INSTITUT HENRI POINCARÉ

Trimester on « Granular Materials »

Cosserat Continum

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Introduction to Cosserat continuum theory with some applications in geomechanics

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Historical note

•**Cosserat, E. and F.** ***Théorie des Corps Déformables*****. A. Hermann et Fils, Paris (1909).**

Fifty years after the first publication of the original work of the brothers **Cosserat, Eugène and François** in 1909, the basic kinematic and static concepts of the “Cosserat” continuum were reworked in a milestone paper Professor **Günther** (1958).

Günther, W. (1958). Zur Statik und Kinematik des Cosseratschen Kontinuums. In: Abhandlungen der Braunscheigschen Wissenschaftlichen Gesellschaft, Göttingen, Verlag E. Goltze, 101, 95-213.

Günther's paper marks the rebirth of continuum micro-mechanics in the late 50’s and early 60's. The state-of-the-art at this time was reflected in the collection of papers presented at the historical IUTAM Symposium on the ***"Mechanics of Generalized Continua*****",** held in Freudenstadt and Stuttgart in 1967

- Schaeffer, H. (1962). Versuch einer Elastizitätstheorie des zweidimensionalen ebenen COSSERAT-Kontinuums. Miszellannenn der Angewandten Mechanik. Akademie Verlag, Berlin.
- Schaeffer, H. (1967). Das Cosserat-Kontinuum. ZAMM, **47/8**, 485-498.
- Kessel, S. (1964). Lineare Elastizitätstheorie des anisotropen Cosserat-Kontinuums. In: Abhandlungen der Braunscheigschen Wissenschaftlichen Gesellschaft, Göttingen, Verlag E. Goltze, **16,** 1-22.
- Koiter, W. T. (1964). Couple-stresses in the Theory of Elasticity. I & II. Proc. Ned. Akad. Uet., **67**, 17-29 & 30-44.
- Mindlin, R.D., (1964). Microstructure in linear elasticity. Arch. Rat. Mech. Anal., **10**, 51-77.
- Mindlin, R.D. and Eshel, N.N. (1968). On first strain-gradient theories in linear Elasticity. Int. J. Solids Structures, **4**,109-124.

Significance of couple stresses and higher-order stresses?

Germain, P. (1973a). La méthode des puissances virtuelles en mécanique des milieux continus. Part I Journal de Mécanique, **12**, 235-274.
Germain, P. (1973b). The Method of virtual power in continuum mechanics. Part 2: Microstructure. SIAM, J. Appl. Math, **25**, 556-575.

Why should we resort to continuum higher order theories?

Limits of application of classical continuum theories
Influence of microstructure

Large scale description (homogeneisation)

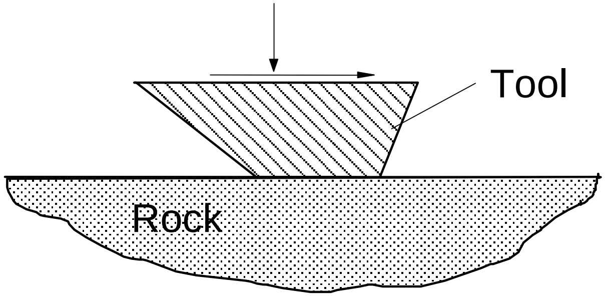
Discontinuous media

Granular materials (soils, rocks, powders)

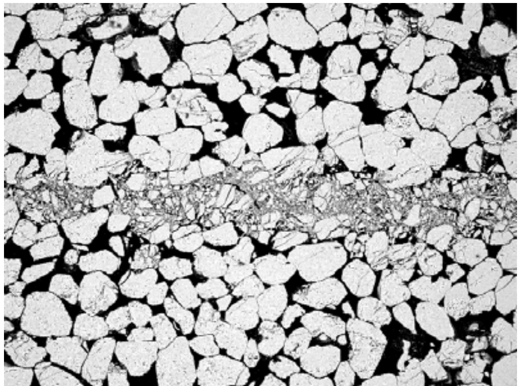
Fractured or stratified rock mass

REV >> size of discontinuities

Limits of classical continuum models in presence of high stress and/or strain gradients



Contact problems
Fracture mechanics
Strain localisation phenomena
Interfaces



According to Mindlin (1963):

For highly inhomogeneous stresses (stress concentration zones)

-stress gradients affect the failure mechanism
-strain gradients affect the apparent strength

"…increasing strain gradients appear to make some materials stronger to a degree that depend upon grain size…"

Necessity of an internal length in the constitutive models

Frame of modelling : generalised (higher order) continuum theories

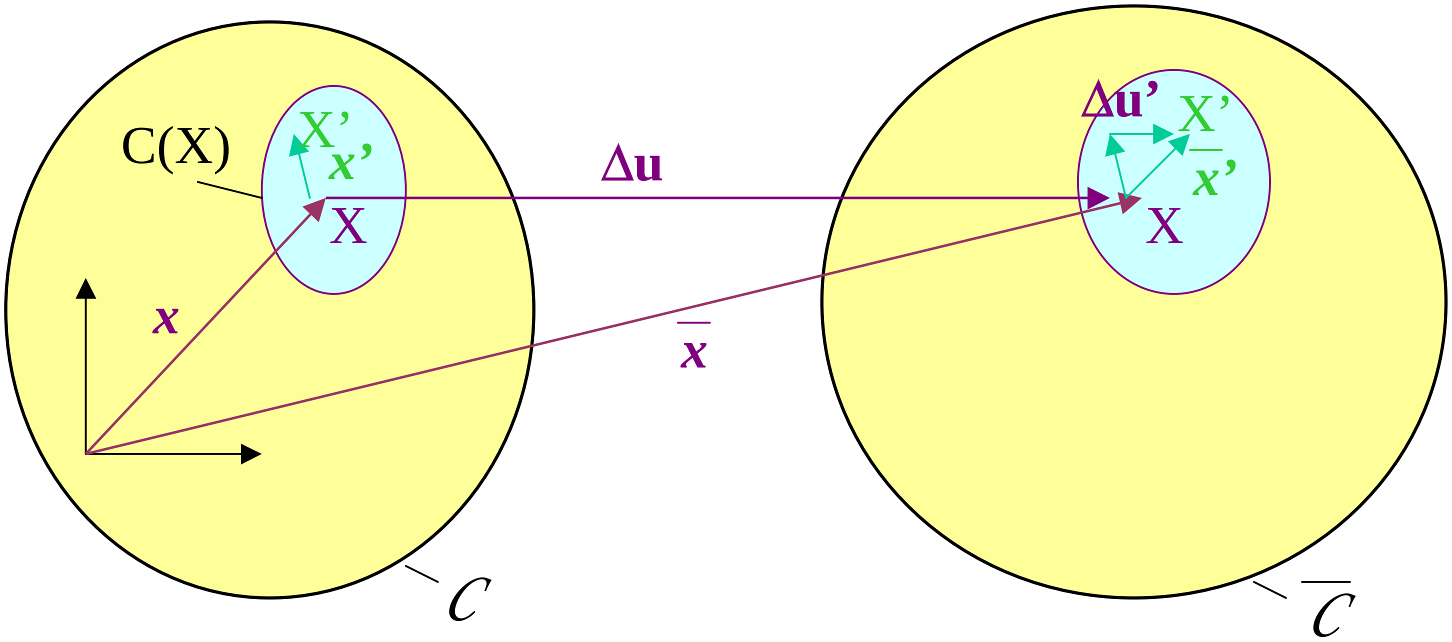
-richer kinematics description
-material length scale related to the fabric length

Basic formalism for Generalised continuum

Mindlin (1964),Germain(1973)

Classical continuum : Assembly material points X

Generalised continuum : a micro-volume C(X) (continuum) is attached to each material points



Macro-déplacement : $\Delta\mathbf{u} = \bar{x} - x$

Micro-déplacement : $\Delta\mathbf{u}' = \bar{x}' - x'$

Kinematics:

macro-deformation : ε symmetric part of the velocity gradient $\nabla\mathbf{v}$ of the macro-particle **X**

macro-rotation : ω antisymmetric part of the velocity gradient ∇v of the macro-particle X

micro-deformation : ψ deformation of the micro-volume C(**X**) (assumed homogeneous in C(**X**) for micromorphic medium of order 1)

relative deformation : $\gamma = \nabla\mathbf{v}\text{-}\psi$ difference between the of the velocity gradient and the micro-deformation

micro-deformation gradient : $\kappa = \nabla\psi$ (3rd order tensor)

Definition of the conjugate static quantities trough the principle of virtual work

Virtual work of internal forces:

$$\boxed{\delta w^{(i)} = \tau_{ij}\delta\varepsilon_{ij} + \alpha_{ij}\gamma_{ij} + \mu_{ijk}\delta\kappa_{ijk}}$$

τ : Cauchy stress tensor (symmetric) dual in energy to the macroscopic strain ε

α : Relative stress tensor dual in energy to the relative deformation γ

μ : Double stress tensor dual in energy to the micro-deformation gradient

total stress tensor : $\sigma = \tau + \alpha$

$$\boxed{\delta w^{(i)} = \sigma_{ij}\partial_i \delta u_j - \alpha_{ij}\delta\psi_{ij} + \mu_{ijk}\delta\kappa_{ijk}}$$

Micromorphic continuum of order 1

Cosserat continuum: The micro-volume C(**X**) moves as a rigid body.

kinematics of a material point : translation and rotation ω^c

micro-deformation : antisymmetric tensor

$$\psi = \omega^c$$

relative deformation $\gamma = \varepsilon + (\omega - \omega^c)$

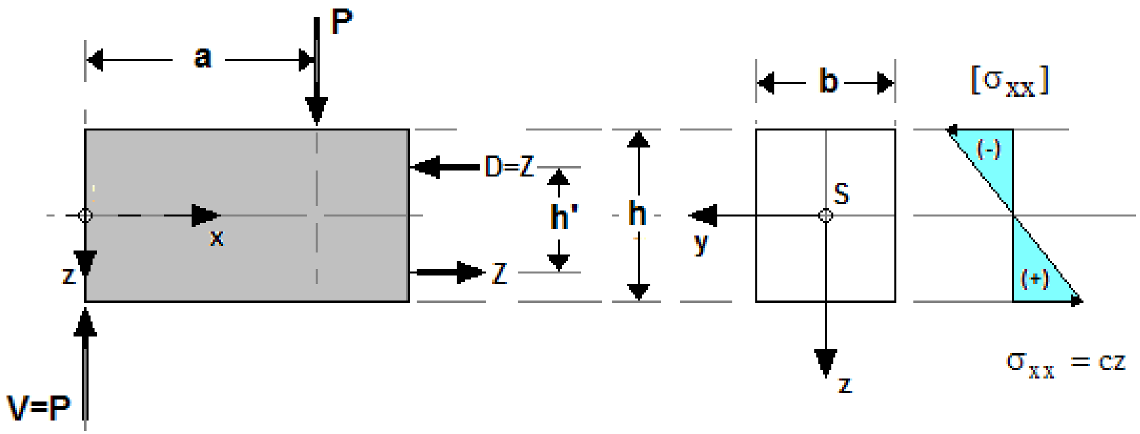
Principle of virtual work

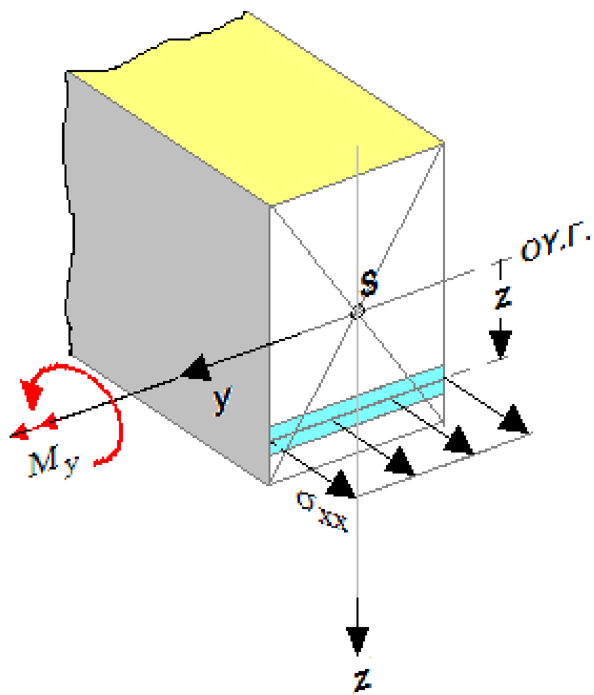
$\varepsilon \leftrightarrow \sigma^s$ symmetric part of the total stress

$\omega - \omega^c \leftrightarrow \sigma^a$ antisymmetric part of the stress

$\nabla\omega^c \leftrightarrow \mu$ couple stress tensor

In classical beam-bending theory the microelement is the cross section of the beam with 2 d.o.f., vertical displacement + rotation.





$$N_x = \int_{(A)} \sigma_{xx}\, dA = c \int_{(A)} z\, dA = 0 \quad (!)$$

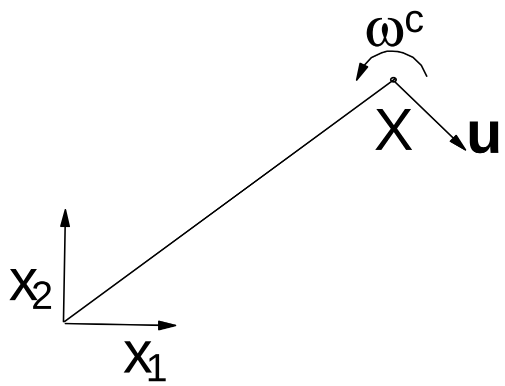
$$M_y = \int_{(A)} z\sigma_{xx}\, dA = c \int_{(A)} z^2 dA = cI_{yy}$$

Basic concepts of a 2D-Cosserat continuum

Kinematics of a 2D-Cosserat continuum:

2 DOF translation u_1 , u_2

1 DOF rotation ω^c

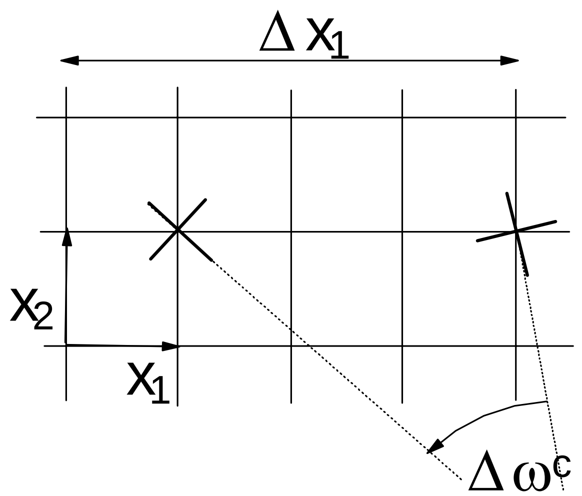


Description of deformation

$$\gamma_{11} = \partial u_1 / \partial x_1 \quad ; \quad \gamma_{12} = \partial u_1 / \partial x_2 + \omega^c$$

$$\gamma_{22} = \partial u_2 / \partial x_2 \quad ; \quad \gamma_{21} = \partial u_2 / \partial x_1 - \omega^c$$

$$\kappa_i = \partial \omega^c / \partial x_i, \; i = 1,2$$



Dual static quantities

4 components for the stress tensor σ_{ij} (non symmetric)

2 couple stresses m_1 , m_2

LOCAL DYNAMIC EQUATIONS

$$\sigma_{11,1} + \sigma_{12,2} - \rho \ddot{u}_1 = 0$$

$$\sigma_{21,1} + \sigma_{22,2} - \rho \ddot{u}_2 = 0$$

$$m_{1,1} + m_{2,2} + \sigma_{21} - \sigma_{12} - I\ddot{\omega}^c = 0$$

BOUNDARY CONDITIONS

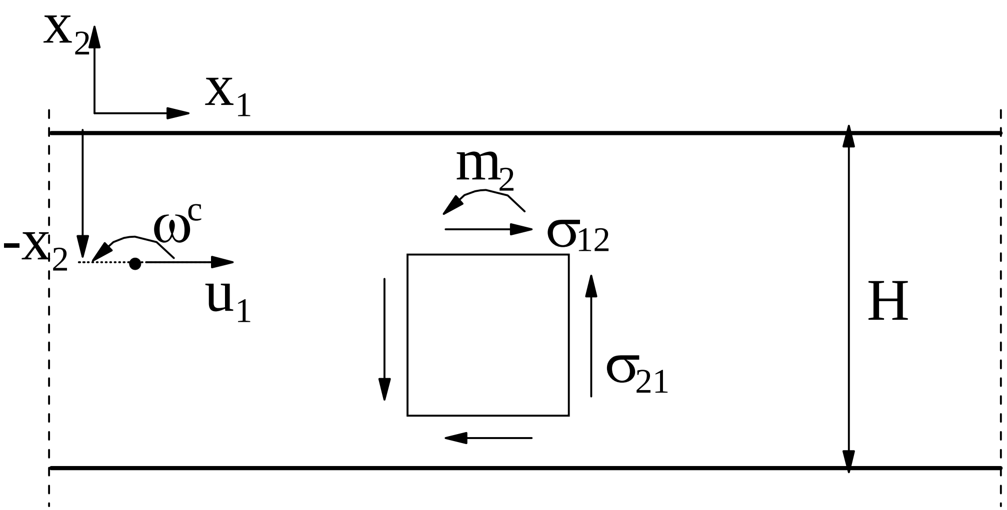
$$\text{on } \partial \mathrm{V_u} \quad v_i = \dot{\delta}_i \quad ; \quad \dot{\omega}^c = \dot{\Omega}^c$$

$$\text{on } \partial \mathrm{V_\sigma} \quad \sigma_{ij} n_j = t_i \quad ; \quad m_i n_i = m$$

Some examples for 2D Cosserat elasticity

The boundary layer effect

Simple shear of a long layer



2D Cosserat elastic stress-strain relation ships

$$\sigma_{11} = (K+G)\varepsilon_{11} + (K-G)\varepsilon_{22}$$
$$\sigma_{22} = (K-G)\varepsilon_{11} + (K+G)\varepsilon_{22}$$
$$\sigma_{(12)} = \sigma_{(21)} = 2G\varepsilon_{12}$$
$$\sigma_{[12]} = -\sigma_{[21]} = -2G^c(\omega - \omega^c)$$
$$m_1 = M\kappa_1; m_2 = M\kappa_2$$

Equilibrium equations

$$\sigma_{11,1} + \sigma_{12,2} = 0$$
$$\sigma_{21,1} + \sigma_{22,2} = 0$$
$$m_{1,1} + m_{2,2} + \sigma_{21} - \sigma_{12} = 0$$

Boundary conditions

$$\text{for } x_2 = 0: \qquad u_1 = u_{10}; \omega^c = \omega_0^c$$
$$\text{for } x_2 = H >> \ell: u_1 = 0; \omega^c = \omega = \omega_H; \omega_H = -\tau/(2G)$$

$$u^* \approx \frac{u_O^*}{1-\eta}\left\{1-\left(\eta/\alpha^2\right)x-\eta e^{-x}\right\}+\frac{\eta\omega_c^0}{1-\eta}\left\{1-\left(\eta/\alpha^2\right)x-e^{-x}\right\}$$

$$\omega^c \approx \omega_H + \left(u_0^* + \omega_0^c\right)\frac{e^{-x}}{1-\eta}$$

Material length $\ell = \sqrt{M/G}$

Coupling number $\alpha = 1/\sqrt{1+G/G^c}$

Scaling factor $\eta = (\alpha\ell)/(2H) << 1$

Dimensionless coordinate $x = -2\alpha x_2/\ell$

Dimensionless imposed displacement $u_0^* = u_{10}/2H$

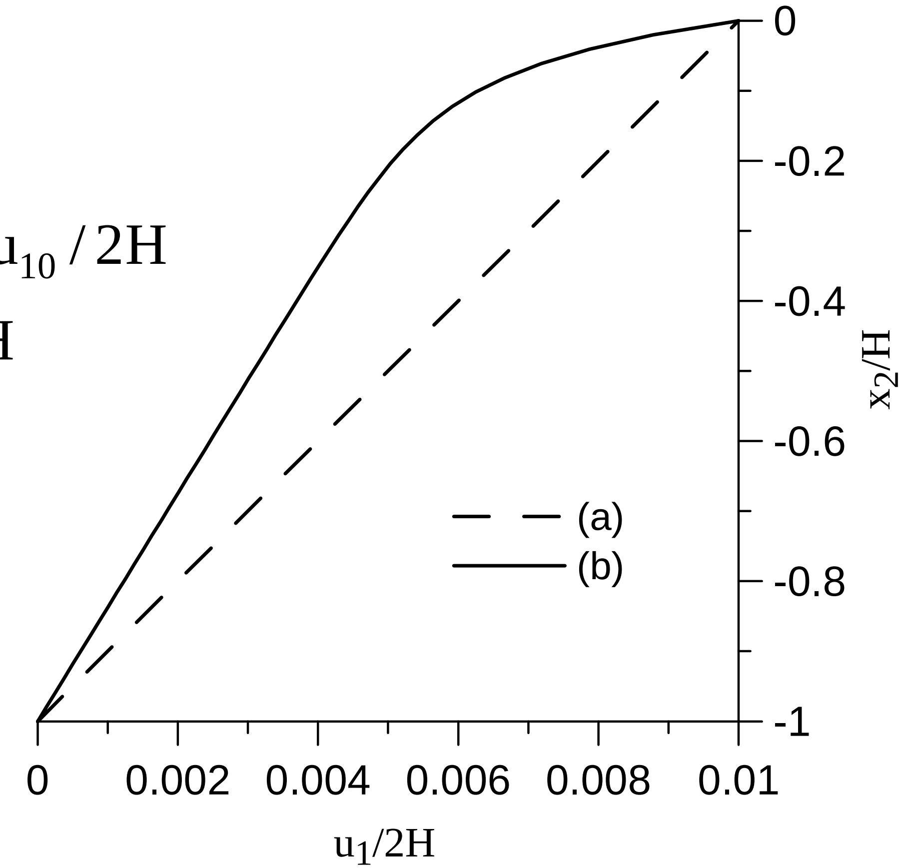
Dimensionless displacement $u^* = u_1/2H$

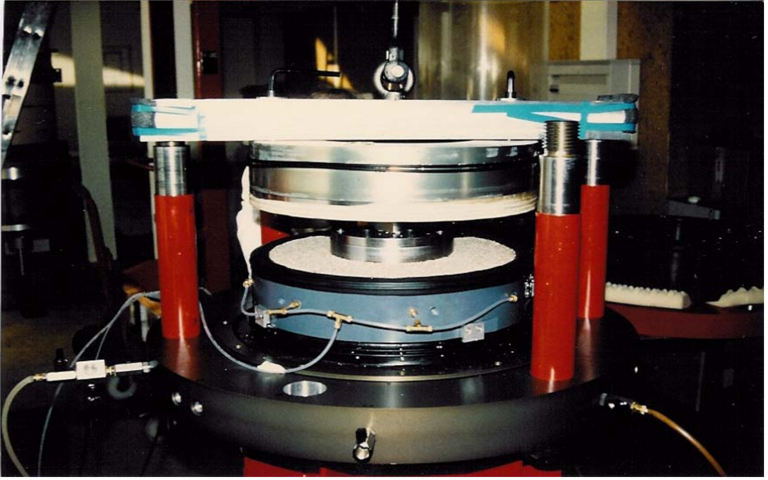
Example:

$\left(G^c/G = 2.0, \ell/H = 0.1\right)$

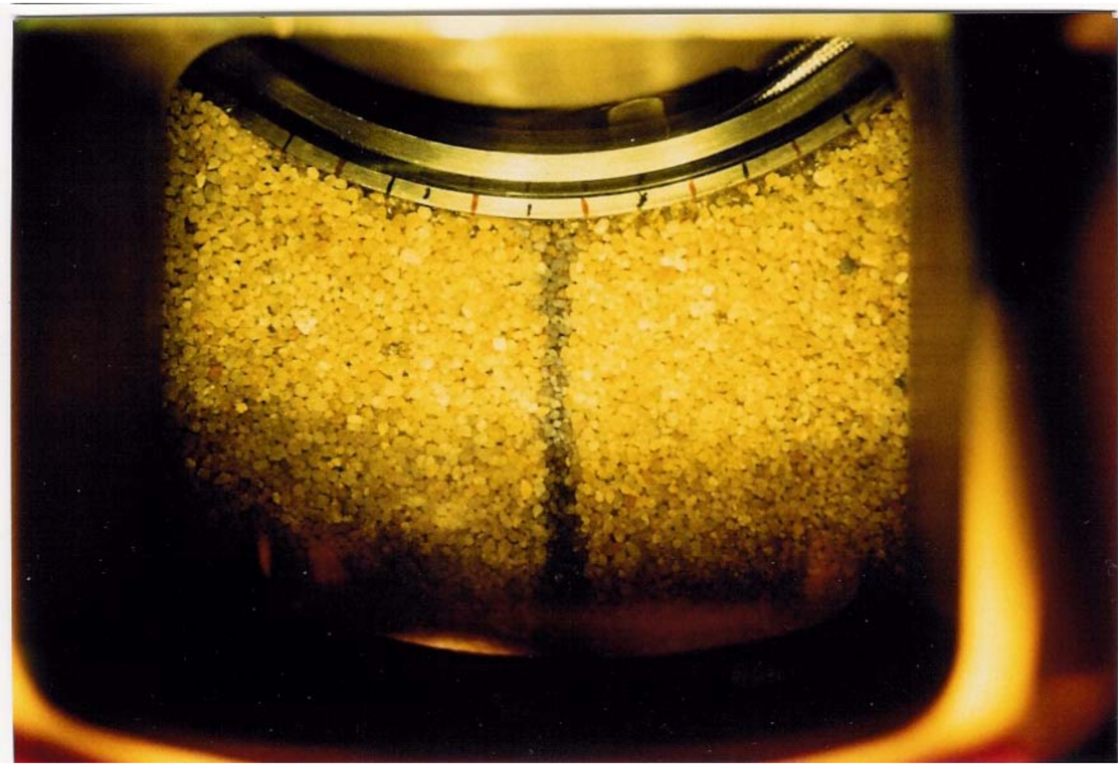
(a) $\omega_c^0 = -u_0^* = -0.01$

(b) $\omega_c^0 = -10u_0^* = -0.1$

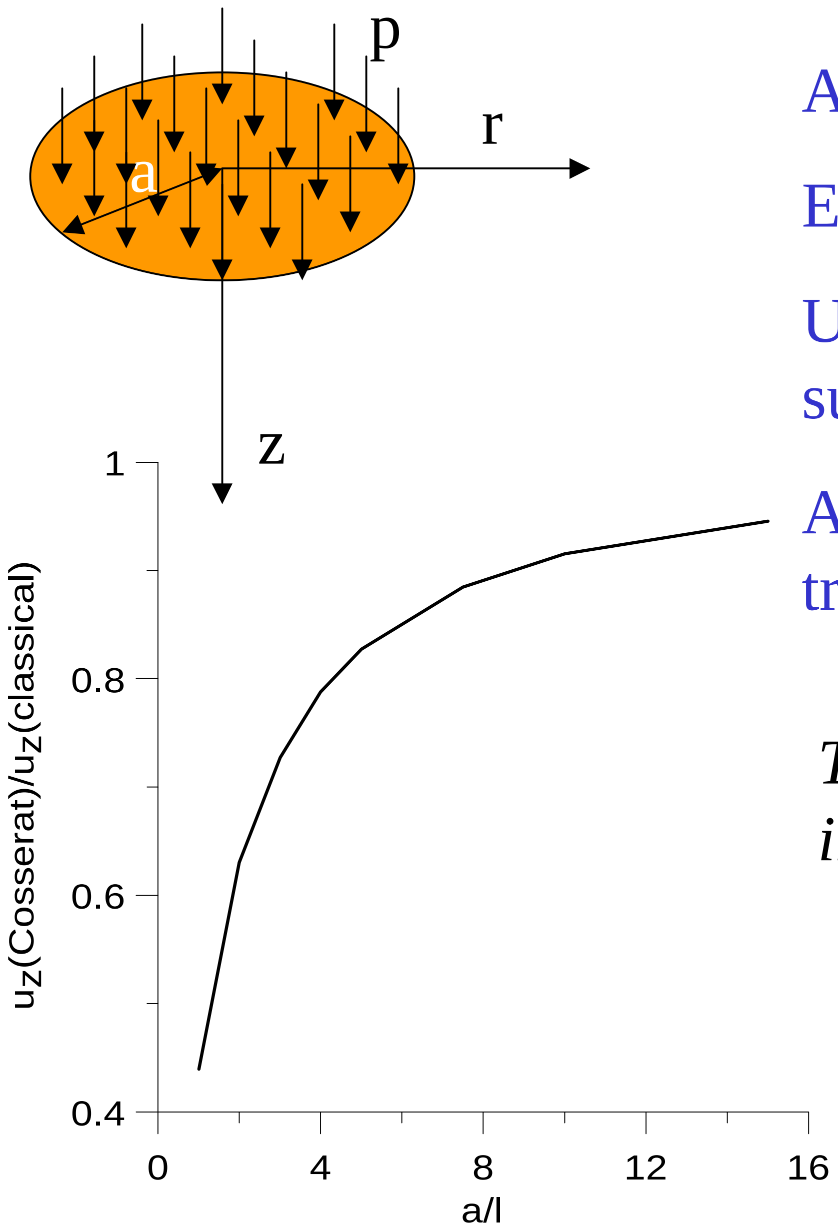




Plane-strain Couette apparatus for sand
(Corfdir, A., Lerat, P. and Vardoulakis, I. A cylinder shear apparatus. ***Geotechnical Testing Journal***, 2004)



Scale effect in indentation tests



Axisymmetric problem:

Elastic Cosserat half-space

Uniform loading on a circular surface of radius a

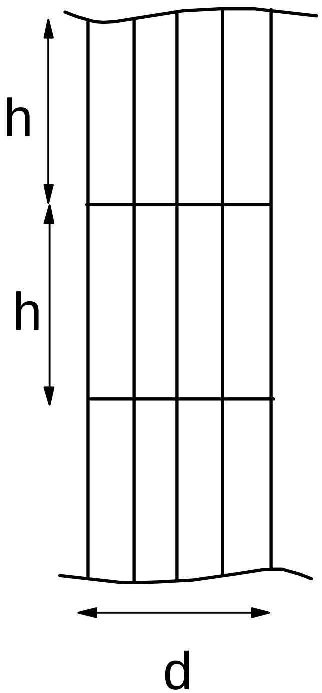
Analytical solution using Hankel transforms

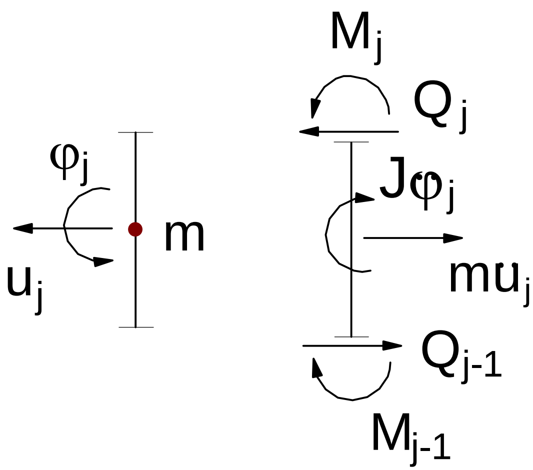
The apparent stiffness of the medium increases with decreasing ratio a/l

From discrete to continuum

1D-Example
Masonry column

Mühlhaus, Sulem, Unterreiner, 1997





Elastic relationships for the blocks

$$Q_j = c_Q \Delta v_j$$

$$\Delta v_j = u_{j+1} - u_j - \frac{h}{2}\left(\varphi_{j+1} + \varphi_j\right)$$

$$M_j = c_M d^2 \kappa_j$$

$$\kappa_j = \varphi_{j+1} - \varphi_j$$

Elastic energy

$$E_j = \frac{Q_j \Delta v_j + M_j \kappa_j}{\pi h d^2 / 4}$$

1D-Cosserat continuum:

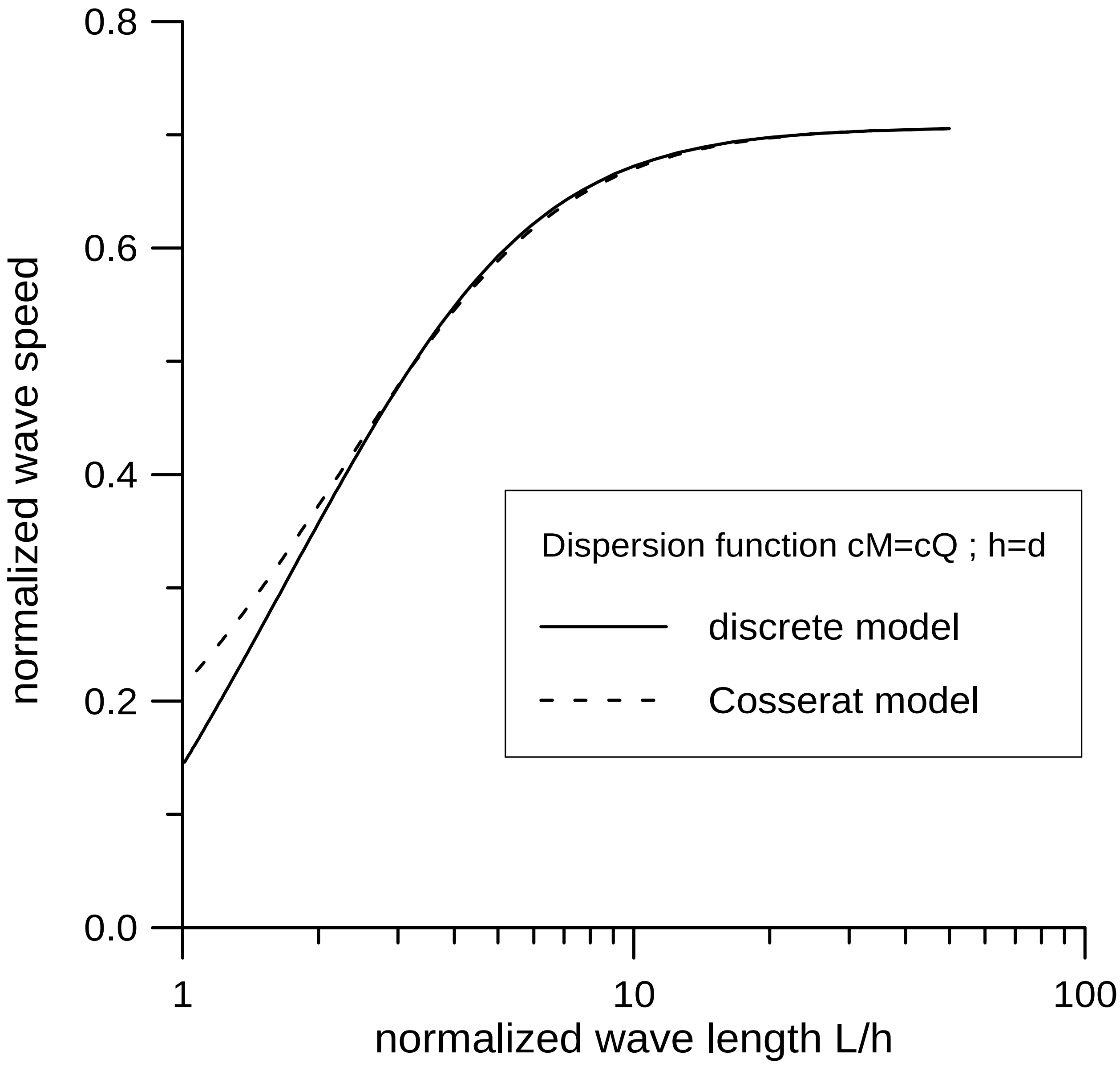
$$Q = G\left(\frac{du}{dx} - \varphi\right) \quad \text{et} \quad M = B\frac{d\varphi}{dx}$$ Elastic constitutive equations

$$E = Q\left(\frac{du}{dx} - \varphi\right) + M\frac{d\varphi}{dx}$$ Elastic energy

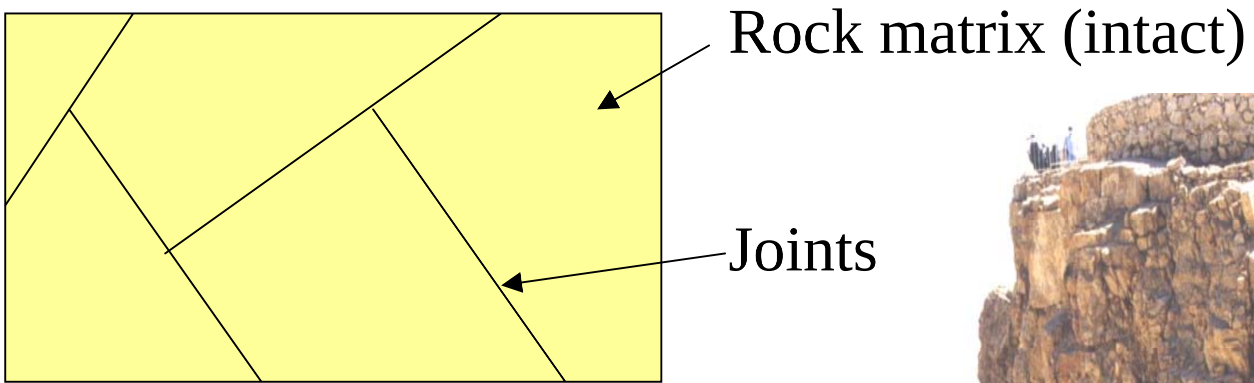
Identification of the elastic energy of the discrete medium
and of the corresponding Cosserat continuum:

$$B = \frac{4}{\pi} c_M h \qquad G = \frac{4}{\pi} c_Q \frac{h}{d^2}$$

Dispersion functions for the continuum and for the discrete structure



DISCONTINUOUS ROCK MASS

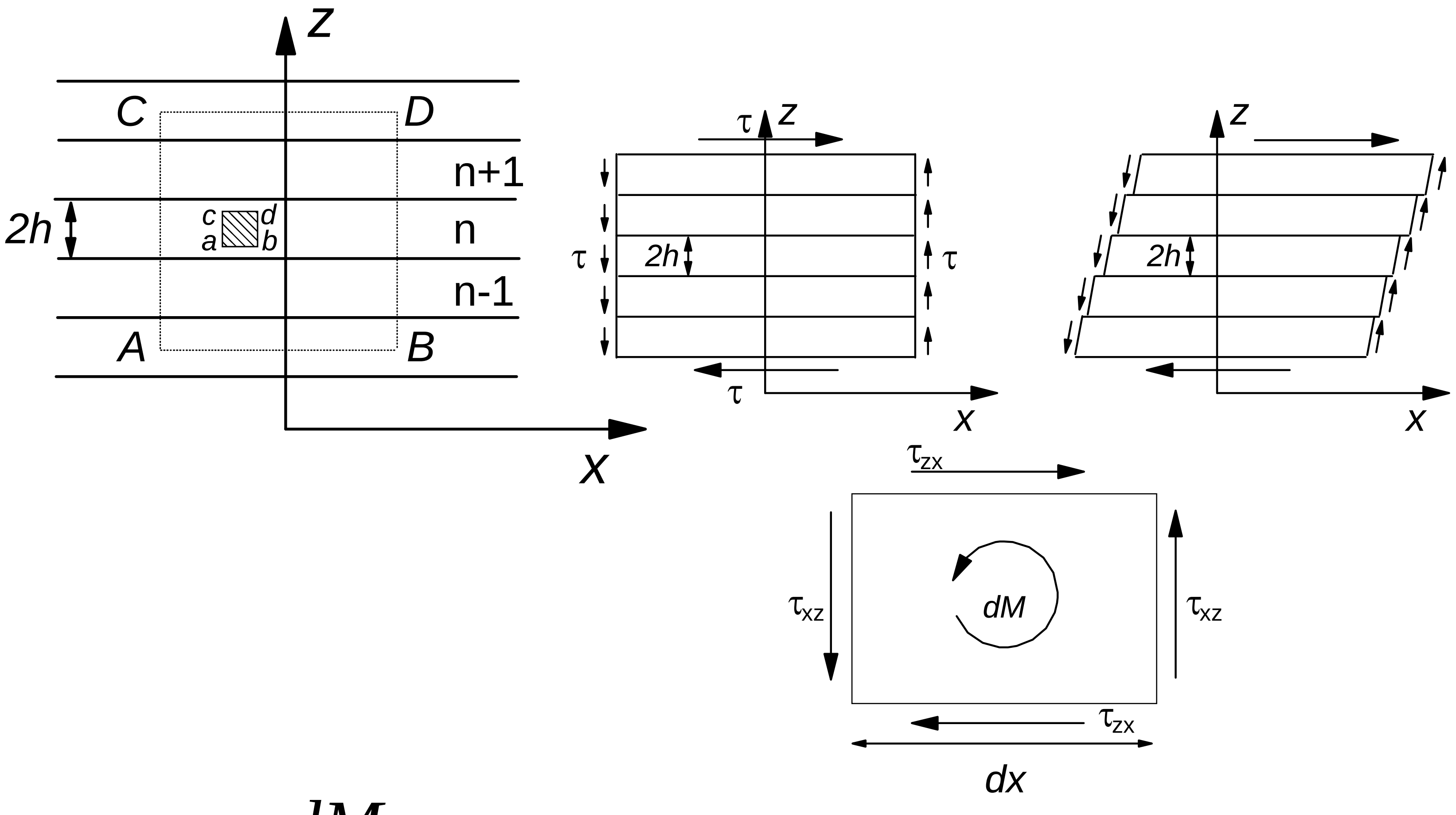


Structure de blocs, Masada, Israël
Dr. Y. Hatzor personnal web page
http://www.bgu.ac.il/geol/hatzor/

- Block structures
- Stratified structures

STRATIFIED ROCK MASS

Interlayer slip and bending moments



$$\frac{dM}{dx} + \tau_{zx} - \tau_{xz} = 0$$

Cosserat representation of stratified media

Biot 1967, Zvolinskii and Shkhinek 1984, Vardoulakis & Sulem 1995, Mühlhaus and co-workers 1988, 1993, 1999

ELASTICITE

Elastic parameters of the equivalent Cosserat continuum

E, ν :Young modulus Posson ratio of the rock matrix

k_n, k_s : Normal and tangential stiffness of the joints

b : layer thickness

Elastic relationships

$$\sigma_{11} = C_{11}\gamma_{11} + C_{12}\gamma_{22}$$

$$\sigma_{22} = C_{21}\gamma_{11} + C_{22}\gamma_{22}$$

$$\sigma_{12} = \left[G + G_c(1-\alpha)\right]\gamma_{12} + \left[G - G_c\right]\gamma_{21}$$

$$\sigma_{21} = \left[G - G_c\right]\gamma_{12} + \left[G + G_c(1+\alpha)\right]\gamma_{21}$$

$$m_1 = M_1\kappa_1$$

$$m_2 = M_2\kappa_2$$

$$C_{11} = \frac{E}{1-\nu^2 - \dfrac{\nu^2(1+\nu)^2}{1-\nu^2 + E/(k_n)}}$$

$$C_{22} = \frac{(1-\nu)E}{(1+\nu)(1-2\nu) + (1-\nu)E/(k_n)}$$

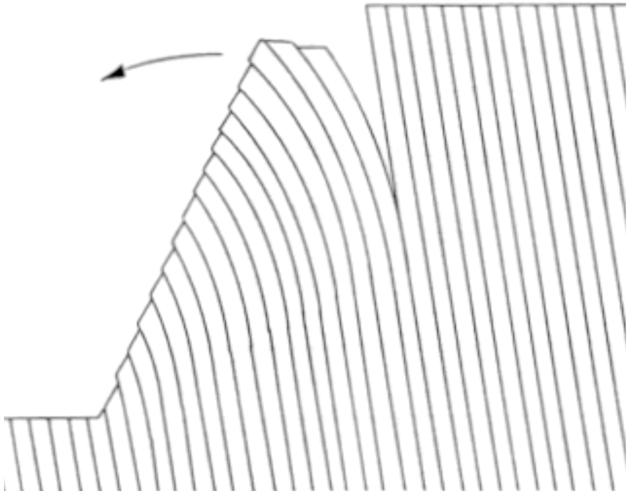
$$C_{12} = C_{21} = \frac{\nu E}{(1+\nu)(1-2\nu) + (1-\nu)E/(k_n)}$$

$$G = \frac{E}{8(1+\nu)}\left[\frac{5k_s + E/(2(1+\nu))}{k_s + E/(2(1+\nu))}\right] ; \; G_c = \frac{E}{8(1+\nu)}; \alpha = 2$$

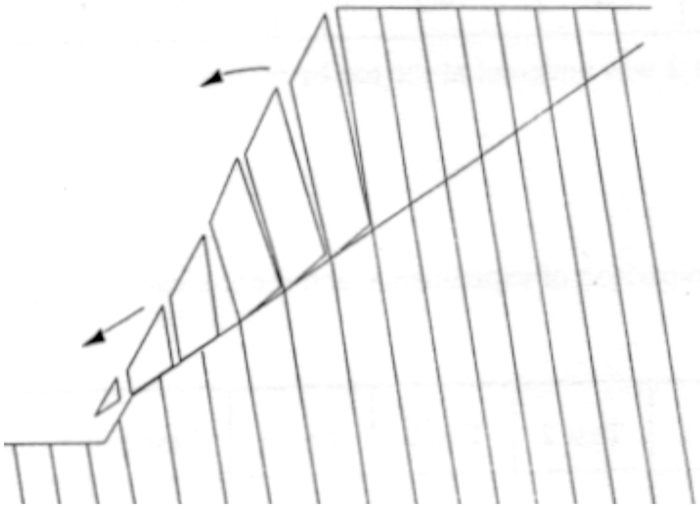
$$M_1 = \frac{Eb^2}{12(1-\nu)}\left[\frac{E/(2(1+\nu))}{k_s + E/(2(1+\nu))}\right] ; \; M_2 = 0$$

TOPPLING FAILURE OF FOLIATED ROCK SLOPE

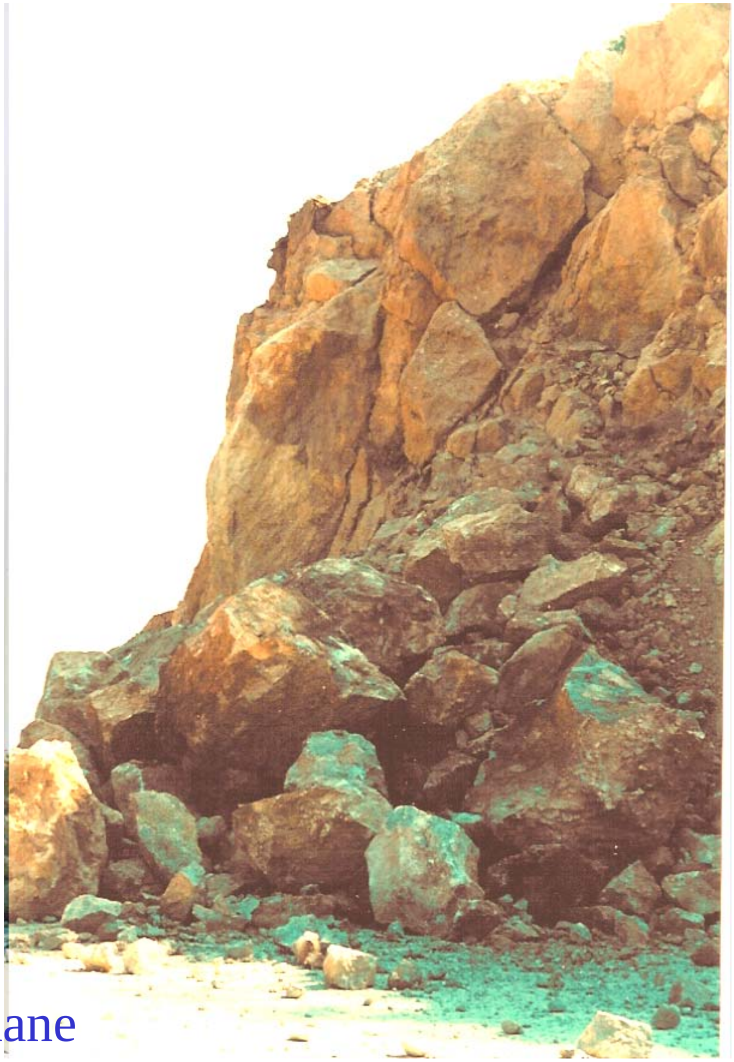
Goodman & Bray 1976, Hoek & Bray 1977, Zanbak 1983, Adhikary et al. 1995, Pritchard & Savigny 1990, Bobet 1999, Merrien-Soukatchoff et al 2001...



Flexural toppling :flexion on slender rock columns
tensile failure



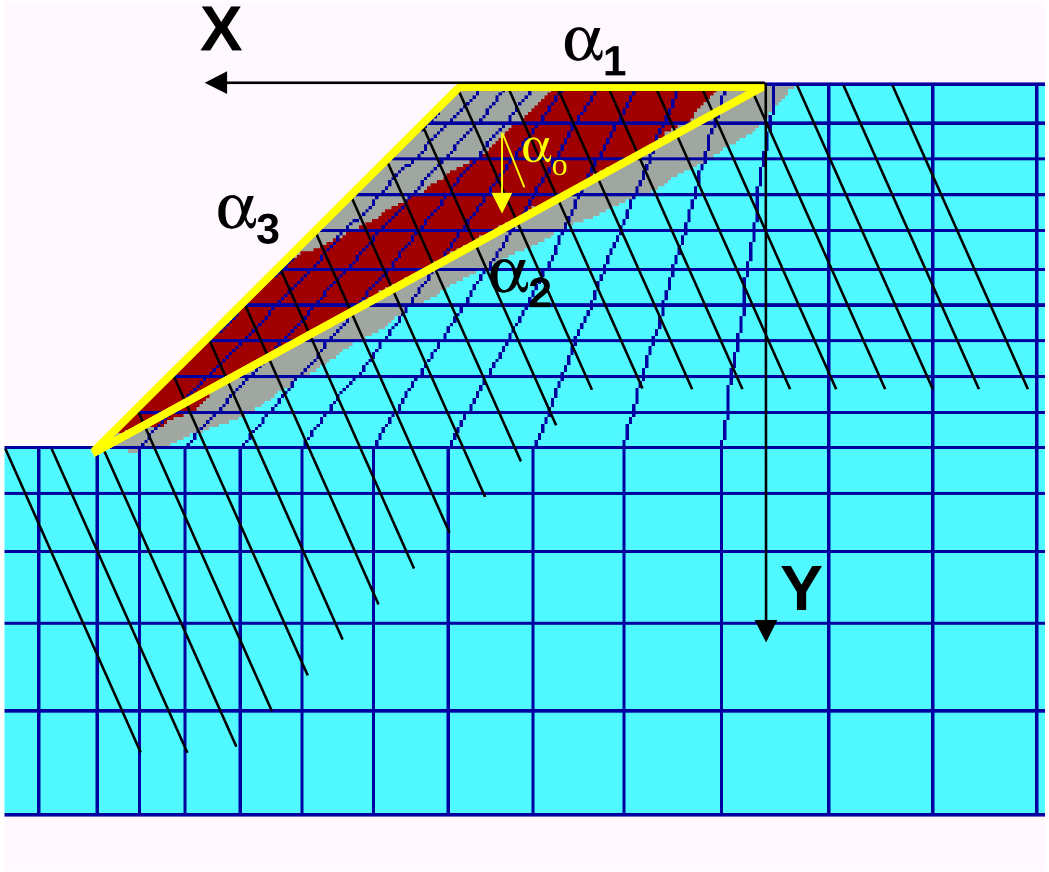
Block toppling and sliding on a pre-existing basal plane



IDENTIFICATION OF A FAILURE SURFACE FOR LIMIT EQUILIBRIUM ANALYSIS

Zones of tensile failure

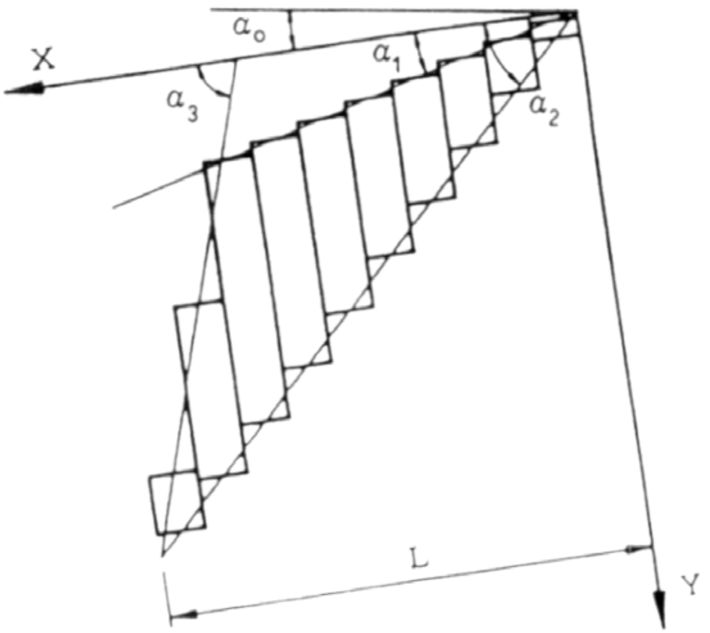
$$\sigma_N = -\frac{N}{A} + \frac{M}{I} y \qquad \sigma_{tensile} = 6\frac{m_{zx}}{b} + \sigma_{xx}$$



m_{zx} is the Cosserat couple stress

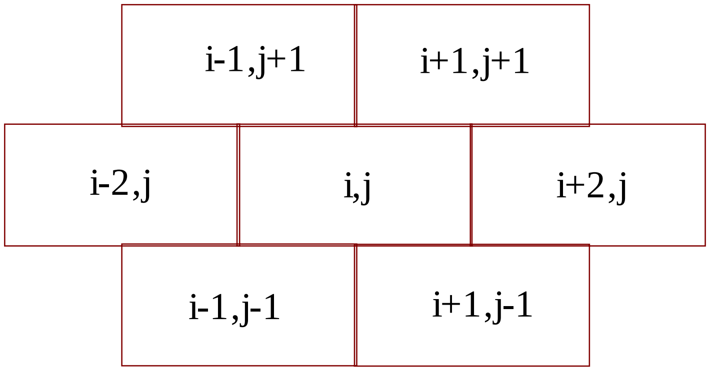
b is the thickness of the layer

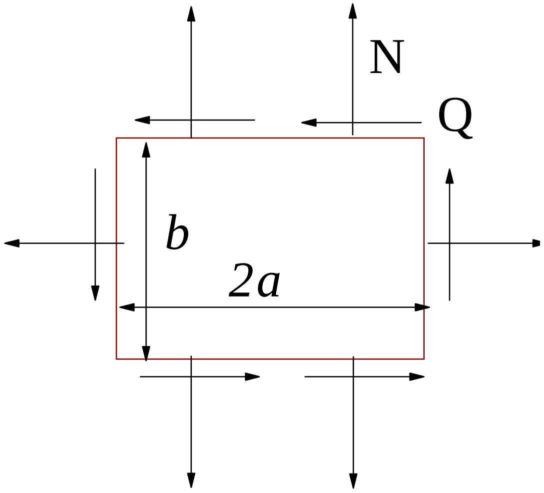
σ_{xx} is the microscopic stress



BLOCKY and MASONRY STRUCTURES

Besdo, 1995, Mühlhaus et al, 1993,1996, Sulem & Mühlhaus 1997





Elastic Interfaces:

$$Q_{kl} = c_Q \Delta u_{kl}$$

$$N_{kl} = c_N \Delta v_{kl}$$

Elastic energy

$$E_d = \frac{1}{4ab} \sum \left(Q_{kl} \Delta u_{kl} + N_{kl} \Delta v_{kl} \right)$$

EQUIVALENT COSSERAT CONTINUUM

Elastic energy of the Cosserat continuum

$$E_c = \sigma_{11}\gamma_{11} + \sigma_{12}\gamma_{12} + \sigma_{21}\gamma_{21} + \sigma_{22}\gamma_{2} + m_1\kappa_1 + m_2\kappa_2$$

Identification of the elastic parameters

$$\sigma_{11} = C_{11}\gamma_{11} + C_{12}\gamma_{22}$$

$$\sigma_{22} = C_{21}\gamma_{11} + C_{22}\gamma_{22}$$

$$\sigma_{12} = \left[G + G_c(1-\alpha)\right]\gamma_{12} + \left[G - G_c\right]\gamma_{21}$$

$$\sigma_{21} = \left[G - G_c\right]\gamma_{12} + \left[G + G_c(1+\alpha)\right]\gamma_{21}$$

$$m_1 = M_1\kappa_1$$

$$m_2 = M_2\kappa_2$$

$$C_{11} = \left(c_Q + 2c_N\right)\frac{a}{b}; C_{22} = c_N\frac{b}{a}$$

$$C_{12} = C_{21} = 0$$

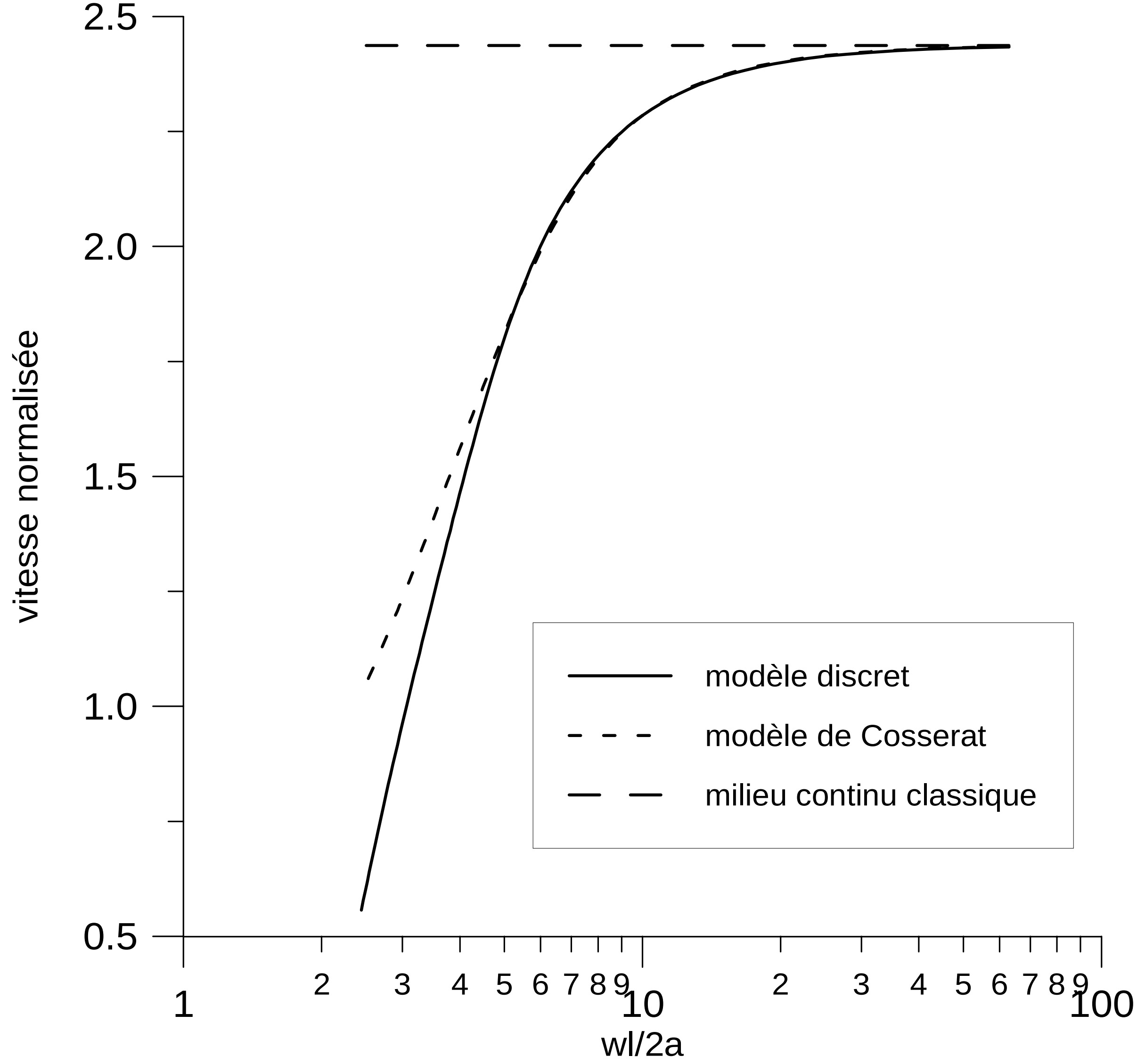
$$G = G_c = \frac{1}{4}\left[c_Q\frac{b}{a} + \left(c_N + 2c_Q\right)\frac{a}{b}\right]$$

$$\alpha = 2\frac{a^2\left(c_N + 2c_Q\right) - c_Q b^2}{a^2\left(c_N + 2c_Q\right) + c_Q b^2}$$

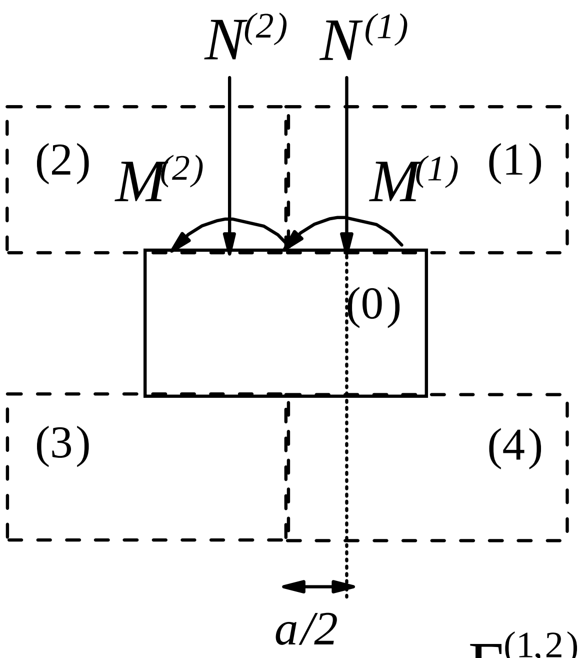
$$M_1 = \frac{a}{b}\left[c_N\frac{a^2}{4} + c_Q\left(\frac{b^2}{4} + 2a^2\right)\right]$$

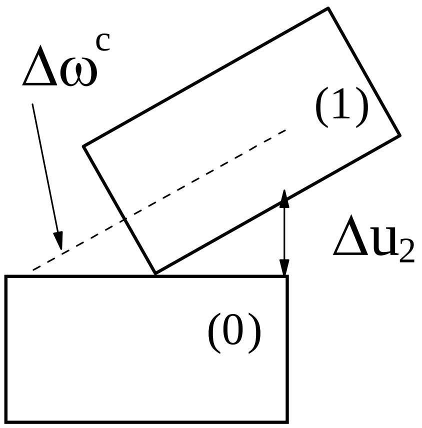
$$M_2 = \frac{b}{a}\left[c_N\frac{a^2}{4} + c_Q\frac{b^2}{4}\right]$$

DISPERSION FUNCTION



EXTENSION TO ELASTO-PLASTIC JOINTS





Block tilting criterion

$$F^{(1,2)} = -N^{(1,2)} + \frac{2}{a}\left|M^{(1,2)}\right| \leq 0$$

$$F^{(1,2)} = \sigma_{22} \pm \frac{b}{a}\sigma_{21} - \frac{2}{a}\left|m_2 \pm \frac{b}{a} m_1\right| \leq 0$$

Block sliding criterion

$$F^{(3)} = |T| \text{ -tan}\phi N \text{ - } C = 0$$

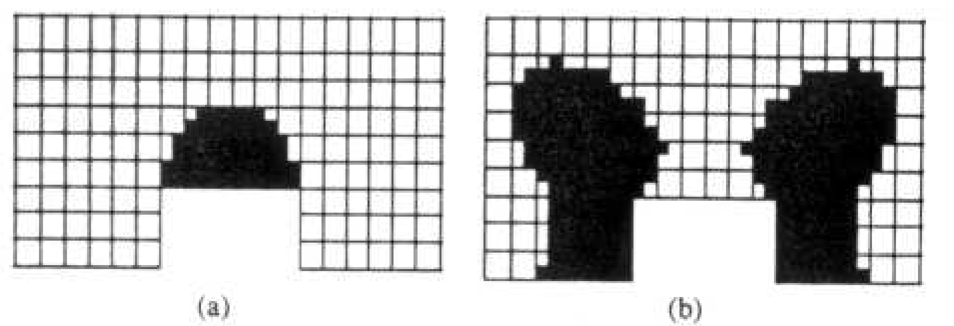
$$F^{3} = \left|\sigma_{12}\right| + \tan\phi\sigma_{22} - c \leq 0$$

EXAMPLES OF APPLICATION

Stability of a rectangular excavation in blocky rock

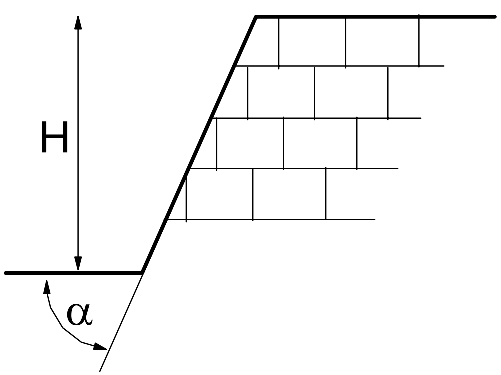
Dai et al, Int. J. Rock Mech Min Sci. 1996

Zones of tilting   Zones of sliding

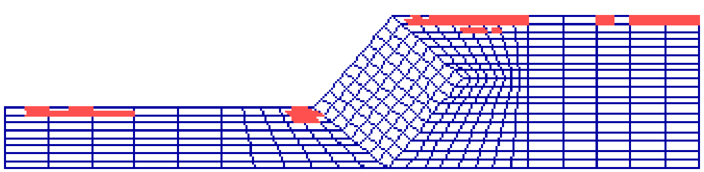


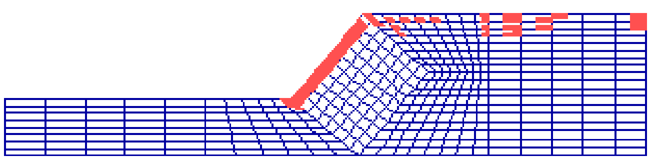
Stabilité de pente dans un massif de blocs

Sulem & Cerrolaza, GeoEng. 2000



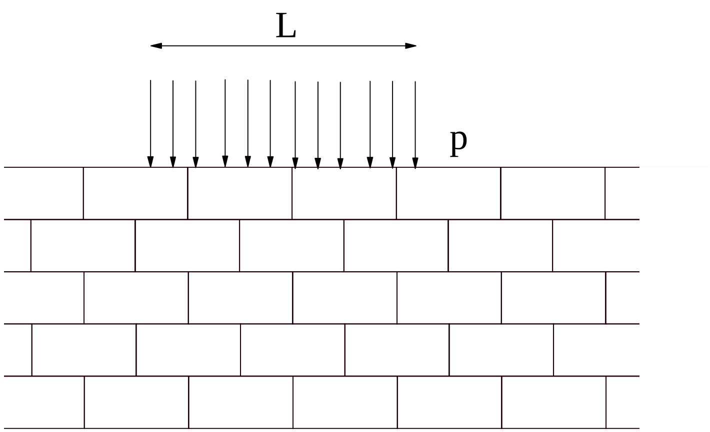
Zones of tilting   Zones of sliding



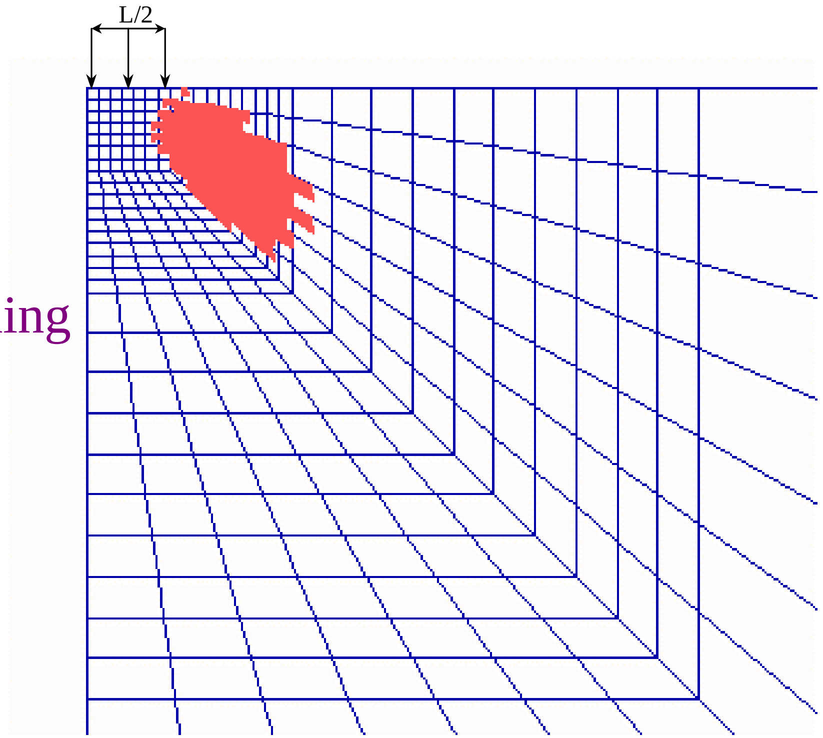


Foundation on blocky rock

Sulem, 2001

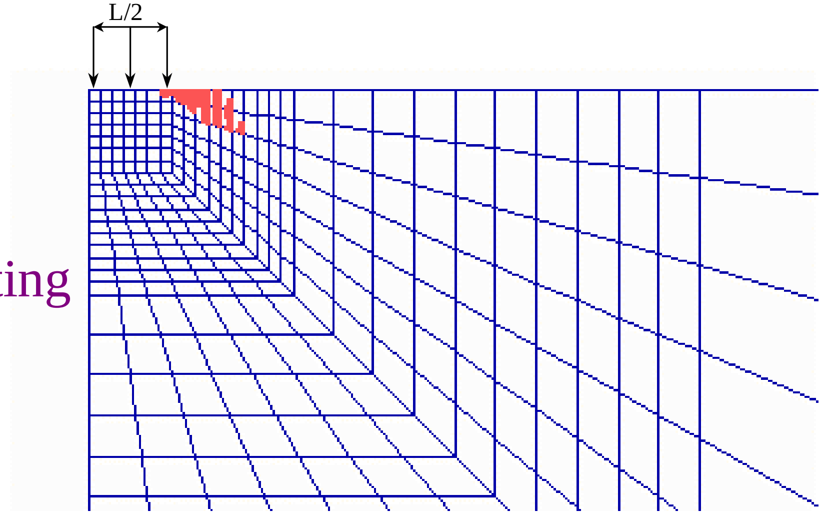


Zones of sliding



(a)

Zones of tilting



(b)

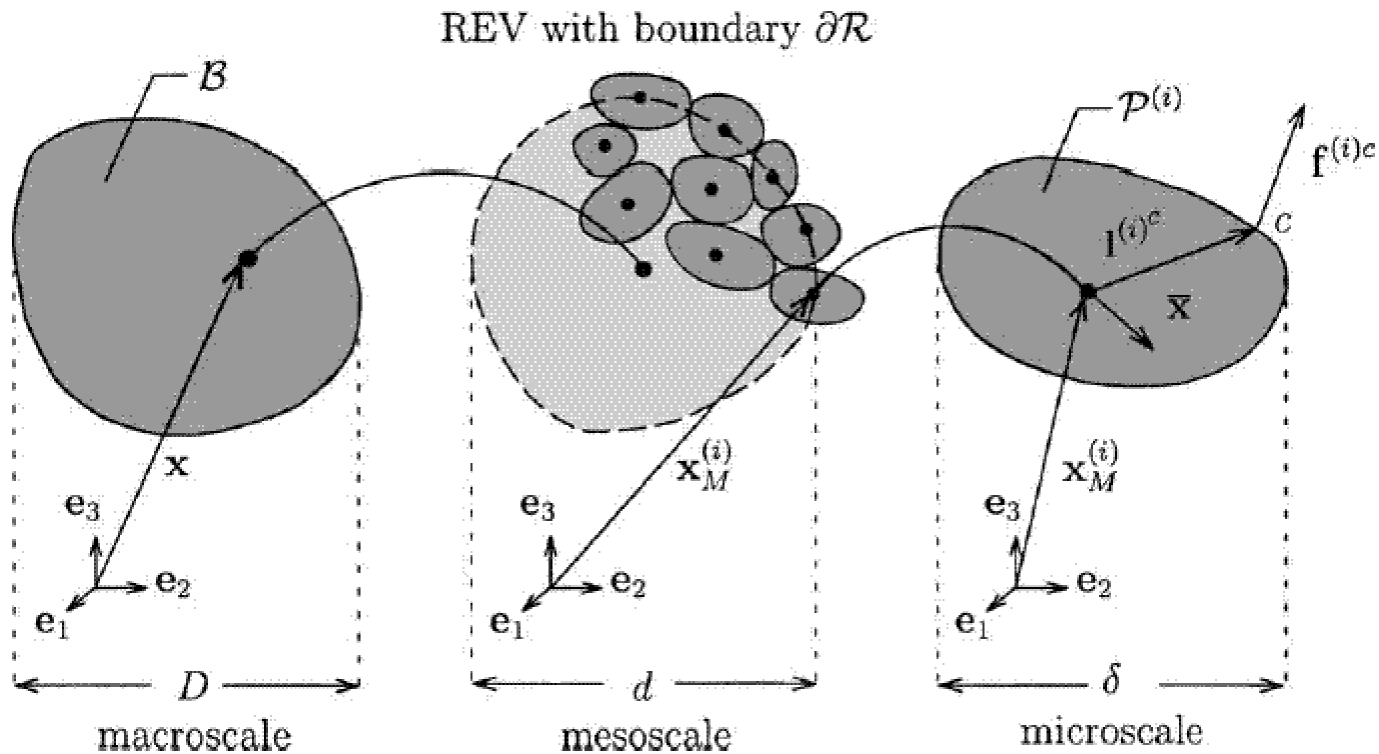
COSSERAT HOMOGENISATION OF GRANULAR MEDIA

Mühlhaus&Vardoulakis, 1987, Vardoulakis & Sulem 1995, Bardet & Vardoulakis, 2001, Ehlers et al, 2003, Kruyt, 2003, Gardiner & Tordesillas, 2004, Chang & Kuhn, 2005

The homogenisation of contact stresses is still an open question

•No unique expression for the expression of the virtual work

•No unique expression for the representative stresses



- Nature of granular contacts ? (anisotropy)
- Homogenisation procedure to relate macroscopic stresses and couple stresses to contact forces
- Stresses are related to the forces acting on the particles
- Couple stresses are related to the moments resulting from the reduction of the contact forces towards the particle centers
- The diameter of the REV should be small enough (5 particles diameter) (otherwise the results from localisation are smeared out)
- Non-symmetric stress tensors and couple stresses in the shear band zones.

LOCALISATION OF DEFORMATIONS

IMPORTANCE OF THE MICROSTRUCTURE

Limitations of classical elasto-plastic theories

- The thickness of the zone of strain localisation is undeterminate
- Ill-posedness of the underlying mathematical problem in the post-bifurcation regime
- Mesh sensitivity of finite element simulations
- Regularization for robust calculations with higher order continuum theories (length scale)
- Important grain rotations inside the shear band (experimental observations, DEM computations)

The Mühlhaus-Vardoulakis (1997) Cosserat plasticity model

Bodganova & Lippmann, 1975

<u>Extension</u> of classical flow theory of plasticity to Cosserat continuum through appropriate <u>generalisation</u> of stress and strain invariants

Example for 2D continuum

$$p = \sigma_{kk} / 2$$

$$\tau = \sqrt{h_1 s_{ij} s_{ij} + h_2 s_{ij} s_{ji} + \frac{h_3}{R^2} m_i m_i} ; h_1 + h_2 = \frac{1}{2}$$

$$\dot{\gamma}^p = \sqrt{g_1 \dot{g}_{ij}^p \dot{g}_{ij}^p + g_2 \dot{g}_{ij}^p \dot{g}_{ji}^p + \frac{g_3}{R^2} \dot{\kappa}_i^p \dot{\kappa}_i^p} ; g_1 + g_2 = 2$$

COULOMB YIELD SURFACE

$$F = \tau + \mu p = 0 \quad (p < 0 \text{ in compression})$$

μ MOBILISED FRICTION COEFFICIENT

PLASTIC POTENTIAL

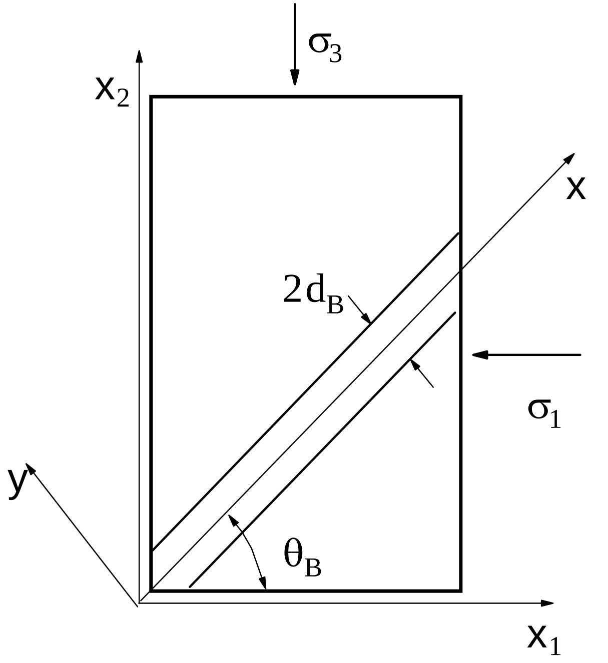
$$Q = \tau + \beta p = 0$$

β MOBILISED DILATANCY COEFFICIENT

SHEAR BAND ANALYSIS IN A COSSERAT CONTINUUM - BIAXIAL TEST

Mühlhaus&Vardoulakis, 1987, Vardoulakis & Sulem 1995

Tordesillas et al 2002



Governing equations

Equations of equilibrium

$$\partial_1 \dot{\sigma}_{11} + \partial_2 \dot{\sigma}_{12} + (\sigma_1 - \sigma_2)\partial_2 \dot{\omega}_{21} = 0$$

$$\partial_1 \dot{\sigma}_{21} + \partial_2 \dot{\sigma}_{22} + (\sigma_1 - \sigma_2)\partial_1 \dot{\omega}_{21} = 0$$

$$\partial_1 \dot{m}_1 + \partial_2 \dot{m}_2 + \dot{\sigma}_{21} - \dot{\sigma}_{12} = 0$$

Incremental constitutive equations

$$\dot{\sigma}_{11} = C_{11}\dot{\gamma}_{11} + C_{12}\dot{\gamma}_{22}$$

$$\dot{\sigma}_{22} = C_{21}\dot{\gamma}_{11} + C_{22}\dot{\gamma}_{22}$$

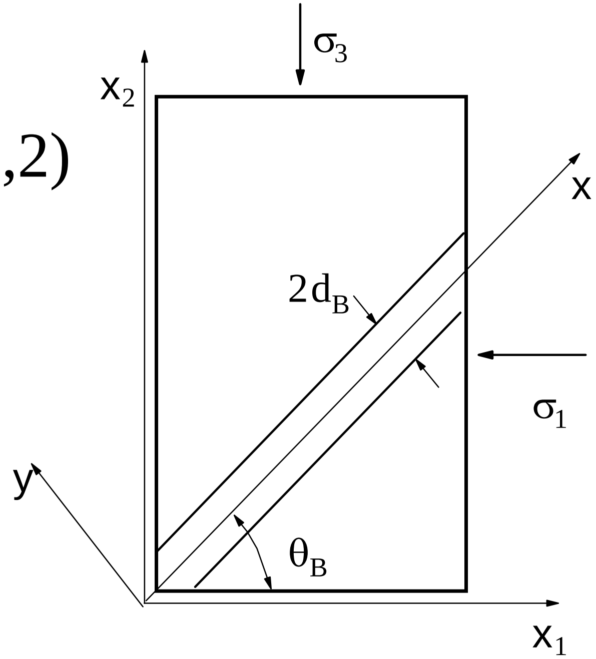
$$\dot{\sigma}_{12} = (G + G^c)\dot{\gamma}_{12} + (G - G^c)\dot{\gamma}_{21}$$

$$\dot{\sigma}_{21} = (G - G^c)\dot{\gamma}_{12} + (G + G^c)\dot{\gamma}_{21}$$

$$\dot{m}_1 = M\dot{\kappa}_1 \;\; ; \;\; \dot{m}_2 = M\dot{\kappa}_2$$

$$\text{with } G^c = \frac{1}{2(h_1 - h_2)} G \;\; ; \;\; M = \frac{1}{h_3} G R^2$$

Initial state : homogeneous rectilinear deformation inside the sample



→ zero couple stress $m_i \equiv 0$ (i=1,2)
→ zero Cosserat rotation $\omega^c \equiv 0$

Shear band solution

We search for periodic solutions

$$\vec{n} = \begin{pmatrix} n_1 \\ n_2 \end{pmatrix} \begin{pmatrix} -\sin\theta \\ \cos\theta \end{pmatrix}$$

$$x = n_2 x_1 - n_1 x_2$$

$$y = n_1 x_1 + n_2 x_2$$

$$v_1 = c_1 d_B \sin(\lambda y / d_B)$$

$$v_2 = c_2 d_B \sin(\lambda y / d_B)$$

$$\dot{\omega}^c = c_3 \cos(\lambda y / d_B)$$

Boundary conditions:

Continuity of couple stress across shear band boundaries
outside the shear-band couple stresses are zero

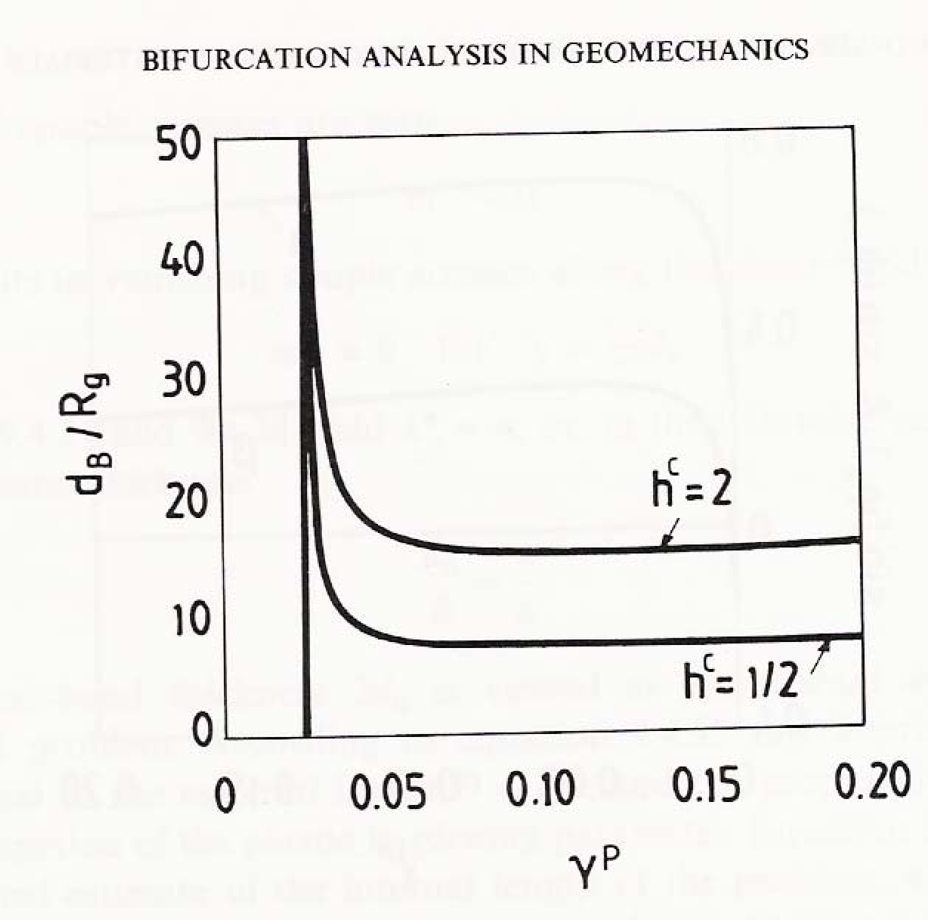
$$\frac{d_B}{R} = \frac{\pi}{\det(\Gamma)}$$

Γ is the acoustic tensor

$\det(\Gamma) = 0$ is the bifurcation condition for the classical approach

The internal length of the Cosserat continuum can be estimated from the thickness of shear bands

Example: Biaxial test on a fine Dutch dune sand



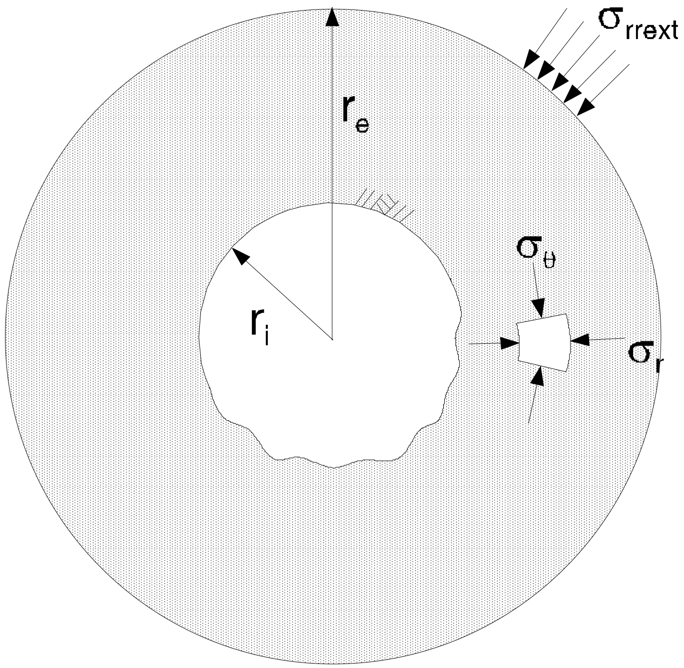
For states past the bifurcation point the deformation localises rapidly

Vardoulakis & Sulem, (1995), Bifurcation analysis in geomechanics

BOREHOLE STABILITY ANALYSIS

Papanastasiou & Vardoulakis, 1992

- Strain localisation
- Surface instability
- Post-localisation analysis
- Scale effect

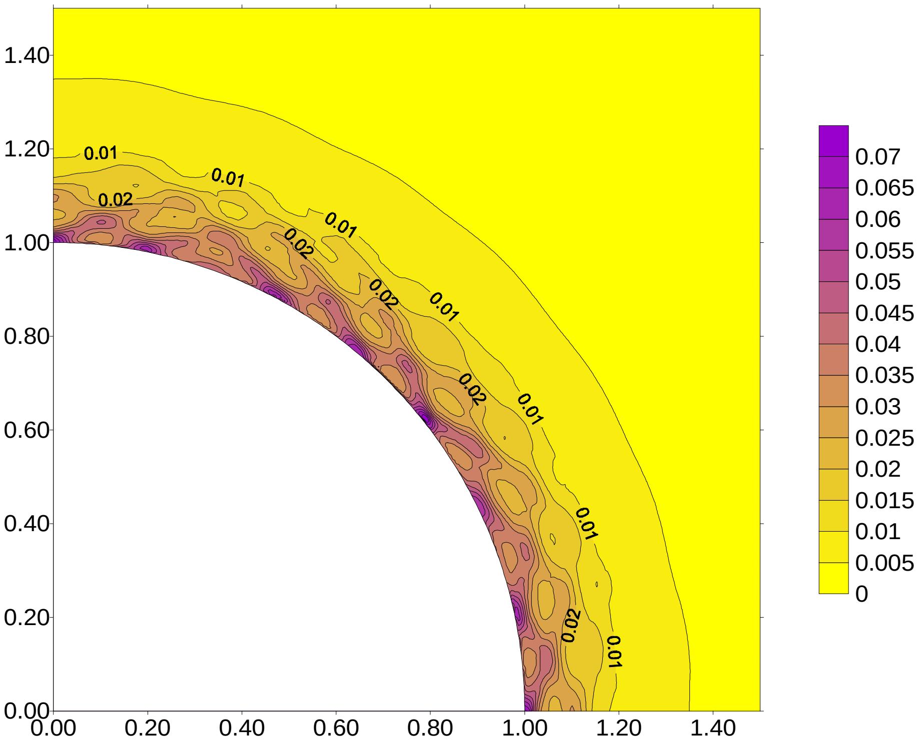


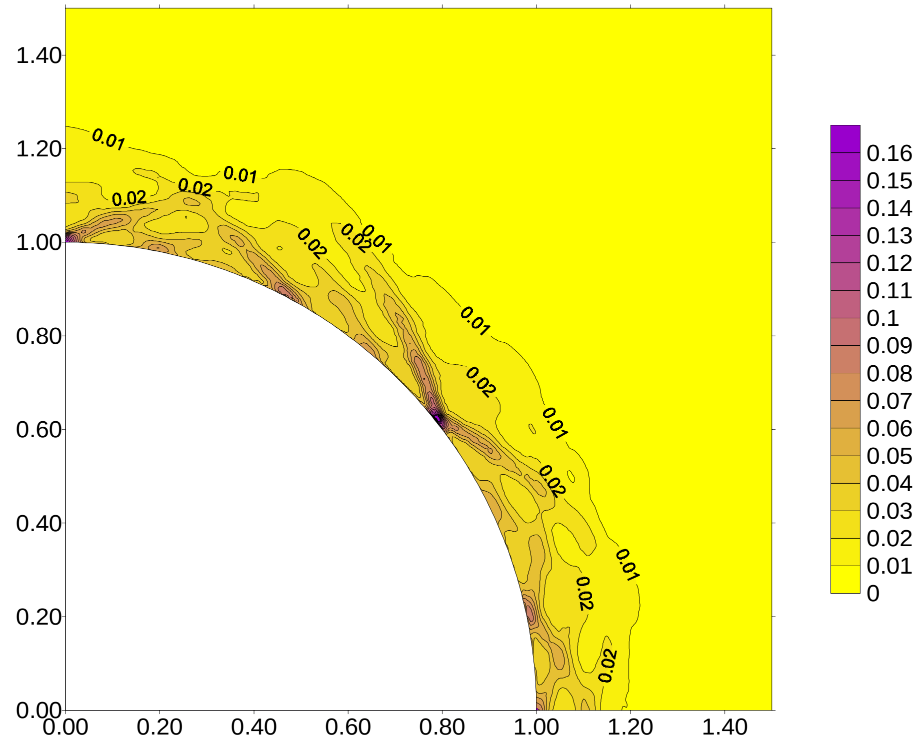
Robust post-localisation computations

(Papamichos & Vardoulakis)

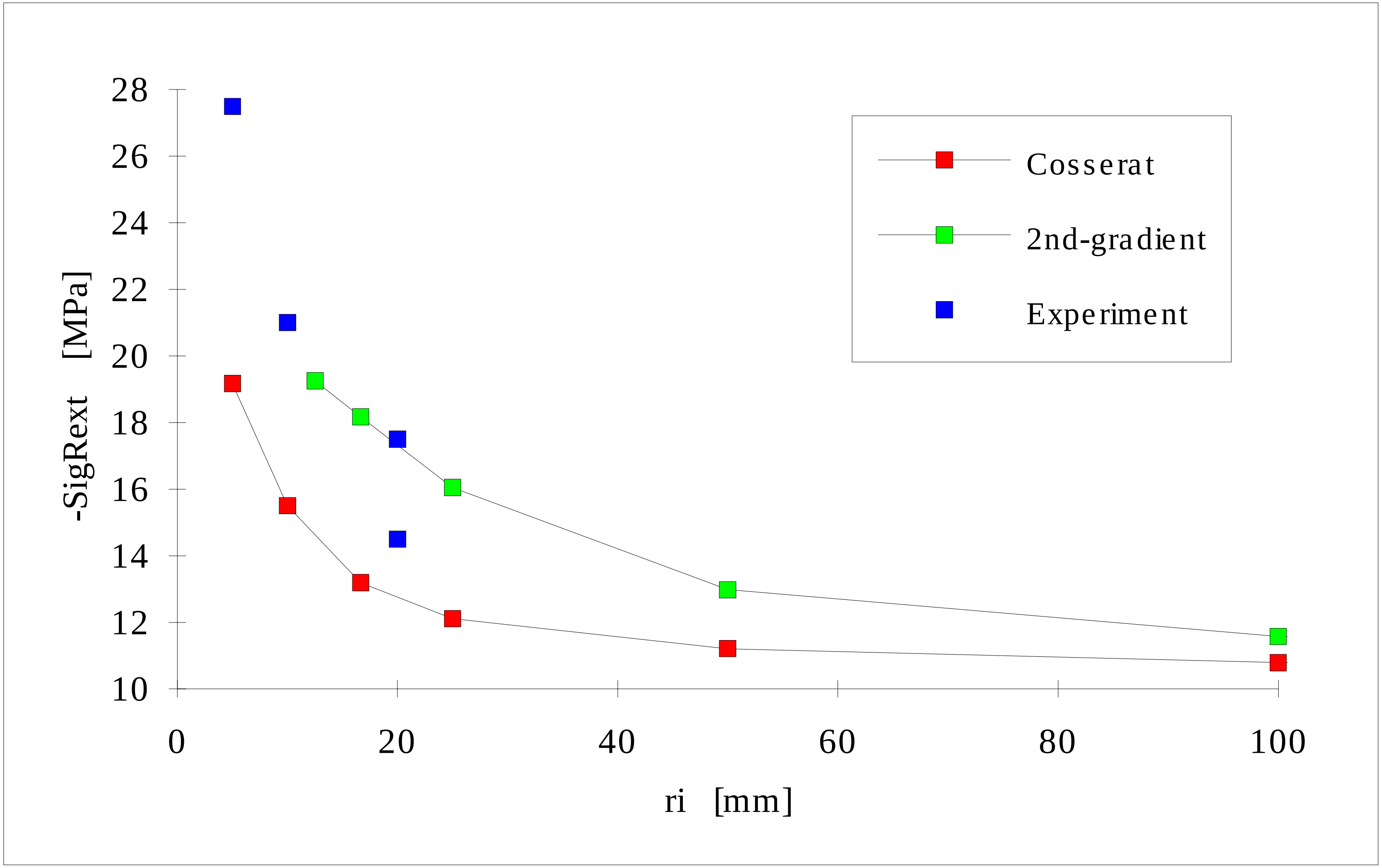
Red Wildmoor sandstone - Cosserat elastoplastic model

(Sulem et al 1999)





Scale effect *(from Papamichos et al 1997)*



CONCLUSIONS

•Important developments in the last 20 years in Cosserat plasticity

•Many applications for structured materials

•Useful tool for localisation and post localisation analysis

•Modelling of scale effect

•Extension of standard FE codes to include Cosserat structures

•Calibration of additional parameters (Cosserat shear modulus, internal length)

•Open questions for the transition between micromechanics of granular ensemble and macroscopic representation with Cosserat continuum