

Midterm I

Examination time: 9:50-11:10 am. No electronic devices, books or notes. Show all your work.

Name _____

Student ID # _____

Problem #	Points/total
1	/5
2	/10
3a	/5
3b	/30
4	/10
5	/20
Total	/80

Name _____

Problem 1 (5pt). Find all complex solutions of the equation $z^4 = -4$. Express them in the form $z = x + iy$ and sketch in the complex plane.

Problem 2 (10pt). Find an analytic function $f(x + iy)$ with $\operatorname{Re}(f) = x^2 - y^2 + 2x$.

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Problem 3a (5pt). Consider the set $D = \{z \in \mathbb{C} \mid 1 < |z| < \sqrt{2}, \operatorname{Im} z < 0, \operatorname{Re} z > 0\}$. Is D a domain? Sketch it. Describe its boundary.

Problem 3b (30pt). Sketch the image of D under the transformation $z \mapsto \frac{1}{z^2 + 1}$.
Hint: it makes sense to represent the transformation as a composition of three transformations.

Name _____

Problem 4 (10pt). Find all complex solutions of the equation $\cos z = 1$.

Name _____

Problem 5 (20pt). Find all complex solutions of the equation $z^i = 1$.