
H. Beirão da Veiga

Dear Friends,

In January 2019 I received a very kind message from our colleagues Darya Apushkinskaya and Alexander Nazarov announcing that, in view of the upcoming 100th anniversary of the birth of Olga Alexandrovna Ladyzhenskaya, they were collecting memoirs about her, hoping to publish them as a book. The goal of the project was not to write a biography, or a description of scientific achievements, but essentially to gather personal impressions of those who knew Olga Ladyzhenskaya in various periods of her life. The message included an invitation to write a contribution to the book. I decided to insert this preface, extracted from their nice invitation letter, for future memory inside the book, and also to give me the possibility of thanking Darya and Alexander for this nice initiative and for the work that they are doing to publish the volume.

Clearly, I was very honored and glad to accept this challenging proposal since I was bound to Olga Ladyzhenskaya by a very strong, old friendship. Fortunately, the invitation included the possibility of reporting facts included here, that are not at all chronological and not particularly organized. At least that's what I deduced, which reassured me a lot. In fact, I'm not famous for a steel memory, I rarely take notes, and I keep photos and written memories in a somewhat chaotic manner. So, I hope to be forgiven in advance.

Olga Ladyzhenskaya was an enchanting person, sometimes rigorous and authoritarian, very sweet in other circumstances. Work and friendships were separate topics, even if they were treated at the same time. Let me give some examples. Our personal acquaintance began when I was a professor in Trento (1976-87). To mention Trento immediately makes me take a leap forward in time, to June 1991 (by that time I was already in Pisa), to recall the CIRM Conference “Partial Differential Equations and Continuum Mechanics”, organized by me and my friend Paolo Secchi, in which Olga played a central role. This memory allows me to include a beautiful photograph (see Photo 1).

I take advantage of this reference to CIRM to leap forward to the conference “Partial Differential Equations in Mathematical Physics – In memory of Olga A. Ladyzhenskaya”, October 2004, this time held in Levico Terme, which I organized with the collaboration of G.A. Seregin, V.A. Solonnikov, N.N. Uraltseva and A. Valli. Again, I include a beautiful photograph (see Photo 2).

I can't talk about Olga without reporting my relationship with the city of St. Petersburg. It is one of the most beautiful and fascinating cities in the World, and I am bound to it by an extraordinary affection. St. Petersburg means LOMI, V.A. Steklov, and Fontanka 27. I have visited this wonderful city several times, but the first time, for better or worse, one never forgets. It was 1992 and I was, as usual, with my wife Iaia (Anna Flavia). Olga teamed me up with Seregin's wife Helena, a very good and safe driver. We had bureaucratic problems upon entering Russia. (I don't recall if they were related to visas or luggage). In any case, Olga's help was providential and welcome.



Photo 1: June 3-7, 1991. Levico Terme (Trento).

People sitting, from the left: Sergey T. Simakov, R. Bruce Kellog, Peter Laurence, Carlo Cercignani, Carlo Marchioro, John A. Nohel, Olga A. Ladyzhenskaya, Anna Zaretti, Tiziana Collini, Maria Luisa Mascarenhas, Hugo Beirão da Veiga, David G. Ebin, Robert Turner, Ling Hsiao, Kyuya Masuda, Olivier Gues, Antonin Novotny, Norbert Ortner.

People standing, from the left: Yoshihiro Shibata, Stefano Panizzi, Enrico Vitali, Giovanni Prouse, Moses A. Boudourides, Sabine Gellrich, Vsevolod A. Solonnikov, Grzegorz Lukaszewicz, Josef Bemelmans, Bruno Salvatore Rubino, Pierangelo Marcati, Juan R. Esteban, Emanuele Callegari, Alfredo Marzocchi, Paolo Maremonti, José Francisco Rodrigues, Josef Malek, Paolo Secchi, Alberto Arosio, Klaus Schilling, Li Ta Tsien.

Remark: Vladimir V. Shelukhin, Hisao Fujita Yashima, Franco Brezzi, Sergio Spagnolo, and Roger Temam also attended this conference, but they are not in the photograph.

I would like to include a copy of one of the many letters exchanged with Olga at that time (see Photo 3). This letter seems particularly interesting to me because, as in other letters, Olga deals with emotional and personal problems along with proposals for work on open problems. However, note the difference in styles in the two cases.

From that time, I became strongly tied to the Steklov (Fontanka 27), for me a myth. I have been invited to visit some of the most famous world mathematical institutions and have been honored by the invitations. These institutions made a strong impression on me, an objective marvel at their power and influence on science. But, at the time of my first visit in 1992 to the “old” Steklov, the whole scene left me with a sort of dreamlike reaction one can have as a child when encountering something new and marvelous. It appeared to me like an artisanal laboratory of a mathematics of the highest level, in continuous

sympiosis with the other sciences, giving and receiving in a fruitful, continuous exchange. I imagined that some of the former frequenters of that venerable institution, so dynamic as to persist outside of time, had come back and were there with me after many years of absence. I could imagine one of the old, pioneering mathematicians at home in the charm of an almost untouched physical environment. At least this was my impression in that distant 1992. I see in this old building a historical memory of a mathematics germinated in the fertile soil of fundamental examples, nurtured with the introduction of new ideas, abstract when necessary, but always reaching for fitting examples, a measure of their real importance. In short, I have a strong memory of the role of first magnitude of the Russian school of mathematics, in particular of the outstanding contribution of Olga A. Ladyzhenskaya.



Photo 2: October 24-30, 2004. Levico Terme (Trento).

People sitting, from the left: Okihiro Sawada, Dongho Chae, Chiara Bottero, Mariarosaria Padula, Claude Bardos, Alexander Nazarov, Yoshikazu Giga, John Heywood, Paolo Secchi.

2nd row: Augusto Micheletti, Hiroko Morimoto, Vivette Girault, Gregory Seregin, Nina Uraltseva, Hugo Beirão da Veiga, Alberto Valli, Adelia Sequeira, Arina A. Arkhipova.

3rd row: Michael Struwe, Kyuya Masuda, Giuseppe Da Prato, Mariano Giaquinta, Sergey Nazarov, Herbert Amann, Carlo Pagani, Vladislav V. Pukhnachev, Endre Suli, Abdolrahman Razani, Alex Mahalov, Basil Nicolaenko.

Last row: Marcello Guidorzi, Stephan Luckhaus, Reimund Rautmann, Hermano Frid, Michela Eleuteri, Antonio Fasano, Atusi Tani, Christian Simader, Timofey Shilkin, Joachim Naumann, Hi Jun Choe, Tetsuro Miyakawa, Pierangelo Marcati.

In the very back: Josef Malek, Luca Pavarino, Luigi Berselli, Max Gunzburger.

Remark: Eduard Feireisl, Gianni Gilardi, Mario Miranda, Francesco Mollica, Alessandro Morando, Giovanni Prouse, Kumbakonam R. Rajagopal, Tommaso Antonio Ruggeri, Michael Ruzicka, Vsevolod Solonnikov, Luciano Tubaro, Augusto Visintin, and Lihe Wang also attended this conference, but they are not in the photograph.

20 rue de Rivoli
Boulet de Monvel

Paris, le

12. 11. 1991

0033-

Equipe de Physique mathématique et Géométrie

Dear Hagen, I am living in the heart of Paris, in a nice quiet apartment, not far from University VII and VI. My home address - rue de Rivoli 20; home telephone (1) 48.87.81.40. As a rule I am home in the mornings and in the evenings. The period May 15-30 of 1992 is good for your visit. I think that it will be interesting for Iain to see Saint-Petersburg with you together. So I invite you both to come and live in my apartment.

It is very modest but it replaces in the old part of city not far from many famous places. When you will write to A.V. Ivanov give him to know that you want to come with the wife and therefore it is convenient for you to have the corresponding invitation (I am not sure that it is necessary to mention the name of your wife in the invitation but may be it will be useful in some aspects).

I am sending to you my small book on Attractors. But most part of it is devoted to abstract semi-groups and there are very few facts concerning the Navier-Stokes eqs. For the latter it is better to see my first publication [1] 1972 referenced in the book.

It would be very nice to see you here, in Paris and to discuss different mathematical problems.

I want to formulate some of them:

1) It is interesting to prove that for the solution u of the problem (1) $\Delta u = f$, $u|_{\partial\Omega} = 0$, $\Omega \subset \mathbb{R}^2$ (Ω -bounded) the derivatives u_x belong to $L^{\frac{2r}{2-r}}(\Omega)$ and

$$(2) \quad \|u_x\|_p \leq C_r \|f\|_r, \quad p = \frac{2r}{2-r} = r^*, \quad r \in (1, 2).$$

The formal "proof": for any $\eta \in H_1 \equiv W_2^1$ we have

$$(3) \quad |(u_x, \eta_x)| = |(f, \eta)| \leq \|f\|_r \|\eta\|_{r'} \leq C_r \|f\|_r \|\eta_x\|_{p'},$$

$$\frac{1}{p'} - \frac{1}{2} = \frac{1}{r'} = 1 - \frac{1}{r} \quad (\text{as } W_{p'}^1 \subset L_{r'})$$

and from (3) „follows“

$$\|u_x\|_p \leq C_n \|f\|_r$$

The ineq. (2) It is desirable to prove for Ω with „very bad“ $\partial\Omega$. For some purpose it is sufficient to have instead of (2) the estimate

$$(4) \quad \|u_x\|_{2+\delta} \leq C_\delta \|f\|_2, \text{ with some } \delta > 0. \text{ for } \Omega \subset \mathbb{R}^n$$

It is desirable ^{also} to have (2) or at least (4), for the solutions of the Stokes problem

$$(5) \quad \Delta u + \nabla p = f, \quad \operatorname{div} u = 0, \quad u|_{\partial\Omega} = 0.$$

For (5) we use usually the estimates

$$(5) \quad \|\partial_x^2 u\|_{q,\Omega} \leq C \|f\|_{q,\Omega}, \quad q > 1,$$

proved by Solonnikov and Cattabriga ^(for $q=2$) for $\partial\Omega \in C^2$ (or, at least, $\partial\Omega \in W_{\tilde{q}}^2$, $\tilde{q} \geq q$, $\tilde{q} > n$). But what we can say of $\partial\Omega$ is not smooth? It is known that (5) is not true for many Ω . For the 2-dimensional Ω we can use the representation $u = (\varphi_{x_2}, -\varphi_{x_1})$ and rewrite (5) in the following form:

$$(6) \quad \|\partial_x^2 \varphi\|_{q,\Omega} \leq C \|\partial_x \phi\|_{q,\Omega}$$

where φ is the solution of the problem


$$(7) \quad \Delta^2 \varphi = \Delta \phi, \quad \varphi|_{\partial\Omega} = \frac{\partial \varphi}{\partial n} \Big|_{\partial\Omega} = 0 \quad \text{with } \phi \in W_2^1(\Omega).$$

It is useful to have (6) at least for $q=2$ for Ω with „bad“ $\partial\Omega$.

As I remember there were publications of Maz'ya on the Stokes operator in Ω with nonsmooth boundary.

It is important to have the proofs of above mentioned estimates not only for the solutions of problem but also for their Galerkin-Faedo approximations.

My best wishes Taja and children

Yours 

Speaking of Steklov, I am immediately reminded of the Euler Institute, and its scientific activities, another great mathematical institution, where many years ago I stayed in a beautiful apartment on its first floor, invited by Vsevolod Alekseevich Solonnikov. I cannot think of St. Petersburg without recalling my old and strong ties with him. Let me just recall an unforgettable event, when Seva invited me to stay overnight in his house to have the opportunity to go together, during the night, to enjoy an incredible event, of light, sound (not music!), motion, and emotion, the famous opening the drawbridges. It left me speechless.

Now I would like to return to my first visit to St. Petersburg and the impressions I had. In the various subsequent visits, the effect became more of a custom. So, I focus on that first visit. Olga hosted us at her home for about ten days. In general, we dined together, with one of her nieces who helped her with the housework. Olga's sweetness, the care for us, the delicacy and extreme availability in practical matters, were above all description – a perfect hostess. I keep with care a teacup which she removed from a glass cupboard and gave me along with her classic book (in Russian) on the Navier–Stokes equations. Her sweetness appeared to me in contrast to the “authoritarian” attitude held during my first seminar at Steklov. I finished the seminar after the classic “one hour”, but Olga said (ordered) something like “Hugo, proofs, the proofs”. There was no possible questioning, just to obey.

In this regard, I remember an amusing incident that happened a few years later. We invited Olga to go to a theatrical spectacle with us at the Mariinsky. It was, if I remember correctly, during Yeltsin's time. At the intermission I would like to have had a pastry, together with the two ladies. At that time, prices in Russia were very low for us. I almost felt like a thief paying ridiculous prices to see shows of a level to which I was not accustomed (at that time, a ticket to the Scala in Milan was perhaps 20 times more expensive). But Olga found that the prices were too high, and she said something like “Hugo, *niet* pastry here”, in a peremptory and definitive way – no way. Another time she accompanied us to the Hermitage to check at the entrance that the ticket's price was the most convenient.

However, Olga was not truly authoritarian. She was simply a great character, authoritative, and engaging. This aspect of her character reminds me of completely opposite situations, and there are many. Let me recall one of these situations. Many years later, having invited Olga to Pisa, we took her to visit Siena. Entering the Palazzo del Campo Olga remained petrified in front of a thirteenth-century Madonna. Both she and Madonna had angelic faces; one no longer understood which of the two was the real Madonna. We did not have the courage to distract her from that contemplation. Olga, in intimate situations, was very sweet, I would say maternal, when she judged or advised. I was also deeply impressed by her capacity of showing her emotions without needing words, such was the expressiveness of her face in such moments, together with a particular

movement of raising her head. For example, I look back at her reaction to the display of our shop windows, or to certain, apparently not very recommendable characters, in the streets of St. Petersburg.

During our first visit to St. Petersburg, one of our strongest memories was the visit to the Summer Palace, taken by car by Nina Uraltseva, a driver with skills on a par with the best Italian drivers. Olga and Nina showed us great availability, sympathy and simplicity. They made us feel at home. This trip reminds me of an outing that happened many years later; it seems to me that it was on the occasion of Olga's 80th birthday. We went with a group of friends and family members of Olga to an outdoor picnic in the *dacha* that Olga owned as a member of the Academia. Olga was happy and simple, as an elderly peasant woman might be in her role as landlady. It was an unforgettable party; such was the great scientist Olga Alexandrovna Ladyzhenskaya.



Photo 4: Friday June 27, 2003, Madeira Island.

People sitting, from the left: Adelia Sequeira, Louis Nirenberg, Anna Flavia (Iaia) Abeasis (Hugo's wife), Hugo Beirão da Veiga, Olga A. Ladyzhenskaya, Takaaki Nishida.
 2nd row: Paolo Secchi, Josef Bemelmans, Christian Simader, José Francisco Rodrigues, Joao Paulo de Carvalho Dias, Arina Arkhipova, Irene Fonseca, David Kinderlehrer, Sadao Miyatake.
 3rd row: Rolf Rannacher, Paola Gervasio.
 4th row: Hermano Frid, Pierangelo Marcati, Robert Turner, Fernando Carapau, Alberto Valli, Motoko Miyatake (Sadao's wife), John Heywood.
 5th row: Ana Barroso, Paula Oliveira, Joaquim Silva (Adelia's husband), Ana Leonor Silvestre, Nadir Arada.
 6th row: José Matias, Carlo Grisanti, Luigi Berselli, Fulvia (Alfio's wife), José Augusto Ferreira.
 7th row: Alfio Quarteroni, Bruno Rubino, Lourenco Beirão da Veiga, Luc Tartar, Anne Robertson, Giovanni Paolo Galdi.
 Haïm Brezis had to leave Madeira before the last day. So, unfortunately, he is not in the photograph.

I must say, with some immodesty, that Olga held me in high esteem. I remember that once, at an important dinner (perhaps also on the occasion of her 80th birthday), she wanted me to sit next to her. About our relationship, friendship and work, I am pleased to include one of the letters exchanged between us, in which Olga proposed mathematical problems that we have later faced. I cannot forget Olga's presence at the conference that was dedicated to me on the occasion of my 60th birthday, on the Island of Madeira, a year before her death. I would like to recall how much Olga and Louis Nirenberg (a dear friend who passed away in 2020), were the central figures, despite the level of all the other participants. Again, I include a photograph (see Photo 4).

Acknowledgements:

I am grateful to the editors and organizers of this challenging initiative, in particular to Darya Apushkinskaya and Alexander Nazarov, for their kind invitation to contribute to the volume. To Robert E.L. Turner, whose deep sensibility, together with his perfect Italian, allows him to translate my Italian-English into real English, keeping alive my emotion (Bob is in two of the above photographs). To Augusto Micheletti for the help concerning the CIRM's photographs and related individuation of people. Augusto is the outstanding CIRM's secretary, in office from the very beginning of the institution, in 1978, until now. I believe that the majority of the people who will have the present book in hand have met Augusto in a CIRM's conference.