The 16th Hilbert problem, a story of mystery, mistakes and solution.

Oleg Viro

April 20, 2007

Read the Sixteenth Hilbert Problem
- **Harnack's inequality**
- Two natures of Harnack inequality
- Relative position of branches
- Harnack's construction
- Hilbert's construction
- Hilbert sextics
- Why impossible?
- Hilbert-Rohn-Gudkov method
- Call for attack
- Solutions
- Solved?

Breakthrough

Post Solution

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- Harnack's inequality
- Two natures of Harnack inequality
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- Hilbert's construction
- Hilbert sextics
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- Solutions
- Solved?

Breakthrough

Post Solution

16. Problem of the topology of algebraic curves and surfaces

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● Harnack's inequality
● Two natures of Harnack inequality
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● Hilbert's construction
● Hilbert sextics
● Why impossible?
● Hilbert-Rohn-Gudkov method
● Call for attack
● Solutions
● Solved?
Breakthrough
Post Solution

Harnack's inequality

16. Problem of the topology of algebraic curves and surfaces

Hilbert started with reminding of a background result:

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- Two natures of Harnack inequality
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- Hilbert's construction
- Hilbert sextics
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- Hilbert-Rohn-Gudkov method
- Call for attack
- Solutions
- Solved?

Breakthrough

Post Solution

16. Problem of the topology of algebraic curves and surfaces

The maximum number of closed and separate branches which a plane algebraic curve of the n-th order can have has been determined by Harnack (Mathematische Annalen, vol. 10).

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Read the Sixteenth Hilbert Problem
- Harnack's inequality
- Two natures of Harnack inequality
- Relative position of branches
- Harnack's construction
- Hilbert's construction
- Hilbert sextics
- Why impossible?
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- Call for attack
- Solutions
- Solved?

Breakthrough

Post Solution

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Here Hilbert referred to the following *Harnack inequality.*

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- **Harnack's inequality**
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- Harnack's construction
- Hilbert's construction
- Hilbert sextics
- Why impossible?
- Hilbert-Rohn-Gudkov method
- Call for attack
- Solutions
- Solved?

Breakthrough

Post Solution

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The words *Harnack inequality* are confusing: there are other, more famous Harnack inequalities concerning values of a positive harmonic function.

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- Hilbert sextics
- Why impossible?
- Hilbert-Rohn-Gudkov method
- Call for attack
- Solutions
- Solved?

Breakthrough

Post Solution

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The number of connected components of a plane projective real algebraic curve of degree n $\leq \frac{(n-1)(n-2)}{2} + 1$.

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Read the Sixteenth Hilbert Problem
- **Harnack's inequality**
- Two natures of Harnack inequality
- Relative position of branches
- Harnack's construction
- Hilbert's construction
- Hilbert sextics
- Why impossible?
- Hilbert-Rohn-Gudkov method
- Call for attack
- Solutions
- Solved?

Breakthrough

Post Solution

Harnack's proof: Let curve A of degree n has $\#(ovals) > M = \frac{(n-1)(n-2)}{2} + 1$.

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Read the Sixteenth Hilbert Problem
- Harnack's inequality
- Two natures of Harnack inequality
- Relative position of branches
- Harnack's construction
- Hilbert's construction
- Hilbert sextics
- Why impossible?
- Hilbert-Rohn-Gudkov method
- Call for attack
- Solutions
- Solved?

Breakthrough

Post Solution

Harnack's proof: Let curve A of degree n has $\#(ovals) > M = \frac{(n-1)(n-2)}{2} + 1$.

- Draw a curve B of degree $n-2$ through M points chosen on M ovals of A and $n-3$ points on one more oval.

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Read the Sixteenth Hilbert Problem
- Harnack's inequality
- Two natures of Harnack inequality
- Relative position of branches
- Harnack's construction
- Hilbert's construction
- Hilbert sextics
- Why impossible?
- Hilbert-Rohn-Gudkov method
- Call for attack
- Solutions
- Solved?

Breakthrough

Post Solution

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A curve of degree $n-2$ is defined by an equation with $\frac{(n-1)n}{2}$ coefficients. Hence it can be drawn through $\frac{(n-1)n}{2} - 1 = \frac{(n-1)(n-2)}{2} + n - 1 - 1 = M + n - 3$ points.

Read the Sixteenth Hilbert Problem
● Harnack's inequality
● Two natures of Harnack inequality
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● Harnack's construction
● Hilbert's construction
● Hilbert sextics
● Why impossible?
● Hilbert-Rohn-Gudkov method
● Call for attack
● Solutions
● Solved?
Breakthrough
Post Solution

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- Estimate the number of intersection points:

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Read the Sixteenth Hilbert Problem
- **Harnack's inequality**
- Two natures of Harnack inequality
- Relative position of branches
- Harnack's construction
- Hilbert's construction
- Hilbert sextics
- Why impossible?
- Hilbert-Rohn-Gudkov method
- Call for attack
- Solutions
- Solved?

Breakthrough

Post Solution

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- Draw a curve B of degree $n-2$ through M points chosen on M ovals of A and $n-3$ points on one more oval.
- Estimate the number of intersection points:

$\geq 2M + n - 3$

Two natures of Harnack inequality

Read the Sixteenth Hilbert Problem
- **Harnack's inequality**
- Two natures of Harnack inequality
- Relative position of branches
- Harnack's construction
- Hilbert's construction
- Hilbert sextics
- Why impossible?
- Hilbert-Rohn-Gudkov method
- Call for attack
- Solutions
- Solved?

Breakthrough

Post Solution

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An oval is met even number of times.

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- Harnack's construction
- Hilbert's construction
- Hilbert sextics
- Why impossible?
- Hilbert-Rohn-Gudkov method
- Call for attack
- Solutions
- Solved?

Breakthrough

Post Solution

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$\geq 2M + n - 3 = (n-1)(n-2) + 2 + n - 3 =$
$n^2 - 2n + 1 > n(n-2)$,

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Read the Sixteenth Hilbert Problem
- **Harnack's inequality**
- Two natures of Harnack inequality
- Relative position of branches
- Harnack's construction
- Hilbert's construction
- Hilbert sextics
- Why impossible?
- Hilbert-Rohn-Gudkov method
- Call for attack
- Solutions
- Solved?

Breakthrough

Post Solution

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$\geq 2M + n - 3 = (n-1)(n-2) + 2 + n - 3 =$
$n^2 - 2n + 1 > n(n-2)$,

- and apply the Bezout Theorem.

Two natures of Harnack inequality

Read the Sixteenth Hilbert Problem

- **Harnack's inequality**
- Two natures of Harnack inequality
- Relative position of branches
- Harnack's construction
- Hilbert's construction
- Hilbert sextics
- Why impossible?
- Hilbert-Rohn-Gudkov method
- Call for attack
- Solutions
- Solved?

Breakthrough

Post Solution

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Two natures of Harnack inequality

Read the Sixteenth Hilbert Problem
- **Harnack's inequality**
- Two natures of Harnack inequality
- Relative position of branches
- Harnack's construction
- Hilbert's construction
- Hilbert sextics
- Why impossible?
- Hilbert-Rohn-Gudkov method
- Call for attack
- Solutions
- Solved?

Breakthrough

Post Solution

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Read the Sixteenth Hilbert Problem
● **Harnack's inequality**
● Two natures of Harnack inequality
● Relative position of branches
● Harnack's construction
● Hilbert's construction
● Hilbert sextics
● Why impossible?
● Hilbert-Rohn-Gudkov method
● Call for attack
● Solutions
● Solved?
Breakthrough
Post Solution

Two natures of Harnack inequality

Klein's proof: apply the following theorem to

- the complexification of the curve and
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Two natures of Harnack inequality

Read the Sixteenth Hilbert Problem
- Harnack's inequality
- Two natures of Harnack inequality
- Relative position of branches
- Harnack's construction
- Hilbert's construction
- Hilbert sextics
- Why impossible?
- Hilbert-Rohn-Gudkov method
- Call for attack
- Solutions
- Solved?

Breakthrough

Post Solution

Klein's proof: apply the following theorem to

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Theorem. Let S be an orientable closed connected surface, $\sigma : S \to S$ an orientation reversing involution, and F the fixed point set of σ.

Two natures of Harnack inequality

Read the Sixteenth Hilbert Problem
- Harnack's inequality
- Two natures of Harnack inequality
- Relative position of branches
- Harnack's construction
- Hilbert's construction
- Hilbert sextics
- Why impossible?
- Hilbert-Rohn-Gudkov method
- Call for attack
- Solutions
- Solved?

Breakthrough

Post Solution

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Read the Sixteenth Hilbert Problem
● Harnack's inequality
● Two natures of Harnack inequality
● Relative position of branches
● Harnack's construction
● Hilbert's construction
● Hilbert sextics
● Why impossible?
● Hilbert-Rohn-Gudkov method
● Call for attack
● Solutions
● Solved?
Breakthrough
Post Solution

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Lemma: $\#$connected components$(S \setminus F) \leq 2$.

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Proof. Let A ba a connected component of $S \smallsetminus F$.

Two natures of Harnack inequality

Read the Sixteenth Hilbert Problem
● Harnack's inequality
● Two natures of Harnack inequality
● Relative position of branches
● Harnack's construction
● Hilbert's construction
● Hilbert sextics
● Why impossible?
● Hilbert-Rohn-Gudkov method
● Call for attack
● Solutions
● Solved?
Breakthrough
Post Solution

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Read the Sixteenth Hilbert Problem
● Harnack's inequality
● Two natures of Harnack inequality
● Relative position of branches
● Harnack's construction
● Hilbert's construction
● Hilbert sextics
● Why impossible?
● Hilbert-Rohn-Gudkov method
● Call for attack
● Solutions
● Solved?
Breakthrough
Post Solution

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Proof. Let A ba a connected component of $S \smallsetminus F$.
Then $\mathrm{Cl}(A) \cup \sigma(A)$ is a closed surface.
Hence $\mathrm{Cl}(A) \cup \sigma(A) = S$.

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Read the Sixteenth Hilbert Problem
- Harnack's inequality
- Two natures of Harnack inequality
- Relative position of branches
- Harnack's construction
- Hilbert's construction
- Hilbert sextics
- Why impossible?
- Hilbert-Rohn-Gudkov method
- Call for attack
- Solutions
- Solved?

Breakthrough

Post Solution

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Two natures of Harnack inequality

Read the Sixteenth Hilbert Problem
- Harnack's inequality
- Two natures of Harnack inequality
- Relative position of branches
- Harnack's construction
- Hilbert's construction
- Hilbert sextics
- Why impossible?
- Hilbert-Rohn-Gudkov method
- Call for attack
- Solutions
- Solved?

Breakthrough

Post Solution

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Read the Sixteenth Hilbert Problem

- **Harnack's inequality**
- Two natures of Harnack inequality
- Relative position of branches
- Harnack's construction
- Hilbert's construction
- Hilbert sextics
- Why impossible?
- Hilbert-Rohn-Gudkov method
- Call for attack
- Solutions
- Solved?

Breakthrough

Post Solution

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Proof of Theorem. A curve with $> genus(S) + x$ components divides S to $> x + 1$ components. □

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Read the Sixteenth Hilbert Problem
- **Harnack's inequality**
- Two natures of Harnack inequality
- Relative position of branches
- Harnack's construction
- Hilbert's construction
- Hilbert sextics
- Why impossible?
- Hilbert-Rohn-Gudkov method
- Call for attack
- Solutions
- Solved?

Breakthrough

Post Solution

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Which proof is better?

Relative position of branches

Read the Sixteenth Hilbert Problem
- **Harnack's inequality**
- Two natures of Harnack inequality
- Relative position of branches
- Harnack's construction
- Hilbert's construction
- Hilbert sextics
- Why impossible?
- Hilbert-Rohn-Gudkov method
- Call for attack
- Solutions
- Solved?

Breakthrough

Post Solution

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Read the Sixteenth Hilbert Problem
● Harnack's inequality
● Two natures of Harnack inequality
● Relative position of branches
● Harnack's construction
● Hilbert's construction
● Hilbert sextics
● Why impossible?
● Hilbert-Rohn-Gudkov method
● Call for attack
● Solutions
● Solved?

Breakthrough

Post Solution

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Read the Sixteenth Hilbert Problem
● Harnack's inequality
● Two natures of Harnack inequality
● Relative position of branches
● Harnack's construction
● Hilbert's construction
● Hilbert sextics
● Why impossible?
● Hilbert-Rohn-Gudkov method
● Call for attack
● Solutions
● Solved?
Breakthrough
Post Solution

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There arises the further question as to the relative position of the branches in the plane.

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Read the Sixteenth Hilbert Problem
- **Harnack's inequality**
- Two natures of Harnack inequality
- Relative position of branches
- Harnack's construction
- Hilbert's construction
- Hilbert sextics
- Why impossible?
- Hilbert-Rohn-Gudkov method
- Call for attack
- Solutions
- Solved?

Breakthrough

Post Solution

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Read the Sixteenth Hilbert Problem
- **Harnack's inequality**
- Two natures of Harnack inequality
- Relative position of branches
- Harnack's construction
- Hilbert's construction
- Hilbert sextics
- Why impossible?
- Hilbert-Rohn-Gudkov method
- Call for attack
- Solutions
- Solved?

Breakthrough

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Read the Sixteenth Hilbert Problem
- **Harnack's inequality**
- Two natures of Harnack inequality
- Relative position of branches
- Harnack's construction
- Hilbert's construction
- Hilbert sextics
- Why impossible?
- Hilbert-Rohn-Gudkov method
- Call for attack
- Solutions
- Solved?

Breakthrough

Post Solution

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Read the Sixteenth Hilbert Problem
- **Harnack's inequality**
- Two natures of Harnack inequality
- Relative position of branches
- Harnack's construction
- Hilbert's construction
- Hilbert sextics
- Why impossible?
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- Call for attack
- Solutions
- Solved?

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Post Solution

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- The depth of each of their nests ≤ 2.

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● **Harnack's inequality**
● Two natures of Harnack inequality
● Relative position of branches
● Harnack's construction
● Hilbert's construction
● Hilbert sextics
● Why impossible?
● Hilbert-Rohn-Gudkov method
● Call for attack
● Solutions
● Solved?
Breakthrough
Post Solution

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- Harnack's inequality
- Two natures of Harnack inequality
- Relative position of branches
- Harnack's construction
- Hilbert's construction
- Hilbert sextics
- Why impossible?
- Hilbert-Rohn-Gudkov method
- Call for attack
- Solutions
- Solved?

Breakthrough

Post Solution

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$\frac{3n^2-6n}{8}+1$ outer and

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● Harnack's construction
● Hilbert's construction
● Hilbert sextics
● Why impossible?
● Hilbert-Rohn-Gudkov method
● Call for attack
● Solutions
● Solved?
Breakthrough
Post Solution

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His curves are very special:
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● Harnack's inequality
● Two natures of Harnack inequality
● Relative position of branches
● Harnack's construction
● Hilbert's construction
● Hilbert sextics
● Why impossible?
● Hilbert-Rohn-Gudkov method
● Call for attack
● Solutions
● Solved?
Breakthrough
Post Solution

Relative position of branches

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His curves are very special:
● The depth of each of their nests ≤ 2.
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In degree 6: 10 outer ovals and 1 inner oval.

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- **Harnack's inequality**
- Two natures of Harnack inequality
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- Harnack's construction
- Hilbert's construction
- Hilbert sextics
- Why impossible?
- Hilbert-Rohn-Gudkov method
- Call for attack
- Solutions
- Solved?

Breakthrough

Post Solution

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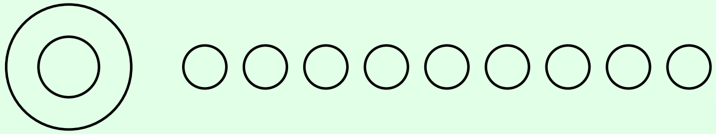
In degree 6:



Harnack's construction

Read the Sixteenth Hilbert Problem

- **Harnack's inequality**
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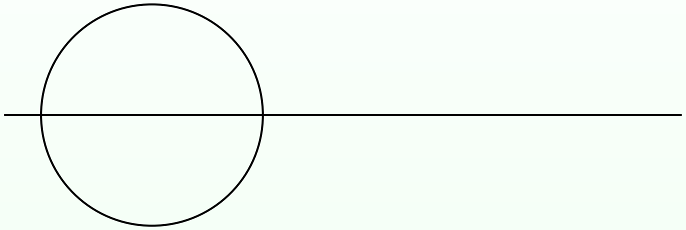
Breakthrough

Post Solution

Take a line and circle: 

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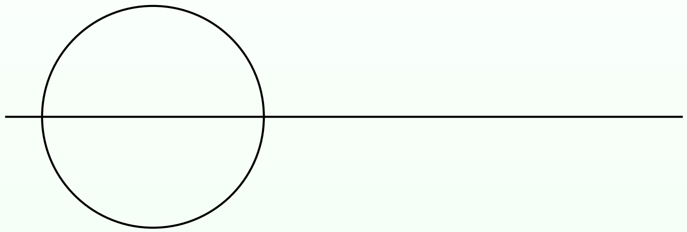
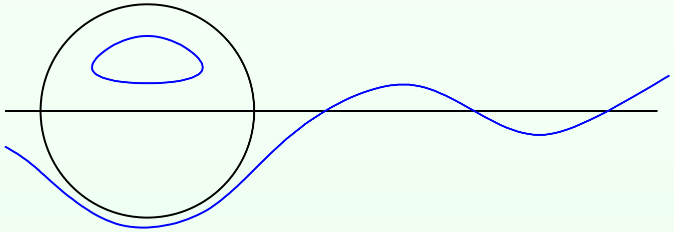


Perturb their union:



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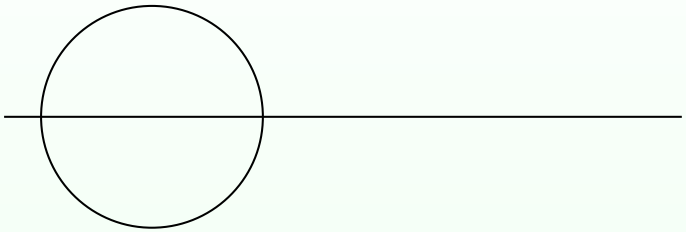

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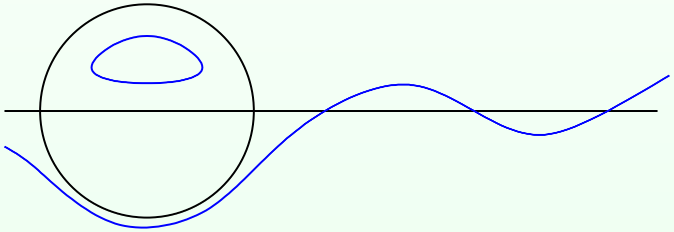

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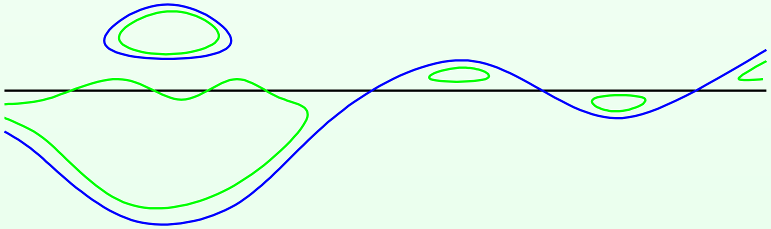

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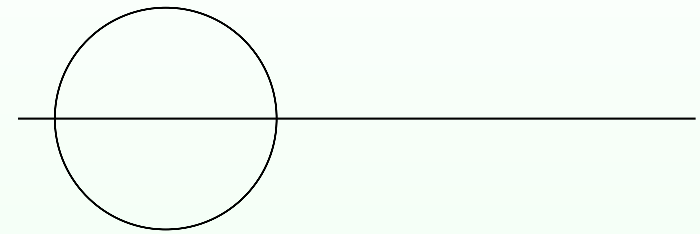

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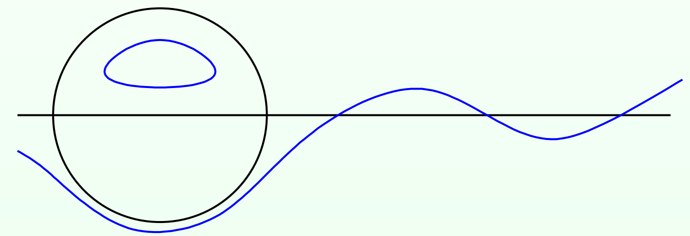

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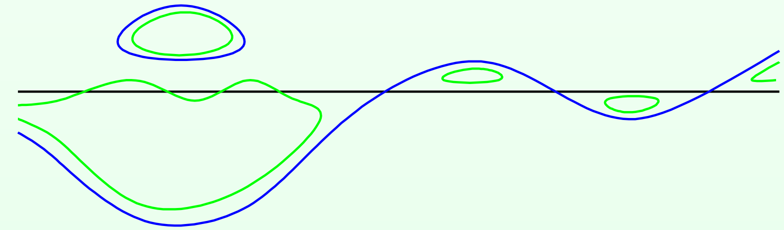

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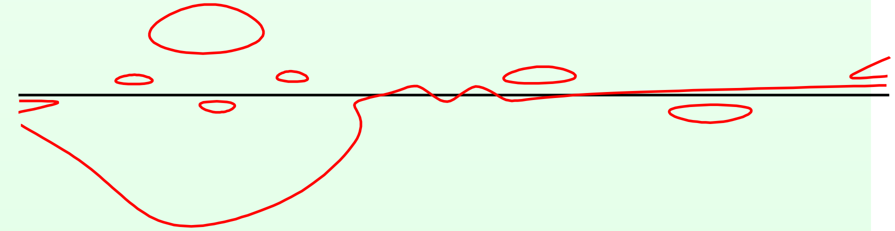

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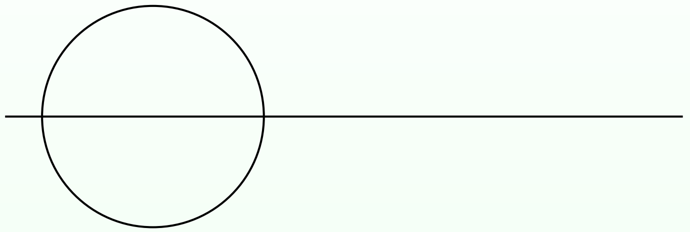

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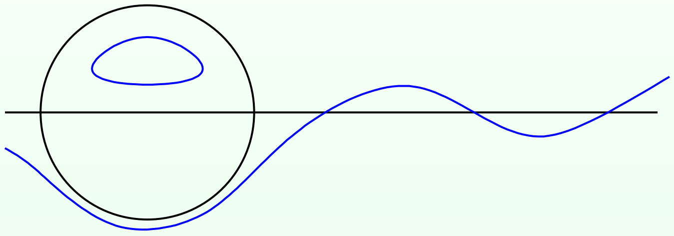

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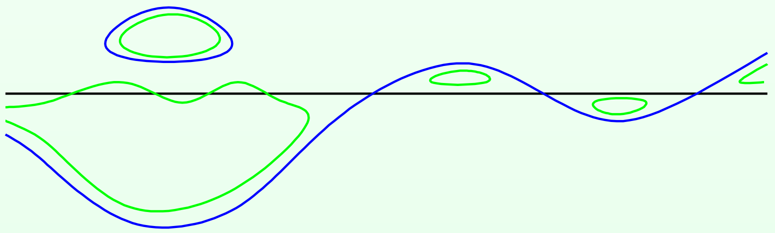

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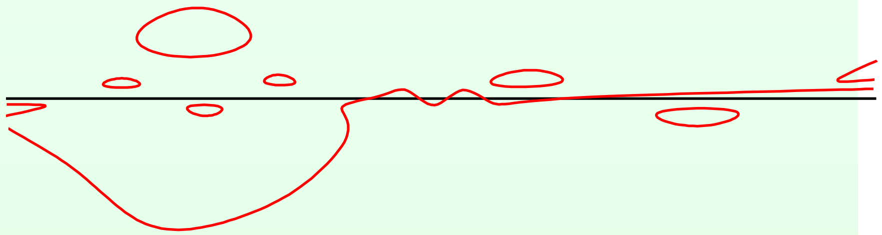

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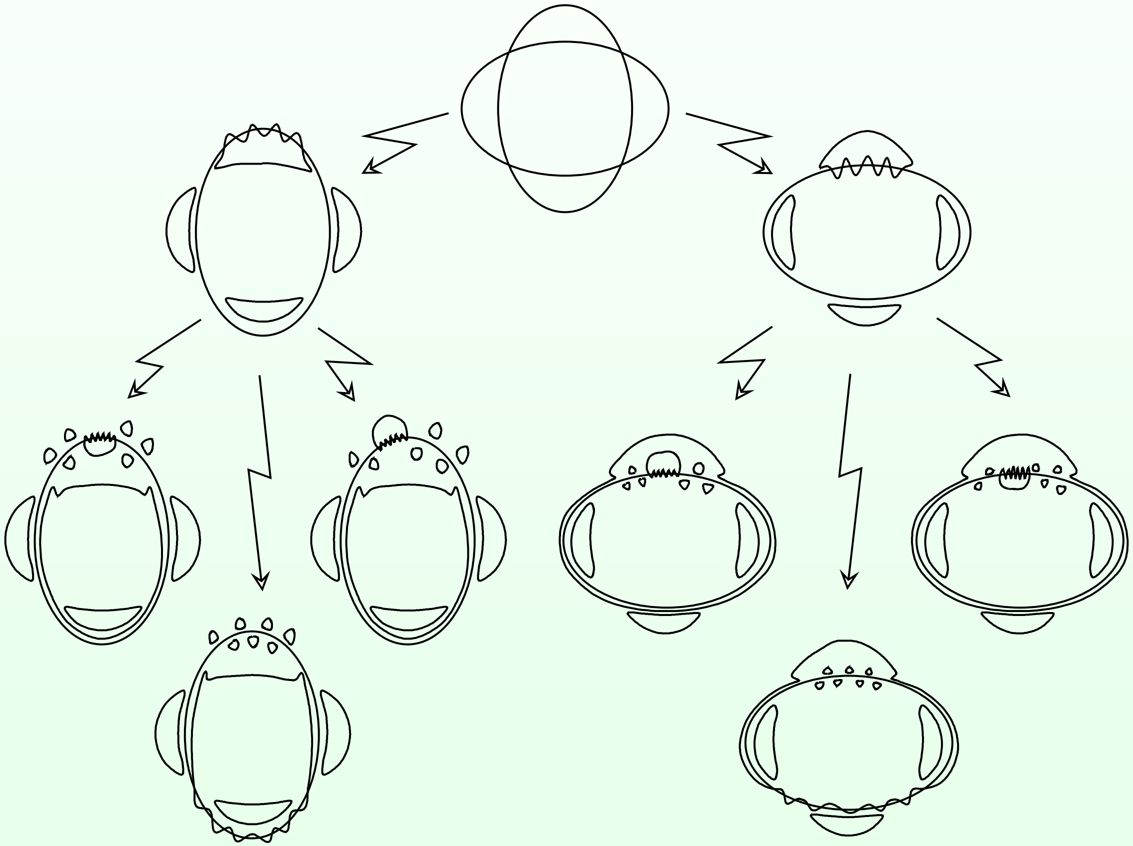

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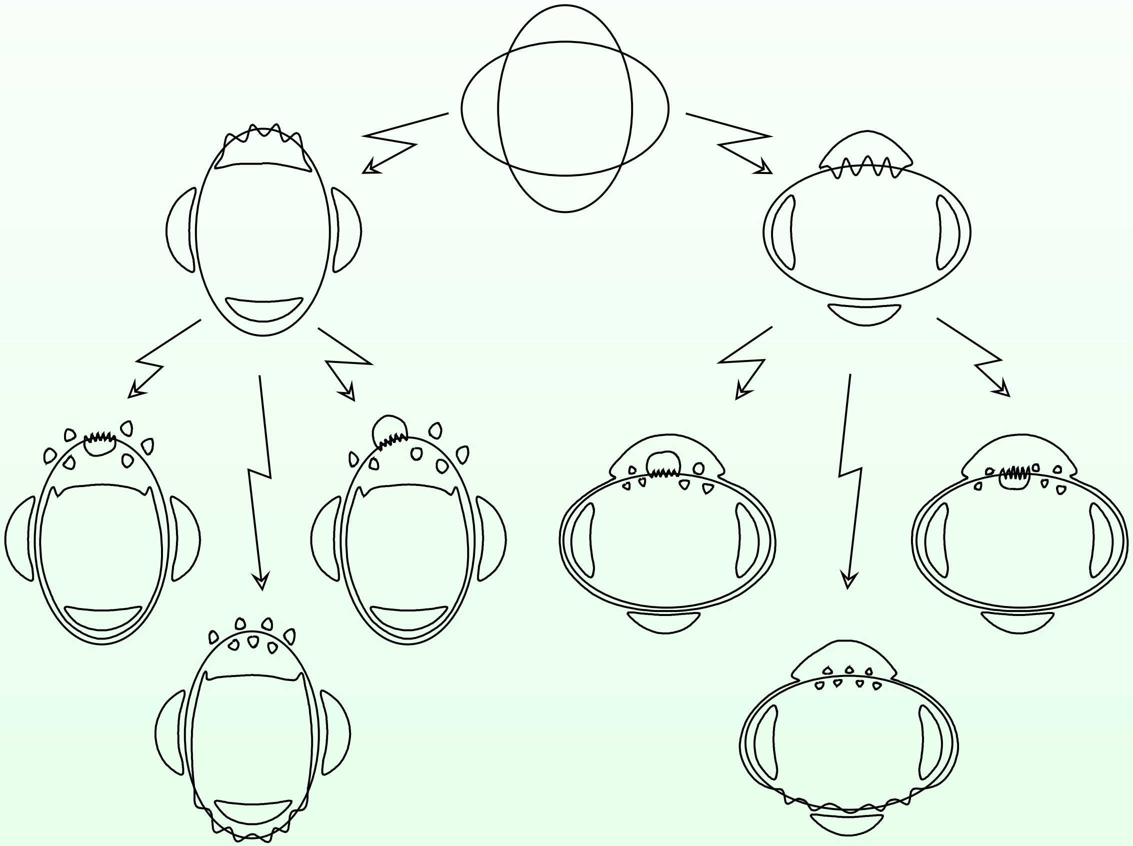

An ellipse does what the line did in Harnack's construction.

Hilbert sextics

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Each Hilbert's curve of degree 6 has one of the following two configurations of ovals:

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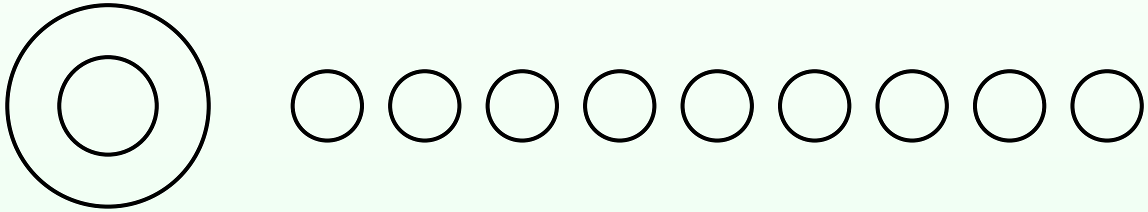

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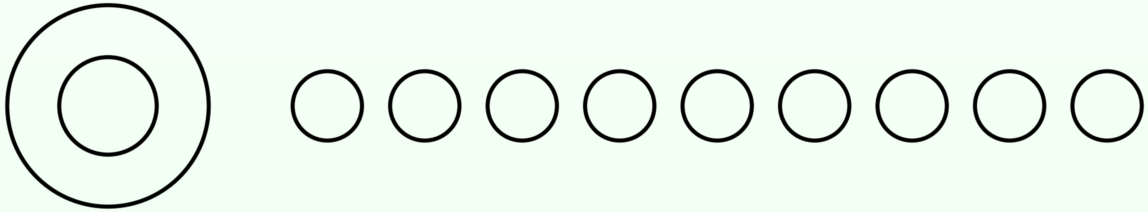

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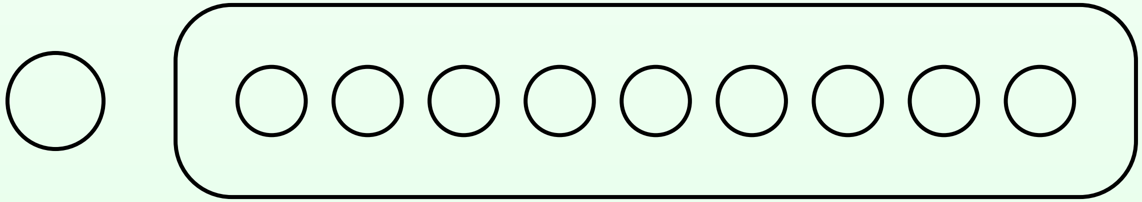

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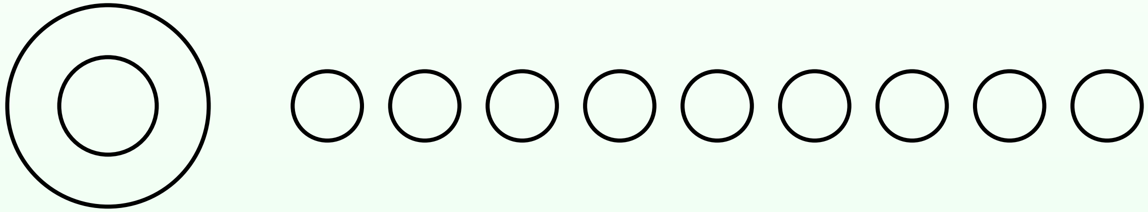

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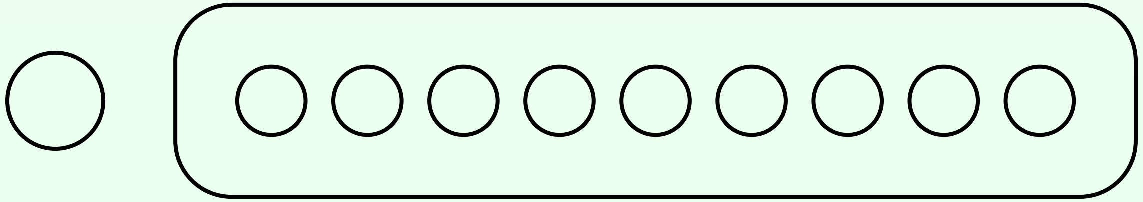

Hilbert worked hard

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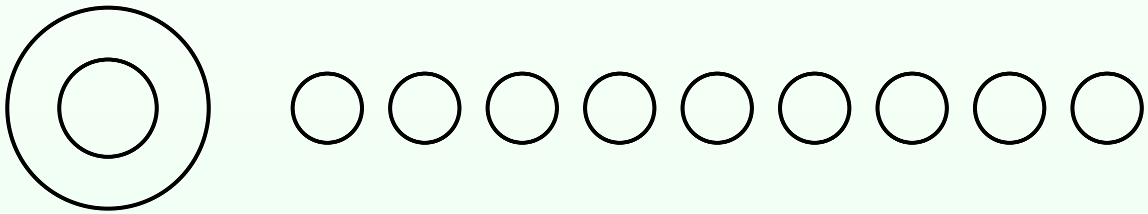

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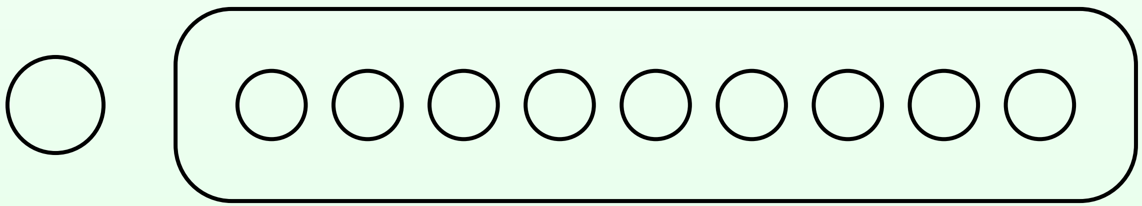

Hilbert worked hard, but could not construct curves of degree 6 with 11 connected components positioned with respect to each other in any other way.

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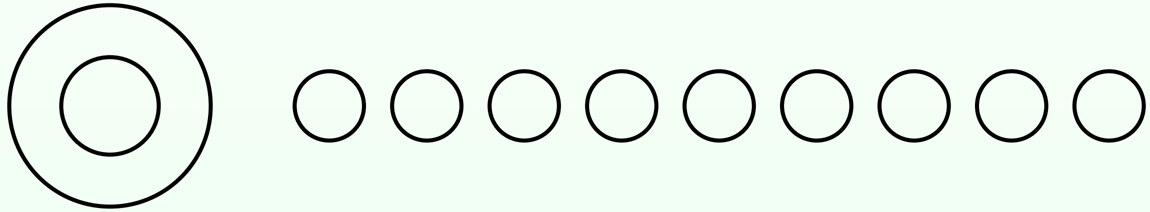

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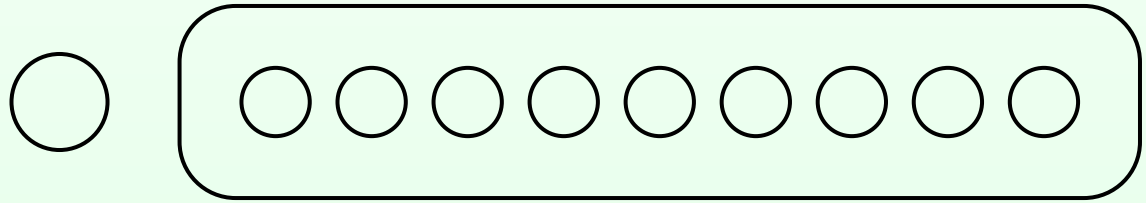

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He concluded that this is impossible.

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As to curves of the 6-th order, I have satisfied myself–by a complicated process, it is true–that of the eleven branches which they can have according to Harnack, by no means all can lie external to one another,

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Now it is called *Hilbert-Rohn-Gudkov method.*

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15 years later, in his Doctor dissertation, Gudkov disproved it and found the final answer.

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A "complicated process" could not really satisfy Hilbert.

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Why did Hilbert distinguish curves with maximal number of branches?

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Hilbert deeply appreciated this paradigm of the calculus of variations.

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- Harnack's construction
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- Call for attack
- Solutions
- Solved?

Breakthrough

Post Solution

Call for attack

A "complicated process" could not really satisfy Hilbert.

Desperately wishing to understand the real reasons of this very mysterious phenomenon, Hilbert called for attack:

A thorough investigation of the relative position of the separate branches when their number is the maximum seems to me to be of very great interest,

Now people (especially, specialists) tend to widen the content of Hilbert's 16th problem.

Call for attack

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To support this view, they cite also the next piece of Hilbert's text:

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A thorough investigation of the relative position of the separate branches when their number is the maximum seems to me to be of very great interest, and not less so the corresponding investigation as to the number, form, and position of the sheets of an algebraic surface in space.

Call for attack

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The word corresponding is crucial here. Without it, this would really be a mere call to study the topology of real algebraic surfaces.

Call for attack

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The word corresponding is crucial here. Without it, this would really be a mere call to study the topology of real algebraic surfaces. So, what is "the corresponding"?

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The word corresponding is crucial here. Without it, this would really be a mere call to study the topology of real algebraic surfaces. So, what is "the corresponding"? Hilbert continues:

Call for attack

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A "complicated process" could not really satisfy Hilbert. Desperately wishing to understand the real reasons of this very mysterious phenomenon, Hilbert called for attack:

A thorough investigation of the relative position of the separate branches when their number is the maximum seems to me to be of very great interest, and not less so the corresponding investigation as to the number, form, and position of the sheets of an algebraic surface in space. Till now, indeed, it is not even known what is the maximum number of sheets which a surface of the 4-th order in three dimensional space can really have (Cf. Rohn, "Flächen vierter Ordnung" 1886).

Solutions

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Post Solution

Now we know that the maximum number of connected components of a quartic surface in the 3-dimensional projective space is 10.

Solutions

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Now we know that the maximum number of connected components of a quartic surface in the 3-dimensional projective space is 10.

This was proven in 1972 by V.M.Kharlamov in his Master thesis

Solutions

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Solutions

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in the breakthrough of 1969-72, which solved the sixteenth Hilbert problem.

All the questions contained, explicitly or implicitly, in the sixteenth problem have been answered by D.A.Gudkov, V.I.Arnold, V.A.Rokhlin and V.M.Kharlamov in this breakthrough.

Solutions

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In 1969, D.A.Gudkov found the final answer to the question about position of real branches of maximal curves of degree 6.

Solutions

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Kharlamov completed by 1976 the "corresponding investigation" of nonsingular quartic surfaces.

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Breakthrough

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Solved?

All in all this gives good reasons to consider the sixteenth Hilbert problem solved.

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Solved?

All in all this gives good reasons to consider the sixteenth Hilbert problem solved.

However, I am not aware about any publication, where it is claimed.

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Unusual?

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The solution was initiated by completion of long difficult technical work.

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It looks like a final point.

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Solved?

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Nobody wanted to dispose of the symbol.

Solved?

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The sixteenth Hilbert problem was the symbol of the breakthrough.
Nobody cared that the puzzle had been solved.

Read the Sixteenth Hilbert Problem

Breakthrough

- Isotopy classification of nonsingular sextics
- Gudkov's M-curve
- Gudkov's conjecture
- Arnold's congruence
- Complexification
- In homology
- Proof of Arnold's congruence
- Gudkov-Rokhlin congruence
- The role of complexification
- Mystery of the 16th Hilbert problem
- Second part
- Second part
- The first part success

Post Solution

Breakthrough

Isotopy classification of nonsingular sextics

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Isotopy classification of nonsingular sextics

In 1969, D.A.Gudkov completed isotopy classification of nonsingular real algebraic plane projective curves of degree 6. The project started in 1948.

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A.A.Andronov proposed to Gudkov: develop theory of degrees of coarseness for real algebraic curves.

Isotopy classification of nonsingular sextics

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Like in the theory of dynamical systems.

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In 1954 Gudkov defended PhD.
About 12-14 years later he prepared publication.

Isotopy classification of nonsingular sextics

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Post Solution

In 1969, D.A.Gudkov completed isotopy classification of nonsingular real algebraic plane projective curves of degree 6.

The summary of results:

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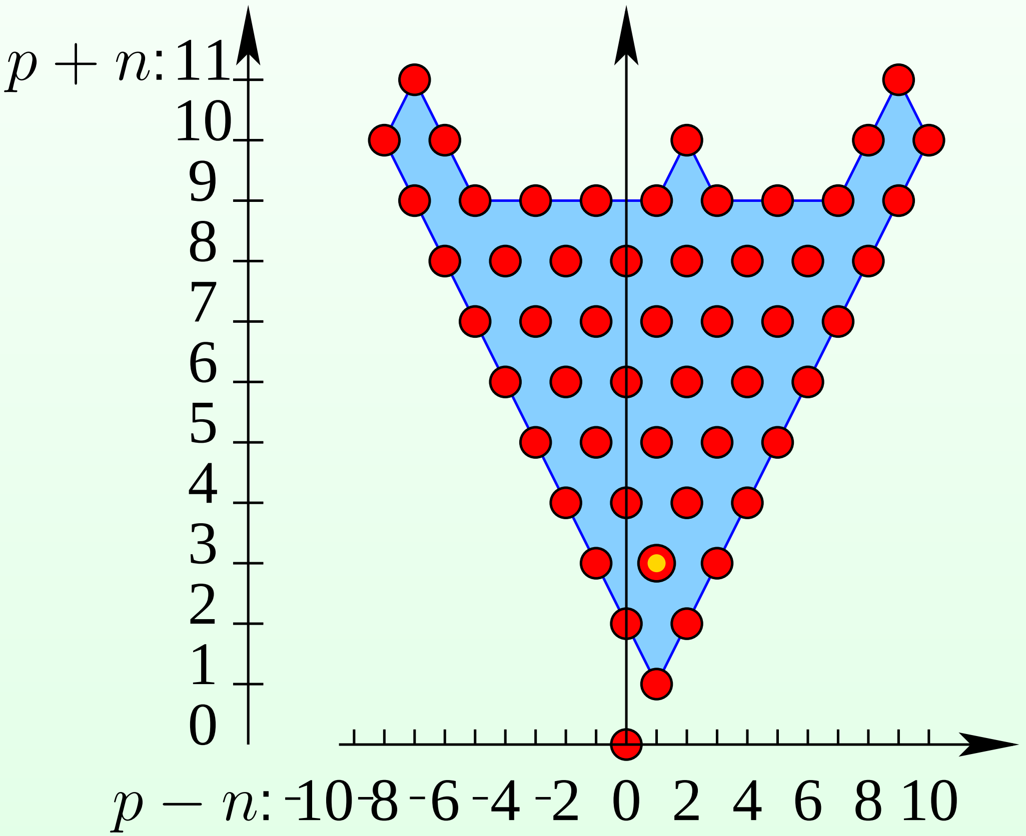

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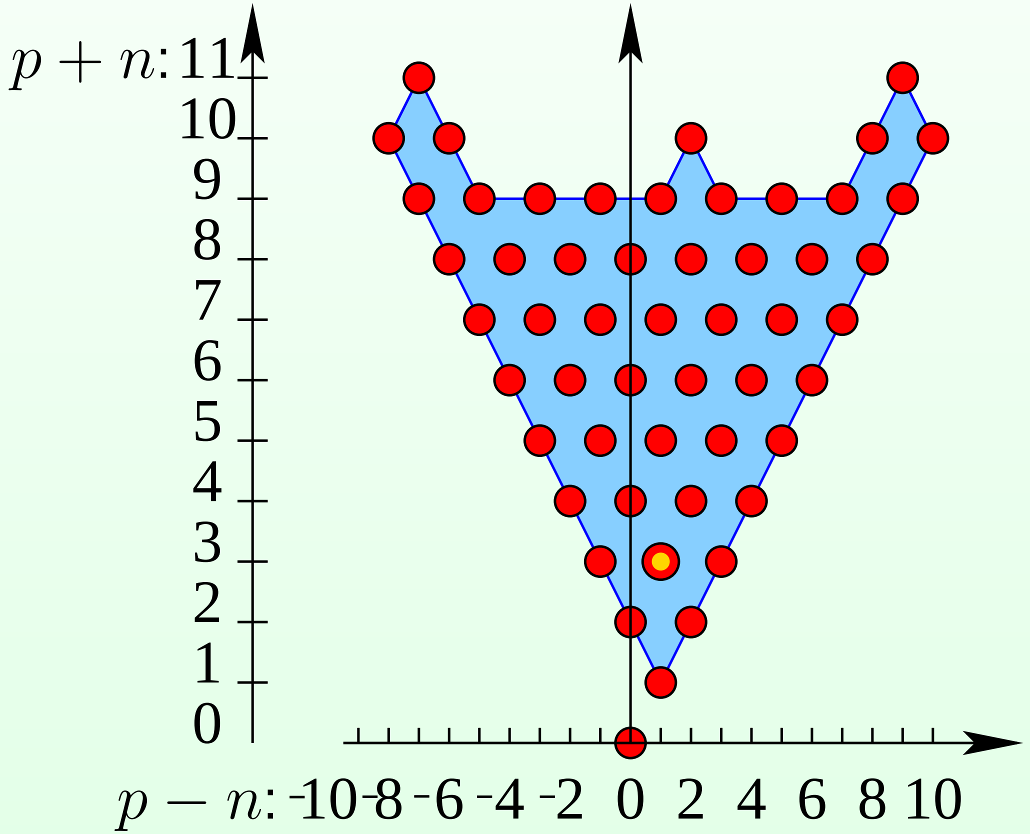

The referee did not like it.

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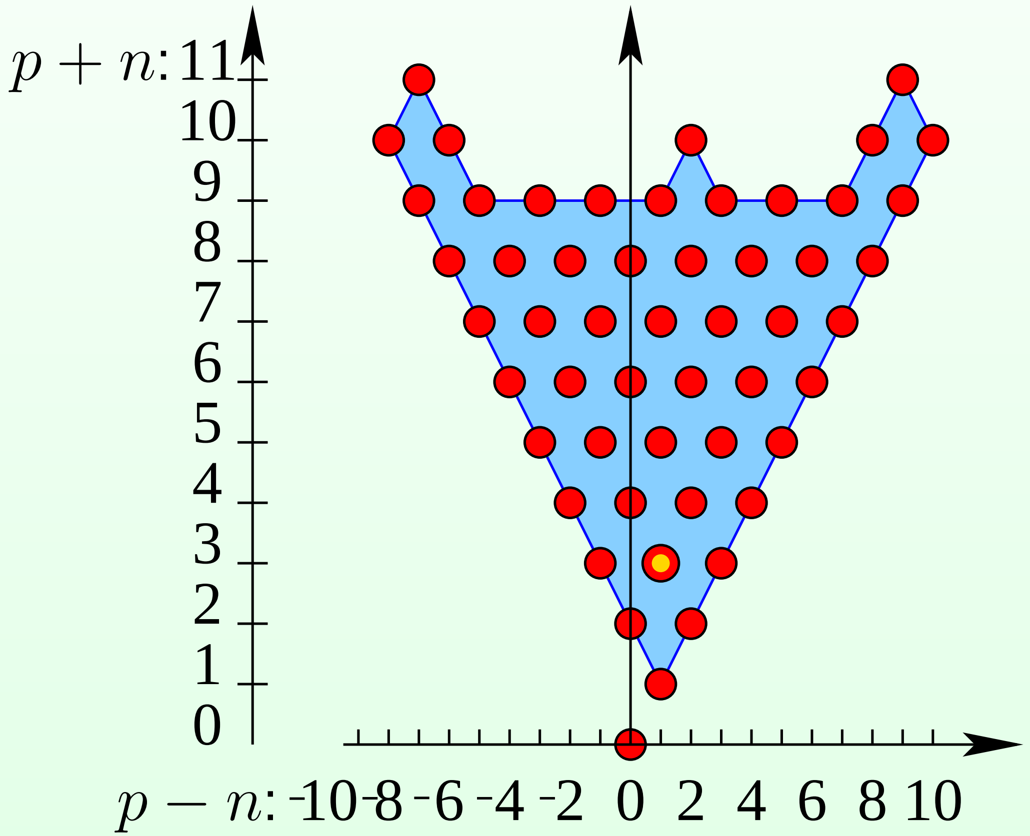

He suggested to make it more symmetric.

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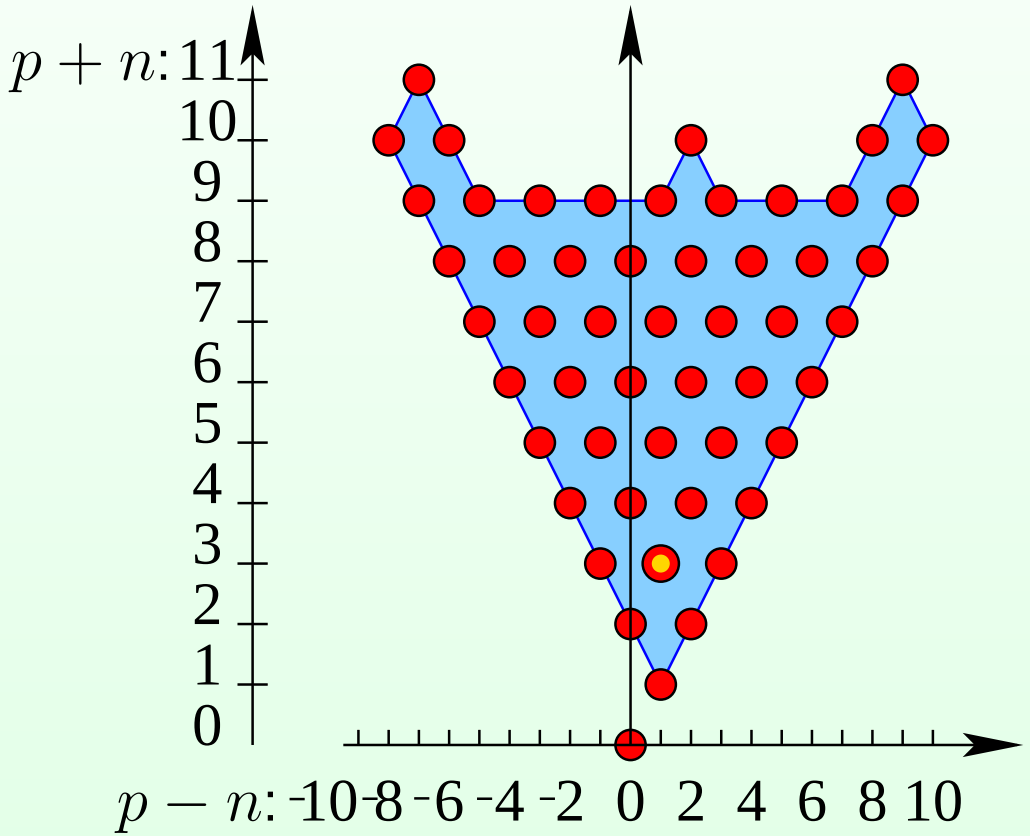

Gudkov found a mistake

Isotopy classification of nonsingular sextics

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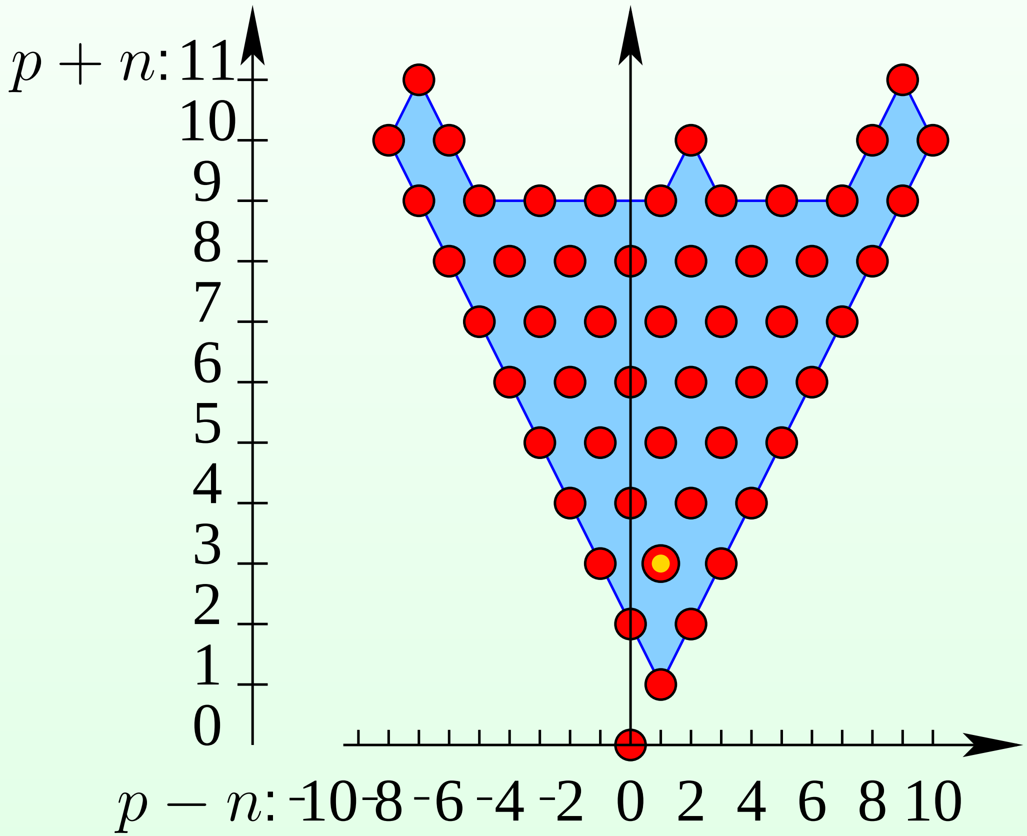

Gudkov found a mistake and the final answer.

Isotopy classification of nonsingular sextics

Read the Sixteenth Hilbert Problem

Breakthrough
- Isotopy classification of nonsingular sextics
- Gudkov's M-curve
- Gudkov's conjecture
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- Complexification
- In homology
- Proof of Arnold's congruence
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- The first part success

Post Solution

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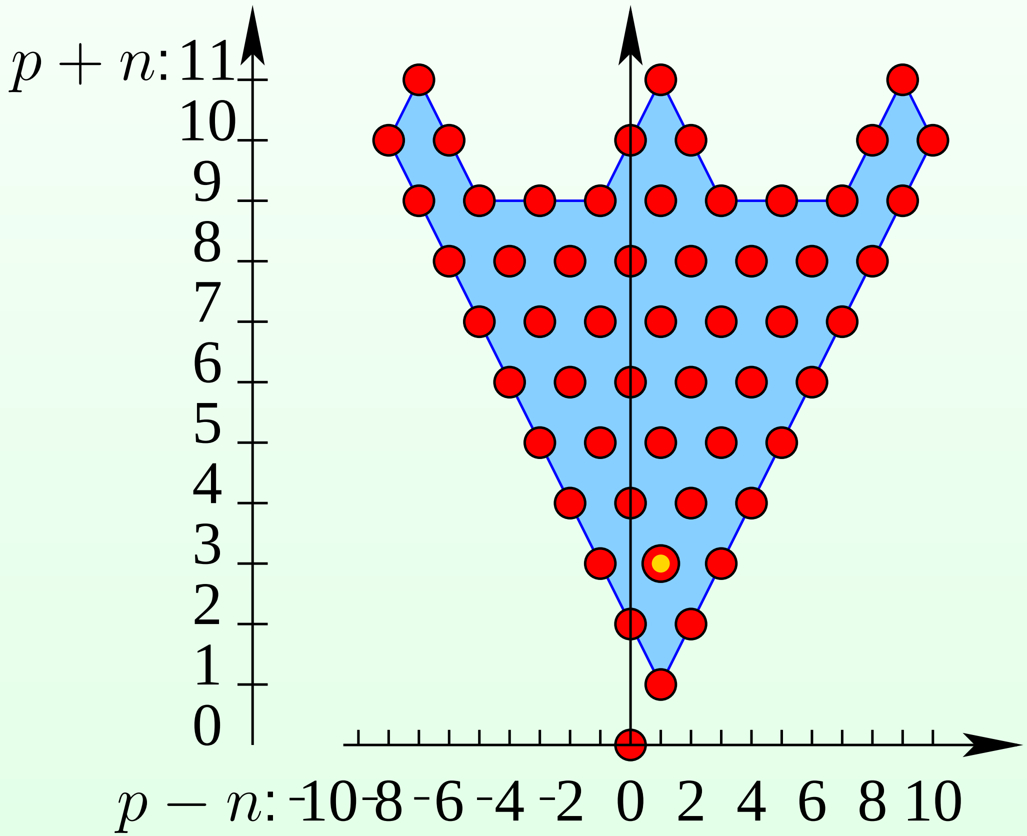

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Breakthrough
- Isotopy classification of nonsingular sextics
- Gudkov's M-curve
- Gudkov's conjecture
- Arnold's congruence
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- In homology
- Proof of Arnold's congruence
- Gudkov-Rokhlin congruence
- The role of complexification
- Mystery of the 16th Hilbert problem
- Second part
- Second part
- The first part success

Post Solution

The missing curve

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Read the Sixteenth Hilbert Problem

Breakthrough
- Isotopy classification of nonsingular sextics
- Gudkov's M-curve
- Gudkov's conjecture
- Arnold's congruence
- Complexification
- In homology
- Proof of Arnold's congruence
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- Mystery of the 16th Hilbert problem
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- Second part
- The first part success

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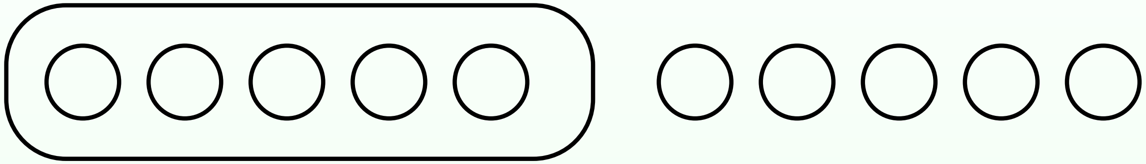
The missing curve



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- Isotopy classification of nonsingular sextics
- Gudkov's M-curve
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- In homology
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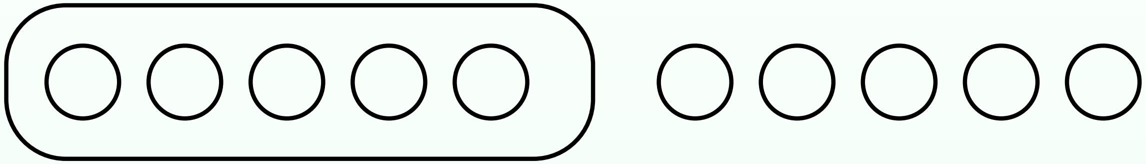

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Breakthrough
- Isotopy classification of nonsingular sextics
- Gudkov's M-curve
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- In homology
- Proof of Arnold's congruence
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- Mystery of the 16th Hilbert problem
- Second part
- Second part
- The first part success

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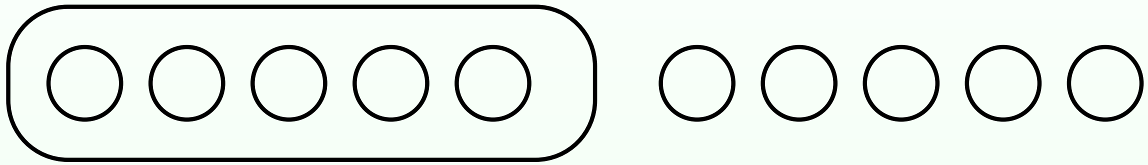

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As to curves of the 6-th order, I have satisfied myself–by a complicated process, it is true–that of the eleven branches which they can have according to Harnack, by no means all can lie external to one another, but that one branch must exist in whose interior one branch and in whose exterior nine branches lie, or inversely.

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Read the Sixteenth Hilbert Problem

Breakthrough
- Isotopy classification of nonsingular sextics
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- Gudkov’s conjecture
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- Proof of Arnold’s congruence
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- Mystery of the 16th Hilbert problem
- Second part
- Second part
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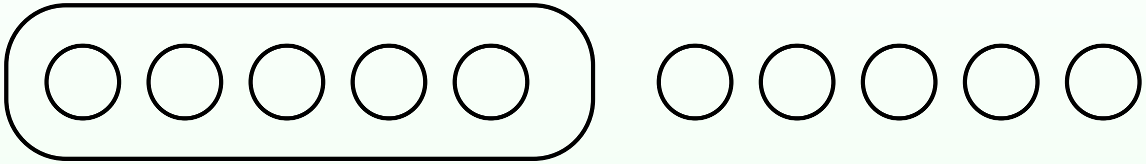

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- Isotopy classification of nonsingular sextics
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- Proof of Arnold's congruence
- Gudkov-Rokhlin congruence
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- Mystery of the 16th Hilbert problem
- Second part
- Second part
- The first part success

Post Solution

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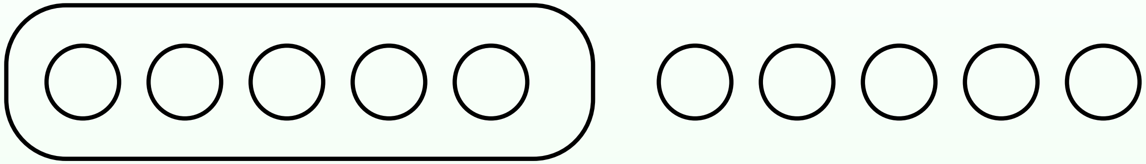

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- Isotopy classification of nonsingular sextics
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- Gudkov-Rokhlin congruence
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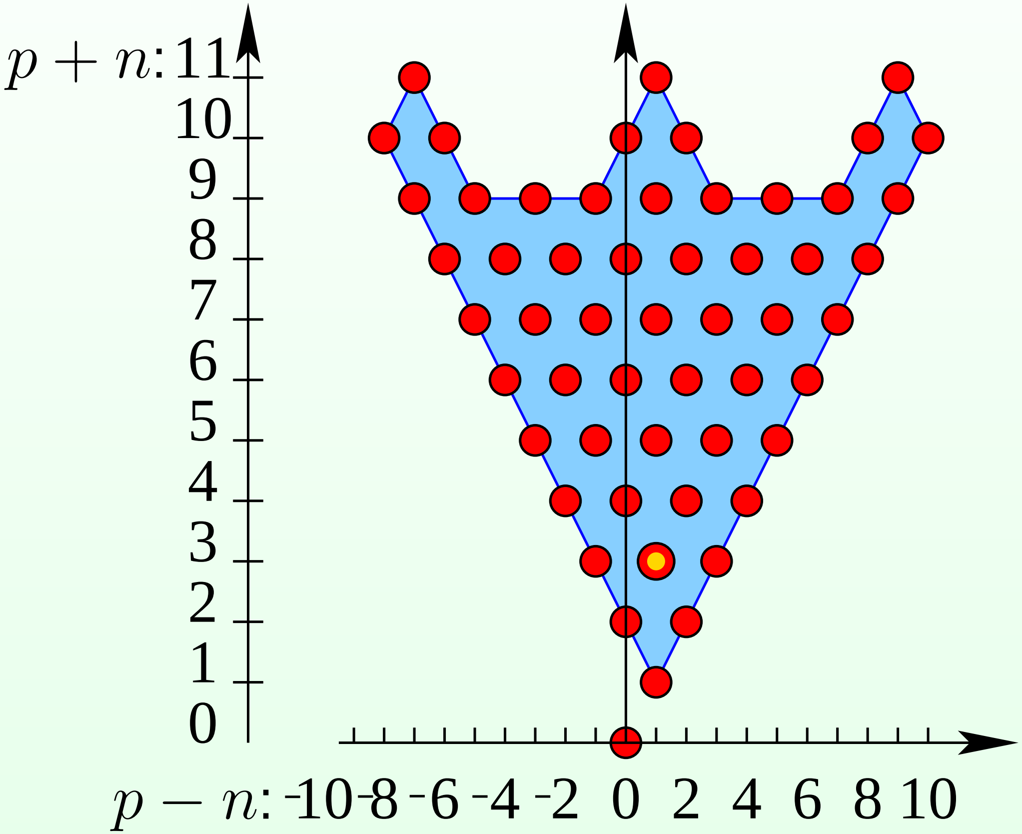
- Mystery of the 16th Hilbert problem
- Second part
- Second part
- The first part success

Post Solution

Gudkov's conjecture

Symmetric top of the table



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Read the Sixteenth Hilbert Problem

Breakthrough
- Isotopy classification of nonsingular sextics
- Gudkov's M-curve
- Gudkov's conjecture
- Arnold's congruence
- Complexification
- In homology
- Proof of Arnold's congruence
- Gudkov-Rokhlin congruence
- The role of complexification
- Mystery of the 16th Hilbert problem
- Second part
- Second part
- The first part success

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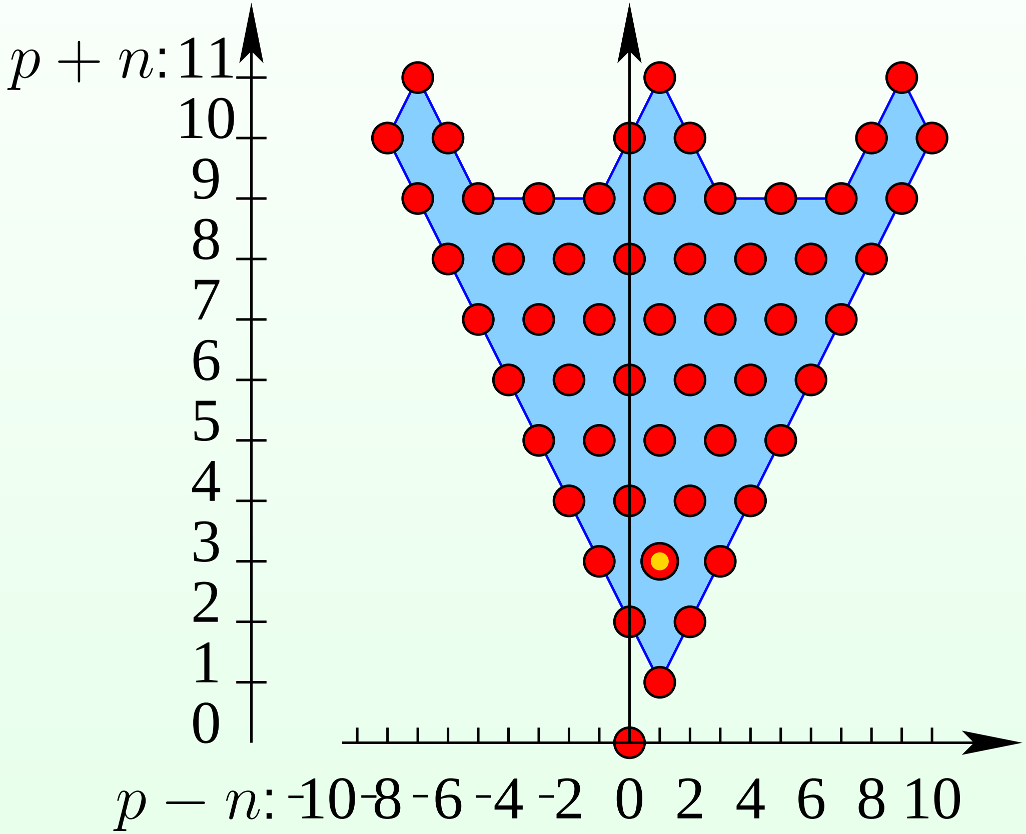

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Gudkov's Conjecture. For any curve of even degree $d = 2k$ with maximal number of ovals, $p - n \equiv k^2 \mod 8$.

Gudkov's conjecture

Read the Sixteenth Hilbert Problem

Breakthrough
- Isotopy classification of nonsingular sextics
- Gudkov's M-curve
- Gudkov's conjecture
- Arnold's congruence
- Complexification
- In homology
- Proof of Arnold's congruence
- Gudkov-Rokhlin congruence
- The role of complexification
- Mystery of the 16th Hilbert problem
- Second part
- Second part
- The first part success

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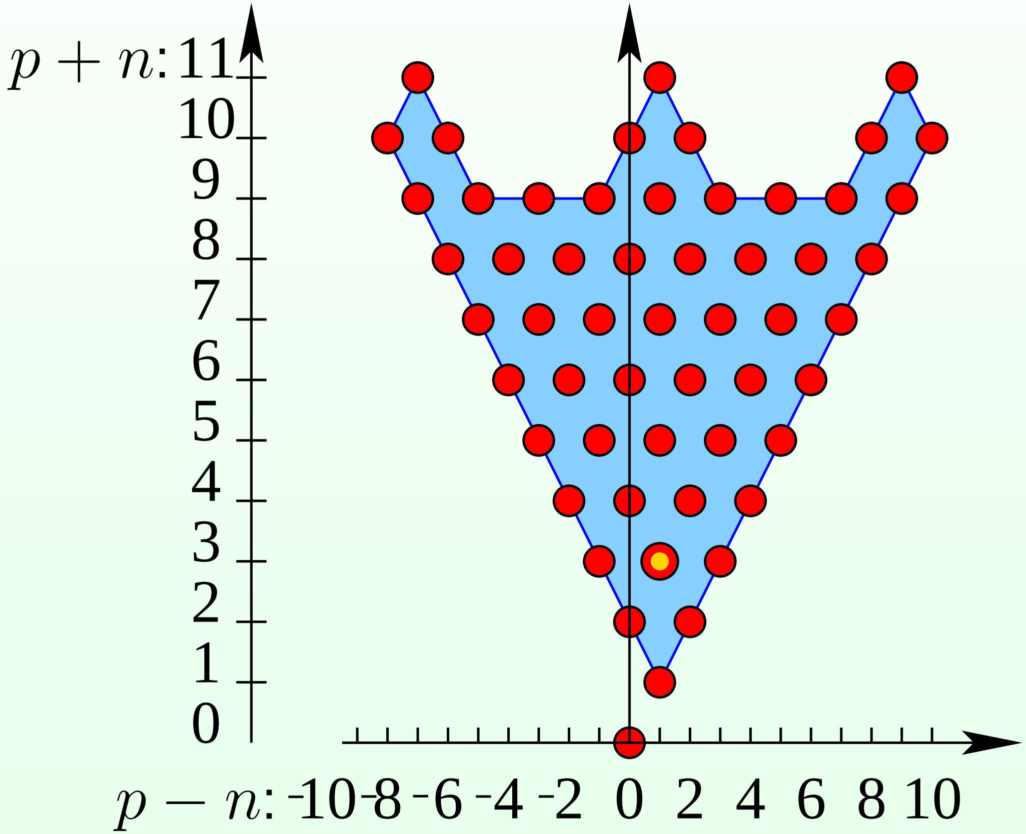

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It was this conjecture that inspired the breakthrough.

Arnold’s congruence

Read the Sixteenth Hilbert Problem

Breakthrough

- Isotopy classification of nonsingular sextics
- Gudkov’s M-curve
- Gudkov’s conjecture
- Arnold’s congruence
- Complexification
- In homology
- Proof of Arnold’s congruence
- Gudkov-Rokhlin congruence
- The role of complexification
- Mystery of the 16th Hilbert problem
- Second part
- Second part
- The first part success

Post Solution

In 1971 Arnold proved a *half* of Gudkov’s conjecture:

Arnold's congruence

Read the Sixteenth Hilbert Problem

Breakthrough

- Isotopy classification of nonsingular sextics
- Gudkov's M-curve
- Gudkov's conjecture
- Arnold's congruence
- Complexification
- In homology
- Proof of Arnold's congruence
- Gudkov-Rokhlin congruence
- The role of complexification
- Mystery of the 16th Hilbert problem
- Second part
- Second part
- The first part success

Post Solution

In 1971 Arnold proved a *half* of Gudkov's conjecture:

What is a half of congruence

$p - n \equiv k^2 \mod 8$?

Arnold's congruence

In 1971 Arnold proved a *half* of Gudkov's conjecture: the same congruence, but modulo 4

Read the Sixteenth Hilbert Problem

Breakthrough

- Isotopy classification of nonsingular sextics
- Gudkov's M-curve
- Gudkov's conjecture
- Arnold's congruence
- Complexification
- In homology
- Proof of Arnold's congruence
- Gudkov-Rokhlin congruence
- The role of complexification
- Mystery of the 16th Hilbert problem
- Second part
- Second part
- The first part success

Post Solution

Arnold's congruence

Read the Sixteenth Hilbert Problem

Breakthrough
- Isotopy classification of nonsingular sextics
- Gudkov's M-curve
- Gudkov's conjecture
- Arnold's congruence
- Complexification
- In homology
- Proof of Arnold's congruence
- Gudkov-Rokhlin congruence
- The role of complexification
- Mystery of the 16th Hilbert problem
- Second part
- Second part
- The first part success

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Read the Sixteenth Hilbert Problem

Breakthrough
- Isotopy classification of nonsingular sextics
- Gudkov's M-curve
- Gudkov's conjecture
- Arnold's congruence
- Complexification
- In homology
- Proof of Arnold's congruence
- Gudkov-Rokhlin congruence
- The role of complexification
- Mystery of the 16th Hilbert problem
- Second part
- Second part
- The first part success

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Arnold's proof works for a larger class of curves:

Read the Sixteenth Hilbert Problem

Breakthrough
- Isotopy classification of nonsingular sextics
- Gudkov's M-curve
- Gudkov's conjecture
- Arnold's congruence
- Complexification
- In homology
- Proof of Arnold's congruence
- Gudkov-Rokhlin congruence
- The role of complexification
- Mystery of the 16th Hilbert problem
- Second part
- Second part
- The first part success

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Arnold's congruence

Read the Sixteenth Hilbert Problem

Breakthrough
- Isotopy classification of nonsingular sextics
- Gudkov's M-curve
- Gudkov's conjecture
- Arnold's congruence
- Complexification
- In homology
- Proof of Arnold's congruence
- Gudkov-Rokhlin congruence
- The role of complexification
- Mystery of the 16th Hilbert problem
- Second part
- Second part
- The first part success

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Arnold's congruence

Read the Sixteenth Hilbert Problem

Breakthrough

- Isotopy classification of nonsingular sextics
- Gudkov's M-curve
- Gudkov's conjecture
- Arnold's congruence
- Complexification
- In homology
- Proof of Arnold's congruence
- Gudkov-Rokhlin congruence
- The role of complexification
- Mystery of the 16th Hilbert problem
- Second part
- Second part
- The first part success

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Arnold's proof relies on the topology of the configuration formed in the complex projective plane $\mathbb{C}P^2$ by the complexification $\mathbb{C}A$ of the curve and the real projective plane $\mathbb{R}P^2$.

Complexification

Curve A of degree $d = 2k$,

Read the Sixteenth Hilbert Problem

Breakthrough
- Isotopy classification of nonsingular sextics
- Gudkov's M-curve
- Gudkov's conjecture
- Arnold's congruence
- Complexification
- In homology
- Proof of Arnold's congruence
- Gudkov-Rokhlin congruence
- The role of complexification
- Mystery of the 16th Hilbert problem
- Second part
- Second part
- The first part success

Post Solution

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Read the Sixteenth Hilbert Problem

Breakthrough
- Isotopy classification of nonsingular sextics
- Gudkov's M-curve
- Gudkov's conjecture
- Arnold's congruence
- Complexification
- In homology
- Proof of Arnold's congruence
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- The role of complexification
- Mystery of the 16th Hilbert problem
- Second part
- Second part
- The first part success

Post Solution

Curve A of degree $d = 2k$, is defined by equation $F(x_0, x_1, x_2) = 0$ on projective plane,

Complexification

Read the Sixteenth Hilbert Problem

Breakthrough

- Isotopy classification of nonsingular sextics
- Gudkov's M-curve
- Gudkov's conjecture
- Arnold's congruence
- Complexification
- In homology
- Proof of Arnold's congruence
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- Mystery of the 16th Hilbert problem
- Second part
- Second part
- The first part success

Post Solution

Curve A of degree $d = 2k$, is defined by equation $F(x_0, x_1, x_2) = 0$ on projective plane, where F is a real homogeneous polynomial of degree d.

Complexification

Read the Sixteenth Hilbert Problem

Breakthrough
- Isotopy classification of nonsingular sextics
- Gudkov's M-curve
- Gudkov's conjecture
- Arnold's congruence
- Complexification
- In homology
- Proof of Arnold's congruence
- Gudkov-Rokhlin congruence
- The role of complexification
- Mystery of the 16th Hilbert problem
- Second part
- Second part
- The first part success

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If F is generic, then $F(x_0, x_1, x_2) = 0$ defines $\mathbb{R}A \subset \mathbb{R}P^2$, a collection of smooth ovals in $\mathbb{R}P^2$

Complexification

Read the Sixteenth Hilbert Problem

Breakthrough
- Isotopy classification of nonsingular sextics
- Gudkov's M-curve
- Gudkov's conjecture
- Arnold's congruence
- Complexification
- In homology
- Proof of Arnold's congruence
- Gudkov-Rokhlin congruence
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- Mystery of the 16th Hilbert problem
- Second part
- Second part
- The first part success

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Complexification

Read the Sixteenth Hilbert Problem

Breakthrough
- Isotopy classification of nonsingular sextics
- Gudkov's M-curve
- Gudkov's conjecture
- Arnold's congruence
- Complexification
- In homology
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- Second part
- Second part
- The first part success

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Complexification

Read the Sixteenth Hilbert Problem

Breakthrough
- Isotopy classification of nonsingular sextics
- Gudkov's M-curve
- Gudkov's conjecture
- Arnold's congruence
- Complexification
- In homology
- Proof of Arnold's congruence
- Gudkov-Rokhlin congruence
- The role of complexification
- Mystery of the 16th Hilbert problem
- Second part
- Second part
- The first part success

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Complexification

Read the Sixteenth Hilbert Problem

Breakthrough

- Isotopy classification of nonsingular sextics
- Gudkov's M-curve
- Gudkov's conjecture
- Arnold's congruence
- Complexification
- In homology
- Proof of Arnold's congruence
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- Mystery of the 16th Hilbert problem
- Second part
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- The first part success

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Complexification

Read the Sixteenth Hilbert Problem

Breakthrough
- Isotopy classification of nonsingular sextics
- Gudkov's M-curve
- Gudkov's conjecture
- Arnold's congruence
- Complexification
- In homology
- Proof of Arnold's congruence
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- Mystery of the 16th Hilbert problem
- Second part
- Second part
- The first part success

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They are well-defined, as $F(\lambda x) = \lambda^{2k} F(x)$.

Complexification

Read the Sixteenth Hilbert Problem

Breakthrough
- Isotopy classification of nonsingular sextics
- Gudkov's M-curve
- Gudkov's conjecture
- Arnold's congruence
- Complexification
- In homology
- Proof of Arnold's congruence
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- Mystery of the 16th Hilbert problem
- Second part
- Second part
- The first part success

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Complexification

Read the Sixteenth Hilbert Problem

Breakthrough
- Isotopy classification of nonsingular sextics
- Gudkov's M-curve
- Gudkov's conjecture
- Arnold's congruence
- Complexification
- In homology
- Proof of Arnold's congruence
- Gudkov-Rokhlin congruence
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- Mystery of the 16th Hilbert problem
- Second part
- Second part
- The first part success

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Complexification

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Breakthrough
- Isotopy classification of nonsingular sextics
- Gudkov's M-curve
- Gudkov's conjecture
- Arnold's congruence
- Complexification
- In homology
- Proof of Arnold's congruence
- Gudkov-Rokhlin congruence
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- Mystery of the 16th Hilbert problem
- Second part
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- The first part success

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p is the number of even ovals

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Breakthrough

- Isotopy classification of nonsingular sextics
- Gudkov's M-curve
- Gudkov's conjecture
- Arnold's congruence
- Complexification
- In homology
- Proof of Arnold's congruence
- Gudkov-Rokhlin congruence
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- Mystery of the 16th Hilbert problem
- Second part
- Second part
- The first part success

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p is the number of even ovals, the number of components of $\mathbb{R}P^2_+$.

Complexification

Read the Sixteenth Hilbert Problem

Breakthrough
- Isotopy classification of nonsingular sextics
- Gudkov's M-curve
- Gudkov's conjecture
- Arnold's congruence
- Complexification
- In homology
- Proof of Arnold's congruence
- Gudkov-Rokhlin congruence
- The role of complexification
- Mystery of the 16th Hilbert problem
- Second part
- Second part
- The first part success

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Read the Sixteenth Hilbert Problem

Breakthrough
- Isotopy classification of nonsingular sextics
- Gudkov's M-curve
- Gudkov's conjecture
- Arnold's congruence
- Complexification
- In homology
- Proof of Arnold's congruence
- Gudkov-Rokhlin congruence
- The role of complexification
- Mystery of the 16th Hilbert problem
- Second part
- Second part
- The first part success

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How to complexify $\mathbb{R}P^2_+$?

Complexification

Read the Sixteenth Hilbert Problem
Breakthrough
- Isotopy classification of nonsingular sextics
- Gudkov's M-curve
- Gudkov's conjecture
- Arnold's congruence
- Complexification
- In homology
- Proof of Arnold's congruence
- Gudkov-Rokhlin congruence
- The role of complexification
- Mystery of the 16th Hilbert problem
- Second part
- Second part
- The first part success

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How to complexify the notion of manifold with boundary?

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Read the Sixteenth Hilbert Problem

Breakthrough
- Isotopy classification of nonsingular sextics
- Gudkov's M-curve
- Gudkov's conjecture
- Arnold's congruence
- Complexification
- In homology
- Proof of Arnold's congruence
- Gudkov-Rokhlin congruence
- The role of complexification
- Mystery of the 16th Hilbert problem
- Second part
- Second part
- The first part success

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How to complexify inequality $F(x) \geq 0$?

Complexification

Read the Sixteenth Hilbert Problem

Breakthrough

- Isotopy classification of nonsingular sextics
- Gudkov's M-curve
- Gudkov's conjecture
- Arnold's congruence
- Complexification
- In homology
- Proof of Arnold's congruence
- Gudkov-Rokhlin congruence
- The role of complexification
- Mystery of the 16th Hilbert problem
- Second part
- Second part
- The first part success

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Arnold: Complexification of inequality is two-fold branched covering!

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Read the Sixteenth Hilbert Problem

Breakthrough

- Isotopy classification of nonsingular sextics
- Gudkov's M-curve
- Gudkov's conjecture
- Arnold's congruence
- Complexification
- In homology
- Proof of Arnold's congruence
- Gudkov-Rokhlin congruence
- The role of complexification
- Mystery of the 16th Hilbert problem
- Second part
- Second part
- The first part success

Post Solution

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Arnold: Complexification of inequality is two-fold branched covering!

Indeed, $F(x) \geq 0 \Leftrightarrow \exists y \in \mathbb{R} : F(x) = y^2$.

Complexification

Read the Sixteenth Hilbert Problem

Breakthrough

- Isotopy classification of nonsingular sextics
- Gudkov's M-curve
- Gudkov's conjecture
- Arnold's congruence
- Complexification
- In homology
- Proof of Arnold's congruence
- Gudkov-Rokhlin congruence
- The role of complexification
- Mystery of the 16th Hilbert problem
- Second part
- Second part
- The first part success

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Read the Sixteenth Hilbert Problem

Breakthrough
- Isotopy classification of nonsingular sextics
- Gudkov's M-curve
- Gudkov's conjecture
- Arnold's congruence
- Complexification
- In homology
- Proof of Arnold's congruence
- Gudkov-Rokhlin congruence
- The role of complexification
- Mystery of the 16th Hilbert problem
- Second part
- Second part
- The first part success

Post Solution

Complexification

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Projection $\mathbb{C}Y \to \mathbb{C}P^2 : [x_0, x_1, x_2, y] \mapsto [x_0 : x_1 : x_2]$ is a two-fold covering branched over $\mathbb{C}A$.

Complexification

Read the Sixteenth Hilbert Problem

Breakthrough
- Isotopy classification of nonsingular sextics
- Gudkov's M-curve
- Gudkov's conjecture
- Arnold's congruence
- Complexification
- In homology
- Proof of Arnold's congruence
- Gudkov-Rokhlin congruence
- The role of complexification
- Mystery of the 16th Hilbert problem
- Second part
- Second part
- The first part success

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It maps $\mathbb{R}Y$ onto $\mathbb{R}P^2_+$.

Read the Sixteenth Hilbert Problem
Breakthrough
- Isotopy classification of nonsingular sextics
- Gudkov's M-curve
- Gudkov's conjecture
- Arnold's congruence
- Complexification
- In homology
- Proof of Arnold's congruence
- Gudkov-Rokhlin congruence
- The role of complexification
- Mystery of the 16th Hilbert problem
- Second part
- Second part
- The first part success

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It maps $\mathbb{R}Y$ onto $\mathbb{R}P^2_+$. Automorphism $\tau : \mathbb{C}Y \to \mathbb{C}Y$, involution with $\operatorname{fix}(\tau) = \mathbb{C}A$.

In homology

$$\pi_1(\mathbb{C}Y) = 0\,.$$

Read the Sixteenth Hilbert Problem

Breakthrough
- Isotopy classification of nonsingular sextics
- Gudkov's M-curve
- Gudkov's conjecture
- Arnold's congruence
- Complexification
- In homology
- Proof of Arnold's congruence
- Gudkov-Rokhlin congruence
- The role of complexification
- Mystery of the 16th Hilbert problem
- Second part
- Second part
- The first part success

Post Solution

In homology

Read the Sixteenth Hilbert Problem

Breakthrough
- Isotopy classification of nonsingular sextics
- Gudkov's M-curve
- Gudkov's conjecture
- Arnold's congruence
- Complexification
- In homology
- Proof of Arnold's congruence
- Gudkov-Rokhlin congruence
- The role of complexification
- Mystery of the 16th Hilbert problem
- Second part
- Second part
- The first part success

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$\pi_1(\mathbb{C}Y) = 0$. This simplifies algebra, makes it commutative.

Read the Sixteenth Hilbert Problem

Breakthrough
- Isotopy classification of nonsingular sextics
- Gudkov's M-curve
- Gudkov's conjecture
- Arnold's congruence
- Complexification
- In homology
- Proof of Arnold's congruence
- Gudkov-Rokhlin congruence
- The role of complexification
- Mystery of the 16th Hilbert problem
- Second part
- Second part
- The first part success

Post Solution

In homology

$\pi_1(\mathbb{C}Y) = 0$. This simplifies algebra, makes it commutative.

$$H_0(\mathbb{C}Y) = H_4(\mathbb{C}Y) = \mathbb{Z}, \qquad H_1(\mathbb{C}Y) = H_3(\mathbb{C}Y) = 0.$$

In homology

Read the Sixteenth Hilbert Problem

Breakthrough
- Isotopy classification of nonsingular sextics
- Gudkov's M-curve
- Gudkov's conjecture
- Arnold's congruence
- Complexification
- In homology
- Proof of Arnold's congruence
- Gudkov-Rokhlin congruence
- The role of complexification
- Mystery of the 16th Hilbert problem
- Second part
- Second part
- The first part success

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Read the Sixteenth Hilbert Problem

Breakthrough

- Isotopy classification of nonsingular sextics
- Gudkov's M-curve
- Gudkov's conjecture
- Arnold's congruence
- Complexification
- In homology
- Proof of Arnold's congruence
- Gudkov-Rokhlin congruence
- The role of complexification
- Mystery of the 16th Hilbert problem
- Second part
- Second part
- The first part success

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Read the Sixteenth Hilbert Problem

Breakthrough
- Isotopy classification of nonsingular sextics
- Gudkov's M-curve
- Gudkov's conjecture
- Arnold's congruence
- Complexification
- In homology
- Proof of Arnold's congruence
- Gudkov-Rokhlin congruence
- The role of complexification
- Mystery of the 16th Hilbert problem
- Second part
- Second part
- The first part success

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decorations: Intersection form

$H_2(\mathbb{C}Y) \times H_2(\mathbb{C}Y) \to \mathbb{Z} : (\alpha, \beta) \mapsto \alpha \circ \beta$

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Read the Sixteenth Hilbert Problem

Breakthrough
- Isotopy classification of nonsingular sextics
- Gudkov's M-curve
- Gudkov's conjecture
- Arnold's congruence
- Complexification
- In homology
- Proof of Arnold's congruence
- Gudkov-Rokhlin congruence
- The role of complexification
- Mystery of the 16th Hilbert problem
- Second part
- Second part
- The first part success

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symmetric bilinear unimodular form.

In homology

Read the Sixteenth Hilbert Problem

Breakthrough

- Isotopy classification of nonsingular sextics
- Gudkov's M-curve
- Gudkov's conjecture
- Arnold's congruence
- Complexification
- In homology
- Proof of Arnold's congruence
- Gudkov-Rokhlin congruence
- The role of complexification
- Mystery of the 16th Hilbert problem
- Second part
- Second part
- The first part success

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Read the Sixteenth Hilbert Problem

Breakthrough
- Isotopy classification of nonsingular sextics
- Gudkov's M-curve
- Gudkov's conjecture
- Arnold's congruence
- Complexification
- In homology
- Proof of Arnold's congruence
- Gudkov-Rokhlin congruence
- The role of complexification
- Mystery of the 16th Hilbert problem
- Second part
- Second part
- The first part success

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Read the Sixteenth Hilbert Problem

Breakthrough
- Isotopy classification of nonsingular sextics
- Gudkov's M-curve
- Gudkov's conjecture
- Arnold's congruence
- Complexification
- In homology
- Proof of Arnold's congruence
- Gudkov-Rokhlin congruence
- The role of complexification
- Mystery of the 16th Hilbert problem
- Second part
- Second part
- The first part success

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Read the Sixteenth Hilbert Problem

Breakthrough

- Isotopy classification of nonsingular sextics
- Gudkov's M-curve
- Gudkov's conjecture
- Arnold's congruence
- Complexification
- In homology
- Proof of Arnold's congruence
- Gudkov-Rokhlin congruence
- The role of complexification
- Mystery of the 16th Hilbert problem
- Second part
- Second part
- The first part success

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Homology class $[\mathbb{C}A] \in H_2(\mathbb{C}Y)$.

In homology

Read the Sixteenth Hilbert Problem

Breakthrough
- Isotopy classification of nonsingular sextics
- Gudkov's M-curve
- Gudkov's conjecture
- Arnold's congruence
- Complexification
- In homology
- Proof of Arnold's congruence
- Gudkov-Rokhlin congruence
- The role of complexification
- Mystery of the 16th Hilbert problem
- Second part
- Second part
- The first part success

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$\pi_1(\mathbb{C}Y) = 0$. This simplifies algebra, makes it commutative.

$H_0(\mathbb{C}Y) = H_4(\mathbb{C}Y) = \mathbb{Z}, \qquad H_1(\mathbb{C}Y) = H_3(\mathbb{C}Y) = 0$.

$H_2(\mathbb{C}Y) = \mathbb{Z}^{4k^2-6k+4}$, our scene of algebraic action;

decorations: Intersection form

$H_2(\mathbb{C}Y) \times H_2(\mathbb{C}Y) \to \mathbb{Z} : (\alpha, \beta) \mapsto \alpha \circ \beta$.

Involution $\tau_* : H_2(\mathbb{C}Y) \to H_2(\mathbb{C}Y)$.

Form of involution τ

$H_2(\mathbb{C}Y) \times H_2(\mathbb{C}Y) \to \mathbb{Z} : (\alpha, \beta) \mapsto \alpha \circ_\tau \beta = \alpha \circ \tau_*(\beta)$

Homology classes $\qquad [\mathbb{R}Y], [\mathbb{C}A] \in H_2(\mathbb{C}Y)$.

We orient $\mathbb{R}Y$.

In homology

Read the Sixteenth Hilbert Problem

Breakthrough
- Isotopy classification of nonsingular sextics
- Gudkov's M-curve
- Gudkov's conjecture
- Arnold's congruence
- Complexification
- In homology
- Proof of Arnold's congruence
- Gudkov-Rokhlin congruence
- The role of complexification
- Mystery of the 16th Hilbert problem
- Second part
- Second part
- The first part success

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$[\infty]$ is the preimage of a generic projective line under $\mathbb{C}Y \to \mathbb{C}P^2$.

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Read the Sixteenth Hilbert Problem

Breakthrough

- Isotopy classification of nonsingular sextics
- Gudkov's M-curve
- Gudkov's conjecture
- Arnold's congruence
- Complexification
- In homology
- Proof of Arnold's congruence
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- The role of complexification
- Mystery of the 16th Hilbert problem
- Second part
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Read the Sixteenth Hilbert Problem

Breakthrough
- Isotopy classification of nonsingular sextics
- Gudkov's M-curve
- Gudkov's conjecture
- Arnold's congruence
- Complexification
- In homology
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- Gudkov-Rokhlin congruence
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- Mystery of the 16th Hilbert problem
- Second part
- Second part
- The first part success

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Because $X \cap \tau(X)$

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Read the Sixteenth Hilbert Problem

Breakthrough
- Isotopy classification of nonsingular sextics
- Gudkov's M-curve
- Gudkov's conjecture
- Arnold's congruence
- Complexification
- In homology
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- Mystery of the 16th Hilbert problem
- Second part
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Because $X \cap \tau(X)$

$= (X \cap \mathbb{C}A) \cup ($even number of points$)$.

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Read the Sixteenth Hilbert Problem

Breakthrough
- Isotopy classification of nonsingular sextics
- Gudkov's M-curve
- Gudkov's conjecture
- Arnold's congruence
- Complexification
- In homology
- Proof of Arnold's congruence
- Gudkov-Rokhlin congruence
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- Second part
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Read the Sixteenth Hilbert Problem

Breakthrough

- Isotopy classification of nonsingular sextics
- Gudkov's M-curve
- Gudkov's conjecture
- Arnold's congruence
- Complexification
- In homology
- Proof of Arnold's congruence
- Gudkov-Rokhlin congruence
- The role of complexification
- Mystery of the 16th Hilbert problem
- Second part
- Second part
- The first part success

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Read the Sixteenth Hilbert Problem

Breakthrough
- Isotopy classification of nonsingular sextics
- Gudkov's M-curve
- Gudkov's conjecture
- Arnold's congruence
- Complexification
- In homology
- Proof of Arnold's congruence
- Gudkov-Rokhlin congruence
- The role of complexification
- Mystery of the 16th Hilbert problem
- Second part
- Second part
- The first part success

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Read the Sixteenth Hilbert Problem

Breakthrough
- Isotopy classification of nonsingular sextics
- Gudkov's M-curve
- Gudkov's conjecture
- Arnold's congruence
- Complexification
- In homology
- Proof of Arnold's congruence
- Gudkov-Rokhlin congruence
- The role of complexification
- Mystery of the 16th Hilbert problem
- Second part
- Second part
- The first part success

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Read the Sixteenth Hilbert Problem

Breakthrough
- Isotopy classification of nonsingular sextics
- Gudkov's M-curve
- Gudkov's conjecture
- Arnold's congruence
- Complexification
- In homology
- Proof of Arnold's congruence
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- The role of complexification
- Mystery of the 16th Hilbert problem
- Second part
- Second part
- The first part success

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Read the Sixteenth Hilbert Problem

Breakthrough

- Isotopy classification of nonsingular sextics
- Gudkov's M-curve
- Gudkov's conjecture
- Arnold's congruence
- Complexification
- In homology
- Proof of Arnold's congruence
- Gudkov-Rokhlin congruence
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- Mystery of the 16th Hilbert problem
- Second part
- Second part
- The first part success

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Any unimodular symmetric bilinear form has a characteristic class.

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Read the Sixteenth Hilbert Problem

Breakthrough
- Isotopy classification of nonsingular sextics
- Gudkov's M-curve
- Gudkov's conjecture
- Arnold's congruence
- Complexification
- In homology
- Proof of Arnold's congruence
- Gudkov-Rokhlin congruence
- The role of complexification
- Mystery of the 16th Hilbert problem
- Second part
- Second part
- The first part success

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Any unimodular symmetric bilinear form has a characteristic class. Any two characteristic classes are congruent modulo 2.

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Read the Sixteenth Hilbert Problem

Breakthrough

- Isotopy classification of nonsingular sextics
- Gudkov's M-curve
- Gudkov's conjecture
- Arnold's congruence
- Complexification
- In homology
- Proof of Arnold's congruence
- Gudkov-Rokhlin congruence
- The role of complexification
- Mystery of the 16th Hilbert problem
- Second part
- Second part
- The first part success

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Lemma. For any two characteristic classes w, w' of a form Φ

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Read the Sixteenth Hilbert Problem

Breakthrough
- Isotopy classification of nonsingular sextics
- Gudkov's M-curve
- Gudkov's conjecture
- Arnold's congruence
- Complexification
- In homology
- Proof of Arnold's congruence
- Gudkov-Rokhlin congruence
- The role of complexification
- Mystery of the 16th Hilbert problem
- Second part
- Second part
- The first part success

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Read the Sixteenth Hilbert Problem

Breakthrough
- Isotopy classification of nonsingular sextics
- Gudkov's M-curve
- Gudkov's conjecture
- Arnold's congruence
- Complexification
- In homology
- Proof of Arnold's congruence
- Gudkov-Rokhlin congruence
- The role of complexification
- Mystery of the 16th Hilbert problem
- Second part
- Second part
- The first part success

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Hence $\Phi(w',w') = \Phi(w,w) + 4\Phi(x,w) + 4\Phi(x,x)$

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Read the Sixteenth Hilbert Problem

Breakthrough
- Isotopy classification of nonsingular sextics
- Gudkov’s M-curve
- Gudkov’s conjecture
- Arnold’s congruence
- Complexification
- In homology
- Proof of Arnold’s congruence
- Gudkov-Rokhlin congruence
- The role of complexification
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- Second part
- Second part
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Read the Sixteenth Hilbert Problem

Breakthrough
- Isotopy classification of nonsingular sextics
- Gudkov's M-curve
- Gudkov's conjecture
- Arnold's congruence
- Complexification
- In homology
- Proof of Arnold's congruence
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- The role of complexification
- Mystery of the 16th Hilbert problem
- Second part
- Second part
- The first part success

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Therefore $\Phi(w',w') \equiv \Phi(w,w) + 8\Phi(x,x) \mod 8$. $\square$

Read the Sixteenth Hilbert Problem

Breakthrough
- Isotopy classification of nonsingular sextics
- Gudkov's M-curve
- Gudkov's conjecture
- Arnold's congruence
- Complexification
- In homology
- Proof of Arnold's congruence
- Gudkov-Rokhlin congruence
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- Mystery of the 16th Hilbert problem
- Second part
- Second part
- The first part success

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Back to $\mathbb{C}Y$ **:**

Proof of Arnold's congruence

Read the Sixteenth Hilbert Problem

Breakthrough
- Isotopy classification of nonsingular sextics
- Gudkov's M-curve
- Gudkov's conjecture
- Arnold's congruence
- Complexification
- In homology
- Proof of Arnold's congruence
- Gudkov-Rokhlin congruence
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- Mystery of the 16th Hilbert problem
- Second part
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- The first part success

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Proof of Arnold's congruence

Read the Sixteenth Hilbert Problem

Breakthrough

- Isotopy classification of nonsingular sextics
- Gudkov's M-curve
- Gudkov's conjecture
- Arnold's congruence
- Complexification
- In homology
- Proof of Arnold's congruence
- Gudkov-Rokhlin congruence
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- Mystery of the 16th Hilbert problem
- Second part
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- The first part success

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Proof of Arnold's congruence

Read the Sixteenth Hilbert Problem

Breakthrough
- Isotopy classification of nonsingular sextics
- Gudkov's M-curve
- Gudkov's conjecture
- Arnold's congruence
- Complexification
- In homology
- Proof of Arnold's congruence
- Gudkov-Rokhlin congruence
- The role of complexification
- Mystery of the 16th Hilbert problem
- Second part
- Second part
- The first part success

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Read the Sixteenth Hilbert Problem

Breakthrough
- Isotopy classification of nonsingular sextics
- Gudkov’s M-curve
- Gudkov’s conjecture
- Arnold’s congruence
- Complexification
- In homology
- Proof of Arnold’s congruence
- Gudkov-Rokhlin congruence
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- Mystery of the 16th Hilbert problem
- Second part
- Second part
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Breakthrough
- Isotopy classification of nonsingular sextics
- Gudkov's M-curve
- Gudkov's conjecture
- Arnold's congruence
- Complexification
- In homology
- Proof of Arnold's congruence
- Gudkov-Rokhlin congruence
- The role of complexification
- Mystery of the 16th Hilbert problem
- Second part
- Second part
- The first part success

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Read the Sixteenth Hilbert Problem

Breakthrough
- Isotopy classification of nonsingular sextics
- Gudkov's M-curve
- Gudkov's conjecture
- Arnold's congruence
- Complexification
- In homology
- Proof of Arnold's congruence
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- Second part
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Read the Sixteenth Hilbert Problem

Breakthrough
- Isotopy classification of nonsingular sextics
- Gudkov's M-curve
- Gudkov's conjecture
- Arnold's congruence
- Complexification
- In homology
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- Second part
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Because multiplication by $\sqrt{-1}$ is antiisomorphism between tangent and normal fibrations of $\mathbb{R}A$ + Poincaré-Hopf.

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Read the Sixteenth Hilbert Problem

Breakthrough
- Isotopy classification of nonsingular sextics
- Gudkov’s M-curve
- Gudkov’s conjecture
- Arnold’s congruence
- Complexification
- In homology
- Proof of Arnold’s congruence
- Gudkov-Rokhlin congruence
- The role of complexification
- Mystery of the 16th Hilbert problem
- Second part
- Second part
- The first part success

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Read the Sixteenth Hilbert Problem

Breakthrough
- Isotopy classification of nonsingular sextics
- Gudkov's M-curve
- Gudkov's conjecture
- Arnold's congruence
- Complexification
- In homology
- Proof of Arnold's congruence
- Gudkov-Rokhlin congruence
- The role of complexification
- Mystery of the 16th Hilbert problem
- Second part
- Second part
- The first part success

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Read the Sixteenth Hilbert Problem

Breakthrough
- Isotopy classification of nonsingular sextics
- Gudkov's M-curve
- Gudkov's conjecture
- Arnold's congruence
- Complexification
- In homology
- Proof of Arnold's congruence
- Gudkov-Rokhlin congruence
- The role of complexification
- Mystery of the 16th Hilbert problem
- Second part
- Second part
- The first part success

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Read the Sixteenth Hilbert Problem

Breakthrough

- Isotopy classification of nonsingular sextics
- Gudkov's M-curve
- Gudkov's conjecture
- Arnold's congruence
- Complexification
- In homology
- Proof of Arnold's congruence
- Gudkov-Rokhlin congruence
- The role of complexification
- Mystery of the 16th Hilbert problem
- Second part
- Second part
- The first part success

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Read the Sixteenth Hilbert Problem

Breakthrough
- Isotopy classification of nonsingular sextics
- Gudkov's M-curve
- Gudkov's conjecture
- Arnold's congruence
- Complexification
- In homology
- Proof of Arnold's congruence
- Gudkov-Rokhlin congruence
- The role of complexification
- Mystery of the 16th Hilbert problem
- Second part
- Second part
- The first part success

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Breakthrough
- Isotopy classification of nonsingular sextics
- Gudkov's M-curve
- Gudkov's conjecture
- Arnold's congruence
- Complexification
- In homology
- Proof of Arnold's congruence
- Gudkov-Rokhlin congruence
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- Mystery of the 16th Hilbert problem
- Second part
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Read the Sixteenth Hilbert Problem

Breakthrough

- Isotopy classification of nonsingular sextics
- Gudkov's M-curve
- Gudkov's conjecture
- Arnold's congruence
- Complexification
- In homology
- Proof of Arnold's congruence
- Gudkov-Rokhlin congruence
- The role of complexification
- Mystery of the 16th Hilbert problem
- Second part
- Second part
- The first part success

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Breakthrough

- Isotopy classification of nonsingular sextics
- Gudkov's M-curve
- Gudkov's conjecture
- Arnold's congruence
- Complexification
- In homology
- Proof of Arnold's congruence
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- The role of complexification
- Mystery of the 16th Hilbert problem
- Second part
- Second part
- The first part success

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Breakthrough
- Isotopy classification of nonsingular sextics
- Gudkov's M-curve
- Gudkov's conjecture
- Arnold's congruence
- Complexification
- In homology
- Proof of Arnold's congruence
- Gudkov-Rokhlin congruence
- The role of complexification
- Mystery of the 16th Hilbert problem
- Second part
- Second part
- The first part success

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Breakthrough
- Isotopy classification of nonsingular sextics
- Gudkov's M-curve
- Gudkov's conjecture
- Arnold's congruence
- Complexification
- In homology
- Proof of Arnold's congruence
- Gudkov-Rokhlin congruence
- The role of complexification
- Mystery of the 16th Hilbert problem
- Second part
- Second part
- The first part success

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Breakthrough

- Isotopy classification of nonsingular sextics
- Gudkov's M-curve
- Gudkov's conjecture
- Arnold's congruence
- Complexification
- In homology
- Proof of Arnold's congruence
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- Second part
- Second part
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Read the Sixteenth Hilbert Problem

Breakthrough

- Isotopy classification of nonsingular sextics
- Gudkov's M-curve
- Gudkov's conjecture
- Arnold's congruence
- Complexification
- In homology
- Proof of Arnold's congruence
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- Second part
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Read the Sixteenth Hilbert Problem

Breakthrough

- Isotopy classification of nonsingular sextics
- Gudkov's M-curve
- Gudkov's conjecture
- Arnold's congruence
- Complexification
- In homology
- Proof of Arnold's congruence
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- Second part
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Read the Sixteenth Hilbert Problem

Breakthrough

- Isotopy classification of nonsingular sextics
- Gudkov's M-curve
- Gudkov's conjecture
- Arnold's congruence
- Complexification
- In homology
- Proof of Arnold's congruence
- Gudkov-Rokhlin congruence
- The role of complexification
- Mystery of the 16th Hilbert problem
- Second part
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($\dim_{\mathbb{Z}_2} H_*(\mathbb{R}A;\mathbb{Z}_2) \le \dim_{\mathbb{Z}_2} H_*(\mathbb{C}A;\mathbb{Z}_2)$ for any A)

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Read the Sixteenth Hilbert Problem

Breakthrough
- Isotopy classification of nonsingular sextics
- Gudkov's M-curve
- Gudkov's conjecture
- Arnold's congruence
- Complexification
- In homology
- Proof of Arnold's congruence
- Gudkov-Rokhlin congruence
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- Mystery of the 16th Hilbert problem
- Second part
- Second part
- The first part success

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Read the Sixteenth Hilbert Problem

Breakthrough

- Isotopy classification of nonsingular sextics
- Gudkov's M-curve
- Gudkov's conjecture
- Arnold's congruence
- Complexification
- In homology
- Proof of Arnold's congruence
- Gudkov-Rokhlin congruence
- The role of complexification
- Mystery of the 16th Hilbert problem
- Second part
- Second part
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Between the two papers by Rokhlin, there was a paper by Kharlamov with the upper bound (=10) for the number of connected components of a quartic surface.

Read the Sixteenth Hilbert Problem

Breakthrough
- Isotopy classification of nonsingular sextics
- Gudkov's M-curve
- Gudkov's conjecture
- Arnold's congruence
- Complexification
- In homology
- Proof of Arnold's congruence
- Gudkov-Rokhlin congruence
- The role of complexification
- Mystery of the 16th Hilbert problem
- Second part
- Second part
- The first part success

Post Solution

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Read the Sixteenth Hilbert Problem

Breakthrough
- Isotopy classification of nonsingular sextics
- Gudkov's M-curve
- Gudkov's conjecture
- Arnold's congruence
- Complexification
- In homology
- Proof of Arnold's congruence
- Gudkov-Rokhlin congruence
- The role of complexification
- Mystery of the 16th Hilbert problem
- Second part
- Second part
- The first part success

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Read the Sixteenth Hilbert Problem

Breakthrough

- Isotopy classification of nonsingular sextics
- Gudkov's M-curve
- Gudkov's conjecture
- Arnold's congruence
- Complexification
- In homology
- Proof of Arnold's congruence
- Gudkov-Rokhlin congruence
- The role of complexification
- Mystery of the 16th Hilbert problem
- Second part
- Second part
- The first part success

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Gudkov's conjecture and its high-dimensional generalization proven by Rokhlin explain all the phenomena which had struck Hilbert and motivated his sixteenth problem.

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Breakthrough
- Isotopy classification of nonsingular sextics
- Gudkov's M-curve
- Gudkov's conjecture
- Arnold's congruence
- Complexification
- In homology
- Proof of Arnold's congruence
- Gudkov-Rokhlin congruence
- The role of complexification
- Mystery of the 16th Hilbert problem
- Second part
- Second part
- The first part success

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Breakthrough
- Isotopy classification of nonsingular sextics
- Gudkov's M-curve
- Gudkov's conjecture
- Arnold's congruence
- Complexification
- In homology
- Proof of Arnold's congruence
- Gudkov-Rokhlin congruence
- The role of complexification
- Mystery of the 16th Hilbert problem
- Second part
- Second part
- The first part success

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Read the Sixteenth Hilbert Problem

Breakthrough
- Isotopy classification of nonsingular sextics
- Gudkov's M-curve
- Gudkov's conjecture
- Arnold's congruence
- Complexification
- In homology
- Proof of Arnold's congruence
- Gudkov-Rokhlin congruence
- The role of complexification
- Mystery of the 16th Hilbert problem
- Second part
- Second part
- The first part success

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Read the Sixteenth Hilbert Problem

Breakthrough
- Isotopy classification of nonsingular sextics
- Gudkov's M-curve
- Gudkov's conjecture
- Arnold's congruence
- Complexification
- In homology
- Proof of Arnold's congruence
- Gudkov-Rokhlin congruence
- The role of complexification
- Mystery of the 16th Hilbert problem
- Second part
- Second part
- The first part success

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Gudkov's conjecture and its high-dimensional generalization proven by Rokhlin explain all the phenomena which had struck Hilbert and motivated his sixteenth problem.

They are *real* manifestations of fundamental topological phenomena located in the *complex*.

Hilbert never showed a slightest sign that he had expected a progress via getting out of the real world into the realm of complex. Felix Klein consciously looked for interaction of real and complex pictures as early as in 1876.

Mystery of the 16th Hilbert problem

Read the Sixteenth Hilbert Problem

Breakthrough
- Isotopy classification of nonsingular sextics
- Gudkov's M-curve
- Gudkov's conjecture
- Arnold's congruence
- Complexification
- In homology
- Proof of Arnold's congruence
- Gudkov-Rokhlin congruence
- The role of complexification
- Mystery of the 16th Hilbert problem
- Second part
- Second part
- The first part success

Post Solution

Read the Sixteenth Hilbert Problem

Breakthrough
- Isotopy classification of nonsingular sextics
- Gudkov's M-curve
- Gudkov's conjecture
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- Proof of Arnold's congruence
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- The role of complexification
- Mystery of the 16th Hilbert problem
- Second part
- Second part
- The first part success

Post Solution

Mystery of the 16th Hilbert problem

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Read the Sixteenth Hilbert Problem

Breakthrough
- Isotopy classification of nonsingular sextics
- Gudkov's M-curve
- Gudkov's conjecture
- Arnold's congruence
- Complexification
- In homology
- Proof of Arnold's congruence
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- Second part
- Second part
- The first part success

Post Solution

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Mystery of the 16th Hilbert problem

Read the Sixteenth Hilbert Problem

Breakthrough

- Isotopy classification of nonsingular sextics
- Gudkov's M-curve
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- Complexification
- In homology
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- Mystery of the 16th Hilbert problem
- Second part
- Second part
- The first part success

Post Solution

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Read the Sixteenth Hilbert Problem

Breakthrough
- Isotopy classification of nonsingular sextics
- Gudkov's M-curve
- Gudkov's conjecture
- Arnold's congruence
- Complexification
- In homology
- Proof of Arnold's congruence
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- Mystery of the 16th Hilbert problem
- Second part
- Second part
- The first part success

Post Solution

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Read the Sixteenth Hilbert Problem

Breakthrough
● Isotopy classification of nonsingular sextics
● Gudkov's M-curve
● Gudkov's conjecture
● Arnold's congruence
● Complexification
● In homology
● Proof of Arnold's congruence
● Gudkov-Rokhlin congruence
● The role of complexification
● Mystery of the 16th Hilbert problem
● Second part
● Second part
● The first part success

Post Solution

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Mystery of the 16th Hilbert problem

Read the Sixteenth Hilbert Problem

Breakthrough

- Isotopy classification of nonsingular sextics
- Gudkov's M-curve
- Gudkov's conjecture
- Arnold's congruence
- Complexification
- In homology
- Proof of Arnold's congruence
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- Second part
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Post Solution

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Mystery of the 16th Hilbert problem

Read the Sixteenth Hilbert Problem

Breakthrough
- Isotopy classification of nonsingular sextics
- Gudkov's M-curve
- Gudkov's conjecture
- Arnold's congruence
- Complexification
- In homology
- Proof of Arnold's congruence
- Gudkov-Rokhlin congruence
- The role of complexification
- Mystery of the 16th Hilbert problem
- Second part
- Second part
- The first part success

Post Solution

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Mystery of the 16th Hilbert problem

Read the Sixteenth Hilbert Problem

Breakthrough

- Isotopy classification of nonsingular sextics
- Gudkov's M-curve
- Gudkov's conjecture
- Arnold's congruence
- Complexification
- In homology
- Proof of Arnold's congruence
- Gudkov-Rokhlin congruence
- The role of complexification
- Mystery of the 16th Hilbert problem
- Second part
- Second part
- The first part success

Post Solution

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Read the Sixteenth Hilbert Problem

Breakthrough
- Isotopy classification of nonsingular sextics
- Gudkov's M-curve
- Gudkov's conjecture
- Arnold's congruence
- Complexification
- In homology
- Proof of Arnold's congruence
- Gudkov-Rokhlin congruence
- The role of complexification
- Mystery of the 16th Hilbert problem
- Second part
- Second part
- The first part success

Post Solution

Second part

Hilbert's sixteenth problem does not stop where I stopped citation, it has the second half:

Second part

Read the Sixteenth Hilbert Problem

Breakthrough
- Isotopy classification of nonsingular sextics
- Gudkov's M-curve
- Gudkov's conjecture
- Arnold's congruence
- Complexification
- In homology
- Proof of Arnold's congruence
- Gudkov-Rokhlin congruence
- The role of complexification
- Mystery of the 16th Hilbert problem
- Second part
- Second part
- The first part success

Post Solution

In connection with this purely algebraic problem, I wish to bring forward a question which, it seems to me, may be attacked by the same method of continuous variation of coefficients, and whose answer is of corresponding value for the topology of families of curves defined by differential equations. This is the question as to the maximum number and position of Poincaré's boundary cycles (cycles limites) for a differential equation of the first order and degree of the form

$$\frac{dy}{dx} = \frac{Y}{X},$$

where X and Y are rational integral functions of the nth degree in x and y.

Second part

Read the Sixteenth Hilbert Problem

Breakthrough
- Isotopy classification of nonsingular sextics
- Gudkov's M-curve
- Gudkov's conjecture
- Arnold's congruence
- Complexification
- In homology
- Proof of Arnold's congruence
- Gudkov-Rokhlin congruence
- The role of complexification
- Mystery of the 16th Hilbert problem
- Second part
- Second part
- The first part success

Post Solution

Written homogeneously, this is

$$X\left(y\frac{dz}{dt} - z\frac{dy}{dt}\right) + Y\left(z\frac{dx}{dt} - x\frac{dz}{dt}\right) + Z\left(x\frac{dy}{dt} - y\frac{dx}{dt}\right) = 0,$$

where X, Y, and Z are rational integral homogeneous functions of the nth degree in x, y, z, and the latter are to be determined as functions of the parameter t.

Read the Sixteenth Hilbert Problem
Breakthrough
- Isotopy classification of nonsingular sextics
- Gudkov's M-curve
- Gudkov's conjecture
- Arnold's congruence
- Complexification
- In homology
- Proof of Arnold's congruence
- Gudkov-Rokhlin congruence
- The role of complexification
- Mystery of the 16th Hilbert problem
- Second part
- Second part
- The first part success

Post Solution

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Read the Sixteenth Hilbert Problem

Breakthrough
- Isotopy classification of nonsingular sextics
- Gudkov's M-curve
- Gudkov's conjecture
- Arnold's congruence
- Complexification
- In homology
- Proof of Arnold's congruence
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- The role of complexification
- Mystery of the 16th Hilbert problem
- Second part
- Second part
- The first part success

Post Solution

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Read the Sixteenth Hilbert Problem

Breakthrough
- Isotopy classification of nonsingular sextics
- Gudkov's M-curve
- Gudkov's conjecture
- Arnold's congruence
- Complexification
- In homology
- Proof of Arnold's congruence
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- The role of complexification
- Mystery of the 16th Hilbert problem
- Second part
- Second part
- The first part success

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Second part

Read the Sixteenth Hilbert Problem

Breakthrough
- Isotopy classification of nonsingular sextics
- Gudkov's M-curve
- Gudkov's conjecture
- Arnold's congruence
- Complexification
- In homology
- Proof of Arnold's congruence
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- The role of complexification
- Mystery of the 16th Hilbert problem
- Second part
- Second part
- The first part success

Post Solution

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There is still almost no progress in the second half of the sixteenth problem. Hilbert's hope for a similarity between the two halves has not realized.

Finiteness for the number of limit cycles for each individual equation has been proven. But even for $n=2$, the maximal number of limit cycles is still unknown.

The first part success

Contrary to this, the first half was extremely successful:

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Read the Sixteenth Hilbert Problem

Breakthrough
- Isotopy classification of nonsingular sextics
- Gudkov's M-curve
- Gudkov's conjecture
- Arnold's congruence
- Complexification
- In homology
- Proof of Arnold's congruence
- Gudkov-Rokhlin congruence
- The role of complexification
- Mystery of the 16th Hilbert problem
- Second part
- Second part
- The first part success

Post Solution

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It contained difficult concrete problems (maximal sextic curves, number of components of a quartic surface) which have been solved.

Read the Sixteenth Hilbert Problem

Breakthrough
- Isotopy classification of nonsingular sextics
- Gudkov's M-curve
- Gudkov's conjecture
- Arnold's congruence
- Complexification
- In homology
- Proof of Arnold's congruence
- Gudkov-Rokhlin congruence
- The role of complexification
- Mystery of the 16th Hilbert problem
- Second part
- Second part
- The first part success

Post Solution

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Breakthrough
- Isotopy classification of nonsingular sextics
- Gudkov's M-curve
- Gudkov's conjecture
- Arnold's congruence
- Complexification
- In homology
- Proof of Arnold's congruence
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- Mystery of the 16th Hilbert problem
- Second part
- Second part
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Read the Sixteenth Hilbert Problem
Breakthrough
- Isotopy classification of nonsingular sextics
- Gudkov's M-curve
- Gudkov's conjecture
- Arnold's congruence
- Complexification
- In homology
- Proof of Arnold's congruence
- Gudkov-Rokhlin congruence
- The role of complexification
- Mystery of the 16th Hilbert problem
- Second part
- Second part
- The first part success

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This direction has little chances to be completed. As a "thorough investigation", the problem can hardly be solved.

Read the Sixteenth Hilbert Problem

Breakthrough

Post Solution
- What has happened since then?
- To what degree?
- Other objects
- Open problems

Post Solution

Read the Sixteenth Hilbert Problem
Breakthrough
Post Solution
- What has happened since then?
- To what degree?
- Other objects
- Open problems

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Use of complexification made possible to find numerous *restrictions on the topology of real algebraic varieties*.

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Read the Sixteenth Hilbert Problem
Breakthrough
Post Solution
- What has happened since then?
- To what degree?
- Other objects
- Open problems

Use of complexification made possible to find numerous *restrictions on the topology of real algebraic varieties*. Besides the congruence modulo 4, Arnold proved in the same paper several inequalities on numerical characteristics of mutual position of ovals.

What has happened since then?

Read the Sixteenth Hilbert Problem

Breakthrough

Post Solution

- What has happened since then?
- To what degree?
- Other objects
- Open problems

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He found several useful ways to translate geometric phenomena in the real domain to the complex domain and back.

Read the Sixteenth Hilbert Problem

Breakthrough

Post Solution
- What has happened since then?
- To what degree?
- Other objects
- Open problems

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What has happened since then?

Read the Sixteenth Hilbert Problem
Breakthrough
Post Solution
- What has happened since then?
- To what degree?
- Other objects
- Open problems

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Read the Sixteenth Hilbert Problem
Breakthrough
Post Solution
- What has happened since then?
- To what degree?
- Other objects
- Open problems

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Read the Sixteenth Hilbert Problem
Breakthrough
Post Solution
- What has happened since then?
- To what degree?
- Other objects
- Open problems

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Read the Sixteenth Hilbert Problem
Breakthrough
Post Solution
- What has happened since then?
- To what degree?
- Other objects
- Open problems

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Read the Sixteenth Hilbert Problem
Breakthrough
Post Solution
- What has happened since then?
- To what degree?
- Other objects
- Open problems

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Read the Sixteenth Hilbert Problem
Breakthrough
Post Solution
- What has happened since then?
- To what degree?
- Other objects
- Open problems

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Read the Sixteenth Hilbert Problem
Breakthrough
Post Solution
● What has happened since then?
● To what degree?
● Other objects
● Open problems

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Read the Sixteenth Hilbert Problem
Breakthrough
Post Solution
- What has happened since then?
- To what degree?
- Other objects
- Open problems

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Read the Sixteenth Hilbert Problem
Breakthrough
Post Solution
- What has happened since then?
- To what degree?
- Other objects
- Open problems

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Breakthrough
Post Solution
- What has happened since then?
- To what degree?
- Other objects
- Open problems

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Read the Sixteenth Hilbert Problem
Breakthrough
Post Solution
- What has happened since then?
- To what degree?
- Other objects
- Open problems

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Only 6 isotopy types are questionable.

Read the Sixteenth Hilbert Problem
Breakthrough
Post Solution
- What has happened since then?
- To what degree?
- Other objects
- Open problems

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Read the Sixteenth Hilbert Problem
Breakthrough
Post Solution
● What has happened since then?
● To what degree?
● Other objects
● Open problems

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Read the Sixteenth Hilbert Problem
Breakthrough
Post Solution
- What has happened since then?
- To what degree?
- Other objects
- Open problems

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Read the Sixteenth Hilbert Problem
Breakthrough
Post Solution
- What has happened since then?
- To what degree?
- Other objects
- Open problems

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Breakthrough
Post Solution
- What has happened since then?
- To what degree?
- Other objects
- Open problems

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Read the Sixteenth Hilbert Problem
Breakthrough
Post Solution
- What has happened since then?
- To what degree?
- Other objects
- Open problems

To what degree?

Often people ask: **To what degree the Hilbert problem has been solved?**

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For $n \le 2$ see textbooks on Analytic Geometry.

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Read the Sixteenth Hilbert Problem
Breakthrough
Post Solution
- What has happened since then?
- To what degree?
- Other objects
- Open problems

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Read the Sixteenth Hilbert Problem
Breakthrough
Post Solution
- What has happened since then?
- To what degree?
- Other objects
- Open problems

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For $n = 4$ in the seventies by Nikulin and Kharlamov.

Read the Sixteenth Hilbert Problem

Breakthrough

Post Solution
- What has happened since then?
- To what degree?
- Other objects
- Open problems

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Read the Sixteenth Hilbert Problem

Breakthrough

Post Solution

- What has happened since then?
- To what degree?
- Other objects
- Open problems

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Read the Sixteenth Hilbert Problem

Breakthrough

Post Solution
- What has happened since then?
- To what degree?
- Other objects
- Open problems

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Read the Sixteenth Hilbert Problem

Breakthrough

Post Solution

- What has happened since then?
- To what degree?
- Other objects
- Open problems

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Read the Sixteenth Hilbert Problem

Breakthrough

Post Solution

- What has happened since then?
- To what degree?
- Other objects
- Open problems

Read the Sixteenth Hilbert Problem

Breakthrough

Post Solution
- What has happened since then?
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- Other objects
- Open problems

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Read the Sixteenth Hilbert Problem
Breakthrough
Post Solution
- What has happened since then?
- To what degree?
- Other objects
- Open problems

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Read the Sixteenth Hilbert Problem

Breakthrough

Post Solution

- What has happened since then?
- To what degree?
- Other objects
- Open problems

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Read the Sixteenth Hilbert Problem

Breakthrough

Post Solution
- What has happened since then?
- To what degree?
- Other objects
- Open problems

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Read the Sixteenth Hilbert Problem

Breakthrough

Post Solution

- What has happened since then?
- To what degree?
- Other objects
- Open problems

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Read the Sixteenth Hilbert Problem

Breakthrough

Post Solution
- What has happened since then?
- To what degree?
- Other objects
- Open problems

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Read the Sixteenth Hilbert Problem

Breakthrough

Post Solution
- What has happened since then?
- To what degree?
- Other objects
- Open problems

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Read the Sixteenth Hilbert Problem

Breakthrough

Post Solution
- What has happened since then?
- To what degree?
- Other objects
- Open problems

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Read the Sixteenth Hilbert Problem

Breakthrough

Post Solution
- What has happened since then?
- To what degree?
- Other objects
- Open problems

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Read the Sixteenth Hilbert Problem

Breakthrough

Post Solution
- What has happened since then?
- To what degree?
- Other objects
- Open problems

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Read the Sixteenth Hilbert Problem
Breakthrough
Post Solution
- What has happened since then?
- To what degree?
- Other objects
- Open problems

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Read the Sixteenth Hilbert Problem
Breakthrough
Post Solution
- What has happened since then?
- To what degree?
- Other objects
- Open problems

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Read the Sixteenth Hilbert Problem
Breakthrough
Post Solution
- What has happened since then?
- To what degree?
- Other objects
- Open problems

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Read the Sixteenth Hilbert Problem
Breakthrough
Post Solution
● What has happened since then?
● To what degree?
● Other objects
● Open problems

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Read the Sixteenth Hilbert Problem
Breakthrough
Post Solution
- What has happened since then?
- To what degree?
- Other objects
- Open problems

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Read the Sixteenth Hilbert Problem
Breakthrough
Post Solution
- What has happened since then?
- To what degree?
- Other objects
- Open problems

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Read the Sixteenth Hilbert Problem

Breakthrough

Post Solution
- What has happened since then?
- To what degree?
- Other objects
- Open problems

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Read the Sixteenth Hilbert Problem
Breakthrough
Post Solution
● What has happened since then?
● To what degree?
● Other objects
● Open problems

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Read the Sixteenth Hilbert Problem

Breakthrough

Post Solution
- What has happened since then?
- To what degree?
- Other objects
- Open problems

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9. Formulate counter-parts of topological questions about real algebraic varieties for varieties over other non algebraically closed fields, and solve them!