

I had not prepared this comparison. As for experience, I taught in the Leningrad University for fifteen years, different subjects, for example, one of them was a Topology course, for first and second year students, then I moved to the United States, to the University of California, Riverside, and I taught the same course in Riverside, for graduate students as a qualifying course with about the same success. Then I moved to Sweden, Uppsala University and found there a similar course on the D level, which is high level undergraduate students mixed with low level graduate students. I tried to move this course down, but it did not work. We had to put it back to the D level.

So levels are different in different countries. And not only levels. What struck me most in my experience was that really these countries are developing in different directions. Say now Sweden is going down in education with a high speed and terrifying acceleration. I cannot explain how much it is. Here are just few points which may give a rough idea about this. About twenty years ago it was quite decent. Now they do not have grades at school until the seventh year. They do not teach geometry at school. I would hardly call mathematics any part of what they teach at high school. Recent graduates of Swedish gymnasiums have a background in mathematics which is about the same as their counterparts in America (laughs...) No, but I mean I like when an American student appears there. We have a lot of exchange of students, I like them, because they make the feedback much better. Swedish students do not like to ask questions, American students love to do this; Swedish students do not appear in the office hours, American students, well ... they do come and ask and ask and ask...

Of course I had to change all my teaching habits. When I arranged my first exam, in the US, nothing to say what happened to my evaluations, but students cried there, just because I asked the same questions and in the same way, as I used to ask in oral exams in Russia. Neither American, nor Swedish undergraduate students experience true oral exams. Their exams are written tests which tend to reduce to collections of standard calculations. They are not required to formulate and prove a theorem, hence for them apparently it does not make sense to learn theorems and proofs.

I do not mean to say that one kind of exams is good and the other is bad. But I think that the exam is one of the major controls, which direct the whole education. Exams play three key roles: 1. organize learning, 2. test knowledge and skills of students, 3. teach students. Unfortunately, only the second role is usually taken into account. The first role can be explained as follows: if a student knows that something will not be needed to pass exam, there is a good chance that this thing would be considered as totally irrelevant. For how we arrange exams, what we ask, determines what students do. If you ask silly questions, they like to answer to the silly questions. If you ask very complicated questions, well, what would they do? They don't know what to do and eventually some of them will be able to answer to complicated questions. The third role may be extremely important, if the exam is oral and includes a detailed conversation with an examiner. Of course, it is up to the examiner to make it valuable.

So, requirements... It is a very important point. In some countries, like say in the US, there are long-playing mechanisms which deteriorate education by pushing down requirements. Let me sketch one of them. As far as I understand it works in the US, being one of the major problems with education People come to a university to buy a diploma, to buy a

degree. They want to pay less. They pay with their knowledge, with their exams, so they try to press teachers to make easier exams, to give less information, to teach them less. And this is a well organized system, it gives a huge drift. You can trace this during the whole twentieth century, now it is about twice less of courses needed for a degree. In more objective units (like number of hours), this difference is maybe greater. I do not know the exact figures. In European countries education is more centralized and is deteriorated more by reforms directed by governments and parliaments.

Sometimes, even bad social environment can play good things. Say in Russia, for a long time, many of the most advanced teachers had a belief, a sort of paradigm, according to which they were set a super-task to teach students to think and considered this even more profound than their main task to train them in doing specific things. This was the main point in our education. It was sort of resistance to the general trend. We had the feeling that we were doing something important, which was not easy to observe, but would not be appreciated by the Soviet authorities. We were proud to do it, you know.

I want to say also a few words about large scale. Traveling from one country to another and asking questions to my colleagues, I got an impression that many members of our community, the world mathematical community, do not feel the historical perspective. What is going on, really? I think that during the twentieth century we, as a mathematical community, lost a couple of very important battles. This is a crucial point. Thirty years ago, an educated person had a feeling that he learned how to think and argue in mathematics class, when studying Geometry. Indeed, a study of Geometry requires a fluent translation from the language of pictures to verbal languages. Axioms, theorems and proofs have appeared in Geometry, there are no crowds of them in the high-school algebra. Of course, these are universal tools, which are needed in each part of Mathematics, but in Geometry the need is felt stronger motivated. Weakening the positions of geometry in the mathematical education implies immediate weakening the logical skills of students due to lack of exercises. Where is Geometry now in most of the countries? There is no geometry. Mathematician, mathematical teacher, was a person who explained what is a proof, what is logic, and now mathematical teacher explains how to calculate, how to work out examples.

In different countries education changes with different rates. France is more resisting to the trend. There are still state exams for teachers. There are Grandes Écoles, and so on, I did not live in France, but my colleagues who live in France say that, well, the trend is in the same direction, but they did not go that far. They are slower, because of more conservative character of society or because of inventions by Napoleon, I don't know. I'm serious, it is not a joke, it is true.

Now we are about to loose Calculus. I don't know how it may happen. As you know, there are many other teachers, not mathematicians, who are able to teach Calculus, easily. Calculus taught by Mathematicians in most countries has not has not adjusted to the change of students' level. It just became boring, since rewriting textbooks by generations of teachers has washed away the best pieces of its contents. However the system of notions has not changed in mathematical sense since late nineteenth century. Still at the beginning of this Calculus there are limits, which are impossible for students who did not study Geometry at school and cannot keep understanding the matter after

they encounter three successive quantors in the definition of limit. Calculus was designed when students could consume limits (well, first of all, it was not the whole population, but its very small part, educated people, now this is almost everybody). Now limits are used throughout the Calculus, as it used to be, and for defining notions which don't require this. Professor Hughes-Hallet, in your new Harvard course, limits are at the beginning, is that right? Integrals are introduced still as limits of the kind which was not studied in the preceding part of Calculus, because these are limits in infinite-dimensional space of subdivisions of the integration interval, and, you know, nobody cares... I mean that mathematicians do not think about this, although they must. They must do something, because as Arnold wrote in his very sharp articles about this: "students who suffer from this Calculus will become ministers and they will care about financing mathematics". So we dig our grave when teaching students in this way. We have to teach them to think, and instead we make them suffer.

Now, in those countries where geometry disappeared from school, universities must take over, because this is a subject where one can teach students to prove, to think, to discover. Why there is no geometry in the first year curricula. Why there is this stupid Calculus, instead?

Another thing. What Calculus really requires is logical skills, which all of mathematicians got somehow, because they became mathematicians. Probably, despite that they attended universities. But it was really a difficult step to move from the colloquial language logic, which is based just on the way how we speak, towards a more accurate way, naive mathematical logic, which is used in mathematics. And I think that it is the role of mathematicians to teach this language. This is what we must do. This is usually done, somewhere, inside mathematical courses. Probably it would be good to understand where and how it is done. Maybe there must be some separate course. I know one experiment of this sort in Saint Petersburg Pedagogical Institute, (it is called now university, but this does not matter). They devote the whole first semester just to a course of mathematical language (although it is called somehow else). They do simple exercises, based on the material of high school mathematics, without theoretical lectures. Students there are very poor, students who come to become teachers in Russia are still very poor because it is a profession which does not pay. Most of them are girls and they have very low preparation from school. (No, I don't mean that it is related, I mean that the girls may think that their husbands will earn money, and they can do something more pleasant.) This is a matter of fact. Somehow a semester of exercises in mathematical language works. I mean that the the students can consume after that some mathematics. Indeed, otherwise it used to be just a disaster.

I think that in many countries it is a government which directs all the education reforms downwards. I think that this is an important task to take away from them the burden of decision what mathematics need to be hidden from children. Now, since we have new technical opportunities with the web, we should apply some efforts to do this. It is possible to create web sites where you and everyone, and, in particular, students can enjoy mathematics. There are some web sites of this sort. Take a look at <http://www.cut-the-knot.com>.

Probably we need to create some infrastructure of knowledge, which would be available

for all students.

I will stop after this point. I want to tell you how mathematicians can do something in spite of the general trend. In Russia, you know, the education for gifted mathematical students of low age is very good. It was created absolutely without any money from the very beginning. It consists of Olimpiads, which detect and catch mathematically gifted students; evening classes, which are taught by students, who like to be professors, who just passed through this structure; mathematical schools, which are usual schools proud to be related to mathematics. This system attracts, creates and is maintained by good students, they like to do this. And this is almost enough. In Saint Petersburg, in Leningrad, we used to get about twenty brilliant students per year. (Well, of course, against two hundred which were not that good.) If mathematicians want to do this, they do this despite disregard and sometimes even resistance from the state. Thank you.