Mathematical Foundations of Fluid Mechanics

Tim Shilkin

General information

- Course title: Mathematical Foundations of Fluid Mechanics
- Lector: Prof. Dr. Tim Shilkin
- Contacts: by e-mail tim dot shilkin at gmail dot com
- Course credits: 4,5 LP
- Course language: English
- Course site: http://www.pdmi.ras.ru/∼shilkin/
- Course format: on-line lectures
- Course schedule: lectures on 19.12, 20.12, 21.12, 22.12, 23.12 (part I);
  lectures and practical classes in January 2023 (TBA, part II)
- Time of lectures: 10:15-11:45 am (daily)
- Link to a zoom conference: please, send a request via e-mail in 'Contacts'
- Final attestation: oral exam
- Homework: several non-obligatory home assignments during the semester, but problems from these home assignments will be included in the final exam as additional questions

Syllabus

One of the goals of this course is to introduce the audience to the formal derivation of such famous models as the Navier-Stokes and the Euler equations from basic principles of fluid mechanics. The Navier-Stokes and the Euler equations for incompressible fluids are considered to be among the most interesting and challenging models in the modern PDE theory. The problem of the global existence of smooth solutions to the 3D Navier-Stokes equations was proposed by the Clay Institute as one of the "Millennium problems".

Another (informal) goal of the course is to provide students with better understanding and the physical interpretation of various facts and concepts traditionally included in the course of Calculus II (vector analysis). We believe that the physical background (which is basically *common sense*) provides students with a major in mathematics with a much better understanding of purely mathematical concepts and develop their intuition in problem analysis.

On the other hand, for students with a major in physics the interpretation of some physical theory from a formal axiomatic point of view and mathematical analysis of the well-posedness of a physical model turns out to be an extremely powerful method in modern physics. Many breakthroughs that led to the creation of fundamentally new physical theories were initially inspired by the purely mathematical analysis of the problem. We can just mention Maxwell's theory of electromagnetic waves, the prediction of positrons based on the analysis of the Dirac equations, the prediction of the Higgs boson, gravitational waves and much more. Knowledge of the mathematical apparatus of vector and tensor analysis, as well as methods for studying PDEs, are necessary skills for any modern physicist.

Information about practical classes

Besides theoretical lectures these course includes several practical classes. These classes will be focused on the mastering by students of the mathematical apparatus of vector and tensor analysis which is commonly used in continuum mechanics. Students will study or improve their skills on vector analysis in both in Cartesian and non-Cartesian (polar, cylindrical and spherical) coordinate systems. Several problem sheets will be assigned during the semester. These homework is non-obligatory, but problems from these home assignments will be included in the final exam as additional questions. Solutions (or hints to solutions) to most of homework problems will be discussed during the practical classes. In the second part of the course we are going to analyze some particular types of flows both for viscous and inviscous incompressible fluids.

Information about attestation for this course

The attestation for this course is based on the final exam. The final exam will be held in oral form (in English) at the end of the course (at the end of February or in the beginning of March 2023). Students well be asked to give a detailed answer on one of questions on the theory covered during the semester. The list of exam questions will be announced at the course site not later than in two weeks before the exam. To receive the highest grade students will be asked also to solve one of the homework problems. In the case of failure with a practical problem the grade is not higher than 2. In the case of minor gaps in the answer on the theoretical question the partial grading is possible (i.e. 1.3, 2.7 etc).

Plan of lectures, part I

- 19.12.22 Overview of tools from vector analysis.
- 20.12.22 Kinematics of continuum media.
- 21.12.22 Fundamental laws of fluid dynamics.
- 22.12.22 Rheological relations. Newtonian and non-Newtonian fluids.
- 23.12.22 Review of the theory. Solutions to selected problems.