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# Quotation Analysis as Based on Sheaf-Theoretic Formal Semantics. I

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**Abstract:** The paper outlines an approach to quotation analysis in a frame of sheaf-theoretic formal semantics of natural language we proposed in a series of papers. The use of well-known in sheaf theory direct and inverse image functors allows us to analyze different types of quotation (use, mention and mixed) that occur in a written type of linguistic communication in some unspecified natural language, say in English.

**Keywords**: sense, fragmentary meaning, phonocentric topology, sheaf of fragmentary meanings, contextual meaning, bundle of contextual meanings, Frege duality, direct image functor, inverse image functor, quotation as mention, quotation as use.

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# Introduction

Firstly, we outline a sheaf-theoretic framework to study the process of interpretation of text written in some unspecified natural language, say in English, considered as a means of communication. Our analysis concerns the only texts written with good grace and intended for human understanding; we call further *admissible*. After a quick technical overview of the sheaf-theoretic semantics which we have introduced in a series of papers (Prosorov, 2005a, 2008, 2011, 2012a), we outline a quotation analysis in this formal framework. The present paper is an extended version of our talk given at the International Workshop *Quotation: Perspectives from Philosophy and Linguistics* (Prosorov, 2012b).

Formally, a text $X$ is a finite sequence $(x_1, x_2, \dots x_n)$ of its constituent sentences, and so it is a graph of the function $i \mapsto x_i$, identified with the set of ordered pairs $\{\langle 1, x_1\rangle, \langle 2, x_2\rangle, \dots \langle n, x_n\rangle\}$. In each pair, the first member indicates the position of the sentence denoted by the second member; the positions are linearly ordered following the adopted writing convention.

When reading a particular distinguished part of the text, we delete mentally all other sentences, but follow the induced order of remaining ones. Important is the induced order of sentences reading and not the concrete index numbers of its occupied places. Thus, any part of text is a subsequence whose graph is a subset of the whole sequence graph. Likewise for a sentence identified with a finite sequence of words, and for a part of sentence considered as a subsequence of the whole sequence.

We distinguish the notions *sense*, *meaning* and *reference*. The term *fragmentary meaning* (or simply *meaning*) of some fragment of a given text is accepted as the communicative content grasped in some particular situation of reading guided by the reader's presuppositions, preferences and prejudices, which we denominate by the term *sense* (or *mode of reading*). In our acceptance, the sense is a kind of semantic intentionality in the interpretative process, and in some degree, it is a secular remake of the term *sense* in the medieval exegesis (St. Thomas). At the level of text, it may be, for example, *literal*, *allegoric*, *moral*, *psychoanalytical*, etc.

In the present work, we assume a total cultural competence of an idealized reader who knows the lexicon of language and follows the rules of common usage. That is why we are less interested in the problem of understanding the *reference* of denotative expressions, and the ontological status of objects thus defined.

# 1 Topologies on Text and Meaningfulness

When reading a text, the understanding is not postponed until the final sentence. So the text should have the meaningful parts. Whether a part of an admissible text $X$ is meaningful or not depends on some accepted *criterion of meaningfulness*. We argue (Prosorov, 2005a, 2008, 2010) that for such a criterion conveying an idealized reader's linguistic competence meant as ability to grasp a communicative content, the set of proper meaningful parts is stable under arbitrary union and finite intersection. So we argue that in agreement with our linguistic intuition:

(i) *an arbitrary union of meaningful parts of an admissible text is meaningful;*

(ii) *a non-empty intersection of two meaningful parts of an admissible text is meaningful.*

The first property expresses the *principle of hermeneutic circle*, which requires to understand the whole (the union) through the understanding of its parts.

The second property expresses the contextual mechanism of understanding. To understand a meaningful part $U$ of text is to understand contextually all its sentences $x \in U$, where the context of a particular sentence $x$ constitutes some meaningful neighborhood of $x$ lying in $U$. Let $U, V \subseteq X$ be two meaningful parts with non-empty intersection. For the least meaningful part $U_x$ containing $x$, we have that $x \in U \cap V$ implies $x \in U_x \subseteq U \cap V$; hence, $U \cap V$ is meaningful as the union $\bigcup_{x \in U \cap V} U_x$ of meaningful parts.

Since an admissible text $X$ is supposed to be meaningful as a whole by the very definition, it remains only to define formally the meaning of its empty part $\varnothing$ (for example, as a singleton) in order to endow $X$ with some *topology* in a mathematical sense, where the set of open sets $\mathfrak{O}(X)$ is defined to be the set of all meaningful parts $U \subseteq X$ (called further *fragments*).

*Remark.* Any explicitly stated *concept of meaning* or *criterion of meaningfulness* satisfying conditions (i) and (ii) allows us to define some type of *semantic topology* on texts. Then we may interpret several tasks of discourse analysis in topological terms (Prosorov, 2006, 2008).

In what follows, we consider only admissible texts endowed with a particular type of semantic topology corresponding to the criterion of meaningfulness conveying an idealized reader's linguistic competence meant as ability to grasp a communicative content. The topology so defined is called *phonocentric*.

The natural process of reading supposes that any sentence $x$ of a text $X$ should be understood on the basis of the text's part already read, because the interpretation cannot be postponed, for "it is compulsive and uncontrollable" according to Rastier (1995). Thus for every pair of distinct sentences $x$, $y$ of a text $X$, there is an open $U$, that contains one of them (to be read first in the natural order $\leq$ of sentences reading) but not the other. Whence a phonocentric topology satisfies the *separation axiom* $T_0$ of Kolmogorov.

# 2 Phonocentric Topology and Partial Order

An admissible text $X$ gives rise to a finite topological space; hence an arbitrary intersection of its open sets is open and so it is an *Alexandrov space*. For a sentence $x \in X$, we define $U_x$ to be the intersection of all the meaningful parts that contain $x$, that is the smallest open neighborhood of $x$. We define the *specialization relation* $x \preceq y$ (read as '$x$ is more special than $y$') on a topological space $X$ by setting that $x \preceq y$ iff $x \in U_y$ or, equally, $U_x \subseteq U_y$. It is equivalent to say that $x \in U_y$ if and only if $y \in \mathrm{cl}(\{x\})$, where $\mathrm{cl}(\{x\})$ denotes the closure of a one-point set $\{x\}$.

It is clear that the specialization relation $\preceq$ on $X$ is reflexive and transitive, because this is true for $\subseteq$. Since a phonocentric topology satisfies the axiom $T_0$, the specialization relation $\preceq$ is also antisymmetric. Hence this relation $\preceq$ is a partial order on $X$.

The following proposition 1 is a linguistic version of the general results from (May, 2003).

**Proposition 1.** *Let X be a text endowed with a phonocentric topology.*

1. *The set of all smallest open neighborhoods $\mathfrak{B} = \{U_x \colon x \in X\}$ is a basis of the phonocentric topology. Each its basis contains $\mathfrak{B}$.*

2. *The initial phonocentric topology can be recovered from this partial order $\preceq$ in a unique way as the topology with the basis made up of all so-called* low *sets $U_x = \{z \colon z \preceq x\}$.*

The following proposition 2 is a criterion of map's continuity in terms of its behavior towards basis sets and specialization order.

**Proposition 2.** *Let X, Y be texts endowed with phonocentric topologies. Then the following conditions are equivalent:*

1. *A map $f \colon X \to Y$ is continuous*;

2. $\forall x \in X \colon f(U_x) \subseteq U_{f(x)}$;

3. *$f$ preserves specialization order*: *$x \preceq y$ implies $f(x) \preceq f(y)$.*

## Examples of Continuous Maps in Linguistic Situation

1. Let $U \subseteq X$ be a meaningful, i.e. open part of a text $X$. Then the map of inclusion $U \hookrightarrow X$ is continuous;

2. Let $Y = (y_1, \dots y_d, \dots y_m)$ be some author's text with a sentence $y_d$ which editor found so obscure that he added a gloss $(x_1, \dots x_n)$ just after $y_d$ to obtain the glossed text $X$. It gives rise to continuous map $f: X \to Y$ which looks like following:

$$\begin{array}{cccccccccc} X & = & (y_1, \dots & y_{d-1}, & y_d, & x_1, \dots & x_n, & y_{d+1}, \dots & y_m) \\ {\scriptstyle f}\big\downarrow & & & \big\downarrow & \big\downarrow & \swarrow & \swarrow & \big\downarrow & \big\downarrow \\ Y & = & (y_1, \dots & y_{d-1}, & y_d, & & & y_{d+1}, \dots & y_m)\,; \end{array}$$

3. Likewise, a multi-glossed text $Z$ obtained this way from the text $Y$ as above gives rise to continuous map $f: Z \to Y$;

4. A continuous map $f_1: X_2 \to X_1$ arises in writing process when an author goes from a first plan $X_1$ of some future text to its detailed plan $X_2$, there a sentence $x_d$ of $X_1$ is substituted by some passage $(x_{d_1}, \dots x_{d_n})$. And so on, in going to more detailed texts $X_3, \dots X_n$, one gets a sequence of continuous maps
$$X_n \xrightarrow{f_{n-1}} X_{n-1}, \dots X_3 \xrightarrow{f_2} X_2 \xrightarrow{f_1} X_1;$$

5. When a text $X$ is resumed in a text $\{a\}$ that consists of only one sentence $a$; it gives rise to continuous function $f: X \to \{a\}$;

6. More generally, suppose that each paragraph $X_i = (x_{i_1}, \dots x_{i_{n_i}})$ of some text $X$ is resumed in a text $\{a_i\}$ that consists of only one sentence $a_i$; then a function $X \to (a_1, \dots a_m): x_{i_j} \mapsto a_i$ is continuous; it represents a passage from the text $X$ of $m$ paragraphs $(X_1, \dots X_m)$ to its abstract $(a_1, \dots a_m)$.

## Graphical Representation of a Finite Poset

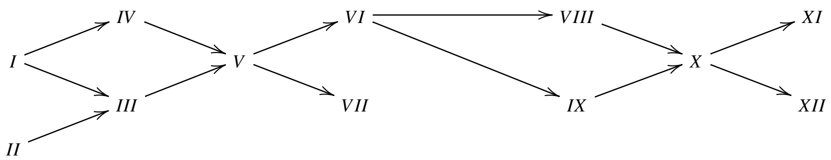
There is a simple intuitive tool for visualization of a finite partially ordered set (or briefly *poset*), called Hasse diagram. For a poset $(X, \preceq)$, the *cover relation* $x \prec y$ (read as '$x$ is covered by $y$') is defined by setting: $x \prec y$ iff $x \preceq y$ and there is no $z$ such that $x \preceq z \preceq y$.

For a given poset $(X, \preceq)$, its Hasse diagram is defined as the graph whose vertices are all the elements of $X$ and whose edges are those pairs $\langle x, y \rangle$ for which $x \prec y$. In the picture, the vertices of Hasse diagram are labeled by the elements of $X$ and the edge $\langle x, y \rangle$ is drawn by an arrow going from $x$ to $y$ (or sometimes by an indirected line connecting $x$ and $y$, but in this case the vertex $y$ is displayed lower than the vertex $x$); moreover, the vertices are displayed in such a way that each line meets only two vertices.

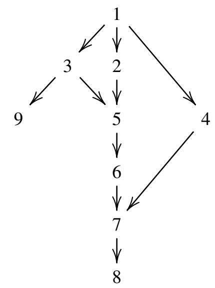
# 3 Phonocentric Topology at the Level of Text

The usage of some kind of Hasse diagram named *Leitfaden* is widely spread in the mathematical textbooks to facilitate the understanding of logical dependence of its chapters or paragraphs. Mostly, the partially ordered set is made up of all chapters of the book. So, in the *Non-commutative Algebraic*

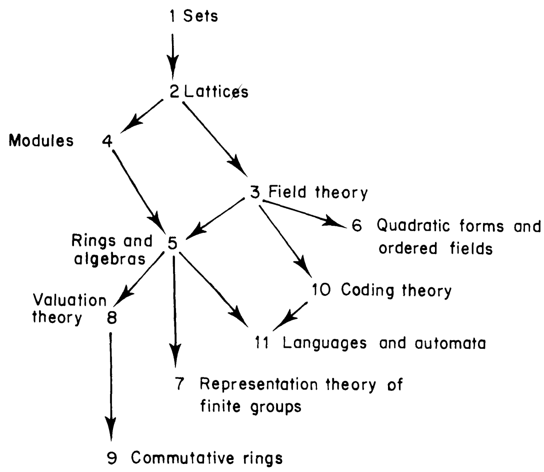
*Geometry* by F. M. J. van Oystaeyen and A. H. M. J. Verschoren (1981, p. 7), there is the following Hasse diagram named as “Leitfaden”:



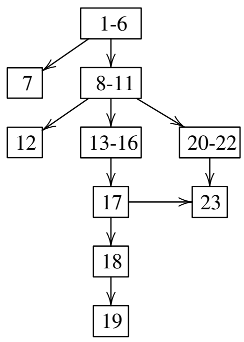

Also, in *Toposes, Triples and Theories* by M. Barr and Ch. Wells (1984, p. xi), there is such a Hasse diagram entitled as “Chapter dependency chart”:




Yet another diagram, whose vertices are labeled with chapter’s number and title, is presented in the vol. 2 of *Algebra* by P. M. Cohn (1989, p. xv) under the title “Table of interdependence of chapters (Leitfaden)”:

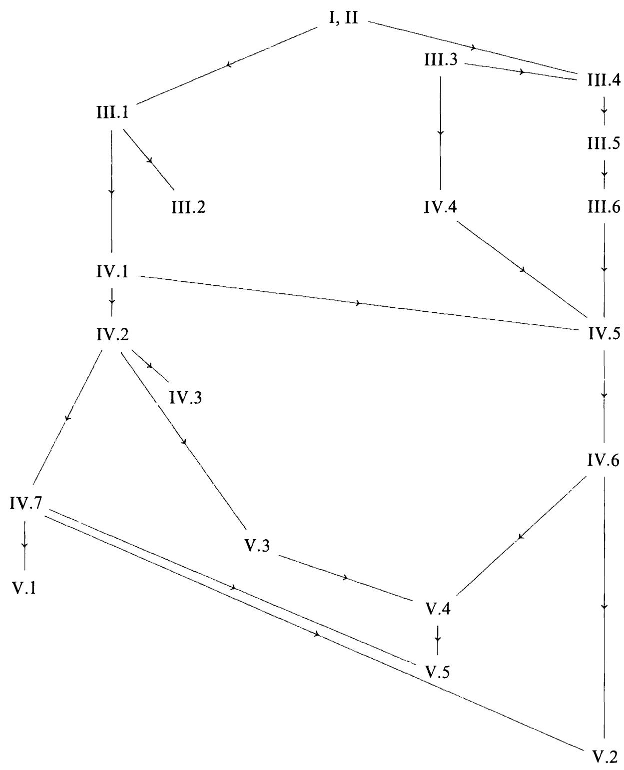



These diagrams surely presuppose the linear reading of sections within each chapter. However, the vertices of any such a diagram may be “split” in order to draw the Leitfaden whose vertices are all the sections like it’s done explicitly in the diagram named “Interdependence of the Sections” borrowed from the *Differential forms in algebraic topology* by R. Bott and L. W. Tu (1982, p. ix):



Here, the authors presuppose indeed the linear reading of sections in each interval 1-6, 8-11, 13-16 and 20-22, but it may be drawn explicitly.

In the *K-Theory* by M. Karoubi (1978, p. xii), there is another example of the Hasse diagram named "Interdependence of Chapters and Sections", the vertices of which are the sections, although the sections have its own consecutive numbering within each chapter.




Certainly, the author assumes here a consecutive dependence between Chapters I and II, as well as consecutive dependence between their sections. The same convention applies to sections called *Exercise* and *Historical Notes* that are included at the end of each Chapter and aren't shown on the diagram.

This way, one may go further and do the next step. For every sentence $x$ of a given admissible text $X$, one can find a basis' open set of the kind of its least open neighborhood $U_x$ in order to define the phonocentric topology at the semantic level of text (where points are sentences), and then to draw the Hasse diagram of the corresponding poset.

# 4 Phonocentric Topology at the Level of Sentence

Likewise, we may go further by doing the next step. In order to define a phonocentric topology at the semantic level of sentence (where points are words), we must distinguish there the meaningful fragments those are similar to meaningful fragments at the level of text. Let $x$, $y$ be any two words such that $x \preceq y$ in the specialization order at the level of sentence that is similar to the specialization order at the level of text. This relation $x \preceq y$ means that the word $x$ should necessary be an element of the least part $U_y$ required to understand the meaning of the word $y$ in the interpreted sentence. So we have $x \leq y$ in the order of writing and there should be some syntactic dependence between them. It means that a grammar in which the notion of dependence between pairs of words plays an essential role will be closer to our topological framework than a grammar of Chomsky's type.

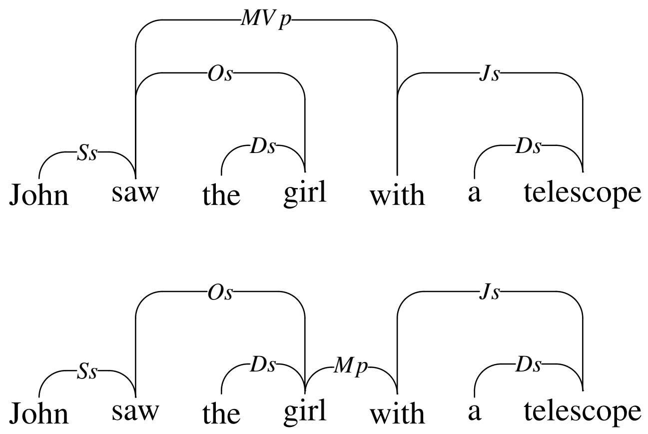
There are many formal grammars focused on links between words. We think that the theoretical approach of a *special link grammar* of D. Sleator and D. Temperley is more relevant to define a phonocentric topology at the level of sentence, because in whose formalism "[t]he grammar is distributed among the words" (1991, p. 3), and "the links are not allowed to form cycles" (1991, p. 13) comparing with *dependency grammars* which draw syntactic structure of sentence as a planar tree with one distinguished root word.

Given a sentence, the link grammar assigns to it a syntactic structure (linkage diagram) which consists of a set of labeled links connecting pairs of words. We use this diagram to define phonocentric topology on a sentence.

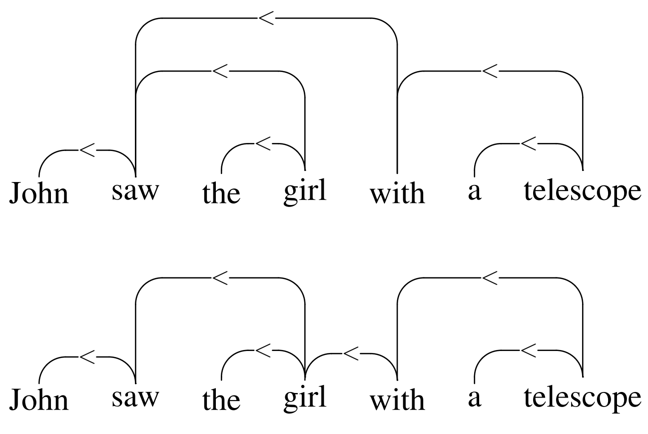
To explain how to do it, let us consider a sentence borrowed from (Werning, 2003, p. 10).

(1) John saw the girl with a telescope.

The analysis of this sentence by the *Link Parser 4.0* of D. Temperley, D. Sleator and J. Lafferty (2008) gives the following two linkage diagrams:

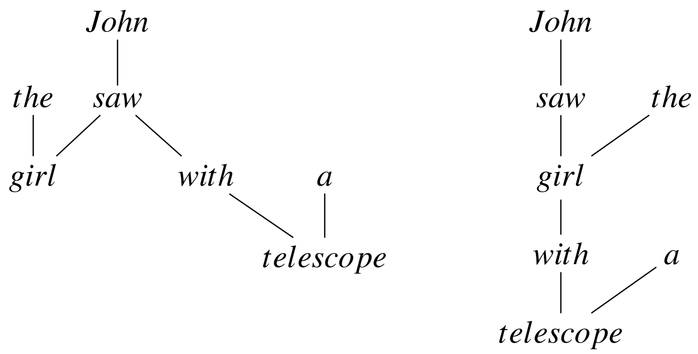



These two diagrams rewritten with arrows that indicate the direction of context dependence in which the connectors match (instead of connector names) have the following appearance:




It is clear that the transitive closure $x \preceq y$ of this relation $<$ between pairs of words defines two partial order structures on the sentence (1).

In recovering the phonocentric topology from this partial order $\preceq$ as the topology with the basis constituted of all $U_x = \{z \colon z \preceq x\}$, we can endow the sentence (1) with a phonocentric topology in two different ways. The Hasse diagrams of corresponding posets are:




To understand the sentence (1), the reader should do an *ambiguity resolution* when arriving to the word $x =$"with" by choosing between basis sets:

$U_x = \{\langle 1, \text{John}\rangle, \langle 2, \text{saw}\rangle, \langle 5, \text{with}\rangle\}$,

$U_x = \{\langle 1, \text{John}\rangle, \langle 2, \text{saw}\rangle, \langle 3, \text{the}\rangle, \langle 4, \text{girl}\rangle, \langle 5, \text{with}\rangle\}$.

In general case, the step by step choice of an appropriate context $U_x \subseteq X$ for each word $x \in X$ results in endowing the interpreted sentence $X$ with a particular phonocentric topology by means of the basis $(U_x)_{x \in X}$ that a reader grasps in the process of reading.

Once the phonocentric topology and the specialization order are determined at a certain semantic level, the systematic interpretation of linguistic concepts in terms of topology and order and their geometric studies is a kind of *formal syntax* at this semantic level, for the word $\sigma\acute{\upsilon}\nu\tau\alpha\xi\iota\zeta$ is derived from $\sigma\acute{\upsilon}\nu$ (together) and $\tau\acute{\alpha}\xi\iota\zeta$ (order). For further details concerning such a geometric approach to syntax, see our papers (Prosorov, 2008, 2011, 2012a).

# 5 Sheaves of Fragmentary Meanings

Let $X$ be an admissible text endowed with a phonocentric topology, and let $\mathscr{F}$ be an adopted sense of reading. In a Platonic manner, for each non-empty open (that is meaningful) part $U \subseteq X$, we collect in the set $\mathscr{F}(U)$ all fragmentary meanings of this part $U$ read in the sense $\mathscr{F}$; also we define $\mathscr{F}(\varnothing)$ to be a singleton $pt$. Thus we are given a map

$$U \mapsto \mathscr{F}(U) \tag{1}$$

defined on the set $\mathfrak{O}(X)$ of all opens of phonocentric topology on $X$.

Following the precept of hermeneutic circle **"to understand a part in accordance with the understanding of the whole"**, for each inclusion $U \subseteq V$ of non-empty opens, $\mathscr{F}$ assigns a restriction map $\text{res}_{V,U} \colon \mathscr{F}(V) \to \mathscr{F}(U)$. Thus we are given a map

$$\{U \subseteq V\} \mapsto \{\text{res}_{V,U} \colon \mathscr{F}(V) \to \mathscr{F}(U)\} \tag{2}$$

with the properties:

(i) identity preserving: $\text{id}_V \mapsto \text{id}_{\mathscr{F}(V)}$, for any open $V$;

(ii) transitivity: $\text{res}_{V,U} \circ \text{res}_{W,V} = \text{res}_{W,U}$, for all nested opens $U \subseteq V \subseteq W$, which means that two consecutive restrictions may be done by one step.

As for the empty part $\varnothing$ of $X$, the restriction maps $\text{res}_{\varnothing,\varnothing}$ and $\text{res}_{V,\varnothing}$ with the same properties are obviously defined.

Thus, any topological space $(X, \mathfrak{O}(X))$ gives rise to a category $\mathbf{Open}(X)$ with opens $U \in \mathfrak{O}(X)$ as objects and their inclusions $U \subseteq V$ as morphisms.

From the mathematical point of view, the assignments (1) and (2) give rise to a presheaf $\mathscr{F}$ defined as a contravariant functor

$$\mathscr{F} : \mathbf{Open}(X) \to \mathbf{Sets}$$

acting as

$$\begin{cases} U \mapsto \mathscr{F}(U) & \text{on objects,} \\ U \subseteq V \mapsto \mathrm{res}_{V,U} : \mathscr{F}(V) \to \mathscr{F}(U) & \text{on morphisms.} \end{cases}$$

In sheaf theory, the elements of $\mathscr{F}(V)$ are called *sections* (*over* $V$); sections over the whole $X$ are said to be *global*.

We consider the reading process of a fragment $U$ as its covering by some family of subfragments $(U_j)_{j \in J}$ already read, that is $U = \bigcup_{j \in J} U_j$.

Following Quine, "There is no entity without identity" (1981). Any reasonable *identity criterion* should define two fragmentary meanings as equal globally if and only if they are equal locally. It motivates the following:

**Claim S (Separability).** *Let $X$ be an admissible text, and let $\mathscr{F}$ be a presheaf of fragmentary meanings over $X$. Suppose that $U$ is an open fragment of $X$ and $s, t \in \mathscr{F}(U)$ are two fragmentary meanings of $U$ and there is an open covering $U = \bigcup_{j \in J} U_j$ such that* $\mathrm{res}_{U,U_j}(s) = \mathrm{res}_{U,U_j}(t)$ *for all fragments $U_j$. Then $s = t$.*

In other words, a kind of *local-global principle* holds for the identity of fragmentary meanings so defined.

According to the precept of hermeneutic circle **"to understand the whole by means of understandings of its parts"**, a presheaf $\mathscr{F}$ of fragmentary meanings should satisfy the following:

**Claim C (Compositionality).** *Let $X$ be an admissible text, and let $\mathscr{F}$ be a presheaf of fragmentary meanings over $X$. Suppose that $U$ is an open fragment of $X$ and $U = \bigcup_{j \in J} U_j$ is an open covering of $U$; suppose we are given a family $(s_j)_{j \in J}$ of fragmentary meanings, $s_j \in \mathscr{F}(U_j)$ for all fragments $U_j$, such that* $\mathrm{res}_{U_i, U_i \cap U_j}(s_i) = \mathrm{res}_{U_j, U_i \cap U_j}(s_j)$. *Then there exists some meaning $s$ of the whole $U$ such that* $\mathrm{res}_{U,U_j}(s) = s_j$ *for all $U_j$.*

Thus a family of locally compatible fragmentary meanings may be composed in a global one. Thus, any presheaf of fragmentary meanings defined as above should satisfy the claims (S) and (C), and so it is a *sheaf* by the very definition. It motivates the following:

**Frege's Generalized Compositionality Principle.** *A presheaf of fragmentary meanings naturally attached to any sense* (*mode of reading*) *of an admissible text is really a sheaf; its sections over a fragment of the text are its fragmentary meanings; its global sections are the meanings of the text as a whole.*

The claim (S) implies the meaning $s$, whose existence is claimed by (C), to be unique as such.

# 6 Category of Schleiermacher

We suppose that any part of text which is meaningful in one sense of reading should remain meaningful after the passage to any other sense of reading. We suppose also that the transfer from the understanding in one sense (e.g., literal) to the understanding in another sense (e.g., moral) commutes with the restriction maps. Formally, this idea is well expressed by the notion of *morphism* of the corresponding sheaves $\phi\colon \mathscr{F} \mapsto \mathscr{F}'$ defined as a family of maps $\phi(V)\colon \mathscr{F}(V) \to \mathscr{F}'(V)$, such that the following diagrams commute for all opens $U \subseteq V$ of $X$:

$$\begin{array}{ccc} \mathscr{F}(V) & \xrightarrow{\phi(V)} & \mathscr{F}'(V) \\ {\scriptstyle \mathrm{res}_{V,U}}\big\downarrow & & \big\downarrow{\scriptstyle \mathrm{res}'_{V,U}} \\ \mathscr{F}(U) & \xrightarrow[\phi(U)]{} & \mathscr{F}'(U) \end{array} .$$

So, for an admissible text $X$, the data of sheaves of fragmentary meanings and its morphisms constitutes a category $\mathbf{Schl}(X)$ in a strict mathematical sense, we call *category of Schleiermacher*.

Note that the class of objects in the category of Schleiermacher $\mathbf{Schl}(X)$ is not limited to a modest list of sheaves corresponding to *literal*, *allegoric*, *moral*, *psychoanalytical* senses mentioned above. In the process of text interpretation, the reader's semantic intentionality changes from time to time, with the result that there is some compositionality (or gluing) of sheaves which are defined only locally (Prosorov, 2008). There is a standard way to name the result of such a gluing as, for example, in the case of *Freudo-Marxist* sense.

# 7 Contextuality in a Sheaf-Theoretic Framework

So far, we have considered only the meanings of open sets in the phonocentric topology. It may happen that a particular point (sentence) $x \in X$ constitutes an open one-point set $\{x\}$, and so the set $\mathscr{F}(\{x\})$ of its fragmentary meanings have yet been defined. But in general, not every singleton is open in $T_0$-topology. Now we describe how to define the meanings of each point in the phonocentric topology.

Two fragmentary meanings $s \in \mathscr{F}(U)$, $t \in \mathscr{F}(V)$ are said to *induce the same contextual meanings of a sentence* $x \in U \cap V$ if there exists some open neighborhood $W$ of $x$, such that $W \subseteq U \cap V$ and $\mathrm{res}_{U,W}(s) = \mathrm{res}_{V,W}(t) \in \mathscr{F}(W)$. This relation is clearly an equivalence relation. Any equivalence class of fragmentary meanings agreeing in some open neighborhood of a sentence $x$ is natural to define as a *contextual meaning* of $x$. The equivalence class defined by a fragmentary meaning $s$ is called a *germ* at $x$ of this $s$ and is denoted by $\mathrm{germ}_x(s)$ (or simply $\mathrm{germ}_x s$).

The set of all equivalence classes is called a *stalk* of $\mathscr{F}$ at $x$ and denoted by $\mathscr{F}_x$.

In other terms, the set $\mathscr{F}_x$ of all contextual meanings of a sentence $x \in X$ is defined as the inductive limit $\mathscr{F}_x = \varinjlim(\mathscr{F}(U), \mathrm{res}_{V,U})_{U,V \in \mathfrak{O}(x)}$, where $\mathfrak{O}(x)$ is a set of all open neighborhoods of $x$.

Note that for an open singleton $\{x\}$, we may canonically identify its contextual meanings with the fragmentary ones, that is $\mathscr{F}_x = \mathscr{F}(\{x\})$.

# 8 Bundles of Contextual Meanings

For the coproduct $F = \bigsqcup_{x \in X} \mathscr{F}_x$, we define now a *projection* map $p\colon F \to X$ by setting $p(\mathrm{germ}_x s) = x$.

Every fragmentary meaning $s \in \mathscr{F}(U)$ determines a genuine function $\dot{s}\colon x \mapsto \mathrm{germ}_x s$ to be well-defined on $U$. It gives rise to a *functional representation* $s \mapsto \dot{s}$ of *fragmentary meanings* which clarifies the nature of abstract entity $s \in \mathscr{F}(U)$ as being represented by a genuine function $\dot{s}$.

We define the topology on $F$ by taking as a basis of open sets all the image sets $\dot{s}(U) \subseteq F$, $U \in \mathfrak{O}(X)$. Given a fragment $U \subseteq X$, a continuous function $t\colon U \to F$ such that $t(x) \in p^{-1}(x)$ for all $x \in U$ is called a *cross-section*. The topology defined on $F$ makes $p$ and every cross-section of the kind of $\dot{s}$ to be continuous. So we have defined topological spaces $F$, $X$ and a continuous map $p\colon F \to X$.

In topology, this data $(F,p)$ is called a *bundle over the base space X*. A *morphism* of bundles from $p\colon F \to X$ to $q\colon G \to X$ is a continuous map $h\colon F \to G$ such that $q \circ h = p$, that is, the following diagram commutes:

$$\begin{tikzcd} F \arrow[rr, "h"] \arrow[dr, "p"'] & & G \arrow[dl, "q"] \\ & X & \end{tikzcd} .$$

We have so defined a category of bundles over $X$.

A bundle $(F,p)$ over $X$ is called *étale* if $p\colon F \to X$ is a local homeomorphism. It is immediately seen that a *bundle of contextual meanings* $(\bigsqcup_{x\in X} \mathscr{F}_x, p)$ constructed as above is étale. Thus, for an admissible text $X$, we have defined the category $\mathbf{Context}(X)$ of étale bundles (of contextual meanings) over $X$ as a framework for the Frege's generalized contextuality principle at the level of text. For further details of contextuality modeling in a frame of sheaf-theoretic semantics, see (Prosorov, 2007).

# 9 Frege Duality

The fundamental theorem of topology states that there is a *duality* between the category of sheaves and the category of étale bundles (Tennison, 1975), (Lambek & Scott, 1986), (Mac Lane & Moerdijk, 1992) established by the pair of adjoint functors $\Lambda$ and $\Gamma$ defined in the following two subsections.

## Germ-functor $\Lambda$

For each sheaf $\mathscr{F}$, it assigns an étale bundle $\Lambda(\mathscr{F}) = (\bigsqcup_{x\in X} \mathscr{F}_x, p)$, where the projection $p$ is defined as above.

For a morphism of sheaves $\phi\colon \mathscr{F} \to \mathscr{F}'$, the induced map of fibers $\phi_x\colon \mathscr{F}_x \to \mathscr{F}'_x$ gives rise to a continuous map $\Lambda(\phi)\colon \bigsqcup_{x\in X} \mathscr{F}_x \to \bigsqcup_{x\in X} \mathscr{F}'_x$ such that $p' \circ \Lambda(\phi) = p$; hence $\Lambda(\phi)$ defines a morphism of bundles.

Given another morphism of sheaves $\psi$, one sees easily that $\Lambda(\psi \circ \phi) = \Lambda(\psi) \circ \Lambda(\phi)$ and $\Lambda(\mathrm{id}_{\mathscr{F}}) = \mathrm{id}_F$.

Thus, we have constructed a germ-functor $\Lambda\colon \mathbf{Schl}(X) \to \mathbf{Context}(X)$.

## Section-functor $\Gamma$

We denote a bundle $(F,p)$ over $X$ simply by $F$. For a bundle $F$, we denote the set of all its cross-sections over $U$ by $\Gamma(U,F)$.

For opens $U \subseteq V$, one has a restriction map $\mathrm{res}_{V,U} : \Gamma(V,F) \to \Gamma(U,F)$ which operates as $s \mapsto s|_U$, where $s|_U(x) = s(x)$ for all $x \in U$. It's clear that $\mathrm{res}_{U,U} = \mathrm{id}_{\Gamma(U,F)}$ for any open $U$, and that the transitivity $\mathrm{res}_{V,U} \circ \mathrm{res}_{W,V} = \mathrm{res}_{W,U}$ holds for all nested opens $U \subseteq V \subseteq W$. So we have constructed obviously a sheaf $(\Gamma(V,F), \mathrm{res}_{V,U})$.

For any given morphism of bundles $h\colon E \to F$, we have a map $\Gamma(h)(U)\colon \Gamma(U,E) \to \Gamma(U,F)$ defined as $\Gamma(h)(U)\colon s \mapsto h \circ s$, which is obviously a morphism of sheaves. Thus, we have constructed a desired section-functor $\Gamma\colon \mathbf{Context}(X) \to \mathbf{Schl}(X)$.

Transferred to linguistics (Prosorov, 2005a), the important duality between the category of sheaves and the category of étale bundles yields at the level of text the following

**Theorem (Frege Duality).** *There is natural duality of categories called* Frege Duality*:*

$$\mathbf{Schl}(X) \underset{\Gamma}{\overset{\Lambda}{\rightleftarrows}} \mathbf{Context}(X)$$

*established by the section-functor $\Gamma$ and the germ-functor $\Lambda$, which are the pair of adjoint functors.*

Due to the functional representation $s \mapsto \dot{s}$ of a fragmentary meaning $s \in \mathscr{F}(U)$, the Frege duality is of a great theoretical importance because it allows us to understand a fragmentary meaning $s \in \mathscr{F}(U)$ as a genuine continuous function $\dot{s}\colon x_i \mapsto \mathrm{germ}_{x_i} s$ which assigns to each sentence $x_i \in U$ its contextual meaning $\mathrm{germ}_{x_i} s$. It allows us to develop a kind of *inductive* or *dynamic theory of meaning* (Prosorov 2005b, 2008, 2011) describing how in reading of the text $X = (x_1, x_2, x_3 \dots x_n)$ the understanding process runs in a discrete time $i = 1, 2, 3 \dots n$ as a sequence of grasped contextual meanings $(\dot{s}(x_1), \dot{s}(x_2), \dot{s}(x_3) \dots \dot{s}(x_n))$ that gives a genuine function $\dot{s}$ on $X$ representing some $s \in \mathscr{F}(X)$ which is one of meanings of the whole text $X$ interpreted in the sense $\mathscr{F}$.

The formal definition of contextual meaning $\dot{s}(x_i)$ follows the idea of Wittgenstein that we understand meanings through usage and context; once a contextual meaning grasped, the reader transcends the latter. “He must so to speak throw away the ladder, after he has climbed up on it.” (Wittgenstein, 1922, 6.54).

# 10 Some Important Semantic Functors

In the general theory of sheaves, one uses the constructions of several important functors associated with a continuous map of topological spaces. We propose to consider these constructions in our sheaf-theoretic formal semantics.

## Induced Sheaf

Let $\mathscr{F}$ be a sheaf of fragmentary meanings over $X$ and $U \hookrightarrow X$ is the map of inclusion of open set $U$ into $X$. The map $V \mapsto \mathscr{F}(V)$ defined on the set of all opens $V \subseteq U$ is a sheaf of fragmentary meanings over $U$, the so-called *induced sheaf* over $U$ by a sheaf $\mathscr{F}$ over $X$, which is denoted by $\mathscr{F}|_U$.

For every morphism $\phi\colon \mathscr{F} \to \mathscr{G}$ of sheaves over $X$, we denote by $\phi|U$ the morphism $\mathscr{F}|_U \to \mathscr{G}|_U$ formed by $\phi|_V$ for $V \subseteq U$.

The functor $\mathscr{F} \to \mathscr{F}|_U$ is a particular case of *inverse image functor*: Let $f\colon X \to Y$ be a continuous map of texts endowed with phonocentric topologies. Then, for any sheaf $\mathscr{G}$ over $Y$, the *inverse image functor* $f^*$ defines a sheaf over $X$ denoted as $f^*\mathscr{F}$.

## Direct Image Functor

Let X and Y be two texts considered as topological spaces endowed with phonocentric topologies, and let $f\colon X \to Y$ be continuous map. Then, for any sheaf $\mathscr{F}$ on $X$, we obtain a sheaf on $Y$ denoted as $f_*\mathscr{F}$, and defined by setting:

$$(f_*\mathscr{F})(V) = \mathscr{F}(f^{-1}(V)) \quad \forall \text{ opens } V \text{ of } Y;$$
$$\mathrm{res}_{*W,V} = \mathrm{res}_{f^{-1}(W), f^{-1}(V)} \quad \forall \text{opens } V, W \text{ of } Y \text{such that } V \subseteq W.$$

In other words, $f_*\mathscr{F}$ is defined as the composition $\mathscr{F} f^{-1}$ of functors.

It is clear that we have defined a sheaf $f_*(\mathscr{F})$ over $Y$, named *direct image of $\mathscr{F}$ by $f$*. The correspondence $\mathscr{F} \mapsto f_*(\mathscr{F})$ defines a functor $f_*\colon \mathbf{Schl}(X) \to \mathbf{Schl}(Y)$ because $(f \circ g)_* = f_* g_*$ and $(\mathrm{id}_X)_* = \mathrm{id}_{\mathscr{F}}$.

### Examples of Direct Image Functors in Linguistic Situation

1. Let $Y$ be some text, $X$ be its glossed version, and $X \to Y$ be the map of glossing. The text $X$ is well understandable in the sense $\mathscr{F}$, that allows us to understand the text $Y$ by transferring $\mathscr{F}$ to $Y$, by means of $f_*$, i.e. by transferring to $Y$ the sheaf of fragmentary meanings over $X$. In fact, there are two sheaves over $Y$: a direct image $f_*\mathscr{F}$ of the sheaf $\mathscr{F}$, and a sheaf $\mathscr{G}$ of meanings

we have grasped in omitting glosses. The clarification of our understanding of an open part $V \subseteq Y$ consists of the fact that several elements of $\mathscr{G}(V)$ (vague fragmentary meanings) are got identified in some one element of $f_*\mathscr{F}(V)$. It results in a map $\phi(V)\colon \mathscr{G}(V) \to f_*\mathscr{F}(V)$ defined for all $V$, such that these $\phi(V)$ obviously commute with restriction maps $\mathrm{res}_{V,U}$. Thus, the following diagram commutes

$$\begin{CD} \mathscr{G}(V) @>{\phi(V)}>> f_*\mathscr{F}(V) \\ @V{\mathrm{res}'_{V,U}}VV @VV{\mathrm{res}_{*V,U}}V \\ \mathscr{G}(U) @>>{\phi(U)}> f_*\mathscr{F}(U)\,. \end{CD}$$

This means that $\phi = (\phi(V))_{V \in \mathfrak{O}(Y)}$ is a morphism of sheaves $\phi\colon \mathscr{G} \to f_*\mathscr{F}$. Such a morphism of sheaves is called *$f$-morphism*. Thus we have a morphism of couples $(f,\phi)\colon\ (X,\mathscr{F}) \to (Y,\mathscr{G})$ where $f\colon X \to Y$ is continuous map of texts and $\phi\colon \mathscr{G} \to f_*\mathscr{F}$ is a $f$-morphism of sheaves. These morphisms are composable in associative manner and the identical morphism defined as $(\mathrm{id}_X, (\mathrm{id}_{\mathscr{F}(U)})_{U \in \mathfrak{O}(X)})$ is obviously among them.

2. The passage $f\colon X \to A$ from some paper $X$ to its abstract $A$ that we considered above at syntactic level may be treated at semantic level as a morphism of couples $(f,\theta)\colon\ (X,\mathscr{F}) \to (A,\mathscr{G})$ made up of a continuous map $f\colon X \to Y$ and some $f$-morphism, $\theta\colon \mathscr{G} \to f_*\mathscr{F}$.

3. The passage from a first plan of some paper to its more detailed plan may be considered in the same manner.

# 11 Sheaf-Theoretic Formal Semantics

The above considerations motivate that the true object of study in text semantics of a natural language $E$ should be a *category of textual spaces* $\mathbf{Logos}_E$; its objects are couples $(X,\mathscr{F})$, where $X$ is a phonocentric topological space attached to a particular admissible text and $\mathscr{F}$ is a sheaf of fragmentary meanings defined over $X$; the morphisms are couples $(f,\theta)\colon\ (X,\mathscr{F}) \to (Y,\mathscr{G})$ made up of a continuous map $f\colon X \to Y$ of texts, and a natural transformation between sheaves, called $f$-morphism, $\theta\colon \mathscr{G} \to f_*\mathscr{F}$, where $f_*$ is a *direct image functor* (Prosorov, 2008).

Given an admissible text $X$ considered as fixed for study, it yields very naturally a subcategory of Schleiermacher $\mathbf{Schl}(X)$ in the category $\mathbf{Logos}$ of all textual spaces. This category $\mathbf{Schl}(X)$ describes the exegesis of some particular text $X$; its objects are couples $(X,\mathscr{F})$ and morphisms are are couples $(\mathrm{id}_X,\theta)$, where $\theta\colon\ \mathscr{F} \to \mathscr{G}$ is a morphism of sheaves.

# 12 Bundle-Theoretic Definition of Morphism

Let $(f,\theta)\colon\ (X,\mathscr{F}) \to (Y,\mathscr{G})$ be a morphism of textual spaces. For a given sentence $x \in X$, a natural transformation of sheaves $\theta\colon \mathscr{G} \to f_*\mathscr{F}$ induces a map $\theta(V)\colon \mathscr{G}(V) \to f_*\mathscr{F}(V)$ for every open neighborhood $V$ of $x$ in $Y$. When $V$ runs all open neighborhoods of $f(x) \in Y$, the $f^{-1}(V)$ runs a subset of the set of all open neighborhoods of $x$ in $X$. By passing to the inductive limit, we obtain a map $\mathscr{G}_{f(x)} = \varinjlim_{V \ni f(x)} \mathscr{G}(V) \to \varinjlim_{f^{-1}(V) \ni x} \mathscr{F}(f^{-1}(V))$. By virtue of the universality of the inductive limit, there is a map $\varinjlim_{f^{-1}(V) \ni x} \mathscr{F}(f^{-1}(V)) \to \varinjlim_{U \ni x} \mathscr{F}(U) = \mathscr{F}_x$, where $U$ runs through all open neighborhoods of $x$. We have so obtained an induced map of the corresponding fibers

$$\theta_x\colon \mathscr{G}_{f(x)} \to \mathscr{F}_x.$$

The family of maps $(\theta_x)_{x\in X}$ gives another (*bundle-theoretic*) definition of a textual spaces morphism $(f,\theta)\colon (X,\mathscr{F}) \to (Y,\mathscr{G})$.

### Inverse Image Functor

Let Y be a text considered as topological space endowed with a phonocentric topology, and let $\mathrm{inj}\colon U \hookrightarrow Y$ be the canonical injection of an open $U \subseteq Y$. In this particular case, for any sheaf $\mathscr{G}$ over $Y$, we obtain a sheaf over $U$ denoted as $\mathrm{inj}^*\mathscr{G}$, where the *inverse image functor* $\mathrm{inj}^*$ is simply defined by setting:

$$
\begin{aligned}
(\mathrm{inj}^*\mathscr{G})(V) &= \mathscr{G}(\mathrm{inj}(V)) && \text{for all opens } V \subseteq U;\\
\mathrm{res}^*{}_{W,V} &= \mathrm{res}_{\mathrm{inj}(W),\,\mathrm{inj}(V)} && \text{for all opens } V,W \text{ in } U \text{ such that } V \subseteq W.
\end{aligned}
$$

It is clear that in the case of injection $\mathrm{inj}_U\colon U \hookrightarrow Y$ of open $U$, the induced sheaf $\mathscr{G}|_U$ is equal to $\mathrm{inj}_U{}^*\mathscr{G}$.

In the general case of a continuous map $f\colon X \to Y$ of admissible texts endowed with phonocentric topologies, the *inverse image functor* $f^*$ is defined in (Prosorov, 2008, p. 151).

## 13 Quotation Analysis

Let us now analyze quotations in the frame of sheaf-theoretic formalism.

### Mention Type of Quotation

The reality is very manifold, and sometimes it is difficult to define verbally some object or situation, and we use then ostensive definition by pointing out the object. In this case, the reality itself, the object or situation, functions in the role of its proper name. One may use a textual label to stick it on the object like manufacturers of gadgets do it. But it is impossible for many other objects, as e.g. for planets, and we give them proper names like “Venus” or use their descriptions like “morning star”, or we use some images to illustrate the text.

The situation simplifies when we need to discuss some linguistic objects, which are eager to be represented in the text. We need only to distinguish somehow the language of linguistic objects under discussion from the language we use in the discussion (metalanguage). If these languages are different, the mistakes are easy to avoid.

But when these language-object and metalanguage are the same, we use some conventional typographic means like quote marks, italics, underlining, etc., to distinguish quotation of this type classified as *mention* from the text in which the discussion takes place.

Consider examples:

(1) a. “John and Mary” is a noun phrase.
 b. “John” and “Mary” are proper nouns.
 c. “John and Mary are happy together.” is a simple sentence in English.

In these sentences, predicative elements apply to linguistic objects, and the quoted materials are taken as referring to such objects. In these examples, the materials inside quotation marks are linguistic objects of different types. Nevertheless, at the semantic level of sentence, the quotation as a whole should be taken as one of primitive elements whose sequence constitutes the containing sentence. Recall that at the level of sentence, we consider its words as primitive elements.

So in (1c) the quoted object is itself a sentence, which takes a slot $x$ of subject in the whole sentence $X$. While reading the sentence inside quotation marks, a reader proceeds the usual process aiming to understand the sentence meaning; but the quotation marks compels to consider only linguistic properties of quoted sentence. Whence, at the level of the whole sentence (1c), the set $\mathscr{F}_x$ of possible contextual meanings of quotation $x$ consists of only linguistic properties of $x$.

Thus, in our approach, a quotation of mention type is treated as a case of ostensive presentation there the quotation marks are used to point out the quoted linguistic object as being its own reference. It seems to be close with demonstrative approach of Cappelen and Lepore (2012) and with Recanati's approach (2001) to what he calls *closed quotation*.

It should be noticed that all examples in (1) assume the ellipsis of a noun phrase in restrictive apposition which defines the linguistic type of quoted material.

So, the above examples may be completed by the first appositive:

(2) a. The expression "John and Mary" is a noun phrase.
    b. The words "John" and "Mary" are proper nouns.
    c. The sentence "John and Mary are happy together." is a simple sentence in English.

Here, each completed sentence presents some reconstruction of presumed original phrase what a reader bears in mind during the process of interpretation. The most possible context that is provided by the restored sentence $X$ during the reading of quoted expression $x$ consists of the interval $(x_1,\dots x)$, i.e., from the beginning till $x$. This interval contains not only expression $x$, but also mentioning of $x$. In effect, the contextual meaning of $x$ involves reference to linguistic properties of quoted expression. Presupposed here in ellipsis, the mentioning of $x$ as linguistic object appears in so-called mixed quotations as determined by some presuppositional context. In the case of mixed quotation, this phenomenon is analyzed in (Geurts & Maier, 2003).

## Quotation as Use

More interesting is the case of quotation classified as *use*, when a text $X$ contains a part $U'$ which coincides, as a sequence of sentences, with a part $U$ of another (source) text $Y$.

This type of quotation is very common in scientific texts and presupposes a dialogue with the author of quoted text $Y$ in order to analyze the content of a quoted fragment $U \subseteq Y$, or to support the arguments developed in the text $X$ by means of a quoted material. The origin of quotation $U'$ is cited in a list of references to publication, and the quotation itself is marked by conventional typographic agreement.

It is clear that $U$ should be a meaningful part of $Y$ as being worth of quotation. When $U'$ is a part of $X$ whose fragmentary meanings in $X$ are inherited from the meanings of the source part $U \subseteq Y$. Namely, for a given textual space $(Y,\mathscr{G})$, the canonical injection $U \subseteq Y$ of open $U$ induces a textual space $(U,\mathscr{G}|_U)$ which is natural to consider as subspace of $(Y,\mathscr{G})$. As we yet noticed, in the case of injection $\mathrm{inj}_U \colon U \hookrightarrow Y$ of open $U$, the induced sheaf is $\mathscr{G}|_U = \mathrm{inj}_U{}^*\mathscr{G}$.

Similarly, for a given textual space $(X,\mathscr{F})$, the canonical injection $\mathrm{inj}_{U'} \colon U' \hookrightarrow X$ defines a subspace $(U',\mathscr{F}|_{U'})$ of $(X,\mathscr{F})$, since the very fact of quotation implies that $U'$ is meaningful and so open in $X$.

The quotation of open part $U \subseteq Y$ gives rise to morphism of corresponding textual subspaces $(\mathrm{id},\theta)\colon (U,\mathscr{G}|_U) \to ((U',\mathscr{F}|_{U'}))$, where $\mathrm{id}\colon U \to U'$ is acting as $\mathrm{id}\colon x \mapsto x$, and $\theta$ is defined by maps of corresponding stalks $\theta_x\colon (\mathscr{F}|U')_{\mathrm{id}(x)} \to (\mathscr{G}|U)_x$ following the bundle-theoretical definition of textual spaces morphism for $U \overset{\mathrm{id}}{\to} U'$.

In the case when the senses $\mathscr{F}$ and $\mathscr{G}$ are of the same kind, (say moral, historical, etc.), we need to have $\theta_x = \mathrm{id}_{\mathscr{F}_x}$; namely, for all $x \in U$, each contextual meaning of $x$ grasped in the context of $X$ is the same as its contextual meaning grasped in the context of $Y$.

The situation is more complicated in the case of quoting out of context, when the cited part $U \subseteq Y$ is not open in $Y$. In this case, we need to consider inverse image $\mathrm{inj}_U{}^*\mathscr{G}$ instead of induced sheaf in order to define subspace $(U,\mathrm{inj}_U{}^*\mathscr{G})$ of $(U,\mathscr{G})$.

In this case, the transfer of meanings from $(U, \mathrm{inj}_U{}^*\mathscr{G})$ to $(X, \mathscr{F})$ by means of direct image functor $\mathrm{id}_*$ deduced from $U \stackrel{\mathrm{id}}{\to} U'$ may produce a fallacy of false understanding in which $y \in U$ is removed out of context in $Y$ that would corrupt the intended meaning of $y$ in the quotation $U' \subseteq X$.

Several types of quotation may be analyzed in the frame of sheaf-theoretic formal semantics. So, we may analyze this way the case of quotation within a quotation and others types of quotation interpreted as different compositions of direct image and inverse image functors defined on the category of textual spaces.

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