Sheaf-Theoretic Formal Semantics

OLEG PROSOROV

Petersburg Department of V. A. Steklov Institute of Mathematics Russian Academy of Sciences

prosorov@pdmi.ras.ru

We outline a sheaf-theoretic framework for a discourse interpretation theory developed in our works (1997-2005). The present sheaf-theoretic formal semantics provides a mathematical account of the text interpretation process while rejecting the attempts to codify interpretative practice as a kind of calculus.

We consider some unspecified Indo-European language, e.g., English, French, Russian, as a means of communication. We deal mostly with a written type of linguistic communication and so its basic units are texts. All the texts we consider are supposed to be written with good grace and intended for a human understanding; we call *admissible* the texts of this kind. A sentence is considered as a sequence of its words and a text as a sequence of its sentences. Likewise any part of a considered whole is simply a subsequence of a given sequence. Any mathematical structure is supposed to be defined on the *graph* of a corresponding sequence.

1 Basic Concepts

In the formal analysis of text understanding, we distinguish the semantic notions *sense, meaning* and *reference*. This triad of concepts formalize a certain distinction that seems to appear in various forms all over the history of language studies. To avoid the possible misunderstanding from the very beginning, we would like to make precise our usage of these key terms and to point out that our distinction *sense/meaning* differs from Frege's classic *Sinn/Bedeutung* distinction, whereas we accept *reference* to be an English translation of Frege's *Bedeutung*. Our aim is not to propose some competitive alternatives to Frege's *Sinn/Bedeutung* distinction but to find some adequate semantic concepts pertinent as instruments for the rigorous formal analysis of the text interpretation process where Frege's classic compositionality and contextuality principles are involved. However, one can find our distinction *sense/meaning* in the different usage of the word 'Sinn' in early writings by Frege before he had formalized the *Sinn/Bedeutung* distinction in his classic work of 1892. We consider *sense* and *meaning* as being the basic notions to be expressed by means of examples and descriptions and instead of their analysis in terms of more basic ones, we seek for key mathematical structures that underlie the process of discourse or text understanding.

We accept the term *fragmentary meaning* of some fragment of a given text to be the content which is grasped when the reader has understood this fragment in some particular situation of reading. But it depends on so many factors such as the personality of the reader, the situation of the reading, many kinds of presuppositions and prejudices summed up in the reader's attitude, etc., which we call *sense* or *mode of reading*; every reading is only an interpretation where the historicity of the reader and the historicity of the text are involved; thus in our usage, a *fragmentary meaning* is immanent not in a given fragment, but in an interpretative process of its reading. In our approach, the notion *sense* (or *mode of reading*) may be considered as a secular remake of the exegetical approach to this notion in the medieval theology. The Fathers of the Church have distinguished the four senses of Sacred Scripture: "Littera gesta docet, quid credas allegoria, moralis quid agas, quo tendas anagogia". In other words, our approach defines the term *sense* as a kind of semantic orientation in the interpretative process which relates to the totality of the message to understand, as some *mode of reading*. At the level of text, it may be

literal, allegoric, moral, eschatological, naïve, psychoanalytical, etc. At the level of sentence, it may be *literal* or *metaphoric (indirect)*.

In our approach, the reader grasps a *fragmentary meaning* in a particular interpretative process guided by some mode of reading or sense adopted in accordance with his attitude and based on the linguistic competence, which is rooted in the social practice of communication with others using the medium of language. Note that, following this terminology, we can read one and the same text in many different senses (moral, historical, etc.) to realize, in result, that we have grasped the different meanings. Likewise for a sentence. It seems that the usage of the key terms *sense*, *meaning* is in accordance with their everyday usage as common English words (likewise for the French terms sens and signification). As for the term sense, it should be mentioned that in French the word 'sens' literally equals 'direction' and as figurative it may be *littéral*, *strict*, *large*, *naif*, *bon*, *platonicien*, *leibnitzéen*, *frégéen*, *kripkéen*, etc. In English, in figurative usage, sense may also be *literal*, narrow, wide, naïve, common, Platonic, Leibnizian, Fregean, Kripkean, etc. In this usage, the term sense deals with the totality of discourse, text, expression or word and involves our subjective premises that what is to be understood constitutes a meaningful whole. In this usage, the term sense or mode of reading concerns the reader's interest in the subject matter of the text; it is a kind of questioning that allows a reader to enter into a dialogue with the author. So our usage of the term sense as a mode of reading is near to that proposed by the exegetic concept of the four senses of the Sacred Scripture, whereas our usage of the term *fragmentary meaning* as the content grasped in some particular situation of reading corresponds rather to the common usage of ordinary English words.

But this fragmentary meaning should not be understood as some mental state of the reader because the mental states of two readers could neither be identified, nor compared in some reasonable way; in contrast, our approach is based on the explicit criterion of equality between the fragmentary meanings we shall formulate later. In our usage, the term *fragmentary meaning* should not be understood in the Tarski/Montague style as the relation between words and world; nor should it be related to any kind of truth-value or truth-conditions because the understanding of e.g. novels or science fictions is achieved regardless of any assumption about verifiability. The understanding of meaning and the knowledge of truth both relate to the world, but in different ways. We observe that a meaning s of some fragment U of a given text X is understood by the reader as an objective result of interpretation of this passage U; its 'objectivity' carries no claim of correspondence to reality, but is grounded in the conviction that this meaning s may be discussed with anybody in some kind of dialogue (actual or imaginary) where such a meaning s may be finally shared by the participants or may be compared with any other meaning t of the same fragment U. We shall later formulate the criterion for such a comparison procedure as some definition of equality (S). This kind of objectivity of meaning is based not only on the shared language, but principally on the shared experience as a common life-world and it deals so with the reality. According to Gadamer, this being-with-each-other is a general building principle both in life and in language. The understanding results from being together in a common world. This understanding as a presumed agreement on 'what this fragment U wants to say' becomes for the reader its meaning s. In this usage, the meaning of an expression is the content that the reader grasps when he understands it; and this can be done regardless of the ontological status of its reference. The process of coming to some fragmentary meaning s of a fragment Umay be thought of as an exercise of the human capacity of naming and understanding; it is a fundamental characteristic of human linguistic behavior.

2 Phonocentric Topology

The reading of text as well as the utterance of discourse is always a process that develops in time, and so it inherits in some way its order structure. From a linguistic point of view, this order structure is known as a notion of *linearity* or that of *word order*. At the level of text, it is a natural linear order \leq of sentences reading the text bears on. It is well-known that any order structure carries several standard topological structures (Erné, 1991) but it isn't a question to graft some topology onto a given text. We argue that any admissible text has an underlying topological structure which arises quite naturally.

In the process of reading, the understanding is not postponed until the final sentence of a given text. So the text should have the meaningful parts and the meanings of these parts determine the meaning of the whole as it is claimed by the principle of *hermeneutic circle*. A central

task of any semantic theory is to explain how these local understandings of the constitutive meaningful parts produce the global understanding of the whole. Whereas a description of some mathematical structure in terms of these constitutive meaningful parts may be treated as a kind of syntactic theory concerned with a considered semantic level. The philological investigations are abound in examples of meaningful fragments quoted from the studied texts. Thus a meaningful part might be a subject of comment or discussion for being considered as worth interpretation. Certainly, not each subsequence of a given text is meaningful, but some meaningful fragment becomes to be understood in the process of reading and rereading. It seems to be quite in agreement with our linguistic intuition that:

(i) an arbitrary union of meaningful parts of an admissible text is meaningful;

(ii) a non-empty intersection of two meaningful parts of an admissible text is meaningful.

For an admissible text is supposed to be meaningful as a whole by definition, it remains only to define the meaning of its empty part (e.g. as a one-element set denoted as usually pt) in order to provide it with some *topology* in a strict mathematical sense, where the open sets are all the meaningful parts. Thus any admissible text may be endowed with a *semantic topology* where the open sets are defined to be all its meaningful parts (Prosorov, 2004). In the following, we often use the term *fragment* as equivalent to that of *open subset* in the case of topological space related to text. Now, however, the question arises, what formal criteria would be given for the meaningfulness of a part $U \subset X$? The concepts of *fragmentary meaning* and *meaningful fragment* are closely related, for they should come together in the matter of natural language text understanding. We have noted at the very beginning, that our theory concerns only the texts referred to as *admissible*, which are supposed to be written for a human understanding. So the meaningful parts are supposed to be those which are intended to convey the communicative content. Therefore, an admissible text should respect good order and arrangement, as each part ought to fall into its right place; because the natural process of reading (from right to left and from top to bottom) supposes that understanding of any sentence x of the text X should be achieved on the base of the text's part already read, because the interpretation cannot be postponed, although it may be made more precise and corrected in further reading and rereading. This is a fundamental feature of a competent reader's linguistic behavior. So the ordinary reading process inherits a natural temporality of phonetic phenomena; it's a reason to call this kind of semantic topology as *phonocentric*. Thus for every pair of distinct sentences x, y of an admissible text X, there exists an open (i.e. meaningful) part of X that contains one of them (to be read first in the natural order \leq of sentences) and doesn't contain the other. Hence the admissible text endowed with the phonocentric topology should satisfy the separation axiom T_0 of Kolmogoroff and so it is a T_0 -space. This characteristic might be posed as a formal definition distinguishing the phonocentric topology between the other semantic topologies. According to our conceptual distinction senselmeaning, we consider sense as a kind of semantic orientation in the interpretative process which relates to the totality of message to understand. Thus we suppose that any part $U \subset X$ which is meaningful in one sense (or mode of reading) should remain meaningful under the passage to some another sense (or mode of reading). It should be noticed that another concept of meaning or criteria of meaningfulness would imply another definition of meaningful fragments and so will define yet another type of *semantic topology*.

Let X be an admissible text. For a sentence $x \in X$, we define U_x to be the intersection of all the meaningful parts that contain x. In other words, for a given sentence x, the part U_x is a smallest open neighborhood of x. It is clear that $x \in U_y$ if and only if $y \in cl(\{x\})$, where $cl(\{x\})$ denotes the closure of the one-element set $\{x\}$. This relation 'x is contained in all open sets that contain y' is usually called a *specialization*, and some authors denote it as $y \leq x$ or $y \leq x$ (e. g. Erné, 1991, p. 59) contrary to others who denote it as $x \leq y$ or $x \leq y$ (e. g. May, 2003, p. 2). As for the notation choice, we follow rather (May, 2003) to define a relation \leq on the text X by setting $x \leq y$ if and only if $x \in U_y$ or, equivalently, $U_x \subset U_y$. Note that in this notation, for all $x, y \in X, x \leq y$ implies that $x \leq y$, where \leq defines the natural order of sentences reading.

Proposition. The set of all open sets of the kind U_x is a basis of a phonocentric topology on X. Moreover, it is the unique minimal basis of a phonocentric topology. The phonocentric topology on an admissible text defines a partial order structure \leq on it by means of specialization; the initial phonocentric topology can be reconstructed from this partial order \leq in a unique way.

These considerations may be repeated with a slight modifications in order to define a phono-

centric topology at each semantic level of a given admissible text. At each level (text, sentence), we distinguish its *primitive elements* which are the points of corresponding topological space considered to be the *whole* at this level. The passage from one semantic level to another immediately superior consists in gluing of the whole space into a point of the higher level space.

As soon as we have defined a phonocentric topology, we may seek to interpret some linguistic notion in the topological terms and then to study it by the topological means.

In the mathematical order theory, there exists a simple intuitive tool for the graphical representation of finite partially ordered set (*poset*), called Hasse diagram (Stanley, 1986). For a poset (X, \preceq) , the *cover relation* \prec is defined by: ' $x \prec y$ if and only if $x \preceq y$ and there exists no element $z \in X$ such that $x \preceq z \preceq y'$. In this case, we say that *y covers x*. For a given poset (X, \preceq) , its Hasse diagram is defined as the graph whose vertices are the elements of *X* and whose edges are those pairs $\{x, y\}$ for which $x \prec y$. In the picture, the vertices of Hasse diagram are labeled by the elements of *X* and the edge $\{x, y\}$ is drawn by an arrow going from *x* to *y* (or sometimes by an indirected line, but in this case the vertex *y* is displayed lower than *x*).

The usage of some kind of Hasse diagram under the name of Leitfaden is widely spread in the mathematical books to facilitate the understanding of logical dependence of the chapters. The poset considered in this usage is always constituted not of all sentences but of all chapters of the book. So, in the introduction to (Serre, 1979) is written: "The logical relations among the different chapters are made more precise in the *Leitfaden* below." and there is the following Hasse diagram:



We cite yet another example of Hasse diagram from (Manin, 1977), where it appears under the title of "Interdependence of Chapters":



These two Leitfadens, as many other their examples, surely presuppose the linear reading of paragraphs within each chapter. Thus, they may be "split" in order to draw the corresponding Leitfadens whose vertices are all the paragraphs, and so on. Given an admissible text, one can, by means of analytical reading or perhaps with the help of the author, define its phonocentric topology at the level of text (find all the basis sets U_x) and then draw the Hasse diagram of the corresponding poset. Certainly, the author has some clear representation of this kind during the writing process. Anyhow, the representations of this kind appear implicitly during the reading process at each semantic level.

3 Sheaves of Fragmentary Meanings

Let X be an admissible text, and let \mathscr{F} be an adopted *sense* or *mode of reading*. For a given fragment $U \subset X$, we collect all the fragmentary meanings of U in the set $\mathscr{F}(U)$. Thus we are given a map $U \mapsto \mathscr{F}(U)$ defined on the set $\mathscr{O}(X)$ of all opens $U \subset X$. Formulated not only for the whole text X but more generally for any meaningful part $V \subset X$, the precept of the

hermeneutic circle 'to understand any part of text in accordance with the understanding of the whole text' defines a family of maps $\operatorname{res}_{V,U}: \mathscr{F}(V) \to \mathscr{F}(U)$, where $U \subset V$, such that $\operatorname{res}_{V,V} = \operatorname{id}_{\mathscr{F}(V)}$ and $\operatorname{res}_{V,U} \circ \operatorname{res}_{W,V} = \operatorname{res}_{W,U}$ for all nested opens $U \subset V \subset W$. From a mathematical point of view, the data $(\mathscr{F}(V), \operatorname{res}_{V,U})_{V,U \in \mathscr{O}(X)}$ is a *presheaf* of fragmentary meanings over *X*.

The reading process of a given fragment U is modeled as its (open) covering by some family of subfragments $(U_i)_{i \in J}$, where each U_i is supposed to be read in a distinct physical act.

According to Quine, there is no entity without identity. The definition of equality that seems to be quite adequate to our linguistic intuition is posed by the following:

Claim S (Separability). Let X be an admissible text, and let U be a fragment of X. Suppose that $s,t \in \mathscr{F}(U)$ are two fragmentary meanings of U and there is an open covering $U = \bigcup_{j \in J} U_j$ such that $\operatorname{res}_{U,U_i}(s) = \operatorname{res}_{U,U_i}(t)$ for all fragments U_j . Then s = t.

Thus an adopted sense (or mode of reading) of an admissible text X determines really some *separated presheaf* \mathscr{F} of fragmentary meanings. Following the precept of the hermeneutic circle 'to understand the whole text by means of understandings of its parts' this separated presheaf \mathscr{F} should satisfy the following:

Claim C (**Compositionality**). Let X be an admissible text, and let U be a fragment of X. Suppose that $U = \bigcup_{j \in J} U_j$ is an open covering of U; suppose we are given a family $(s_j)_{j \in J}$ of fragmentary meanings, $s_j \in \mathscr{F}(U_j)$ for all fragments U_j , such that $\operatorname{res}_{U_i, U_i \cap U_j}(s_i) = \operatorname{res}_{U_j, U_i \cap U_j}(s_j)$. Then there exists some meaning s of the whole fragment U such that $\operatorname{res}_{U, U_j}(s) = s_j$ for all U_j .

A separated presheaf satisfying the claim (C) is called a *sheaf*. It imposes the following: **Definition (Frege's Generalized Compositionality Principle).** A separated presheaf of fragmentary meanings naturally attached to any sense (mode of reading) of an admissible text is really a sheaf; its sections over any fragment of the text are the fragmentary meanings; its global sections are the meanings of the whole text.

Recall that the elements of $\mathscr{F}(U)$ are usually called *sections over* U. We note that the claim (S) guarantees the meaning s, whose existence is claimed by (C), to be unique as such. A *morphism* of sheaves $\phi : \mathscr{F} \mapsto \mathscr{F}'$ over the same text X is a family of maps $(\phi(V))_{V \in \mathscr{O}(X)}$,

A morphism of sheaves $\phi : \mathscr{F} \mapsto \mathscr{F}'$ over the same text X is a family of maps $(\phi(V))_{V \in \mathscr{O}(X)}$, where each $\phi(V) : \mathscr{F}(V) \to \mathscr{F}'(V)$ represents a transfer from the understanding of V in the sense \mathscr{F} to its understanding in the sense \mathscr{F}' which is compatible with the restriction maps, i.e. $\phi(U) \circ \operatorname{res}_{V,U} = \operatorname{res}'_{V,U} \circ \phi(V)$ for all $U \subset V$.

Thus, given an admissible text X, the data of all sheaves \mathscr{F} of fragmentary meanings together with all its morphisms constitutes a category **Schl**(X), called *category of Schleiermacher*. This category supplies a mathematical framework for the part-whole structure in the text understanding formulated by Schleiermacher as the theoretical principle of *hermeneutic circle*.

4 Contextuality

So far, we have defined only a notion of fragmentary meaning. To consider at each semantic level not only the meanings of fragments but also the meanings of points of a corresponding topological space, we define a notion of *contextual meaning*. Let U, V be two neighborhoods of x and let \mathscr{F} be an adopted sense. Two fragmentary meanings $s \in \mathscr{F}(U)$ and $t \in \mathscr{F}(V)$ are said to induce the same contextual meaning at x if there exists some smaller open neighborhood W of x, such that $W \subset U \cap V$ and $\operatorname{res}_{U,W}(s) = \operatorname{res}_{V,W}(t) \in \mathscr{F}(W)$. This relation "induce the same contextual meaning at x" is an equivalence relation, and any equivalence class of fragmentary meanings agreeing in some neighborhood of x is called a *contextual meaning* of x. The set of all equivalence classes is called a *stalk* of \mathscr{F} at x and denoted by \mathscr{F}_x . The equivalence class of a fragmentary meaning $s \in \mathscr{F}(U)$ in \mathscr{F}_x is called the *germ of s at x* and denoted by $\operatorname{germ}_x s$. Recalling the construction of *inductive limit*, we postulate at the level of text the following

Definition (Frege's Generalized Contextuality Principle). A sentence x within a fragment U of an admissible text X has a contextual meaning defined as the germ at x of some fragmentary meaning $s \in \mathscr{F}(U)$, where the sheaf \mathscr{F} is the adopted sense (mode of reading); the set \mathscr{F}_x of all contextual meanings of a sentence $x \in X$ is defined as the stalk of \mathscr{F} at x, i. e. as the inductive limit $\mathscr{F}_x = \varinjlim(\mathscr{F}(U), \operatorname{res}_{U,V})_{U,V}$ neighborhoods of x.

The contextuality principle proposed above is an explicit definition of contextual meaning for a given sentence at the semantic level of text. The similar definition may be formulated at each semantic level. This one formulated at the level of sentence renders Frege's classic contextuality principle. As soon as the semantic level is fixed, the definition of a contextual meaning for a point x a corresponding topological space X is given as germ_xs, where s is some fragmentary meaning defined on some neighborhood U of x.

According to a well-known *inductive limit characterizing theorem* (Tennison, 1975, th. 3.8, p. 5), this contextuality principle stated at the level of text is equivalent to the conjunction (Ct)&(E) of two claims (Ct) and (E) formulated in (Prosorov, 2003). The claim (Ct) is a generalization in the narrow sense of the Frege's classic contextuality principle; it may be paraphrased as "ask for the meaning of a sentence only in the context of some fragment of a given text". The claim (E) is an *explicit criterion of equality* between contextual meanings of a given sentence in the context of a given text. Stated explicitly, the notion of contextual meaning allows, for any admissible text X, to define the category **Context**(X) of étale bundles of contextual meanings over X as a framework for the generalized contextuality principle at the level of text.

5 Frege Duality

The fundamental theorem of topology states that the section-functor Γ and the germ-functor Λ establish a *dual adjunction* between the category of presheaves and the category of bundles (over the same topological space); this dual adjunction restricts to a *dual equivalence* of categories (or *duality*) between corresponding full subcategories of sheaves and of étale bundles (Lambek & Scott, 1986, p. 179; Mac Lane & Moerdijk, 1992, p. 89). In the linguistic situation, this important result yields the following:

Theorem (Frege Duality). *The generalized compositionality and contextuality principles are formulated in terms of categories that are in natural duality*

$$\mathbf{Schl}(X) \xrightarrow[\Gamma]{\Lambda} \mathbf{Context}(X)$$

established by the section-functor Γ and the germ-functor Λ which are the pair of adjoint functors.

Obtained by the same reasoning as many of well-known classic dualities such as Stone, Gelfand-Naimark, and Pontrjagin-van Kampen ones, Frege Duality gives rise to the functional representation of fragmentary meanings at each semantic level that permits to establish an *inductive theory of meaning* (Prosorov, 2004, 2005) describing how runs the process of text understanding. At the level of sentence, the same considerations generalize the classic Frege's compositionality and contextuality principles, but with words as primitive elements and syntagmas as meaningful fragments.

6 Sheaf-Theoretic Semantics

Thus the true object of study in the natural language semantics should be a pair (X, \mathscr{F}) , i. e. a text with a sheaf of its fragmentary meanings; any such a couple is called a *textual space*. But this representation is possible only in the realm of a language following the famous slogan of Wittgenstein "to understand a text is to understand a language". Rigorously, this claim may be formulated in the frame of category theory. Likewise the present sheaf-theoretic formal semantics describes a natural language in the *category of textual spaces* **Logos**. The objects of this category are couples (X, \mathscr{F}) , where X is a topological space attached naturally to an admissible text and \mathscr{F} is a sheaf of fragmentary meanings defined on X; the morphisms are couples $(f, \theta): (X, \mathscr{F}) \to (Y, \mathscr{G})$ made of a continuous map $f: X \to Y$ and an f-morphism θ which respects the given sheaves, i.e. $\theta: \mathscr{G} \to f_* \mathscr{F}$, where f_* is a well-known *direct image* functor. All these notions are discussed at length in our works (2001-2005).

Given any admissible text X considered as fixed forever, it yields very naturally a full subcategory Schl(X) in the category Logos of all textual spaces. This category of Schleiermacher Schl(X) describes the situation when the reader is interested in the exegesis of some particular text as, for example, Sacred Scripture.

References

- Erné, M. (1991). The ABC of order and topology. In H. Herrlich & H.-E. Porst (Eds.), *Research and exposition in mathematics Vol. 18. Category theory at work* (pp. 57–83). Berlin: Heldermann.
- Lambek, J., & Scott, P. S. (1986). *Introduction to higher order categorical logic*. Cambridge: Cambridge University Press.
- Mac Lane, S., & Moerdijk, I. (1992). *Sheaves in geometry and logic. A first introduction to topos theory*. New York: Springer-Verlag.
- Manin, Y. I. (1977). A course in mathematical logic. New York Heidelberg Berlin: Springer-Verlag.
- May, J. P. (2003). *Finite topological spaces*. Retrieved January 31, 2005, from http://www.math .uchicago.edu/~may/MISC/FiniteSpaces.pdf. (Notes for REU)
- Prosorov, O. (1997). *Critique de la raison herméneutique. Esquisse d'une herméneutique formelle* [Critique of hermeneutical reason. Outline of formal hermeneutics]. Unpublished master's thesis, Collège universitaire français, St. Petersburg.
- Prosorov, O. (2001). Esquisse d'une herméneutique formelle [Outline of formal hermeneutics]. *Echos du Collège : Dialogue franco-russe*, 2, 9–29.
- Prosorov, O. (2002). *Herméneutique formelle et principe de Frege généralisé* [Formal hermeneutics and Frege's principle generalized]. Available from the Web site on texts semantics *texto* ! http://www.revue-texto.net/Inedits/Prosorov_Principe.pdf. Paris. (Published with the participation of Ferdinand de Saussure Institute)
- Prosorov, O. (2003, Mar 12). Formal hermeneutics and Frege duality (PDMI preprint No. 5/2003). St. Petersburg: Steklov Mathematical Institute. Available from Steklov Mathematical Institute Web site, ftp://ftp.pdmi.ras.ru/pub/publicat/preprint/2003/05-03 .ps.gz.
- Prosorov, O. (2004, Jun 7). Compositionnalité et contextualité, deux principes de Frege en adjonction [Compositionality and contextuality, two Frege's principles in adjunction] (PDMI preprint No. 8/2004). St. Petersburg: Steklov Mathematical Institute. Available from Steklov Mathematical Institute Web site, ftp://ftp.pdmi.ras.ru/pub/publicat/preprint/2004/ 08-04.ps.gz.
- Prosorov, O. (2005, Apr). Formal hermeneutics based on Frege duality. In E. Maier, C. Bary, & J. Huitink (Eds.), *Proceedings of SuB9* (pp. 286–298). Nijmegen: NCS.
- Serre, J.-P. (1979). *Local fields* (M. J. Greenberg, Trans.). New York: Springer-Verlag. (Original work published 1962)

Stanley, R. P. (1986). Enumerative combinatorics, Vol. 1. Monterey, CA: Wadsworth & Brooks/Cole.

Tennison, B. R. (1975). Sheaf theory. Cambridge: Cambridge University Press.