A gap in the spectrum of the Neumann-Laplacian on infinite cylinder with a periodic family of small voids.

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We study spectral properties of the Neumann-Laplacian on the periodic singularly perturbed quasicylinder $\Omega(\epsilon)$:

$$\begin{aligned} -\Delta u^\epsilon(x) &= \lambda^\epsilon u^\epsilon(x),\ x \in \Omega(\epsilon) \\ \partial_n u^\epsilon(x) &= 0,\ x \in \partial\Omega(\epsilon). \end{aligned}$$

This spectral problem should be interpreted as a singular perturbation of the same problem in the straight cylinder $\Omega = \omega \times \mathbb{R} \subset \mathbb{R}^3$. The quasicylinder $\Omega(\epsilon)$ is a periodic set which depends on a small parameter $\epsilon > 0$. It will be obtained from the straight cylinder Ω by a periodic nucleation of small voids with diameter of order ϵ.

In the literature the existence of spectral gaps is mainly investigated for periodic media which is infinite in all directions while the coefficients of differential operators, both in scalar and matrix case, are usually assumed to be contrasting (see [2], [3] and many others). Much less results are obtained for periodic waveguides, which are infinite in one direction only. In this case spectral gaps ought to be opened by varying the shape of the periodicity cell only which is in accordance with the results in the known engineering practice.

We utilizes the asymptotic analysis of the spectrum for the associated model problem in the periodicity cell. This approach has been realized for the Dirichlet Laplacian in papers [4], [2], [5] and others. We emphasize that our results for the Neumann-Laplacian are completely new and require different techniques.

We prove that introducing a periodic family of small voids in straight cylinder opens a gap in the spectrum of the Neumann-Laplacian in the case that the period is large enough. We calculate asymptotically the position and the width of the spectral gap. The mixed boundary value problem is under consideration as well and a similar result is obtained.

References

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