-genus dispersion surfaces.

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Traveling wave dynamics in a coupled nutrient-bacteria diffusive system

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Quantifying bacterial traveling wave behavior is essential for understanding the dynamics of population evolution, and mathematical modeling serves as the primary tool for this analysis. In natural environments, bacterial diffusion and proliferation are regulated by multiple factors, including metabolism and nutrient availability, leading to non-smooth and non-uniform characteristics of traveling wave propagation. These features pose challenges for accurate modeling. While the classical Fisher–KPP equation can describe smooth traveling wave expansion of bacterial colonies in homogeneous environments, its idealized assumptions fail to account for practical conditions such as nutrient limitation, precluding accurate capture of the non-smooth characteristics of bacterial traveling waves. To address this limitation, we construct a coupled reaction-diffusion model for bacterial density and nutrient concentration. The model captures the coupled dynamics of bacterial waves and nutrient evolution through a generalized nonlinear reaction term. The governing equations are as follows:

$$\frac{\partial B}{\partial t} = \nabla \cdot (D_B \cdot R(\mathbf{x}) \nabla B) + \frac{aN^n B^2 (b-B)}{N_{th}^n + N^n}, \quad \mathbf{x} \in \Omega,\ t \in (0,T], \tag{1}$$

$$\frac{\partial N}{\partial t} = D_N \Delta N - h\frac{\partial B}{\partial t} - vBN, \quad \mathbf{x} \in \Omega,\ t \in (0,T], \tag{2}$$

where Ω is the spatial domain; $T > 0$ is the final simulation time; $B(\mathbf{x}, t)$ is the normalized bacterial density; $N(\mathbf{x}, t)$ is the normalized nutrient concentration; $R(\mathbf{x})$ is a spatially correlated random field with unit mean, introduced to capture environmental heterogeneity; D_B, D_N are the diffusion coefficients; a, b, N_{th}, h, v are positive model parameters.

The governing equations are supplemented by initial and boundary conditions:

$$B(\mathbf{x}, 0) = B_0 \exp\left(-\frac{|\mathbf{x} - \mathbf{x_0}|^2}{q^2}\right), \quad N(\mathbf{x}, 0) = N_0, \quad \mathbf{x} \in \Omega, \tag{3}$$

$$\left.\frac{\partial B}{\partial \mathbf{n}}\right|_{\partial\Omega} = 0, \quad \left.\frac{\partial N}{\partial \mathbf{n}}\right|_{\partial\Omega} = 0, \quad t \in (0,T], \tag{4}$$

where B_0, N_0 are the initial values; $\partial\Omega$ denotes the boundary of the domain; $\mathbf{n}$ is the outward unit normal vector.

The nonlinear reaction term combines three biologically relevant mechanisms. The first factor represents logistic growth limitation by the carrying capacity, a standard formulation in population dynamics. The second factor describes nutrient-limited growth through a Monod–Hill dependence, where the Hill coefficient allows for cooperative or switch-like responses to nutrient availability. The third factor introduces a quadratic dependence on bacterial density, accounting for cooperative interactions among bacterial cells, such as quorum sensing-mediated activation of growth or matrix production. This form is consistent with models of Allee effects in microbial populations, where growth is enhanced at higher population densities due to collective behaviors.

We developed a MATLAB-based numerical platform employing the Yanenko method (operator splitting) to solve the two-dimensional partial differential equations, enabling efficient resolution of the coupled reaction-diffusion system. Computational experiments focus on characterizing wave speed while incorporating the dynamic changes in nutrient availability during bacterial growth.

The effect of multiplicity change for a system of Maxwell's equations with a localized electromagnetic field source

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In [1], an algorithm for constructing an asymptotic solution was described for a wide class of pseudodifferential systems with a localized right-hand side. Outside the neighborhood of the source, the asymptotic solution is constructed as a sum of canonical operators on Lagrangian manifolds, which are obtained by union of semi-infinite trajectories of the Hamiltonian system, issued from the zeros of the so-called effective Hamiltonians (eigenvalues of the leading part of the matrix-valued symbol on the left-hand side of the equation). It is required that all effective Hamiltonians be distinct (i.e., that no change in multiplicity occur).

In our talk, using the Maxwell system of equations as an example, we will consider cases where a change in multiplicity occurs. In these cases, modifications to the asymptotic formulas will be presented.

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Far-field behavior of nonlinear waves in a non-ideal interstellar medium

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In this paper, the far-field behavior of a governing system describing the one-dimensional motion of an inviscid, self-gravitating, and spherically symmetric non-ideal interstellar van der Waals gas cloud is investigated. A generalized Burgers' equation is derived to describe the evolution of waves in the far-field regime using the method of slowly varying stretched coordinates. The resulting nonlinear equation is solved using the Homotopy Analysis Method (HAM). An advanced squared residual error technique is employed to determine the optimal convergence-control parameter. The solutions obtained via HAM are further validated through comparison with the exact solution and are found to be in good agreement. All computational work has been carried out using MATLAB.

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Dynamics of the three-dimensional localized wave packet generated by the spherical Bessel function $\mathbf{j}_0$

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We discuss an approach based on the theory of Maslov canonical operator [1], which allows us to study the dynamics of wave packets and obtain asymptotic representations for them in the terms of special functions.

We consider a Bessel-type packet generated by the spherical Bessel function $\mathbf{j}_0$, which propagation is described by a three-dimensional wave equation with constant velocity. We assume that the initial beam has a large initial momentum vector, which allows us to pass to the paraxial approximation for the wave equation. The initial condition is represented as a canonical operator on a certain non-compact three-dimensional Lagrangian manifold of a special type [2]. The initial manifold contains only a singularity of the three-dimensional degenerate fold type (which corresponds to the asymptotics in the form of the spherical Bessel function).

However, over time, the packet spreads out, and the singularity on the manifold decays. In particular, singularities of the fold type appear on the shifted manifold, which corresponds to the asymptotics in terms of the Airy function. Also we obtain a cusp type singularity, which corresponds to the asymptotics in terms of the Pearcey function. A non-generic singularity also appears on the shifted manifold, in the neighborhood of which the asymptotics is determined by the error function. Studying the types of singularity on the resulting manifold allows us to obtain an effective asymptotics of the package.

This talk is based on the joint work with S. Yu. Dobrokhotov.

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Acousto-optic conversion of higher-order vector modes in circular optical fibers induced by a torsional acoustic wave

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This work investigates the acousto-optic conversion of higher-order optical modes in a circularly anisotropic optical fiber endowed with a propagating fundamental torsional acoustic wave. Analytical expressions for the resonant optical eigenmodes and their corresponding propagation constants are derived for the first time. The mutual conversion of linearly polarized optical vortices is theoretically demonstrated with allowance for the optical frequency shift. An inversion of the sign of the spin angular momentum of a circularly polarized optical vortex is established. Mutual conversion between higher-order vector HE and EH modes is demonstrated; this conversion is accompanied by a change in both the mode type and its parity. A practically important property of the fiber system is identified, namely its ability to perform efficient parallel mode conversion for modes with different values of the azimuthal index.

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Numerical simulation of seismic wave propagation in a planetary model on overset and multi-block grids

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Natural seismicity is one of the key processes attracting the attention of the scientific community. In recent decades, the intensification of mineral extraction has led to the occurrence of large natural and induced earthquakes and rock bursts in previously weakly seismic and aseismic regions. Studying the propagation of wave disturbances in planetary-scale models is a critical task in modern seismology, as wavefield analysis enables the interpretation of seismic data and the explanation of geodynamic processes. This problem is being actively investigated; in particular, the spectral element method has become widely used for wavefield modeling [1]. At the same time, the grid-characteristic method has proven effective for simulating the propagation of dynamic wave disturbances.

This paper presents a numerical study of wave propagation in planetary-scale models using the grid-characteristic method. The approach employs two structured mesh strategies: overset grids and multi-block grids. A two-dimensional planetary model with internal structure and surface topography is implemented. The governing equations are solved within the framework of linear elasticity theory for a homogeneous medium with a Ricker wavelet source. The problem considered is analogous to that described in [2]. Verification is performed by comparison with reference solutions obtained using the SPECFEM2D software package [1], which is based on the spectral element method. The study confirms the applicability of multi-block and overset grid techniques for planetary-scale wave simulations and establishes a basis for further investigations of heterogeneous media and topographic effects.

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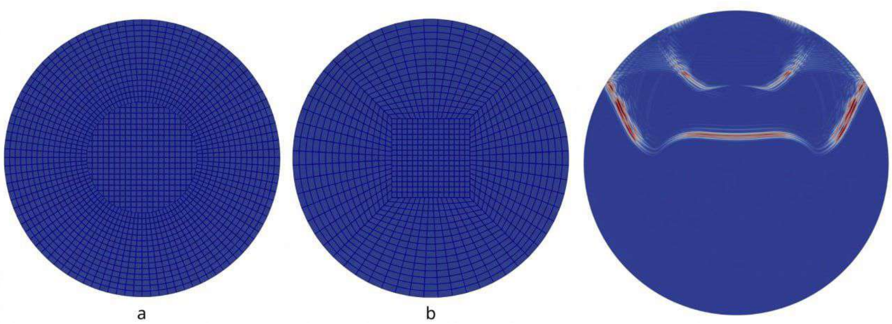

Fig. 1: Overset grid (a), multiblock grid (b) and wave propagation.

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Application of the ray method in photoacoustic tomography for layered media

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Photoacoustic tomography is a medical imaging technique that combines the high contrast of optical methods with the spatial resolution of ultrasound [1, 2]. A short laser pulse illuminates biological tissue, and regions with higher optical absorption (such as malignant cells) generate acoustic waves due to thermoelastic expansion [3]. These waves are measured on the tissue surface. The inverse problem is to reconstruct the initial pressure distribution (i.e., the acoustic source) from measurements of the acoustic field on the boundary [4, 6].

Many reconstruction methods rely on the Green's function of the acoustic wave equation [5]. In most standard approaches, the medium is assumed to be homogeneous, which enables the use of inversion formulas based on the Radon transform, incorporating the travel time and amplitude of the Green's function [6, 8, 9].

In this work, we consider a more realistic model in which the medium is layered and acoustically inhomogeneous. The Green's function for such a medium is constructed using the ray method, which accounts for the refraction of acoustic waves [7, 10]. The classical inversion formula is modified to incorporate the effects of inhomogeneity. Numerical calculations with realistic tissue parameters show that accounting for layering significantly affects wave travel times compared to the homogeneous approximation.

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Exact solutions and series expansions for parabolic wavefront diffraction in aberration metrology

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Modern photolithography requires utmost precision in the control of optical systems, which makes it necessary to measure wavefront aberrations directly at the actinic wavelength. Several such metrological systems, developped for EUV lithgrophy, are known: ILIAS [1] (used in the ASML systems), LSI (Lateral Shearing Interferometer), Lawrence Berkeley National Laboratory [2] and MISTI, developed at Nikon [3] (for a general overview, see [4]). The measurement setup is designed to measure wavefront aberrations of the projection system in the virtual pupil, as seen from the focal plane at the wafer side (see Fig. 1). Since there is only one wavefront in the exit pupil, a diffraction grid is used to split it into several diffraction orders, where the zeroth order waverfont acts as a reference wavefront, against which the higher diffraction orders are compared by inspecting the image produced at the detector plane, see again Fig. 1. Here we analyze the case when the diffraction grid is placed at a significant distance from the focal plane, which is close to the setups used in [2] and [3].

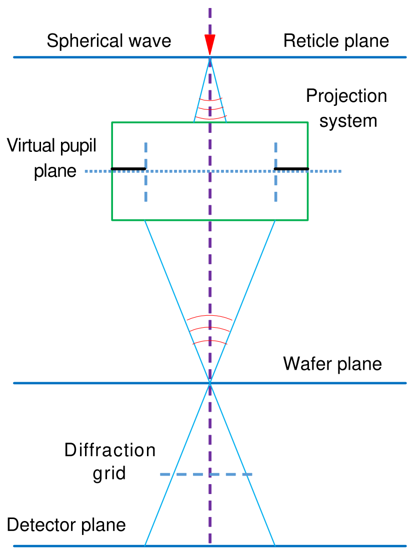

Fig. 1: Schematic of the wavefront aberration measurement setup.

In our work, we modelize the measurement scheme described above as follows: we assume that an abberated diverging wavefront [5] is diffracted at a grid for which we can compute the solution via the angular spectrum method with Kirchhoff boundary [6]. Under the assumptions presented above, it is possible to use the paraxial approximation, for which we calculate explicit solutions of the diffraction problem. In this study, we have found exact solutions for a parabolic wavefront in the Fresnel diffraction approximation [6]. The diffraction integral can also be evaluated for a perturbed parabolic wavefront under the assumption that the diffraction grating period is much smaller than the characteristic scale of the field variation, so it suffices to retain only the first-order Taylor series expansion of the perturbation wavefront. Note that for measurements setups discussed in [7] and used in [1], this assumption is in general not valid.

We use systematic and controlled series expansions to assess and, if necessary, to increase the computation accuracy of the propagation integrals, first, to go beyond the paraxial approximation and second, to take into account the deviation of the spherical wavefront from the parabolic one.

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Excitation of surface plasmon by incident Gaussian beam localised near a flat interface between dielectic medium and Drude metal

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The problem of excitation of surface electro-magnetic waves excited by electric dipoles located near to the interface separating dielectric medium and metal with finite conductivity has been popular since the beginning of the twentieth century (see first of all [1]) in various areas of radio-science in both theoretical activity and engineering. In this paper we study a similar problem of excitation of surface plasmon localised near interface between dielectic and Drude metal by normally incident linearly polarised Gaussian beam that is vital in optics and infrared light physics (see [2]). In the problem the high-frequency Gaussian beam propagating in dielectric medium reflects normally from the flat interface of Drude metal with well-known dispersion law. It is analysed on the basis of the Sommerfeld techniques ([1]) resulting in a total description of reflecting back and propagating inside the metal electro-magnetic fields in the form of plane wave decompositions of integral representations. We pay a particular attention to the short-wave asymptotic analysis of the wave field inside the metallic medium in the far field zone, applying the steepest descend method to the corresponding wave field integral representations. Thus, inside Drude metal we obtain asymptotic expansions of the decaying bulky spherical waves as well as the poles contributions of excited surface waves of plasmons. It is worth remarking that the obtained expression for the pole looks the same as the well-known in the plasmonic literature (see [2]) for the surface plasmon homogeneous problem. Thus, developing the asymptotic analysis we derive approximate formulas for the surface plasmonic waves excitation coefficients incorporating the incident Gaussian beam parameters. In the numerical analysis we pay a particular attention to the behaviour of the radiation pattern and total power of the surface plasmonic wave generated by the incident beam.

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Micrometer-scale structured light with simultaneous longitudinal and transverse control

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Frozen Waves [1] provide an efficient framework for tailoring the longitudinal intensity profile of nondiffracting beams. More recently, an extension of this method [2,3] introduced an additional degree of freedom through a phase modulation of the morphological function, enabling dynamic control of the transverse beam structure along propagation. However, this approach has been primarily developed within the paraxial regime and is thus inherently associated with long longitudinal distances and transverse dimensions much larger than the wavelength.

In this work, we extend the concept of transversally and longitudinally structured FW beams to the micrometer scale. In this regime, a continuous spectral formulation is more appropriate [4], and the scalar paraxial approximation must be replaced by exact vectorial solutions of Maxwell's equations. Within this context, by incorporating a complex morphological function whose amplitude defines the desired longitudinal intensity pattern and whose phase is engineered from a prescribed transverse evolution, we demonstrate that it is possible to simultaneously control both longitudinal and transverse beam structures over micrometer spatial regions.

The proposed approach enables the generation of highly non-paraxial structured light with customizable three-dimensional intensity distributions, opening new possibilities for applications in optical trapping, micromanipulation, and high-resolution photonic structuring.

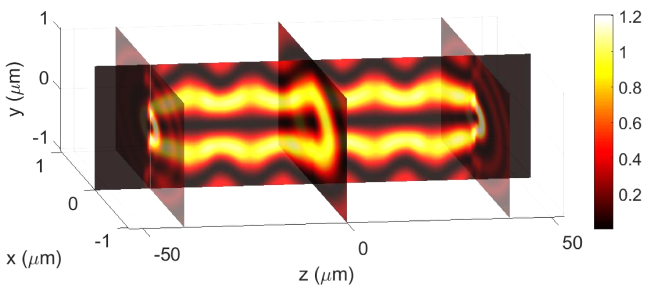

Fig. 1: Three-dimensional intensity distribution of a micrometer-scale structured beam obtained with the proposed method, illustrating the simultaneous control of both longitudinal and transverse features within a highly non-paraxial regime.

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Concerning a property of roots of the dispersion relation

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If $\pm\lambda_0$, $\pm\lambda_1$, $\pm\lambda_2$ are roots of the dispersion relation $\lambda^6 + \lambda^2(\rho_1 g - \rho h\omega^2)/D - \rho_1\omega^2/(Dh_1) = 0$, than the formula

$$\sum_{n=0}^{2} \frac{\lambda_n^{2k}}{3\lambda_n^4 + (\rho_1 g - \rho h\omega^2)/D} = \begin{cases} 0, & \text{for } k = 0, 1, 3, \\ 1, & \text{for } k = 2 \end{cases}$$

is true. This statement occurs in studies [1] of interaction of flexural-gravity waves in water with hydraulic structures under assumption of a thin fluid layer and is proved here on the basis of the generalized Vieta formulas.

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The Malyuzhinets–Popov-type diffraction problem

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We consider diffraction of a plane wave e^{ikx} with a large wavenumber k by an obstacle (with the Neumann boundary condition) shown in Fig. 1. Its boundary consist of two half-lines and a semicircle of radius a. The curvature of the boundary jumps at the junction points $A_1(0, a)$ and $A_2(0, -a)$. In small neighborhoods of $A_{1,2}$, for $x > 0$, the wavefield was earlier investigated in [1, 2] in the framework of the parabolic equation method (see, e.g., [3]). Short-wavelength asymptotic formulas were constructed applicable at distance $x \ll a$. We aim at extending these results to a larger distance $x \sim a$ and $x \gg a$. For this purpose, developing the ideas of [3–5], we match the expressions obtained in [1, 2] with anzatzes constructed using parabolic equations in appropriate ray coordinates.

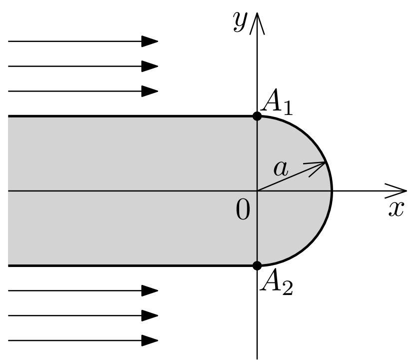

Fig. 1: Problem under consideration.

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Author index